Preserving Absolute Simultaneity with the Lorentz Transformation

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Preserving Absolute Simultaneity with the Lorentz Transformation

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Abstract

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Keywords: *Simultaneity - Lorentz Transformation - Special Relativity - Ritz Emission Theory - Fizeau Experiment - Stellar Aberration*

1. Introduction

It is known that the two major theories of Physics, Relativity and Quantum Mechanics, are fundamentally incompatible, in particular in the way they conceive and deal with time which is not absolute in the framework of Relativity. This idea emerged with the development of the Theory of Special Relativity, where time is considered dependent from the motion of the observer. This theory has been derived by Einstein[1] on the basis of two postulates: the invariance of the laws of nature for any inertial reference frame and the invariance of the speed of light.

The first postulate is commonly considered as an extension of the Galilean Principle of Relativity to electromagnetic phenomena and its formulation is frequently associated to the invariance of Maxwell equations under the application of the Lorentz transformation. However, as already demonstrated by Somigliana[2], it is possible to show that the D'Alembert equation, which represents the propagation of various kind of waves, not only of electromagnetic nature, is invariant with respect to a more general class of coordinate transformations that include also, as a particular case, the specific form of the Lorentz transformation. This led Somigliana, after having discussed in his work the properties of these general functions from a mathematical point of view, to raise the question of what is the physical meaning of such coordinate transformations in the context of classical Newtonian mechanics, and to answer to this question with the interpretation provided by Voigt[3] which is also discussed by A. Pagano and E.V. Pagano in a note on the relationship between the Lorentz transformation and the notion of simultaneity[4].

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In the present work a different interpretation of the physical meaning of Lorentz-like transformations is presented. This interpretation preserves the concept of absolute simultaneity and absolute time despite making use of the same mathematics of the Lorentz transformation. The starting point that leads to this alternative view is provided by the Galilean Principle of Relativity and by its implications for the propagation of isotropic phenomena with respect to a given observer.

In his formulation of the Principle, as presented in the Dialogue[5], Galilei in fact noted that for an observer located inside the cabin of a ship and stationary with respect to it, any physical phenomenon which is characterized by an isotropic propagation speed for a given state of motion of the ship, will maintain this property also when the entire system is moving with constant and uniform velocity with respect to its original condition. This invariance property is applicable both to wave-like phenomena, that need a propagation medium to occur, and to particle-like or corpuscular phenomena, that can occur also in vacuum and that involve the motion of physical objects, provided that these corpuscular physical entities are emitted by the source with the same constant speed in all directions. However, the same physical phenomena will have different propagation speed along different directions for an observer that is moving inside the ship's cabin with uniform and constant speed. Both observers, the stationary and the moving one, being in a state of rectilinear and uniform motion, represent an inertial reference frame. The difference between them lies in the property of conserving, or not conserving, the isotropy of propagation of the phenomena. This characteristics therefore splits the class of the inertial frames into two groups, and shows that the mathematical laws describing the evolution of the physical phenomena should not be the same for *all* inertial reference frames, and for the associated observers. The mathematical laws of the same physical phenomenon must take, in general, a different form in an inertial frame for which the isotropy of propagation is conserved and into another inertial frame that is in relative motion with respect to the first one and for which the isotropy of propagation is not conserved.

On the basis of the above considerations, in the next Section 2, isotropically propagating signals are employed to derive a criterion to assess the simultaneity of events. This criterion is similar to the one used by Einstein in his 1905 paper[1], but differs from it because it is based not only and not just on the use of light signals to synchronize distant clocks, but on isotropically propagating signals of generic nature and because it must be satisfied *for any* finite value of their characteristic propagation speed. The application of this criterion allows to establish a common time basis for all the points of space in a given stationary reference frame.

In Section 3, a class of Lorentz-like coordinate transformations is derived by imposing the invariance of a scalar quantity, named "characteristic interval", that represents a generalization of the relativistic interval to the case of signals having characteristic propagation speed v_c different from c - the speed of light in vacuum. It is shown that the simultaneity criterion derived into the previous section is mathematically equivalent to the condition of equality of the characteristic intervals between the events of emission and detection of the synchronization signals. Thanks to the invariance of the characteristic interval under Lorentz-like transformations, it is thus possible to preserve, also mathematically, the simultaneity of events between two uniformly moving reference frames. In the Appendix these results are applied to an example of a physical system for which the simultaneity of events is imposed by the symmetry of the configuration. On the basis of what above it is therefore possible to conclude that the transformed time variable τ' does not represent time.

A consequence of the above conclusion regarding the physical meaning of the Lorentz transformed variables is that space-time distortion cannot be invoked to explain the results of physical experiences and that an alternative interpretation of the experimental results should be sought. In

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Section 4 the well-known Fizeau experiment on light propagation is analyzed, with the intent to show that this experiment can be explained without invoking any alteration of the properties of space-time, but rather through the use of the emission theory proposed by W. Ritz[6]. In particular it is shown that the law derived by Fizeau from his experiments with counter-propagating light beams traveling into a stream of moving water can also be obtained in the context of the emission theory by introducing the hypothesis that the presence of turbulence in the flow regime alters the index of refraction of the fluid, hypothesis that appears compatible with the Galilean Principle of Relativity since the presence of turbulence is associated with a not uniform state of motion of the liquid particles. If this alternative explanation of the Fizeau Experiment proves valid, this could remove one of the deficiencies highlighted by Fox [7] in the correlation between Ritz's emission theory and experimental results.

In Section 5 a new test case is presented with the purpose of determining, on experimental basis, which is the right velocity composition formula between the classical Galilean vector sum and the Relativistic rule. This test case is based on the analysis of the phenomenon of Stellar Aberration and is characterized by two distinctive features: a) it is a mutually exclusive test that should allow, at least in principle, to discard either one or the other velocity composition formula; b) it is a purely kinematical test that does not require to measure neither the time, nor the distance traveled by light, being based solely on the analysis of the measured positions of the celestial objects over the course of one or more orbits around the Sun of a terrestrial, or space based, observer.

2. Simultaneity and Time Intervals

In order to describe the governing laws of physical phenomena by means of mathematical expressions it is necessary to define a set of coordinates to associate each event being analyzed to a position in space and to a time of occurrence. The spatial position where the event occurs can be established by means of rigid rulers whilst the determination of time requires the use of clocks that must be synchronized in order to provide a consistent time basis. The synchronization of two clocks which are located in the same spatial position can be done by directly comparing their time readouts. However, when we consider two non coincident clocks, the application of this synchronization method is not straightforward, because of the delay associated with the transport of the information from one location to the other one. It is therefore necessary to establish a method to determine when two events, the readout of the two clocks, occurring at different space locations are simultaneous. Such method can provide the basis to properly synchronize couple of distant clocks and can thus allow to synchronize any clock with a reference, or master, clock in such a way that a common time basis can be established and used to describe the time of occurrence of the physical events being investigated.

Let us consider two generic events occurring at two space locations A and B. Suppose that at the time of occurrence of each event some kind of synchronization signals are emitted from the two points of space where the events occur and that such synchronization signals travel with uniform and constant speed in all directions, i.e. that they propagate isotropically into a reference frame K that is stationary with respect to the two geometric locations A and B. Let v_c be the finite *characteristic speed* of propagation of such signals, which is assumed to be equal in all directions into the K frame. Since each signal propagates isotropically, the locus of the points that is reached after a given interval of time from its emission is a sphere, having the center at the location of the corresponding originating event.

If the two events being considered are simultaneous, i.e. if they both occur at the same time $t_A = t_B = t_o$, then the radii of the two spheres with centers in A and B are equal for every instant

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of time, since the two signals have the same propagation speed v_c. Therefore, the signal coming from A will encounter the other signal coming from B exactly at the midpoint M of segment AB.

