The stochastic nature of the transport of coarse material in rain-impacted flows

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January 17, 2023

Abstract

Traditionally, so-called physically based models of erosion by rain-impacted flow focus on the conservation of mass. Raindrop induced saltation and rolling produce random intermittent movement of soil particles and it seems that the traditional approach to modelling sediment transport in rain-impacted flows does not account for the effects these transport processes well. Arguably, the traditional approach needs to be replaced by one that is better able the deal with the stochastic nature of the transport of coarse material in rain-impacted flows.

The stochastic nature of the transport of coarse material in rain-impacted flows

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Updated 12/01/2023

Abstract

Traditionally, so-called physically based models of erosion by rain-impacted flow focus on the conservation of mass. Raindrop induced saltation and rolling produce random intermittent movement of soil particles and it seems that the traditional approach to modelling sediment transport in rain-impacted flows does not account for the effects these transport processes well. Arguably, the traditional approach needs to be replaced by one that is better able the deal with the stochastic nature of the transport of coarse material in rain-impacted flows.

Introduction

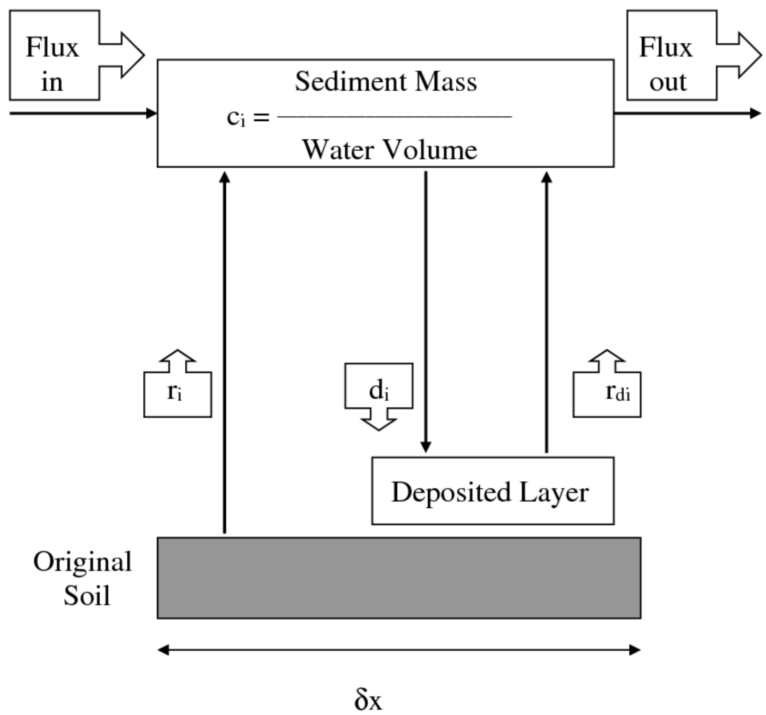


Fig.1. Schematic for the raindrop driven sediment transport model in rain-impacted flow developed by Hairsine and Rose (1991) c_i is the sediment concentration for particles of size class i, q is the water flux, r_i is the detachment rate for the cohesive soil surface, r_{di} is the "redetachment" rate from the layer on loose particles, and d_i is the rate of deposition produced by particles falling back to the soil surface

Many so-called physical models of erosion are based on equations for the conservation of mass. For example, based on the scheme shown in Fig.1, the Hairsine and Rose (1991) model uses

$$c_i \frac{\mathrm{d}q}{\mathrm{d}x} + q\frac{\mathrm{d}c_i}{\mathrm{d}x} = r_i + r_{di} - d_i \tag{1}$$

where c_i is the sediment concentration for particles of size class i, q is the water flux, r_i is the detachment rate for the cohesive soil surface, r_{di} is the "redetachment" rate from the layer on loose particles, and d_i is the rate of deposition produced by particles falling back to the soil surface.

