

Estimating particle size and coercivity distributions of pigmentary hematite in red chert with thermal fluctuation tomography

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Abstract

Pigmentary hematite carries important signals in paleomagnetic and paleoenvironmental studies. However, weak magnetism and the assumption that it has high magnetic coercivity prevents routine identification of the size distribution of pigmentary hematite, especially for fine particle sizes. We present a strategy for estimating joint hematite particle volume and microcoercivity ($f(V, H_k0)$) distributions from low-temperature demagnetization curves and thermal fluctuation tomography (TFT) of pigmentary hematite in bulk samples of Triassic-Jurassic Inuyama red chert, Japan. The coercivity of the pigmentary hematite increases exponentially with decreasing temperature, following a modified Kneller's law, where microcoercivity has a wide but approximately symmetric distribution in logarithmic space from ~ 1 tesla to tens of tesla. All of the red chert samples contain stable single domain (SSD) hematite with 35 - 160 nm diameter; a significant superparamagnetic (SP) hematite population with sizes down to several nanometers also occurs in Jurassic samples. The SP/SSD threshold size is estimated to be 8 - 18 nm in these samples. The fine particle size of the pigmentary hematite is evident in its low median unblocking temperature (194 °C to 529 °C) and, thus, this hematite may contribute to all four paleomagnetic components identified in published thermal magnetization studies of the Inuyama red chert. In this work, uniaxial anisotropy and magnetization switching via coherent rotation are assumed. Uniaxial anisotropy is often dominant in fine-grained hematite, although the dominant anisotropy type should be evaluated before using TFT. This approach is applicable to studies that require knowledge of coercivity and size distributions of hematite pigments.

1 **Estimating particle size and coercivity distributions of pigmentary hematite in red**
2 **chert with thermal fluctuation tomography**

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12
13 **Key Points:**

- 14 • Thermal fluctuation tomography is applied to red cherts to estimate grain size and
15 microcoercivity distributions of pigmentary hematite
- 16 • Fine hematite particles (35 - 160 nm) occur in all samples including superparamagnetic
17 hematite down to a few nanometers
- 18 • Temperature-dependent coercivity variations in pigmentary hematite follow Kneller's law
19

20 **Plain language Summary**

21 Pigmentary hematite widely presents in rocks and sediment and is crucial for paleomagnetic and
22 paleoenvironmental studies because they can record ancient earth magnetic field and past climate
23 signals. As the most important properties in paleomagnetic and paleoenvironmental applications,
24 the coercivity and grain size distribution of natural pigmentary hematite is poorly constrained
25 due to the weak magnetism of hematite and the small size. In this study, we provide a strategy
26 using low-temperature demagnetization curves for estimating joint particle volume and
27 microcoercivity distribution of pigmentary hematite in Inuyama red chert samples. The hematite
28 coercivity increases exponentially with decreasing temperature. Hematite microcoercivity
29 without thermal fluctuation has a wide but approximately symmetric distribution in logarithmic

30 space from ~1 tesla to tens of tesla. The grain size of hematite varies from several nanometers to
31 about 160 nm. The fine particle size of these hematite results in low unblocking temperature,
32 which makes them suitable to record remagnetization in geological time.

33

34 **Abstract**

35 Pigmentary hematite carries important signals in paleomagnetic and paleoenvironmental studies.
36 However, weak magnetism and the assumption that it has high magnetic coercivity prevents
37 routine identification of the size distribution of pigmentary hematite, especially for fine particle
38 sizes. We present a strategy for estimating joint hematite particle volume and microcoercivity (f
39 (V , H_{k0})) distributions from low-temperature demagnetization curves and thermal fluctuation
40 tomography (TFT) of pigmentary hematite in bulk samples of Triassic-Jurassic Inuyama red chert,
41 Japan. The coercivity of the pigmentary hematite increases exponentially with decreasing
42 temperature, following a modified Kneller's law, where microcoercivity has a wide but
43 approximately symmetric distribution in logarithmic space from ~1 tesla to tens of tesla. All of the
44 red chert samples contain stable single domain (SSD) hematite with 35 - 160 nm diameter; a
45 significant superparamagnetic (SP) hematite population with sizes down to several nanometers
46 also occurs in Jurassic samples. The SP/SSD threshold size is estimated to be 8 - 18 nm in these
47 samples. The fine particle size of the pigmentary hematite is evident in its low median unblocking
48 temperature (194 °C to 529 °C) and, thus, this hematite may contribute to all four paleomagnetic
49 components identified in published thermal magnetization studies of the Inuyama red chert. In this
50 work, uniaxial anisotropy and magnetization switching via coherent rotation are assumed. Uniaxial
51 anisotropy is often dominant in fine-grained hematite, although the dominant anisotropy type
52 should be evaluated before using TFT. This approach is applicable to studies that require
53 knowledge of coercivity and size distributions of hematite pigments.

54

55 **1. Introduction**

56 Hematite is abundant in sedimentary rocks, especially red beds. It occurs commonly as a fine-
57 grained chemically precipitated pigment and as coarser detrital or specular hematite (Cornell and
58 Schwertmann, 2003; Lepre and Olsen, 2021; Jiang et al., 2022; Swanson-Hysell et al., 2019; Tauxe

59 et al., 1980). Poorly crystalline pigmentary hematite can be the dominant iron oxide in many red
60 soils and sediments and is responsible for their characteristic red color. Both specular and
61 pigmentary hematite can carry magnetic remanence. Specular hematite carries a detrital remanent
62 magnetization (DRM), which is often assumed to be a primary or near-primary magnetization.
63 Widely observed red bed remagnetizations tend to be associated with late diagenetic pigmentary
64 hematite formation. Debate about whether red beds record a primary DRM or a secondary
65 chemical remanent magnetization (CRM) led to the “red bed controversy” (Beck et al., 2003;
66 Butler, 1992; Van Der Voo & Torsvik, 2012). Identification of CRM acquisition in pigmentary
67 hematite can enable more accurate paleomagnetic interpretations in regional tectonic studies
68 (Abrajevitch et al., 2018; Jiang et al., 2017; Swanson-Hysell et al., 2019). As the most abundant
69 surficial iron oxide on Earth resulting from near-surface processes, hematite is also an excellent
70 recorder of paleoenvironmental signals. Its formation via authigenic chemical processes means
71 that pigmentary hematite is used as an indicator of hydration conditions, acidity of aqueous
72 environments, and monsoon evolution (e.g., Larrasoña et al., 2003; Abrajevitch et al., 2013; Lepre
73 and Olsen, 2021). Despite the usefulness of pigmentary hematite as a paleoclimatic indicator or as
74 a carrier of paleomagnetic records, it is often described vaguely as a “fine hematite population”,
75 with poorly constrained coercivity and grain size distributions.

76
77 Characterizing the grain size and coercivity of pigmentary hematite is challenging because it is
78 necessary to overcome the combined difficulty of detecting weakly magnetic hematite when it co-
79 occurs with other magnetic minerals and characterizing poorly crystalline nanoparticles. Magnetite
80 has a spontaneous magnetization that is more than 200 times stronger than hematite, so even small
81 amounts of magnetite can overwhelm the magnetic contribution of hematite (Dekkers, 1990; Frank
82 and Nowaczyk, 2008; Roberts et al., 2020). In practice, hematite detection in natural samples often
83 relies on its high coercivity and distinctive color (Roberts et al., 2020; Jiang et al., 2022 and
84 references therein). However, the small size and often poorly crystalline nature of hematite
85 nanoparticles means that hematite concentrations can be difficult to determine with many
86 spectroscopic approaches. Such particles will also be responsible for a substantial low-coercivity
87 distribution that is not usually attributed to hematite in mineral magnetic studies, especially when
88 using magnetic parameters with cut-off fields of 300 mT (Roberts et al., 2020). Isothermal
89 remanent magnetization (IRM) component analysis appears to be the most suitable magnetic

90 method for detecting hematite because it enables estimation of continuous, non-truncated
91 coercivity distributions (Hu et al., 2021; Roberts et al., 2020). However, superparamagnetic (SP)
92 pigmentary hematite, which is abundant in natural environments (Collinson, 1969; Schwertmann,
93 1991), will not be evident in room temperature IRM results. Color and diffuse reflectance methods
94 also have limitations for detecting or quantifying hematite because they depend strongly on grain
95 size and crystallinity. Decreasing grain size tends to reduce the reflectance wavelength and
96 changes the color from purple-red to yellow-red (Cornell and Schwertmann, 2003; Jiang et al.,
97 2022). Evaluating grain size distributions for pigmentary hematite is, therefore, difficult because
98 grain size influences most proxies used to estimate hematite properties.

99

100 Microscopy observations reveal the existence of nano-sized hematite with sizes from ~20 nm to a
101 few hundred nanometers in soils and banded iron formations (Eggseder et al., 2018; Hyodo et al.,
102 2020; Sun et al., 2015). A more systematic relationship between hematite grain size and
103 unblocking temperature has been established by Swanson-Hysell et al. (2011, 2019) using Néel
104 (1949) relaxation theory. The grain size range of remanence-carrying hematite can be inferred
105 using unblocking temperatures from thermal demagnetization experiments, although this approach
106 cannot be used to estimate SP particles because they do not carry a stable remanence at room
107 temperature.

108

109 Grain volume is a key variable in Néel (1949) theory. Dunlop (1965) pointed out that it is possible
110 to combine field- and temperature-dependent measurements to determine the joint grain volume
111 (V) and microcoercivity at absolute zero (H_{k0}) distribution; $f(V, H_{k0})$. Jackson et al. (2006)
112 developed a procedure to estimate $f(V, H_{k0})$ for particle assemblages that contain both SP and
113 stable single domain (SSD) magnetite based on backfield remanence curves measured over a range
114 of temperatures, which they called “thermal fluctuation tomography” (TFT). This method was
115 used to reconstruct the grain size distribution of magnetite in both synthetic and natural tuff and
116 paleosol samples. Theoretically, TFT can also be used for weakly magnetic minerals like hematite
117 that have a wider size range for SSD behaviour (Banerjee, 1971; Kletetschka and Wasilewski,
118 2002; Özdemir and Dunlop, 2014). Here, we present a TFT procedure to estimate the grain size
119 and microcoercivity distribution for pigmentary hematite in natural red chert samples based on the
120 approach of Jackson et al. (2006). Multiple low-temperature magnetic measurements are integrated

121 to constrain the magnetic mineralogy of natural hematite-magnetite-bearing samples. Our results
122 also provide new insights into the nature of pigmentary hematite in red sedimentary rocks.

123

124 **2. Materials and Methods**

125 ***2.1. Inuyama red chert***

126 Red chert is a distinctive hematite-rich biosiliceous sedimentary rock, which was a common
127 pelagic marine sediment type from the Ordovician to the early Late Cretaceous (Jones and
128 Murchey, 1986). The Inuyama red chert crops out along the Kiso River about 30 km north of
129 Nagoya, Japan. Red chert, gray chert, and siliceous claystone were deposited alternately over
130 thicknesses of several hundred meters in the middle Triassic to early Jurassic (Oda and Suzuki,
131 2000). Hematite occurs as a finely dispersed pigment of chemical origin in red cherts (Jones and
132 Murchey, 1986; Matsuo et al., 2003). The Inuyama red chert contains variable mixtures of
133 magnetite and pigmentary hematite (Oda and Suzuki, 2000; Abrajevitch et al., 2011; Hu et al.,
134 2021). Four representative samples were selected from three red bedded chert sites (KA1, KA6,
135 UN2) in the Inuyama area. Biostratigraphic, paleomagnetic, and rock magnetic results for the same
136 sample set have been published by Oda and Suzuki (2000) and Hu et al. (2021). Radiolarian fossils
137 indicate that the KA1 and UN2 samples have middle (Anisian) and late Triassic (Norian) ages
138 while two KA6 samples are of early Jurassic age (Oda and Suzuki, 2000).

139

140 ***2.2. Low temperature magnetic measurements***

141 Samples were cut into 4 mm × 4 mm × 3 mm pieces and were measured with a Quantum Design
142 Magnetic Properties Measurement System (MPMS) at the Black Mountain Paleomagnetism
143 Laboratory, Australian National University. First, an isothermal remanent magnetization (IRM)
144 was imparted at 10 K in a 5 T field (LTSIRM) after cooling in zero field (ZFC) and then
145 demagnetized by ramping the superconducting magnet down in oscillation mode from 100 to 0
146 mT to simulate an alternating field (AF) demagnetization at 10 K (Lagroix and Guyodo, 2017).
147 The resulting magnetization was then measured from 10 K to 300 K to obtain LTIRM_{@AF100}
148 warming curves. Following the same protocol, LTIRM_{@AF300} warming curves were also measured
149 for the same sample by ramping the magnet down from 300 to 0 mT after imparting a LTSIRM.

150 In this way, magnetizations carried over different coercivity ranges (>300 mT, 100-300 mT, and
 151 <100 mT) were separated to identify their low temperature characteristics. In this study,
 152 $LTIRM_{>300 \text{ mT}}$ is represented by $LTIRM_{@AF300}$, $LTIRM_{100-300 \text{ mT}}$ is given by $LTIRM_{@AF100} -$
 153 $LTIRM_{@AF300}$, and $LTIRM_{<100 \text{ mT}}$ is calculated as $LTSIRM - LTIRM_{@AF100}$. Second, a 5 T field
 154 was imparted again at 10 K after ZFC and IRM was measured in zero field during warming to 300
 155 K to obtain ZFC-LTSIRM curves. Samples were then field-cooled (FC) to 10 K in a 5 T field and
 156 measured during warming back to 300 K after removing the field (FC-LTSIRM). Third, backfield
 157 demagnetization curves were measured for the same four samples at 50 logarithmically spaced
 158 steps from 0.001 T to 5 T after being saturated with an initial 5 T field at eight temperatures: 300
 159 K, 250 K, 200 K, 150 K, 100 K, 80 K, 50 K, and 10 K. These curves were later decomposed into
 160 skew-normal coercivity components using the fitting software MAX UnMix (Maxbauer et al.,
 161 2016). Finally, to test for goethite contributions that will be fully demagnetized at 400 K, samples
 162 were given a room temperature IRM (RTSIRM) in a 5 T field and were demagnetized by ramping
 163 the magnet from 300 to 0 mT in oscillation mode. The remaining remanence $RTSIRM_{@AF300}$ was
 164 measured in zero field during cooling to 150 K and then during warming to 400 K and then during
 165 cooling back to 150 K.

166

167 **3. Thermal fluctuation tomography theory for hematite**

168 We adapted the tomographic imaging method of Jackson et al. (2006) to estimate $f(V, H_{k0})$
 169 distributions for SP and SDD hematite grains. The procedure is described briefly below, with focus
 170 on modifications made for hematite. For a detailed explanation and derivation of TFT theory, see
 171 Jackson et al. (2006) and Dunlop (1965).

172

173 The TFT approach involves using backfield remanence data to estimate the blocking field (H_B).
 174 For hematite at a given temperature, we evaluate $H_B = H_{cr} - H_q$, where H_{cr} is the coercivity of
 175 remanence, and H_q is the thermal fluctuation field. For a randomly oriented population of
 176 identical grains, H_q is expressed as (equation 8 of Jackson et al. (2006)):

$$177 \quad H_q = 0.801 \left(\frac{kT\sqrt{H_k(T)}}{\mu_0 VM_S(T)} \right)^{\frac{2}{3}} \ln^{\frac{2}{3}} \left[\frac{\tau_{exp}}{\tau_0 \mu_0 \Delta H_{DC} \sqrt{\mu_0 H_k(T)}} \times \left(\frac{kT}{VM_S(T)} \right)^{\frac{2}{3}} \right], \quad (1)$$

178 where M_s (T) and H_k (T) are the saturation magnetization and microcoercivity as a function of
 179 absolute temperature, T, respectively, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m), k is
 180 Boltzmann's constant (1.38×10^{-23} J/K), V is the hematite particle volume, and τ_0 is a characteristic
 181 time related to the natural frequency of gyromagnetic precession. For nanosized hematite, τ_0 is
 182 found to be 10^{-12} - 10^{-11} s (Henrik, 2014 and references therein). We here assume τ_0 10^{-12} s. The
 183 exposure time τ_{exp} for the backfield treatments is assigned as 300 s, and ΔH_{DC} is the applied field
 184 difference between successive backfields. We assume here that the saturation magnetization at
 185 absolute zero (M_{S0}) is 2500 A/m for hematite (Dunlop and Özdemir, 1997), then $M_S(T)$ can be
 186 represented using Bloch's 3/2 law (Bloch, 1930):

$$187 \quad M_S(T) = M_{S0} \times \left(1 - B \times T^{\frac{3}{2}}\right), \quad (2)$$

188 where B is the Bloch constant. B is determined by the spin-wave stiffness constant; we adopt $B =$
 189 10^{-5} for hematite nanoparticles (Martínez et al., 1996).

190

191 The next step is to describe microcoercivity (H_k) as the function of temperature. Two analytic
 192 models have been used previously to describe the temperature dependence of the coercive force.

193 1. By taking $\frac{H_k(T)}{H_{k0}} = \left(\frac{M_S(T)}{M_{S0}}\right)^n$ (Dunlop and Özdemir, 2000; Jackson et al., 2006; Menyeh
 194 and O'Reilly, 1995), where n depends on the dominant anisotropy, we can calculate
 195 $H_k(T)$ based on Bloch's 3/2 law:

$$196 \quad H_k(T) = H_{k0} \left(1 - B \times T^{\frac{3}{2}}\right)^n, \quad (3)$$

197 However, hematite anisotropy can be complex and published n values for fine-grained
 198 hematite below room temperature are rare. Study of synthetic nano-sized hematite
 199 reveals that temperature has a minimal impact on $M_S(T)$ while coercivity increases
 200 significantly at low temperature due to frozen canted spins (Satheesh et al., 2017).
 201 Therefore, n should be large because of the significant $H_k(T)$ change compared to
 202 minimal $M_S(T)$ change. Satheesh et al. (2017) reported M_s and H_c for a 64 nm hematite
 203 sample at both 5 K and 300 K, to give a calculated n value of ~10.

204 2. The temperature dependence of coercivity, H_c (T), can also be expressed by Kneller's
 205 law (Kneller and Luborsky, 1963):

$$H_c(T) = H_{c0} \left(1 - \left(\frac{T}{T_B} \right)^\alpha \right), \quad (4)$$

where T_B is the blocking temperature for SP particles, α is Kneller's exponent and H_{c0} is the coercivity at absolute zero. For non-interacting single domain nanoparticles with uniaxial anisotropy, α usually takes a value of 0.5 (Kuncser et al., 2020; Maaz et al., 2010; Osman and Moyo, 2015). However, α can deviate from 0.5 due to finite size effects at the nanoscale as well as due to variations in volume distribution, randomness of anisotropy axes, and interparticle interactions (Nayek et al., 2017). Similar to the n value in equation (3), α for hematite nanoparticles is poorly constrained.

To establish a thermally dependent coercivity model for pigmentary hematite in Inuyama red chert, we compare the hematite median H_{cr} values obtained from backfield curve decomposition using both models. First, we need to clarify the relationship among different coercivity forms, H_c , H_{cr} and H_k . Experimental H_{cr}/H_c ratios for SSD hematite are almost constant at ~ 1.5 (Martin-Hernandez and Guerrero-Suarez, 2012; Peters and Dekkers, 2003; Özdemir and Dunlop, 2014; Roberts et al., 2021). The relationship between H_{cr} and H_k depends largely on the dominant anisotropy type. For randomly oriented identical particles with uniaxial anisotropy, Stoner and Wohlfarth (1948) theory gives $H_{cr}/H_k = 0.524$. Multiaxial anisotropy, such as cubic or hexagonal anisotropy, can increase H_{cr} (Harrison et al., 2019) and therefore raise this ratio close to 1. The high magnetostriction of hematite and weak M_s suggests a high sensitivity to magnetostrictive strain in hematite (Banerjee, 1963); this strain-related anisotropy is taken to be uniaxial. FORC diagrams for the studied red chert samples have “ridge-type” distributions for hematite up to 1.2 T (Hu et al., 2021), which is typical of uniaxial SSD particle assemblages (Egli et al., 2010). Therefore, by assuming a dominant uniaxial anisotropy and relatively constant H_{cr}/H_c ratios for SP/SSD hematite in the Inuyama red chert, we adopt linear relationships among H_c , H_{cr} and H_k . Then, for model 1, we fit the hematite median H_{cr} data using equation (3). Under the assumption of a common n value for all samples, we estimate both H_{cr0} and n using Bayesian regression. Similarly, by assuming a common α value, we fit the hematite H_{cr} data using equation (4) and obtain median posterior estimates of H_{cr0} , T_B , and α via Bayesian regression for model 2 (see section 4.3 for details). By selecting an appropriate model based on our experimental data (section

235 4.3) and assigning $H_{cr0} = 0.524H_{k0}$, we can estimate $H_k(T)$. Then $H_B(T)$ is obtained by substituting
 236 $H_B = 0.524H_k - H_q$ into equation (1).

237
 238 After constructing field blocking contours for hematite, we describe each hematite grain using two
 239 essential attributes, V and H_{k0} . A saturating field applied and removed isothermally at temperature
 240 T_1 magnetizes the entire thermally stable population at that temperature (Figure 1, blue shaded
 241 region), which corresponds to grains with (V, H_{k0}) that plot above and to the right of the zero-field
 242 blocking contour for T_1 (Figure 1). Subsequent application and removal of a reverse DC field, H_1 ,
 243 flips the magnetic moments of grains with (V, H_{k0}) that plot below and to the left of the blocking
 244 contour for (T_1, H_1) (Figure 1, hatched area). Each backfield reverses the moments of grains that
 245 plot in a region on the Néel diagram (Dunlop, 1965; Néel, 1949) bounded by two blocking field
 246 contours for a specified temperature. The change in remanence ΔM_R produced by each DC
 247 backfield treatment can, therefore, be expressed as (equation 11 of Jackson et al. (2006)):

$$248 \quad \Delta M_R = \int f(V, H_{k0}) d\Omega, \text{ and}$$

$$249 \quad \Omega = \left\{ V, H_{k0} \mid H_{i-1} \leq H_B \leq H_i \right\} \quad (5)$$

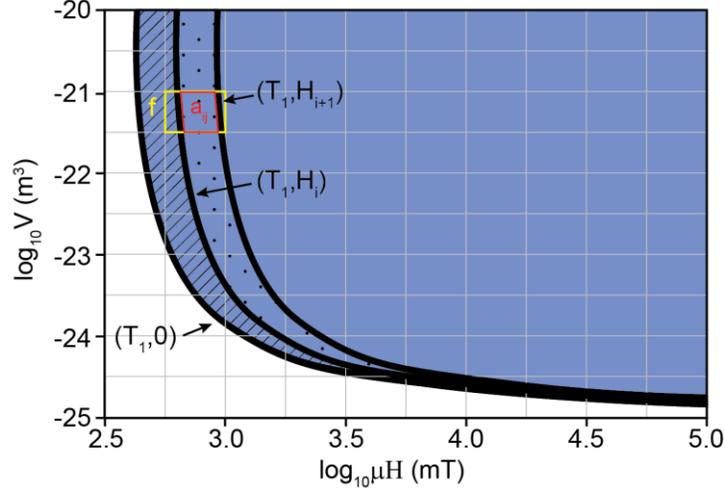
250 where Ω represents the region bounded by two blocking field contours and H_i represents a reverse
 251 DC field treatment. Therefore, the procedure is essentially an inverse problem involving $f(V, H_{k0})$
 252 estimation from a series of DC backfield remanence curves for hematite. Details of procedures
 253 used to obtain hematite backfield remanence curves are explained in section 4.2.

254
 255 To estimate $f(V, H_{k0})$, we divided the Néel diagram into a rectilinear grid of cells in which f is
 256 uniform in each cell (Figure 1, yellow cell). The discrete equivalent of equation (5) is:

$$257 \quad \Delta M_{Ri} = \sum_{j=1}^{n_{\text{cells}}} f_j a_{ij}, \quad (6)$$

258 Where a_{ij} is the area of cell j within the area bounded by the blocking contours for a given
 259 temperature and applied field used when measuring ΔM_{Ri} (Figure 1, red region). Each temperature
 260 and applied field, H_{app} , pair (T, H_{app}) corresponds to a unique blocking contour, defined as the
 261 locus of (V, H_{k0}) for which $H_B(T, V, H_{k0}) = H_{app}$, so we can calculate intersection points of the

262 contours with the grid lines by piecewise linear interpolation between nodes and approximate the
 263 contours by straight-line segments between these intersection points to estimate the areas a_{ij} .



264

265 **Figure 1** Schematic illustration of the TFT technique, modified from Jackson et al. (2006). A
 266 strong field IRM imparted at temperature T_1 is carried by the entire thermally stable population
 267 (blue shaded area); a backfield, H_1 , applied and removed at temperature T_1 , reverses the moments
 268 of grains in the hatched area; a larger backfield, H_2 , further reverses the moments of grains in the
 269 dotted area. a_{ij} (red area) represents the area bounded by the blocking contours for T_1 and applied
 270 fields H_{i+1} and H_i when measuring ΔM_{Ri} . The yellow rectangle represents the j^{th} cell, f_j is the value
 271 of $f(V, H_{k0})$ for the j^{th} cell.

272

273 We employ an initialization of $f = 0$ at all points to generate a forward model based on equation
 274 (6). Residuals are then calculated as the difference between the measured and model remanence
 275 data:

$$277 \quad R_i = \Delta M_{Ri, \text{measured}} - \Delta M_{Ri, \text{model}} \quad (7)$$

276 The model is then adjusted by “back-projecting” the residuals:

278

$$279 \quad {}^s \Delta f_{ij} = \frac{R_i a_{ij}}{\sum_{k=1}^{n_{\text{cells}}} a_{ik}^2}. \quad (8)$$

280 The adjustment for cell j is proportional to R_i and the area a_{ij} bounded by the blocking contours.
 281 n_{cells} represents the number of cells and s represents the current simulation. Stepwise updates are
 282 applied after all calculations for each iteration:

283

$$f_j^{s+1} = f_j^s + \frac{C}{n_{\text{measurements}}} \sum_{i=1}^{n_{\text{measurements}}} \Delta f_{ij} (j = 1 \dots n_{\text{cells}}). \quad (9)$$

C is a dimensionless constant used to control the rate of convergence, where higher values cause more rapid convergence, but excessive values can cause the process to become unstable and diverge. Our aim is to reduce the fitting error to ~10% within 100 iterations; after multiple attempts, we found that a C value of 50 generally meets our requirement.

