

Numerical Analysis of the Effect of Heterogeneity on CO₂ Dissolution Enhanced by Gravity Driven Convection

yufei wang¹, Daniel Fernandez Garcia², and Maarten W. Saaltink³

¹Universitat Politècnica de Catalunya

²Technical University of Catalonia

³Department of Civil and Environmental Engineering, Universitat Politècnica de Catalunya (UPC)

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Abstract

Dissolution trapping of CO₂ in brine can reduce the risk of leakage of supercritical CO₂ during long-term Geological Carbon Sequestration (GCS). The dissolution of overlying gaseous CO₂ into brine increases the density of brine in its upper portion, which causes gravity-driven convection (GDC) and thus significantly increases the rate of CO₂ dissolution. To date, most studies on GDC-driven dissolution are based on homogeneous media and only few studies exist on the effect of heterogeneity on GDC-driven dissolution. Here, we study the effect of heterogeneity on GDC-driven dissolution rate by using numerical simulations with randomly obtained permeability fields. Dissolution rates calculated by these simulations are related to properties of the permeability field by using least-squares regression. We obtained two empirical formulas for predicting the asymptotic GDC-driven dissolution rate. In the first formula the dissolution rate is almost linearly proportional to the dimensionless equivalent vertical permeability. In the second one the dissolution rate is linearly proportional to a dimensionless vertical finger-tip velocity. Both formulas show a non-linear relation between dissolution and anisotropy with higher anisotropy giving lower dissolution rates.

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29 1 Introduction

30 Geological Carbon Sequestration (GCS) has proved to be a feasible and necessary approach
 31 to mitigate CO₂ emissions [Vilarrasa *et al.*, 2013, 2014; *European Commission*, 2014; Vilarrasa
 32 and Carrera, 2015; Tutolo *et al.*, 2014, 2015a,b; Celia *et al.*, 2015]. GCS consists of sequestering
 33 CO₂, separated from other gases at large point sources (e.g., thermal power plants), into ge-
 34 ological brine formations [Vilarrasa *et al.*, 2010; Brainard, 2018; Maticic, 2018]. The injected
 35 CO₂, usually in supercritical state under reservoir condition, is expected to flow over the denser
 36 brine during the process of injection, and continuously move upwards and accumulate beneath
 37 a low permeability caprock after injection. Because the less dense CO₂ is immiscible but can dis-
 38 solve in the resident brine, the injected CO₂ is sequestered by four mechanisms: (1) by being sealed
 39 under the low permeability caprock (hydrodynamic trapping), (2) by being trapped by capillar-
 40 ity (residual trapping), (3) by dissolving into the brine at the CO₂-brine interface (dissolution trap-
 41 ping) and (4) by reacting with rock formation (mineral trapping) [Kumar *et al.*, 2005; Riaz *et al.*,
 42 2006; Bachu *et al.*, 2007; Gasda *et al.*, 2011, 2012; Macminn and Juanes, 2013; Soltanian *et al.*,
 43 2017]. Hydrodynamic trapping is the fastest mechanism but is unstable, because the relatively
 44 light CO₂ can escape from the reservoir in case of seismic activity, faults or failing wellbore cas-
 45 ings [Vilarrasa, 2012]. Mineral trapping is most stable but slow and usually negligible [Baines
 46 and Worden, 2004; Saaltink *et al.*, 2013; Sathaye *et al.*, 2014]. During the later stage of injec-
 47 tion, the safety of the GCS is governed by the dissolution trapping, which is not affected by the
 48 non-integrity (e.g., faults) of the formation [Strandli and Benson, 2013; Nicot, 2008]. Therefore,
 49 the dissolution of CO₂ into brine is an important indicator to evaluate the safety of long-term CO₂
 50 sequestration [Anbar and Akin, 2011; Xiao *et al.*, 2019].

51 The dissolution process is enhanced by Gravity-Driven Convection (GDC) [Lindeberg and
 52 Wessel-Berg, 1997; Lindeberg and Bergmo, 2003; Lindeberg and Wessel-Berg, 2011; Tsai *et al.*,
 53 2013; Sathaye *et al.*, 2014]. Dissolution of the overlying CO₂ into the brine increases the brine
 54 density in the upper portion of the saline formation creating an unstable situation with more dense
 55 on top of less dense brine. This causes nonuniform GDC in the brine phase and enhances the down-
 56 ward movement of CO₂-saturated brine [Weir *et al.*, 1996; Vella and Hupper, 2006; Pritchard,
 57 2007; Pruess and Zhang, 2008]. Moreover, the nonuniform downward flux of CO₂-saturated brine
 58 is accompanied by an upward flux of CO₂-unsaturated brine, increasing the contact between CO₂-
 59 rich phase and unsaturated brine, which further accelerates CO₂ dissolution [Pruess, 2005; Gil-
 60 fillan *et al.*, 2009; Elenius and Gasda, 2021]. In contrast to pure molecular diffusion, which quickly
 61 fades away as the concentration profile reaches a Gaussian distribution, the GDC can fuel the ver-
 62 tical mass exchange at a high constant rate for a certain long time until the whole domain is close
 63 to full CO₂ saturation [Pau *et al.*, 2010; Slim, 2014]. Because the dissolution rate enhanced by

64 GDC can significantly reduce the amount of supercritical CO₂, thus reducing the possibility of
65 leakage, a good understanding of this dissolution is essential [Elenius et al., 2015].

66 Currently, a large quantity of studies exists on GDC-driven dissolution, ranging from the-
67 oretical analysis [e.g., Elenius et al., 2012; Emami-Meybodi, 2017], laboratory experiments [e.g.,
68 Neufeld et al., 2010; Rasmusson et al., 2017; Salibindla et al., 2018], numerical simulations [e.g.,
69 Chen et al., 2013; Elenius et al., 2015] to field observations [e.g., Sathaye et al., 2014]. It is found
70 that GDC depends to a large extent on properties of the formation (e.g., the permeability and poros-
71 ity) and of the brine phase (e.g., the relation between brine density and concentration of dissolved
72 CO₂, viscosity and molecular diffusion coefficient) [Hassanzadeh et al., 2007, 2008; Neufeld et al.,
73 2010; Elenius and Johannsen, 2012; Emami-Meybodi and Hassanzadeh, 2015]. For instance,
74 researchers find a linear relation between the dissolution rate and the intrinsic permeability for
75 isotropic homogeneous media [Rasmusson et al., 2015]. However, most of the current researches
76 and conclusions are limited to (isotropic or anisotropic) homogeneous fields [e.g. Ennis-King and
77 Paterson, 2005; Pruess and Nordbotten, 2011; Cheng et al., 2012; De Paoli et al., 2017; Ran-
78 ganathan et al., 2012; Taheri et al., 2012].

79 Studies on heterogeneous fields are usually limited to qualitative analysis of the effect of
80 heterogeneity [Agartan et al., 2015; Lin et al., 2016; Soltanian et al., 2017; Kim et al., 2019; Yan
81 et al., 2019; Wang et al., 2021; Elgahawy and Azaiez, 2021]. Heterogeneity in permeability plays
82 an important role in onset, growth, maintenance and decay of the GDC and the dissolution rate
83 [Schincariol et al., 1997; Simmons et al., 2001]. Frykman and Wessel-Berg [2014] show that we
84 may overestimate the effects of instability in a real heterogeneous field simply by conducting nu-
85 merical simulations using a homogeneous field of equivalent permeability. Prasad and Simmons
86 [2003] find that while it can trigger early instability, heterogeneity can reduce the instability by
87 dissipating the density fingers through disordered high permeability pathways at a later stage.

88 Although several researches attempt to quantitatively analyze the relation between GDC
89 and field properties for heterogeneous media, only a few offer explicit formulas between the GDC-
90 driven dissolution rates and permeability heterogeneity. For instance, Farajzadeh et al. [2011]
91 and Kong and Saar [2013] conducted numerical simulations of GDC in isotropic heterogeneous
92 media, but did not offer a quantitative formula between CO₂ dissolution rates and heterogene-
93 ity measures. Therefore, conclusions from these researches cannot be directly applied to estimate
94 the dissolution rate in real reservoirs. Several researches offer quantitative formulas for predict-
95 ing dissolution rates but only consider simplified binary heterogeneous media [Elenius and Gasda,
96 2013; Green and Ennis-King, 2014] or homogeneous media with anisotropic permeability [Er-
97 fani et al., 2022]. In these researches, different conclusions are given regarding the effect of per-
98 meability anisotropy on the dissolution rate.

99 Overall, we have a solid understanding of the GDC-driven dissolution process in isotropic
100 homogeneous media, but the GDC-driven dissolution in heterogeneous media needs further study.
101 Especially, we need to quantitatively clarify the impact of the anisotropy ratio on the effective dis-
102 solution rate. Moreover, the current predictors are all based on the (equivalent) permeability, and
103 it remains unclear whether we can predict the dissolution rate based on other formation proper-
104 ties or field observations, such as the finger-tip velocity.

105 Therefore the objective of this work is to quantitatively analyze the effect of permeability
106 heterogeneity on the GDC-driven dissolution rate in a wide range of (isotropic and anisotropic)
107 heterogeneous fields with varying degrees of heterogeneity. We do this in two steps. The first step
108 consists of performing numerical simulations over a large number of heterogeneous fields of dif-
109 ferent permeability distributions. Numerical simulations are carried out by a finite-difference nu-
110 merical program developed by Wang [2022]. Permeability fields are generated with the sequen-
111 tial Gaussian simulation method implemented in the SGSIM code [Journal and Huijbregts, 1976].
112 In the second step the results of the simulations are analyzed to find relations between the GDC-
113 driven dissolution rate and heterogeneity of permeability, and we compare our results against those
114 given in literature. In this step, ordinary-least-squares linear regressions are used.

115 The rest of this paper is organized as follows. Section 2 gives a concise description of GDC.
 116 In section 3, we review existing formulas for GDC driven dissolution rates. Section 4 describes
 117 the computational approach. Section 5 gives the results and discussions. Major conclusions are
 118 listed in section 6.

119 **2 Gravity-Driven Convection**

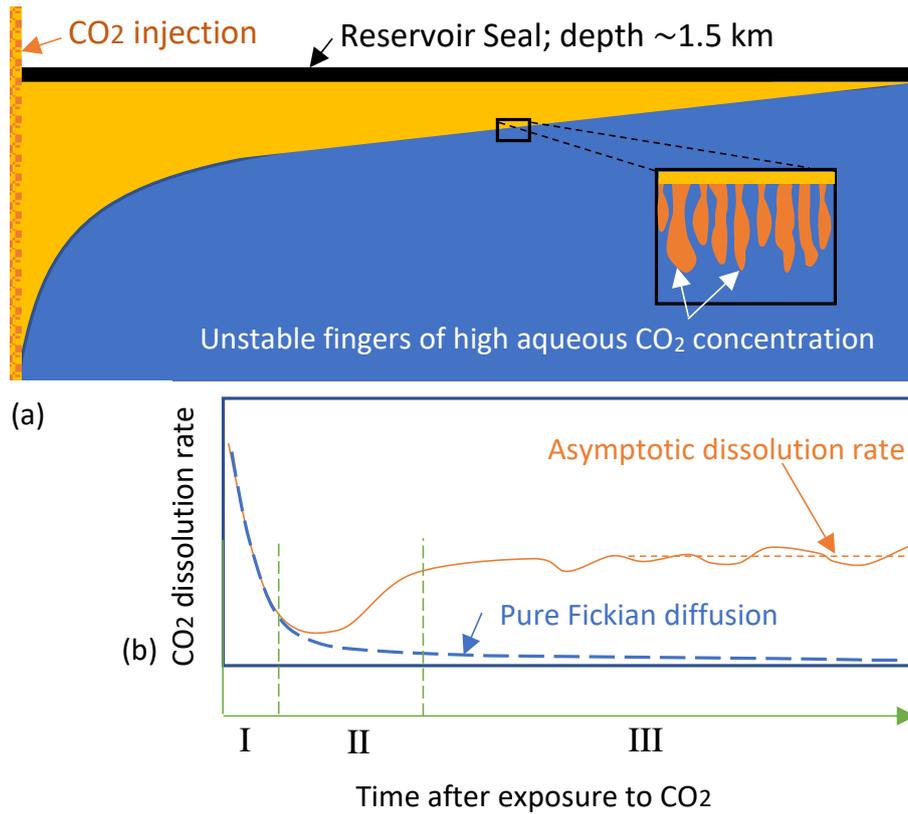


Figure 1: (a) Sketch of the appearance of unstable fingers at the interface of brine and CO₂-rich phase, and (b) enhanced dissolution rate due to vertical mass exchange driven by unstable fingers (I: diffusive regime; II: flux-growth regime; and III: constant-flux regime).

120 During the later stage of injection, the injected less dense CO₂-rich phase floats over the
 121 brine, as shown in Figure 1. This CO₂ moves upwards until it reaches a sealing caprock that traps
 122 the CO₂. The trapped CO₂ remains stored in the reservoir over extended periods, contributing
 123 to long-term carbon storage. Far from the injection site, a long quasi horizontal interface forms
 124 between the brine phase and the CO₂-rich phase. This interface represents a boundary of constant
 125 CO₂ concentration for the underlying brine, from which the overlying supercritical CO₂ dis-
 126 solves into the brine. Importantly, brine with high CO₂ concentration has ~ 1.0% higher den-
 127 sity than the underlying brine of very low CO₂ concentration, triggering Gravity-Driven Con-
 128 vection (GDC) that remarkably increases the CO₂ dissolution rate (see Figure 1). GDC contin-
 129 ues until the whole reservoir is fully saturated with CO₂, which may take a long time. The dis-

130 solution process can be divided into 4 regimes [*Slim, 2014; Tsinober et al., 2022*]. The first regime
 131 is called ‘diffusive regime’, in which the mass flux is controlled by Fickian diffusion that fades
 132 away exponentially, as can be seen from Figure 1. After this, GDC starts to develop and disso-
 133 lution switches from diffusion controlled to convection controlled. During this regime, which is
 134 called ‘flux-growth regime’, the dissolution rate starts to increase. Once the GDC is fully devel-
 135 oped, the dissolution rate reaches a quasi constant value for a relatively long time. This regime
 136 is known as ‘constant-flux regime’. The last regime (not shown in Figure 1) is referred to as ‘shut-
 137 down regime’ or flux decay regime. In this regime, the dissolution rate decreases as the reser-
 138 voir becomes fully saturated with CO₂ [*Elenius et al., 2015*].

139 In this work, we focus on the dissolution rate taking place in the ‘constant-flux regime’. This
 140 regime controls the overall behavior of the dissolution efficiency during the geological carbon
 141 sequestration [*Tsinober et al., 2022*]. In this ‘constant-flux regime’, the dissolution rate can be
 142 approximated by an asymptotic dissolution rate, as shown in Figure 1.

143 3 Review of Formulas for GDC-Driven Dissolution Rates

144 In isotropic homogeneous porous media, the asymptotic dissolution rate [kg·m⁻²·s⁻¹] can
 145 be written as [e.g., *Hesse, 2008; Pau et al., 2010; Slim, 2014*],

$$F_{\infty} = \gamma X_0^C \rho_0 \frac{\Delta \rho g \kappa}{\bar{\mu}}, \quad (1)$$

146 where γ [-] is a scaling coefficient, X_0^C [kg·kg⁻¹] and ρ_0 [kg·m⁻³] respectively represent the CO₂
 147 mass fraction in brine and the brine density at the interface of brine and CO₂-rich phase, $\Delta \rho$ [kg·m⁻³]
 148 is the density increase when aqueous CO₂ mass fraction increases from the initial very small back-
 149 ground value to X_0^C , g [m·s⁻²] is the gravitational acceleration, κ [m²] is the intrinsic permeabil-
 150 ity, and $\bar{\mu}$ [pa·s] is the mean viscosity of brine. Different researches have obtained different val-
 151 ues for the scaling coefficient γ that varies from ~ 0.015 to ~ 0.075 , and a summary of γ is given
 152 in Table 1.

153 In heterogeneous porous media, the intrinsic permeability can vary by several orders of mag-
 154 nitude [*Elenius and Johannsen, 2012; Elenius et al., 2012*]. Therefore, it is important to under-
 155 stand the effect of permeability heterogeneity on the GDC dissolution. Although there exists a
 156 large amount of studies on GDC dissolution in heterogeneous porous media [e.g. *Green and Ennis-
 157 King, 2010; Elenius and Gasda, 2013; Green and Ennis-King, 2014; Taheri et al., 2018; Mahyapour
 158 et al., 2022*], only a few researches offer concise formulas for predicting the dissolution rate. By
 159 conducting numerical simulations in homogeneous medium with embedded horizontal barriers,
 160 *Elenius and Gasda [2013]* proposed that the asymptotic dissolution rate can be estimated by

$$F_{\infty} = \gamma X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}} \left(\frac{\kappa_z^e}{\kappa_g} \right), \quad (2)$$

161 where κ_z^e is the equivalent vertical intrinsic permeability of the heterogeneous medium, and κ_g
 162 is the geometric mean of the permeability field. Essentially, these authors proposed to replace
 163 the intrinsic permeability by its equivalent quantity in heterogeneous media. Subsequent stud-
 164 ies analyze whether and how the anisotropic effect of the permeability distribution affects the GDC,
 165 but the results are inconsistent [*Xu et al., 2006; Green and Ennis-King, 2010; Cheng et al., 2012;
 166 Chen et al., 2013; Green and Ennis-King, 2014; Kim, 2014; Soltanian et al., 2017; Elgahawy and
 167 Azaiez, 2021*]. Inspired by the result for the isotropic heterogeneous field, researchers propose
 168 that the dissolution rate in the anisotropic heterogeneous field can be expressed by

$$F_{\infty} = \gamma X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}} \left(\frac{\kappa_z^e}{\kappa_g} \right) \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{\eta}, \quad (3)$$

Table 1: Published data for the scaling coefficient γ in isotropic homogeneous field.

γ	Top boundary	Method	Reference
0.015-0.017	diff. only ^[a]	Num.	<i>Pruess and Zhang</i> [2008]
0.017	diff. only	Num.	<i>Hesse</i> [2008]
0.017-0.018	diff. only	Num.	<i>Pau et al.</i> [2010]
$0.12Ra^{-0.16}$ ^[e]	-	Num.,Exp.	<i>Neufeld et al.</i> [2010]
$0.045Ra^{-0.24}$	Permeable	Exp.	<i>Backhaus et al.</i> [2011]
0.017	diff. only	Num.	<i>Cheng et al.</i> [2012]
0.02	diff. only	Num.	<i>Elenius and Johannsen</i> [2012]
0.02	diff. only	Num.	<i>Elenius et al.</i> [2012]
0.075	CTZ ^[b]	Num.	<i>Elenius et al.</i> [2012]
$0.037Ra^{-0.16}$	Permeable	Exp.	<i>Tsai et al.</i> [2013]
$0.0794Ra^{-0.168}$	diff. only	Theor.	<i>Farajzadeh et al.</i> [2013]
0.017	diff. only	Num.	<i>Slim</i> [2014]
0.025	(0.2κ) ^[c]	Num.	<i>Slim</i> [2014]
0.044	(0.6κ) ^[d]	Num.	<i>Slim</i> [2014]
0.065	CTZ	Num.	<i>Martinez and Hesse</i> [2016]
0.018-0.019	diff. only	Num.	<i>Martinez and Hesse</i> [2016]
$Ra^{-0.2154}$, 0.06	Permeable	Exp.	<i>Rasmussen et al.</i> [2017]
0.09	Permeable	Num.	This study

^[a] The top boundary only allows mass to go through the boundary via diffusion.

^[b] The top boundary is capillary transition zone.

^[c] The permeability of the top boundary is 0.2 the permeability of the domain.

^[d] The permeability of the top boundary is 0.6 the permeability of the domain.

^[e] Ra is Rayleigh number (cf. [*Neufeld et al.*, 2010]).