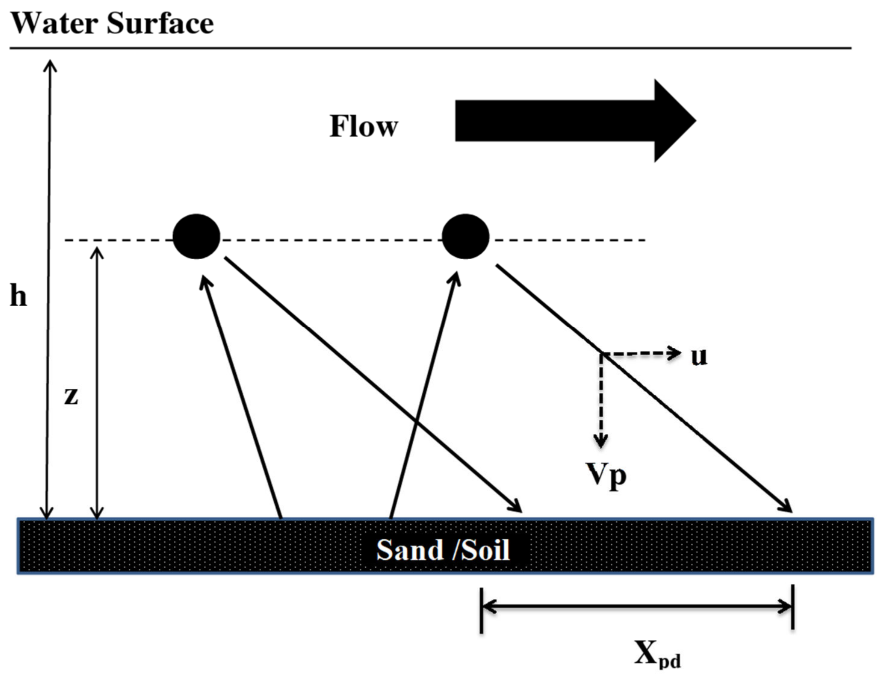


Figure 2. Schematic of the uplift and downstream movement of particles generated by raindrop induced saltation. X_{pd} **= distance travelled by particles while moving laterally in the flow after a drop impact.** x_c **= diameter of the area where a drop impact lift particles into the flow,** x_{cz} **= diameter of the volume of particles lifted into the flow by a drop impact.** V_p **= the settling velocity of the particle, u = flow velocity**

In rain-impacted flows, particles detached from the cohesive soil surface by drop impacts may travel laterally in complete suspension, by raindrop induced saltation, by raindrop induced rolling, by flow driven saltation and by flow drive rolling (see video via https://osf.io/j3caz). With raindrop induced saltation, soil material lifted into the flow from a surface eroding under rain-impacted flow returns to the soil surface after moving laterally at a velocity that is determined by the velocity of the flow (Fig. 2). As a result, raindrop induced saltation is a pulsed transport system that operates randomly in time and space because raindrops impact flows randomly in time and space. Likewise, raindrop induced rolling is a pulsed transport system that operates randomly in time and space. Cheraghi et al. (2016) applied the version of Hairsine and Rose model developed for unsteady conditions by Sander et al. (1996) to an experiment using simulated rainfall over a 6 m flume containing a loamy agricultural soil and observed that the Hairsine and Rose model results agreed well with the total eroded soil and particles < 50 µm. However, the agreement declined for particles > 50 µm and was poor for particles > 1000 µm. They concluded that result for particles > 1000 µm resulted from the fact the Hairsine and Rose model was not designed to model raindrop driven rolling. However, it is apparent that the Hairsine and Rose model did not model sediment transport by raindrop induced saltation well as particle size increase from 50 µm.

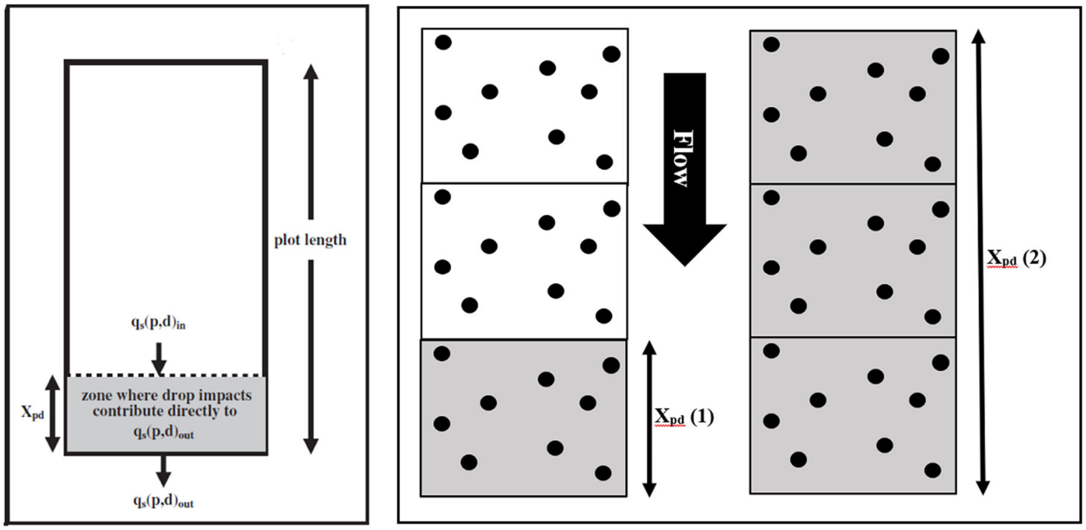


Figure 3. Diagram showing how the distance particles travel during a raindrop induced saltation affect sediment discharge. X_{pd} = distance travelled by particles while moving laterally in the flow after a drop impact. X_{pd}(2) is 3 times X_{pd} (1) so that 3 times as many drop impacts cause sediment discharge in the X_{pd}(2) case than the X_{pd}(1) case