4. Results

4.1. Unblocking of pigmentary hematite

LTSIRM variations of different coercivity fractions for both Triassic and Jurassic red chert samples are shown in Figure 2. The Verwey transition for magnetite is clearly evident at ~120 K for particles with coercivity < 300 mT, which disappears or becomes less noticeable in $LTIRM_{>300}$ curves for the high coercivity component (Figure 2a, 2b), and demonstrating that the coercivity of magnetite is mostly less than 300 mT. For Jurassic specimens, $LTIRM_{>300}$ warming curves decay steeply compared to the relatively flat $LTIRM_{<100}$ curves (Figure 2b), which indicates a wide unblocking temperature distribution of a SP hematite content. A concave shape around 200 K is present in $LTIRM_{>300}$ curves, but not in the low coercivity component, which indicates a likely Morin transition that was not completely smeared out by progressive unblocking of fine hematite (Figure 2b). The $LTIRM_{100-300}$ warming curve contains both a Verwey transition and marked low temperature unblocking, which suggests a mixture of magnetite and finer hematite in this coercivity range. Hematite unblocking is less significant for Triassic samples, which indicates a smaller SP hematite contribution (Figure 2a). However, $LTIRM_{>300}$ curves for both Triassic and Jurassic samples have comparable magnetizations despite the fact that magnetite has a much stronger magnetization, which indicates that hematite dominates the red chert magnetism by mass.

A Verwey transition is clearly present in both ZFC-LTSIRM and FC-LTSIRM curves, while a Morin transition is likely smeared by progressive unblocking of SP hematite (Figure 2c, 2d). ZFC-LTSIRM and FC-LTSIRM curves are not widely separated, which contrasts with the behavior of

312 goethite-rich samples (Guyodo et al., 2003; Liu et al., 2006; Huang et al., 2019). Given that the
313 curves almost overlap (Figure 2c, 2d) it is inferred that any goethite contribution is insignificant.

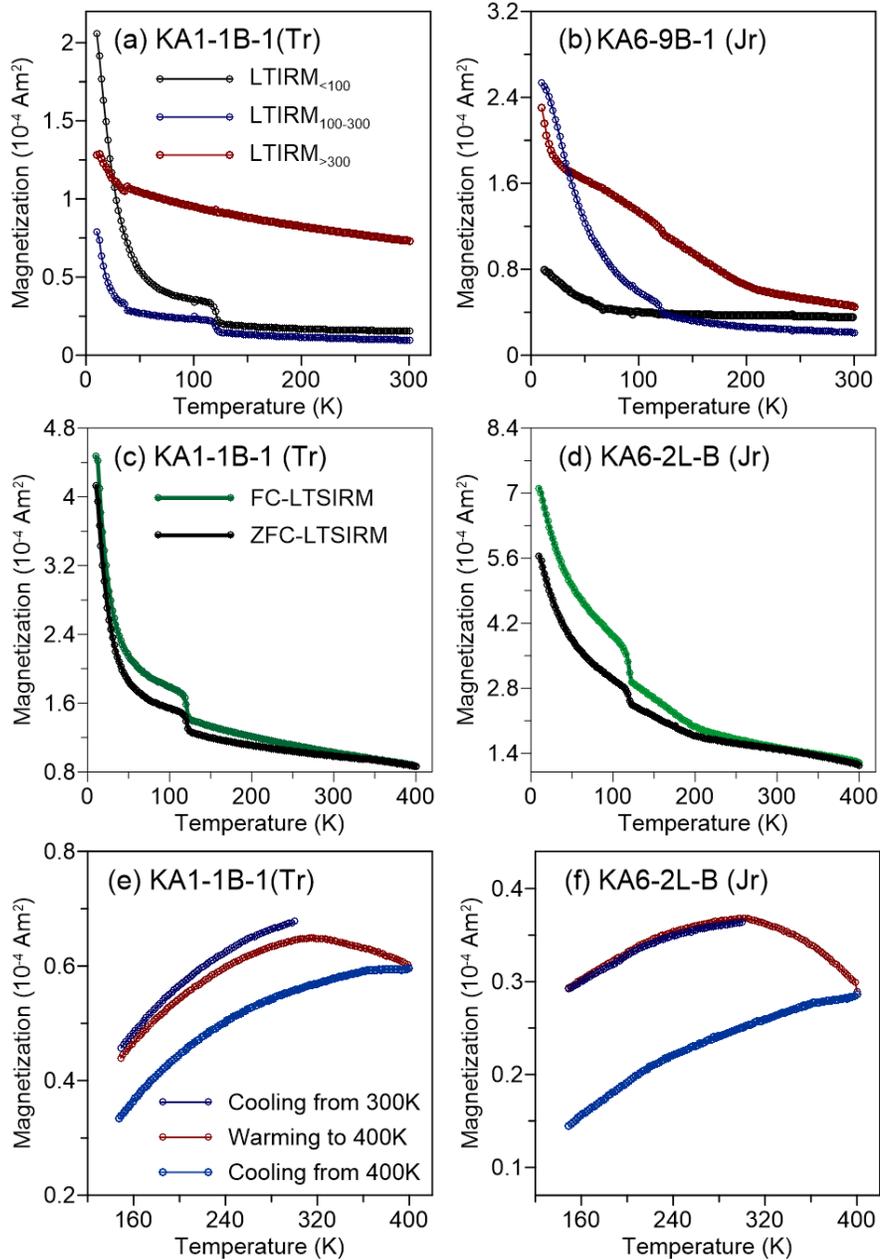
314

315 After removing the low coercivity contribution by applying a 300 mT AF, $RTSIRM_{@AF300}$
316 warming curves decrease gradually from 300 K to 400 K with a net remanence loss during re-
317 cooling (Figure 2e, 2f). No sharp drop is seen at the Néel temperature for goethite. The gradual
318 decrease in warming curves above 300 K is likely due to unblocking of slightly larger hematite
319 particles near the SP/SSD size threshold at room temperature.

320

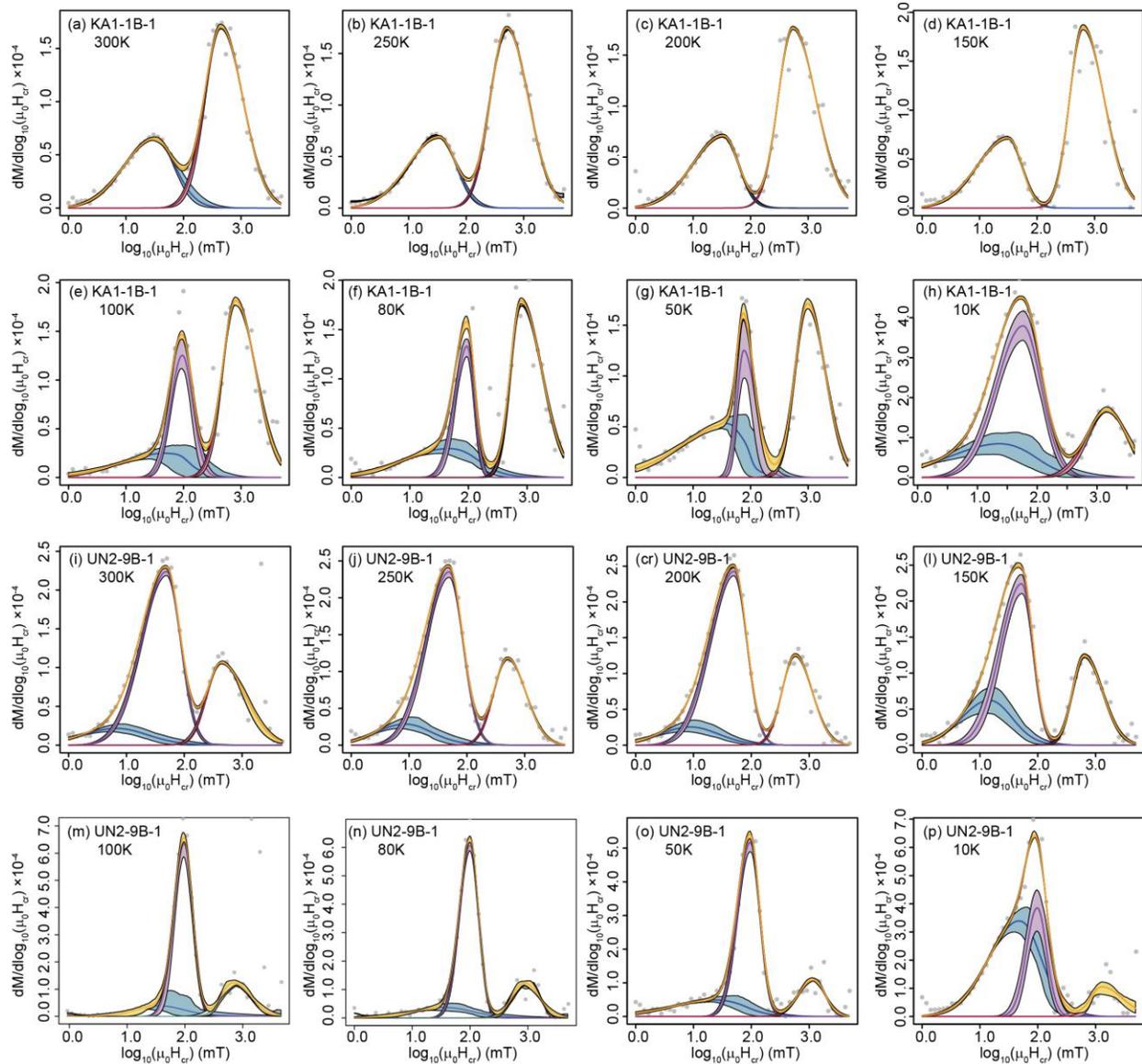
321 ***4.2. Coercivity distributions for pigmentary hematite***

322 The results shown in Figure 2 indicate that goethite is not magnetically important in the Inuyama
323 red chert and that magnetite is mostly confined to the low coercivity component (< 300 mT). We
324 further examine coercivity spectra at eight temperatures from 10 K to 300 K. At room temperature,
325 most Triassic and Jurassic specimens are well fitted with two skew-normal distributions (Figures
326 3a, 4a, 4i). One distribution has a 19-35 mT median coercivity and extends from 0 to ~ 500 mT
327 (based on ± 3 standard deviations from the median coercivity). The other distribution has a higher
328 median coercivity of 413-598 mT and extends from ~ 60 mT to ~ 6 T, which is likely to be due to
329 SSD hematite. Triassic sample UN2-9B-1 has an additional lowest-coercivity contribution with a
330 broad distribution that extends to ~ 200 mT (Figure 3i). At room temperature there is only a small
331 overlap between the low- and high-coercivity components. With decreasing temperature, the
332 overlap is reduced and finally disappears or becomes insignificant below 100 K. This behavior is
333 consistent with hematite coercivity increasing with decreasing temperature (equations (3) and (4)),
334 while magnetite has a much less dramatic coercivity change with temperature (Özdemir et al.,
335 2002). The low-temperature dividing point of the low-coercivity component appears at ~ 250 mT,
336 which is consistent with the Verwey transition being significant in the $IRM_{<300}$ component (Figure
337 2a, 2b). Therefore, for both Triassic and Jurassic specimens, the hematite population can be well
338 separated from magnetite based on their coercivity distributions.



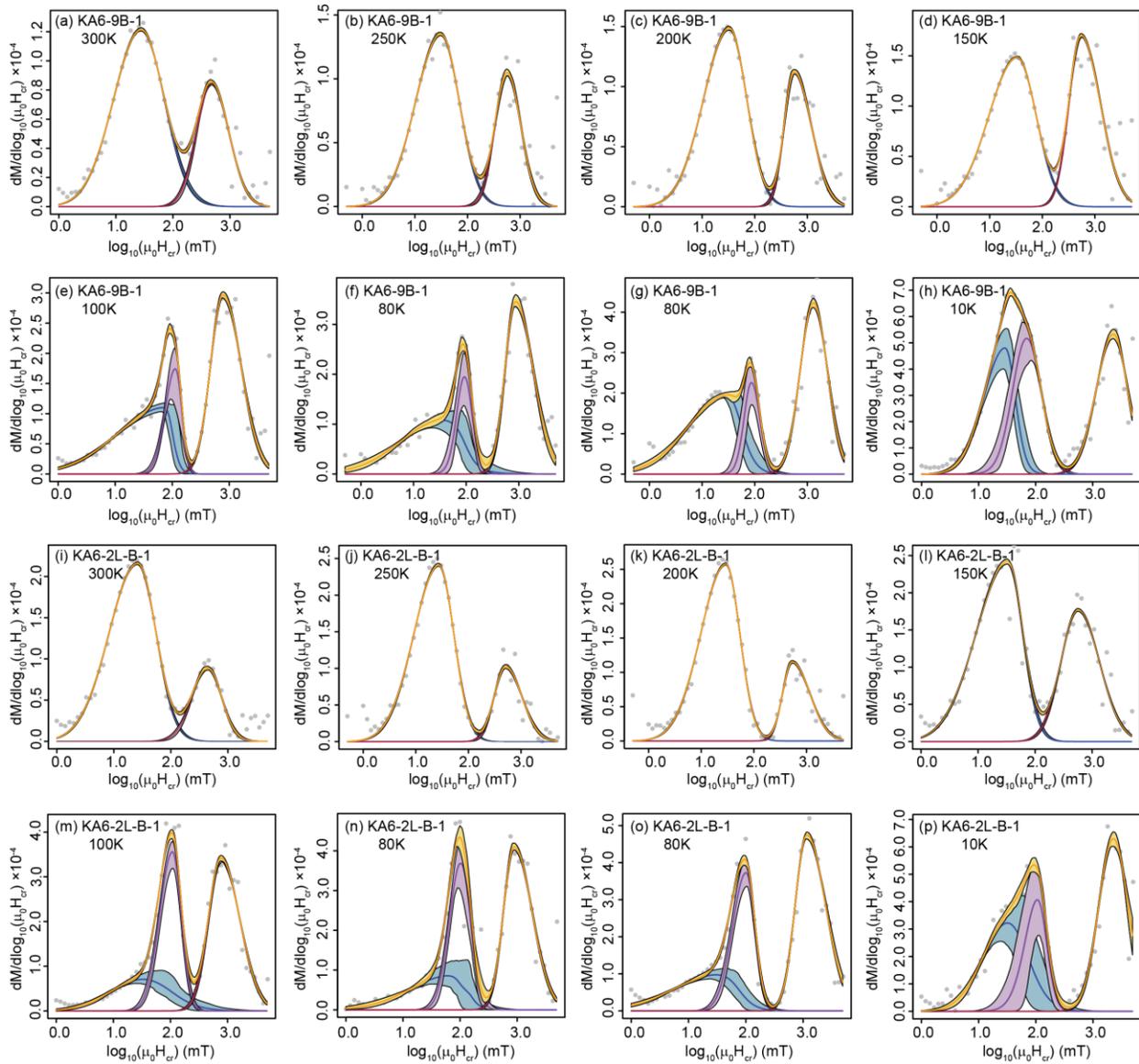
339

340 **Figure 2** LTSIRM variations versus temperature. (a, b) Samples were given a SIRM in a 5 T field
 341 at 10 K and then AF demagnetized in peak fields of 100 and 300 mT, respectively. Then the
 342 LTSIRM was measured during warming for components with coercivity ranges of < 100 mT
 343 (black), between 100 and 300 mT (blue line), and > 300 mT (red). (c, d) ZFC (black) and FC
 344 (green) LTSIRM curves. (e, f) Samples were saturated in a 5 T field at 300 K and then AF
 345 demagnetized in a 300 mT peak field. The $RTSIRM_{@AF300}$ was then measured during cooling to
 346 150 K (dark blue), warming to 400 K (red), and then cooling back to 150 K (light blue). Tr =
 347 Triassic; Jr = Jurassic.



348

349 **Figure 3** Coercivity spectra from backfield SIRM demagnetization curves for two Triassic
 350 specimens (KA1-1B-1, Anisian; UN2-9B-1, upper Norian). The data were fitted using skew-
 351 normal distributions with the Max Unmix software (Maxbauer et al., 2016). We fitted data with a
 352 minimum number of components. At eight temperatures, the data can be fitted with 2-3
 353 components: the lowest coercivity component is shown in blue, the intermediate coercivity
 354 distribution in purple, and the highest coercivity distribution in red. Yellow lines represent the sum
 355 of all components, while grey dots represent the data. Shaded areas are 95% confidence intervals
 356 for each component.

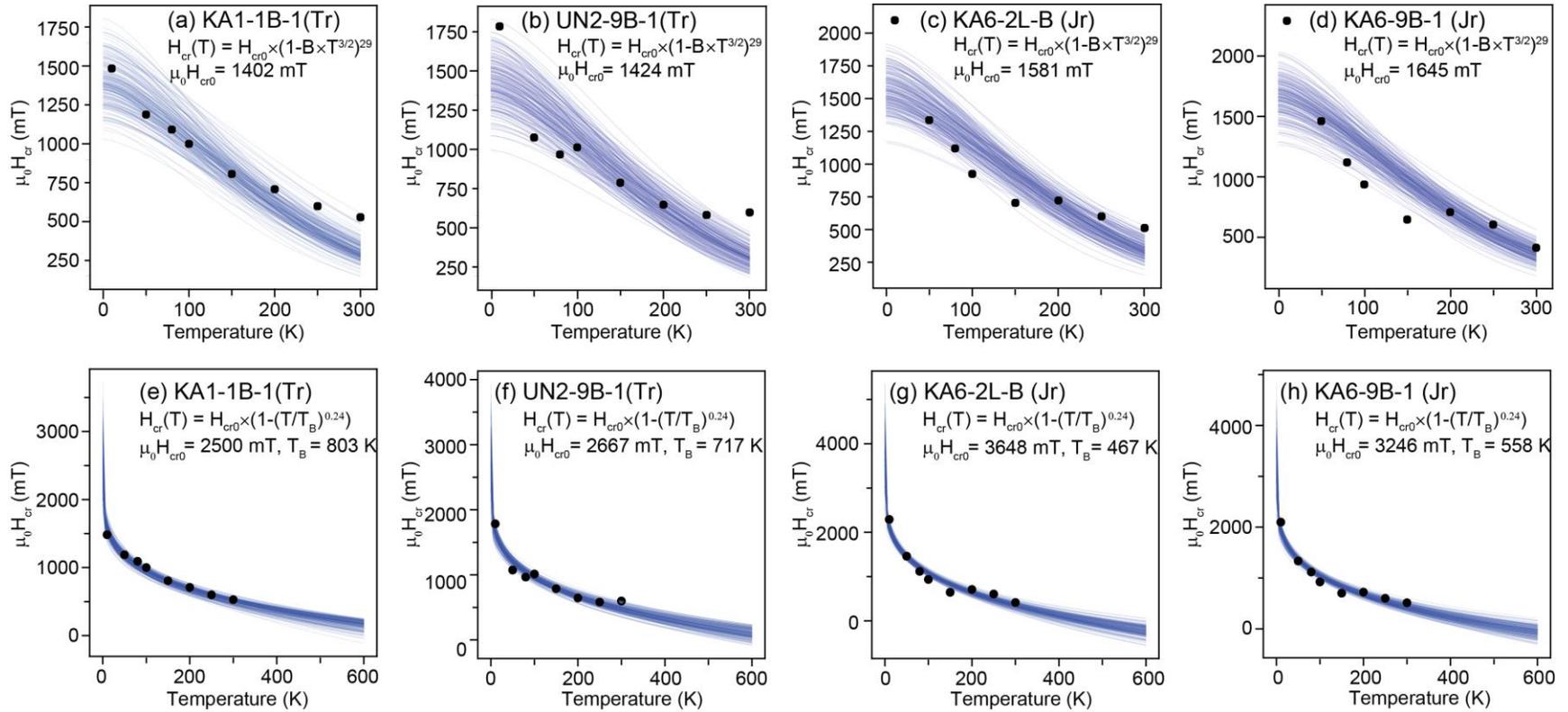


357

358 **Figure 4** Coercivity spectra from backfield SIRM demagnetization curves for two Jurassic
 359 specimens (KA6-9B-1; KA6-2L-B-1 from early Jurassic). Formatting is the same as Figure 3.

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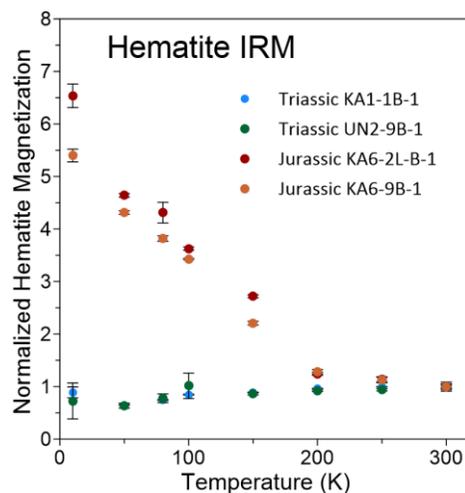
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369

Figure 5 Hematite median remanent coercivity variation with temperature. (a-d) Bayesian posterior distribution of fitted curves based on equation (3) given the priors listed in Appendix A and assuming a common n value for all samples. (e-h) Bayesian posterior distribution of fitted curves based on equation (4) given the Bayesian priors listed in Appendix A and assuming a common value of Kneller's α exponent for all samples. The n and α parameters, T_B shown in the equations are the median values from the parameter posterior distributions. Standard deviations and other posterior distribution statistics are provided in Appendix A.

370 Increases in the coercivity of hematite with decreasing temperature are illustrated in Figure 5.
 371 Below ~ 150 K, the increase is steeper; it triples for Triassic specimens and increases five-fold for
 372 Jurassic specimens at 10 K compared to room temperature. The coercivity-temperature fits in
 373 Figures 5a-5d and 5e-5h were made using equations (3) and (4), respectively, using Bayesian
 374 regression (Appendix A). In this study, we assume that the exponent parameter n or α is constant
 375 for pigmentary hematite in Triassic/Jurassic Inuyama red chert. Under this assumption, we
 376 combine all 32 data points from four specimens at eight temperatures to estimate a common n and
 377 α posterior distribution and individual posterior distributions of H_{cr0} and T_B for each specimen by
 378 Bayesian regression (see Appendix A for details). As expected, large n values of 22 - 36 (97%
 379 high density interval) are obtained, which demonstrates that the hematite coercivity increases more
 380 strongly than M_s . However, these fits are less satisfying at low- and room-temperature. The fits
 381 tend to underestimate H_{cr0} and the coercivity close to room temperature due to the flatness of the
 382 fitted curves, which largely comes from the $3/2$ exponent. Fits based on equation (4) achieve better
 383 results (Figure 5e-5h). The posterior α ranges from 0.151 to 0.339 (97% high density interval) with
 384 median value of 0.24. Triassic red chert samples have lower H_{cr0} and higher T_B than Jurassic red
 385 cherts. Low T_B values of ~ 194 °C and ~ 285 °C are predicted for hematite in Jurassic red cherts,
 386 which suggests they have a fine grain size.



387
 388 **Figure 6** Hematite IRM variation with temperature. Data are normalized by hematite IRM at 300
 389 K for each sample. Error bars represent fitting errors for the hematite component.

390

391 Distinctively IRM intensity changes for hematite with temperature are shown for Triassic and
392 Jurassic samples, respectively, in Figure 6. The hematite remanence remains relatively constant
393 for the Triassic specimens (blue and green dots), which indicates that almost all of the hematite is
394 in the SSD state at room temperature. In contrast, hematite remanence increases exponentially with
395 decreasing temperature for the Jurassic samples (brown and red dots), which indicates a significant
396 SP contribution with a wide blocking temperature range.

397

398 ***4.4. Tomographic Analysis***

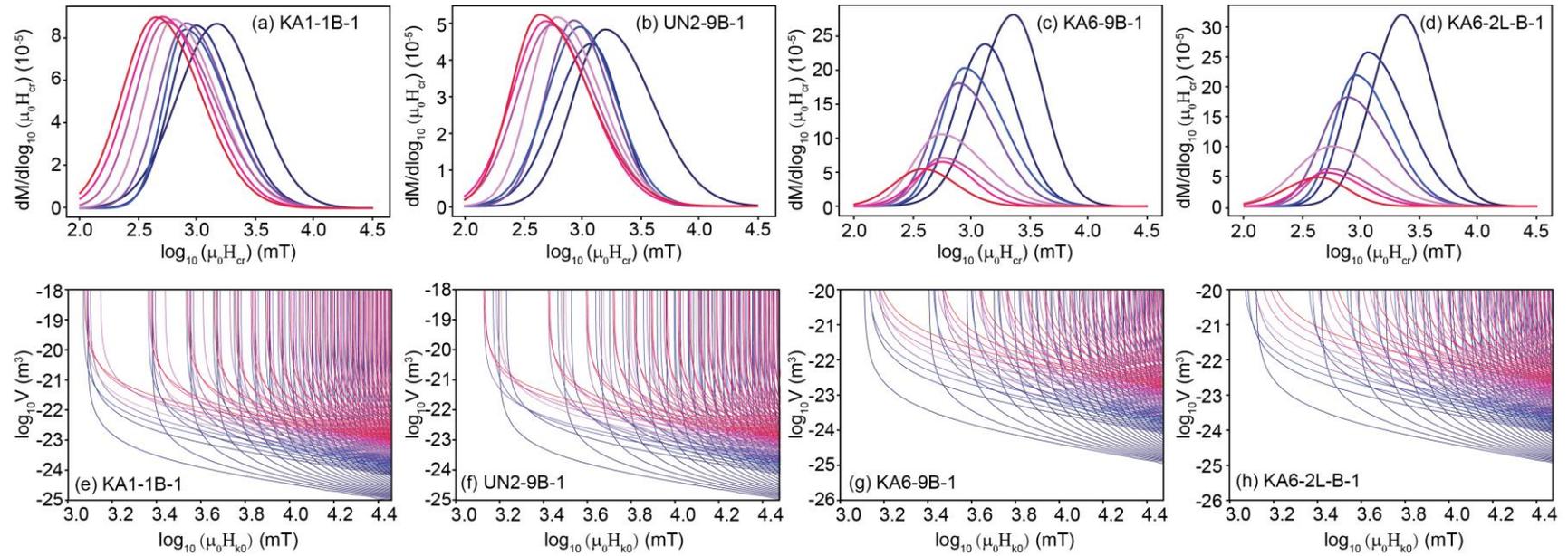
399 Based on the above results, for our tomographic analysis we adopted Kneller's law as a coercivity-
400 temperature relationship with $\alpha = 0.24$ and $H_{cr} = 0.524H_k$. The median T_B values shown in Figure
401 5e-5f are used for each specimen to calculate hematite blocking contours. Hematite coercivity
402 distributions extracted from the high-coercivity component fitting in Figures 3 and 4 are shown in
403 Figure 7a-7d. Each dataset contains 808 backfield remanence data points (101 field steps at each
404 of eight temperatures). These data are combined with equations (1) and (4) and are mapped into
405 blocking contours (Figure 7e-7h).

406

407 Upon cooling to 10 K, all backfield derivative curves shift progressively to higher coercivities as
408 expected. Peak heights are roughly constant for Triassic samples but increase significantly upon
409 cooling for Jurassic samples (Figure 7). This indicates a greater SP hematite content that blocks
410 gradually with cooling in Jurassic red chert, while the Triassic red chert is dominated by coarser
411 SSD hematite. The blocking contour density is nonuniform, so poor resolution is expected for
412 particles smaller than ~ 10 nm with microcoercivities less than ~ 1 T.

413

414 After determining the blocking contours, we start the iterative process to calculate the joint grain
415 size and microcoercivity distribution of hematite particles. Best-fit backfield derivative curves
416 reproduce large-scale features of the measured spectra, while still containing higher frequency
417 deviations (Figures A2-A5). Fitting errors are below 15% for all samples.