Conversely, if two isotropically propagating signals having the same characteristic speed v_c meet each other at the midpoint M of segment AB, then, since the distance traveled by each one is the same by construction, being equal to half the length of AB, and since they both have the same propagation speed, the time elapsed from the emission to the encounter of each signal is the same for both, thus the two originating events A and B are simultaneous.

In order to mathematically derive the above results, let us define a reference frame with the x-axis directed from A to B. Let M be the midpoint of segment AB, and 2L be its length. Let x_1 and x_2 be the x-axis position reached at time t by the two isotropic signals: $x_1(t) = x_A + v_c(t - t_A)$ and $x_2(t) = x_B - v_c(t - t_B)$. Taking into account that $t_A = t_B = t_o$ and that $x_A = x_M - L$ and $x_B = x_M + L$, we can calculate the time of encounter of the signals by solving $x_1(t) = x_2(t)$, which gives $t = t_o + L/v_c$. Inserting this value of t into the expression of x_1 and x_2 gives: $x_1 = x_A + L = x_M$ and $x_2 = x_B - L = x_M$, i.e. the signal coming from A encounters the other signal coming from B at the midpoint M of segment AB.

Conversely, let us assume that two isotropic signals having the same propagation speed v_c meet each other at the midpoint M of segment AB, at some time $t = t_M$, i.e. that $x_1(t_M) = x_2(t_M) = x_M$. The time of occurrence of the two originating events A and B can be calculated by solving for t the two equations: $x_1(t) = x_M + v_c(t - t_M) = x_A$ and $x_2(t) = x_M - v_c(t - t_M) = x_B$, from which it results: $t_A = t_B = t_M - L/v_c$, thus showing that the time of occurrence of the two events is the same, i.e. that the two events are simultaneous.

The same considerations are valid also for any other couple of isotropically propagating signals, emitted from A and B at the same time of the first two ones, but which are characterized by a different value v'_c of their finite propagation speed. Therefore, the criterion of simultaneity of events can be formulated in the following way:

Two events are simultaneous if and only if two isotropically propagating synchronization signals, emitted from the points A and B at the time of occurrence of the corresponding events, meet each other at the mid-point M of segment AB, **for any** *finite value of the characteristic speed v_c of the selected signals.* [1]

This shows that the simultaneity of events is a characteristics that must be invariant with respect to the speed of propagation v_c of the synchronization signals, thus resulting independent from the specific kind of signal being selected.

According to the above criterion, if an event A is simultaneous with a second event B and also with a third event C, then also the two events B and C are simultaneous. The synchronization signals selected to assess the mutual simultaneity between the three events can be different for each couple of events, the outcome of the process will be the same.

The physical nature of the specific signals being used for the synchronization is not relevant for the method, they could be particle-like or wave-like phenomena, nor the value of their characteristic propagation speed v_c, which is only assumed to be finite and equal in all directions. The only assumption required for the validity of the method is that the selected synchronization signals propagate isotropically with respect to the reference frame K. For example, in vacuum one could imagine to employ small particles, emitted in every direction with the same relative speed with respect to the source by a spring-loaded launching device, or one could consider to perturb an ideal string tensioned between its endpoints A and B and use the propagation of the resultant waveform as synchronization signal. In both cases we can ideally imagine of being able to tune

[1] In general, the location of the control point M need only to be selected in such a way that it is equidistant from A and B, i.e. it can be located at the center of a sphere having points A and B on its surface. The midpoint of the segment AB represents the minimum distance choice.

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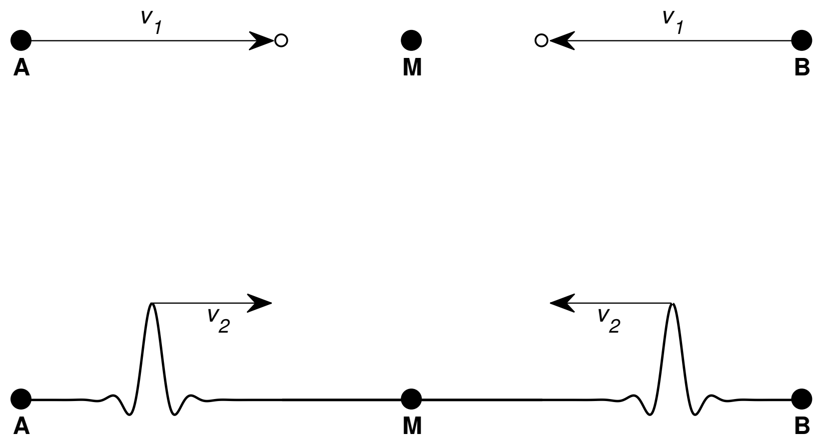

Figure 1: *Simultaneity assessment by means of particle-like (top) or wave-like (bottom) isotropic synchronization signals traveling with characteristic speed $v_c = v_1$ and $v_c = v_2$, respectively. M represents the midpoint of segment AB.*

the value of the signal speed to whatever finite value v_c, by properly adjusting the governing parameters of the selected physical phenomenon (string tension, spring and mass values). In presence of a homogeneous and isotropic medium, other kind of signals could also be employed like, for example, acoustic waves traveling in the air at the speed of sound.

In order to guarantee the isotropy of propagation of the synchronization signals, according to what stated by the Galilean Principle of Relativity, it is necessary that the frame of reference K identified to represent the coordinates of the two events A and B is stationary both with the source of the signals and with the propagation medium (for those phenomena that require a medium to propagate). In the above examples this means that the spring-loaded launcher of the particle-like objects, in one case, and the entire ideal string, in the other case, must be stationary with respect to the frame K.

The process can be applied to any pair of geometrical points in the space and to the corresponding couple of events. In such a way, it can be used to check the synchronization of pairs of clocks placed at distinct space locations. Without losing generality we can assume that the origin of the reference frame K is coincident with one of the two points selected as the source of the synchronization signals. By using this method, therefore, it is possible to verify the synchronization of a *"master"* clock located in the origin of the reference frame K with a clock placed at any point of the entire space domain. This synchronization of the clocks guarantees also that the two clocks run at the same pace, spanning the same time intervals at the two different locations, i.e. it allows to state that $\Delta t_B = \Delta t_A$.

Repeating the same process for all points of the entire space domain it is possible to synchronize all the clocks located at the different geometrical locations of K with the reference time basis of the master clock located in the origin. All clocks will therefore beat in unison, spanning the same time intervals of the master clock. In this manner it is thus possible to associate, in a unique way and consistently with the Galilean Principle of Relativity, the space and time coordinates, expressed into the reference frame K, to any event occurring into the system being observed, and the process is not dependent neither on the type of physical signal used to perform the synchronization nor

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on its characteristics speed v_c, the only requirement for the validity of the synchronization method being that such signals are isotropically propagating with respect to the K frame.

The synchronization check procedure described above, and the related considerations, are valid also when light signals are used to establish the simultaneity of events, provided that the light sources being considered and the transparent light propagation medium, if present, are both stationary with respect to the reference frame of the observer and with the clocks that are being synchronized. The emission hyphothesis formulated by W. Ritz[6], that assumes that light is emitted in all directions with the same relative speed with respect to its source, being fully consistent with the Galilean Principle of Relativity and therefore also compliant with the above requirements of the synchronization procedure, justifies the usage of light signals to synchronize the clocks also into the moving reference frame K'.

3. Lorentz-like transformations of coordinates between moving frames

In this paragraph it will be shown how the absolute nature of simultaneity can be consistently assessed by a moving observer through the use of a class of Lorentz-like transformations of coordinates and by applying the simultaneity criterion defined in the previous Section.

Let us consider two events occurring at two distinct locations A and B of the space and be K a reference frame stationary with respect to the points A and B. Let v_c be the characteristic propagation speed of the isotropic signals that have been selected to synchronize the clocks into this reference frame. According to the previously described synchronization method, the two events are simultaneous if the synchronization signals emitted from A and B at the time of occurrence of the corresponding events meet each other at the midpoint of segment AB.