Particles moving by raindrop induced saltation move limited distances from the point of drop impact during a saltation event. That distance, (X_{pd}) varies with the time the particles are moving with the flow following a drop impact (t_{pd}) and flow velocity (u) (Kinnell, 1990).

$$X_{pd} = t_{pd}\, u \tag{2}$$

In order for a drop impact to cause particles to pass across any given boundary it must impact within the zone that extends upstream no further than X_{pd} from that boundary (Fig 3). If the amount of material mobilized by each impact within the distance X_{pd} of the boundary is M_{pd}, then the sediment discharge (q_s (p,d), mass/width/time) is given by

$$q_s(p,d) = M_{pd} F_d\, X_{pd} \tag{3}$$

where F_d is the spatial averaged impact frequency of drops of size d falling on the surface. As can be perceived from Fig. 3, if X_{pd} is tripled when M_{pd} and F_d are constant, then sediment discharge increases by a factor of 3.

Sediment concentration in erosion is the mass of soil material discharged per unit volume of water discharged. Consequently, the mass of soil discharge and the volume of water discharged are independent variables in respect to the sediment concentration for the sediment discharged in the outflow irrespective of the transport mechanisms involved. Flow discharge (q_w, volume/width/time) is given by the product of flow depth (h) and flow velocity

$$q_w = h\,u \tag{4}$$

Combining Eqs. 2 and 3 gives

$$q_s(p,d) = F_d\ [M_{pd}\,t_{pd}\,]\,u \tag{5}$$

As can be seen from Eq.4 and 5, q_w and $q_s(p, d)$ covary when flow velocity varies so that when raindrop induced saltation occurs, the sediment concentration in the outflow

$$c_s(p,d) = F_d\ \left[M_{pd}\,t_{pd}\,\right] / \,h \tag{6}$$

remains constant as flow velocity varies. However, both M_{pd} and t_{pd} are influenced by flow depth in ways that influence the sediment concentration in the outflow. Fig. 4 shows the effect of flow depth on the product of M_{pd} and t_{pd} when particle size varied the experiments reported by Kinnell (1991) using 2.7 mm drops falling at near terminal velocity. Fig 5 shows the effect of flow depth and drop size when 0.2 mm sand was eroded in the experiments reported by Kinnell (1991) and Moss and Green (1983) using the apparatus shown in Fig 6.. The product of M_{pd} and t_{pd} peaks at a drop size dependent flow depth. The manner in which the product declines with flow depth after those peak changes when the flow depth is about 3d. After about 3d, the cavity carved in the flow by moderate to large raindrops no longer reaches the eroding surface at its fullest extent. Although, the effect of flow depth on sediment concentration can be determined using Eq. 6, approach used to develop Eq. 3 facilitates the modelling erosion by rain-impacted flows in a manner that gives more consideration to the stochastic nature of raindrop impact than in the commonly used conservation of mass approach.