418

419 **Figure 7** Hematite backfield remanence data and blocking contours for Triassic and Jurassic red cherts. (a-d) Hematite coercivity
 420 distributions extracted from backfield LTSIRM decomposition at eight temperatures from 10 K to 300 K. (e-h) Blocking contours for
 421 the fields and temperatures in the corresponding datasets above. The equations in Figure 5e-5h are used for each respective specimen.
 422 Color variations indicate temperature changes from 10 K (blue) to 300 K (red).

423

424

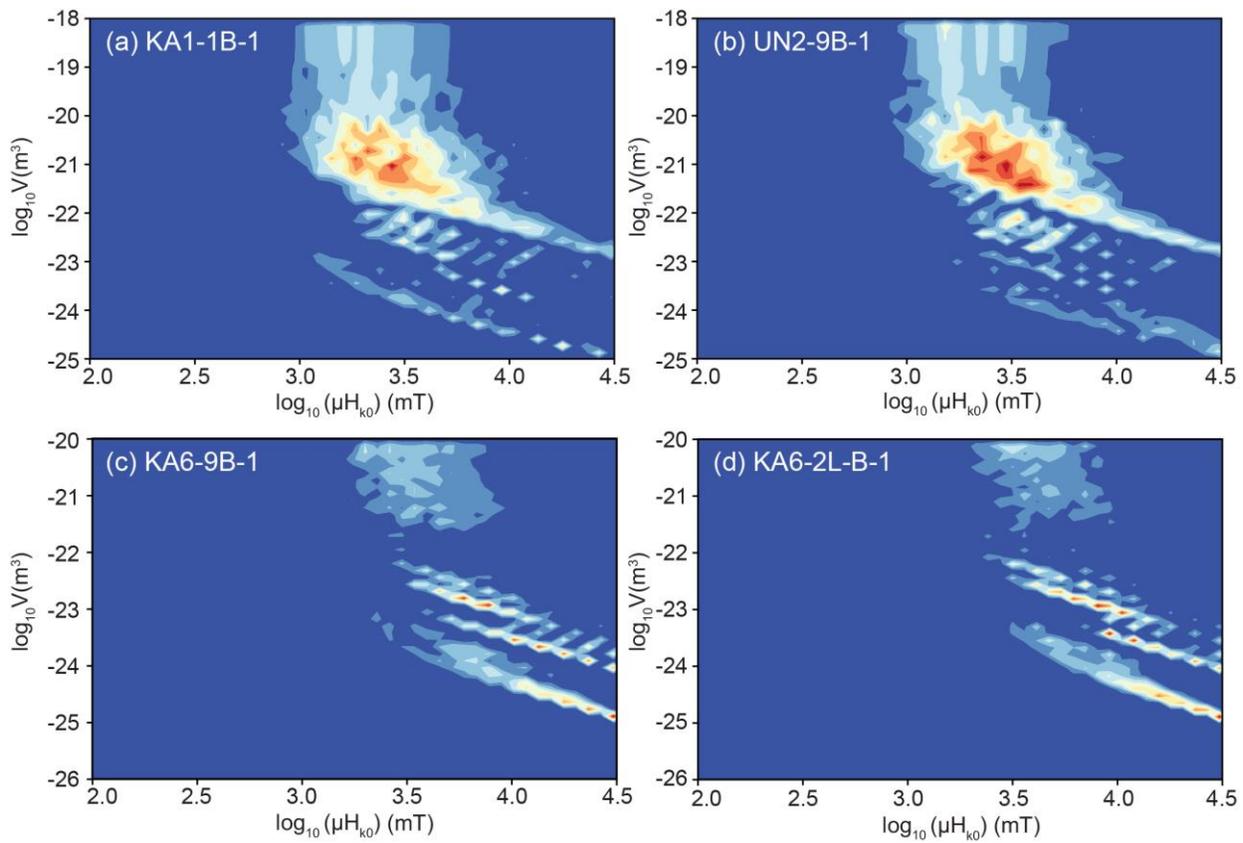
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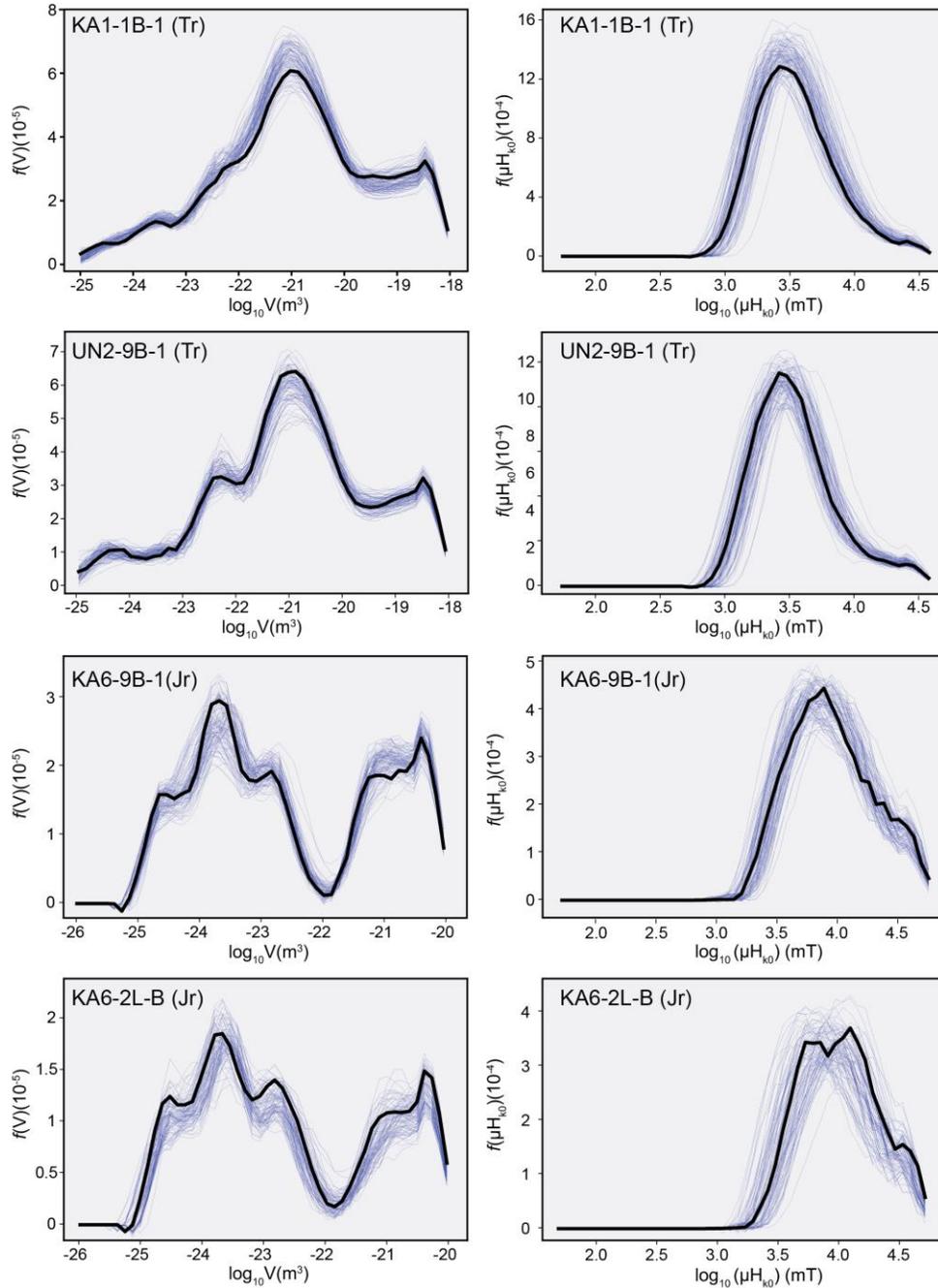
428

429 Estimated $f(V, H_{k0})$ distributions for Triassic samples have a continuous feature centering at
 430 volumes around $1 \times 10^{-21} \text{ m}^3$ and microcoercivities between 1 T and 10 T (Figure 8a, 8b). The
 431 central volumes are equivalent to spherical hematite particles with $\sim 75 \text{ nm}$ diameters. In Jurassic
 432 samples, the hematite particles are smaller but magnetically harder (Figure 8c, 8d), with more
 433 discrete distributions centered around volumes of $\sim 1 \times 10^{-25} \text{ m}^3$, $\sim 1 \times 10^{-23.5} \text{ m}^3$, $\sim 1 \times 10^{-22.5} \text{ m}^3$, and
 434 $> 1 \times 10^{-22} \text{ m}^3$, which correspond to diameters of $\sim 3 \text{ nm}$, $\sim 11 \text{ nm}$, $\sim 24 \text{ nm}$, and $> 35 \text{ nm}$ for
 435 spherical hematite. The microcoercivity of Jurassic hematite ranges from 3 T to $> 30 \text{ T}$. There is
 436 clear elongation of the distribution toward the lower right, along with the dominant blocking
 437 contour orientation, which may be an artifact of the inversion process (Jackson et al., 2006).
 438



439
 440
 441 **Figure 8** Estimated $f(V, H_{k0})$ for (a, b) Triassic and (c, d) Jurassic red chert samples from the
 442 data in Figure 6a-d. Contour interval = $f_{\text{max}}/30$.

443
 444



445

446 **Figure 9** Volume and microcoercivity distributions obtained by summing the rows and columns
 447 of the 2D model. The data were smoothed with a Savitzky-Golay filter with window length of 5
 448 in ‘nearest’ mode using the Python Scipy.signal package. Thick black lines represent the median
 449 value; light blue lines represent calculations based on 100 randomly drawn T_B and α values from
 450 the Bayesian posterior distribution in Figure A1, which are used to indicate the uncertainty on
 451 the calculation of the volume and microcoercivity distribution.

452 Bayesian modeling was used to calculate the volume and microcoercivity distributions shown in
 453 Figure 9. Thick black lines represent the median volume and microcoercivity distribution for each
 454 sample based on median T_B and α values. The light blue lines represent calculations based on 100
 455 randomly drawn T_B and α values from their Bayesian posterior distribution in Figure A1, which
 456 indicates the uncertainty on the volume and microcoercivity distribution calculation. The
 457 marginalized microcoercivities are nearly lognormally distributed. Additional high
 458 microcoercivity contributions (> 10 T) are more evident in the Jurassic hematite than Triassic
 459 hematite. By contrast, volume distributions are asymmetrical and more complex. Triassic hematite
 460 populations have a small peak at 1×10^{-23} to 1×10^{-22} m^3 and then gradually increase to a major peak
 461 at $\sim 1 \times 10^{-21}$ m^3 (Figure 9a). Additional coarse particles with volume larger than 1×10^{-20} m^3 are also
 462 present. The Jurassic hematite population has a roughly bimodal distribution separated at around
 463 1×10^{-22} m^3 . The larger particle population has a broad peak from 1×10^{-22} m^3 to 1×10^{-20} m^3 . The
 464 smaller particle population has a major peak at $\sim 1 \times 10^{-24}$ m^3 with two smaller peaks at $1 \times 10^{-24.6}$ m^3
 465 and 1×10^{-23} m^3 , which correspond to their discrete components in Figure 8c, 8d. The discrete
 466 nature of the distribution is most likely due to the limited numbers of temperatures used here
 467 because the distributions are all elongated along the unblocking contours. Nevertheless, two grain
 468 size populations are evident in Jurassic samples; the finer fraction ranges from a few nanometers
 469 to ~ 35 nm in diameter while the coarser fraction is from ~ 35 nm to ~ 160 nm in diameter and is
 470 comparable with Triassic samples.

471

472 **5. Discussion**

473 ***5.1. Coercivity of pigmentary hematite in red chert***

474 Early studies of the Inuyama red chert reported large saturating fields of up to several tesla for
 475 pigmentary hematite from IRM acquisition curves (Oda and Suzuki, 2000; Shibuya and Sasajima,
 476 1986). In our results, room temperature $\mu_0 H_{cr}$ ranges from ~ 60 mT to ~ 6 T in Triassic red chert
 477 and from ~ 70 mT to ~ 3 T in Jurassic red chert (Figures 3 and 4), which is comparable to recent
 478 studies (Abrajevitch et al., 2013; Hu et al., 2021). Published data for pigmentary hematite in red
 479 beds have a similarly wide range of $\mu_0 H_{cr}$ values. In the Deer Lake Group red beds of western
 480 Newfoundland, hematite remanent coercivity ranges from ~ 60 mT to 3 T (Bilardello and Kodama,
 481 2010a), and for red beds from the Maritime provinces of Canada, it varies from ~ 40 mT to 5 T and

482 beyond (Bilardello and Kodama, 2010b). Hematite in Triassic red beds from South China has
483 remanent coercivity values from ~60 mT to 3 T (Jiang et al., 2017). For zebra rock in Western
484 Australia, strong fields up to 3 T are needed to saturate hematite (Abrajevitch et al., 2018). North
485 American red siltstone intraclasts have remanent coercivity of ~100 mT to 1.8 T and beyond
486 (Swanson-Hysell et al., 2019). Thus, $\mu_0 H_{cr}$ values of ~60 mT to ~3 T are typical of natural
487 pigmentary hematite in red beds, although values up to even ~6 T are sometimes observed. This
488 $\mu_0 H_{cr}$ range gives an idea of the remanent coercivity distribution of natural SSD pigmentary
489 hematite.

490

491 We further illustrate remanent coercivity variations with temperature for pigmentary hematite. H_{cr}
492 increases exponentially with decreasing temperature, following the T^α law, where $\alpha = 0.24$ is the
493 median posterior value for red chert samples in this study (Figure 5e, 5f, 5g, 5h). This behavior
494 can be understood by considering thermal fluctuation effects of blocked moments across an
495 anisotropy barrier (Maaz et al., 2010). For natural pigmentary hematite, this simple thermal
496 activation model appears to be applicable from 10 K to 300 K. The significant H_{cr} increase at low
497 temperatures also provides a way to separate a hematite component from magnetite. Based on
498 results in Figures 2 and 3, there is almost no overlap between magnetite and hematite components
499 below 100 K.

500

501 Quintupled hematite M_{rs} values at 10 K compared to room temperature confirms the presence of a
502 large SP hematite population in the Jurassic red chert (Figure 6). The steep M_{rs} rise below 200 K
503 indicates that the blocking temperature of most SP hematite is below 200 K. Our results
504 demonstrate that decomposition of low temperature backfield curves reveals and potentially
505 enables quantification of entire pigmentary hematite populations, especially SP particles. Although
506 SP signatures are detected in remanent FORC diagrams (Hu et al., 2021), these signals tend to be
507 dominated by magnetite because FORC diagrams reflect bulk signals and the magnetization of
508 hematite is more than two hundred times lower than magnetite (Dunlop and Özdemir, 1997).

509

510 Compared to room-temperature H_{cr} , which only represents SSD populations, H_{k0} provides a
511 measure of the entire hematite population, including SP particles. In the calculations of Jackson et
512 al. (2006), $\mu_0 H_{k0}$ values for magnetite in a Tiva Canyon Tuff sample can extend to 300 mT, which

513 is the upper limit for prolate spheroids with magnetite-like magnetizations. According to our
514 calculations, $\mu_0 H_{k0}$ for the pigmentary hematite can be much higher, and varies from 1 T to ~ 10 T
515 in Triassic red chert and from ~ 1.5 T to > 30 T in Jurassic red chert (Figure 9). As is the case for
516 magnetite, microcoercivity distributions for hematite are nearly symmetric in logarithmic space
517 (Figure 9).

518

519 The high coercivity of pigmentary hematite cannot be explained by shape anisotropy (Banerjee,
520 1971). Özdemir and Dunlop (2014) concluded by studying MD hematite that both magnetoelastic
521 and magnetocrystalline effects contribute to coercivity. However, magnetocrystalline anisotropy
522 causes a gradual coercivity decrease during cooling below room temperature (Liu et al., 2010;
523 Özdemir et al., 2002), which is opposite to the temperature dependence of coercivity observed
524 here. The increasing coercivity trend with cooling in pigmentary hematite is interpreted to be due
525 to magnetoelastic anisotropy, which arises from crystal defects, dislocations, or internal strain
526 (Sunagawa and Flanders, 1965; Sunagawa, 1960; Liu et al., 2010).

527

528 ***5.2. Grain size distribution and unblocking temperature of pigmentary hematite in red chert***

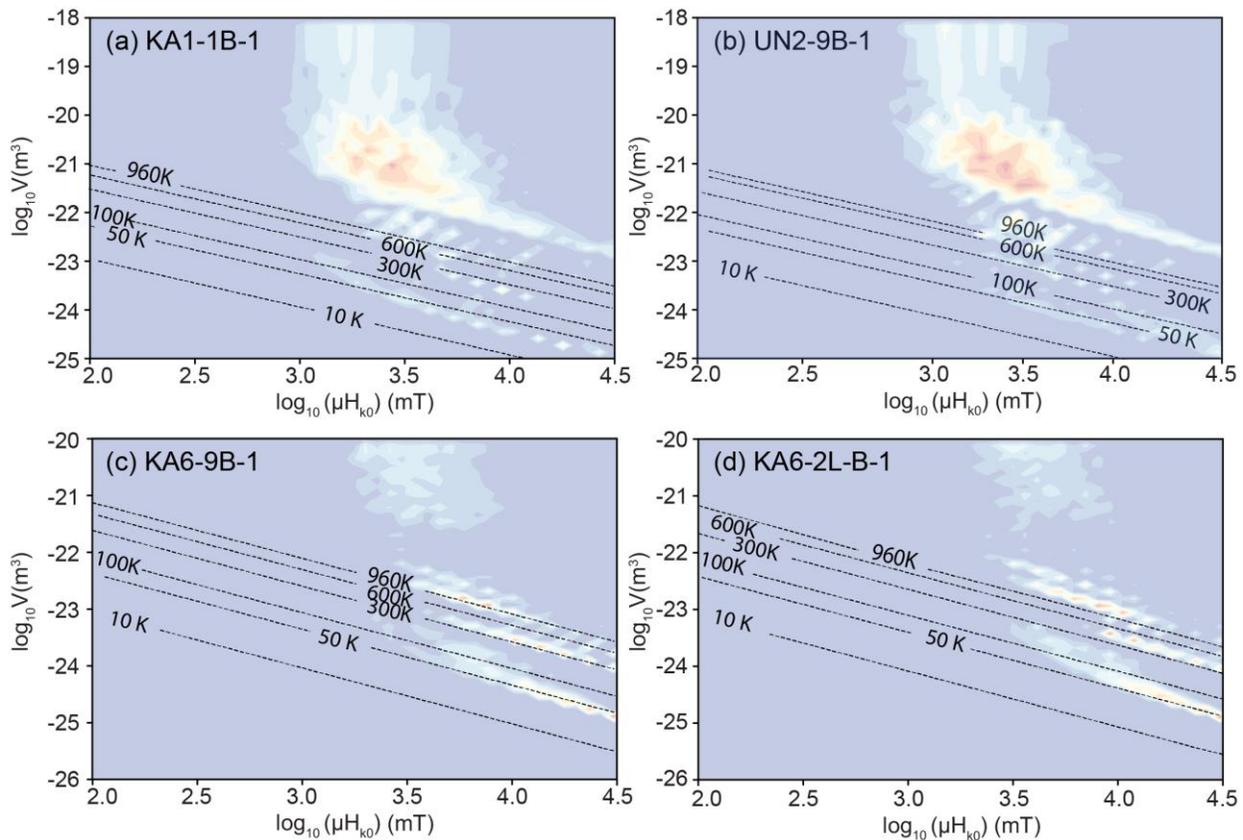
529 Our analysis reveals that both Triassic and Jurassic red chert samples have hematite population
530 with median size of ~ 75 nm. Additional large amounts of finer hematite $< \sim 35$ nm occurs in the
531 Jurassic red chert (Figure 9). The median T_B for each red chert sample is 529 °C (KA1-1B-1),
532 438 °C (UN2-9B-1), 194 °C (KA6-2L-B-1), and 285 °C (KA6-9B-1). The low unblocking
533 temperatures are also consistent with the small particle size of pigmentary hematite. According to
534 the unblocking temperature and grain size model of Swanson-Hysell et al. (2011, 2019), the
535 median unblocking temperatures in red chert correspond to grain sizes of 100-160 nm, which is
536 within the range of our calculated grain size distributions. The broad unblocking temperature
537 distribution reflects the wide size distribution of natural pigmentary hematite.

538

539 We further calculated T_B contours based on the volume and microcoercivity distributions obtained
540 here (Figure 10). Almost the entire pigmentary hematite population in the Triassic red cherts is in
541 the SSD state at room temperature, with unblocking temperatures generally above 300 K (Figure
542 10a, b). For the Jurassic red cherts, a significant part of the hematite is in the SP state with

543 unblocking temperatures extending below 100 K. The SP/SSD threshold size for stoner-wohlfarth
 544 hematite has been estimated at 25-30 nm (Banerjee, 1971; Özdemir & Dunlop, 2014) and at 17
 545 nm for Al-hematite (Jiang et al., 2014). From our T_B calculations, the SP/SSD threshold size for
 546 stoichiometric hematite in the Jurassic red cherts is 7-18 nm, which is close to estimates for Al-
 547 hematites.

548



549

550 **Figure 10** Calculated blocking temperature contours from 10 K to 960 K with $f(V, H_{k0})$

551 distributions for (a, b) Triassic and (c, d) Jurassic samples.

552

553 Detailed paleomagnetic studies have been conducted on the Inuyama red chert, with four
 554 remanence components identified from stepwise thermal demagnetization (Oda and Suzuki, 2000;
 555 Shibuya and Sasajima, 1986). The four components are labeled A (70-200 °C), B (200-350 °C), C
 556 (350-530 °C), and D (530-680 °C). Components B and C were thought to be Late Cretaceous
 557 remagnetizations, although the remagnetization mechanism has been debated. Partial thermal
 558 demagnetization (ThD) after alternating field (AF) demagnetization at 80 mT reveals the same

559 four components (Oda and Suzuki, 2000), which indicates that hematite was at least partly
560 responsible for all four components. This is consistent with the wide unblocking temperature range
561 for hematite in our calculations. Paleomagnetic studies of the Inuyama red chert have all suggested
562 that magnetite is the dominant remanence carrier for components A, B, and C and, thus, magnetite
563 carries the remagnetizations. However, pigmentary hematite should also contribute to the
564 remagnetizations due to its low unblocking temperature, especially for Component B. We suggest
565 this because the unblocking temperature range of component B (200-350 °C) and C (350-530 °C)
566 spans the median hematite unblocking temperature range. Meanwhile, little magnetization for
567 component B was lost between 200 and 350 °C in the original ThD experiment (Figure 6a from
568 Oda and Suzuki (2000)), but the magnetization drops significantly after AF demagnetization of 80
569 mT (Figure 6c in Oda and Suzuki (2000)). According to our analysis on the magnetite remanent
570 coercivity distribution of the Inuyama red chert at room temperature (Figure 3 and Figure 4), an
571 alternating field of 80 mT will demagnetize most of the magnetite, which suggests that the more
572 significant unblocking feature of component B after AF demagnetization is likely due to
573 pigmentary hematite.

574 ***5.3. Limitations of the TFT method for reconstructing hematite grain size distributions***

575 There are two main limitations of the TFT technique for hematite. First, it has the same limitation
576 for hematite as it does for magnetite (Jackson et al., 2006). Reconstructed size distributions are
577 elongated toward the upper left and lower right-hand sides of the Néel diagram, along the blocking
578 contour orientation. This elongation can be due to artifacts in the inversion method, the physical
579 model, and the assumption made, as shown by Jackson et al. (2006) and Dunlop (1965). Resolution
580 limits also remain for inversion due to the restricted orientation distribution of integration paths.
581 Areas of marginal resolution are sampled sparsely by subparallel contours, such as within the small
582 V , high H_{k0} region that is stable only at the lowest temperatures. At the same time, given the small
583 numbers of temperatures employed, the resolution of our results is imperfect, and stripes are
584 evident in the $f(V, H_{k0})$ distribution rather than a smooth continuous distribution (Figure 8c, d).
585 Increasing the number of temperature steps will improve the resolution, but will also be time and
586 helium expensive, especially for multiple specimens.

587

588 Second, compared to magnetite, an additional challenge when modeling hematite is the complexity
589 of its anisotropy. An important assumption in our calculation is that we assign $H_{cr} = 0.524H_K$,
590 which is based on Stoner and Wohlfarth (1948) theory for identical randomly oriented uniaxial
591 particles. Harrison et al. (2019) simulated remanence FORC diagrams for particles with uniaxial,
592 cubic, and hexagonal anisotropy. Among these anisotropy types, randomly oriented, non-
593 interacting particles with uniaxial anisotropy have the lowest H_{cr} values, while H_{cr} for cubic and
594 hexagonal anisotropy is approximately 1.05 and 1.8 times larger, respectively. Therefore,
595 multiaxial anisotropy will produce $0.5 < H_{cr}/H_K < 1$. Magnetic minerals with uniaxial, cubic, and
596 hexagonal anisotropy produce distinctive FORC diagram types (e.g., Egli, 2021; Roberts et al.,
597 2021), which provides a useful way to evaluate the dominant anisotropy type before undertaking
598 TFT analyses. Conventional and remanence FORC diagrams for the Inuyama red chert all have a
599 central ridge up to 1.2 T. Although magnetite dominates the FORC signatures, hematite is
600 responsible for central ridges with coercivities > 300 mT. Ridge-type signatures for conventional
601 and remanence FORC diagrams are explained as a manifestation of uniaxial SD magnetic behavior
602 based on simulations (Harrison et al., 2019). However, instead of being shape dominated, as is the
603 case for magnetite, ridge-type FORC signals for hematite nanoparticles reflect stress-induced
604 uniaxial anisotropy that dominates the intrinsic magnetocrystalline anisotropy (Roberts et al.,
605 2021). This magnetostrictive anisotropy associated with uniaxial internal stress controls the
606 coercivity of hematite, which is consistent with the rapid H_{cr} increase at low temperatures (Figure
607 5) that is usually enhanced in nanoparticles (Muench et al., 1985; Bruzzone and Ingalls, 1983).
608 Therefore, considering the ridge-type signals observed in both conventional and remanence FORC
609 diagrams for the red chert (see Figure 2 and 3 in Hu et al. (2021)) and H_{cr} variation with
610 temperature, we predict that uniaxial anisotropy dominates pigmentary hematite in red chert and
611 adopt $H_{cr}/H_K = 0.524$. Increased H_{cr}/H_K will result in decreased microcoercivity, increased T_B
612 estimates, and slightly decreased grain size estimates. Thus, when considering variations in the
613 dominant anisotropy type, our calculations at least provide an upper limit of microcoercivity and
614 grain size and a lower T_B limit.