169 where κ_x^e is the equivalent intrinsic permeability along the horizontal direction, and the exponent
170 η describes the impact of anisotropy. The last term represents the anisotropic effect described by
171 the horizontal to vertical equivalent permeability ratio. By conducting numerical simulations of
172 GDC dissolution in homogeneous media with embedded horizontal barriers, which is similar to
173 the aforementioned method used by *Elenius and Gasda* [2013], *Green and Ennis-King* [2014]
174 found that $\eta = 0.5$, which indicates that dissolution increases with horizontal to vertical equiv-
175 alent permeability ratio. Note that when $\eta = 0.5$, the dissolution rate is actually proportional
176 to the geometric mean of the permeabilities $\sqrt{\kappa_x^e \kappa_z^e}$. In contrast, *Erfani et al.* [2022] give $\eta = 0.21$.
177 However, the results from *Soltanian et al.* [2017] and *Elgahawy and Azaiez* [2021] show that the
178 increasing κ_x^e at fixed κ_z^e can reduce the asymptotic dissolution rate, implying that η may be neg-
179 ative.

180 4 Computational Approach

181 4.1 Model Setup

182 We conduct numerical simulations of Gravity-Driven Convection (GDC) over two-dimensional
183 vertical fields of various permeability heterogeneity, which varies in space as a function of the
184 horizontal and vertical distances. We assume that the interface between the CO₂-rich phase and
185 brine is horizontal under buoyancy force, and that the brine at the interface is always saturated
186 with CO₂. We only study the portion below the interface and therefore all simulations are con-
187 ducted with a single-phase model. Initially, the CO₂ concentration in brine is very low and the
188 system is at static state. CO₂ enters into the domain through the top boundary that has fixed high
189 CO₂ mass fraction. Brine is represented by a high-concentration Sodium Chloride (NaCl) so-
190 lution. The objective is to get a quantitative relation between representative properties of the het-
191 erogeneous field and the CO₂ dissolution rate through the top boundary.

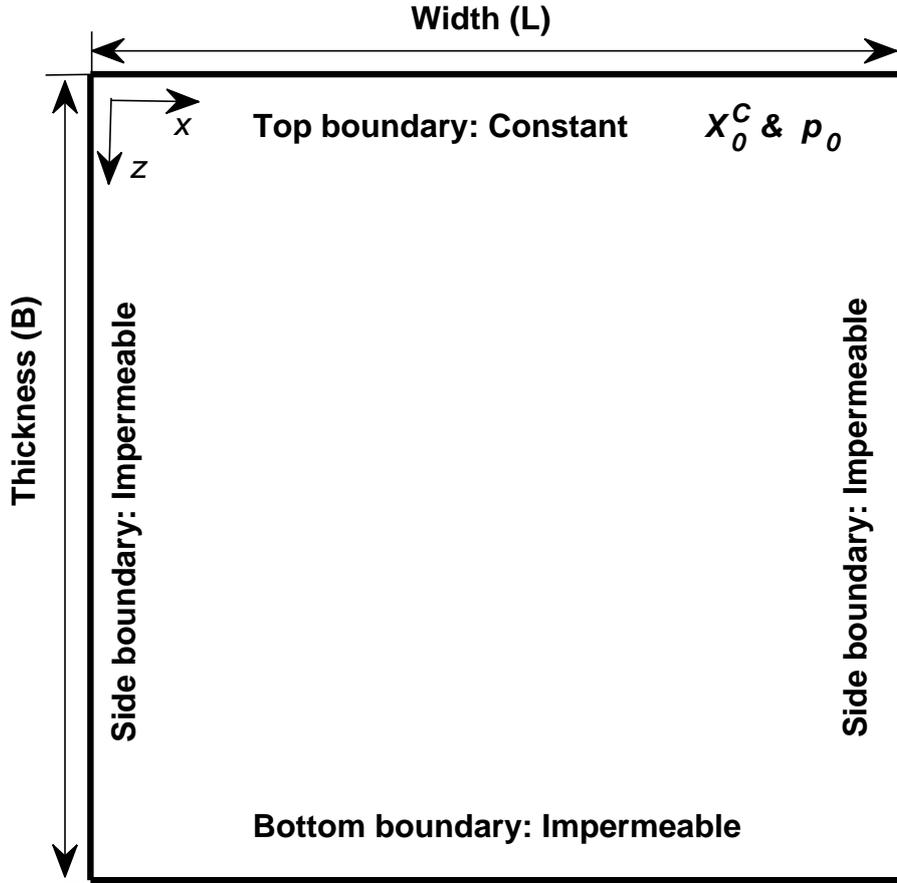


Figure 2: Sketch of setup design.

192 Figure 2 shows the two-dimensional vertical domain used to simulate the development of
 193 dissolution process enhanced by GDC. Detailed parameter settings are as follows. The top bound-
 194 ary, which represents the interface between brine and CO₂-rich phase, has constant liquid pres-
 195 sure $p_0 = 150$ [bar] and constant CO₂ mass fraction $X_0^C = 0.041$ [kg·kg⁻¹]. The bottom and
 196 lateral boundaries are no-flow boundaries. Initially, the brine has a very low background CO₂ mass
 197 fraction $X_{bg}^C \approx 0.0006$ [kg·kg⁻¹]. An initial brine pressure of $p_0 = 150$ [bar] is imposed at the
 198 top layer and increases downwards at a gradient of ρg , which means that the brine is initially at
 199 hydrostatic state. The temperature (T_c) is fixed to 60 [°C], and the salinity of brine (m^S) is con-
 200 stant 0.5 [molal]. When the mass fraction of aqueous CO₂ increases from initial X_{bg}^C to X_0^C on
 201 the top boundary, the density of brine increases by $\Delta\rho = 8.2$ [kg·m⁻³]. We note that the vis-
 202 cosity of the brine slightly changes from 1.0 [mpa·s] at the minimum background CO₂ mass frac-
 203 tion (X_{bg}^C) to 0.9 [mpa·s] at the maximum CO₂ mass fraction (X_0^C). Thus, the mean viscosity is
 204 approximated by $\bar{\mu} = 0.95$ [mpa·s]. Table 2 summarizes the parameters that are used in the nu-
 205 merical simulations.

206 We note that the top boundary condition is different from most of those used in literature
 207 where the convection of the CO₂-saturated layer is either totally or partially suppressed (see Ta-
 208 ble 1). For instance, in *Pau et al.* [2010], the top constant concentration boundary only allows
 209 CO₂ to enter the domain via diffusion; this top boundary condition generates a much lower dis-

Table 2: Summary of the parameters adopted during numerical simulations.

Parameters	Symbol	Units	Values
Domain size	$L \times B$	[m × m]	7.5×7.5
Grid discretization	$n_x \times n_z$	[-]	100 × 100
Porosity	ϕ	[-]	0.15 ^{c,f}
Geometric mean permeability	κ_g	[m ²]	10^{-12} ^{b,i}
Initial liquid pressure at top layer	p_0	[bar]	150 ^h
Initial background CO ₂ (aq) mass fraction	X_{bg}^C	[kg · kg ⁻¹]	~0.0006 ^{g,j}
CO ₂ (aq) mass fraction at top boundary	X_0^C	[kg · kg ⁻¹]	0.041
Brine density at top boundary	ρ_0	[kg · m ⁻³]	1017
Brine density increase due to increased CO ₂ (aq) mass fraction	$\Delta\rho$	[kg · m ⁻³]	8.2
Salinity	m^S	[molal]	0.5 ^{e,i}
Mean viscosity	$\bar{\mu}$	[mpa · s]	0.95
Molecular diffusion coefficient	D	[m ² · s ⁻¹]	2×10^{-9} ^{d,i}
Temperature	T_c	[°C]	60 ^{a,k}

Reference: ^a*Spycher et al.* [2003]; ^b*Chadwick et al.* [2004]; ^c*Maldal and Tappel* [2004]; ^d*Tewes and Boury* [2005]; ^e*Spycher and Pruess* [2005]; ^f*Mathieson et al.* [2009]; ^g*Xu et al.* [2007]; ^h*Iding and Ringrose* [2010]; ⁱ*Elenius and Johannsen* [2012]; ^j*Saaltink et al.* [2013]; ^k*Strandli and Benson* [2013].

210 solution rate. In some studies the top boundary is partially permeable and larger dissolution rates
 211 are obtained [*Hesse, 2008; Elenius et al., 2014; Rasmusson et al., 2015*]. Although it is more re-
 212 alistic to add a capillary transition zone beneath the top boundary [*Elenius et al., 2015*], exper-
 213 imental results with totally permeable top boundaries [*Rasmusson et al., 2017*] show only little
 214 discrepancy from the numerical results obtained in a model that includes the capillary transition
 215 zone [*Martinez and Hesse, 2016*]. Therefore, we employ the single phase flow model with per-
 216 meable top boundary in this study, although our model is capable of two-phase flow simulations
 217 [*Wang et al., 2022*].

218 An initial perturbation of the initial CO₂ mass fraction on the top boundary is added to stim-
 219 ulate the onset of instability at the beginning of simulation. We added a white noise that follows
 220 an uncorrelated Gaussian distribution [*Fu et al., 2013*]. The magnitude of the noise is 1% of the
 221 initial mass fraction. *Hidalgo and Carrera* [2009] show that instability can be generated by the
 222 numerical error without introducing any external noise. Certainly, even though larger noise strength
 223 tends to accelerate the appearance of instability fingers, it is however unlikely to change the statis-
 224 tic behavior of the dissolution rate once the instability has fully developed [*Hidalgo and Carrera,*
 225 *2009; Elenius and Johannsen, 2012*].

226 4.2 Governing Equations

227 On the basis of the mass balances of water and CO₂ species, the two governing transport
 228 equations are constructed as,

$$229 \quad 0 = \frac{\partial(\phi\rho X^H)}{\partial t} + \nabla \cdot (\rho X^H \mathbf{q}) - \nabla \cdot (\phi \mathbf{D} \rho \nabla X^H), \quad (4)$$

$$0 = \frac{\partial(\phi\rho X^C)}{\partial t} + \nabla \cdot (\rho X^C \mathbf{q}) - \nabla \cdot (\phi \mathbf{D} \rho \nabla X^C), \quad (5)$$

230 where ϕ [-] is the porosity of the saline formation, ρ [$\text{kg}\cdot\text{m}^{-3}$] represents the density of brine, X
 231 [$\text{kg}\cdot\text{kg}^{-1}$] is the mass fraction, superscripts (H, C) represent the water and aqueous CO_2 species,
 232 respectively, t [s] denotes the time, $\mathbf{D} = D\mathbf{I}_d$ [$\text{m}^2\cdot\text{s}^{-1}$] denotes the dispersion tensor, which is
 233 assumed locally constant. Local dispersion has little impact on the asymptotic dissolution rates,
 234 which is the objective of this work [Prasad and Simmons, 2003; Hidalgo and Carrera, 2009].
 235 The discharge rate (\mathbf{q}) is given by Darcy's law

$$\mathbf{q} = -\frac{\kappa}{\mu}(\nabla p - \rho g \nabla z), \quad (6)$$

236 where κ [m^2] is the intrinsic permeability, μ [$\text{pa}\cdot\text{s}$] is the viscosity, p [pa] is the liquid pressure,
 237 and z [m] is the depth. Besides, we have the following constraints:

$$X^S = 0.05844X^H m^S, \quad (7)$$

238 and

$$X^H + X^C + X^S = 1, \quad (8)$$

239 where m^S denotes the molality of salt. Here, we assume that the salt comprises only NaCl, and
 240 the molality of NaCl (m^S) is fixed. Define $\omega = (1 + 0.05844m^S)$ and then Equations (7) and
 241 (8) merge to

$$\omega X^H + X^C = 1. \quad (9)$$

242 Under isotherm and isosalinity condition, ρ and μ are only governed by liquid pressure and
 243 aqueous CO_2 mass fraction (see Appendix A).

244 4.3 Dimensionless variables

245 In order to facilitate the analysis, results are presented using the following dimensionless
 246 variables, which are defined based on the works of *Ennis-King and Paterson* [2003] and *Rasmus-*
 247 *son et al.* [2017],

$$X^{C*} = \frac{X^C}{X_0^C}, \quad X^{H*} = \frac{X^H}{X_0^H}, \quad (10)$$

248 and

$$t^* = \frac{t}{t_c}, \quad x^* = \frac{x}{L_c}, \quad z^* = \frac{z}{L_c}, \quad (11)$$

249 where X_0^C and X_0^H are, respectively, the maximum CO_2 and water mass fractions, and t_c and L_c
 250 are the characteristic time and length scale of the gravity-driven convection problem defined as

$$t_c = \frac{(\bar{\mu}\phi)^2 D}{(\Delta\rho g \kappa_g)^2}, \quad L_c = \frac{\bar{\mu}\phi D}{\Delta\rho g \kappa_g}. \quad (12)$$

251 The characteristic time t_c has been found to be closely related to the onset time of gravity-
 252 driven convection, and the characteristic length L_c closely related to the earliest finger width. In
 253 our simulations, we found that the earliest finger width, denoted as ℓ_c , can be approximated by
 254 $\ell_c \approx 70L_c$. The governing equations are expressed in dimensionless form in the Appendix B.

255 Importantly, by expressing Darcy's law and the global dissolution rate in dimensionless form, we
 256 obtain that

$$q^* = \frac{q}{q_c}, \quad q_c = \frac{\Delta \rho g \kappa_g}{\bar{\mu}}, \quad (13)$$

$$F^* = \frac{F}{F_c}, \quad F_c = X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}}. \quad (14)$$

257 The characteristic velocity $v_c = q_c / \phi$ is closely related to the finger-tip velocity (see *Ele-*
 258 *nious and Johannsen* [2012]). We note that although the vertical length scale (i.e., domain thick-
 259 ness) and related dimensionless number (e.g., Rayleigh number) have been typically used to study
 260 gravity instability in the literature [*Rasmusson et al.*, 2017, and references therein], herein we do
 261 not use it because the vertical domain size has little impact on the asymptotic enhanced disso-
 262 lution rate driven by instability fingers [*Elenius et al.*, 2015; *Tsinober et al.*, 2022]. This defini-
 263 tion of the dimensionless length scale without using the domain thickness indicates that the in-
 264 stability is a random statistic behavior that does not change with the domain size provided that
 265 the domain is large enough to accommodate sufficient number of density fingers. In the support-
 266 ing information, we have shown that increasing the vertical domain size employed in this work
 267 does not systematically affect the asymptotic dissolution rate [*Elenius and Johannsen*, 2012]. *Sim-*
 268 *mons et al.* [2001] give a detailed discussion of the limitation of using Rayleigh number based
 269 on the domain thickness.

270 4.4 Heterogeneity

271 The intrinsic permeability is the only aquifer property considered to vary in space. We rep-
 272 resent the natural logarithm of the intrinsic permeability, denoted as $Y = \ln \kappa$, by a random space
 273 function to create multiple realizations of the aquifer permeability distribution. The random space
 274 function model is characterized by an exponential covariance function with mean (\bar{Y}), variance
 275 (σ_Y^2), horizontal correlation length (λ_h) and vertical correlation length (λ_v). The geometric mean
 276 of the intrinsic permeability is fixed to $\kappa_g = 10^{-12} \text{ m}^2$. Different degrees of heterogeneity and
 277 correlation anisotropy $\Omega = \lambda_h / \lambda_v$ are explored with σ_Y^2 ranging between highly homogeneous,
 278 $\sigma_Y^2 = 0.1$, to relatively highly heterogeneous, $\sigma_Y^2 = 4$, and Ω ranging between isotropic, $\Omega = 1$, and
 279 perfectly stratified, $\Omega = \infty$. The perfectly stratified random field is formed by separate horizon-
 280 tal layers of constant properties. The vertical correlation length is fixed to $\lambda_z = 2L_c$, and the
 281 horizontal correlation length is $\lambda_x = \Omega \cdot \lambda_z$. For comparison purposes, we also considered a
 282 homogeneous medium with $\kappa = \kappa_g$. In total, we conduct GDC simulations with 365 realiza-
 283 tions. Random fields were generated using the sequential Gaussian simulation method implemented
 284 in the SGSIM code of GSLIB [*Journel and Huijbregts*, 1976; *Deutsch et al.*, 1992]. Table 3 sum-
 285 marizes the statistical properties of the random fields. An illustrative realization of each random
 286 field type is shown in Figure 4.

Table 3: Permeability heterogeneity adopted for the numerical simulations. The vertical correlation length is fixed to $\lambda_z = 2L_c$, and the horizontal correlation length is $\lambda_x = \Omega \cdot \lambda_z$.

Case	Ω	σ_Y^2
Homogeneous	-	-
Isotropic	1	1
Isotropic	1	4
Anisotropic	2	0.1
Anisotropic	2	1
Anisotropic	4	0.1
Anisotropic	4	1
Anisotropic	4	4
Anisotropic	∞	1

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4.5 Global Measures

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Simulation results are analyzed based on two global measures of the dissolution behavior. The global dissolution rate (F [$\text{kg}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$]) is defined as the rate at which CO_2 dissolves from the upper boundary at $z=0$. This can be expressed as [Hidalgo and Carrera, 2009]

$$F(t) = \frac{1}{L} \int_0^L \left[\rho X^C q_z - \phi D \rho \frac{\partial X^C}{\partial z} \right]_{z=0} dx. \quad (15)$$

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Initially, the domain is stable, the convection flux is zero, and only molecular diffusion transports downwards the dissolved CO_2 . The density-driven unstable convection does not emerge until the CO_2 mass fraction distribution reaches a critical point. After this, the dissolution rate rapidly increases to a quasi constant value until the domain is almost totally saturated with aqueous CO_2 . In our simulations, the asymptotic value of the global dissolution rate F_∞ is estimated as the temporal average of $F(t)$ over the period of $[t_b/3, t_b]$, where t_b is the time when the earliest finger of aqueous CO_2 reaches the bottom (time when the maximum bottom CO_2 mass fraction exceeds 25% of X_0^C).

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Another important global parameter that describes the vertical migration or the penetration depth of the CO_2 plume is the vertical finger-tip velocity [Prasad and Simmons, 2003]. The representative vertical finger-tip velocity of the CO_2 plume is estimated as,

$$v(t) = \max_{0 < z < B} \left\{ \frac{1}{L} \int_0^L \frac{1}{\phi} |q_z| dx \right\}. \quad (16)$$

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Figure 3 illustrates the concept of vertical finger-tip velocity in our simulations. Similar to the global dissolution rate behavior, the vertical finger-tip velocity also reaches an asymptotic value [Elenius and Johannsen, 2012] in the constant-flux regime and remains at that value until the field is almost saturated. The asymptotic vertical finger-tip velocity v_∞ is also estimated by the temporal average of $v(t)$ over the time interval $[t_b/3, t_b]$.

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In order to characterize the overall hydraulic behavior of the permeability field, we estimated the equivalent permeability along the horizontal κ_x^e and vertical κ_z^e direction for each realization of the random fields. For this, to estimate κ_i^e ($i = x, y$), we neglect gravity and saturate the porous medium with only water. We then set the domain sides perpendicular to the i th direction as impermeable, and we impose a pressure decrement $|\Delta_i p|$ along the i th direction. κ_i^e is estimated by the total volumetric flow Q_i passing through the system in the i th direction as $\kappa_i^e = \mu Q_i L_i / (A_i |\Delta_i p|)$, where L_i is the domain size along the i th direction and A_i the corresponding cross-sectional area [Knudby and Carrera, 2005; Wang, 2022].

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4.6 Numerical Features

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The model is implemented in a Matlab reservoir simulator toolbox designed for CO_2 storage [Wang, 2022]. The program is based on the finite volume method. The two governing equations (4) and (5) are solved simultaneously with an implicit Newton-Raphson method. Two-point flux approximation with up-winding scheme is used to calculate mass transport. Although the unconditional stable implicit method is employed, we should still control the time step in the numerical simulation to avoid significant numerical dispersion. In this work, the time step is limited by either advection ($\Delta t < \Delta l \phi / (\mathbf{q} \cdot \mathbf{n})$) or diffusion ($\Delta t < \sqrt{\Delta l / (2D\phi)}$), whichever is smaller. Here, Δl and \mathbf{n} are respectively the distance between the centroids of two adjacent cells and the unitary perpendicular vector to their interface. Detailed implementation of this numerical solution and related source code can be found in our previous works [Wang et al., 2022, 2023].