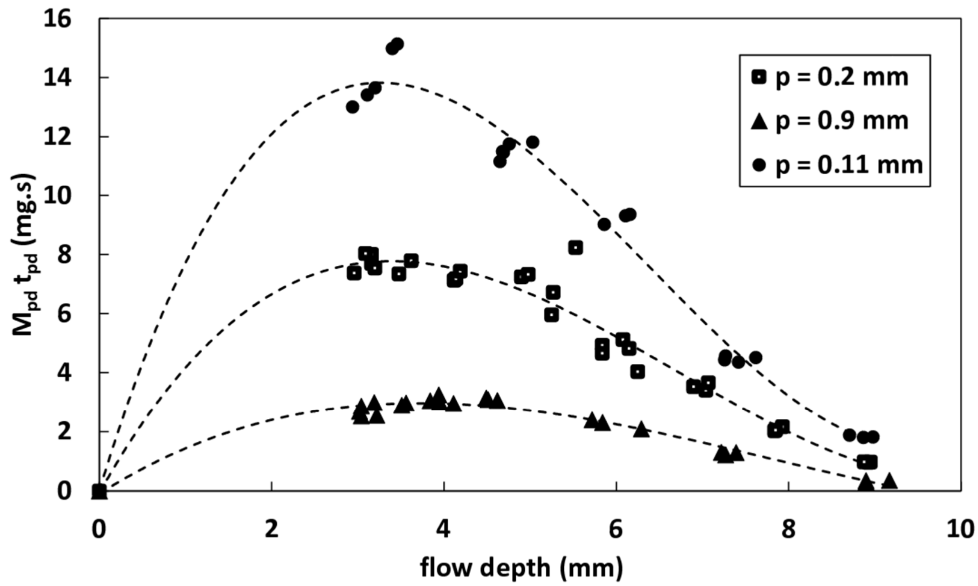
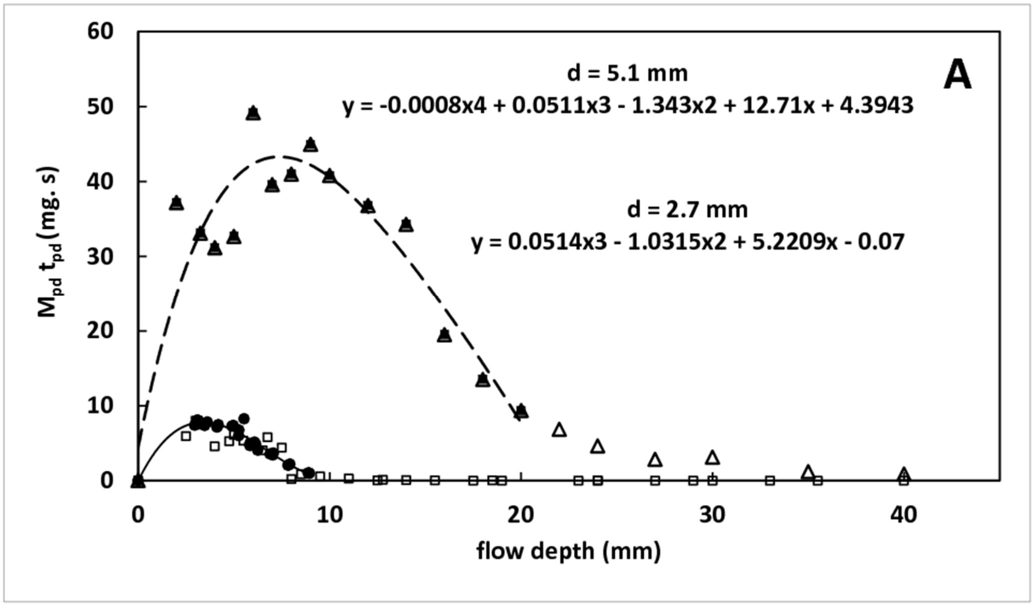


Figure 4 . The effect of flow depth on M_{pd} t_{pd} for 0.11, 0.2 and 0.9 mm sands obtained for rain produced by 2.7 mm drops falling 11.2 m using the apparatus shown in Fig.6 The trend lines are produced by polynomials for the data including the 0,0 case. Relationships are based on data from experiments reported by Kinnell (1991)