615

616 **6. Conclusions**

617 Reconstructions of particle microcoercivity and volume distributions were performed with the
618 method of Jackson et al. (2006) for SP/SSD hematite assemblages assuming a dominant uniaxial

619 anisotropy. The median temperature variation of H_{cr} for pigmentary hematite in Triassic/Jurassic
620 red chert over eight temperatures from 300 K to 10 K follows a modified Kneller's law;
621 $H_{cr}(T) = H_{cr0} \left(1 - \left(\frac{T}{T_B} \right)^{0.24} \right)$. The coercivity of hematite increases more rapidly than for
622 magnetite with decreasing temperature, and coercivity distribution overlap between the two
623 minerals starts to disappear below 100 K. Microcoercivity distributions that are nearly symmetric
624 in logarithmic space vary from 1 T to ~10 T in Triassic red chert and from ~3 T to 30 T in
625 Jurassic red chert. Both Triassic and Jurassic red chert have wide hematite grain size
626 distributions. Most of the Triassic hematite ranges between ~35 nm to ~160 nm in diameter
627 while the Jurassic red chert has a coarser hematite particles fraction similar to the Triassic
628 hematite and a finer hematite fraction with grain size from a few nanometers to ~35 nm in
629 diameter. Calculated median T_B varies from ~194 °C to 529 °C for red chert samples and T_B
630 contours indicate that most of the Triassic hematite is in the SSD state with a significant SP
631 particle content with $T_B < 300$ K. The SP/SSD threshold size for pigmentary hematite in Jurassic
632 red chert is estimated to be 8-18 nm. Considering the low and broad T_B distribution in the
633 Inuyama red chert, we propose that pigmentary hematite has a significant contribution to a
634 secondary early Cenozoic thermoviscous magnetization rather than magnetite (Component B
635 defined by Oda and Suzuki (2000)).

636
637 Our work demonstrates that the main features of SP/SSD hematite $f(V, H_{k0})$ distributions can be
638 recovered using the TFT technique, although details should be interpreted judiciously. Smearing
639 of results due to low measurement resolution, artifacts, and variations in dominant anisotropy of
640 hematite should be considered on a case-by-case basis. Ridge-like FORC signatures for red chert
641 are interpreted here to indicate a dominant uniaxial anisotropy. If multiaxial anisotropy is instead
642 dominant in hematite samples (e.g., Roberts et al., 2021), this would increase T_B estimates and
643 decrease microcoercivity and grain size estimates.

644

645 **Data Availability Statement**

646 All low-temperature magnetic data used here will be uploaded to the Magnetic Information
647 Consortium rock magnetic portal (MagIC; www.earthref.org).

648

649

650 **Acknowledgments**

651 We thank Mike Jackson for providing the original TFFT code. This work was supported by the
 652 National Institute of Advanced Industrial Science and Technology, Ministry of Economy, Trade
 653 and Industry, Japan, and the Australian Research Council through grants DP160100805 and
 654 DP200100765.

655

656

657 **Appendix A**

658 Bayesian regression was performed using the PyMC3 Python package (Salvatier et al., 2016).

659 The parameterizations of the Bayesian prior distributions for H_{cr0} , T_B , α and fitting error σ are
 660 provided below.

661 **1. H_{cr0}**

662 The prior for $\mu_0 H_{cr0}$ follows a log normal distribution with median value of 0 and variance of
 663 1. Thus, the probability density function can be expressed as:

664

$$665 \quad p(\mu_0 H_{cr0}) = \frac{1}{\mu_0 H_{cr0} \sqrt{2\pi}} \exp\left(-\frac{\ln(\mu_0 H_{cr0})^2}{2}\right) \quad (A1)$$

666

667

668

669 **2. T_B**

670 Theoretically, T_B can vary between 0 and 960 K for hematite. For better calculation efficiency,
 671 we set all variable values between 0 and 1 in the Bayesian regression. Therefore, instead of T_B , we
 672 use $T_B/1000$ to follow a Beta distribution with $\alpha = 2$ and $\beta = 2$, the probability density function can
 673 be expressed as:

$$674 \quad p\left(\frac{T_B}{1000}\right) = \frac{\frac{T_B}{1000} \times \left(1 - \frac{T_B}{1000}\right)}{B(2,2)}; \text{ and} \quad (A2)$$

675

676

$$677 \quad B(2,2) = \frac{\Gamma(2)\Gamma(2)}{\Gamma(4)} \quad (A3)$$

678

679 Where Γ is the Gamma function.

680

681 **3. α and n**

682 α and n follow a Beta distribution with $\alpha = 8$ and $\beta = 8$, the probability density function can be
 683 expressed as:

$$684 \quad p(\alpha) = \frac{\alpha^7 \times (1 - \alpha)^7}{B(8,8)} \text{ or } p\left(\frac{n}{50}\right) = \frac{\left(\frac{n}{50}\right)^7 \times \left(1 - \left(\frac{n}{50}\right)\right)^7}{B(8,8)}, \text{ with} \quad (A4, A5)$$

$$685 \quad B(8,8) = \frac{\Gamma(8)\Gamma(8)}{\Gamma(16)} \quad (A6)$$

688
 689 Based on previous studies, n rarely exceeds the value of 50. Therefore, we normalized n by 50
 690 for it to vary from 0 to 1. Γ in equation (A6) is the Gamma function.

691

692 **4. Fitting error σ**

693 σ follows a half-Cauchy distribution with location parameter of 0 and scale parameter of 1.
 694 The probability density function can be expressed as:

$$695 \quad f(\sigma, 1) = \frac{2}{\pi(1 + \sigma)} \quad (A7)$$

697

698 The, no U-Turn Sampler (NUTS) from the Python Pymc3 package was used for sampling, in
 699 where we set both the sampling number and iteration number to 4000.

700

701 Statistical results of Bayesian regression using equation (3) and equation (4), respectively are
 702 listed. Two hundreds regression curves for each sample are shown in Figure 5. In the following
 703 tables, hdi_3% and hdi_97% are the lower and upper bounds of the 97% high density interval.
 704 $R_{\hat{}}$ is the Gelman-Rubin convergence statistic, where a value of 1 indicates that the chains have
 705 converged.

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714 Table A1 Posterior statistics of Bayesian regression of equation (3)

715

	Median	Standard deviation	hdi_3% ¹	hdi_97% ²	R_hat ³
n	28.650	3.700	21.650	35.600	1
$\mu_0 H_{cr0}$ (T) (KA1-1B-1)	1.402	0.137	1.133	1.649	1
$\mu_0 H_{cr0}$ (T) (UN2-9B-1)	1.424	0.139	1.166	1.682	1
$\mu_0 H_{cr0}$ (T) (KA6-9B-1)	1.581	0.145	1.308	1.853	1
$\mu_0 H_{cr0}$ (T) (KA6-2L-B-1)	1.645	0.145	1.371	1.916	1
σ	0.253	0.037	0.189	0.321	1

716

717 Table A2 Posterior statistics of Bayesian regression of equation (4)

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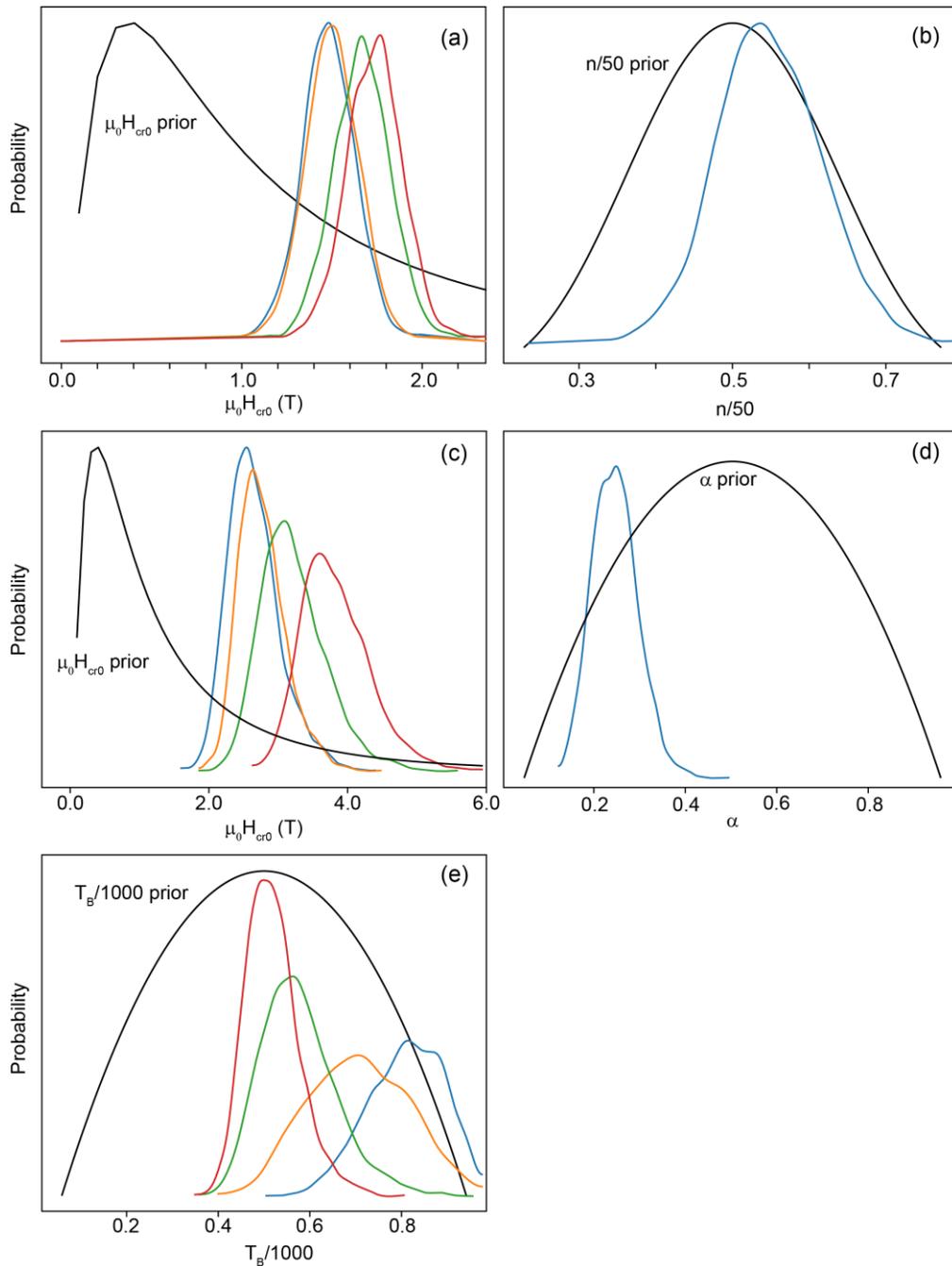
731

732

	mean	Standard deviation	hdi_3% ¹	hdi_97% ²	R_hat ³
α	0.240	0.052	0.151	0.339	1
$\mu_0 H_{cr0}$ (T) (KA1-1B-1)	2.500	0.383	1.829	3.211	1
$\mu_0 H_{cr0}$ (T) (UN2-9B-1)	2.667	0.405	1.947	3.446	1
$\mu_0 H_{cr0}$ (T) (KA6-9B-1)	3.246	0.494	2.335	4.164	1
$\mu_0 H_{cr0}$ (T) (KA6-2L-B-1)	3.648	0.561	2.627	4.664	1
$T_B/100$ (K) (KA1-1B-1)	0.803	0.099	0.621	0.972	1
$T_B/1000$ (K) (UN2-9B-1)	0.717	0.103	0.534	0.919	1
$T_B/1000$ (K) (KA6-9B-1)	0.558	0.076	0.425	0.701	1
$T_B/1000$ (K) (KA6-2L-B-1)	0.467	0.054	0.370	0.567	1
σ	0.111	0.019	0.078	0.148	1

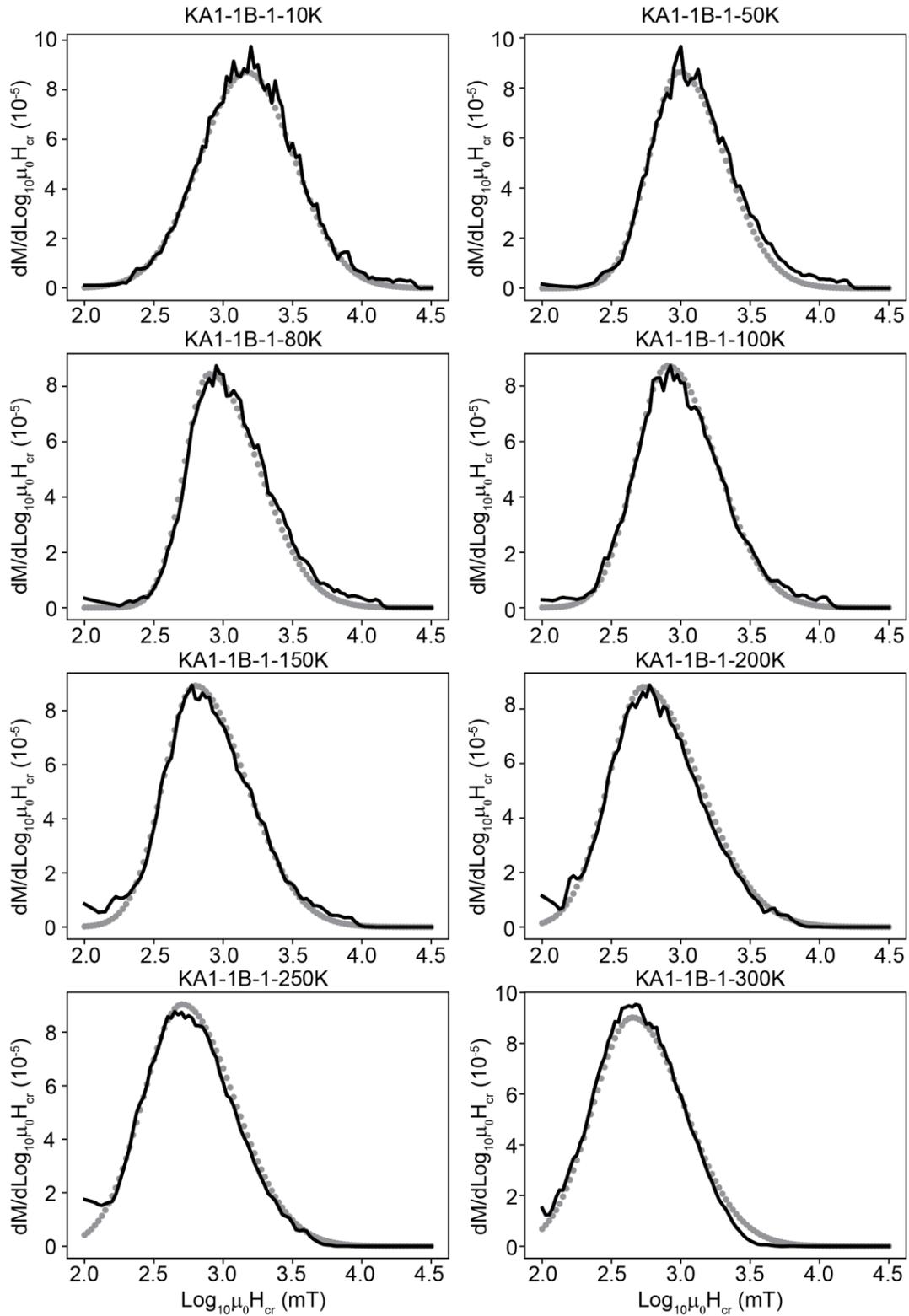
¹ hdi_3% represents the lower bounds of the 97% high density interval of the corresponding posterior distribution.² hdi_97% represents the upper bounds of the 97% high density interval of the corresponding posterior distribution³ R_hat is the Gelman-Rubin convergence statistic, which estimates the degree of convergence of a random Markov Chain. Values close to one indicate convergence to the underlying distribution.

733



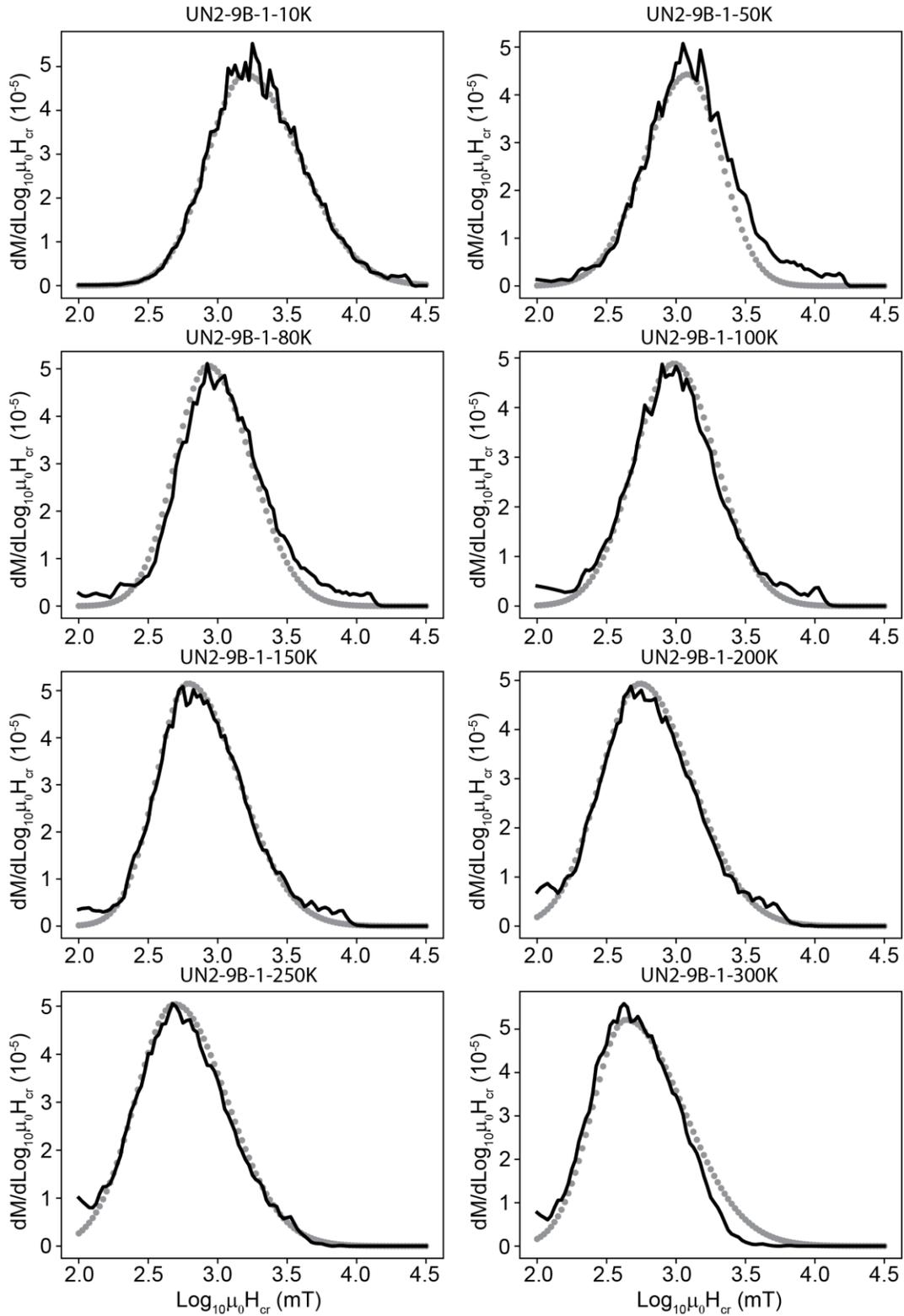
734

735 **Figure A1** The prior distribution (black lines) and posterior distribution (colored lines) of
 736 parameters in the Bayesian model for (a-b) equation (3) and (c-e) equation (4). Blue, orange, green,
 737 and red lines represent corresponding parameters for sample KA1-1B-1, UN2-9B-1, KA6-9B-1
 738 and KA6-2L-B-1, respectively.



739

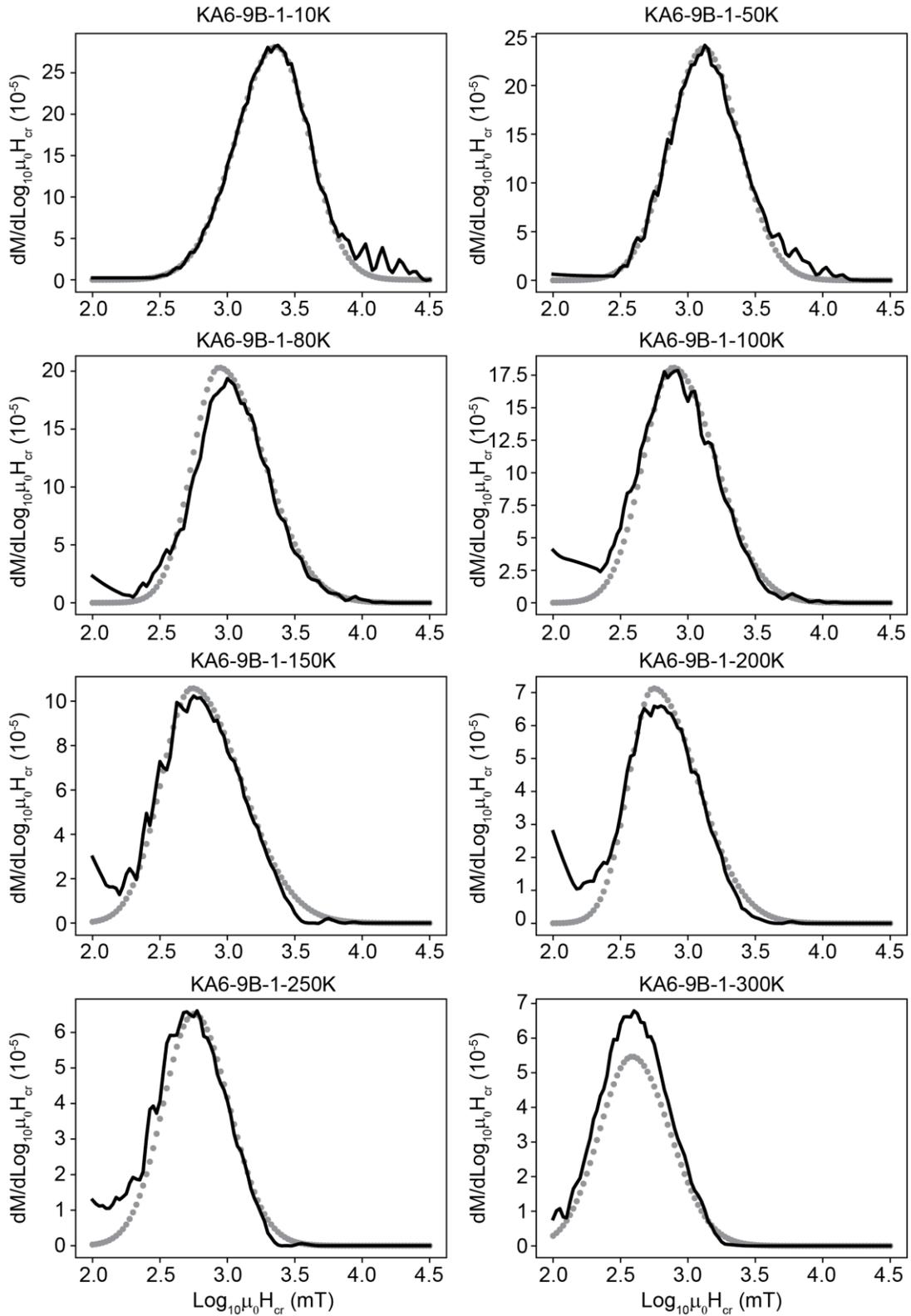
740 **Figure A2** Reconstructed (black solid lines) and measured backfield remanence curve derivatives
 741 (gray dots) for Triassic sample KA1-1B-1.



742

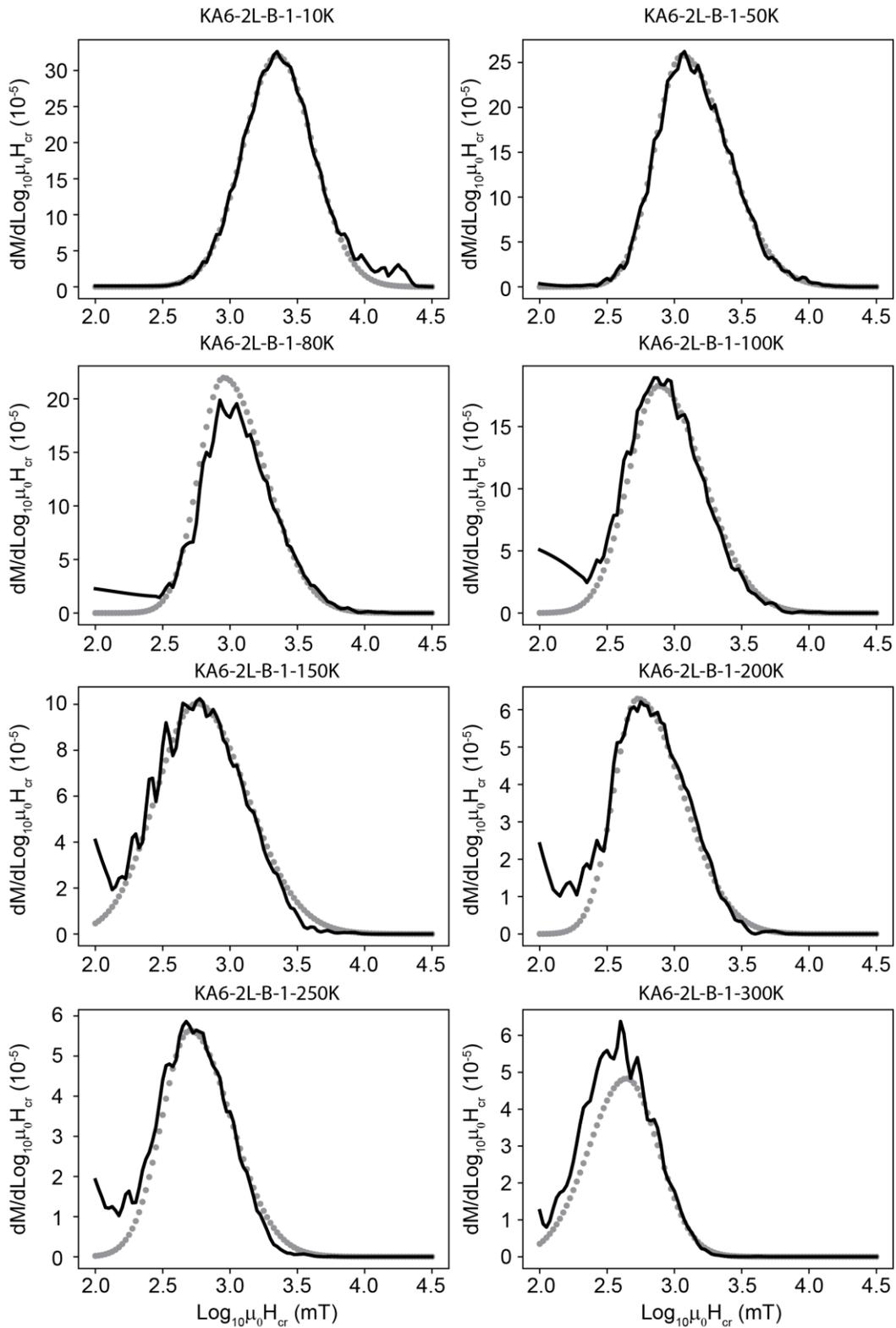
743 **Figure A3** Reconstructed (black solid lines) and measured backfield remanence curve derivatives

744 (gray dots) for Triassic sample UN2-9B-1.