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The numerical discretization consists in 100×100 squared cells of equal size. The cell size is chosen to strike a balance between capturing small instability fingers and managing computational expenses without compromising accuracy following Lindeberg and Bergmo [2003] and

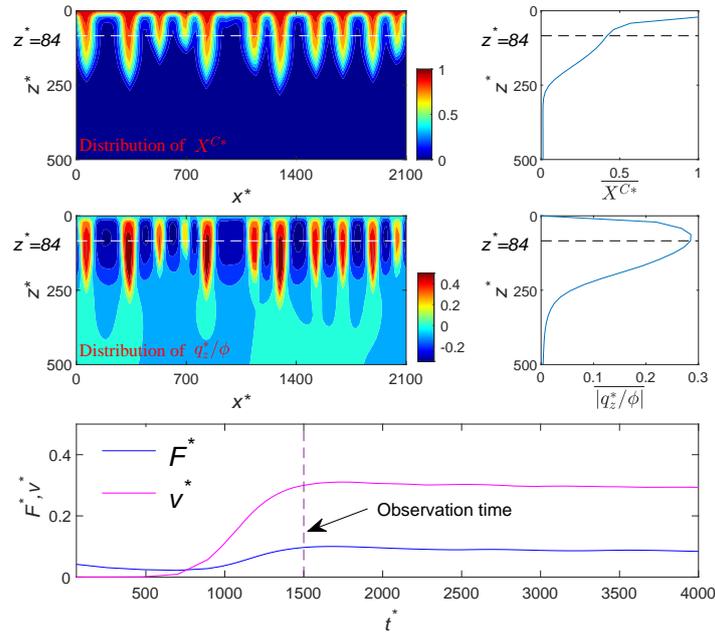


Figure 3: Maps of the dimensionless CO_2 mass fraction (X^{C*}) and its horizontal average ($\overline{X^{C*}}$), the dimensionless vertical flow velocity (q_z^*/ϕ) and the horizontal average of absolute dimensionless vertical flow velocity ($\overline{|q_z^*/\phi|}$), and temporal the development of dimensionless dissolution rate (F^*) and finger velocity (v^*) in an illustrative realization. The dimensionless finger velocity is represented by $v^* = \max |q_z^*/\phi|$.

329 *Elenius et al.* [2015]. The grid size is smaller than the finger width, $\Delta = 0.3\ell_c = 21L_c$, and at
 330 the same time the shortest correlation length is described by at least 7 grid cells. We note that
 331 the finger width ℓ_c is close to the approximate value $4\pi^2L_c$ needed for the generation of insta-
 332 bility [Lapwood, 1948].

333 4.7 Effective Asymptotic Dissolution Models

334 Two effective dissolution models are proposed here based on previous results reported in
 335 literature (see Section 3) with the objective to offer a simplified representation of the overall asymp-
 336 totic dissolution behavior in naturally occurring heterogeneous porous media. In the first effec-
 337 tive model, we have extended the formula (3) to a more general expression that incorporates the
 338 effect of anisotropy. In dimensionless form, the model expresses that

$$F_\infty^* = \gamma_1 \left(\frac{\kappa_z^e}{\kappa_g} \right)^{\alpha_1} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{\beta_1}. \quad (17)$$

339 Here, the dimensionless asymptotic dissolution rate is $F_\infty^* = F_\infty/F_c$. The last term on the right
 340 hand side of this expression represents the anisotropy of the permeability field, defined as $a_f =$
 341 κ_x^e/κ_z^e .

342 Existing effective asymptotic dissolution models rely exclusively on equivalent permeabil-
 343 ity values. Thus, it remains uncertain whether predictions of the dissolution rate can be made us-
 344 ing field observations, such as the finger-tip velocity v_∞ . For this, in the second model, we have
 345 considered the following relationship written in dimensionless form as

$$F_\infty^* = \gamma_2 \left(\frac{v_\infty}{v_c} \right)^{\alpha_2} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{\beta_2}. \quad (18)$$

346 Essentially, this expression replaces the equivalent vertical permeability with the vertical finger
 347 velocity, which seems to offer a more direct description of the CO₂ plume migration. $\gamma_1, \gamma_2, \alpha_1,$
 348 α_2, β_1 and β_2 are tuning coefficients of the two effective asymptotic dissolution models.

349 5 Results and Discussion

350 5.1 Impact of Heterogeneity

351 We first provide a general description of the impact of heterogeneity on the vertical migra-
 352 tion of the CO₂ plume and dissolution rates. We focused on the influence of anisotropy in the cor-
 353 relation structure of permeability Ω and the degree of heterogeneity σ_Y^2 . Figure 4 shows the tem-
 354 poral evolution of the spatial distribution of CO₂ mass fraction for different types of heterogene-
 355 ity. For illustrative purposes, we chose a representative permeability realization for each case.
 356 These realizations are shown in the first panel of Figure 4. In all cases, results show that insta-
 357 bility CO₂ fingers grow, merge and re-initiate as also observed in laboratory experiments [Ras-
 358 musson *et al.*, 2017; Liyanage, 2018; Tsinober *et al.*, 2022] and numerical simulations [Elenius
 359 *et al.*, 2015]. Importantly, from Figure 4, we can see that CO₂ fingering is strongly affected by
 360 heterogeneity. In particular, the presence of vertical well-connected high permeability zones (pref-
 361 erential channels) facilitates the initiation and growth of the instability fingers (see for instance
 362 the second column of Figure 4). Actually, the white randomness of the top CO₂ mass fraction
 363 (needed in homogeneous media to create instabilities) is redundant in heterogeneous porous me-
 364 dia as instabilities are controlled by these vertical preferential channels. In accordance with *Sim-*
 365 *mons et al.* [2001], we also see that heterogeneity dissipates vertical finger growth through dis-
 366 persive mixing. This effect increases with Ω , i.e., when horizontal well-connected high-permeability
 367 structures exist. This is strongly manifested in perfectly stratified media with $\sigma_Y^2=1$ and $\Omega=\infty$ (fourth
 368 column of Figure 4).

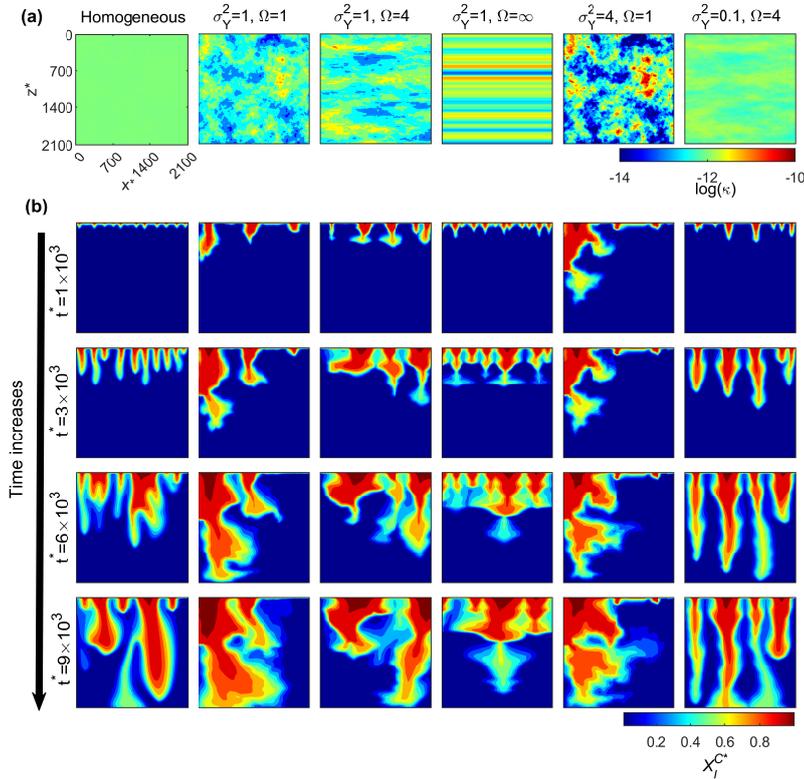


Figure 4: (a) Logarithm permeability ($\log(\kappa)$) distribution. (b) Development of dimensionless CO_2 mass fraction distribution (X_l^{C*}) due to gravity-driven convection.

369 For completeness, we also show the temporal evolution of the dissolution rate as a func-
 370 tion of σ_Y^2 and Ω in Figure 5. Results are presented in terms of the ensemble average and the co-
 371 efficient of variation of $F^*(t^*)$. As expected, in homogeneous media, the dissolution process, shows
 372 the three well-known stages: an initial diffusion-controlled decrease, followed by an onset of non-
 373 linear growth at $t^* = 500$ due to instability, and eventually stabilizing at an approximately constant
 374 dissolution rate. However, in heterogeneous media, results show in all cases that the early-time
 375 evolution of $F^*(t^*)$ in heterogeneous media is remarkably different than that in homogeneous
 376 media. The diffusive and the flux-growth regimes cannot be distinguished anymore and the sys-
 377 tem seems to be controlled by the interaction between gravity-driven convection and heterogene-
 378 ity, indicating that heterogeneity helps triggering the onset of instability. This is consistent with
 379 results reported by *Schincariol et al.* [1997] and *Simmons et al.* [2001]. Of course, this also in-
 380 dicates that caution is needed when using the onset time of instability for homogeneous media
 381 [*Ennis-King and Paterson, 2005; Pruess and Zhang, 2008*] in real applications.

382 The influence of σ_Y^2 and Ω can also be seen from Figure 5. Interestingly, in statistically isotropic
 383 heterogeneous media, the degree of heterogeneity σ_Y^2 significantly influences the early behavior
 384 of $F^*(t^*)$, eventually converging to a similar asymptotic dissolution rate at large times. This sug-
 385 gests that the asymptotic dissolution rate might be governed by the existence of well-connected
 386 high-permeability zones, regardless of the specific high value of permeability. This effect is not
 387 seen in anisotropic heterogeneous media where we found that the higher the σ_Y^2 the lower is the

388 asymptotic dissolution rate. We attribute this to the fact that, when $\Omega > 1$, an increase in σ_Y^2
 389 produces stronger well-connected permeability layers that inhibits gravity-driven convection. For
 390 the same reason, for equal σ_Y^2 , the higher anisotropy Ω the less significant is the asymptotic dis-
 391 solution rate. We also report in this figure a measure of the uncertainty in $F^*(t^*)$ given by the
 392 coefficient of variation (CV). We can observe that the coefficient of variation reaches a similar
 393 asymptotic value for all cases, regardless of the degree of heterogeneity. A similar trend is also
 394 observed for the vertical finger velocity, as shown in the second column of Figure 5, indicating
 395 a close relation between the dissolution rate and the vertical finger velocity.

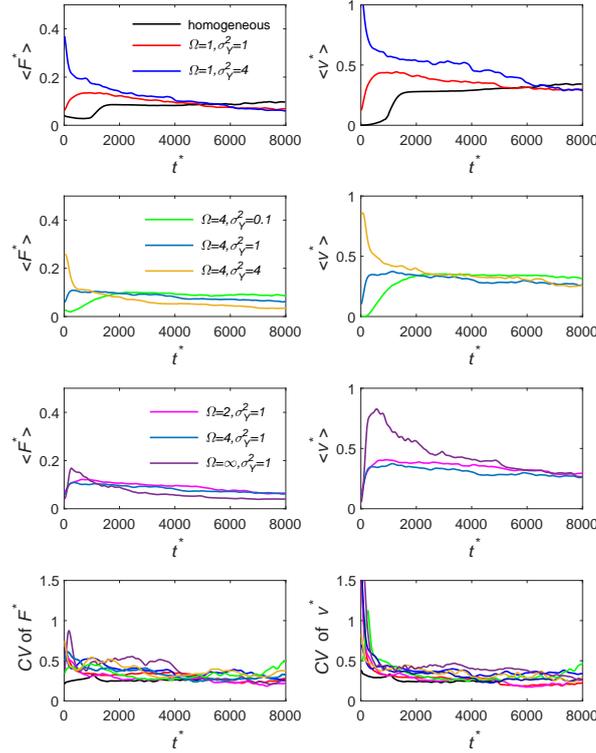


Figure 5: The temporal evolution of the ensemble average of the dimensionless dissolution rate ($\langle F^* \rangle$) through the top boundary and ensemble average of the dimensionless finger velocity ($\langle v^* \rangle$) for all the test cases listed in Table 3. Here, we also give the coefficient of variation (CV), which is the ratio of standard deviation to the ensemble average.

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5.2 The Effective Asymptotic Dissolution Rate

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398 The tuning coefficients of the two effective asymptotic dissolution models were independ-
 399 ently adjusted by regression analysis of all simulation data obtained from the 365 realizations
 400 of the permeability distributions. These realizations involved random fields with different cor-
 401 relation structures of permeability and degrees of heterogeneity. To do this, we used Ordinary
 402 Least Squares (OLS) regression of the natural logarithm of the dissolution rate models (17) and
 403 (18). We respectively obtained a coefficient of determination R^2 of 0.71 and 0.84. The signif-
 404 icance of all regression coefficients was below 0.05, meaning that both models can properly ex-
 plain dissolution rates. Table 4 provides a summary of the regression analysis. Similar to the val-

ues reported in the literature, we find that $\alpha_1 = 1.1$, which is close to unity, meaning that the effective dissolution efficiency is almost linearly proportional to the equivalent vertical permeability. Moreover, we find that, in anisotropic heterogeneous media, the anisotropy of the equivalent permeability ($a_f = \kappa_x^e/\kappa_z^e$) can reduce the effectiveness of dissolution with a power law behavior given by $\beta_1 = -0.2$. This contradicts previous results obtained in homogeneous media with embedded horizontal impermeable inclusions [Green and Ennis-King, 2014; Erfani et al., 2022], which indicated that $\beta_1 > 0$.

Table 4: Coefficients for effective asymptotic dissolution models obtained from OLS regression of all simulation data; the results from literature are also listed for comparison.

Predictors	Coefficients	Values	MAE	RMSE	Source
(17)	$(\gamma_1, \alpha_1, \beta_1)$	(0.08, 1.1, -0.2)	27%	22%	This work
(18)	$(\gamma_2, \alpha_2, \beta_2)$	(0.33, 1.0, -0.3)	20%	19%	This work
(17)	$(\gamma_1, \alpha_1, \beta_1)$	(0.09*, 1.0, 0)	40%	24%	<i>Elenius and Gasda</i> [2013]
(17)	$(\gamma_1, \alpha_1, \beta_1)$	(0.09*, 1.0, 0.5)	76%	44%	<i>Green and Ennis-King</i> [2014]

* Here, we update the value for the reference dissolution rate in the homogeneous case, because the original value was around 0.02 based on the impermeable top boundary (cf. Table 1).

Figure 6 compares the asymptotic dissolution rate predicted by the proposed asymptotic dissolution rate models, expressions (17) and (18), with corresponding simulated values. For completeness, we also show the performance of the reported expressions given by *Elenius and Gasda* [2013] and *Green and Ennis-King* [2014]. We visually differentiate between isotropic and anisotropic random fields. We can see that the predictor given by *Green and Ennis-King* [2014] significantly overestimates the dissolution rate in anisotropic random fields. Actually, the expression by *Green and Ennis-King* [2014] does not seem to improve the prediction given by *Elenius and Gasda* [2013]. The second effective dissolution model given by equation (18), proposed here based on the vertical finger-tip velocity, shows the best performance, indicating that the dissolution rate has a strong relationship with the finger-tip velocity. The Mean Absolute Error (MAE) for formulas (17) and (18) are 27% and 20%, respectively. These error are well accepted considering that even in homogeneous fields, the dissolution rate can fluctuate around 15% [*Pau et al.*, 2010].

In this work, we found a negative impact of the anisotropy of the permeability field a_f on dissolution rates. This can be physically explained in the following manner. Instabilities in the form of fingers exhibit nonuniform periodic high concentration distributions along the horizontal direction. When the spatial continuity of permeability values in the horizontal plane is substantial, any nonuniform concentration in this direction is rapidly eradicated by the enhancement of horizontal mixing induced by the introduction of companion horizontal flows. Consequently, the formation of finger-like instabilities becomes more challenging, especially in scenarios with high horizontal permeability values. In accordance, *Schincariol et al.* [1997] show that increasing the horizontal correlation length of the permeability distribution (increase in κ_x^e), can effectively inhibit instability growth and stabilize perturbations. Through numerical simulations in a wide variety of heterogeneous fields, *Simmons et al.* [2001], *Soltanian et al.* [2017] and *Elgahawy and Azaiez* [2021] also conclude that instability is dampened when κ_x^e/κ_z^e is relatively large. Recent studies by *Tsinober et al.* [2022] and *Hansen et al.* [2023] have also highlighted that the introduction of a minor horizontal background flow in geological carbon sequestration fields enhances horizontal mixing. This enhancement of mixing disrupts the growth of fingers, consequently leading to a reduction in the dissolution rate. All these works also suggest that horizontal flows have the potential to decrease nonuniform instabilities. To better illustrate this, Figure 7 shows simulated and estimated effective dissolution rates as a function aquifer properties $\{\kappa_z^e, \kappa_x^e/\kappa_z^e, \text{ and } v_\infty^*\}$. The figure shows a clear negative dependence of the asymptotic dissolution rates with permeability anisotropy.

444 We acknowledge that our numerical simulations have not covered cases with $a_f < 1$, which
 445 are uncommon in natural sediment formations. Studies conducted by *Simmons et al.* [2001] have
 446 demonstrated that vertically stratified structures with $a_f < 1$ can encourage vertical unstable
 447 convection by diminishing horizontal dissipation in instability fingers. Thus, it is observed that
 448 instability becomes more pronounced when a_f is small, aligning with the findings of our study.

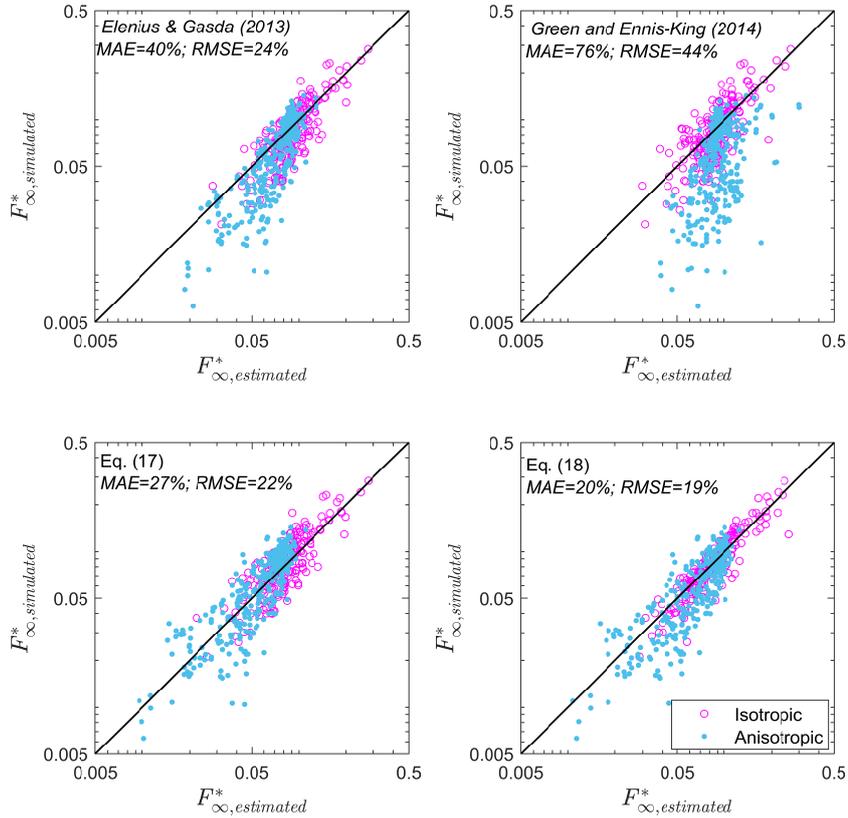


Figure 6: Comparison of the performances of the predictors given by *Elenius and Gasda* [2013], *Green and Ennis-King* [2014] and this work (cf. Table 4).