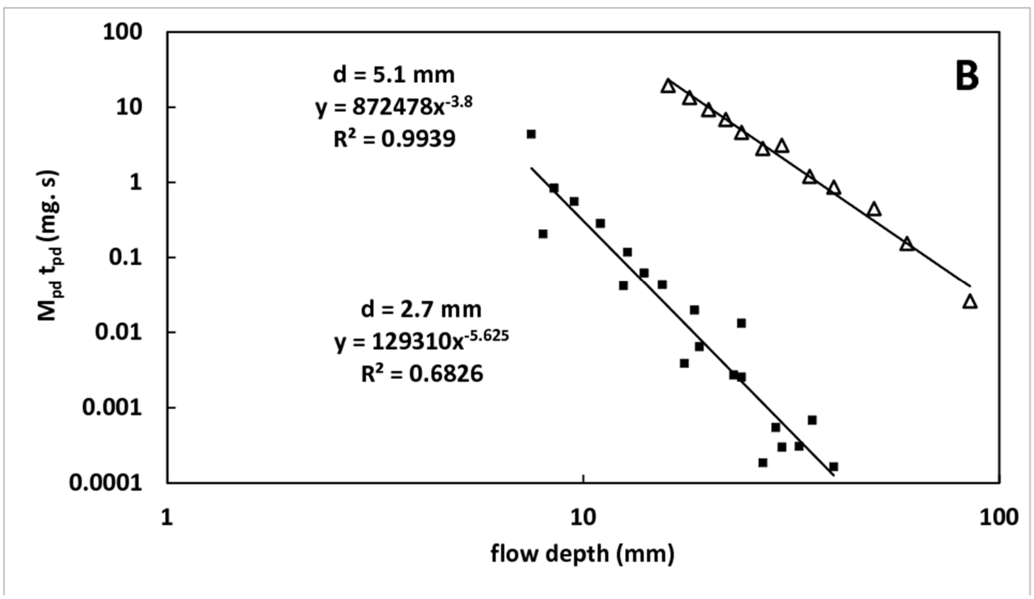


Figure 5 (A) The effect of drop size and flow depth on M_{pd} t_{pd} obtained for rain produced by 2.7 mm drops and 5.1 mm drops falling 11.2 m using the apparatus shown in Fig.6 are based on data from experiments reported by Kinnell (1991) and Moss and Green (1983). The regressions are for data shown as solid points, (B). The effect of drop size and flow depth on M_{pd} t_{pd} obtained for rain produced by 2.7 mm drops and 5.1 mm drops falling 11.2 m using the apparatus shown in Fig.1 for flow depths greater than 3*d* based on data from experiments reported by Moss and Green (1983).

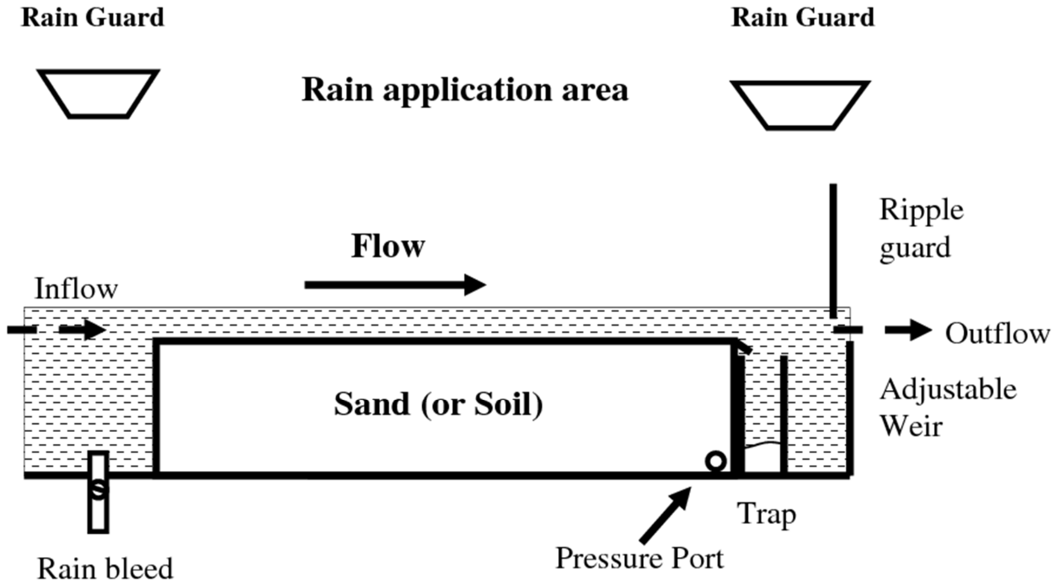


Figure 6: Apparatus that enables flow depth and velocity to be controlled when raindrop impact erodes sand or soil surfaces under a flow over a horizontal bed. It is a modification of the apparatus developed by Moss and Green (1983). The ripple guard prevents ripples generated by drop impacts for affecting how the weir control flow depth. The rain bleed compensates for the input of rain water and the pressure port provides for the hydrostatic measurement of flow depth during rain.