745

746 **Figure A4** Reconstructed (black solid lines) and measured backfield remanence curve derivatives
 747 (gray dots) for Jurassic sample KA6-9B-1.



748

749 **Figure A5** Reconstructed (black solid lines) and measured backfield remanence curve derivatives

750 (gray dots) for Jurassic sample KA6-2L-B-1.

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928

1 **Estimating particle size and coercivity distributions of pigmentary hematite in red**
2 **chert with thermal fluctuation tomography**

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12
13 **Key Points:**

- 14 • Thermal fluctuation tomography is applied to red cherts to estimate grain size and
15 microcoercivity distributions of pigmentary hematite
- 16 • Fine hematite particles (35 - 160 nm) occur in all samples including superparamagnetic
17 hematite down to a few nanometers
- 18 • Temperature-dependent coercivity variations in pigmentary hematite follow Kneller's law
19

20 **Plain language Summary**

21 Pigmentary hematite widely presents in rocks and sediment and is crucial for paleomagnetic and
22 paleoenvironmental studies because they can record ancient earth magnetic field and past climate
23 signals. As the most important properties in paleomagnetic and paleoenvironmental applications,
24 the coercivity and grain size distribution of natural pigmentary hematite is poorly constrained
25 due to the weak magnetism of hematite and the small size. In this study, we provide a strategy
26 using low-temperature demagnetization curves for estimating joint particle volume and
27 microcoercivity distribution of pigmentary hematite in Inuyama red chert samples. The hematite
28 coercivity increases exponentially with decreasing temperature. Hematite microcoercivity
29 without thermal fluctuation has a wide but approximately symmetric distribution in logarithmic

30 space from ~1 tesla to tens of tesla. The grain size of hematite varies from several nanometers to
31 about 160 nm. The fine particle size of these hematite results in low unblocking temperature,
32 which makes them suitable to record remagnetization in geological time.

33

34 **Abstract**

35 Pigmentary hematite carries important signals in paleomagnetic and paleoenvironmental studies.
36 However, weak magnetism and the assumption that it has high magnetic coercivity prevents
37 routine identification of the size distribution of pigmentary hematite, especially for fine particle
38 sizes. We present a strategy for estimating joint hematite particle volume and microcoercivity (f
39 (V , H_{k0})) distributions from low-temperature demagnetization curves and thermal fluctuation
40 tomography (TFT) of pigmentary hematite in bulk samples of Triassic-Jurassic Inuyama red chert,
41 Japan. The coercivity of the pigmentary hematite increases exponentially with decreasing
42 temperature, following a modified Kneller's law, where microcoercivity has a wide but
43 approximately symmetric distribution in logarithmic space from ~1 tesla to tens of tesla. All of the
44 red chert samples contain stable single domain (SSD) hematite with 35 - 160 nm diameter; a
45 significant superparamagnetic (SP) hematite population with sizes down to several nanometers
46 also occurs in Jurassic samples. The SP/SSD threshold size is estimated to be 8 - 18 nm in these
47 samples. The fine particle size of the pigmentary hematite is evident in its low median unblocking
48 temperature (194 °C to 529 °C) and, thus, this hematite may contribute to all four paleomagnetic
49 components identified in published thermal magnetization studies of the Inuyama red chert. In this
50 work, uniaxial anisotropy and magnetization switching via coherent rotation are assumed. Uniaxial
51 anisotropy is often dominant in fine-grained hematite, although the dominant anisotropy type
52 should be evaluated before using TFT. This approach is applicable to studies that require
53 knowledge of coercivity and size distributions of hematite pigments.

54

55 **1. Introduction**

56 Hematite is abundant in sedimentary rocks, especially red beds. It occurs commonly as a fine-
57 grained chemically precipitated pigment and as coarser detrital or specular hematite (Cornell and
58 Schwertmann, 2003; Lepre and Olsen, 2021; Jiang et al., 2022; Swanson-Hysell et al., 2019; Tauxe

59 et al., 1980). Poorly crystalline pigmentary hematite can be the dominant iron oxide in many red
60 soils and sediments and is responsible for their characteristic red color. Both specular and
61 pigmentary hematite can carry magnetic remanence. Specular hematite carries a detrital remanent
62 magnetization (DRM), which is often assumed to be a primary or near-primary magnetization.
63 Widely observed red bed remagnetizations tend to be associated with late diagenetic pigmentary
64 hematite formation. Debate about whether red beds record a primary DRM or a secondary
65 chemical remanent magnetization (CRM) led to the “red bed controversy” (Beck et al., 2003;
66 Butler, 1992; Van Der Voo & Torsvik, 2012). Identification of CRM acquisition in pigmentary
67 hematite can enable more accurate paleomagnetic interpretations in regional tectonic studies
68 (Abrajevitch et al., 2018; Jiang et al., 2017; Swanson-Hysell et al., 2019). As the most abundant
69 surficial iron oxide on Earth resulting from near-surface processes, hematite is also an excellent
70 recorder of paleoenvironmental signals. Its formation via authigenic chemical processes means
71 that pigmentary hematite is used as an indicator of hydration conditions, acidity of aqueous
72 environments, and monsoon evolution (e.g., Larrasoña et al., 2003; Abrajevitch et al., 2013; Lepre
73 and Olsen, 2021). Despite the usefulness of pigmentary hematite as a paleoclimatic indicator or as
74 a carrier of paleomagnetic records, it is often described vaguely as a “fine hematite population”,
75 with poorly constrained coercivity and grain size distributions.

76
77 Characterizing the grain size and coercivity of pigmentary hematite is challenging because it is
78 necessary to overcome the combined difficulty of detecting weakly magnetic hematite when it co-
79 occurs with other magnetic minerals and characterizing poorly crystalline nanoparticles. Magnetite
80 has a spontaneous magnetization that is more than 200 times stronger than hematite, so even small
81 amounts of magnetite can overwhelm the magnetic contribution of hematite (Dekkers, 1990; Frank
82 and Nowaczyk, 2008; Roberts et al., 2020). In practice, hematite detection in natural samples often
83 relies on its high coercivity and distinctive color (Roberts et al., 2020; Jiang et al., 2022 and
84 references therein). However, the small size and often poorly crystalline nature of hematite
85 nanoparticles means that hematite concentrations can be difficult to determine with many
86 spectroscopic approaches. Such particles will also be responsible for a substantial low-coercivity
87 distribution that is not usually attributed to hematite in mineral magnetic studies, especially when
88 using magnetic parameters with cut-off fields of 300 mT (Roberts et al., 2020). Isothermal
89 remanent magnetization (IRM) component analysis appears to be the most suitable magnetic

90 method for detecting hematite because it enables estimation of continuous, non-truncated
91 coercivity distributions (Hu et al., 2021; Roberts et al., 2020). However, superparamagnetic (SP)
92 pigmentary hematite, which is abundant in natural environments (Collinson, 1969; Schwertmann,
93 1991), will not be evident in room temperature IRM results. Color and diffuse reflectance methods
94 also have limitations for detecting or quantifying hematite because they depend strongly on grain
95 size and crystallinity. Decreasing grain size tends to reduce the reflectance wavelength and
96 changes the color from purple-red to yellow-red (Cornell and Schwertmann, 2003; Jiang et al.,
97 2022). Evaluating grain size distributions for pigmentary hematite is, therefore, difficult because
98 grain size influences most proxies used to estimate hematite properties.

99

100 Microscopy observations reveal the existence of nano-sized hematite with sizes from ~20 nm to a
101 few hundred nanometers in soils and banded iron formations (Eggseder et al., 2018; Hyodo et al.,
102 2020; Sun et al., 2015). A more systematic relationship between hematite grain size and
103 unblocking temperature has been established by Swanson-Hysell et al. (2011, 2019) using Néel
104 (1949) relaxation theory. The grain size range of remanence-carrying hematite can be inferred
105 using unblocking temperatures from thermal demagnetization experiments, although this approach
106 cannot be used to estimate SP particles because they do not carry a stable remanence at room
107 temperature.

108

109 Grain volume is a key variable in Néel (1949) theory. Dunlop (1965) pointed out that it is possible
110 to combine field- and temperature-dependent measurements to determine the joint grain volume
111 (V) and microcoercivity at absolute zero (H_{k0}) distribution; $f(V, H_{k0})$. Jackson et al. (2006)
112 developed a procedure to estimate $f(V, H_{k0})$ for particle assemblages that contain both SP and
113 stable single domain (SSD) magnetite based on backfield remanence curves measured over a range
114 of temperatures, which they called “thermal fluctuation tomography” (TFT). This method was
115 used to reconstruct the grain size distribution of magnetite in both synthetic and natural tuff and
116 paleosol samples. Theoretically, TFT can also be used for weakly magnetic minerals like hematite
117 that have a wider size range for SSD behaviour (Banerjee, 1971; Kletetschka and Wasilewski,
118 2002; Özdemir and Dunlop, 2014). Here, we present a TFT procedure to estimate the grain size
119 and microcoercivity distribution for pigmentary hematite in natural red chert samples based on the
120 approach of Jackson et al. (2006). Multiple low-temperature magnetic measurements are integrated

121 to constrain the magnetic mineralogy of natural hematite-magnetite-bearing samples. Our results
122 also provide new insights into the nature of pigmentary hematite in red sedimentary rocks.

123

124 **2. Materials and Methods**

125 ***2.1. Inuyama red chert***

126 Red chert is a distinctive hematite-rich biosiliceous sedimentary rock, which was a common
127 pelagic marine sediment type from the Ordovician to the early Late Cretaceous (Jones and
128 Murchey, 1986). The Inuyama red chert crops out along the Kiso River about 30 km north of
129 Nagoya, Japan. Red chert, gray chert, and siliceous claystone were deposited alternately over
130 thicknesses of several hundred meters in the middle Triassic to early Jurassic (Oda and Suzuki,
131 2000). Hematite occurs as a finely dispersed pigment of chemical origin in red cherts (Jones and
132 Murchey, 1986; Matsuo et al., 2003). The Inuyama red chert contains variable mixtures of
133 magnetite and pigmentary hematite (Oda and Suzuki, 2000; Abrajevitch et al., 2011; Hu et al.,
134 2021). Four representative samples were selected from three red bedded chert sites (KA1, KA6,
135 UN2) in the Inuyama area. Biostratigraphic, paleomagnetic, and rock magnetic results for the same
136 sample set have been published by Oda and Suzuki (2000) and Hu et al. (2021). Radiolarian fossils
137 indicate that the KA1 and UN2 samples have middle (Anisian) and late Triassic (Norian) ages
138 while two KA6 samples are of early Jurassic age (Oda and Suzuki, 2000).

139

140 ***2.2. Low temperature magnetic measurements***

141 Samples were cut into 4 mm × 4 mm × 3 mm pieces and were measured with a Quantum Design
142 Magnetic Properties Measurement System (MPMS) at the Black Mountain Paleomagnetism
143 Laboratory, Australian National University. First, an isothermal remanent magnetization (IRM)
144 was imparted at 10 K in a 5 T field (LTSIRM) after cooling in zero field (ZFC) and then
145 demagnetized by ramping the superconducting magnet down in oscillation mode from 100 to 0
146 mT to simulate an alternating field (AF) demagnetization at 10 K (Lagroix and Guyodo, 2017).
147 The resulting magnetization was then measured from 10 K to 300 K to obtain LTIRM_{@AF100}
148 warming curves. Following the same protocol, LTIRM_{@AF300} warming curves were also measured
149 for the same sample by ramping the magnet down from 300 to 0 mT after imparting a LTSIRM.

150 In this way, magnetizations carried over different coercivity ranges (>300 mT, 100-300 mT, and
 151 <100 mT) were separated to identify their low temperature characteristics. In this study,
 152 $LTIRM_{>300 \text{ mT}}$ is represented by $LTIRM_{@AF300}$, $LTIRM_{100-300 \text{ mT}}$ is given by $LTIRM_{@AF100} -$
 153 $LTIRM_{@AF300}$, and $LTIRM_{<100 \text{ mT}}$ is calculated as $LTSIRM - LTIRM_{@AF100}$. Second, a 5 T field
 154 was imparted again at 10 K after ZFC and IRM was measured in zero field during warming to 300
 155 K to obtain ZFC-LTSIRM curves. Samples were then field-cooled (FC) to 10 K in a 5 T field and
 156 measured during warming back to 300 K after removing the field (FC-LTSIRM). Third, backfield
 157 demagnetization curves were measured for the same four samples at 50 logarithmically spaced
 158 steps from 0.001 T to 5 T after being saturated with an initial 5 T field at eight temperatures: 300
 159 K, 250 K, 200 K, 150 K, 100 K, 80 K, 50 K, and 10 K. These curves were later decomposed into
 160 skew-normal coercivity components using the fitting software MAX UnMix (Maxbauer et al.,
 161 2016). Finally, to test for goethite contributions that will be fully demagnetized at 400 K, samples
 162 were given a room temperature IRM (RTSIRM) in a 5 T field and were demagnetized by ramping
 163 the magnet from 300 to 0 mT in oscillation mode. The remaining remanence $RTSIRM_{@AF300}$ was
 164 measured in zero field during cooling to 150 K and then during warming to 400 K and then during
 165 cooling back to 150 K.

166

167 3. Thermal fluctuation tomography theory for hematite

168 We adapted the tomographic imaging method of Jackson et al. (2006) to estimate $f(V, H_{k0})$
 169 distributions for SP and SDD hematite grains. The procedure is described briefly below, with focus
 170 on modifications made for hematite. For a detailed explanation and derivation of TFT theory, see
 171 Jackson et al. (2006) and Dunlop (1965).

172

173 The TFT approach involves using backfield remanence data to estimate the blocking field (H_B).
 174 For hematite at a given temperature, we evaluate $H_B = H_{cr} - H_q$, where H_{cr} is the coercivity of
 175 remanence, and H_q is the thermal fluctuation field. For a randomly oriented population of
 176 identical grains, H_q is expressed as (equation 8 of Jackson et al. (2006)):

$$177 H_q = 0.801 \left(\frac{kT\sqrt{H_k(T)}}{\mu_0 VM_S(T)} \right)^{\frac{2}{3}} \ln^{\frac{2}{3}} \left[\frac{\tau_{exp}}{\tau_0 \mu_0 \Delta H_{DC} \sqrt{\mu_0 H_k(T)}} \times \left(\frac{kT}{VM_S(T)} \right)^{\frac{2}{3}} \right], \quad (1)$$

178 where M_s (T) and H_k (T) are the saturation magnetization and microcoercivity as a function of
 179 absolute temperature, T, respectively, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m), k is
 180 Boltzmann's constant (1.38×10^{-23} J/K), V is the hematite particle volume, and τ_0 is a characteristic
 181 time related to the natural frequency of gyromagnetic precession. For nanosized hematite, τ_0 is
 182 found to be 10^{-12} - 10^{-11} s (Henrik, 2014 and references therein). We here assume τ_0 10^{-12} s. The
 183 exposure time τ_{exp} for the backfield treatments is assigned as 300 s, and ΔH_{DC} is the applied field
 184 difference between successive backfields. We assume here that the saturation magnetization at
 185 absolute zero (M_{S0}) is 2500 A/m for hematite (Dunlop and Özdemir, 1997), then $M_S(T)$ can be
 186 represented using Bloch's 3/2 law (Bloch, 1930):

$$187 \quad M_S(T) = M_{S0} \times \left(1 - B \times T^{\frac{3}{2}}\right), \quad (2)$$

188 where B is the Bloch constant. B is determined by the spin-wave stiffness constant; we adopt $B =$
 189 10^{-5} for hematite nanoparticles (Martínez et al., 1996).

190

191 The next step is to describe microcoercivity (H_k) as the function of temperature. Two analytic
 192 models have been used previously to describe the temperature dependence of the coercive force.

193 1. By taking $\frac{H_k(T)}{H_{k0}} = \left(\frac{M_S(T)}{M_{S0}}\right)^n$ (Dunlop and Özdemir, 2000; Jackson et al., 2006; Menyeh
 194 and O'Reilly, 1995), where n depends on the dominant anisotropy, we can calculate
 195 $H_k(T)$ based on Bloch's 3/2 law:

$$196 \quad H_k(T) = H_{k0} \left(1 - B \times T^{\frac{3}{2}}\right)^n, \quad (3)$$

197 However, hematite anisotropy can be complex and published n values for fine-grained
 198 hematite below room temperature are rare. Study of synthetic nano-sized hematite
 199 reveals that temperature has a minimal impact on $M_S(T)$ while coercivity increases
 200 significantly at low temperature due to frozen canted spins (Satheesh et al., 2017).
 201 Therefore, n should be large because of the significant $H_k(T)$ change compared to
 202 minimal $M_S(T)$ change. Satheesh et al. (2017) reported M_s and H_c for a 64 nm hematite
 203 sample at both 5 K and 300 K, to give a calculated n value of ~10.

204 2. The temperature dependence of coercivity, H_c (T), can also be expressed by Kneller's
 205 law (Kneller and Luborsky, 1963):

$$H_c(T) = H_{c0} \left(1 - \left(\frac{T}{T_B} \right)^\alpha \right), \quad (4)$$

206 where T_B is the blocking temperature for SP particles, α is Kneller's exponent and H_{c0} is
 207 the coercivity at absolute zero. For non-interacting single domain nanoparticles with
 208 uniaxial anisotropy, α usually takes a value of 0.5 (Kuncser et al., 2020; Maaz et al., 2010;
 209 Osman and Moyo, 2015). However, α can deviate from 0.5 due to finite size effects at the
 210 nanoscale as well as due to variations in volume distribution, randomness of anisotropy
 211 axes, and interparticle interactions (Nayek et al., 2017). Similar to the n value in equation
 212 (3), α for hematite nanoparticles is poorly constrained.
 213

214
 215 To establish a thermally dependent coercivity model for pigmentary hematite in Inuyama red chert,
 216 we compare the hematite median H_{cr} values obtained from backfield curve decomposition using
 217 both models. First, we need to clarify the relationship among different coercivity forms, H_c , H_{cr}
 218 and H_k . Experimental H_{cr}/H_c ratios for SSD hematite are almost constant at ~ 1.5 (Martin-
 219 Hernandez and Guerrero-Suarez, 2012; Peters and Dekkers, 2003; Özdemir and Dunlop, 2014;
 220 Roberts et al., 2021). The relationship between H_{cr} and H_k depends largely on the dominant
 221 anisotropy type. For randomly oriented identical particles with uniaxial anisotropy, Stoner and
 222 Wohlfarth (1948) theory gives $H_{cr}/H_k = 0.524$. Multiaxial anisotropy, such as cubic or hexagonal
 223 anisotropy, can increase H_{cr} (Harrison et al., 2019) and therefore raise this ratio close to 1. The
 224 high magnetostriction of hematite and weak M_s suggests a high sensitivity to magnetostrictive
 225 strain in hematite (Banerjee, 1963); this strain-related anisotropy is taken to be uniaxial. FORC
 226 diagrams for the studied red chert samples have “ridge-type” distributions for hematite up to 1.2
 227 T (Hu et al., 2021), which is typical of uniaxial SSD particle assemblages (Egli et al., 2010).
 228 Therefore, by assuming a dominant uniaxial anisotropy and relatively constant H_{cr}/H_c ratios for
 229 SP/SSD hematite in the Inuyama red chert, we adopt linear relationships among H_c , H_{cr} and H_k .
 230 Then, for model 1, we fit the hematite median H_{cr} data using equation (3). Under the assumption
 231 of a common n value for all samples, we estimate both H_{cr0} and n using Bayesian regression.
 232 Similarly, by assuming a common α value, we fit the hematite H_{cr} data using equation (4) and
 233 obtain median posterior estimates of H_{cr0} , T_B , and α via Bayesian regression for model 2 (see
 234 section 4.3 for details). By selecting an appropriate model based on our experimental data (section

235 4.3) and assigning $H_{cr0} = 0.524H_{k0}$, we can estimate $H_k(T)$. Then $H_B(T)$ is obtained by substituting
 236 $H_B = 0.524H_k - H_q$ into equation (1).

237
 238 After constructing field blocking contours for hematite, we describe each hematite grain using two
 239 essential attributes, V and H_{k0} . A saturating field applied and removed isothermally at temperature
 240 T_1 magnetizes the entire thermally stable population at that temperature (Figure 1, blue shaded
 241 region), which corresponds to grains with (V, H_{k0}) that plot above and to the right of the zero-field
 242 blocking contour for T_1 (Figure 1). Subsequent application and removal of a reverse DC field, H_1 ,
 243 flips the magnetic moments of grains with (V, H_{k0}) that plot below and to the left of the blocking
 244 contour for (T_1, H_1) (Figure 1, hatched area). Each backfield reverses the moments of grains that
 245 plot in a region on the Néel diagram (Dunlop, 1965; Néel, 1949) bounded by two blocking field
 246 contours for a specified temperature. The change in remanence ΔM_R produced by each DC
 247 backfield treatment can, therefore, be expressed as (equation 11 of Jackson et al. (2006)):

$$248 \quad \Delta M_R = \int f(V, H_{k0}) d\Omega, \text{ and}$$

$$249 \quad \Omega = \left\{ V, H_{k0} \mid H_{i-1} \leq H_B \leq H_i \right\} \quad (5)$$

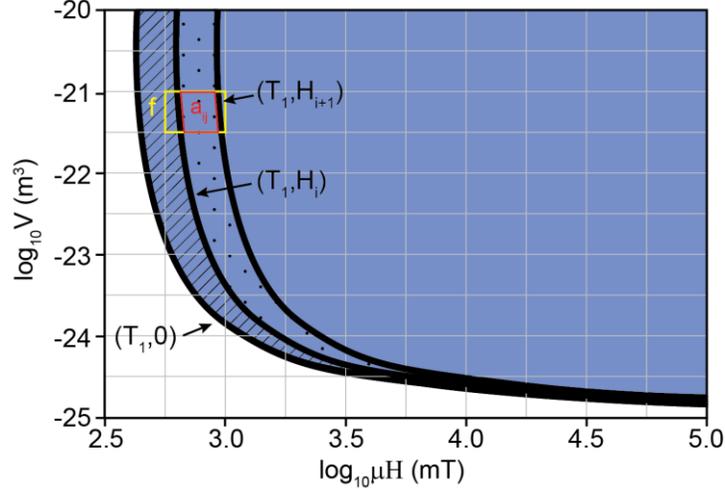
250 where Ω represents the region bounded by two blocking field contours and H_i represents a reverse
 251 DC field treatment. Therefore, the procedure is essentially an inverse problem involving $f(V, H_{k0})$
 252 estimation from a series of DC backfield remanence curves for hematite. Details of procedures
 253 used to obtain hematite backfield remanence curves are explained in section 4.2.

254
 255 To estimate $f(V, H_{k0})$, we divided the Néel diagram into a rectilinear grid of cells in which f is
 256 uniform in each cell (Figure 1, yellow cell). The discrete equivalent of equation (5) is:

$$257 \quad \Delta M_{Ri} = \sum_{j=1}^{n_{\text{cells}}} f_j a_{ij}, \quad (6)$$

258 Where a_{ij} is the area of cell j within the area bounded by the blocking contours for a given
 259 temperature and applied field used when measuring ΔM_{Ri} (Figure 1, red region). Each temperature
 260 and applied field, H_{app} , pair (T, H_{app}) corresponds to a unique blocking contour, defined as the
 261 locus of (V, H_{k0}) for which $H_B(T, V, H_{k0}) = H_{app}$, so we can calculate intersection points of the

262 contours with the grid lines by piecewise linear interpolation between nodes and approximate the
 263 contours by straight-line segments between these intersection points to estimate the areas a_{ij} .



264

265 **Figure 1** Schematic illustration of the TFT technique, modified from Jackson et al. (2006). A
 266 strong field IRM imparted at temperature T_1 is carried by the entire thermally stable population
 267 (blue shaded area); a backfield, H_1 , applied and removed at temperature T_1 , reverses the moments
 268 of grains in the hatched area; a larger backfield, H_2 , further reverses the moments of grains in the
 269 dotted area. a_{ij} (red area) represents the area bounded by the blocking contours for T_1 and applied
 270 fields H_{i+1} and H_i when measuring ΔM_{Ri} . The yellow rectangle represents the j^{th} cell, f_j is the value
 271 of $f(V, H_{k0})$ for the j^{th} cell.

272

273 We employ an initialization of $f = 0$ at all points to generate a forward model based on equation
 274 (6). Residuals are then calculated as the difference between the measured and model remanence
 275 data:

$$276 \quad R_i = \Delta M_{Ri, \text{measured}} - \Delta M_{Ri, \text{model}} \quad (7)$$

277 The model is then adjusted by “back-projecting” the residuals:

278

$$279 \quad {}^s \Delta f_{ij} = \frac{R_i a_{ij}}{\sum_{k=1}^{n_{\text{cells}}} a_{ik}^2}. \quad (8)$$

280 The adjustment for cell j is proportional to R_i and the area a_{ij} bounded by the blocking contours.
 281 n_{cells} represents the number of cells and s represents the current simulation. Stepwise updates are
 282 applied after all calculations for each iteration:

283

$$f_j^{s+1} = f_j^s + \frac{C}{n_{\text{measurements}}} \sum_{i=1}^{n_{\text{measurements}}} \Delta f_{ij} \quad (j = 1 \dots n_{\text{cells}}). \quad (9)$$

C is a dimensionless constant used to control the rate of convergence, where higher values cause more rapid convergence, but excessive values can cause the process to become unstable and diverge. Our aim is to reduce the fitting error to ~10% within 100 iterations; after multiple attempts, we found that a C value of 50 generally meets our requirement.

4. Results

4.1. Unblocking of pigmentary hematite

LTSIRM variations of different coercivity fractions for both Triassic and Jurassic red chert samples are shown in Figure 2. The Verwey transition for magnetite is clearly evident at ~120 K for particles with coercivity < 300 mT, which disappears or becomes less noticeable in $LTIRM_{>300}$ curves for the high coercivity component (Figure 2a, 2b), and demonstrating that the coercivity of magnetite is mostly less than 300 mT. For Jurassic specimens, $LTIRM_{>300}$ warming curves decay steeply compared to the relatively flat $LTIRM_{<100}$ curves (Figure 2b), which indicates a wide unblocking temperature distribution of a SP hematite content. A concave shape around 200 K is present in $LTIRM_{>300}$ curves, but not in the low coercivity component, which indicates a likely Morin transition that was not completely smeared out by progressive unblocking of fine hematite (Figure 2b). The $LTIRM_{100-300}$ warming curve contains both a Verwey transition and marked low temperature unblocking, which suggests a mixture of magnetite and finer hematite in this coercivity range. Hematite unblocking is less significant for Triassic samples, which indicates a smaller SP hematite contribution (Figure 2a). However, $LTIRM_{>300}$ curves for both Triassic and Jurassic samples have comparable magnetizations despite the fact that magnetite has a much stronger magnetization, which indicates that hematite dominates the red chert magnetism by mass.