449 6 Conclusions

450 Geological Carbon Sequestration (GCS) in saline aquifers reduces the release of CO_2 into
 451 the atmosphere, thereby mitigating its impact on climate change. Once CO_2 is injected, the less
 452 dense CO_2 -rich phase floats over the brine and gets trapped beneath an impermeable geological
 453 formation. At the interface between the brine phase and the CO_2 -rich phase, CO_2 slowly dissolves
 454 into the brine, thereby reducing the risk of CO_2 leakage. Estimating CO_2 dissolution rates in this
 455 context is complex, as it requires characterizing the enhancement of dissolution due to Gravity-
 456 Driven Convection (GDC). GDC creates instability fingers that transfer the high CO_2 concen-
 457 tration brine downwards. While many studies offer a deep understanding of GDC in homogeneous
 458 porous media, less is known about dissolution rates in more realistic heterogeneous porous me-

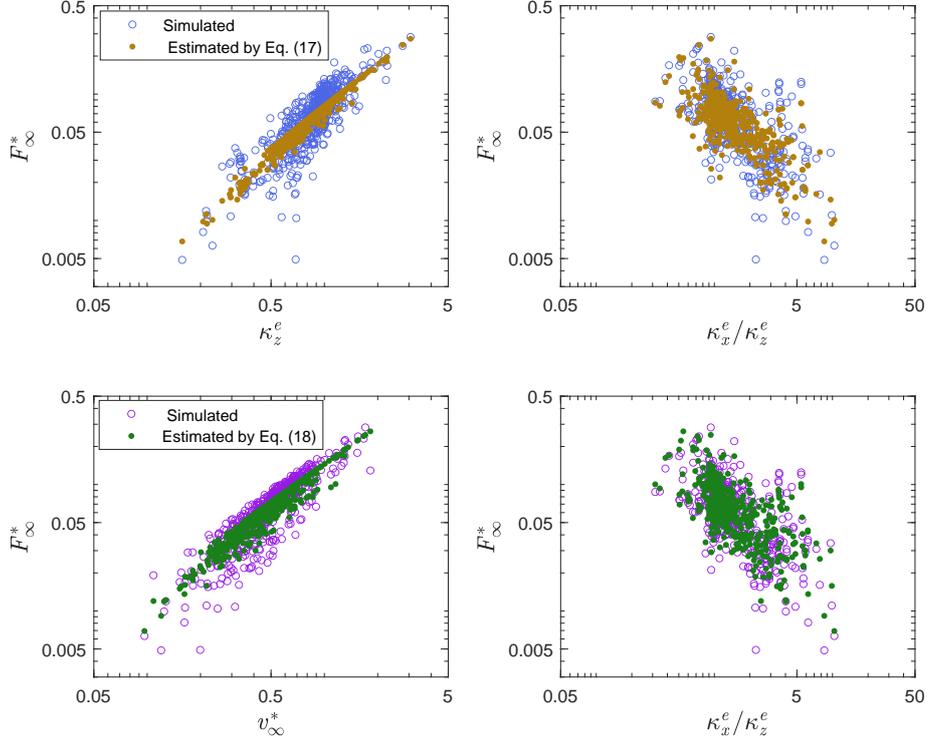


Figure 7: Simulated and estimated effective dissolution rates as a function aquifer properties.

459 dia. In this work, we have systematically analyzed the effect of heterogeneity on GDC-driven dis-
 460 solution rates during GCS. To achieve this, we have conducted numerical simulations of GDC
 461 in multiple aquifer realizations of various permeability distributions. These distributions follow
 462 a random space function that exhibits distinct correlation structures (anisotropy) and degrees of
 463 heterogeneity. Based on these simulations, we have developed two effective asymptotic disso-
 464 lution rate models derived from regression analysis of all the simulated data. The first model es-
 465 timates the asymptotic dissolution rate using the aquifer’s general hydraulic properties. For this
 466 model, we achieved a coefficient of determination (R^2) of 0.71, indicating a strong correlation
 467 between the variables compared to previous effective dissolution models [Elenius and Gasda,
 468 2013; Green and Ennis-King, 2014],

$$F_{\infty} = 0.08 X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}} \left(\frac{\kappa_z^e}{\kappa_g} \right)^{1.1} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{-0.2}. \quad (19)$$

469 This model requires some knowledge of the equivalent permeability value along the x and z di-
 470 rections $\{\kappa_x^e, \kappa_z^e\}$ and fluid properties. The equivalent permeability can be estimated by a wide
 471 variety of methods, including hydraulic tests (see Renard and de Marsily [1997]; Sanchez-Vila
 472 *et al.* [2006] for a review). Alternatively, direct observations of the CO_2 finger-tip velocity v can
 473 be used to estimate asymptotic dissolution rates. This velocity refers to the rate at which the fin-
 474 gers or plumes of dissolved CO_2 move downwards through the subsurface. Determining the finger-

tip velocity of CO₂ in the field involves the use of tracer tests, or geophysical or wellbore monitoring [Würdemann *et al.*, 2010; Jenkins *et al.*, 2015; Michel-Meyer *et al.*, 2020, and references therein]. Results have demonstrated that the finger-tip velocity offers a better estimate of dissolution rates with a coefficient of determination (R^2) of 0.84,

$$F_{\infty}^* = 0.33 X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}} \left(\frac{v_{\infty}}{v_c} \right)^{1.0} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{-0.3}. \quad (20)$$

These effective asymptotic dissolution rate models express that the anisotropy of the permeability field (last term of the expressions) negatively affects dissolution rates. When permeability values have substantial spatial continuity horizontally, the corresponding increase in horizontal mixing inhibits nonuniform concentrations, making it harder for instabilities to form. These results differ from those presented by Green and Ennis-King [2014] in homogeneous media with horizontal barriers, where the dissolution rate is proposed to be enhanced by anisotropy with an exponent of 0.5. In accordance with our work, Soltanian *et al.* [2017] and Elgahawy and Azaiez [2021] demonstrated that increasing κ_x^e at fixed κ_z^e can reduce the asymptotic dissolution rate.

We have also explored the impact of heterogeneity on the temporal evolution of dissolution rates. We have shown that in heterogeneous porous media, vertical preferential channels, formed by well-connected high-permeability zones, play a significant role in initiating and developing instability fingers. Moreover, the presence of horizontal well-connected high-permeability structures inhibits the vertical growth of fingers by favoring dispersive mixing. Consequently, in anisotropic heterogeneous porous media, an increase in the degree of heterogeneity leads to a decrease in the asymptotic dissolution rate.

We note that the results from this study may have potential application to other common gravity-driven convection problems, such as contaminant migration, geothermal exploitation, salt-water intrusion and mineral precipitation/dissolution, where density differences may exist [Zhang and Schwartz, 1995; Simmons *et al.*, 1999; Nield *et al.*, 2008].

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A Density and Viscosity

In the numerical model, instead of using a simplified linear expression of the brine density based on the aqueous CO₂ concentration [e.g., Elenius *et al.*, 2015; Martinez and Hesse, 2016], we incorporated a more realistic brine density expression derived from [Vilarrasa, 2012], given the sensitivity of instability to the fluid property [Jafari Raad *et al.*, 2016; Rasmusson *et al.*, 2015, 2017]. In this model, the brine density, ρ [kg·m⁻³], depends on brine phase pressure, temperature, molality of NaCl and CO₂ concentration. The expression for brine is given by Garcia [2003]

$$\rho = \rho_{lr} + cM^C - c\rho_{lr}V_{\phi}, \quad (A.1)$$

516 where c [$\text{mol}\cdot\text{m}^{-3}$] is the number of moles of CO_2 per unit volume of brine phase; M^C [$\text{kg}\cdot\text{mol}^{-1}$]
 517 is the molar mass of CO_2 ; and V_ϕ [$\text{m}^3\cdot\text{mol}^{-1}$] is the apparent molar volume of CO_2 given by

$$V_\phi = 3.751 \times 10^{-5} - 9.585 \times 10^{-8}T_c + 8.74 \times 10^{-10}T_c^2 - 5.044 \times 10^{-13}T_c^3, \quad (\text{A.2})$$

518 where T_c [$^\circ\text{C}$] is temperature in Celsius; ρ_{lr} is the brine density when there is no CO_2 dissolu-
 519 tion, calculated by *Phillips et al.* [1982]

$$\rho_{lr} = -3.033405 \times 10^3 + 1.0128163 \times 10^4\iota - 8.750567 \times 10^3\iota^2 + 2.66310 \times 10^3\iota^3, \quad (\text{A.3})$$

520 with

$$\begin{aligned} \iota = & -9.9595 \exp(-4.539 \times 10^{-3}m^S) + 7.0845 \exp(-1.638 \times 10^{-4}T_c) \\ & + 3.9093 \exp(2.551 \times 10^{-10}p), \end{aligned} \quad (\text{A.4})$$

521 where m^S [molal] is the molality of NaCl and p [pa] is the pressure of brine. Equation (A.1) ap-
 522 plies to $5 < T_c < 297$ [$^\circ\text{C}$] and $p_{sv} < p < 300$ [bar]. Equation (A.3) applies to $10 < T_c <$
 523 350 $^\circ\text{C}$, $0.25 < m_l^S < 5$ [molal] and $p_{sv} < p < 500$ [bar] [*Phillips et al.*, 1982]. Here, p_{sv} is
 524 saturated vapor pressure. Rearranging Equation (A.1), we have [*Vilarrasa*, 2012]

$$\rho = \rho_{lr} \frac{1}{1 - X^C f_\delta} \approx \rho_{lr} (1 + X^C f_\delta), \quad (\text{A.5})$$

525 with

$$f_\delta = 1 - \rho_{lr} \frac{V_\phi}{M^C}; \quad (\text{A.6})$$

526 here X^C denotes the mass fraction of CO_2 . The viscosity of brine is calculated following the works
 527 of *Garcia* [2003] and *Kumagai and Yokoyama* [1999]

$$\begin{aligned} \mu = & (3.85971 - 1.32561 \times 10^{-2}T_k)m^S + (-5.37539 + 1.90621 \times 10^{-2}T_k)(m^S)^{1/2} \\ & + (8.79552 - 3.17229 \times 10^{-2}T_k)m^C + (-7.22796 + 2.64498 \times 10^{-2}T_k)(m^C)^2 \\ & + 1.69956 \times 10^{-9}(p - 1 \times 10^5) + \mu_w(T_k, p = 10^5[\text{Pa}]), \end{aligned} \quad (\text{A.7})$$

528 where T_k [K] is temperature in Kelvin, m^C [molal] is the molality of CO_2 , and μ_w [mPa·s] is
 529 the viscosity of pure water.

530 B Dimensionless Governing Equations

531 Given the dimensionless variables defined in section 4.2, the governing mass balance equa-
 532 tions (4) and (5) can be written in dimensionless form as

$$\frac{\partial X^{C*}}{\partial t^*} = -\nabla^* \cdot (X^{C*} \mathbf{q}^*) + \nabla^* \cdot (\nabla^* X^{C*}), \quad (\text{B.1})$$

$$\frac{\partial X^{H*}}{\partial t^*} = -\nabla^* \cdot (X^{H*} \mathbf{q}^*) + \nabla^* \cdot (\nabla^* X^{H*}), \quad (\text{B.2})$$

533 where Darcy's law is expressed as

$$\mathbf{q}^* = -\exp(Y') (\nabla^* p^* - \rho^* \nabla^* z^*), \quad (\text{B.3})$$

534 and $\nabla^* = [\partial/\partial x^*, \partial/\partial z^*]$. Y' is the deviation of the natural log of the intrinsic permeability from
 535 the mean, i.e., $Y' = Y - \langle Y \rangle$. The geometric mean permeability is $\kappa_g = \exp(\langle Y \rangle)$. The fluid
 536 pressure and the density are normalized by

$$p^* = \frac{p\kappa_g}{\mu\phi D}, \text{ and } \rho^* = \frac{\rho}{\Delta\rho}. \quad (\text{B.4})$$

537 Open Research

538 This work has no data-sharing issues, because all of the results are obtained by numerical
 539 solutions and have been provided in the figures and tables.

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14 Abstract

15 Dissolution trapping of CO₂ in brine can reduce the risk of leakage of supercritical CO₂ during
 16 long-term Geological Carbon Sequestration (GCS). The dissolution of overlying gaseous CO₂
 17 into brine increases the density of brine in its upper portion, which causes gravity-driven con-
 18 vection (GDC) and thus significantly increases the rate of CO₂ dissolution. To date, most stud-
 19 ies on GDC-driven dissolution are based on homogeneous media and only few studies exist on
 20 the effect of heterogeneity on GDC-driven dissolution. Here, we study the effect of heterogene-
 21 ity on GDC-driven dissolution rate by using numerical simulations with randomly obtained per-
 22 meability fields. Dissolution rates calculated by these simulations are related to properties of the
 23 permeability field by using least-squares regression. We obtained two empirical formulas for pre-
 24 dicting the asymptotic GDC-driven dissolution rate. In the first formula the dissolution rate is
 25 almost linearly proportional to the dimensionless equivalent vertical permeability. In the second
 26 one the dissolution rate is linearly proportional to a dimensionless vertical finger-tip velocity. Both
 27 formulas show a non-linear relation between dissolution and anisotropy with higher anisotropy
 28 giving lower dissolution rates.

29 1 Introduction

30 Geological Carbon Sequestration (GCS) has proved to be a feasible and necessary approach
 31 to mitigate CO₂ emissions [Vilarrasa *et al.*, 2013, 2014; *European Commission*, 2014; Vilarrasa
 32 and Carrera, 2015; Tutolo *et al.*, 2014, 2015a,b; Celia *et al.*, 2015]. GCS consists of sequestering
 33 CO₂, separated from other gases at large point sources (e.g., thermal power plants), into ge-
 34 ological brine formations [Vilarrasa *et al.*, 2010; Brainard, 2018; Maticic, 2018]. The injected
 35 CO₂, usually in supercritical state under reservoir condition, is expected to flow over the denser
 36 brine during the process of injection, and continuously move upwards and accumulate beneath
 37 a low permeability caprock after injection. Because the less dense CO₂ is immiscible but can dis-
 38 solve in the resident brine, the injected CO₂ is sequestered by four mechanisms: (1) by being sealed
 39 under the low permeability caprock (hydrodynamic trapping), (2) by being trapped by capillar-
 40 ity (residual trapping), (3) by dissolving into the brine at the CO₂-brine interface (dissolution trap-
 41 ping) and (4) by reacting with rock formation (mineral trapping) [Kumar *et al.*, 2005; Riaz *et al.*,
 42 2006; Bachu *et al.*, 2007; Gasda *et al.*, 2011, 2012; Macminn and Juanes, 2013; Soltanian *et al.*,
 43 2017]. Hydrodynamic trapping is the fastest mechanism but is unstable, because the relatively
 44 light CO₂ can escape from the reservoir in case of seismic activity, faults or failing wellbore cas-
 45 ings [Vilarrasa, 2012]. Mineral trapping is most stable but slow and usually negligible [Baines
 46 and Worden, 2004; Saaltink *et al.*, 2013; Sathaye *et al.*, 2014]. During the later stage of injec-
 47 tion, the safety of the GCS is governed by the dissolution trapping, which is not affected by the
 48 non-integrity (e.g., faults) of the formation [Strandli and Benson, 2013; Nicot, 2008]. Therefore,
 49 the dissolution of CO₂ into brine is an important indicator to evaluate the safety of long-term CO₂
 50 sequestration [Anbar and Akin, 2011; Xiao *et al.*, 2019].

51 The dissolution process is enhanced by Gravity-Driven Convection (GDC) [Lindeberg and
 52 Wessel-Berg, 1997; Lindeberg and Bergmo, 2003; Lindeberg and Wessel-Berg, 2011; Tsai *et al.*,
 53 2013; Sathaye *et al.*, 2014]. Dissolution of the overlying CO₂ into the brine increases the brine
 54 density in the upper portion of the saline formation creating an unstable situation with more dense
 55 on top of less dense brine. This causes nonuniform GDC in the brine phase and enhances the down-
 56 ward movement of CO₂-saturated brine [Weir *et al.*, 1996; Vella and Hupper, 2006; Pritchard,
 57 2007; Pruess and Zhang, 2008]. Moreover, the nonuniform downward flux of CO₂-saturated brine
 58 is accompanied by an upward flux of CO₂-unsaturated brine, increasing the contact between CO₂-
 59 rich phase and unsaturated brine, which further accelerates CO₂ dissolution [Pruess, 2005; Gil-
 60 fillan *et al.*, 2009; Elenius and Gasda, 2021]. In contrast to pure molecular diffusion, which quickly
 61 fades away as the concentration profile reaches a Gaussian distribution, the GDC can fuel the ver-
 62 tical mass exchange at a high constant rate for a certain long time until the whole domain is close
 63 to full CO₂ saturation [Pau *et al.*, 2010; Slim, 2014]. Because the dissolution rate enhanced by

64 GDC can significantly reduce the amount of supercritical CO₂, thus reducing the possibility of
65 leakage, a good understanding of this dissolution is essential [Elenius et al., 2015].

66 Currently, a large quantity of studies exists on GDC-driven dissolution, ranging from the-
67 oretical analysis [e.g., Elenius et al., 2012; Emami-Meybodi, 2017], laboratory experiments [e.g.,
68 Neufeld et al., 2010; Rasmusson et al., 2017; Salibindla et al., 2018], numerical simulations [e.g.,
69 Chen et al., 2013; Elenius et al., 2015] to field observations [e.g., Sathaye et al., 2014]. It is found
70 that GDC depends to a large extent on properties of the formation (e.g., the permeability and poros-
71 ity) and of the brine phase (e.g., the relation between brine density and concentration of dissolved
72 CO₂, viscosity and molecular diffusion coefficient) [Hassanzadeh et al., 2007, 2008; Neufeld et al.,
73 2010; Elenius and Johannsen, 2012; Emami-Meybodi and Hassanzadeh, 2015]. For instance,
74 researchers find a linear relation between the dissolution rate and the intrinsic permeability for
75 isotropic homogeneous media [Rasmusson et al., 2015]. However, most of the current researches
76 and conclusions are limited to (isotropic or anisotropic) homogeneous fields [e.g. Ennis-King and
77 Paterson, 2005; Pruess and Nordbotten, 2011; Cheng et al., 2012; De Paoli et al., 2017; Ran-
78 ganathan et al., 2012; Taheri et al., 2012].

79 Studies on heterogeneous fields are usually limited to qualitative analysis of the effect of
80 heterogeneity [Agartan et al., 2015; Lin et al., 2016; Soltanian et al., 2017; Kim et al., 2019; Yan
81 et al., 2019; Wang et al., 2021; Elgahawy and Azaiez, 2021]. Heterogeneity in permeability plays
82 an important role in onset, growth, maintenance and decay of the GDC and the dissolution rate
83 [Schincariol et al., 1997; Simmons et al., 2001]. Frykman and Wessel-Berg [2014] show that we
84 may overestimate the effects of instability in a real heterogeneous field simply by conducting nu-
85 merical simulations using a homogeneous field of equivalent permeability. Prasad and Simmons
86 [2003] find that while it can trigger early instability, heterogeneity can reduce the instability by
87 dissipating the density fingers through disordered high permeability pathways at a later stage.

88 Although several researches attempt to quantitatively analyze the relation between GDC
89 and field properties for heterogeneous media, only a few offer explicit formulas between the GDC-
90 driven dissolution rates and permeability heterogeneity. For instance, Farajzadeh et al. [2011]
91 and Kong and Saar [2013] conducted numerical simulations of GDC in isotropic heterogeneous
92 media, but did not offer a quantitative formula between CO₂ dissolution rates and heterogene-
93 ity measures. Therefore, conclusions from these researches cannot be directly applied to estimate
94 the dissolution rate in real reservoirs. Several researches offer quantitative formulas for predict-
95 ing dissolution rates but only consider simplified binary heterogeneous media [Elenius and Gasda,
96 2013; Green and Ennis-King, 2014] or homogeneous media with anisotropic permeability [Er-
97 fani et al., 2022]. In these researches, different conclusions are given regarding the effect of per-
98 meability anisotropy on the dissolution rate.