The qualitative physical model

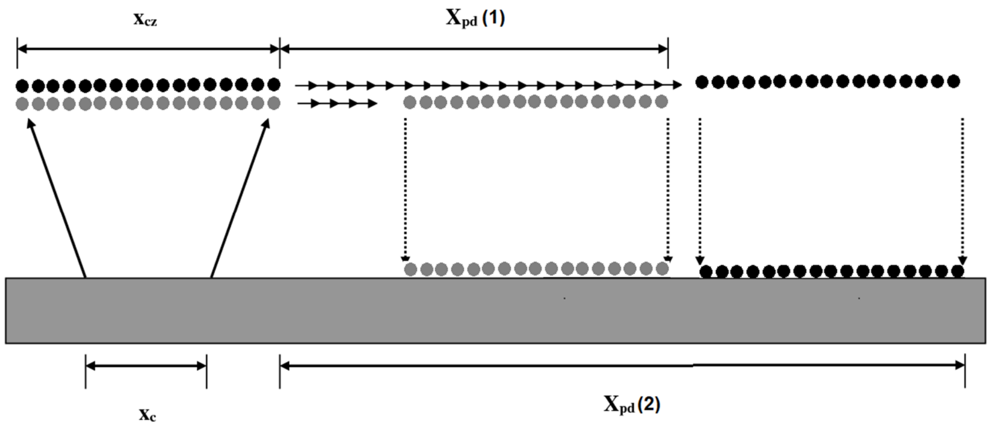


Figure 7 . Schematic of the uplift and downstream movement of particles generated by raindrop induced saltation in the qualitative model of raindrop induced saltation developed by Kinnell (1994). ***X_{pd}*** **= distance travelled by particles while moving laterally in the flow after a drop impact.** ***x_c*** **= diameter of the area where a drop impact lift particles into the flow,** ***x_{cz}*** **= diameter of the volume of particles lifted into the flow by a drop impact.**

Kinnell (1994) developed a qualitatively model of raindrop induced saltation to demonstrate the development of the layer of loose particles on top of the cohesive soil surface. For simplicity, the material detached from the cohesive soil surface forms a single layer of particles above the soil surface (Fig.7) and the layer moves horizontally the distance X_{pd} before falling to the soil surface. The model is designed to operate on a horizontal rectangular surface using a 1mm square grid. Consequently, the projected areas for the layer and surface disturbed by the drop impact are square rather than circular. Initially, M_{pd}, the mass of material moved laterally as the result of a drop impact is equal to $M_{pd.cs}$, the mass detached from the cohesive soil surface when no loose material is present. Loose material sitting on the surface provide a degree of protection (H). H varies from 0 when no loose particles exist on the surface to 1.0 when the loose particles fully protect the soil surface. As a result

$$M_{pd} = H\, M_{pd.LL} + (1 - H)\, M_{pd.CS} \tag{7}$$

where $M_{pd.LL}$ is the mass lifted into the flow when $H = 1$. When $H < 1.0$ all the loose material sitting on the surface in the impact area is lifted into the flow in addition to some material that is detached from the cohesive soil surface. Consequently, H increases along the line of flow as the sequence of lateral movements moves particles towards the downstream boundary. The model demonstrated that over time, H increases from 0 at the upstream end towards a value of 1.0 at the downstream boundary with the value at the downstream boundary being closer to

1.0 when the detachability of the cohesive surface was high rather than low. The development of the layer of loose material results from the random nature of drop impacts producing long wait times between successive downstream movements of the loose particles.

A feature of erosion by rain-impacted flows where raindrop induced saltation and perhaps rolling occur is the fact that the particle size distribution is initially finer than the original soil but then coarsens with time to eventually equal that of the original surface. This feature was observed in experiments with unsorted sand in a 3 m flume under artificial rainfall reported by Walker et al. (1978). This feature was also observed with two soils in planar surfaces ranging from 2.9 to 5.8 m under artificial rainfall in experiments reported by Proffitt et al. (1991). The Hairsine and Rose model for rain-impacted flows in the absence of flow driven processes (Hairsine and Rose, 1991) was based on the results of these experiments. One of the reasons for the coarsening of the particle size distribution in the sediment discharge is that particles of different sizes and densities travelling by raindrop induced saltation move laterally over the soil surfaces with effective velocities that vary among other things with their size and density. Small less dense particles move faster than bigger more dense particles. If, for example, a cohesive soil contained equal amounts of just two particles that differ in size and/or density, it follows from Fig. 3 and the combination of Eqs. 5 and 7,

$$q_s(p,d) = F_d \; [\, H \, M_{pd.LL} \; + \; (1-H) \, M_{pd.CS} \,] \, t_{pd} \; u \qquad (8)$$

that initially, the discharged sediment would contain more than 50 % of the material that has the lower settling velocity. In early times, the feed of particles into the area where drop impact cause particles to pass over the down slope boundary results in the loose layer containing more than 50 % of the material that has the lower settling velocity but, as time goes by, the proportion of the slower moving particles increases so that the slower moving particles become dominant in the loose layer at the downstream end of the eroding surface. At the steady state which occurs when the slowest moving particle initially mobilized at the most upstream point passes over the boundary, the particle size composition of the sediment discharge will be the same as that of the original soil if all the particles traveling by raindrop induced saltation are stable. The qualitative model developed Kinnell (1994) can model the change in the particle compositions in both the loose layer and the sediment discharge.