A Verwey transition is clearly present in both ZFC-LTSIRM and FC-LTSIRM curves, while a Morin transition is likely smeared by progressive unblocking of SP hematite (Figure 2c, 2d). ZFC-LTSIRM and FC-LTSIRM curves are not widely separated, which contrasts with the behavior of

312 goethite-rich samples (Guyodo et al., 2003; Liu et al., 2006; Huang et al., 2019). Given that the
313 curves almost overlap (Figure 2c, 2d) it is inferred that any goethite contribution is insignificant.

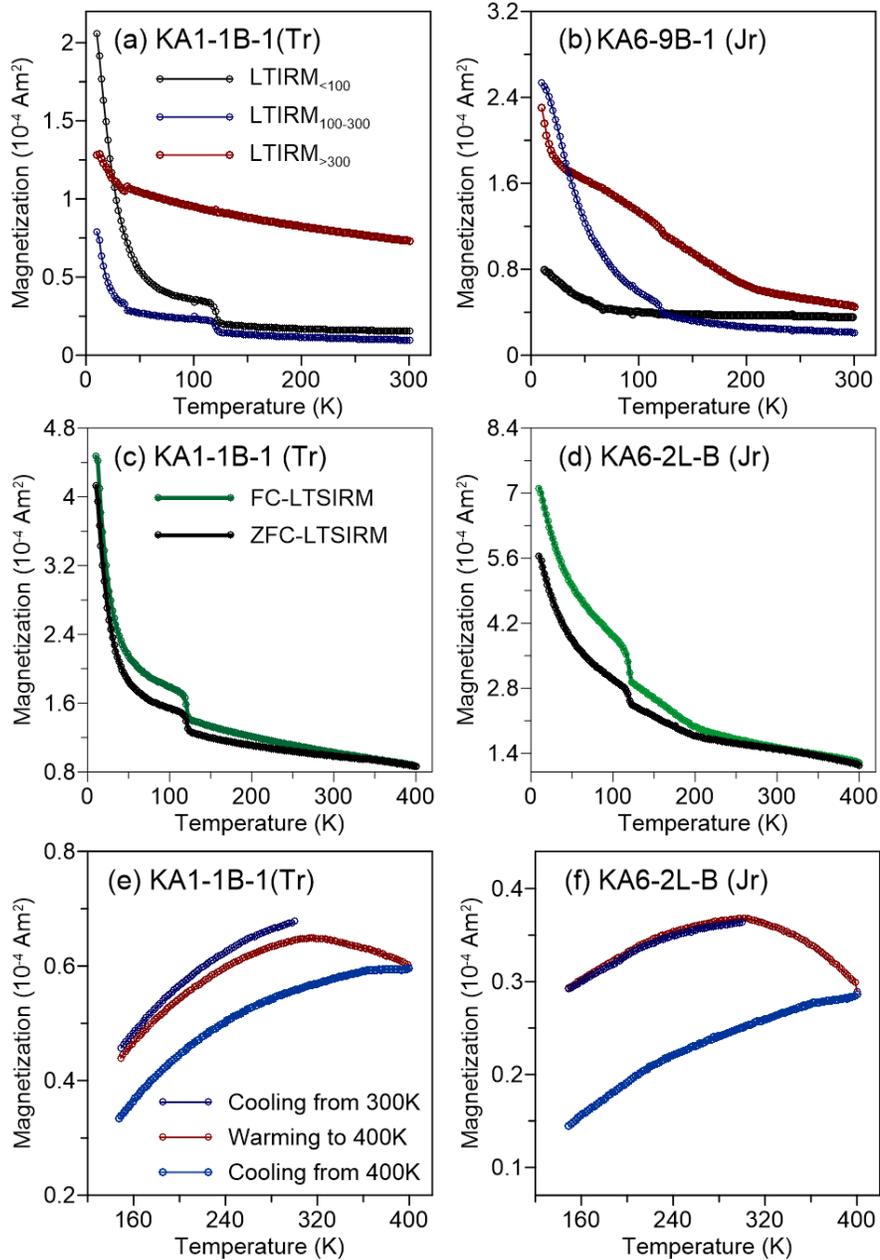
314

315 After removing the low coercivity contribution by applying a 300 mT AF, $RTSIRM_{@AF300}$
316 warming curves decrease gradually from 300 K to 400 K with a net remanence loss during re-
317 cooling (Figure 2e, 2f). No sharp drop is seen at the Néel temperature for goethite. The gradual
318 decrease in warming curves above 300 K is likely due to unblocking of slightly larger hematite
319 particles near the SP/SSD size threshold at room temperature.

320

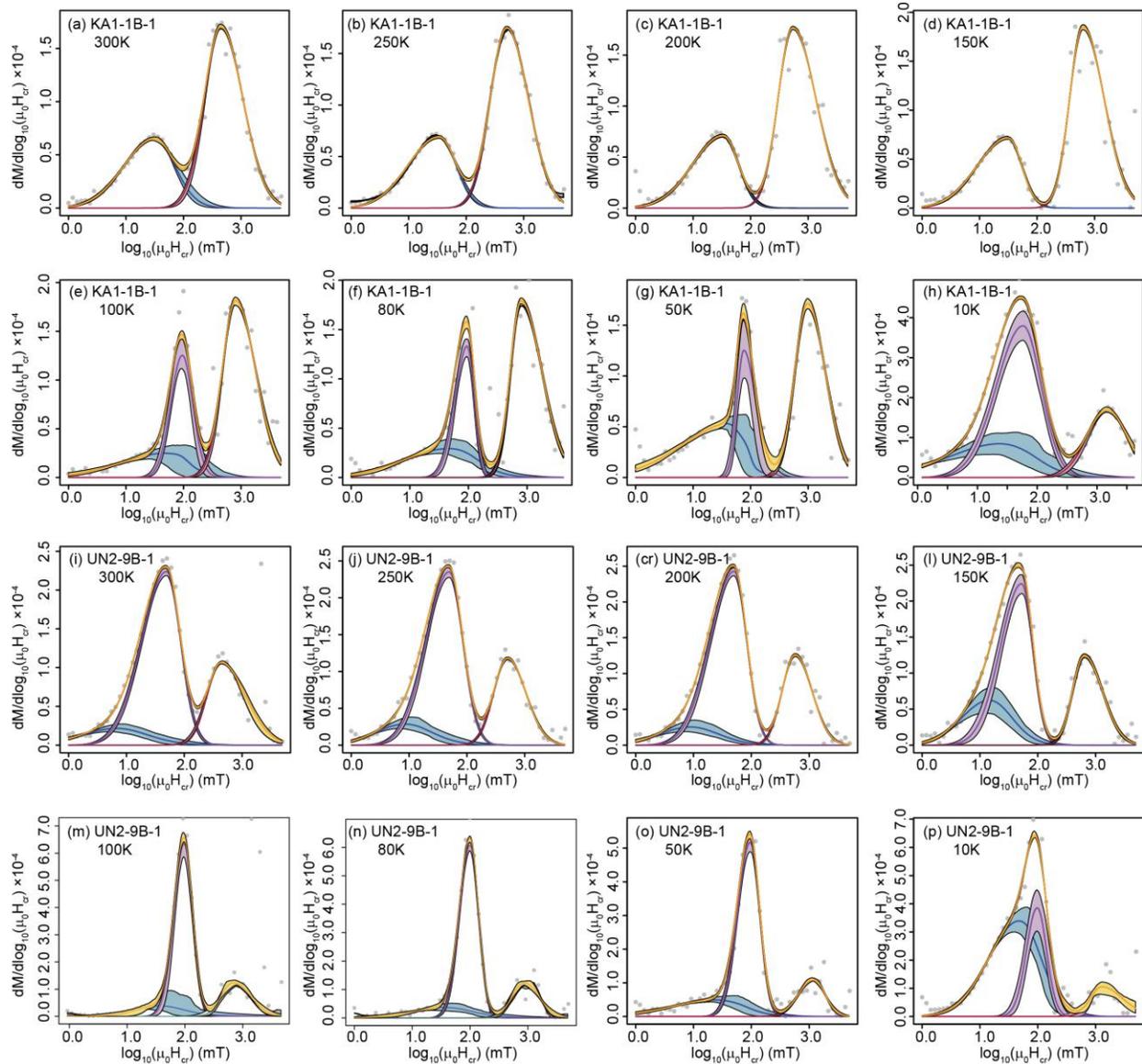
321 ***4.2. Coercivity distributions for pigmentary hematite***

322 The results shown in Figure 2 indicate that goethite is not magnetically important in the Inuyama
323 red chert and that magnetite is mostly confined to the low coercivity component (< 300 mT). We
324 further examine coercivity spectra at eight temperatures from 10 K to 300 K. At room temperature,
325 most Triassic and Jurassic specimens are well fitted with two skew-normal distributions (Figures
326 3a, 4a, 4i). One distribution has a 19-35 mT median coercivity and extends from 0 to ~ 500 mT
327 (based on ± 3 standard deviations from the median coercivity). The other distribution has a higher
328 median coercivity of 413-598 mT and extends from ~ 60 mT to ~ 6 T, which is likely to be due to
329 SSD hematite. Triassic sample UN2-9B-1 has an additional lowest-coercivity contribution with a
330 broad distribution that extends to ~ 200 mT (Figure 3i). At room temperature there is only a small
331 overlap between the low- and high-coercivity components. With decreasing temperature, the
332 overlap is reduced and finally disappears or becomes insignificant below 100 K. This behavior is
333 consistent with hematite coercivity increasing with decreasing temperature (equations (3) and (4)),
334 while magnetite has a much less dramatic coercivity change with temperature (Özdemir et al.,
335 2002). The low-temperature dividing point of the low-coercivity component appears at ~ 250 mT,
336 which is consistent with the Verwey transition being significant in the $IRM_{<300}$ component (Figure
337 2a, 2b). Therefore, for both Triassic and Jurassic specimens, the hematite population can be well
338 separated from magnetite based on their coercivity distributions.



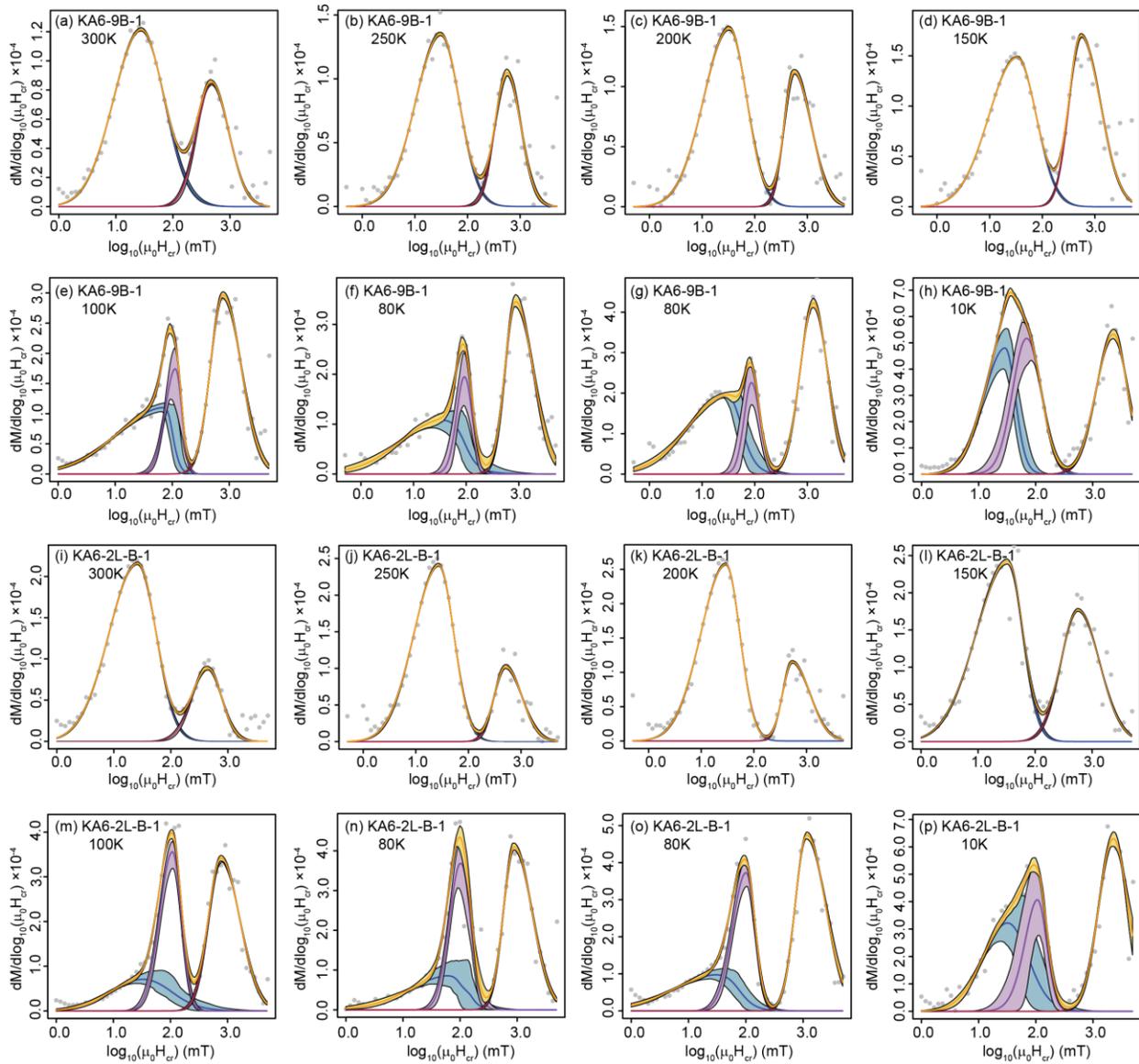
339

340 **Figure 2** LTSIRM variations versus temperature. (a, b) Samples were given a SIRM in a 5 T field
 341 at 10 K and then AF demagnetized in peak fields of 100 and 300 mT, respectively. Then the
 342 LTSIRM was measured during warming for components with coercivity ranges of < 100 mT
 343 (black), between 100 and 300 mT (blue line), and > 300 mT (red). (c, d) ZFC (black) and FC
 344 (green) LTSIRM curves. (e, f) Samples were saturated in a 5 T field at 300 K and then AF
 345 demagnetized in a 300 mT peak field. The $RTSIRM_{@AF300}$ was then measured during cooling to
 346 150 K (dark blue), warming to 400 K (red), and then cooling back to 150 K (light blue). Tr =
 347 Triassic; Jr = Jurassic.



348

349 **Figure 3** Coercivity spectra from backfield SIRM demagnetization curves for two Triassic
 350 specimens (KA1-1B-1, Anisian; UN2-9B-1, upper Norian). The data were fitted using skew-
 351 normal distributions with the Max Unmix software (Maxbauer et al., 2016). We fitted data with a
 352 minimum number of components. At eight temperatures, the data can be fitted with 2-3
 353 components: the lowest coercivity component is shown in blue, the intermediate coercivity
 354 distribution in purple, and the highest coercivity distribution in red. Yellow lines represent the sum
 355 of all components, while grey dots represent the data. Shaded areas are 95% confidence intervals
 356 for each component.

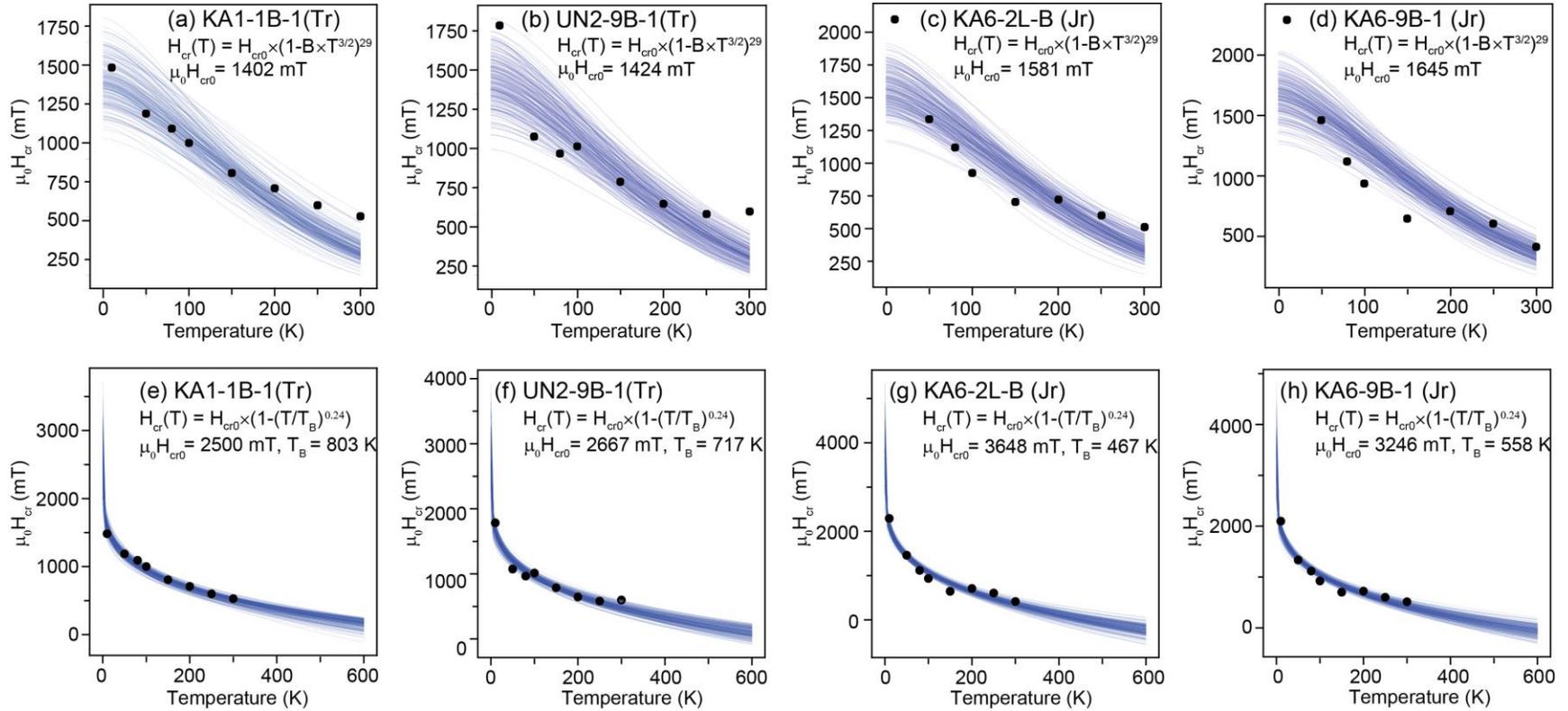


357

358 **Figure 4** Coercivity spectra from backfield SIRM demagnetization curves for two Jurassic
 359 specimens (KA6-9B-1; KA6-2L-B-1 from early Jurassic). Formatting is the same as Figure 3.

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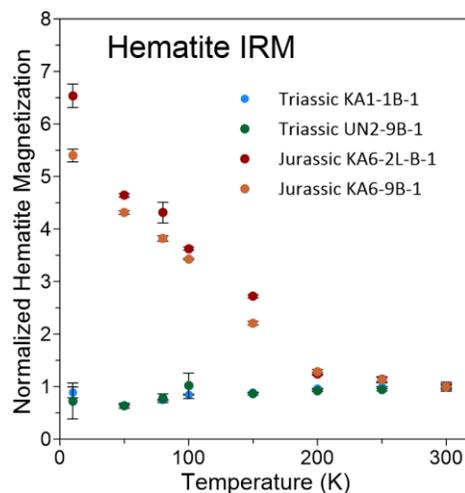
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368

369

Figure 5 Hematite median remanent coercivity variation with temperature. (a-d) Bayesian posterior distribution of fitted curves based on equation (3) given the priors listed in Appendix A and assuming a common n value for all samples. (e-h) Bayesian posterior distribution of fitted curves based on equation (4) given the Bayesian priors listed in Appendix A and assuming a common value of Kneller's α exponent for all samples. The n and α parameters, T_B shown in the equations are the median values from the parameter posterior distributions. Standard deviations and other posterior distribution statistics are provided in Appendix A.

370 Increases in the coercivity of hematite with decreasing temperature are illustrated in Figure 5.
 371 Below ~ 150 K, the increase is steeper; it triples for Triassic specimens and increases five-fold for
 372 Jurassic specimens at 10 K compared to room temperature. The coercivity-temperature fits in
 373 Figures 5a-5d and 5e-5h were made using equations (3) and (4), respectively, using Bayesian
 374 regression (Appendix A). In this study, we assume that the exponent parameter n or α is constant
 375 for pigmentary hematite in Triassic/Jurassic Inuyama red chert. Under this assumption, we
 376 combine all 32 data points from four specimens at eight temperatures to estimate a common n and
 377 α posterior distribution and individual posterior distributions of H_{cr0} and T_B for each specimen by
 378 Bayesian regression (see Appendix A for details). As expected, large n values of 22 - 36 (97%
 379 high density interval) are obtained, which demonstrates that the hematite coercivity increases more
 380 strongly than M_s . However, these fits are less satisfying at low- and room-temperature. The fits
 381 tend to underestimate H_{cr0} and the coercivity close to room temperature due to the flatness of the
 382 fitted curves, which largely comes from the $3/2$ exponent. Fits based on equation (4) achieve better
 383 results (Figure 5e-5h). The posterior α ranges from 0.151 to 0.339 (97% high density interval) with
 384 median value of 0.24. Triassic red chert samples have lower H_{cr0} and higher T_B than Jurassic red
 385 cherts. Low T_B values of ~ 194 °C and ~ 285 °C are predicted for hematite in Jurassic red cherts,
 386 which suggests they have a fine grain size.



387
 388 **Figure 6** Hematite IRM variation with temperature. Data are normalized by hematite IRM at 300
 389 K for each sample. Error bars represent fitting errors for the hematite component.

390

391 Distinctively IRM intensity changes for hematite with temperature are shown for Triassic and
392 Jurassic samples, respectively, in Figure 6. The hematite remanence remains relatively constant
393 for the Triassic specimens (blue and green dots), which indicates that almost all of the hematite is
394 in the SSD state at room temperature. In contrast, hematite remanence increases exponentially with
395 decreasing temperature for the Jurassic samples (brown and red dots), which indicates a significant
396 SP contribution with a wide blocking temperature range.

397

398 ***4.4. Tomographic Analysis***

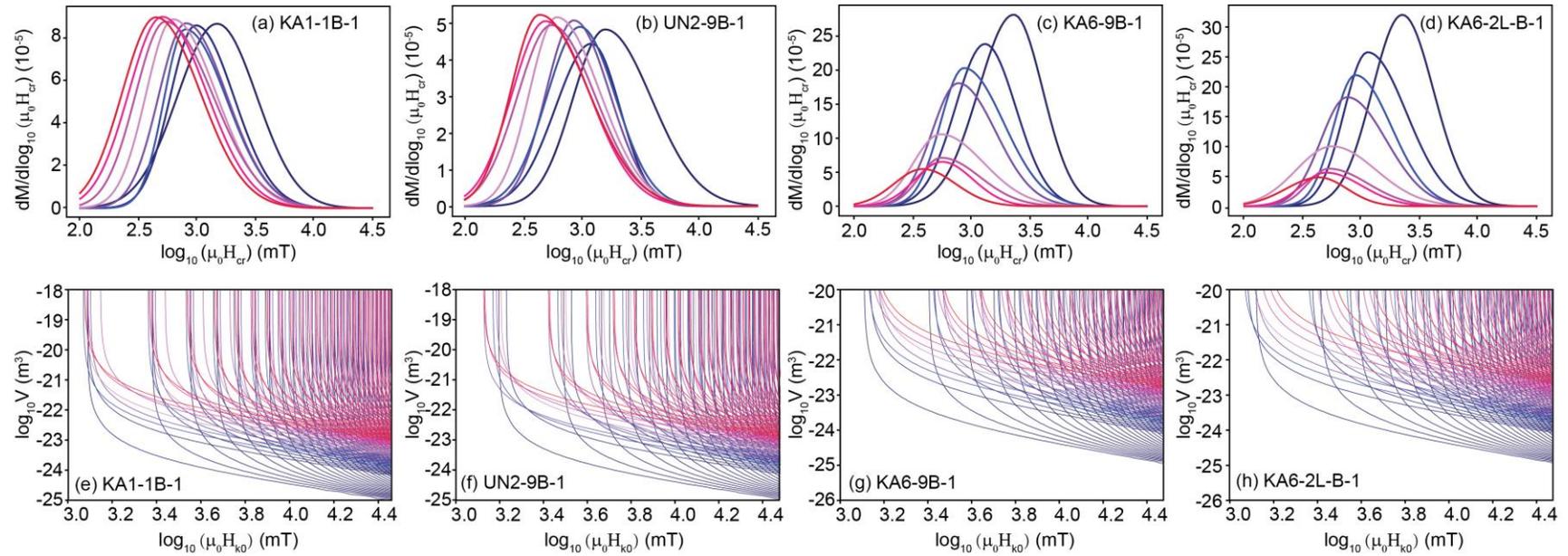
399 Based on the above results, for our tomographic analysis we adopted Kneller's law as a coercivity-
400 temperature relationship with $\alpha = 0.24$ and $H_{cr} = 0.524H_k$. The median T_B values shown in Figure
401 5e-5f are used for each specimen to calculate hematite blocking contours. Hematite coercivity
402 distributions extracted from the high-coercivity component fitting in Figures 3 and 4 are shown in
403 Figure 7a-7d. Each dataset contains 808 backfield remanence data points (101 field steps at each
404 of eight temperatures). These data are combined with equations (1) and (4) and are mapped into
405 blocking contours (Figure 7e-7h).

406

407 Upon cooling to 10 K, all backfield derivative curves shift progressively to higher coercivities as
408 expected. Peak heights are roughly constant for Triassic samples but increase significantly upon
409 cooling for Jurassic samples (Figure 7). This indicates a greater SP hematite content that blocks
410 gradually with cooling in Jurassic red chert, while the Triassic red chert is dominated by coarser
411 SSD hematite. The blocking contour density is nonuniform, so poor resolution is expected for
412 particles smaller than ~ 10 nm with microcoercivities less than ~ 1 T.

413

414 After determining the blocking contours, we start the iterative process to calculate the joint grain
415 size and microcoercivity distribution of hematite particles. Best-fit backfield derivative curves
416 reproduce large-scale features of the measured spectra, while still containing higher frequency
417 deviations (Figures A2-A5). Fitting errors are below 15% for all samples.



418

419 **Figure 7** Hematite backfield remanence data and blocking contours for Triassic and Jurassic red cherts. (a-d) Hematite coercivity
 420 distributions extracted from backfield LTSIRM decomposition at eight temperatures from 10 K to 300 K. (e-h) Blocking contours for
 421 the fields and temperatures in the corresponding datasets above. The equations in Figure 5e-5h are used for each respective specimen.
 422 Color variations indicate temperature changes from 10 K (blue) to 300 K (red).

