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A Short Note on "Two-layered Model (Casson-Newtonian) for Blood Flow Through an Arterial Stenosis: Axially Variable Slip Velocity at the Wall"

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Abstract

The purpose of this note is to analyze the work done by Ponalagusamy and Tamilselvi [1]. Ponalagusamy and Tamilselvi [1] have obtained the analytic expression for an axially variable slip velocity at the wall by assuming the flow rate for the case of one-layered model with slip velocity is equal to that of two-layered model with zero slip velocity. This assumption may, in general, not be valid. Keeping this in view, the flow rate for the case of one-layered model with slip velocity is generally considered to be equal to the product of a constant(C_1) and the flow rate of two-layered model without slip velocity, where $0 < C_1$. Slip velocity at the wall has been computed for different values of C_1 for tube diameters 40 μm and 66.6 μm. It is observed from the present investigation that a series of experiments on blood flow is to be performed to determine the proper values of C_1 under various values of flow parameters and radius of the arterial stenosis.

Key words: New formulas, Blood flow, Axially variable slip velocity, Stenosed arteries, A two-fluid model, Casson fluid, Different shapes of stenoses

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Discussion and Conclusion

In the paper [1], it is mentioned that the introduction of a thin solvent layer near the wall produces the same effect as that of the slip at the wall (Bennett [2]). In the case of one layered model ($R = R_1$) with slip at the wall, the flow rate Q_{1L} is given as (for more detail, refer [1]):

Q_{1L} =

$$\frac{\rho^* q(z)\,\mathrm{Re}}{8\beta} R^4 + R^2 u_s(z) + \frac{\mathrm{Re}\,\theta\rho^*}{3}\{R^3 - R_p{}^3\} - \frac{2\sqrt{2}\rho^*\,\mathrm{Re}}{7\sqrt{\beta}}\{\theta q(z)\}^{1/2}\{R^{7/2} - R_p{}^{7/2}\}\ ,$$

…(1)

where $\rho^* = \dfrac{\bar{\rho}_p}{\bar{\rho}}$, $\mathrm{Re} = \dfrac{\bar{\rho}\bar{U}_0\bar{R}_0}{\bar{\mu}^*}$ and $\bar{\rho}$ and $\bar{\mu}^*$ are the density and viscosity of the fluid when the flow is one-layered. For the two-layered model without slip at the wall ($u_s = 0$), the flow rate Q_{2L} is given as

Q_{2L} =

$$\frac{q(z)R_{ep}R^4}{8\beta}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right] + \frac{\mu R_{ep}\theta R^3}{3}\{(1-\frac{\delta(z)}{R})^3 - (\frac{R_p}{R})^3\}$$
$$-\frac{2\sqrt{2}\mu R_{ep}}{7\sqrt{\beta}}\{\theta q(z)R^7\}^{1/2}[\{1-\frac{\delta(z)}{R}\}^{7/2} - (\frac{R_p}{R})^{7/2}]$$

…. (2)

where $\delta(z)=\bar{\delta}(\bar{z})/\bar{R}_0$ is the non-dimensional peripheral layer thickness which is a function of axial distance z. Since the two models (one-layered with slip and two-layered without slip) represent the same phenomena and reported by(Bennett[2]), the flow rates can be equated as

$$Q_{1L} \;=\; C_1\, Q_{2L}$$

$$\frac{\rho^* q(z)\,\mathrm{Re}}{8\beta}R^4 + R^2 u_s(z) + \frac{\mathrm{Re}\,\theta\rho^*}{3}\{R^3 - R_p{}^3\} - \frac{2\sqrt{2}\rho^*\,\mathrm{Re}}{7\sqrt{\beta}}\{\theta q(z)\}^{1/2}\{R^{7/2} - R_p{}^{7/2}\} =$$

$$C_1\ \{\frac{q(z)R_{ep}R^4}{8\beta}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right] + \frac{\mu R_{ep}\theta R^3}{3}\{(1-\frac{\delta(z)}{R})^3 - (\frac{R_p}{R})^3\}$$
$$-\frac{2\sqrt{2}\mu R_{ep}}{7\sqrt{\beta}}\{\theta q(z)R^7\}^{1/2}[\{1-\frac{\delta(z)}{R}\}^{7/2} - (\frac{R_p}{R})^{7/2}]\ \}$$

Where $Q_{2L} \;=\; Q*$.

