Predicting Coronal Mass Ejection Arrival Times with Thirty-Minute Accuracy

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Abstract

J. Schmidt and Cairns (2019) have recently shown that they can predict Coronal Mass Ejection (CME) arrival times with an accuracy of 0.9+-1.9 hours for four separate events. They also showed that the accuracy gets better with increased grid resolution. Here, we further improve these results by using the Richardson extrapolation (Richardson and Gaunt, 1927), which is a standard technique in computational fluid dynamics, and predict the CME arrival time with 0.2+-0.26 hours accuracy. The CME arrival time errors of the new model lie in a 95% confidence interval [-0.21,0.61] h. We also show that the probability of getting these accurate arrival time predictions with a model with a standard deviation exceeding 2 hours is less than 0.1%, indicating that the excellent results cannot be due to random chance, and the Richardson extrapolation has indeed improved the original model by J. Schmidt and Cairns (2019). This unprecedented accuracy is about 40 times better than the current state-of-the-art prediction of CME arrival times with an average error of about +-10 hours. The new model uses information available within a few hours after the CME eruption and it can run much faster than real-time on a couple of CPU cores. Based on the result, we recommend the new model to be transitioned to operations as soon as possible to better protect our space-born and ground-based assets from the harmful effects of space weather.

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²³ 1 Introduction

Predicting the propagation of Coronal Mass Ejections (CMEs) and their arrival time 24 at Earth has been a major goal of space weather prediction for decades. The ENLIL model 25 (Odstrčil & Pizzo, 1999a, 1999b), for example, solves the ideal magnetohydrodynamic 26 (MHD) equations from about $20 R_s$ (solar radii) to the Earth orbit and beyond. The in-27 ner boundary conditions are provided by the Wang-Sheeley-Arge (WSA) model (Arge 28 & Pizzo, 2000). CMEs are initiated with the empirical CONE model based on flare ob-29 servations and coronal white light images. Another approach is followed by the Alfvén 30 Wave Solar atmosphere Model (AWSoM) (van der Holst et al., 2014) that is based on 31 the BATS-R-US MHD code (Powell et al., 1999; Toth et al., 2012), also widely used to 32 model the solar corona, the heliosphere and the eruption and propagation of CMEs from 33 the surface of the Sun (initiated by a flux rope model) to Earth and beyond. AWSoM 34 solves the MHD equations extended with solar wind heating and acceleration due to Alfvén 35 wave turbulence, radiative cooling and heat conduction. However, these first-principles 36

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Looking at Figure 4 in (J. Schmidt & Cairns, 2019), reproduced here as Figure 1, we have noticed that the distances between the observations (diamonds) and the model predictions obtained on two different computational grids (squares and stars) form a distinctive pattern: the distances between the three symbols appear to be approximately



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the same for all four events displayed. The main idea in this paper is that this fact can
be exploited to obtain an even more accurate estimate of the CME arrival time.

The new arrival time estimate based on the Richardson extrapolation (Richardson 70 & Gaunt, 1927) has a bias and standard deviation of 0.2 ± 0.26 hours, which is signif-71 icantly better than the 0.9 ± 1.9 hours obtained by J. Schmidt and Cairns (2019). We 72 will also show that the agreement between observations and simulations cannot be at-73 tributed to luck. Since the four events happened in different years and/or have very dif-74 ferent arrival times covering a wide range from about 40 hours to 72 hours, the technique 75 must be applicable to most CMEs. This means that the model should provide extremely 76 reliable and accurate information for operational space weather forecasters, which is im-77 portant for our national security and human safety. 78

⁷⁹ 2 Predicting CME arrival times

To perform a quantitative evaluation of the results presented in Figure 4 of (J. Schmidt & Cairns, 2019), we have digitized the figure and put the observed and simulated arrival times (relative to the eruption time) into Table 1. These values were also used to produce Figure 1 confirming that the values were extracted correctly.