423

424

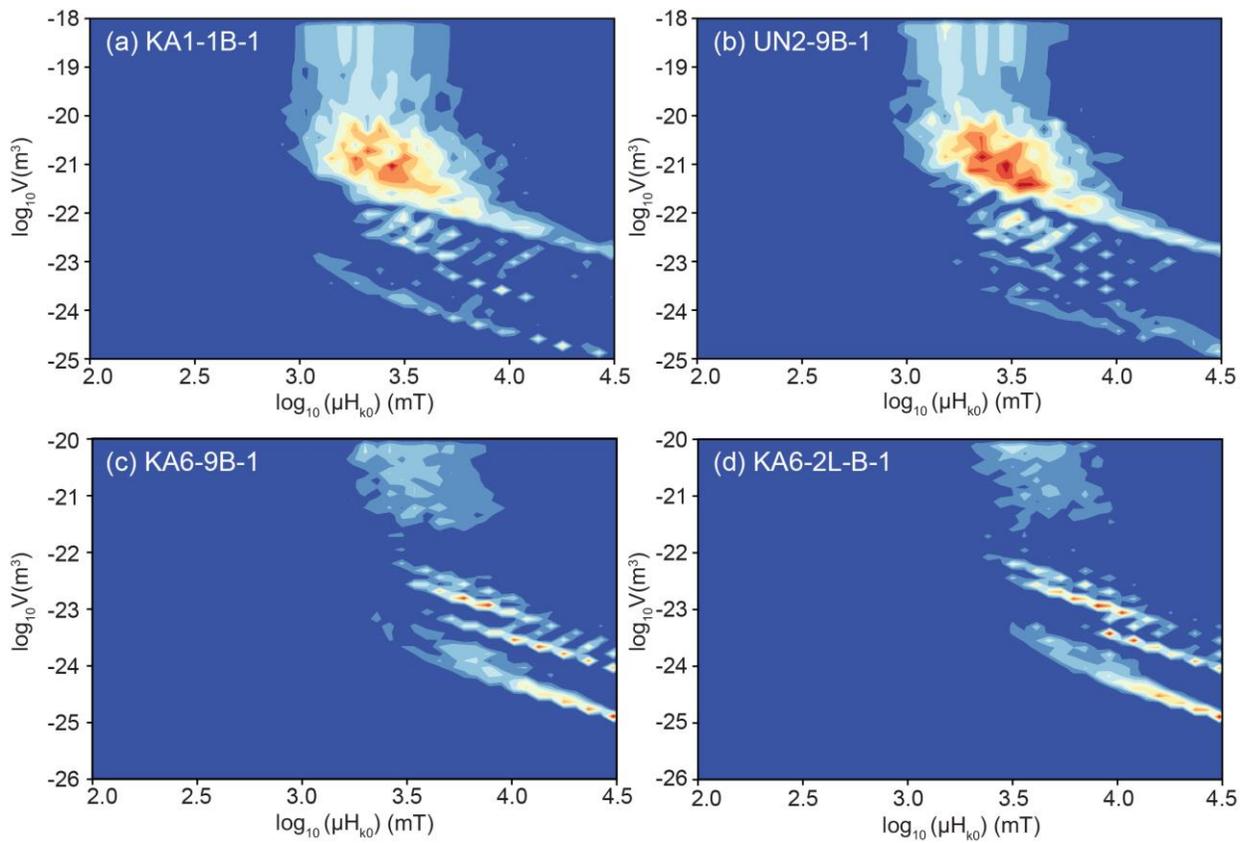
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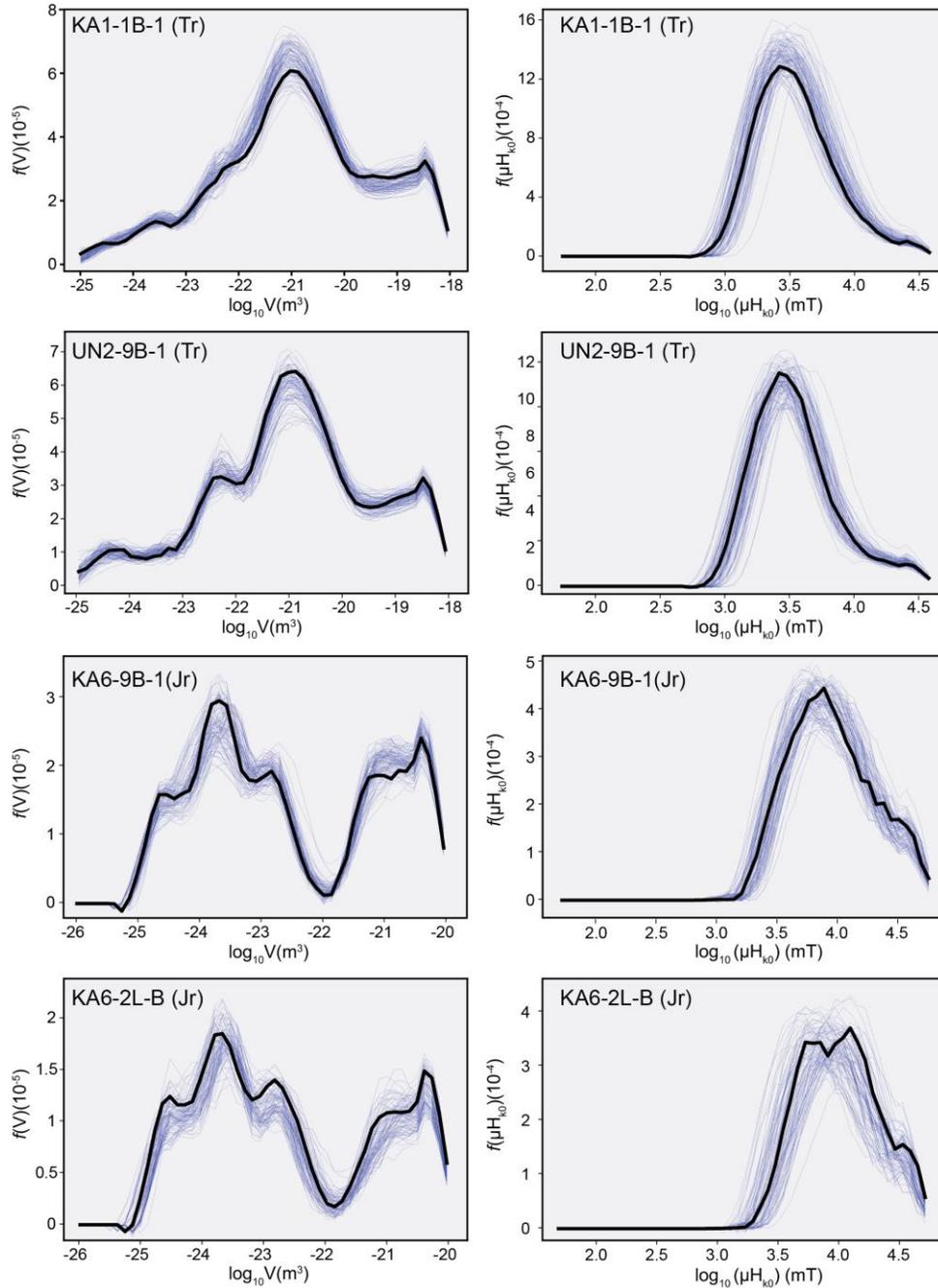
428

429 Estimated $f(V, H_{k0})$ distributions for Triassic samples have a continuous feature centering at
 430 volumes around $1 \times 10^{-21} \text{ m}^3$ and microcoercivities between 1 T and 10 T (Figure 8a, 8b). The
 431 central volumes are equivalent to spherical hematite particles with $\sim 75 \text{ nm}$ diameters. In Jurassic
 432 samples, the hematite particles are smaller but magnetically harder (Figure 8c, 8d), with more
 433 discrete distributions centered around volumes of $\sim 1 \times 10^{-25} \text{ m}^3$, $\sim 1 \times 10^{-23.5} \text{ m}^3$, $\sim 1 \times 10^{-22.5} \text{ m}^3$, and
 434 $> 1 \times 10^{-22} \text{ m}^3$, which correspond to diameters of $\sim 3 \text{ nm}$, $\sim 11 \text{ nm}$, $\sim 24 \text{ nm}$, and $> 35 \text{ nm}$ for
 435 spherical hematite. The microcoercivity of Jurassic hematite ranges from 3 T to $> 30 \text{ T}$. There is
 436 clear elongation of the distribution toward the lower right, along with the dominant blocking
 437 contour orientation, which may be an artifact of the inversion process (Jackson et al., 2006).
 438



439
 440
 441 **Figure 8** Estimated $f(V, H_{k0})$ for (a, b) Triassic and (c, d) Jurassic red chert samples from the
 442 data in Figure 6a-d. Contour interval = $f_{\text{max}}/30$.

443
 444



445

446 **Figure 9** Volume and microcoercivity distributions obtained by summing the rows and columns
 447 of the 2D model. The data were smoothed with a Savitzky-Golay filter with window length of 5
 448 in ‘nearest’ mode using the Python Scipy.signal package. Thick black lines represent the median
 449 value; light blue lines represent calculations based on 100 randomly drawn T_B and α values from
 450 the Bayesian posterior distribution in Figure A1, which are used to indicate the uncertainty on
 451 the calculation of the volume and microcoercivity distribution.

452 Bayesian modeling was used to calculate the volume and microcoercivity distributions shown in
 453 Figure 9. Thick black lines represent the median volume and microcoercivity distribution for each
 454 sample based on median T_B and α values. The light blue lines represent calculations based on 100
 455 randomly drawn T_B and α values from their Bayesian posterior distribution in Figure A1, which
 456 indicates the uncertainty on the volume and microcoercivity distribution calculation. The
 457 marginalized microcoercivities are nearly lognormally distributed. Additional high
 458 microcoercivity contributions (> 10 T) are more evident in the Jurassic hematite than Triassic
 459 hematite. By contrast, volume distributions are asymmetrical and more complex. Triassic hematite
 460 populations have a small peak at 1×10^{-23} to 1×10^{-22} m³ and then gradually increase to a major peak
 461 at $\sim 1 \times 10^{-21}$ m³ (Figure 9a). Additional coarse particles with volume larger than 1×10^{-20} m³ are also
 462 present. The Jurassic hematite population has a roughly bimodal distribution separated at around
 463 1×10^{-22} m³. The larger particle population has a broad peak from 1×10^{-22} m³ to 1×10^{-20} m³. The
 464 smaller particle population has a major peak at $\sim 1 \times 10^{-24}$ m³ with two smaller peaks at $1 \times 10^{-24.6}$ m³
 465 and 1×10^{-23} m³, which correspond to their discrete components in Figure 8c, 8d. The discrete
 466 nature of the distribution is most likely due to the limited numbers of temperatures used here
 467 because the distributions are all elongated along the unblocking contours. Nevertheless, two grain
 468 size populations are evident in Jurassic samples; the finer fraction ranges from a few nanometers
 469 to ~ 35 nm in diameter while the coarser fraction is from ~ 35 nm to ~ 160 nm in diameter and is
 470 comparable with Triassic samples.

471

472 **5. Discussion**

473 ***5.1. Coercivity of pigmentary hematite in red chert***

474 Early studies of the Inuyama red chert reported large saturating fields of up to several tesla for
 475 pigmentary hematite from IRM acquisition curves (Oda and Suzuki, 2000; Shibuya and Sasajima,
 476 1986). In our results, room temperature $\mu_0 H_{cr}$ ranges from ~ 60 mT to ~ 6 T in Triassic red chert
 477 and from ~ 70 mT to ~ 3 T in Jurassic red chert (Figures 3 and 4), which is comparable to recent
 478 studies (Abrajevitch et al., 2013; Hu et al., 2021). Published data for pigmentary hematite in red
 479 beds have a similarly wide range of $\mu_0 H_{cr}$ values. In the Deer Lake Group red beds of western
 480 Newfoundland, hematite remanent coercivity ranges from ~ 60 mT to 3 T (Bilardello and Kodama,
 481 2010a), and for red beds from the Maritime provinces of Canada, it varies from ~ 40 mT to 5 T and

482 beyond (Bilardello and Kodama, 2010b). Hematite in Triassic red beds from South China has
483 remanent coercivity values from ~60 mT to 3 T (Jiang et al., 2017). For zebra rock in Western
484 Australia, strong fields up to 3 T are needed to saturate hematite (Abrajevitch et al., 2018). North
485 American red siltstone intraclasts have remanent coercivity of ~100 mT to 1.8 T and beyond
486 (Swanson-Hysell et al., 2019). Thus, $\mu_0 H_{cr}$ values of ~60 mT to ~3 T are typical of natural
487 pigmentary hematite in red beds, although values up to even ~6 T are sometimes observed. This
488 $\mu_0 H_{cr}$ range gives an idea of the remanent coercivity distribution of natural SSD pigmentary
489 hematite.

490

491 We further illustrate remanent coercivity variations with temperature for pigmentary hematite. H_{cr}
492 increases exponentially with decreasing temperature, following the T^α law, where $\alpha = 0.24$ is the
493 median posterior value for red chert samples in this study (Figure 5e, 5f, 5g, 5h). This behavior
494 can be understood by considering thermal fluctuation effects of blocked moments across an
495 anisotropy barrier (Maaz et al., 2010). For natural pigmentary hematite, this simple thermal
496 activation model appears to be applicable from 10 K to 300 K. The significant H_{cr} increase at low
497 temperatures also provides a way to separate a hematite component from magnetite. Based on
498 results in Figures 2 and 3, there is almost no overlap between magnetite and hematite components
499 below 100 K.

500

501 Quintupled hematite M_{rs} values at 10 K compared to room temperature confirms the presence of a
502 large SP hematite population in the Jurassic red chert (Figure 6). The steep M_{rs} rise below 200 K
503 indicates that the blocking temperature of most SP hematite is below 200 K. Our results
504 demonstrate that decomposition of low temperature backfield curves reveals and potentially
505 enables quantification of entire pigmentary hematite populations, especially SP particles. Although
506 SP signatures are detected in remanent FORC diagrams (Hu et al., 2021), these signals tend to be
507 dominated by magnetite because FORC diagrams reflect bulk signals and the magnetization of
508 hematite is more than two hundred times lower than magnetite (Dunlop and Özdemir, 1997).

509

510 Compared to room-temperature H_{cr} , which only represents SSD populations, H_{k0} provides a
511 measure of the entire hematite population, including SP particles. In the calculations of Jackson et
512 al. (2006), $\mu_0 H_{k0}$ values for magnetite in a Tiva Canyon Tuff sample can extend to 300 mT, which

513 is the upper limit for prolate spheroids with magnetite-like magnetizations. According to our
514 calculations, $\mu_0 H_{k0}$ for the pigmentary hematite can be much higher, and varies from 1 T to ~ 10 T
515 in Triassic red chert and from ~ 1.5 T to > 30 T in Jurassic red chert (Figure 9). As is the case for
516 magnetite, microcoercivity distributions for hematite are nearly symmetric in logarithmic space
517 (Figure 9).

518

519 The high coercivity of pigmentary hematite cannot be explained by shape anisotropy (Banerjee,
520 1971). Özdemir and Dunlop (2014) concluded by studying MD hematite that both magnetoelastic
521 and magnetocrystalline effects contribute to coercivity. However, magnetocrystalline anisotropy
522 causes a gradual coercivity decrease during cooling below room temperature (Liu et al., 2010;
523 Özdemir et al., 2002), which is opposite to the temperature dependence of coercivity observed
524 here. The increasing coercivity trend with cooling in pigmentary hematite is interpreted to be due
525 to magnetoelastic anisotropy, which arises from crystal defects, dislocations, or internal strain
526 (Sunagawa and Flanders, 1965; Sunagawa, 1960; Liu et al., 2010).

527

528 ***5.2. Grain size distribution and unblocking temperature of pigmentary hematite in red chert***

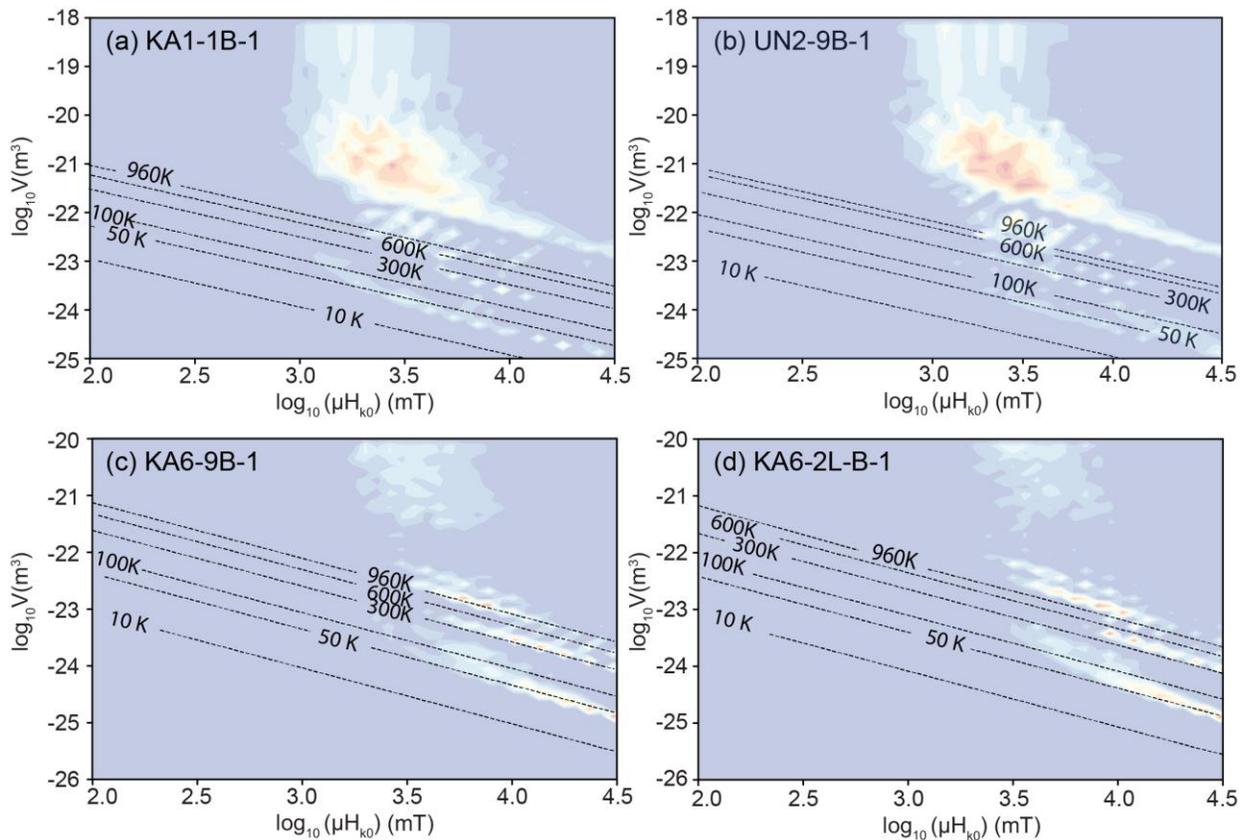
529 Our analysis reveals that both Triassic and Jurassic red chert samples have hematite population
530 with median size of ~ 75 nm. Additional large amounts of finer hematite $< \sim 35$ nm occurs in the
531 Jurassic red chert (Figure 9). The median T_B for each red chert sample is 529 °C (KA1-1B-1),
532 438 °C (UN2-9B-1), 194 °C (KA6-2L-B-1), and 285 °C (KA6-9B-1). The low unblocking
533 temperatures are also consistent with the small particle size of pigmentary hematite. According to
534 the unblocking temperature and grain size model of Swanson-Hysell et al. (2011, 2019), the
535 median unblocking temperatures in red chert correspond to grain sizes of 100-160 nm, which is
536 within the range of our calculated grain size distributions. The broad unblocking temperature
537 distribution reflects the wide size distribution of natural pigmentary hematite.

538

539 We further calculated T_B contours based on the volume and microcoercivity distributions obtained
540 here (Figure 10). Almost the entire pigmentary hematite population in the Triassic red cherts is in
541 the SSD state at room temperature, with unblocking temperatures generally above 300 K (Figure
542 10a, b). For the Jurassic red cherts, a significant part of the hematite is in the SP state with

543 unblocking temperatures extending below 100 K. The SP/SSD threshold size for stoner-wohlfarth
 544 hematite has been estimated at 25-30 nm (Banerjee, 1971; Özdemir & Dunlop, 2014) and at 17
 545 nm for Al-hematite (Jiang et al., 2014). From our T_B calculations, the SP/SSD threshold size for
 546 stoichiometric hematite in the Jurassic red cherts is 7-18 nm, which is close to estimates for Al-
 547 hematites.

548



549

550 **Figure 10** Calculated blocking temperature contours from 10 K to 960 K with $f(V, H_{k0})$

551 distributions for (a, b) Triassic and (c, d) Jurassic samples.

552

553 Detailed paleomagnetic studies have been conducted on the Inuyama red chert, with four
 554 remanence components identified from stepwise thermal demagnetization (Oda and Suzuki, 2000;
 555 Shibuya and Sasajima, 1986). The four components are labeled A (70-200 °C), B (200-350 °C), C
 556 (350-530 °C), and D (530-680 °C). Components B and C were thought to be Late Cretaceous
 557 remagnetizations, although the remagnetization mechanism has been debated. Partial thermal
 558 demagnetization (ThD) after alternating field (AF) demagnetization at 80 mT reveals the same

559 four components (Oda and Suzuki, 2000), which indicates that hematite was at least partly
560 responsible for all four components. This is consistent with the wide unblocking temperature range
561 for hematite in our calculations. Paleomagnetic studies of the Inuyama red chert have all suggested
562 that magnetite is the dominant remanence carrier for components A, B, and C and, thus, magnetite
563 carries the remagnetizations. However, pigmentary hematite should also contribute to the
564 remagnetizations due to its low unblocking temperature, especially for Component B. We suggest
565 this because the unblocking temperature range of component B (200-350 °C) and C (350-530 °C)
566 spans the median hematite unblocking temperature range. Meanwhile, little magnetization for
567 component B was lost between 200 and 350 °C in the original ThD experiment (Figure 6a from
568 Oda and Suzuki (2000)), but the magnetization drops significantly after AF demagnetization of 80
569 mT (Figure 6c in Oda and Suzuki (2000)). According to our analysis on the magnetite remanent
570 coercivity distribution of the Inuyama red chert at room temperature (Figure 3 and Figure 4), an
571 alternating field of 80 mT will demagnetize most of the magnetite, which suggests that the more
572 significant unblocking feature of component B after AF demagnetization is likely due to
573 pigmentary hematite.

574 ***5.3. Limitations of the TFT method for reconstructing hematite grain size distributions***

575 There are two main limitations of the TFT technique for hematite. First, it has the same limitation
576 for hematite as it does for magnetite (Jackson et al., 2006). Reconstructed size distributions are
577 elongated toward the upper left and lower right-hand sides of the Néel diagram, along the blocking
578 contour orientation. This elongation can be due to artifacts in the inversion method, the physical
579 model, and the assumption made, as shown by Jackson et al. (2006) and Dunlop (1965). Resolution
580 limits also remain for inversion due to the restricted orientation distribution of integration paths.
581 Areas of marginal resolution are sampled sparsely by subparallel contours, such as within the small
582 V , high H_{k0} region that is stable only at the lowest temperatures. At the same time, given the small
583 numbers of temperatures employed, the resolution of our results is imperfect, and stripes are
584 evident in the $f(V, H_{k0})$ distribution rather than a smooth continuous distribution (Figure 8c, d).
585 Increasing the number of temperature steps will improve the resolution, but will also be time and
586 helium expensive, especially for multiple specimens.

587

588 Second, compared to magnetite, an additional challenge when modeling hematite is the complexity
589 of its anisotropy. An important assumption in our calculation is that we assign $H_{cr} = 0.524H_K$,
590 which is based on Stoner and Wohlfarth (1948) theory for identical randomly oriented uniaxial
591 particles. Harrison et al. (2019) simulated remanence FORC diagrams for particles with uniaxial,
592 cubic, and hexagonal anisotropy. Among these anisotropy types, randomly oriented, non-
593 interacting particles with uniaxial anisotropy have the lowest H_{cr} values, while H_{cr} for cubic and
594 hexagonal anisotropy is approximately 1.05 and 1.8 times larger, respectively. Therefore,
595 multiaxial anisotropy will produce $0.5 < H_{cr}/H_K < 1$. Magnetic minerals with uniaxial, cubic, and
596 hexagonal anisotropy produce distinctive FORC diagram types (e.g., Egli, 2021; Roberts et al.,
597 2021), which provides a useful way to evaluate the dominant anisotropy type before undertaking
598 TFT analyses. Conventional and remanence FORC diagrams for the Inuyama red chert all have a
599 central ridge up to 1.2 T. Although magnetite dominates the FORC signatures, hematite is
600 responsible for central ridges with coercivities > 300 mT. Ridge-type signatures for conventional
601 and remanence FORC diagrams are explained as a manifestation of uniaxial SD magnetic behavior
602 based on simulations (Harrison et al., 2019). However, instead of being shape dominated, as is the
603 case for magnetite, ridge-type FORC signals for hematite nanoparticles reflect stress-induced
604 uniaxial anisotropy that dominates the intrinsic magnetocrystalline anisotropy (Roberts et al.,
605 2021). This magnetostrictive anisotropy associated with uniaxial internal stress controls the
606 coercivity of hematite, which is consistent with the rapid H_{cr} increase at low temperatures (Figure
607 5) that is usually enhanced in nanoparticles (Muench et al., 1985; Bruzzone and Ingalls, 1983).
608 Therefore, considering the ridge-type signals observed in both conventional and remanence FORC
609 diagrams for the red chert (see Figure 2 and 3 in Hu et al. (2021)) and H_{cr} variation with
610 temperature, we predict that uniaxial anisotropy dominates pigmentary hematite in red chert and
611 adopt $H_{cr}/H_K = 0.524$. Increased H_{cr}/H_K will result in decreased microcoercivity, increased T_B
612 estimates, and slightly decreased grain size estimates. Thus, when considering variations in the
613 dominant anisotropy type, our calculations at least provide an upper limit of microcoercivity and
614 grain size and a lower T_B limit.

615

616 **6. Conclusions**

617 Reconstructions of particle microcoercivity and volume distributions were performed with the
618 method of Jackson et al. (2006) for SP/SSD hematite assemblages assuming a dominant uniaxial

619 anisotropy. The median temperature variation of H_{cr} for pigmentary hematite in Triassic/Jurassic
620 red chert over eight temperatures from 300 K to 10 K follows a modified Kneller's law;
621 $H_{cr}(T) = H_{cr0} \left(1 - \left(\frac{T}{T_B} \right)^{0.24} \right)$. The coercivity of hematite increases more rapidly than for
622 magnetite with decreasing temperature, and coercivity distribution overlap between the two
623 minerals starts to disappear below 100 K. Microcoercivity distributions that are nearly symmetric
624 in logarithmic space vary from 1 T to ~10 T in Triassic red chert and from ~3 T to 30 T in
625 Jurassic red chert. Both Triassic and Jurassic red chert have wide hematite grain size
626 distributions. Most of the Triassic hematite ranges between ~35 nm to ~160 nm in diameter
627 while the Jurassic red chert has a coarser hematite particles fraction similar to the Triassic
628 hematite and a finer hematite fraction with grain size from a few nanometers to ~35 nm in
629 diameter. Calculated median T_B varies from ~194 °C to 529 °C for red chert samples and T_B
630 contours indicate that most of the Triassic hematite is in the SSD state with a significant SP
631 particle content with $T_B < 300$ K. The SP/SSD threshold size for pigmentary hematite in Jurassic
632 red chert is estimated to be 8-18 nm. Considering the low and broad T_B distribution in the
633 Inuyama red chert, we propose that pigmentary hematite has a significant contribution to a
634 secondary early Cenozoic thermoviscous magnetization rather than magnetite (Component B
635 defined by Oda and Suzuki (2000)).