For any couple of events we can now introduce, into the reference frame K, a *characteristic interval* s_c, that is a scalar quantity dependent from the space and time coordinates of the two events A and B and that is defined by the following relation containing the value of the signal propagation speed v_c as a constant parameter:

$$s_c^2 = v_c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \tag{1}$$

where $\Delta t = (t_B - t_A)$, and $\Delta x = (x_B - x_A)$, $\Delta y = (y_B - y_A)$, $\Delta z = (z_B - z_A)$.

Let us now consider a second reference frame K', having its axes parallel to those of K, and let K' be translating with constant speed V along the positive direction of the x axes with respect to frame K. We will call K the stationary frame and K' the moving frame. Let us also introduce, into the moving frame K', a new set of four *generalized* space-time coordinates, that will be indicated with $(\varepsilon', \eta', \zeta', \tau')$, and that are functions of the (x, y, z, t) coordinates of the stationary reference frame K:

$$(\varepsilon', \eta', \zeta', \tau') = f(x, y, z, t) \tag{2}$$

In what follows it will be shown that it is possible to select the functions f that defines the primed generalized space-time coordinates $(\varepsilon', \eta', \zeta', \tau')$ in such a way that the characteristic interval between two events results invariant in the passage from K to K', and viceversa, i.e. to select f in such a way that it results:

$$\left[s_c'(\varepsilon', \eta', \zeta', \tau')\right]^2 = \left[s_c(x, y, z, t)\right]^2 \tag{3}$$

Since the y and z axes of the two reference frames are parallel by construction and are not mutually traslating along the respective directions, the corresponding coordinates of the two

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frames can be set equal to each other: $\eta' = y$ and $\zeta' = z$, from which it follows: $\Delta\eta' = \Delta y$ and $\Delta\zeta' = \Delta z$. With this choice the problem reduces to that of finding the relations between the (ε', τ') and (x, t) coordinates. We are therefore looking for the specific form of the transformations of coordinates that gives:

$$(v_c \Delta\tau')^2 - (\Delta\varepsilon')^2 = (v_c \Delta t)^2 - (\Delta x)^2 \tag{4}$$

This relation can be satisfied by putting:

$$\varepsilon' = \gamma_c (x - Vt); \qquad \tau' = \gamma_c \big(t - \frac{V}{v_c^2} x\big) \tag{5}$$

with $\gamma_c = 1/\sqrt{(1 - V^2/v_c^2)}$, as it can be verified by substituting these expressions into eq. (4). Therefore, the coordinate transformations that satisfy the invariance property (3) of the characteristic interval have the following form:

$$\varepsilon' = \frac{x - Vt}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \quad \eta' = y; \quad \zeta' = z; \qquad \tau' = \frac{t - \frac{V}{v_c^2} x}{\sqrt{1 - \frac{V^2}{v_c^2}}} \tag{6}$$

and the inverse transformations, from the generalized space-time coordinates of K′ to K, are:

$$x = \frac{\varepsilon' + V\tau'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \quad y = \eta'; \quad z = \zeta'; \qquad t = \frac{\tau' + \frac{V}{v_c^2} \varepsilon'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{7}$$

When light signals propagating in vacuum are chosen as synchronization signals, the characteristic speed is equal to the speed of light in vacuum, $v_c = c$, and the above transformations of coordinates coincide with the Lorentz transformation.

It can be noted that the transformations (6), and the corresponding inverse (7), are not defined for $V = v_c$, whereas for $V > v_c$ the two generalized coordinates ε' and τ' become complex, having a non null imaginary part. Even in this case, these complex coordinates still preserve the invariance of the characteristic interval s_c, as it can be verified by direct substitution of (6) into equation (4). The invariance of the characteristic interval is therefore verified for all values of $V \neq v_c$ and it holds true for any finite value of the characteristic speed v_c of the selected isotropic signal used to synchronize the clocks into the stationary frame K.

When the relative speed V of the moving reference frame is very small with respect to the characteristic speed v_c of the selected synchronization signals, the transformation of coordinates of eqs.(6) can be approximated, to first order in V/v_c, with:

$$\varepsilon' = x - Vt; \quad \eta' = y; \quad \zeta' = z; \qquad \tau' = t - \frac{V}{v_c} \frac{x}{v_c} \tag{8}$$

This expression is not coincident with the form of the Galileian transformation, that is given by eq. (9), because of the term dependent from V and x which is present into the transformed time variable τ' and that vanishes only for the trivial case $V = 0$ and for the non physical case of infinite signal propagation speed $v_c = \infty$.

$$x' = x - Vt; \qquad y' = y; \qquad z' = z; \qquad t' = t \tag{9}$$

Let us now consider, into frame K, two simultaneous events A and B occurring at two generic points of the space and let (x_A, y_A, z_A) and (x_B, y_B, z_B) be the coordinates of the geometrical locations of the two events and $t_A = t_B$ the corresponding time of occurrence. According to the

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definition of simultaneity given in the previous Section, the synchronization signals emitted by A and B will both reach at the same time $t_M > t_A$ the midpoint M of segment AB, with M having coordinates:

$$(x_M, y_M, z_M) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2} \right) \tag{10}$$

The two characteristic intervals, into frame K, between the two simultaneous events A and B being considered and the event O of detection of the arrival of their synchronization signals at the midpoint M are given by:

$$s_{OA}^2 = (v_c \Delta t_{AM})^2 - L_{AM}^2; \qquad s_{OB}^2 = (v_c \Delta t_{BM})^2 - L_{BM}^2 \tag{11}$$

where:

$$\Delta t_{AM} = (t_M - t_A); \qquad \Delta t_{BM} = (t_M - t_B) \tag{12}$$

$$L_{AM}^2 = (x_A - x_M)^2 + (y_A - y_M)^2 + (z_A - z_M)^2 \tag{13}$$

$$L_{BM}^2 = (x_B - x_M)^2 + (y_B - y_M)^2 + (z_B - z_M)^2 \tag{14}$$

Since in the stationary frame K the two points A and B are located symmetrically with respect to the midpoint of the segment, it is $L_{BM} = L_{AM} = L/2$, where L is the length of segment AB, and since the time of emission of the signals is the same, $t_A = t_B$, it follows that $\Delta t_{AM} = \Delta t_{BM}$ and therefore it results:

$$s_{OA}^2 = s_{OB}^2 \tag{15}$$

This shows that, in the stationary frame K, two distinct events A and B are simultaneous when they are separated by the same characteristic interval from the event of the arrival of their synchronization signals at the midpoint of segment AB. Expression (15) can thus be considered as the mathematical formulation of the simultaneity criterion described in the previous section and based on isotropic signals propagating with characteristic speed v_c into frame K. The same considerations can be repeated for any other finite value of the parameter v_c, leading always to the same result expressed by relation (15), thus showing that simultaneity is invariant with respect to the propagation speed v_c of the selected synchronization signals. Moreover, being valid for any finite value of v_c this condition is applicable also to the case of light signals ($v_c = c$) and to the corresponding relativistic intervals[2].

Since the characteristic interval s_c is invariant under the generalized coordinate transformations (6) defined above, we have $(s'_{OA})^2 = (s_{OA})^2$ and $(s'_{OB})^2 = (s_{OB})^2$. If we now consider two simultaneous events into the stationary frame K, for these two events it is $s_{OA}^2 = s_{OB}^2$, and therefore it will also be:

$$(s'_{OA})^2 = (s'_{OB})^2 \tag{16}$$

[2]This can be further generalized by considering a reference event Q, occurring at a given time t_Q at a point of space belonging to the midplane of segment AB. Q is therefore equidistant from A and from B: $L_{AQ} = L_{BQ}$. Since the two events A and B are simultaneous, it is $t_A = t_B$, and therefore we have also $(\Delta t_{AQ})^2 = (t_A - t_Q)^2 = (t_B - t_Q)^2 = (\Delta t_{BQ})^2$ for every time of occurrence of the event Q. Therefore, also in this case it results: $s_{OA}^2 = (v_c \Delta t_{AQ})^2 - L_{AQ}^2 = (v_c \Delta t_{BQ})^2 - L_{BQ}^2 = s_{OB}^2$ and this shows that the simultaneity of two distinct events can be established by evaluating and comparing the two characteristic intervals that separate events A and B from a generic reference event Q occurring at a location that is equidistant from the two events being considered. The two events A and B are simultaneous when the two intervals are equal.