99 Overall, we have a solid understanding of the GDC-driven dissolution process in isotropic
100 homogeneous media, but the GDC-driven dissolution in heterogeneous media needs further study.
101 Especially, we need to quantitatively clarify the impact of the anisotropy ratio on the effective dis-
102 solution rate. Moreover, the current predictors are all based on the (equivalent) permeability, and
103 it remains unclear whether we can predict the dissolution rate based on other formation proper-
104 ties or field observations, such as the finger-tip velocity.

105 Therefore the objective of this work is to quantitatively analyze the effect of permeability
106 heterogeneity on the GDC-driven dissolution rate in a wide range of (isotropic and anisotropic)
107 heterogeneous fields with varying degrees of heterogeneity. We do this in two steps. The first step
108 consists of performing numerical simulations over a large number of heterogeneous fields of dif-
109 ferent permeability distributions. Numerical simulations are carried out by a finite-difference nu-
110 merical program developed by Wang [2022]. Permeability fields are generated with the sequen-
111 tial Gaussian simulation method implemented in the SGSIM code [Journal and Huijbregts, 1976].
112 In the second step the results of the simulations are analyzed to find relations between the GDC-
113 driven dissolution rate and heterogeneity of permeability, and we compare our results against those
114 given in literature. In this step, ordinary-least-squares linear regressions are used.

115 The rest of this paper is organized as follows. Section 2 gives a concise description of GDC.
 116 In section 3, we review existing formulas for GDC driven dissolution rates. Section 4 describes
 117 the computational approach. Section 5 gives the results and discussions. Major conclusions are
 118 listed in section 6.

119 **2 Gravity-Driven Convection**

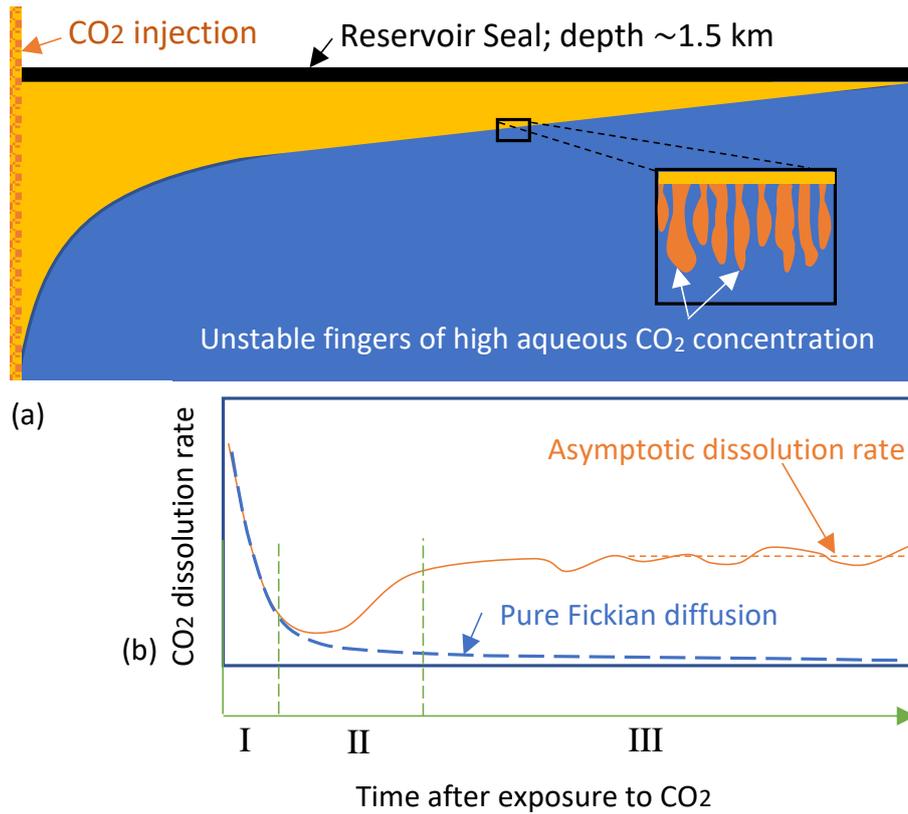


Figure 1: (a) Sketch of the appearance of unstable fingers at the interface of brine and CO₂-rich phase, and (b) enhanced dissolution rate due to vertical mass exchange driven by unstable fingers (I: diffusive regime; II: flux-growth regime; and III: constant-flux regime).

120 During the later stage of injection, the injected less dense CO₂-rich phase floats over the
 121 brine, as shown in Figure 1. This CO₂ moves upwards until it reaches a sealing caprock that traps
 122 the CO₂. The trapped CO₂ remains stored in the reservoir over extended periods, contributing
 123 to long-term carbon storage. Far from the injection site, a long quasi horizontal interface forms
 124 between the brine phase and the CO₂-rich phase. This interface represents a boundary of constant
 125 CO₂ concentration for the underlying brine, from which the overlying supercritical CO₂ dis-
 126 solves into the brine. Importantly, brine with high CO₂ concentration has ~ 1.0% higher den-
 127 sity than the underlying brine of very low CO₂ concentration, triggering Gravity-Driven Con-
 128 vection (GDC) that remarkably increases the CO₂ dissolution rate (see Figure 1). GDC contin-
 129 ues until the whole reservoir is fully saturated with CO₂, which may take a long time. The dis-

130 solution process can be divided into 4 regimes [*Slim, 2014; Tsinober et al., 2022*]. The first regime
 131 is called ‘diffusive regime’, in which the mass flux is controlled by Fickian diffusion that fades
 132 away exponentially, as can be seen from Figure 1. After this, GDC starts to develop and disso-
 133 lution switches from diffusion controlled to convection controlled. During this regime, which is
 134 called ‘flux-growth regime’, the dissolution rate starts to increase. Once the GDC is fully devel-
 135 oped, the dissolution rate reaches a quasi constant value for a relatively long time. This regime
 136 is known as ‘constant-flux regime’. The last regime (not shown in Figure 1) is referred to as ‘shut-
 137 down regime’ or flux decay regime. In this regime, the dissolution rate decreases as the reser-
 138 voir becomes fully saturated with CO₂ [*Elenius et al., 2015*].

139 In this work, we focus on the dissolution rate taking place in the ‘constant-flux regime’. This
 140 regime controls the overall behavior of the dissolution efficiency during the geological carbon
 141 sequestration [*Tsinober et al., 2022*]. In this ‘constant-flux regime’, the dissolution rate can be
 142 approximated by an asymptotic dissolution rate, as shown in Figure 1.

143 3 Review of Formulas for GDC-Driven Dissolution Rates

144 In isotropic homogeneous porous media, the asymptotic dissolution rate [kg·m⁻²·s⁻¹] can
 145 be written as [e.g., *Hesse, 2008; Pau et al., 2010; Slim, 2014*],

$$F_{\infty} = \gamma X_0^C \rho_0 \frac{\Delta \rho g \kappa}{\bar{\mu}}, \quad (1)$$

146 where γ [-] is a scaling coefficient, X_0^C [kg·kg⁻¹] and ρ_0 [kg·m⁻³] respectively represent the CO₂
 147 mass fraction in brine and the brine density at the interface of brine and CO₂-rich phase, $\Delta \rho$ [kg·m⁻³]
 148 is the density increase when aqueous CO₂ mass fraction increases from the initial very small back-
 149 ground value to X_0^C , g [m·s⁻²] is the gravitational acceleration, κ [m²] is the intrinsic permeabil-
 150 ity, and $\bar{\mu}$ [pa·s] is the mean viscosity of brine. Different researches have obtained different val-
 151 ues for the scaling coefficient γ that varies from ~ 0.015 to ~ 0.075 , and a summary of γ is given
 152 in Table 1.

153 In heterogeneous porous media, the intrinsic permeability can vary by several orders of mag-
 154 nitude [*Elenius and Johannsen, 2012; Elenius et al., 2012*]. Therefore, it is important to under-
 155 stand the effect of permeability heterogeneity on the GDC dissolution. Although there exists a
 156 large amount of studies on GDC dissolution in heterogeneous porous media [e.g. *Green and Ennis-
 157 King, 2010; Elenius and Gasda, 2013; Green and Ennis-King, 2014; Taheri et al., 2018; Mahyapour
 158 et al., 2022*], only a few researches offer concise formulas for predicting the dissolution rate. By
 159 conducting numerical simulations in homogeneous medium with embedded horizontal barriers,
 160 *Elenius and Gasda [2013]* proposed that the asymptotic dissolution rate can be estimated by

$$F_{\infty} = \gamma X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}} \left(\frac{\kappa_z^e}{\kappa_g} \right), \quad (2)$$

161 where κ_z^e is the equivalent vertical intrinsic permeability of the heterogeneous medium, and κ_g
 162 is the geometric mean of the permeability field. Essentially, these authors proposed to replace
 163 the intrinsic permeability by its equivalent quantity in heterogeneous media. Subsequent stud-
 164 ies analyze whether and how the anisotropic effect of the permeability distribution affects the GDC,
 165 but the results are inconsistent [*Xu et al., 2006; Green and Ennis-King, 2010; Cheng et al., 2012;
 166 Chen et al., 2013; Green and Ennis-King, 2014; Kim, 2014; Soltanian et al., 2017; Elgahawy and
 167 Azaiez, 2021*]. Inspired by the result for the isotropic heterogeneous field, researchers propose
 168 that the dissolution rate in the anisotropic heterogeneous field can be expressed by

$$F_{\infty} = \gamma X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}} \left(\frac{\kappa_z^e}{\kappa_g} \right) \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^\eta, \quad (3)$$

Table 1: Published data for the scaling coefficient γ in isotropic homogeneous field.

γ	Top boundary	Method	Reference
0.015-0.017	diff. only ^[a]	Num.	<i>Pruess and Zhang</i> [2008]
0.017	diff. only	Num.	<i>Hesse</i> [2008]
0.017-0.018	diff. only	Num.	<i>Pau et al.</i> [2010]
$0.12Ra^{-0.16}$ ^[e]	-	Num.,Exp.	<i>Neufeld et al.</i> [2010]
$0.045Ra^{-0.24}$	Permeable	Exp.	<i>Backhaus et al.</i> [2011]
0.017	diff. only	Num.	<i>Cheng et al.</i> [2012]
0.02	diff. only	Num.	<i>Elenius and Johannsen</i> [2012]
0.02	diff. only	Num.	<i>Elenius et al.</i> [2012]
0.075	CTZ ^[b]	Num.	<i>Elenius et al.</i> [2012]
$0.037Ra^{-0.16}$	Permeable	Exp.	<i>Tsai et al.</i> [2013]
$0.0794Ra^{-0.168}$	diff. only	Theor.	<i>Farajzadeh et al.</i> [2013]
0.017	diff. only	Num.	<i>Slim</i> [2014]
0.025	(0.2κ) ^[c]	Num.	<i>Slim</i> [2014]
0.044	(0.6κ) ^[d]	Num.	<i>Slim</i> [2014]
0.065	CTZ	Num.	<i>Martinez and Hesse</i> [2016]
0.018-0.019	diff. only	Num.	<i>Martinez and Hesse</i> [2016]
$Ra^{-0.2154}$, 0.06	Permeable	Exp.	<i>Rasmussen et al.</i> [2017]
0.09	Permeable	Num.	This study

^[a] The top boundary only allows mass to go through the boundary via diffusion.

^[b] The top boundary is capillary transition zone.

^[c] The permeability of the top boundary is 0.2 the permeability of the domain.

^[d] The permeability of the top boundary is 0.6 the permeability of the domain.

^[e] Ra is Rayleigh number (cf. [*Neufeld et al.*, 2010]).

169 where κ_x^e is the equivalent intrinsic permeability along the horizontal direction, and the exponent
170 η describes the impact of anisotropy. The last term represents the anisotropic effect described by
171 the horizontal to vertical equivalent permeability ratio. By conducting numerical simulations of
172 GDC dissolution in homogeneous media with embedded horizontal barriers, which is similar to
173 the aforementioned method used by *Elenius and Gasda* [2013], *Green and Ennis-King* [2014]
174 found that $\eta = 0.5$, which indicates that dissolution increases with horizontal to vertical equiv-
175 alent permeability ratio. Note that when $\eta = 0.5$, the dissolution rate is actually proportional
176 to the geometric mean of the permeabilities $\sqrt{\kappa_x^e \kappa_z^e}$. In contrast, *Erfani et al.* [2022] give $\eta = 0.21$.
177 However, the results from *Soltanian et al.* [2017] and *Elgahawy and Azaiez* [2021] show that the
178 increasing κ_x^e at fixed κ_z^e can reduce the asymptotic dissolution rate, implying that η may be neg-
179 ative.

180 4 Computational Approach

181 4.1 Model Setup

182 We conduct numerical simulations of Gravity-Driven Convection (GDC) over two-dimensional
183 vertical fields of various permeability heterogeneity, which varies in space as a function of the
184 horizontal and vertical distances. We assume that the interface between the CO₂-rich phase and
185 brine is horizontal under buoyancy force, and that the brine at the interface is always saturated
186 with CO₂. We only study the portion below the interface and therefore all simulations are con-
187 ducted with a single-phase model. Initially, the CO₂ concentration in brine is very low and the
188 system is at static state. CO₂ enters into the domain through the top boundary that has fixed high
189 CO₂ mass fraction. Brine is represented by a high-concentration Sodium Chloride (NaCl) so-
190 lution. The objective is to get a quantitative relation between representative properties of the het-
191 erogeneous field and the CO₂ dissolution rate through the top boundary.

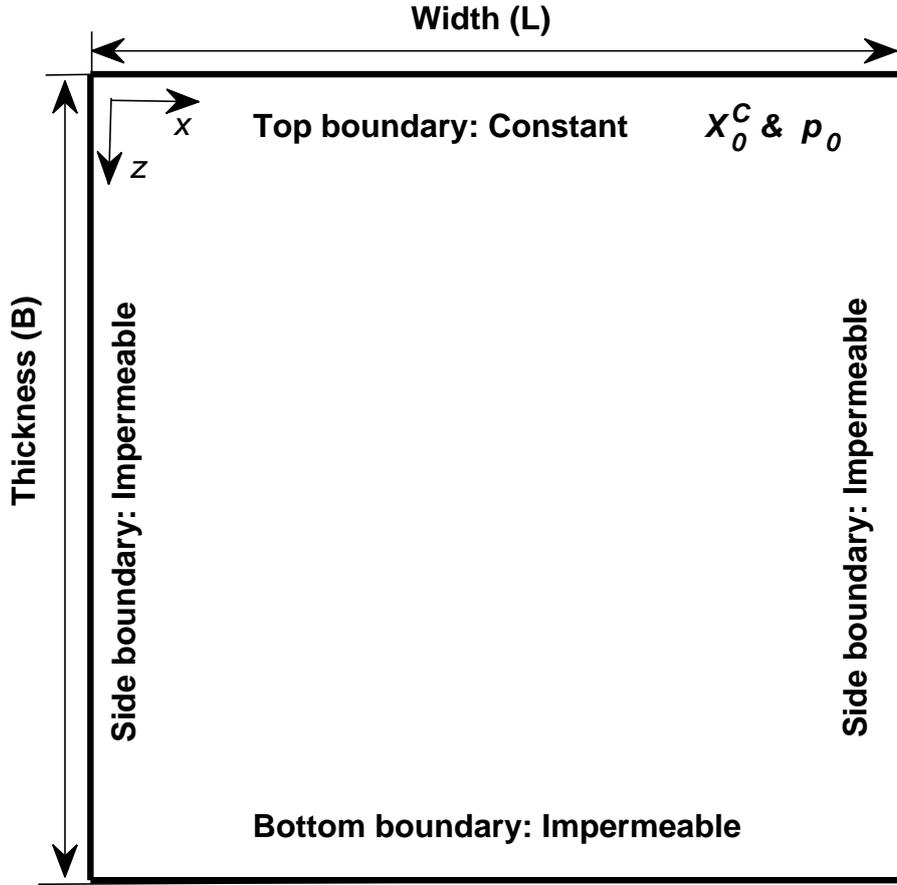


Figure 2: Sketch of setup design.

192 Figure 2 shows the two-dimensional vertical domain used to simulate the development of
 193 dissolution process enhanced by GDC. Detailed parameter settings are as follows. The top bound-
 194 ary, which represents the interface between brine and CO₂-rich phase, has constant liquid pres-
 195 sure $p_0 = 150$ [bar] and constant CO₂ mass fraction $X_0^C = 0.041$ [kg·kg⁻¹]. The bottom and
 196 lateral boundaries are no-flow boundaries. Initially, the brine has a very low background CO₂ mass
 197 fraction $X_{bg}^C \approx 0.0006$ [kg·kg⁻¹]. An initial brine pressure of $p_0 = 150$ [bar] is imposed at the
 198 top layer and increases downwards at a gradient of ρg , which means that the brine is initially at
 199 hydrostatic state. The temperature (T_c) is fixed to 60 [°C], and the salinity of brine (m^S) is con-
 200 stant 0.5 [molal]. When the mass fraction of aqueous CO₂ increases from initial X_{bg}^C to X_0^C on
 201 the top boundary, the density of brine increases by $\Delta\rho = 8.2$ [kg·m⁻³]. We note that the vis-
 202 cosity of the brine slightly changes from 1.0 [mpa·s] at the minimum background CO₂ mass frac-
 203 tion (X_{bg}^C) to 0.9 [mpa·s] at the maximum CO₂ mass fraction (X_0^C). Thus, the mean viscosity is
 204 approximated by $\bar{\mu} = 0.95$ [mpa·s]. Table 2 summarizes the parameters that are used in the nu-
 205 merical simulations.

206 We note that the top boundary condition is different from most of those used in literature
 207 where the convection of the CO₂-saturated layer is either totally or partially suppressed (see Ta-
 208 ble 1). For instance, in *Pau et al.* [2010], the top constant concentration boundary only allows
 209 CO₂ to enter the domain via diffusion; this top boundary condition generates a much lower dis-

Table 2: Summary of the parameters adopted during numerical simulations.

Parameters	Symbol	Units	Values
Domain size	$L \times B$	[m × m]	7.5×7.5
Grid discretization	$n_x \times n_z$	[-]	100 × 100
Porosity	ϕ	[-]	0.15 ^{c,f}
Geometric mean permeability	κ_g	[m ²]	10^{-12} ^{b,i}
Initial liquid pressure at top layer	p_0	[bar]	150 ^h
Initial background CO ₂ (aq) mass fraction	X_{bg}^C	[kg · kg ⁻¹]	~0.0006 ^{g,j}
CO ₂ (aq) mass fraction at top boundary	X_0^C	[kg · kg ⁻¹]	0.041
Brine density at top boundary	ρ_0	[kg · m ⁻³]	1017
Brine density increase due to increased CO ₂ (aq) mass fraction	$\Delta\rho$	[kg · m ⁻³]	8.2
Salinity	m^S	[molal]	0.5 ^{e,i}
Mean viscosity	$\bar{\mu}$	[mpa · s]	0.95
Molecular diffusion coefficient	D	[m ² · s ⁻¹]	2×10^{-9} ^{d,i}
Temperature	T_c	[°C]	60 ^{a,k}

Reference: ^a*Spycher et al.* [2003]; ^b*Chadwick et al.* [2004]; ^c*Maldal and Tappel* [2004]; ^d*Tewes and Boury* [2005]; ^e*Spycher and Pruess* [2005]; ^f*Mathieson et al.* [2009]; ^g*Xu et al.* [2007]; ^h*Iding and Ringrose* [2010]; ⁱ*Elenius and Johannsen* [2012]; ^j*Saaltink et al.* [2013]; ^k*Strandli and Benson* [2013].