As a situation where the qualitative model can be used to demonstrate the changes in the particle compositions in both the loose layer and the sediment discharge over time, consider a horizontal surface made up of 50% 0.46 mm sand and 50% 0.46 mm coal under a flow with a velocity and depth that remain constant in space and time. Rainfall generating raindrop induced saltation produces 2. 7mm drops that impact about their terminal velocity with a rainfall intensity of 50 mm h^{-1}. x_c = 5 mm, and x_{cz} = 11 mm are values which are consistent with visual observation of 2.7 mm drops impacting flows of about this depth. $M_{pd.CS}$ for each material is arbitrarily set to 0.025 times 37 mg/drop, the $M_{pd.LL}$ for 0.46 mm sand when covered by 4 mm deep water. Fig. 8 shows the modelled sediment discharges and masses in loose layer in bottom 20 mm for 4 mm deep flows over the 0.5 m long artificial cohesive surface during one hour of rainfall. When flow velocity was 20 mm s^{-1}, the steady state where both 0. 46 mm coal and 0.46 mm sand were discharged at the same rate was not achieved and consequently, coal was enriched throughout the 1 hour simulated. The steady state was close to being achieved when the flow velocity was 60 mm s^{-1}. It was achieved during the one hour when flow velocity was 100 mm s^{-1}. The ratios for 0.46 mm coal to 0.46 mm sand discharged during the hour varied from 2.16 for 20 mm s^{-1} flow, to 1.46 for 60 mm s^{-1} flow, to 1.22 for 100 mm s^{-1} flow.

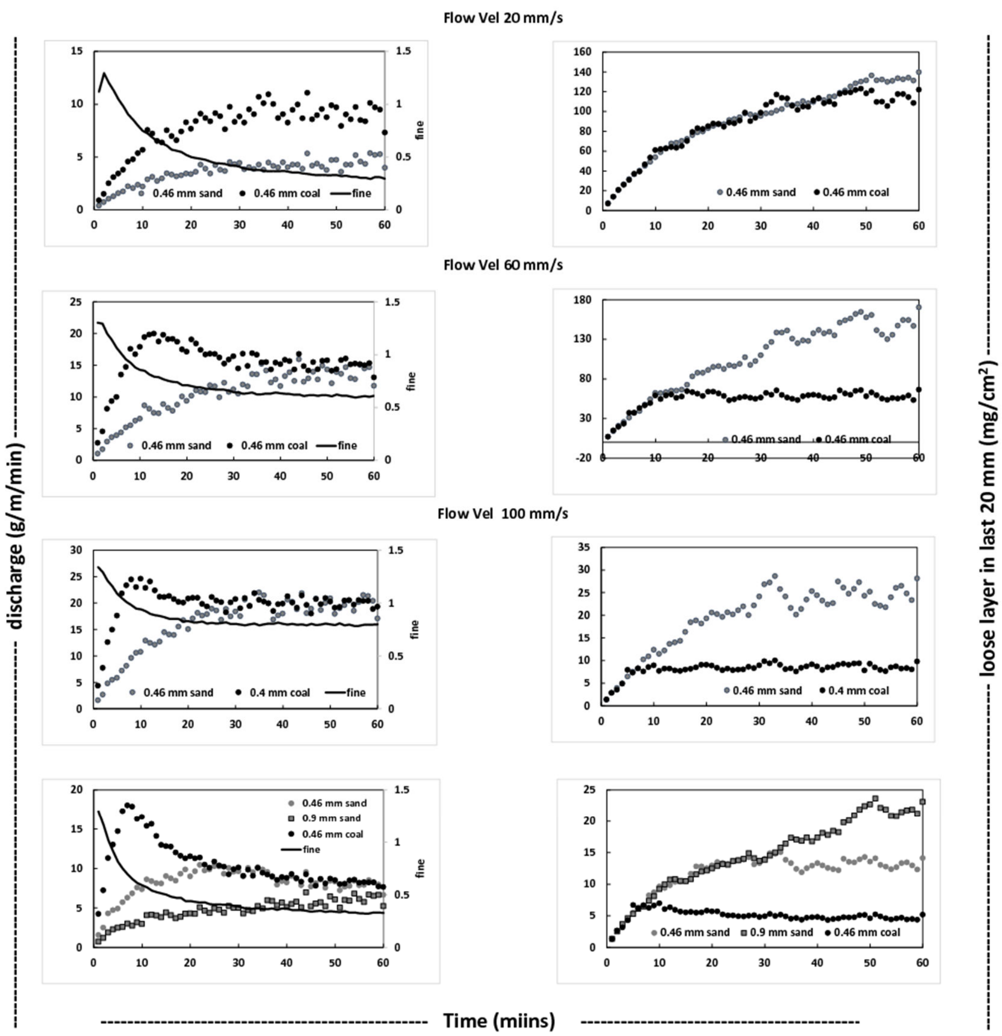


Figure 8. Modelled sediment discharges and masses in loose layer in bottom 20 mm for 4 mm deep flows over the 0.5 m long artificial cohesive surface during one hour of rainfall at 50 mm h^{-1}.