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Thus, according to the same criterion established before, based on the equality of the characteristic intervals, two events that are simultaneous in the stationary frame K are simultaneous also in the moving frame K', and viceversa. This invariance of simultaneity holds true for any value of the relative speed V between the two moving frames and for any kind of physical signals selected to synchronize the clocks, so it holds true for any finite value of their characteristic speed v_c and is thus consistent with the definition of simultaneity given in the previous section. It appears therefore that simultaneity is an absolute characteristic of the events, that can be defined and assessed univocally by different observers that are in a state of uniform relative motion one with respect to the other, by applying the same general criterion of equality of the characteristic intervals with respect to an equidistant reference event.

Let us now calculate, in the moving frame K', the generalized time coordinate τ' of the two simultaneous events A and B. According to (6), it is:

$$\tau'_A = \frac{t_A - \frac{V}{v_c^2} x_A}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad \tau'_B = \frac{t_B - \frac{V}{v_c^2} x_B}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{17}$$

Taking into account that $t_B = t_A$ it thus results:

$$\tau'_B = \tau'_A - \frac{V}{v_c^2} \frac{(x_B - x_A)}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{18}$$

This relation shows that the generalized coordinate τ' of two simultaneous events A and B, evaluated in the moving frame K', has not the same value for the two events, being, in general:

$$\tau'_B \neq \tau'_A \tag{19}$$

In other words, two simultaneous events A and B turn out as being characterized by a different value of the corresponding generalized coordinate τ' which, therefore, cannot be used by the moving observer to represent the time of occurrence of the events, i.e. τ' is not time.[3]

We can now consider the governing laws that describe the isotropic propagation of the signals selected to synchronize the clocks. In particular let us consider the case of the tensioned ideal string. It is known that for small amplitudes, the transverse displacement u of the points of the string is determined by the solution of the d'Alembert equation:

$$v_s^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0 \qquad \textit{with} \qquad u = u(x,t) \tag{20}$$

where $v_s = \sqrt{N/\lambda}$ gives the speed of propagation of the perturbations along the string as a function of the applied axial tension N and linear mass density λ of the string.

According to the Galilean Principle of Relativity, any experimental determination of the string properties and of its response will give identical results when the same characterization tests are repeated into two different laboratories that are uniformly translating one with respect to the other. Therefore, the string behaviour will be represented by the same governing laws in both cases, i.e. the same equation (20) will be determined both by the observer of the stationary laboratory and by the observer of the moving one, and the propagation of the perturbations along the string will remain isotropic and will have the same characteristic speed v_s in both reference frames.

[3] The only particular case for which $\tau'_B = \tau'_A$ occurs when $x_B = x_A$, i.e. when the two simultaneous events A and B are located in a plane orthogonal to the direction of the velocity vector V of frame K', and therefore in a plane orthogonal to the x axis of the K frame, being it parallel to V by construction.

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The situation is different if we consider, into a given laboratory, a moving observer with its associated moving reference frame. Let K be a reference frame stationary with the laboratory, and stationary also with respect to the string, and let K' be another reference frame translating with velocity V parallel to the string axis. For this frame, which is in relative motion with respect to the string, the perturbations on the string will be no more propagating isotropically, their speed being greater than v_s along one direction and lower than v_s in the opposite direction. Correspondingly, also the governing laws of the string will change when expressed into the moving frame K'. In this case therefore the governing law of the string, expressed by equation (20) for the stationary observer, should not be invariant in the transformation from the stationary frame K to the moving frame K'.

Let us now see how the wave equation (20) transforms in the moving frame K' when the generalized coordinates with characteristic speed v_c, as defined by (6), are used. The d'Alembert equation (20) can be reformulated as:

$$\left(v_s\frac{\partial}{\partial x}+\frac{\partial}{\partial t}\right)\left(v_s\frac{\partial}{\partial x}-\frac{\partial}{\partial t}\right)u=0 \tag{21}$$

and the two generalized coordinates (ε', τ') can be written in a more compact form as:

$$\varepsilon'=\gamma_c(x-Vt); \qquad \tau'=\gamma_c(t-Vx/v_c^2) \tag{22}$$

where $\gamma_c = 1/\sqrt{1-V^2/v_c^2}$. This change of variables can be applied to the d'Alembert equation by taking into account that:

$$\frac{\partial}{\partial x}=\frac{\partial\varepsilon'}{\partial x}\frac{\partial}{\partial\varepsilon'}+\frac{\partial\tau'}{\partial x}\frac{\partial}{\partial\tau'}=\gamma_c\left(\frac{\partial}{\partial\varepsilon'}-\frac{V}{v_c^2}\frac{\partial}{\partial\tau'}\right) \tag{23}$$

$$\frac{\partial}{\partial t}=\frac{\partial\varepsilon'}{\partial t}\frac{\partial}{\partial\varepsilon'}+\frac{\partial\tau'}{\partial t}\frac{\partial}{\partial\tau'}=\gamma_c\left(\frac{\partial}{\partial\tau'}-V\frac{\partial}{\partial\varepsilon'}\right) \tag{24}$$

In this way the wave equation (21) takes the form:

$$\gamma_c^2\left(A\,\frac{\partial^2 u}{\partial\tau'^2}+B\,\frac{\partial u}{\partial\varepsilon'}\frac{\partial u}{\partial\tau'}-C\,\frac{\partial^2 u}{\partial\varepsilon'^2}\right)=0 \tag{25}$$

where the three terms A, B and C are given by:

$$A=\left(1-\frac{V^2v_s^2}{v_c^4}\right); \qquad B=2V\left(\frac{v_s^2}{v_c^2}-1\right); \qquad C=(v_s^2-V^2) \tag{26}$$

From these expressions it can be noted that when $v_s = v_c$ it results $B=0$, and $A=1/\gamma_c^2$, $C=v_c^2-V^2$. Substituting these terms into (25), gives:

$$v_c^2\frac{\partial^2 u}{\partial\varepsilon'^2}-\frac{\partial^2 u}{\partial\tau'^2}=0 \quad with \quad u=u(\varepsilon',\tau') \tag{27}$$

Therefore, when the parameter v_c contained into the generalized coordinate transformation (6) is equal to the speed of propagation v_s of the specific phenomenon being described, the corresponding equation governing the evolution of the perturbations along the string is invariant in the passage from the stationary frame K to the moving frame K'. This invariance property, however, is no longer verified when the characteristic speed v_c used in the coordinate transformation is different from v_s. In this case, in fact, the term B is not null, and the equation resulting from the change of coordinates has no longer the same form of the original wave equation.

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This peculiar invariance property of the generalized coordinates is valid not only for the monodimensional case of the string equation that has been considered here, but also for the tridimensional case of the wave equation that has the same form of (20). Also in this more general case, the invariance of the governing equations is satisfied only when the parameter v_c that appears in the definition of the coordinate transformation has the same value of the characteristic speed of the isotropically propagating phenomenon being represented. If some of the physical properties characterizing the phenomenon change, thereby changing the corresponding physical speed of propagation, then the wave equation will take a different form in the passage from the stationary to the moving observer and its solutions will be different along different directions.