210 solution rate. In some studies the top boundary is partially permeable and larger dissolution rates
 211 are obtained [*Hesse, 2008; Elenius et al., 2014; Rasmusson et al., 2015*]. Although it is more re-
 212 alistic to add a capillary transition zone beneath the top boundary [*Elenius et al., 2015*], exper-
 213 imental results with totally permeable top boundaries [*Rasmusson et al., 2017*] show only little
 214 discrepancy from the numerical results obtained in a model that includes the capillary transition
 215 zone [*Martinez and Hesse, 2016*]. Therefore, we employ the single phase flow model with per-
 216 meable top boundary in this study, although our model is capable of two-phase flow simulations
 217 [*Wang et al., 2022*].

218 An initial perturbation of the initial CO₂ mass fraction on the top boundary is added to stim-
 219 ulate the onset of instability at the beginning of simulation. We added a white noise that follows
 220 an uncorrelated Gaussian distribution [*Fu et al., 2013*]. The magnitude of the noise is 1% of the
 221 initial mass fraction. *Hidalgo and Carrera* [2009] show that instability can be generated by the
 222 numerical error without introducing any external noise. Certainly, even though larger noise strength
 223 tends to accelerate the appearance of instability fingers, it is however unlikely to change the statis-
 224 tic behavior of the dissolution rate once the instability has fully developed [*Hidalgo and Carrera,*
 225 *2009; Elenius and Johannsen, 2012*].

226 4.2 Governing Equations

227 On the basis of the mass balances of water and CO₂ species, the two governing transport
 228 equations are constructed as,

$$229 \quad 0 = \frac{\partial(\phi\rho X^H)}{\partial t} + \nabla \cdot (\rho X^H \mathbf{q}) - \nabla \cdot (\phi \mathbf{D} \rho \nabla X^H), \quad (4)$$

$$0 = \frac{\partial(\phi\rho X^C)}{\partial t} + \nabla \cdot (\rho X^C \mathbf{q}) - \nabla \cdot (\phi \mathbf{D} \rho \nabla X^C), \quad (5)$$

230 where ϕ [-] is the porosity of the saline formation, ρ [$\text{kg}\cdot\text{m}^{-3}$] represents the density of brine, X
 231 [$\text{kg}\cdot\text{kg}^{-1}$] is the mass fraction, superscripts (H, C) represent the water and aqueous CO_2 species,
 232 respectively, t [s] denotes the time, $\mathbf{D} = D\mathbf{I}_d$ [$\text{m}^2\cdot\text{s}^{-1}$] denotes the dispersion tensor, which is
 233 assumed locally constant. Local dispersion has little impact on the asymptotic dissolution rates,
 234 which is the objective of this work [Prasad and Simmons, 2003; Hidalgo and Carrera, 2009].
 235 The discharge rate (\mathbf{q}) is given by Darcy's law

$$\mathbf{q} = -\frac{\kappa}{\mu}(\nabla p - \rho g \nabla z), \quad (6)$$

236 where κ [m^2] is the intrinsic permeability, μ [$\text{pa}\cdot\text{s}$] is the viscosity, p [pa] is the liquid pressure,
 237 and z [m] is the depth. Besides, we have the following constraints:

$$X^S = 0.05844X^H m^S, \quad (7)$$

238 and

$$X^H + X^C + X^S = 1, \quad (8)$$

239 where m^S denotes the molality of salt. Here, we assume that the salt comprises only NaCl, and
 240 the molality of NaCl (m^S) is fixed. Define $\omega = (1 + 0.05844m^S)$ and then Equations (7) and
 241 (8) merge to

$$\omega X^H + X^C = 1. \quad (9)$$

242 Under isotherm and isosalinity condition, ρ and μ are only governed by liquid pressure and
 243 aqueous CO_2 mass fraction (see Appendix A).

244 4.3 Dimensionless variables

245 In order to facilitate the analysis, results are presented using the following dimensionless
 246 variables, which are defined based on the works of *Ennis-King and Paterson* [2003] and *Rasmus-*
 247 *son et al.* [2017],

$$X^{C*} = \frac{X^C}{X_0^C}, \quad X^{H*} = \frac{X^H}{X_0^H}, \quad (10)$$

248 and

$$t^* = \frac{t}{t_c}, \quad x^* = \frac{x}{L_c}, \quad z^* = \frac{z}{L_c}, \quad (11)$$

249 where X_0^C and X_0^H are, respectively, the maximum CO_2 and water mass fractions, and t_c and L_c
 250 are the characteristic time and length scale of the gravity-driven convection problem defined as

$$t_c = \frac{(\bar{\mu}\phi)^2 D}{(\Delta\rho g \kappa_g)^2}, \quad L_c = \frac{\bar{\mu}\phi D}{\Delta\rho g \kappa_g}. \quad (12)$$

251 The characteristic time t_c has been found to be closely related to the onset time of gravity-
 252 driven convection, and the characteristic length L_c closely related to the earliest finger width. In
 253 our simulations, we found that the earliest finger width, denoted as ℓ_c , can be approximated by
 254 $\ell_c \approx 70L_c$. The governing equations are expressed in dimensionless form in the Appendix B.

255 Importantly, by expressing Darcy's law and the global dissolution rate in dimensionless form, we
 256 obtain that

$$q^* = \frac{q}{q_c}, \quad q_c = \frac{\Delta \rho g \kappa_g}{\bar{\mu}}, \quad (13)$$

$$F^* = \frac{F}{F_c}, \quad F_c = X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}}. \quad (14)$$

257 The characteristic velocity $v_c = q_c / \phi$ is closely related to the finger-tip velocity (see *Ele-*
 258 *nious and Johannsen* [2012]). We note that although the vertical length scale (i.e., domain thick-
 259 ness) and related dimensionless number (e.g., Rayleigh number) have been typically used to study
 260 gravity instability in the literature [*Rasmusson et al.*, 2017, and references therein], herein we do
 261 not use it because the vertical domain size has little impact on the asymptotic enhanced disso-
 262 lution rate driven by instability fingers [*Elenius et al.*, 2015; *Tsinober et al.*, 2022]. This defini-
 263 tion of the dimensionless length scale without using the domain thickness indicates that the in-
 264 stability is a random statistic behavior that does not change with the domain size provided that
 265 the domain is large enough to accommodate sufficient number of density fingers. In the support-
 266 ing information, we have shown that increasing the vertical domain size employed in this work
 267 does not systematically affect the asymptotic dissolution rate [*Elenius and Johannsen*, 2012]. *Sim-*
 268 *mons et al.* [2001] give a detailed discussion of the limitation of using Rayleigh number based
 269 on the domain thickness.

270 4.4 Heterogeneity

271 The intrinsic permeability is the only aquifer property considered to vary in space. We rep-
 272 resent the natural logarithm of the intrinsic permeability, denoted as $Y = \ln \kappa$, by a random space
 273 function to create multiple realizations of the aquifer permeability distribution. The random space
 274 function model is characterized by an exponential covariance function with mean (\bar{Y}), variance
 275 (σ_Y^2), horizontal correlation length (λ_h) and vertical correlation length (λ_v). The geometric mean
 276 of the intrinsic permeability is fixed to $\kappa_g = 10^{-12} \text{ m}^2$. Different degrees of heterogeneity and
 277 correlation anisotropy $\Omega = \lambda_h / \lambda_v$ are explored with σ_Y^2 ranging between highly homogeneous,
 278 $\sigma_Y^2 = 0.1$, to relatively highly heterogeneous, $\sigma_Y^2 = 4$, and Ω ranging between isotropic, $\Omega = 1$, and
 279 perfectly stratified, $\Omega = \infty$. The perfectly stratified random field is formed by separate horizon-
 280 tal layers of constant properties. The vertical correlation length is fixed to $\lambda_z = 2L_c$, and the
 281 horizontal correlation length is $\lambda_x = \Omega \cdot \lambda_z$. For comparison purposes, we also considered a
 282 homogeneous medium with $\kappa = \kappa_g$. In total, we conduct GDC simulations with 365 realiza-
 283 tions. Random fields were generated using the sequential Gaussian simulation method implemented
 284 in the SGSIM code of GSLIB [*Journel and Huijbregts*, 1976; *Deutsch et al.*, 1992]. Table 3 sum-
 285 marizes the statistical properties of the random fields. An illustrative realization of each random
 286 field type is shown in Figure 4.

Table 3: Permeability heterogeneity adopted for the numerical simulations. The vertical correlation length is fixed to $\lambda_z = 2L_c$, and the horizontal correlation length is $\lambda_x = \Omega \cdot \lambda_z$.

Case	Ω	σ_Y^2
Homogeneous	-	-
Isotropic	1	1
Isotropic	1	4
Anisotropic	2	0.1
Anisotropic	2	1
Anisotropic	4	0.1
Anisotropic	4	1
Anisotropic	4	4
Anisotropic	∞	1

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4.5 Global Measures

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Simulation results are analyzed based on two global measures of the dissolution behavior. The global dissolution rate (F [$\text{kg}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$]) is defined as the rate at which CO_2 dissolves from the upper boundary at $z=0$. This can be expressed as [Hidalgo and Carrera, 2009]

$$F(t) = \frac{1}{L} \int_0^L \left[\rho X^C q_z - \phi D \rho \frac{\partial X^C}{\partial z} \right]_{z=0} dx. \quad (15)$$

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Initially, the domain is stable, the convection flux is zero, and only molecular diffusion transports downwards the dissolved CO_2 . The density-driven unstable convection does not emerge until the CO_2 mass fraction distribution reaches a critical point. After this, the dissolution rate rapidly increases to a quasi constant value until the domain is almost totally saturated with aqueous CO_2 . In our simulations, the asymptotic value of the global dissolution rate F_∞ is estimated as the temporal average of $F(t)$ over the period of $[t_b/3, t_b]$, where t_b is the time when the earliest finger of aqueous CO_2 reaches the bottom (time when the maximum bottom CO_2 mass fraction exceeds 25% of X_0^C).

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Another important global parameter that describes the vertical migration or the penetration depth of the CO_2 plume is the vertical finger-tip velocity [Prasad and Simmons, 2003]. The representative vertical finger-tip velocity of the CO_2 plume is estimated as,

$$v(t) = \max_{0 < z < B} \left\{ \frac{1}{L} \int_0^L \frac{1}{\phi} |q_z| dx \right\}. \quad (16)$$

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Figure 3 illustrates the concept of vertical finger-tip velocity in our simulations. Similar to the global dissolution rate behavior, the vertical finger-tip velocity also reaches an asymptotic value [Elenius and Johannsen, 2012] in the constant-flux regime and remains at that value until the field is almost saturated. The asymptotic vertical finger-tip velocity v_∞ is also estimated by the temporal average of $v(t)$ over the time interval $[t_b/3, t_b]$.

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In order to characterize the overall hydraulic behavior of the permeability field, we estimated the equivalent permeability along the horizontal κ_x^e and vertical κ_z^e direction for each realization of the random fields. For this, to estimate κ_i^e ($i = x, y$), we neglect gravity and saturate the porous medium with only water. We then set the domain sides perpendicular to the i th direction as impermeable, and we impose a pressure decrement $|\Delta_i p|$ along the i th direction. κ_i^e is estimated by the total volumetric flow Q_i passing through the system in the i th direction as $\kappa_i^e = \mu Q_i L_i / (A_i |\Delta_i p|)$, where L_i is the domain size along the i th direction and A_i the corresponding cross-sectional area [Knudby and Carrera, 2005; Wang, 2022].

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4.6 Numerical Features

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The model is implemented in a Matlab reservoir simulator toolbox designed for CO_2 storage [Wang, 2022]. The program is based on the finite volume method. The two governing equations (4) and (5) are solved simultaneously with an implicit Newton-Raphson method. Two-point flux approximation with up-winding scheme is used to calculate mass transport. Although the unconditional stable implicit method is employed, we should still control the time step in the numerical simulation to avoid significant numerical dispersion. In this work, the time step is limited by either advection ($\Delta t < \Delta l \phi / (\mathbf{q} \cdot \mathbf{n})$) or diffusion ($\Delta t < \sqrt{\Delta l / (2D\phi)}$), whichever is smaller. Here, Δl and \mathbf{n} are respectively the distance between the centroids of two adjacent cells and the unitary perpendicular vector to their interface. Detailed implementation of this numerical solution and related source code can be found in our previous works [Wang et al., 2022, 2023].

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The numerical discretization consists in 100×100 squared cells of equal size. The cell size is chosen to strike a balance between capturing small instability fingers and managing computational expenses without compromising accuracy following Lindeberg and Bergmo [2003] and

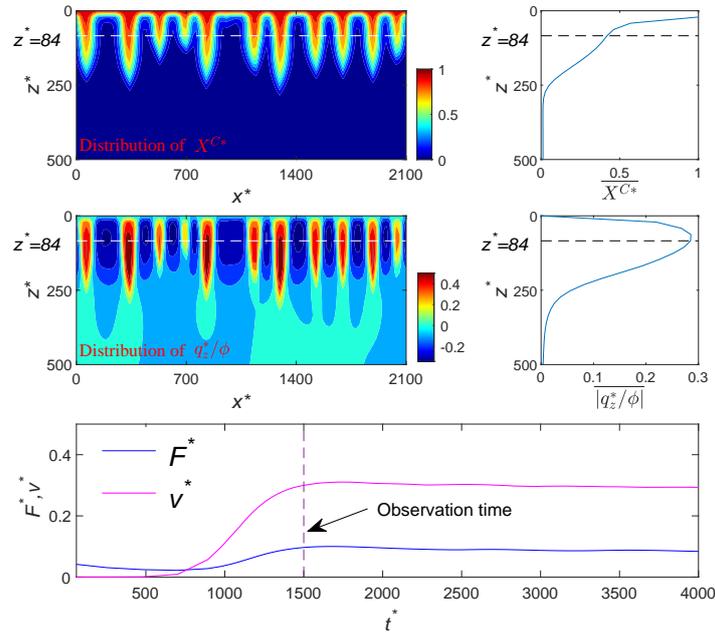


Figure 3: Maps of the dimensionless CO₂ mass fraction (X^{C*}) and its horizontal average ($\overline{X^{C*}}$), the dimensionless vertical flow velocity (q_z^*/ϕ) and the horizontal average of absolute dimensionless vertical flow velocity ($\overline{|q_z^*/\phi|}$), and temporal the development of dimensionless dissolution rate (F^*) and finger velocity (v^*) in an illustrative realization. The dimensionless finger velocity is represented by $v^* = \max |q_z^*/\phi|$.

329 *Elenius et al.* [2015]. The grid size is smaller than the finger width, $\Delta = 0.3\ell_c = 21L_c$, and at
 330 the same time the shortest correlation length is described by at least 7 grid cells. We note that
 331 the finger width ℓ_c is close to the approximate value $4\pi^2L_c$ needed for the generation of insta-
 332 bility [Lapwood, 1948].

333 4.7 Effective Asymptotic Dissolution Models

334 Two effective dissolution models are proposed here based on previous results reported in
 335 literature (see Section 3) with the objective to offer a simplified representation of the overall asymp-
 336 totic dissolution behavior in naturally occurring heterogeneous porous media. In the first effec-
 337 tive model, we have extended the formula (3) to a more general expression that incorporates the
 338 effect of anisotropy. In dimensionless form, the model expresses that

$$F_\infty^* = \gamma_1 \left(\frac{\kappa_z^e}{\kappa_g} \right)^{\alpha_1} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{\beta_1}. \quad (17)$$

339 Here, the dimensionless asymptotic dissolution rate is $F_\infty^* = F_\infty/F_c$. The last term on the right
 340 hand side of this expression represents the anisotropy of the permeability field, defined as $a_f =$
 341 κ_x^e/κ_z^e .

342 Existing effective asymptotic dissolution models rely exclusively on equivalent permeabil-
 343 ity values. Thus, it remains uncertain whether predictions of the dissolution rate can be made us-
 344 ing field observations, such as the finger-tip velocity v_∞ . For this, in the second model, we have
 345 considered the following relationship written in dimensionless form as

$$F_\infty^* = \gamma_2 \left(\frac{v_\infty}{v_c} \right)^{\alpha_2} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{\beta_2}. \quad (18)$$

346 Essentially, this expression replaces the equivalent vertical permeability with the vertical finger
 347 velocity, which seems to offer a more direct description of the CO₂ plume migration. $\gamma_1, \gamma_2, \alpha_1,$
 348 α_2, β_1 and β_2 are tuning coefficients of the two effective asymptotic dissolution models.

349 5 Results and Discussion

350 5.1 Impact of Heterogeneity

351 We first provide a general description of the impact of heterogeneity on the vertical migra-
 352 tion of the CO₂ plume and dissolution rates. We focused on the influence of anisotropy in the cor-
 353 relation structure of permeability Ω and the degree of heterogeneity σ_Y^2 . Figure 4 shows the tem-
 354 poral evolution of the spatial distribution of CO₂ mass fraction for different types of heterogene-
 355 ity. For illustrative purposes, we chose a representative permeability realization for each case.
 356 These realizations are shown in the first panel of Figure 4. In all cases, results show that insta-
 357 bility CO₂ fingers grow, merge and re-initiate as also observed in laboratory experiments [Ras-
 358 musson *et al.*, 2017; Liyanage, 2018; Tsinober *et al.*, 2022] and numerical simulations [Elenius
 359 *et al.*, 2015]. Importantly, from Figure 4, we can see that CO₂ fingering is strongly affected by
 360 heterogeneity. In particular, the presence of vertical well-connected high permeability zones (pref-
 361 erential channels) facilitates the initiation and growth of the instability fingers (see for instance
 362 the second column of Figure 4). Actually, the white randomness of the top CO₂ mass fraction
 363 (needed in homogeneous media to create instabilities) is redundant in heterogeneous porous me-
 364 dia as instabilities are controlled by these vertical preferential channels. In accordance with *Sim-*
 365 *mons et al.* [2001], we also see that heterogeneity dissipates vertical finger growth through dis-
 366 persive mixing. This effect increases with Ω , i.e., when horizontal well-connected high-permeability
 367 structures exist. This is strongly manifested in perfectly stratified media with $\sigma_Y^2=1$ and $\Omega=\infty$ (fourth
 368 column of Figure 4).

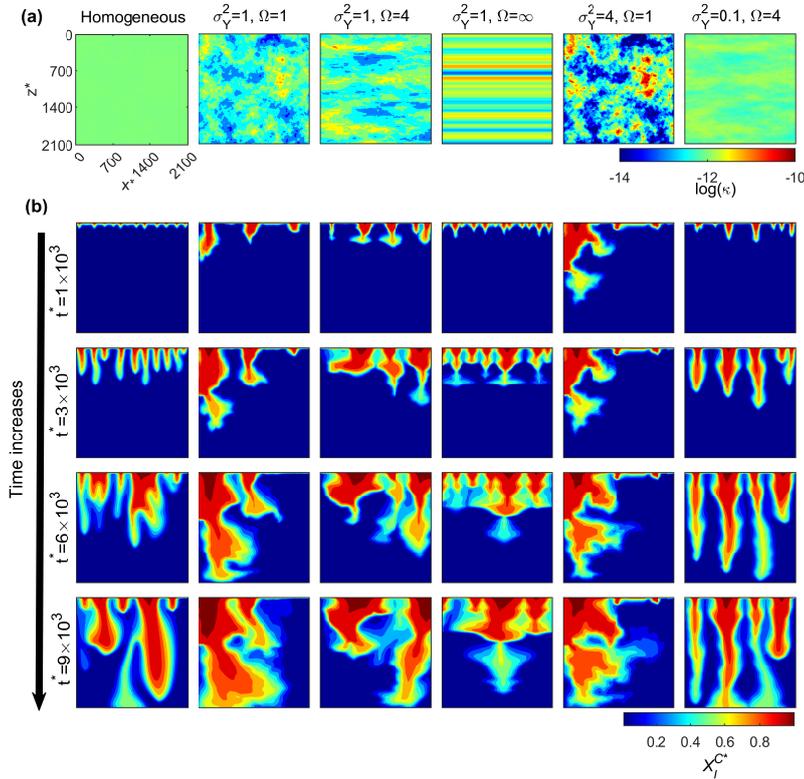


Figure 4: (a) Logarithm permeability ($\log(\kappa)$) distribution. (b) Development of dimensionless CO_2 mass fraction distribution (X_l^{C*}) due to gravity-driven convection.