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ID	Date	Observed	Model1	Model2	Error1	Error2	Error1/Error2
1	Sep 04, 2017	52.676	48.873	50.845	3.803	1.831	2.077
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4	Nov 29, 2013	50.422	46.901	48.943	3.521	1.479	2.381
Average				3.891	1.8735	2.094	

The errors Error1 and Error2 of the two models Model1 and Model2, corresponding to Refinement Level 2 and 5 in (J. Schmidt & Cairns, 2019), are remarkably constant across the four events, and the ratio of the errors is approximately 2.1. Using the idea of the Richardson extrapolation, which improves the numerical accuracy by estimating the exact solution from numerical solutions at two different grid resolutions, we construct the following formula for the extrapolated arrival time:

$$T_R = 2T_2 - T_1 \tag{1}$$

where T_1 and T_2 are the arrival times predicted by models 1 and 2. This provides the smallest error compared to observations and it is also consistent with the Richardson extrapolation for a first order accurate scheme. Indeed, the numerical scheme for discontinuities, like the CME shock, is only first order accurate.

⁹⁴ 3 Statistical Analysis and Probability Estimates

Table 2 shows that the mean absolute error of the extrapolated arrival time is about 0.218 hours, which is useful information, but not suitable for statistical analysis. To bet-

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Table 2. Observed and extrapolated CME arrival times for four events. The times are measured in hours from the eruption time. The last column is the absolute value of the error.

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$$B = \frac{1}{N} \sum_{i=1}^{N} (T_{i,R} - T_i) = 0.198 h$$
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$$S = \sqrt{\frac{\sum_{i=1}^{N} (T_{i,R} - T_i - B)^2}{N - 1}} = 0.257 \,\mathrm{h} \tag{3}$$

where T_i is the observed arrival time for event *i* and $T_{i,R}$ is the Richardson extrapolated time calculated from Equation 1. The 95% confidence interval for the error $T_R - T$ is $B \pm tS/\sqrt{N}$, where t = 3.182 from the T-distribution for p = 0.025 and N - 1 = 3degrees of freedom:

$$(T_R - T) \in [-0.211, 0.607] \,\mathrm{h} \tag{4}$$

We conclude that there is a 95% chance that the model will produce arrival time predictions with errors less than 37 minutes, while the average error is only 12 minutes.

Finally, it is important to check if the small errors in Table 2 are statistically significant, or they can be attributed to simple luck. Let us assume that the new model with the extrapolation has no bias, $\mu = 0$, and its standard deviation is $\sigma = 2$ h. The quantity

$$X^{2} = \frac{\sum_{i=1}^{N} (T_{i,R} - T_{i})^{2}}{\sigma^{2}} = 0.0887$$
(5)

follows the $\chi^2(N, p)$ distribution since the mean value is assumed to be known. For N =4, we find that there is only p = 0.1% chance that $X^2 \le 0.0887$ by pure luck. If σ was larger than 2 hours, this probability would be even less. We can safely conclude that the
model is indeed capable of predicting the CME arrival time with high accuracy, even higher
than the original J. Schmidt and Cairns (2019) model.

116 4 Conclusions

In this paper we have further improved the work of J. Schmidt and Cairns (2019), who achieved an excellent 0.9 ± 1.9 h accuracy predicting the CME arrival times. Using the standard Richardson extrapolation technique, we have further improved the accuracy of the model to an average error to 0.2 ± 0.26 hours. We showed that the predictions are in the range 0.2 ± 0.4 hours with 95% confidence, and it is practically impossible that the good agreement between observations and simulation results obtained by J. Schmidt and Cairns (2019) was simply a lucky coincidence.

Given the low computational cost of the model and the fact that it relies on readily available real time observations, we believe that this break-through result can improve the current CME arrival prediction accuracy by more than an order of magnitude, and provide reliable and timely forecast for the space weather affected infrastructure operators, as well as enthusiasts of aurora observations.

¹²⁹ 5 Open Research

All data used in this paper are contained in Table 1. The Space Weather Modeling Framework including (BATS-R-US/AWSoM) is an open-source code available at https://github.com/MSTEM-QUDA with a full version history.

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We conclude that there is a 95% chance that the model will produce arrival time predictions with errors less than 37 minutes, while the average error is only 12 minutes.

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