Let us now analyze the relationship between the generalized velocity $\mathbf{w}'$, evaluated into frame K′ on the basis of the coordinate transformation defined by (6), and the expression of the velocity into frame K. This can be done by evaluating, from eqs. (7), the differentials:

$$dx = \frac{d\varepsilon' + V d\tau'}{\sqrt{1-\frac{V^2}{v_c^2}}}; \quad dy = d\eta'; \quad dz = d\zeta'; \qquad dt = \frac{d\tau' + \frac{V}{v_c^2} d\varepsilon'}{\sqrt{1-\frac{V^2}{v_c^2}}}; \tag{28}$$

Through the definition of the velocity in the stationary frame:

$$\mathbf{v} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) \tag{29}$$

and, by analogy, of the generalized velocity in the moving frame:

$$\mathbf{w}' = \left(\frac{d\varepsilon'}{d\tau'}, \frac{d\eta'}{d\tau'}, \frac{d\zeta'}{d\tau'}\right) \tag{30}$$

it follows that:

$$v_x = \frac{w'_x + V}{1 + \frac{V w'_x}{v_c^2}}; \qquad v_y = \frac{w'_y \sqrt{1-(V^2/v_c^2)}}{1 + \frac{V w'_x}{v_c^2}}; \qquad v_z = \frac{w'_z \sqrt{1-(V^2/v_c^2)}}{1 + \frac{V w'_x}{v_c^2}} \tag{31}$$

The expression of the generalized velocity $\mathbf{w}'$ into frame K′ is found by inverting the above relations, obtaining:

$$w'_x = \frac{v_x - V}{1 - \frac{V v_x}{v_c^2}}; \qquad w'_y = \frac{v_y \sqrt{1-(V^2/v_c^2)}}{1 - \frac{V v_x}{v_c^2}}; \qquad w'_z = \frac{v_z \sqrt{1-(V^2/v_c^2)}}{1 - \frac{V v_x}{v_c^2}} \tag{32}$$

From these expressions it turns out that when the magnitude of the velocity in the stationary frame K is equal to the characteristic speed, i.e. when $|\mathbf{v}| = v_c$, then also the magnitude of the generalized velocity in the moving frame K′ has the same value: $|\mathbf{w}'| = v_c$. In fact, considering the case $v^2 = v_x^2 + v_y^2 + v_z^2 = v_c^2$ and evaluating the magnitude of $\mathbf{w}'$ from equations (32), it results:

$$|\mathbf{w}'|^2 = v_c^2 \frac{[1 - 2\frac{v_x V}{v_c^2} + \frac{v_x^2 V^2}{v_c^4}]}{(1 - v_x V/v_c^2)^2} = v_c^2 \tag{33}$$

The transformations of coordinates defined by equations (6) represent therefore a change of variables that leaves invariant the characteristic speed. Considering the case of particle-like signals used to synchronize the clocks (for example the spring-loaded launcher device considered in the previous section), and applying the characteristic coordinate transformation (6) to calculate the generalized speed of the particles into the moving frame K′, it turns out that also these particle-like

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signals, that propagate isotropically with speed v_c in the stationary frame K, will have the same value v_c of the generalized speed w' along every direction, also in the moving frame K'. This invariance of the characteristic speed is valid for any finite value of the speed of the specific synchronization signal being considered. When the value of characteristic speed is equal to the speed of light in vacuum, $v_c = c$, the above result corresponds to the invariance of the speed of light under the Lorentz coordinate transformations.

Let us now consider the case of a signal, particle-like or wave-like, traveling along the x axis of the stationary frame K with constant speed v, that is: $v_x = v$ and $v_y = v_z = 0$. Let K' be a moving reference frame which is also traveling along the direction of the x axis of K with uniform constant speed equal to v. According to the Galilean rule of speed composition, the velocity of the signal with respect to such moving frame K' is null, since it is given by $v' = v - V = v - v = 0$ for any value of the common speed v of both the signal and the reference frame. We want now to evaluate the generalized velocity of this signal into the moving frame K' according to the formulas (32) previously established. By putting $v_x = V = v$ and $v_y = v_z = 0$ it results:

$$w'_y = w'_z = 0; \qquad w'_x = \frac{v_x - V}{1 - \frac{V v_x}{v_c^2}} = \frac{v - v}{1 - \frac{v^2}{v_c^2}} = v_c \frac{\beta - \beta}{1 - \beta^2} \tag{34}$$

where $\beta = v/v_c$. Therefore it results $w'_x = 0 \quad \forall\, v \neq v_c$ whilst in the case $v = v_c$, for which $\beta = 1$, the previous expression gives an undetermined form of the type $0/0$, expression that can however be evaluated by applying the rule of de l'Hopital. Putting $f(\beta) = \beta - \beta$ and $g(\beta) = 1 - \beta^2$ it gives:

$$\lim_{\beta \to 1} w'_x = \lim_{\beta \to 1} v_c \frac{f}{g} = \lim_{\beta \to 1} v_c \frac{f'}{g'} = v_c \frac{0}{-2} = 0 \tag{35}$$

Thus, also for the case $v = v_c$ the generalized speed of a signal traveling with speed v, as evaluated by an observer comoving with it at the same speed, $V = v$, is zero. When applied to the case of a light signal, or a photon, traveling in vacuum with velocity $v = c$, this result shows that the generalized speed of a light signal evaluated by a luminal observer, i.e. the speed of light evaluated by a reference frame moving at the same speed of light in vacuum, is null. This result appears in contrast with the postulate of invariance of the speed of light that is at the basis of the Special Theory of Relativity[1], since it shows that there is at least one observer, the luminal observer, for which the speed of light, calculated according to the rules determined by the theory itself, is zero instead of being equal to c as required by the postulate.

In summary, the generalized coordinate transformations defined by (6) are characterized by the following peculiar properties in the passage from a reference frame K to another frame K' that is translating with constant velocity V:

1. they make invariant the characteristic interval s_c defined by equation (1);

2. they leave invariant the constitutive laws representing isotropic propagation of a phenomenon having characteristic speed v_c in the stationary frame K;

3. they maintain, for the generalized speed of propagation in the moving frame K', the same value of the characteristic speed v_c that such phenomenon has in the stationary frame K.

The above properties are verified for any finite value of the characteristic speed v_c and correspond, for the case $v_c = c$, to the same properties of the Lorentz transformation that are valid

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for the propagation of light in vacuum and for the corresponding governing laws as described by the Maxwell equations of electromagnetism. [4]

Being valid for any value of the selected characteristic propagation speed v_c, these invariance properties of the generalized coordinate transformation (6) can be considered as a peculiar mathematical characteristics of this type of coordinate transformations, rather than a dependency of the properties of space and time from the motion of the observer.

It has been shown above by relation (19) that for a moving reference frame K′, two simultaneous events do not have, in general, the same value of the generalized coordinate τ' which therefore cannot be used as a time identification of the events. This coordinate, instead, can be interpreted in a different way as follows. Let us consider, in frame K, a generic event P occurring at a given time t and at a given point (x, y, z) of the space, and let us consider a second reference frame K′, moving with uniform velocity V along the x axis, and having its origin O' coincident with the origin O of frame K at time $t = 0$. The amount of time needed by the synchronization signal emitted from P in order to reach the $x = 0$ plane of frame K is $\Delta t = x/v_c$. In this same amount of time, the origin of frame K′ will have traveled a distance, along the x axis of frame K, equal to $\Delta x = V\,\Delta t = Vx/v_c$. A synchronization signal, traveling with characteristic speed v_c in frame K, would take a time interval equal to $\Delta t_c = \Delta x/v_c = Vx/v_c^2$ in order to cover such distance. It is therefore possible to write the expression of the generalized coordinate τ' in the following way:

$$\tau' = \gamma_c\,(t - \Delta t_c) \tag{36}$$

where $\Delta t_c = Vx/v_c^2$ and $\gamma_c = 1/\sqrt{1 - V^2/v_c^2}$. Equation (36) shows that τ' represents a *retarded* (or *advanced*, depending on the sign of the characteristic time delay Δt_c) and *scaled* generalized time coordinate which is a function of the position x of the event along the direction of motion of the moving frame K′, of its velocity V, and of the characteristic speed v_c of the specific synchronization signal that has been considered. Looking now to the definition of the generalized coordinate ε' associated to the moving frame K′, it turns out that it can be interpreted as a *scaled* version of the position $x' = (x - Vt)$ of the event P along the x axis of frame K′, that uses the same value of the non-dimensional scaling factor γ_c that enters into the definition of the generalized coordinate τ', that is:

$$\varepsilon' = \gamma_c\,x' \tag{37}$$

Let us now consider an event occurring, for the stationary observer K, at a certain space location with coordinates $(x_o; y_o; z_o)$ and at a given time t_o. Let us indicate this event with the quadrivector $\underline{\mathbf{x}}_\mathbf{o} = (x_o; y_o; z_o; t_o)$. In the four-dimensional domain of the (x, y, z, t) variables, all events that are simultaneous with the event $\underline{\mathbf{x}}_\mathbf{o}$ are characterized by having the same time of occurrence $t = t_o$. Therefore all simultaneous events belong to the hyperplane defined by the following equation:

$$(\underline{\mathbf{x}} - \underline{\mathbf{x}}_\mathbf{o}) \cdot \underline{\mathbf{n}} = 0 \tag{38}$$

where $\underline{\mathbf{n}} = (0; 0, 0; 1)$ and the dot product between two quadrivectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ is defined as: $\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} = x_a x_b + y_a y_b + z_a z_b + t_a t_b$. Vector $\underline{\mathbf{n}}$ represents the normal to the plane and is parallel to the axis of time t. If we consider a series of events regularly spaced in time, i.e. separated by a constant time interval Δt, the intercept of the corresponding simultaneity hyperplanes with the x-t plane form a series of horizontal lines, all parallel between themselves and separated by Δt along the direction of the t axis, as shown in the left part of Figure 2.

[4] The above described invariance properties of the Lorentz transformation are true only for the vacuum case, for which the speed of light is equal to c. In any other transparent medium, for which the speed of light is lower than c, the Lorentz transformation does not verify any more these invariance properties.

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Let us now evaluate the generalized coordinates of all the events simultaneous with $\underline{\mathbf{x_o}}$, by applying the Lorentz-like transformation defined by (6) to the coordinates of the above events:

$$\varepsilon' = \gamma_c(x - Vt_o); \quad \eta' = y; \quad \zeta' = z; \qquad \tau' = \gamma_c(t_o - \frac{V}{v_c^2}x) \qquad \forall x, y, z \in \mathbb{R}^3 \tag{39}$$

Eliminating x from the above equations gives the following relation between ε' and τ':

$$\frac{V}{v_c^2}\,\varepsilon' + \tau' = \sqrt{1 - \beta_c^2}\,t_o \qquad \textit{where} \qquad \beta_c = V/v_c \tag{40}$$

Into the four-dimensional domain of the generalized variables $(\varepsilon'; \eta'; \zeta'; \tau')$ this expression represents the equation of an hyperplane orthogonal to the vector $\underline{\mathbf{n}}' = (V/v_c^2; 0, 0; 1)$, vector which in this case is not aligned with the τ' axis having a non null component along the ε' axis. The equation of this hyperplane can be reformulated more compactly in the following way:

$$(\underline{\mathbf{x}}' - \underline{\mathbf{x}}'_{\mathbf{o}}) \cdot \underline{\mathbf{n}}' = 0 \tag{41}$$

where the primed quadrivectors are defined as follows: $\underline{\mathbf{x}}' = (\varepsilon'; \eta'; \zeta'; \tau')$ and $\underline{\mathbf{x}}'_{\mathbf{o}} = (\varepsilon'_o; \eta'_o; \zeta'_o; \tau'_o)$.

This equation provides a convenient and more immediate way to assess event simultaneity when operating with the Lorentz-like transformed coordinates. Given the generalized coordinates of a reference event $(\varepsilon'_o; \eta'_o; \zeta'_o; \tau'_o)$, all $\underline{\mathbf{x}}'$ quadrivectors that satisfy relation (41) are simultaneous with the given reference event. In fact, applying the inverse transformation given by (7) to the coordinates of the quadrivector $\underline{\mathbf{x}}'_{\mathbf{o}}$ we can calculate the time of occurrence of the reference event:

$$t_o = \gamma_c(\tau'_o + \frac{V}{v_c^2}\varepsilon'_o) \tag{42}$$

Inserting the components of $\underline{\mathbf{n}}'$ into Equation (41) gives:

$$(\tau' + \frac{V}{v_c^2}\varepsilon') = (\tau'_o + \frac{V}{v_c^2}\varepsilon'_o) \tag{43}$$

and applying also to this expression the inverse Lorentz-like transformation given by (7), we get:

$$t = \gamma_c(\tau' + \frac{V}{v_c^2}\varepsilon') = \gamma_c(\tau'_o + \frac{V}{v_c^2}\varepsilon'_o) = t_o \tag{44}$$

that shows that all events belonging to the simultaneity hyperplane defined by Equation (41) occur at the same time $t = t_o$, i.e. that they are all simultaneous.

From this analysis it is possible to see that there is a biunivocal relation between the simultaneity planes of the $(x; y; z; t)$ domain and the corresponding simultaneity planes into the domain of the generalized coordinates $(\varepsilon'; \eta'; \zeta'; \tau')$. As shown in Figure 2, the differences between the two cases is that in the generalized four-dimensional domain the simultaneity planes are not orthogonal to the axis of the generalized time τ', and that the separation between two successive planes is given by $\Delta\tau' = \sqrt{(1 - \beta_c^2)}\Delta t$. The amount of tilt of the orthogonal vector $\underline{\mathbf{n}}'$ with respect to the τ' axis depends from the ratio of the relative speed V between the two moving reference frames and from the square of characteristic speed v_c. For $V > 0$ this tilt is therefore always not null for any finite value of the characteristic speed v_c.

Since simultaneous events are those that belong to these hyperplanes, for any tilt, this demonstrates the independence of events simultaneity from the choice of the characteristic speed v_c of

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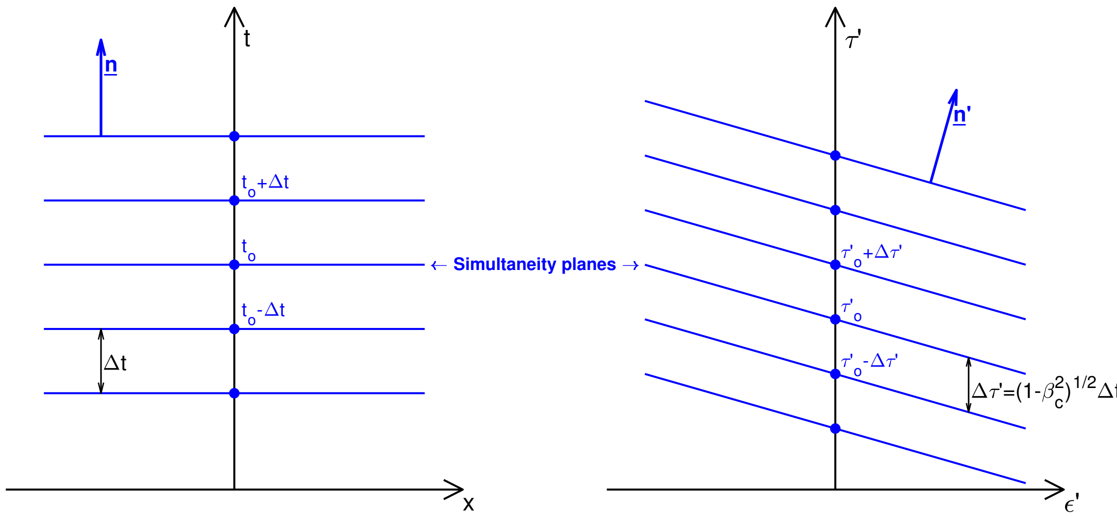

Figure 2: *Intersection with the x-t and ε'-τ' plane of uniformly separated simultaneity hyperplanes.The tilt and the vertical separation of the hyperplanes vary with the velocity V of the observer and with the characteristic speed v_c of the synchronization signals. Each simultaneity hyperplane represents the separation between past and future events.*

the signals and from the speed V of the reference frame and also shows that event simultaneity is independent from their spatial location, holding true for any point belonging to a given hyperplane.