369 For completeness, we also show the temporal evolution of the dissolution rate as a func-
 370 tion of σ_Y^2 and Ω in Figure 5. Results are presented in terms of the ensemble average and the co-
 371 efficient of variation of $F^*(t^*)$. As expected, in homogeneous media, the dissolution process, shows
 372 the three well-known stages: an initial diffusion-controlled decrease, followed by an onset of non-
 373 linear growth at $t^* = 500$ due to instability, and eventually stabilizing at an approximately con-
 374 stant dissolution rate. However, in heterogeneous media, results show in all cases that the early-
 375 time evolution of $F^*(t^*)$ in heterogeneous media is remarkably different than that in homogeneous
 376 media. The diffusive and the flux-growth regimes cannot be distinguished anymore and the sys-
 377 tem seems to be controlled by the interaction between gravity-driven convection and heterogene-
 378 ity, indicating that heterogeneity helps triggering the onset of instability. This is consistent with
 379 results reported by *Schincariol et al.* [1997] and *Simmons et al.* [2001]. Of course, this also in-
 380 dicates that caution is needed when using the onset time of instability for homogeneous media
 381 [*Ennis-King and Paterson, 2005; Pruess and Zhang, 2008*] in real applications.

382 The influence of σ_Y^2 and Ω can also be seen from Figure 5. Interestingly, in statistically isotropic
 383 heterogeneous media, the degree of heterogeneity σ_Y^2 significantly influences the early behavior
 384 of $F^*(t^*)$, eventually converging to a similar asymptotic dissolution rate at large times. This sug-
 385 gests that the asymptotic dissolution rate might be governed by the existence of well-connected
 386 high-permeability zones, regardless of the specific high value of permeability. This effect is not
 387 seen in anisotropic heterogeneous media where we found that the higher the σ_Y^2 the lower is the

388 asymptotic dissolution rate. We attribute this to the fact that, when $\Omega > 1$, an increase in σ_Y^2
 389 produces stronger well-connected permeability layers that inhibits gravity-driven convection. For
 390 the same reason, for equal σ_Y^2 , the higher anisotropy Ω the less significant is the asymptotic dis-
 391 solution rate. We also report in this figure a measure of the uncertainty in $F^*(t^*)$ given by the
 392 coefficient of variation (CV). We can observe that the coefficient of variation reaches a similar
 393 asymptotic value for all cases, regardless of the degree of heterogeneity. A similar trend is also
 394 observed for the vertical finger velocity, as shown in the second column of Figure 5, indicating
 395 a close relation between the dissolution rate and the vertical finger velocity.

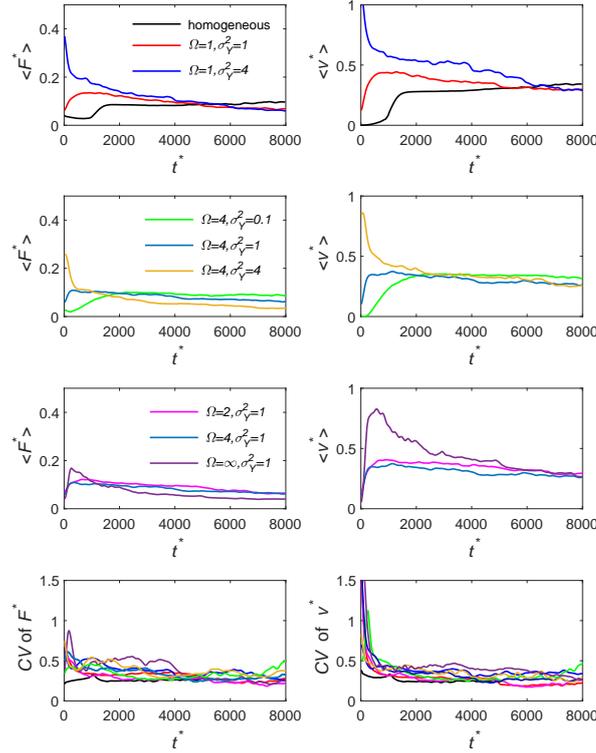


Figure 5: The temporal evolution of the ensemble average of the dimensionless dissolution rate ($\langle F^* \rangle$) through the top boundary and ensemble average of the dimensionless finger velocity ($\langle v^* \rangle$) for all the test cases listed in Table 3. Here, we also give the coefficient of variation (CV), which is the ratio of standard deviation to the ensemble average.

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5.2 The Effective Asymptotic Dissolution Rate

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The tuning coefficients of the two effective asymptotic dissolution models were independently adjusted by regression analysis of all simulation data obtained from the 365 realizations of the permeability distributions. These realizations involved random fields with different correlation structures of permeability and degrees of heterogeneity. To do this, we used Ordinary Least Squares (OLS) regression of the natural logarithm of the dissolution rate models (17) and (18). We respectively obtained a coefficient of determination R^2 of 0.71 and 0.84. The significance of all regression coefficients was below 0.05, meaning that both models can properly explain dissolution rates. Table 4 provides a summary of the regression analysis. Similar to the val-

ues reported in the literature, we find that $\alpha_1 = 1.1$, which is close to unity, meaning that the effective dissolution efficiency is almost linearly proportional to the equivalent vertical permeability. Moreover, we find that, in anisotropic heterogeneous media, the anisotropy of the equivalent permeability ($a_f = \kappa_x^e/\kappa_z^e$) can reduce the effectiveness of dissolution with a power law behavior given by $\beta_1 = -0.2$. This contradicts previous results obtained in homogeneous media with embedded horizontal impermeable inclusions [Green and Ennis-King, 2014; Erfani et al., 2022], which indicated that $\beta_1 > 0$.

Table 4: Coefficients for effective asymptotic dissolution models obtained from OLS regression of all simulation data; the results from literature are also listed for comparison.

Predictors	Coefficients	Values	MAE	RMSE	Source
(17)	$(\gamma_1, \alpha_1, \beta_1)$	(0.08, 1.1, -0.2)	27%	22%	This work
(18)	$(\gamma_2, \alpha_2, \beta_2)$	(0.33, 1.0, -0.3)	20%	19%	This work
(17)	$(\gamma_1, \alpha_1, \beta_1)$	(0.09*, 1.0, 0)	40%	24%	<i>Elenius and Gasda</i> [2013]
(17)	$(\gamma_1, \alpha_1, \beta_1)$	(0.09*, 1.0, 0.5)	76%	44%	<i>Green and Ennis-King</i> [2014]

* Here, we update the value for the reference dissolution rate in the homogeneous case, because the original value was around 0.02 based on the impermeable top boundary (cf. Table 1).

Figure 6 compares the asymptotic dissolution rate predicted by the proposed asymptotic dissolution rate models, expressions (17) and (18), with corresponding simulated values. For completeness, we also show the performance of the reported expressions given by *Elenius and Gasda* [2013] and *Green and Ennis-King* [2014]. We visually differentiate between isotropic and anisotropic random fields. We can see that the predictor given by *Green and Ennis-King* [2014] significantly overestimates the dissolution rate in anisotropic random fields. Actually, the expression by *Green and Ennis-King* [2014] does not seem to improve the prediction given by *Elenius and Gasda* [2013]. The second effective dissolution model given by equation (18), proposed here based on the vertical finger-tip velocity, shows the best performance, indicating that the dissolution rate has a strong relationship with the finger-tip velocity. The Mean Absolute Error (MAE) for formulas (17) and (18) are 27% and 20%, respectively. These error are well accepted considering that even in homogeneous fields, the dissolution rate can fluctuate around 15% [Pau et al., 2010].

In this work, we found a negative impact of the anisotropy of the permeability field a_f on dissolution rates. This can be physically explained in the following manner. Instabilities in the form of fingers exhibit nonuniform periodic high concentration distributions along the horizontal direction. When the spatial continuity of permeability values in the horizontal plane is substantial, any nonuniform concentration in this direction is rapidly eradicated by the enhancement of horizontal mixing induced by the introduction of companion horizontal flows. Consequently, the formation of finger-like instabilities becomes more challenging, especially in scenarios with high horizontal permeability values. In accordance, *Schincariol et al.* [1997] show that increasing the horizontal correlation length of the permeability distribution (increase in κ_x^e), can effectively inhibit instability growth and stabilize perturbations. Through numerical simulations in a wide variety of heterogeneous fields, *Simmons et al.* [2001], *Soltanian et al.* [2017] and *Elgahawy and Azaiez* [2021] also conclude that instability is dampened when κ_x^e/κ_z^e is relatively large. Recent studies by *Tsinober et al.* [2022] and *Hansen et al.* [2023] have also highlighted that the introduction of a minor horizontal background flow in geological carbon sequestration fields enhances horizontal mixing. This enhancement of mixing disrupts the growth of fingers, consequently leading to a reduction in the dissolution rate. All these works also suggest that horizontal flows have the potential to decrease nonuniform instabilities. To better illustrate this, Figure 7 shows simulated and estimated effective dissolution rates as a function aquifer properties $\{\kappa_z^e, \kappa_x^e/\kappa_z^e, \text{ and } v_\infty^*\}$. The figure shows a clear negative dependence of the asymptotic dissolution rates with permeability anisotropy.

444 We acknowledge that our numerical simulations have not covered cases with $a_f < 1$, which
 445 are uncommon in natural sediment formations. Studies conducted by *Simmons et al.* [2001] have
 446 demonstrated that vertically stratified structures with $a_f < 1$ can encourage vertical unstable
 447 convection by diminishing horizontal dissipation in instability fingers. Thus, it is observed that
 448 instability becomes more pronounced when a_f is small, aligning with the findings of our study.

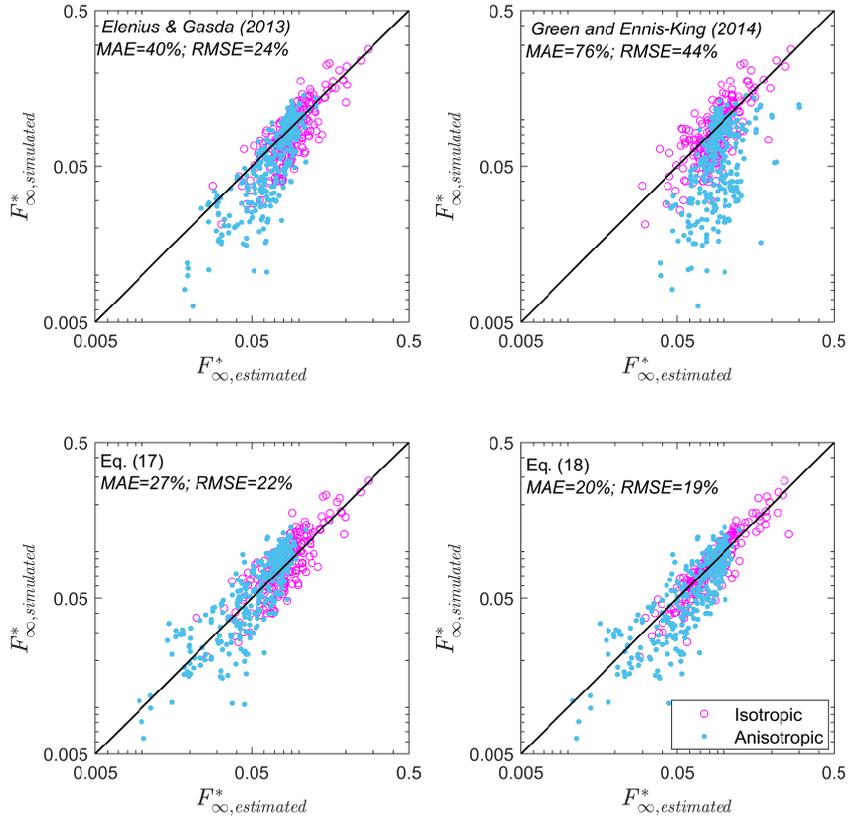


Figure 6: Comparison of the performances of the predictors given by *Elenius and Gasda* [2013], *Green and Ennis-King* [2014] and this work (cf. Table 4).

449 6 Conclusions

450 Geological Carbon Sequestration (GCS) in saline aquifers reduces the release of CO_2 into
 451 the atmosphere, thereby mitigating its impact on climate change. Once CO_2 is injected, the less
 452 dense CO_2 -rich phase floats over the brine and gets trapped beneath an impermeable geological
 453 formation. At the interface between the brine phase and the CO_2 -rich phase, CO_2 slowly dissolves
 454 into the brine, thereby reducing the risk of CO_2 leakage. Estimating CO_2 dissolution rates in this
 455 context is complex, as it requires characterizing the enhancement of dissolution due to Gravity-
 456 Driven Convection (GDC). GDC creates instability fingers that transfer the high CO_2 concen-
 457 tration brine downwards. While many studies offer a deep understanding of GDC in homogeneous
 458 porous media, less is known about dissolution rates in more realistic heterogeneous porous me-

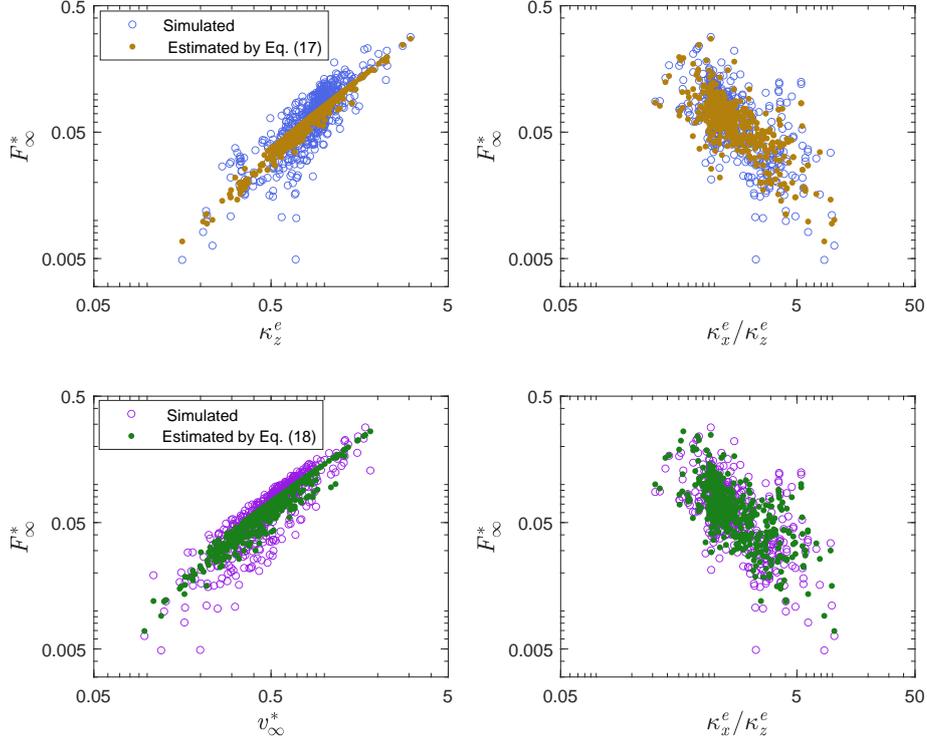


Figure 7: Simulated and estimated effective dissolution rates as a function aquifer properties.

459 dia. In this work, we have systematically analyzed the effect of heterogeneity on GDC-driven dis-
 460 solution rates during GCS. To achieve this, we have conducted numerical simulations of GDC
 461 in multiple aquifer realizations of various permeability distributions. These distributions follow
 462 a random space function that exhibits distinct correlation structures (anisotropy) and degrees of
 463 heterogeneity. Based on these simulations, we have developed two effective asymptotic disso-
 464 lution rate models derived from regression analysis of all the simulated data. The first model es-
 465 timates the asymptotic dissolution rate using the aquifer’s general hydraulic properties. For this
 466 model, we achieved a coefficient of determination (R^2) of 0.71, indicating a strong correlation
 467 between the variables compared to previous effective dissolution models [Elenius and Gasda,
 468 2013; Green and Ennis-King, 2014],

$$F_{\infty} = 0.08 X_0^C \rho_0 \frac{\Delta \rho g \kappa_g}{\bar{\mu}} \left(\frac{\kappa_z^e}{\kappa_g} \right)^{1.1} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{-0.2}. \quad (19)$$

469 This model requires some knowledge of the equivalent permeability value along the x and z di-
 470 rections $\{\kappa_x^e, \kappa_z^e\}$ and fluid properties. The equivalent permeability can be estimated by a wide
 471 variety of methods, including hydraulic tests (see Renard and de Marsily [1997]; Sanchez-Vila
 472 *et al.* [2006] for a review). Alternatively, direct observations of the CO_2 finger-tip velocity v can
 473 be used to estimate asymptotic dissolution rates. This velocity refers to the rate at which the fin-
 474 gers or plumes of dissolved CO_2 move downwards through the subsurface. Determining the finger-

tip velocity of CO₂ in the field involves the use of tracer tests, or geophysical or wellbore monitoring [Würdemann *et al.*, 2010; Jenkins *et al.*, 2015; Michel-Meyer *et al.*, 2020, and references therein]. Results have demonstrated that the finger-tip velocity offers a better estimate of dissolution rates with a coefficient of determination (R^2) of 0.84,

$$F_{\infty}^* = 0.33X_0^C \rho_0 \frac{\Delta\rho g \kappa_g}{\bar{\mu}} \left(\frac{v_{\infty}}{v_c} \right)^{1.0} \left(\frac{\kappa_x^e}{\kappa_z^e} \right)^{-0.3}. \quad (20)$$

These effective asymptotic dissolution rate models express that the anisotropy of the permeability field (last term of the expressions) negatively affects dissolution rates. When permeability values have substantial spatial continuity horizontally, the corresponding increase in horizontal mixing inhibits nonuniform concentrations, making it harder for instabilities to form. These results differ from those presented by Green and Ennis-King [2014] in homogeneous media with horizontal barriers, where the dissolution rate is proposed to be enhanced by anisotropy with an exponent of 0.5. In accordance with our work, Soltanian *et al.* [2017] and Elgahawy and Azaiez [2021] demonstrated that increasing κ_x^e at fixed κ_z^e can reduce the asymptotic dissolution rate.

We have also explored the impact of heterogeneity on the temporal evolution of dissolution rates. We have shown that in heterogeneous porous media, vertical preferential channels, formed by well-connected high-permeability zones, play a significant role in initiating and developing instability fingers. Moreover, the presence of horizontal well-connected high-permeability structures inhibits the vertical growth of fingers by favoring dispersive mixing. Consequently, in anisotropic heterogeneous porous media, an increase in the degree of heterogeneity leads to a decrease in the asymptotic dissolution rate.

We note that the results from this study may have potential application to other common gravity-driven convection problems, such as contaminant migration, geothermal exploitation, salt-water intrusion and mineral precipitation/dissolution, where density differences may exist [Zhang and Schwartz, 1995; Simmons *et al.*, 1999; Nield *et al.*, 2008].