....(3)

From Eq.(3), one can obtain u_s (z) as

$$u_s(z)= \frac{q(z)R^2}{8\beta}\left[\ C_1R_{ep}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right] - \rho^*\,\mathrm{Re}\right]$$

$$+\frac{R\theta}{3}[\ C_1\mu R_{ep}\{(1-\frac{\delta(z)}{R})^3 - (\frac{R_p}{R})^3\} - \rho^*\,\mathrm{Re}\{1-(\frac{R_p}{R})^3\}] + \frac{2}{7}\{2\theta q(z)R^3/\beta\}^{1/2}[\rho^*\,\mathrm{Re}\{1-(\frac{Rp}{R})^{7/2}\}$$
$$-\ C_1\mu R_{ep}\{(1-\frac{\delta(z)}{R})^{7/2} - (\frac{R_p}{R})^{7/2}\}]$$

...(4)

From Eq.(4), the dimensional form of the slip velocity $\bar{u}_s$ is obtained as

$$\bar{u}_s = \frac{\bar{q}_0 (\bar{R}_0)^2}{8} [\frac{C_1}{\bar{\mu}_p} \{1 - \{1 - \frac{\bar{\delta}_0}{R_0}\}^4 (1-\mu)\} - \frac{1}{\bar{\mu}^*}] + \frac{\bar{\tau}_0 \bar{R}_0}{3} [\frac{C_1}{\bar{\mu}_c} \{(1 - \frac{\bar{\delta}_0}{\bar{R}_0})^3 - (\frac{\bar{R}_p}{\bar{R}_0})^3\} - \frac{1}{\bar{\mu}^*} \{1 - (\frac{\bar{R}_p}{\bar{R}_0})^3\}]$$

$$+ \frac{8}{7} \{\frac{\bar{\tau}_0 \bar{Q}^* \bar{\rho}_p}{\pi \bar{R}_0^{\ 2}}\}^{1/2} [(\frac{1}{\bar{\mu}^*})^{1/2} \{1 - (\frac{\bar{R}_p}{\bar{R}_0})^{7/2}\} - C_1 \mu (\frac{1}{\bar{\mu}_p})^{1/2} \{(1 - \frac{\bar{\delta}_0}{\bar{R}_0})^{7/2} - (\frac{\bar{R}_p}{\bar{R}_0})^{7/2}\}] \qquad \ldots\ldots(5)$$

It is to be mentioned that ‘-‘over a letter denotes the corresponding dimensional quantity.

The slip velocities at the wall with $C_1 = 1$ for tube diameters $40\,\mu m$ and $66.6\,\mu m$ are mistakenly mentioned in the paper [1] and the corresponding corrected values are given in Table-I.

Tube Diameter	$\bar{u}_s$ $cm/\sec$			
	$C_1 = 0.25$	$C_1 = 0.5$	$C_1 = 0.75$	$C_1 = 1$
$40\,\mu m$	0.30248	1.06002	2.01756	2.87511
$66.6\,\mu m$	0.06103	0.36307	0.66512	0.96716

Table-I Slip Velocity $\bar{u}_s$ $cm/\sec$

It is observed from Table-I that the slip velocity at the wall decreases as the value of C_1 decreases. The value of C_1 should be less than unity due to the fact that the slip velocity cannot be greater than the velocity at the centre of the tube for tube diameter 40 μm. Whereas for the case of tube diameter 66.6 μm, the value of C_1 may be greater than unity. Hence, it is concluded that a series of experiments on blood flow is to be carried out to determine the proper values of C_1 under various values of flow parameters.

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