Each simultaneity hyperplane splits the four dimensional domain into two non intersecting regions. All events lying below a given hyperplane occur at an earlier time with respect the time of the events that lie on the simultaneity plane, whilst all events above the same plane occur later than that time, i.e. all events below a given plane belong to the past, whilst all events above the plane belong to the future.

In what follows it will be shown that events simultaneity can be consistently established, also in the domain of the transformed coordinates, by means of the direct application of the simultaneity criterion presented into Section II. Let us consider two events A and B occurring, at time $t = 0$, at two spatial positions along the x-axis of frame K and simmetrically located with respect to the origin, so that the midpoint M of segment AB coincides with the origin. Let $\underline{\mathbf{x}}_{\mathbf{a}} = (-L; 0; 0; 0)$ and $\underline{\mathbf{x}}_{\mathbf{b}} = (L; 0; 0; 0)$ be the quadrivectors corresponding to these two events. As it has been shown in Section II, in the stationary frame K two isotropic signals emitted from A and B at the time of occurrence of the two events meet each other at the midpoint M, for any value of their propagation speed. Graphically this corresponds to the situation illustrated in Figure 3a) for two different couples of isotropic signals, one couple of signals having propagation speed equal to the characteristic speed v_c that enter into the transformation of coordinates defined by (6), the other couple characterized by a different propagation speed w. In both cases the space-time lines describing the propagation of the two couple of signals intersect themselves along the vertical dotted line that represent the locus of all events occurring at midpoint M, and this holds true also for any other value of the isotropic signal propagation speed.

Let us now calculate the transformed $(\varepsilon'; \tau')$ coordinates of the two events A and B. From (6)

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we have:

$$\varepsilon'_a = -\gamma_c L; \qquad \tau'_a = \gamma_c \frac{V}{v_c^2} L \tag{45}$$

$$\varepsilon'_b = \gamma_c L; \qquad \tau'_b = -\gamma_c \frac{V}{v_c^2} L \tag{46}$$

In the reference frame K, let w be the propagation speed of the two isotropic signals emitted from A and B, so that $|\mathbf{w_a}| = |\mathbf{w_b}| = w$. Applying the velocity composition rule defined by (32) we can calculate, into the moving frame K', their generalized velocity along the ε' axis:

$$w'_{ax} = \frac{w - V}{1 - \frac{wV}{v_c^2}}; \qquad w'_{bx} = \frac{-w - V}{1 + \frac{wV}{v_c^2}}; \tag{47}$$

With these two expression it is now possible to determine the space-time trajectories $\varepsilon'_a(\tau')$ and $\varepsilon'_b(\tau')$ of the two signals, by imposing the passage from the coordinates of corresponding originating events A and B:

$$\varepsilon'_a = v_c^2 \frac{w - V}{v_c^2 - wV} \tau' - \gamma_c L \frac{v_c^2 - V^2}{v_c^2 - wV} \tag{48}$$

$$\varepsilon'_b = -v_c^2 \frac{w + V}{v_c^2 + wV} \tau' + \gamma_c L \frac{v_c^2 - V^2}{v_c^2 + wV} \tag{49}$$

The encounter of the two signals occurs when $\varepsilon'_a(\tau') = \varepsilon'_b(\tau')$, from which it results:

$$\overline{\tau}' = \gamma_c \frac{L}{w} \tag{50}$$

Substituting this value of τ' into the expression of the ε' coordinate of the two signals gives:

$$\varepsilon'_a = \varepsilon'_b = \overline{\varepsilon}' = -\gamma_c \frac{LV}{w} \tag{51}$$

Let us now calculate, in reference frame K', the transformed coordinates of all the events that occur at the midpoint M for any time of occurrence. Into the stationary frame K, these events are given by $x = 0 \ \forall t$, and they all lie on a vertical line parallel to the t axis, as shown in Figure 3a). Transforming these events by means of (6)

$$\varepsilon' = -\gamma_c V t; \qquad \tau' = \gamma_c t \tag{52}$$

and eliminating t allows to find the relation between the ε' and τ' coordinates of the midpoint:

$$\varepsilon' = -V\tau'; \tag{53}$$

Into the $(\varepsilon';\tau')$ plane this is the equation of a straight line that is not parallel to the τ' axis, as shown in Figure 3b), which accounts for the fact that in the K' frame the position of the midpoint M is no longer constant but is continuously changing because of the uniform velocity V of frame K' with respect to K. It is thus verified that the generalized coordinates $(\overline{\varepsilon}';\overline{\tau}')$ of the point of encounter of the two isotropic signals, given by equations (50) and (51), belongs to the space-time trajectory of the midpoint M, for any value of the propagation speed w, thus showing, in agreement with the criterion of Section II that the two events A and B are simultaneous also for the moving observer K'.

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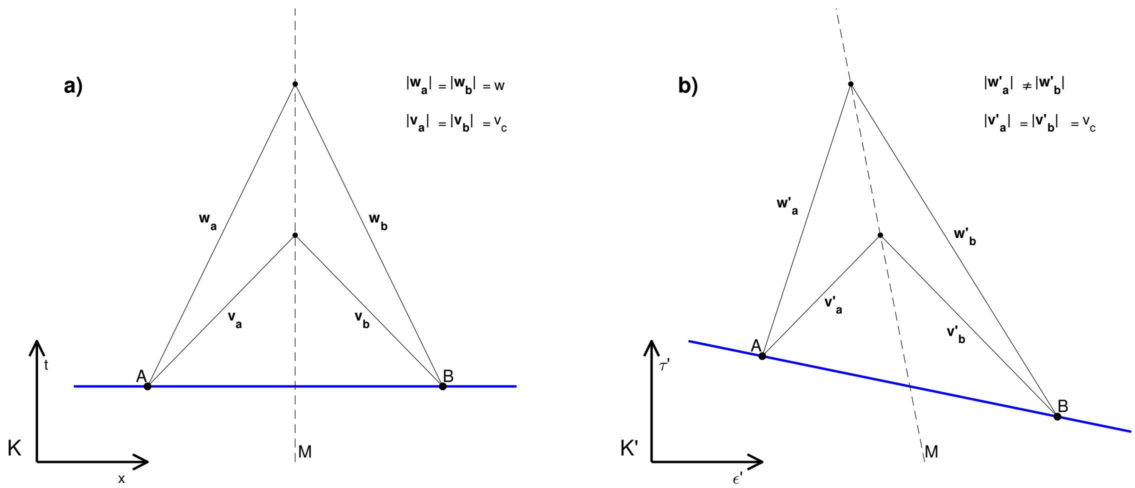

Figure 3: *Space-time representation of the simultaneity criterion: (a) for a stationary observer K, (b) for a moving observer K′ using the Lorentz-like coordinates. The blue line represents the plane of all events simultaneous with A and B. The dashed line represents the locus of all events occurring at midpoint M of segment AB. In both reference frames the two lines representing the propagation of isotropic signals emitted from A and B intersect themselves in correspondence of the midpoint M, for any finite value of the propagation speed. Only when $v = v_c$ the generalized speed of the signal into the moving frame is equal to the characteristic speed, $v'_a = v'_b = v_c$, i.e. it is invariant and remains also isotropic in the passage from K to K′.*

When the coordinates of the events into the moving frame K′ are calculated by means of the Galilean transformation (9), that assumes $t' = t$, the situation depicted in Figures 2 and 3 is similar, with the difference that the simultaneity planes are now horizontal and separated by the same interval Δt. In this case therefore the simultaneity of events in the moving frame is determined by the equality of the transformed time variable, i.e. in the frame K′ event A is simultaneous with event B when $t'_A = t'_B$. It can be shown that the simultaneity criterion of Section II holds true also in this case since every couple of isotropic signals meet along the space-time trajectory of midpoint M for both reference frames K and K′ and for any value of the signal speed.