Figure 9 shows ratios of 0.46 mm coal to 0.46 mm sand in the modelled sediment discharges for 4 mm deep flows over 0.5 m to 3m long artificial cohesive surface containing 50 % 0.46 mm coal and 50 % 0.46 mm sand. As slope length increases, the duration of non-steady state conditions increased. The steady state never occurred in 10 hours of rain when flow velocity was 20 mm s^{-1} on the 3 m long surface. As to be expected from the results presented in Fig. 7, the duration of non-steady state conditions decreased with flow velocity

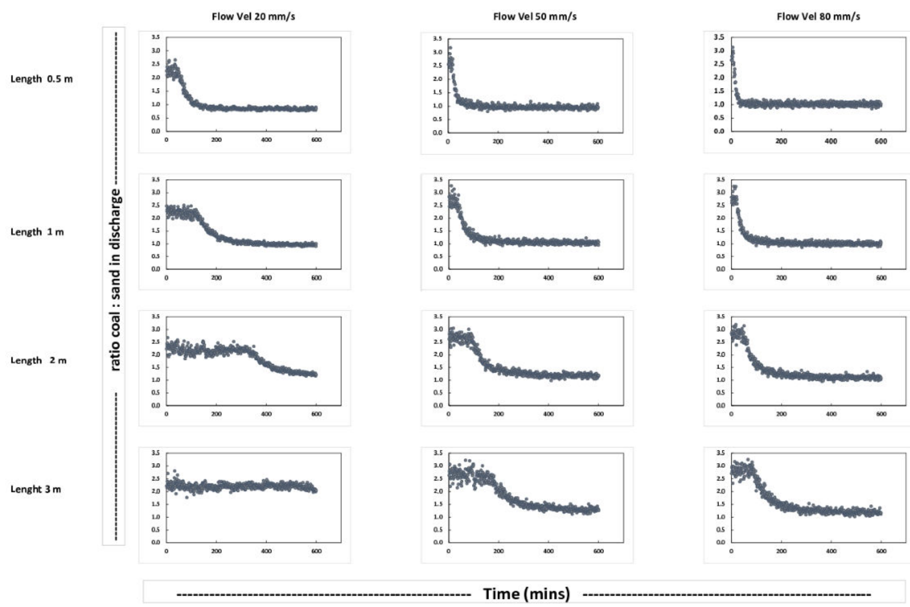


Figure 9. Ratios of 0.46 mm coal to 0.46 mm sand in modelled sediment discharges for 4 mm deep flows over 0.5 m to 3m long artificial cohesive surface containing 50 % 0.46 mm coal and 50 % 0.46 mm sand. Rainfall intensity = 50 mm h^{-1}.

The qualitative model developed by Kinnell (1994) has been extended to model the effects of not just raindrop induced saltation on sediment discharge but also flow driven sediment transport by complete suspension and flow driven saltation (Kinnell, 2009). The examples shown above are selected to demonstrate how the random nature of sediment transport by rain-impacted flows is important to determining not just the mass of material discharged but also its particle composition.

Mathematical modelling of raindrop induced saltation

The qualitative model developed by Kinnell (1994) is not designed to predict soil loss in experiments like those reported by Cheraghi et al. (2016). However, it is clear the approach to the modelling erosion by rain-impacted flows using the classical mass conservation approach does not capture how particles are detached and transported in rain-impacted flow well. Arguably, an alternate approach to developing so-called physically based models of erosion by rain-impacted flow needs to focus more specifically on the stochastic nature of the transport of coarse material in rain-impacted flows. This has been recognised by Lisle et al. (1998) who developed a stochastic model governing the transport of particles that alternated between resting on the bed and being transported in the flowing surface water. They observed that a suitable averaging of the statistical particle motions in their model gave rise to the deterministic erosion differential of Hairsine and Rose (1991). However, a stochastic sediment transport model of soil erosion in rain impacted flows which takes better account of the effects of rain, flow and sediment characteristics observed in experiments where flow and rain factors are well controlled has yet to be developed.

Conclusion

Raindrop driven saltation and rolling transport soil particles limited distances upon initiation by raindrop impacts that occur randomly in time and space. It seems that the traditional approach to modelling sediment transport in rain-impacted flows does not account for the effects these transport processes well. Consequently, the traditional approach needs to be replaced by one that is better able the deal with the stochastic nature of the transport of coarse material in rain-impacted flows.

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