636
637 Our work demonstrates that the main features of SP/SSD hematite $f(V, H_{k0})$ distributions can be
638 recovered using the TFT technique, although details should be interpreted judiciously. Smearing
639 of results due to low measurement resolution, artifacts, and variations in dominant anisotropy of
640 hematite should be considered on a case-by-case basis. Ridge-like FORC signatures for red chert
641 are interpreted here to indicate a dominant uniaxial anisotropy. If multiaxial anisotropy is instead
642 dominant in hematite samples (e.g., Roberts et al., 2021), this would increase T_B estimates and
643 decrease microcoercivity and grain size estimates.

644

645 **Data Availability Statement**

646 All low-temperature magnetic data used here will be uploaded to the Magnetic Information
647 Consortium rock magnetic portal (MagIC; www.earthref.org).

648

649

650 **Acknowledgments**

651 We thank Mike Jackson for providing the original TFFT code. This work was supported by the
 652 National Institute of Advanced Industrial Science and Technology, Ministry of Economy, Trade
 653 and Industry, Japan, and the Australian Research Council through grants DP160100805 and
 654 DP200100765.

655

656

657 **Appendix A**

658 Bayesian regression was performed using the PyMC3 Python package (Salvatier et al., 2016).

659 The parameterizations of the Bayesian prior distributions for H_{cr0} , T_B , α and fitting error σ are
 660 provided below.

661 **1. H_{cr0}**

662 The prior for $\mu_0 H_{cr0}$ follows a log normal distribution with median value of 0 and variance of
 663 1. Thus, the probability density function can be expressed as:

664

$$665 \quad p(\mu_0 H_{cr0}) = \frac{1}{\mu_0 H_{cr0} \sqrt{2\pi}} \exp\left(-\frac{\ln(\mu_0 H_{cr0})^2}{2}\right) \quad (A1)$$

666

667

668

669 **2. T_B**

670 Theoretically, T_B can vary between 0 and 960 K for hematite. For better calculation efficiency,
 671 we set all variable values between 0 and 1 in the Bayesian regression. Therefore, instead of T_B , we
 672 use $T_B/1000$ to follow a Beta distribution with $\alpha = 2$ and $\beta = 2$, the probability density function can
 673 be expressed as:

$$674 \quad p\left(\frac{T_B}{1000}\right) = \frac{\frac{T_B}{1000} \times \left(1 - \frac{T_B}{1000}\right)}{B(2,2)}; \text{ and} \quad (A2)$$

675

676

$$677 \quad B(2,2) = \frac{\Gamma(2)\Gamma(2)}{\Gamma(4)} \quad (A3)$$

678

679 Where Γ is the Gamma function.

680

681 **3. α and n**

682 α and n follow a Beta distribution with $\alpha = 8$ and $\beta = 8$, the probability density function can be
 683 expressed as:

$$684 \quad p(\alpha) = \frac{\alpha^7 \times (1 - \alpha)^7}{B(8,8)} \text{ or } p\left(\frac{n}{50}\right) = \frac{\left(\frac{n}{50}\right)^7 \times \left(1 - \left(\frac{n}{50}\right)\right)^7}{B(8,8)}, \text{ with} \quad (A4, A5)$$

$$685 \quad B(8,8) = \frac{\Gamma(8)\Gamma(8)}{\Gamma(16)} \quad (A6)$$

688
 689 Based on previous studies, n rarely exceeds the value of 50. Therefore, we normalized n by 50
 690 for it to vary from 0 to 1. Γ in equation (A6) is the Gamma function.

691

692 **4. Fitting error σ**

693 σ follows a half-Cauchy distribution with location parameter of 0 and scale parameter of 1.
 694 The probability density function can be expressed as:

$$695 \quad f(\sigma, 1) = \frac{2}{\pi(1 + \sigma)} \quad (A7)$$

697

698 The, no U-Turn Sampler (NUTS) from the Python Pymc3 package was used for sampling, in
 699 where we set both the sampling number and iteration number to 4000.

700

701 Statistical results of Bayesian regression using equation (3) and equation (4), respectively are
 702 listed. Two hundreds regression curves for each sample are shown in Figure 5. In the following
 703 tables, hdi_3% and hdi_97% are the lower and upper bounds of the 97% high density interval.
 704 $R_{\hat{}}$ is the Gelman-Rubin convergence statistic, where a value of 1 indicates that the chains have
 705 converged.

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714 Table A1 Posterior statistics of Bayesian regression of equation (3)

715

	Median	Standard deviation	hdi_3% ¹	hdi_97% ²	R_hat ³
n	28.650	3.700	21.650	35.600	1
$\mu_0 H_{cr0}$ (T) (KA1-1B-1)	1.402	0.137	1.133	1.649	1
$\mu_0 H_{cr0}$ (T) (UN2-9B-1)	1.424	0.139	1.166	1.682	1
$\mu_0 H_{cr0}$ (T) (KA6-9B-1)	1.581	0.145	1.308	1.853	1
$\mu_0 H_{cr0}$ (T) (KA6-2L-B-1)	1.645	0.145	1.371	1.916	1
σ	0.253	0.037	0.189	0.321	1

716

717 Table A2 Posterior statistics of Bayesian regression of equation (4)

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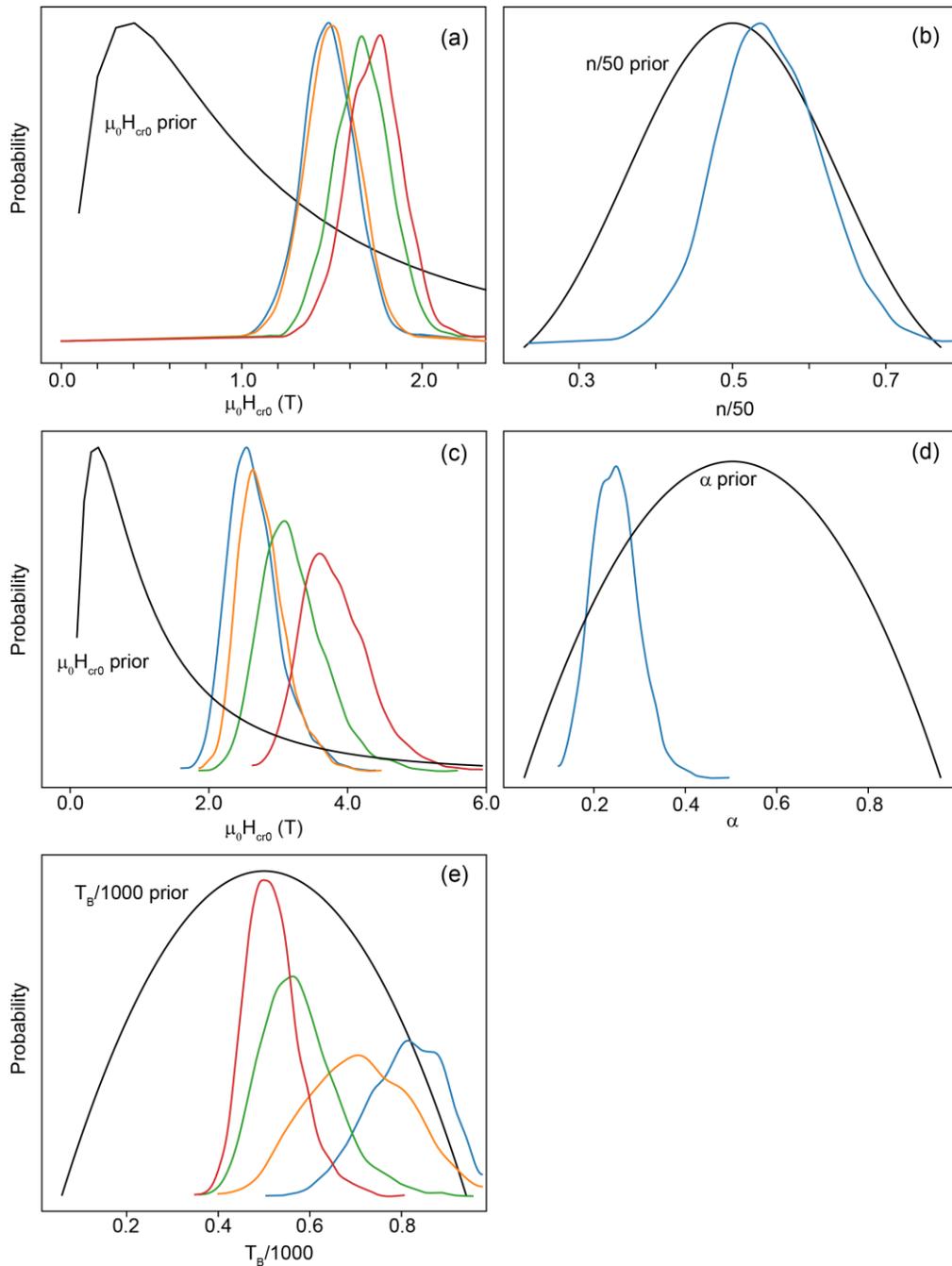
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732

	mean	Standard deviation	hdi_3% ¹	hdi_97% ²	R_hat ³
α	0.240	0.052	0.151	0.339	1
$\mu_0 H_{cr0}$ (T) (KA1-1B-1)	2.500	0.383	1.829	3.211	1
$\mu_0 H_{cr0}$ (T) (UN2-9B-1)	2.667	0.405	1.947	3.446	1
$\mu_0 H_{cr0}$ (T) (KA6-9B-1)	3.246	0.494	2.335	4.164	1
$\mu_0 H_{cr0}$ (T) (KA6-2L-B-1)	3.648	0.561	2.627	4.664	1
$T_B/100$ (K) (KA1-1B-1)	0.803	0.099	0.621	0.972	1
$T_B/1000$ (K) (UN2-9B-1)	0.717	0.103	0.534	0.919	1
$T_B/1000$ (K) (KA6-9B-1)	0.558	0.076	0.425	0.701	1
$T_B/1000$ (K) (KA6-2L-B-1)	0.467	0.054	0.370	0.567	1
σ	0.111	0.019	0.078	0.148	1

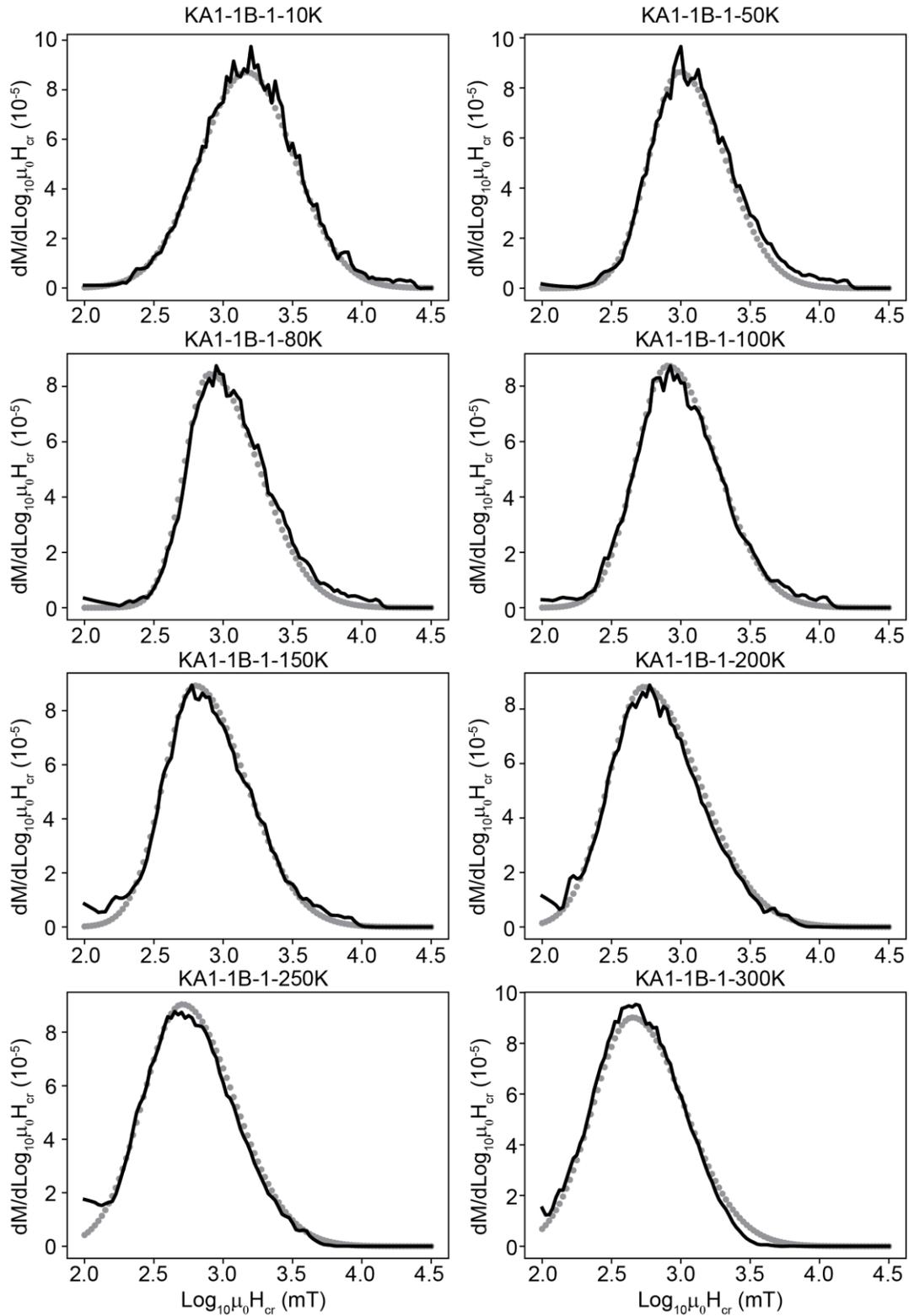
¹ hdi_3% represents the lower bounds of the 97% high density interval of the corresponding posterior distribution.² hdi_97% represents the upper bounds of the 97% high density interval of the corresponding posterior distribution³ R_hat is the Gelman-Rubin convergence statistic, which estimates the degree of convergence of a random Markov Chain. Values close to one indicate convergence to the underlying distribution.

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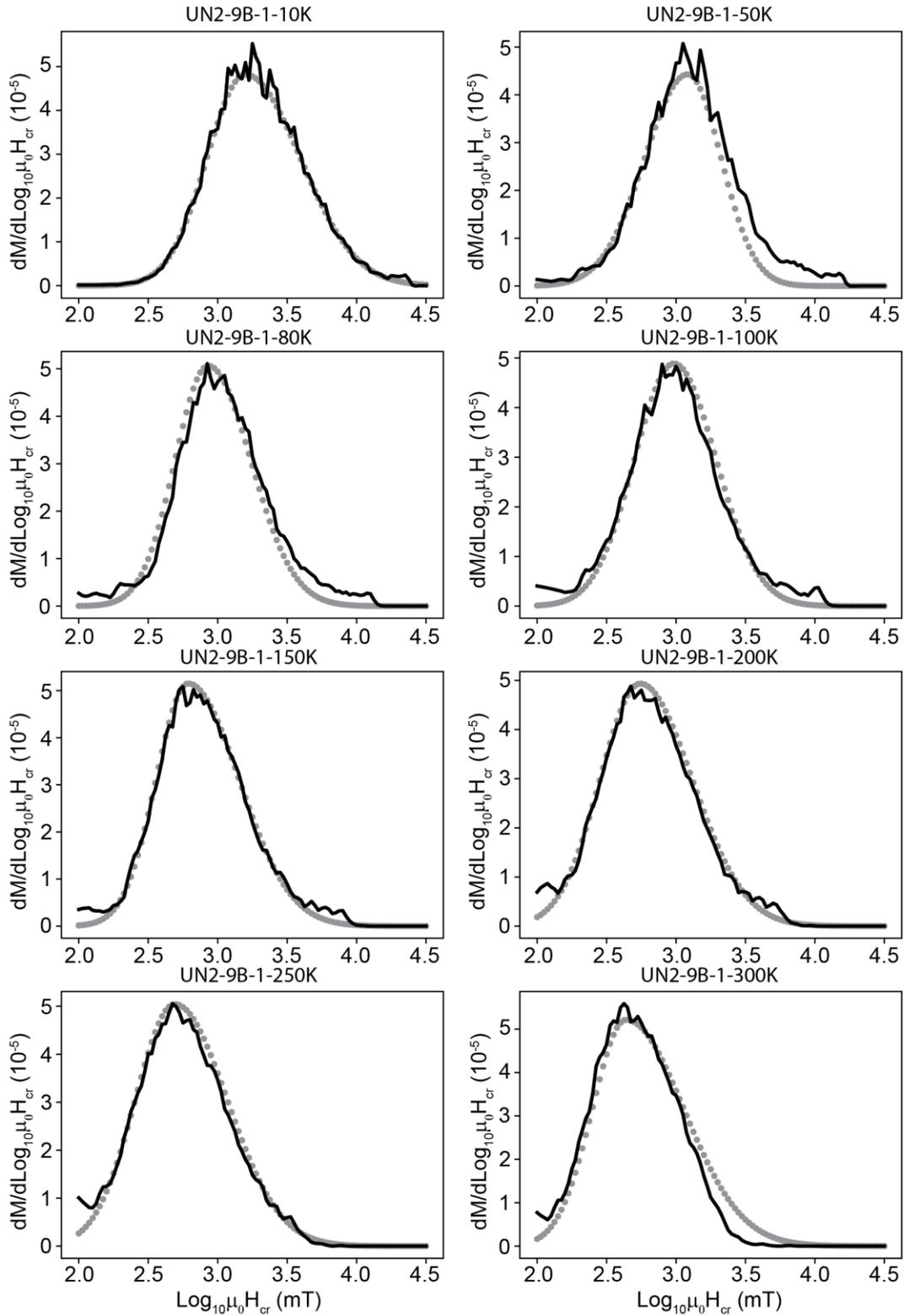
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735 **Figure A1** The prior distribution (black lines) and posterior distribution (colored lines) of
 736 parameters in the Bayesian model for (a-b) equation (3) and (c-e) equation (4). Blue, orange, green,
 737 and red lines represent corresponding parameters for sample KA1-1B-1, UN2-9B-1, KA6-9B-1
 738 and KA6-2L-B-1, respectively.



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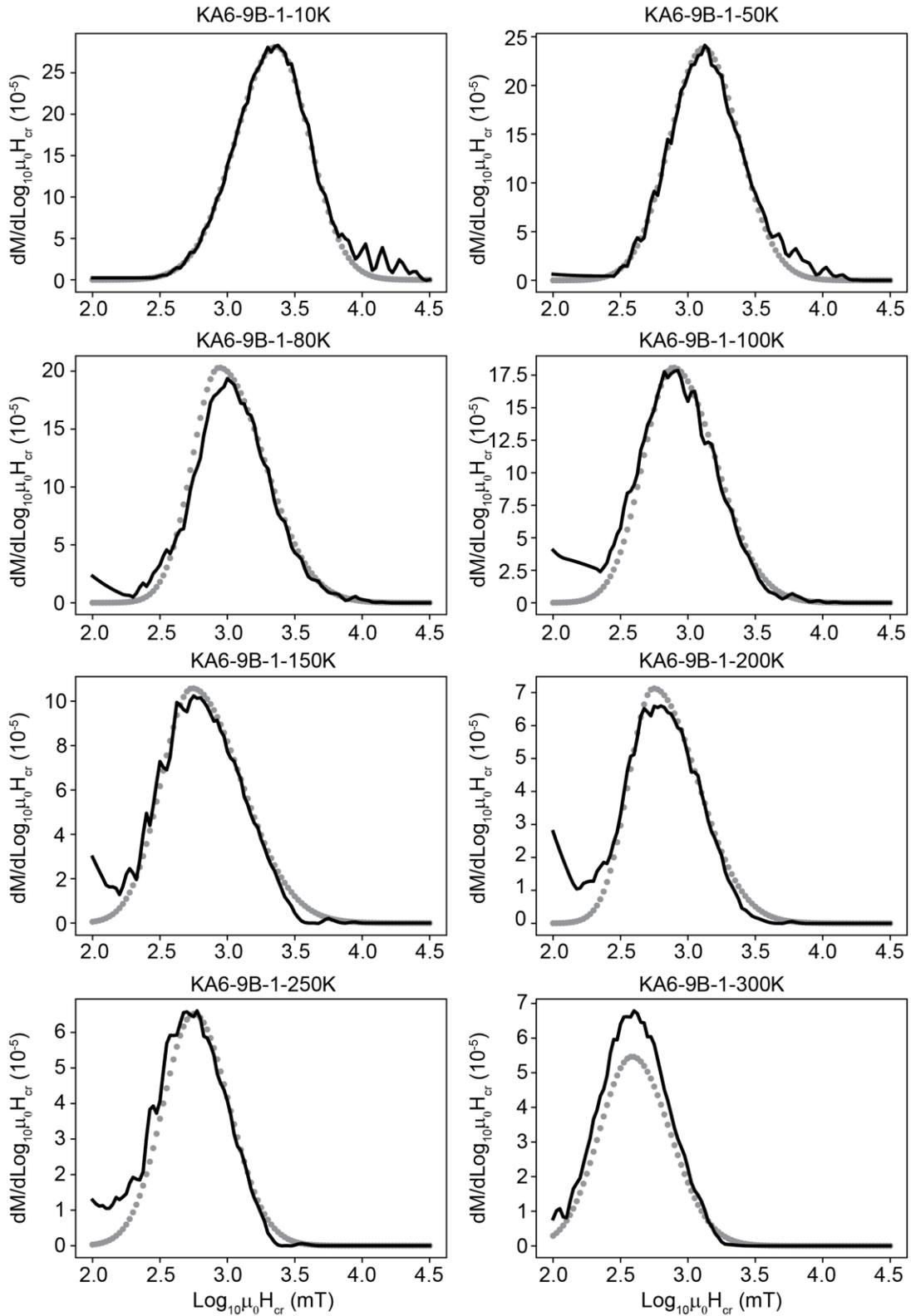
740 **Figure A2** Reconstructed (black solid lines) and measured backfield remanence curve derivatives
 741 (gray dots) for Triassic sample KA1-1B-1.



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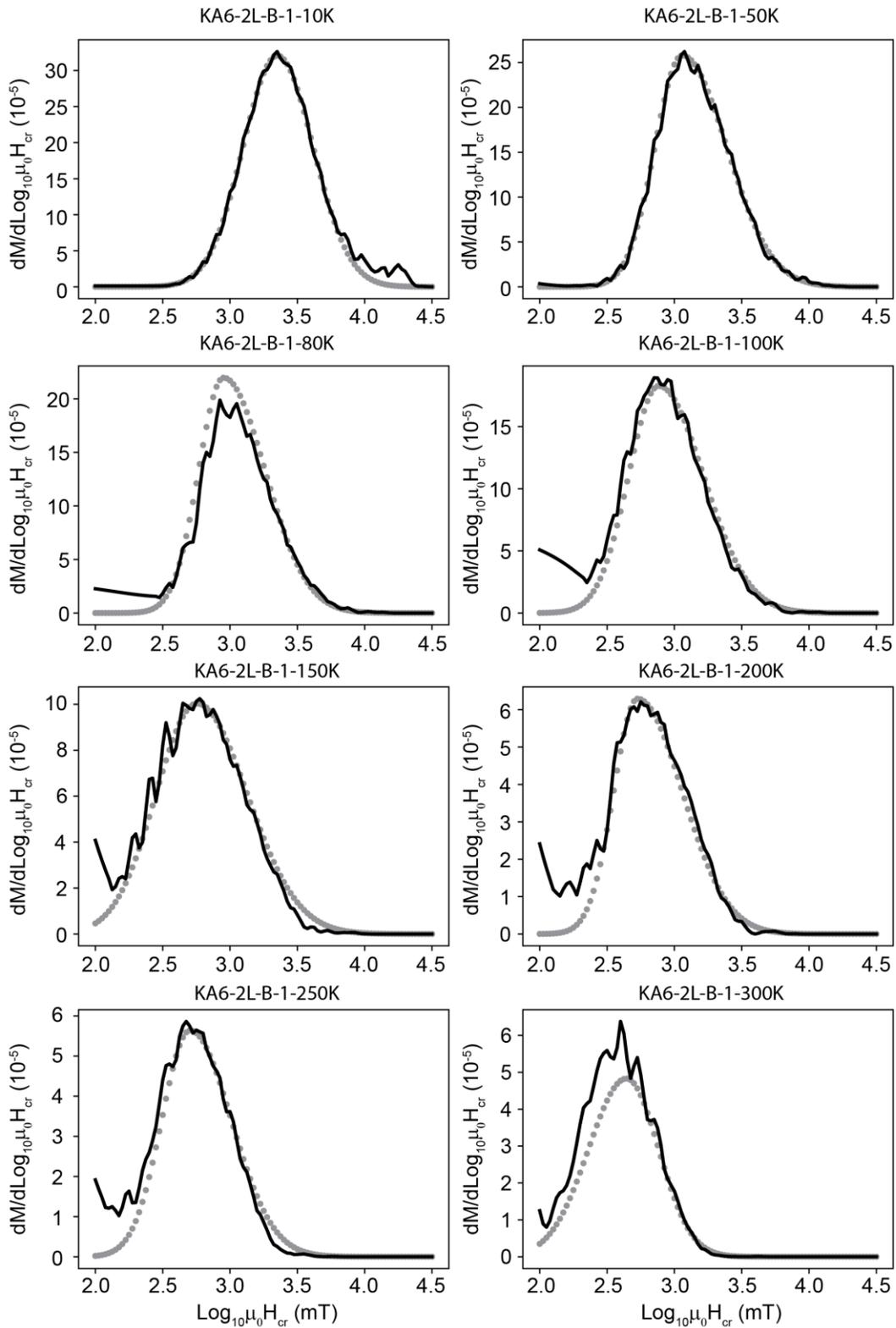
743 **Figure A3** Reconstructed (black solid lines) and measured backfield remanence curve derivatives

744 (gray dots) for Triassic sample UN2-9B-1.



745

746 **Figure A4** Reconstructed (black solid lines) and measured backfield remanence curve derivatives
 747 (gray dots) for Jurassic sample KA6-9B-1.



748

749 **Figure A5** Reconstructed (black solid lines) and measured backfield remanence curve derivatives

750 (gray dots) for Jurassic sample KA6-2L-B-1.

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