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A Density and Viscosity

In the numerical model, instead of using a simplified linear expression of the brine density based on the aqueous CO₂ concentration [e.g., Elenius *et al.*, 2015; Martinez and Hesse, 2016], we incorporated a more realistic brine density expression derived from [Vilarrasa, 2012], given the sensitivity of instability to the fluid property [Jafari Raad *et al.*, 2016; Rasmusson *et al.*, 2015, 2017]. In this model, the brine density, ρ [kg·m⁻³], depends on brine phase pressure, temperature, molality of NaCl and CO₂ concentration. The expression for brine is given by Garcia [2003]

$$\rho = \rho_{lr} + cM^C - c\rho_{lr}V_{\phi}, \quad (A.1)$$

516 where c [$\text{mol}\cdot\text{m}^{-3}$] is the number of moles of CO_2 per unit volume of brine phase; M^C [$\text{kg}\cdot\text{mol}^{-1}$]
 517 is the molar mass of CO_2 ; and V_ϕ [$\text{m}^3\cdot\text{mol}^{-1}$] is the apparent molar volume of CO_2 given by

$$V_\phi = 3.751 \times 10^{-5} - 9.585 \times 10^{-8}T_c + 8.74 \times 10^{-10}T_c^2 - 5.044 \times 10^{-13}T_c^3, \quad (\text{A.2})$$

518 where T_c [$^\circ\text{C}$] is temperature in Celsius; ρ_{lr} is the brine density when there is no CO_2 dissolu-
 519 tion, calculated by *Phillips et al.* [1982]

$$\rho_{lr} = -3.033405 \times 10^3 + 1.0128163 \times 10^4\iota - 8.750567 \times 10^3\iota^2 + 2.66310 \times 10^3\iota^3, \quad (\text{A.3})$$

520 with

$$\begin{aligned} \iota = & -9.9595 \exp(-4.539 \times 10^{-3}m^S) + 7.0845 \exp(-1.638 \times 10^{-4}T_c) \\ & + 3.9093 \exp(2.551 \times 10^{-10}p), \end{aligned} \quad (\text{A.4})$$

521 where m^S [molal] is the molality of NaCl and p [pa] is the pressure of brine. Equation (A.1) ap-
 522 plies to $5 < T_c < 297$ [$^\circ\text{C}$] and $p_{sv} < p < 300$ [bar]. Equation (A.3) applies to $10 < T_c <$
 523 350 $^\circ\text{C}$, $0.25 < m_l^S < 5$ [molal] and $p_{sv} < p < 500$ [bar] [*Phillips et al.*, 1982]. Here, p_{sv} is
 524 saturated vapor pressure. Rearranging Equation (A.1), we have [*Vilarrasa*, 2012]

$$\rho = \rho_{lr} \frac{1}{1 - X^C f_\delta} \approx \rho_{lr} (1 + X^C f_\delta), \quad (\text{A.5})$$

525 with

$$f_\delta = 1 - \rho_{lr} \frac{V_\phi}{M^C}; \quad (\text{A.6})$$

526 here X^C denotes the mass fraction of CO_2 . The viscosity of brine is calculated following the works
 527 of *Garcia* [2003] and *Kumagai and Yokoyama* [1999]

$$\begin{aligned} \mu = & (3.85971 - 1.32561 \times 10^{-2}T_k)m^S + (-5.37539 + 1.90621 \times 10^{-2}T_k)(m^S)^{1/2} \\ & + (8.79552 - 3.17229 \times 10^{-2}T_k)m^C + (-7.22796 + 2.64498 \times 10^{-2}T_k)(m^C)^2 \\ & + 1.69956 \times 10^{-9}(p - 1 \times 10^5) + \mu_w(T_k, p = 10^5[\text{Pa}]), \end{aligned} \quad (\text{A.7})$$

528 where T_k [K] is temperature in Kelvin, m^C [molal] is the molality of CO_2 , and μ_w [mPa·s] is
 529 the viscosity of pure water.

530 B Dimensionless Governing Equations

531 Given the dimensionless variables defined in section 4.2, the governing mass balance equa-
 532 tions (4) and (5) can be written in dimensionless form as

$$\frac{\partial X^{C*}}{\partial t^*} = -\nabla^* \cdot (X^{C*} \mathbf{q}^*) + \nabla^* \cdot (\nabla^* X^{C*}), \quad (\text{B.1})$$

$$\frac{\partial X^{H*}}{\partial t^*} = -\nabla^* \cdot (X^{H*} \mathbf{q}^*) + \nabla^* \cdot (\nabla^* X^{H*}), \quad (\text{B.2})$$

533 where Darcy's law is expressed as

$$\mathbf{q}^* = -\exp(Y') (\nabla^* p^* - \rho^* \nabla^* z^*), \quad (\text{B.3})$$

534 and $\nabla^* = [\partial/\partial x^*, \partial/\partial z^*]$. Y' is the deviation of the natural log of the intrinsic permeability from
 535 the mean, i.e., $Y' = Y - \langle Y \rangle$. The geometric mean permeability is $\kappa_g = \exp(\langle Y \rangle)$. The fluid
 536 pressure and the density are normalized by

$$p^* = \frac{p\kappa_g}{\mu\phi D}, \text{ and } \rho^* = \frac{\rho}{\Delta\rho}. \quad (\text{B.4})$$

537 Open Research

538 This work has no data-sharing issues, because all of the results are obtained by numerical
 539 solutions and have been provided in the figures and tables.

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Supporting Information for “ Numerical Analysis of the Effect of Heterogeneity on CO₂ Dissolution Enhanced by Gravity Driven Convection”

Yufei Wang^{1,2} Daniel Fernández-García^{1,2} Maarten W. Saaltink^{1,2}

¹Dept. of Civil and Environmental Engineering, Universitat Politècnica de Catalunya, Jordi Girona 1-3, 08034 Barcelona, Spain

²Associated Unit: Hydrogeology Group (UPC-CSIC)

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Introduction

This supporting information is comprised of two sections. The first section (S1) shows how to choose the grid size for numerical simulation, and the second section (S2) gives detailed results on gravity driven convection in homogeneous fields.

Text S1: Optimum Setup Design

An optimum setup design could represent the Gravity Driven Convection (GDC) with smallest number of grids and shortest simulation time. To get a good setup design we need to know both the behavior of the GDC and the properties of the field and the fluid.

First, two basic features of GDC are used as references in designing the simulation setup. We use characteristic wave length (or earliest finger width) ℓ_c to assist designing the grid and domain sizes, and use the characteristic onset time τ_c to assist setting the simulation time. They are, respectively, given as

$$\ell_c = \ell_c^* \frac{\mu\phi D}{\Delta\rho g \kappa_g} \quad (1)$$

and

$$\tau_c = \tau_c^* \frac{(\mu\phi)^2 D}{(\Delta\rho g \kappa_g)^2}, \quad (2)$$

where, ℓ_c^* and τ_c^* are, respectively, dimensionless characteristic wavelength and onset time, which are constants obtained from theoretical, numerical or experimental studies; μ is the viscosity; ϕ is the porosity; D is the diffusion coefficient; $\Delta\rho$ is the maximum density increase due to CO₂ dissolution; g is the gravitational acceleration; κ_g is the geometric mean permeability. For homogeneous media, the empirical values for ℓ_c^* and τ_c^* are listed in Table S1, which also contains the results from this study.

The domain size should be large enough to cover enough number of convection fingers, while the grid size should be smaller than or close to the size of convection fingers. From our testing simulations (see Figures S1 and S2), we find that the domain size of $30L_c \times 30L_c$ and grid size of $0.3L_c \times 0.3L_c$ are enough to describe the GDC. Increasing the domain size to $60L_c \times 30L_c$ and $30L_c \times 60L_c$ or reducing the grid size to $0.15L_c \times 0.3L_c$, $0.3L_c \times 0.15L_c$

and $0.15L_c \times 0.075L_c$ does not systematically affect the onset time and dissolution rate. We notice that reducing or increasing the (especially vertical) grid size could significantly affect the characteristic wave length (i.e., the width of earliest unstable fingers) but this does not affect the statistic dissolution rate. Results from Elenius et al. (2015) show that the dissolution can be correctly simulated even if the grid size is several times larger than the characteristic wave length, although the concentration profile is not precisely represented. For homogeneous fields, a simulation time of $6\tau_c$ is enough to obtain the asymptotic dissolution rate. For heterogeneous fields, the simulation is not terminated until the fast unstable finger touches the bottom, to make sure the finger has ergodically traveled the heterogeneous domain; here, we define the finger has touched the bottom when the maximum $\text{CO}_2(\text{aq})$ concentration on the bottom increases to 25% of the maximum $\text{CO}_2(\text{aq})$ concentration.

Second, for numerical simulations in heterogeneous fields, we need also to take into account the correlation length scale. The domain size should be, if possible, 4 times larger than the correlation length scale which should be at least 5 times larger than the grid size. Because the domain size is $30L_c$, the correlation length should be less than $10L_c$. Because the grid size is $0.3L_c$, the minimum correlation length is $1.5L_c$. Here, maximum employed correlation length is $8L_c$ and minimum employed correlation length is $2L_c$. Additionally, the horizontally layered formation is used to represent the infinite correlation length in the horizontal direction.

Text S2: Gravity-Driven Convection in Reference Homogeneous Fields

We begin the analysis of Gravity-Driven Convection (GDC) from the results in homogeneous fields. The analysis of GDC in homogeneous field is given for two reasons. First, the analysis on GDC in homogeneous field serves as benchmark of the numerical algorithm, since it has been well studied in literature. Second, it shares the general behavior of GDC, and thus it serves as reference for the heterogeneous field.

From Figure S3, we can see that the dimensionless dissolution rate in homogeneous media (F^*) initially follows the pure diffusion function, until when $t^* \approx 500$, after that, the dissolution rate increases due to the gravity driven convection enhances the dissolution rate. The dissolution rate finally reaches the asymptotic value of around 0.09. This dissolution rate ($F^* \approx 0.09$) obtained in this work is slightly larger than 0.075 in Elenius & Johannsen (2012) and 0.065 in Martinez & Hesse (2016); in the latter the Capillary Transition Zone (CTZ) boundary is employed. If we compare the dissolution rate to the vertical and horizontal finger velocities, we can see that the increase of dissolution rate is accompanied by the increase of the vertical and horizontal finger velocities. This implies that the dissolution rate is dominated by advection when gravity-driven instability appears. The vertical finger velocity is around 0.30, which is larger than 0.21 obtained in Elenius & Johannsen (2012), because impermeable top boundary is employed in the latter study.

Here, we specially note that the onset time for the enhanced dissolution is defined as nonlinear onset time, which is the time when instability fingers emerge (Elenius & Johannsen, 2012). If we observe the middle panel of Figure S3, we can find that the vertical finger velocity first decreases and then increases at around $t^* = 250$. Hidalgo & Carrera

(2009) also show the flow rate initially decreases and then increases to a constant value. The time at the inflection point is defined as linear onset time (Elenius & Johannsen, 2012). Here, we only consider the nonlinear onset time because it is more related to the dissolution efficiency in practice. The nonlinear onset time is affected by the initial perturbation, the field heterogeneity and the computational accuracy. For the real heterogeneous field, the nonlinear onset time is negligibly small, as shown in the paper supported by this document.

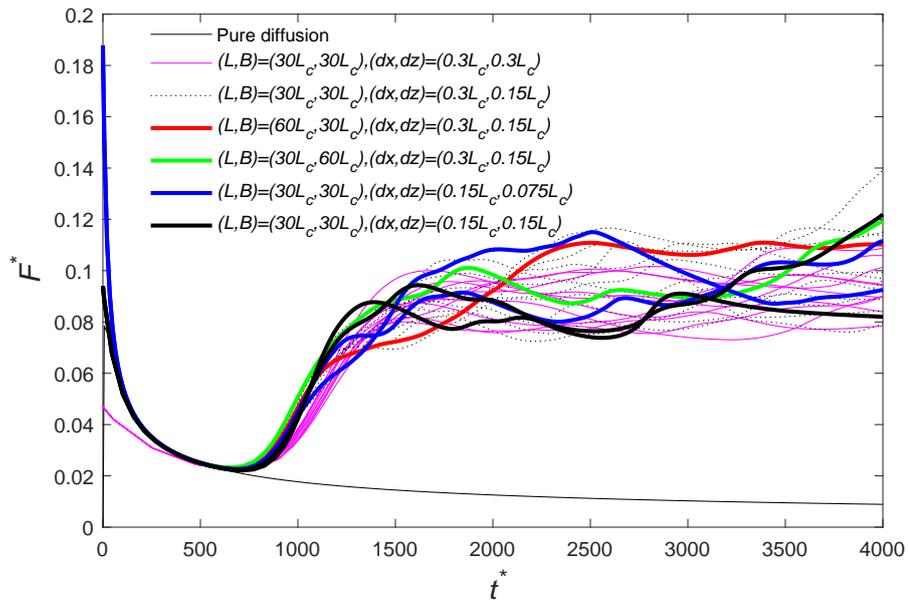


Figure S1: Analysis of the effect of grid size on the the dissolution rate.

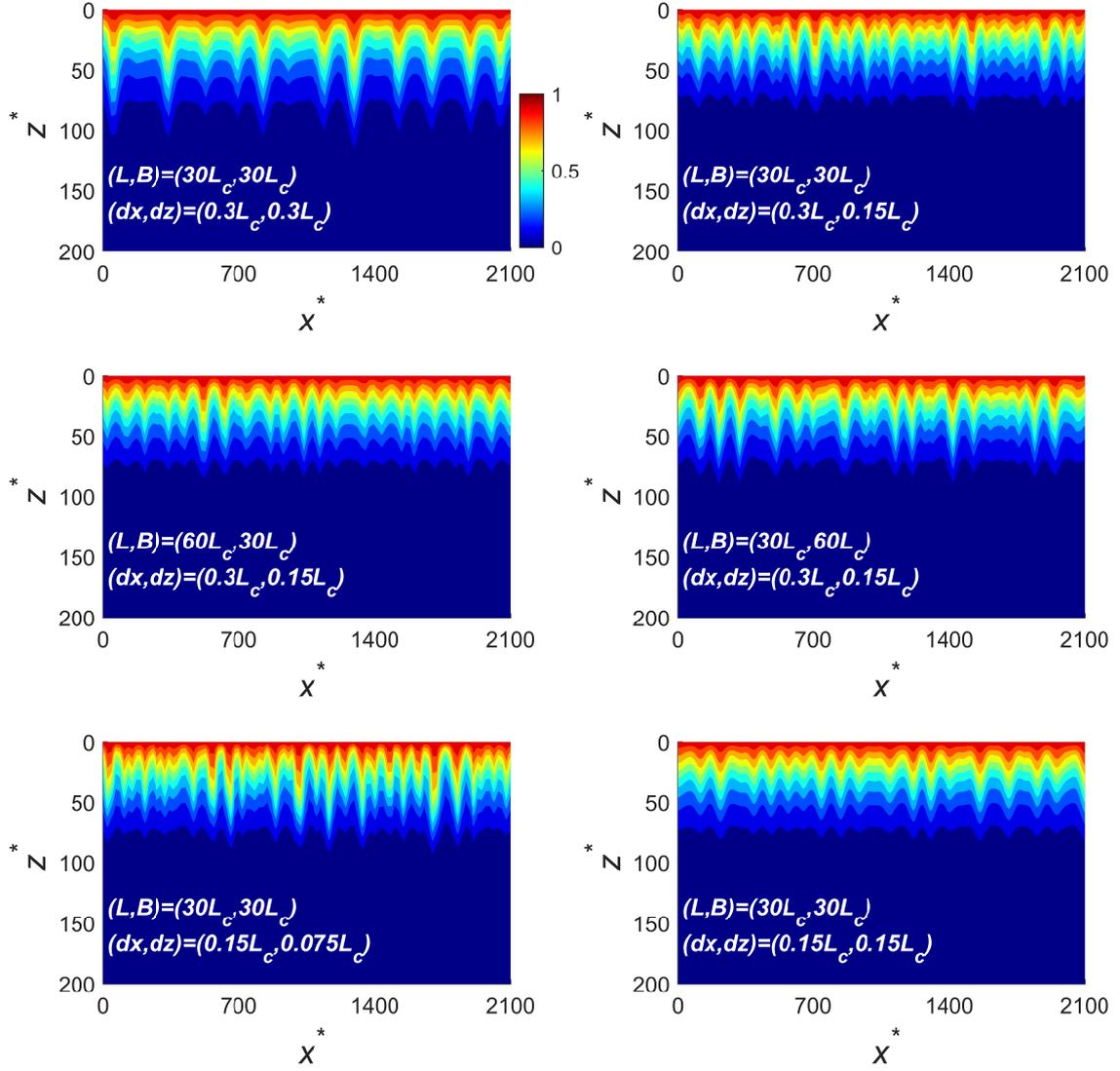


Figure S2: Analysis of the effect of grid size on the finger size (for the case of $(L, B) = (60L_c, 30L_c)$, $(dx, dz) = (0.3L_c, 0.15L_c)$, we only show $x \in (0, 30L_c)$).

Table S1: Published data for dimensionless onset time (τ_c^*) and dimensionless wave length (ℓ_c^*) for (isotropic) homogeneous field.

Onset time (τ_c^*)	Wave length (ℓ_c^*)	Top boundary	Method	Reference
30,75	95	-	Theor.	Ennis-King et al. (2005)
78	92	-	Theor.	Ennis-King & Paterson (2005)
75	96	-	Theor.	Xu et al. (2006)
1,156- 1,412	34-38	diff. only ^a	Num.	Pruess & Zhang (2008)
146	90	-	Theor.	Riaz et al. (2006)
1,000- 5,000	-	diff. only	Num.	Pau et al. (2010)
47.9	115.3	-	Theor.	Cheng et al. (2012)
3,500	-	diff. only	Num.	Cheng et al. (2012)
4,860	170	diff. only	Num.	Elenius & Johannsen (2012)
31	73	-	Theor.	Elenius et al. (2012)
5800	-	diff. only	Num.	Elenius et al. (2012)
2500	-	CTZ ^b	Num.	Elenius et al. (2012)
845	-	diff. only	Num.	Azin et al. (2013)
1,200	80-140	diff. only	Num.	Slim (2014)
56	-	-	Theor.	Jafari Raad et al. (2016)
$Ra^{0.8573}$	-	Permeable	Exp.	Rasmusson et al. (2017)
500	70	Permeable	Num.	This study

^a The top boundary only allows mass to go through the boundary via diffusion.

^b The top boundary is capillary transition zone.

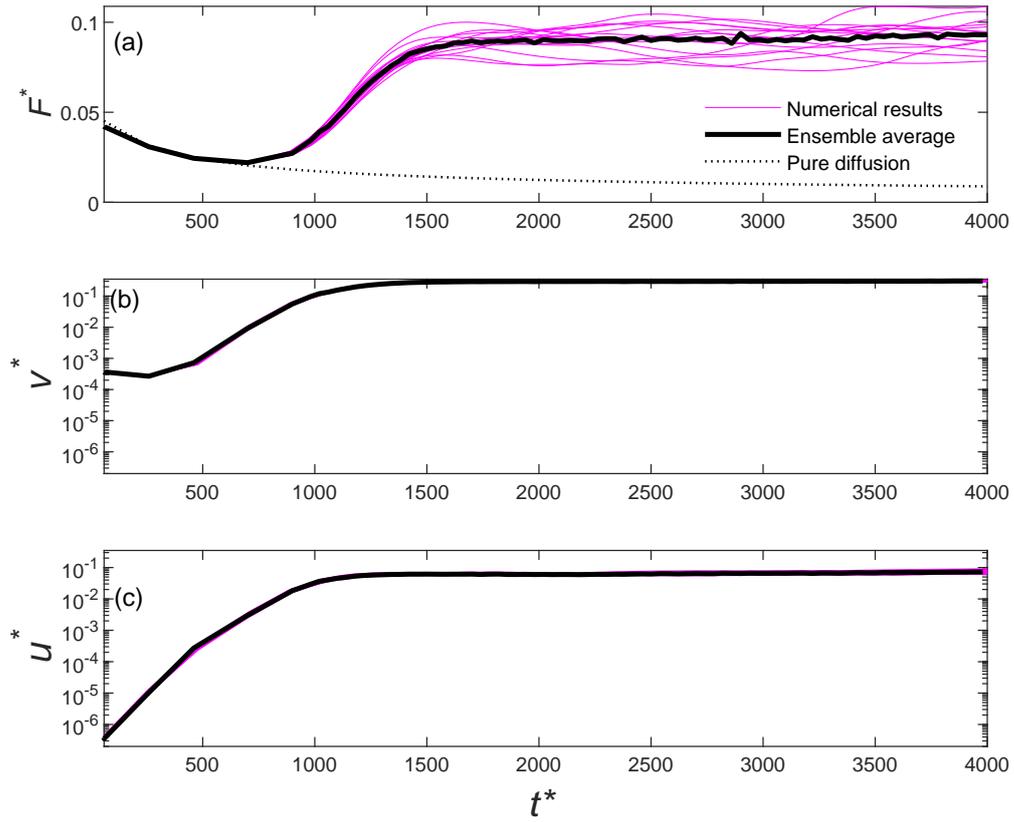


Figure S3: Homogeneous case: temporal developments of dimensionless dissolution rate (top), dimensionless vertical finger velocity (middle) and dimensionless horizontal finger velocity (bottom).

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