4. The Fizeau experiment

In the Fizeau experience[8], the propagation of light into a stream of water flowing within pipes has been investigated by analyzing the fringe patterns generated at the recombination of two light beams that are counter-propagating in the fluid stream. The analysis of the test results obtained by Fizeau with its original apparatus led to the following expression for the relative velocity W_F of the light with respect to the stationary system of the laboratory:

$$W_F = w + v(1 - \frac{1}{n^2}); \tag{54}$$

where v is the velocity of the fluid flowing into the pipes having circular cross-section, and $w = c/n$ is the speed of light into the fluid being utilized for the test, characterized by an index of refraction equal to n when such fluid is stationary.

We want now to investigate if the experimental result expressed by eq. (54) that was derived by Fizeau, can be obtained by applying the Ritz emission hypothesis and the associated Galilean vector sum of the velocities of light into the propagation medium and the velocity of the fluid.

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This requires the determination of the actual value of the speed v of the fluid flow to be used in the vector sum formula, since in this kind of experiment the propagation medium used, typically water, is not characterized by a common and uniform state of motion of all its particles inside the volume occupied. The motion of the fluid in fact cannot be represented as a pure rigid body translation with constant speed, therefore there is not just a single value of the velocity for the entire fluid, but a rather complex velocity field with a distribution that varies from point to point inside the volume of the pipes.

In addition, the specific geometrical layout of the Fizeau test setup introduces several factors that can affect the characteristics of the interference fringes formed at the recombination of the two counter propagating beams, in particular:

1. the non null area of the optical path, because of which interference fringes can arise also in absence of fluid motion (and actually also without any fluid) due to the Sagnac effect associated to the Earth's rotation;

2. the radial shape of the velocity profile of the fluid motion at the various sections of the pipes;

3. the axial speed of the fluid flow which varies along the pipe length and therefore along the optical path;

4. the non-axial components of the fluid velocity associated to a turbulent regime of the flow.

The Sagnac effect can be considered as a constant bias, since both the angular velocity of the laboratory where the experiment is performed and the area of the optical path, determined by the geometrical layout of the test setup are not varied during the execution of the measures, therefore the product ΩA remains constant.

The radial distribution of the axial velocity profile of the fluid flow can cause a distortion of the shape of the incident wavefront of the light beam. For the beam propagating in the same direction of the fluid stream an incident planar wavefront could be deformed in a way similar to that of a plane-concave lens, since the equivalent optical length of the light rays closer to the centerline of the pipes will be shortened by the dragging effect due to the fluid flow more than that of the rays travelling farther from the pipe centerline. Conversely, the wavefront deformation associated to the light beam traveling against the fluid stream should be similar to the one generated by a plane-convex lens. On the recombining plane, the interference pattern generated by the two counter-propagating beams would be affected by the actual shape of the two distorted wavefront and in particular this effect could generate a variation of the fringe spacing if the shape of the radial velocity profile changes as a result of changes of the average flow rate. This effect should be more pronounced comparing the distortion associated with a laminar flow regime, characterized by a parabolic velocity profile, with respect to that of a turbulent flow regime, where the velocity profile in the central portion of the pipes is more flat. The two different shapes of the velocity profiles for laminar and turbulent regimes are shown qualitatively in Figure 4

The axial velocity of the flow is also not constant along the length of the pipes, and therefore along the optical path traveled by the light beams into the moving fluid. The total amount of dragging effect to be added, or subtrated, to the speed of propagation of the light would be dependent from the integral, across the entire length of the optical path, of all the local values of the axial fluid velocity on every section of the pipes. This calculation is not straightforward, being the actual velocity field quite complex, especially in the transition regions close to the end of the tubes. Using longer tubes could help in reducing the sensitivity of the results to the local effects concentrated at the ends of the tubes, but even along the central straight portion of the pipes the flow regime would reach a stable, fully developed state with a constant radial velocity profile

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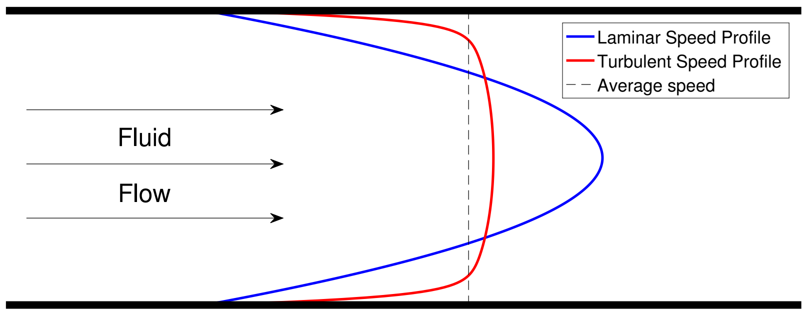

Figure 4: *Laminar vs turbulent velocity profiles inside circular pipes*

distribution, only after some distance along the pipe. Overall, this variation of the speed profile along the length of the pipes will have the effect of reducing the average axial speed seen by the light beam at the center of the pipes, with respect to the value determined at a specific section, typically located towards the exit end of the pipes, where the actual velocity profile measurement is performed and the value of the velocity at the center of the flow is determined.

In the case of turbulent flow regime, which is the actual flow regime used for the measures in the original Fizeau experiment, the velocity field of the fluid flow is characterized by having also radial components of the fluid velocities in addition to the axial ones. These radial components are associated to the presence of fluid vortices, typical of the turbulent flow regime, having different scales, and which can be randomic and non-stationary. In this regime, the motion of the fluid particles with respect to the stationary frame of the laboratory does not corresponds to a pure axial motion along the axis of the pipes, but contains also circular components, due to the vorticity of the flow, with the associated accelerations. Under these conditions the invariance of the physical phenomena asserted by the Galilean Principle of Relativity is no longer applicable, therefore it may be possible that the physical properties of the entities involved in the test are somehow affected by the accelerated state of motion of the particles that constitute the system being observed, thereby changing to some extent the values of their physical characteristics with respect to the corresponding values determined under stationary conditions. In particular, in the case of the Fizeau's experiment, the specific state of motion associated to turbulence could have an impact on the light propagation inside the transparent medium flowing into the pipes. On average it could introduce an additional *"dragging"* term, generated by the circular motion of the fluid into the turbulent vortices, that creates an additional delay of the axial propagation of the light beam. In other terms, the turbulent motion of the fluid, with the associated vortices, can have the effect of reducing the average equivalent propagation speed of the light beam inside the fluid which therefore would have a greater index of refraction in turbulent conditions with respect to the stationary case.

In order to separate this term from the ones associated to the variation of the axial components of the fluid velocity, it can be taken into account by including into the equation an *"equivalent"* index of refraction n^*, which would be dependent on the level of turbulence of the fluid, and would take values greater than the one corresponding to the stationary fluid, i.e. $n^* \geq n$. Being associated to the presence of a turbulent flow regime, such equivalent index of refraction can be expressed as a function of the Reynolds number Re that is used to characterize the level of turbulence of the flow: $n^* = n^*(Re)$. For low values of the Reynolds number, within the laminar range, n^* would be equal to the index of refraction of the stationary fluid, whereas for Reynolds

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number values greater than the threshold corresponding to the onset of turbulent flow, an increase of n^* with the Reynolds number could be expected.

Taking into account of these effects, the expression of the relative speed of light W with respect to the stationary observer, calculated on the basis of the classical Galilean rule of vector sum, can be written in the following form:

$$W = w^* + \bar{v} = \frac{c}{n^*} + \frac{1}{L}\int_0^L v(x)\,dx; \tag{55}$$

where the first term accounts for the effect of variation of the refraction index, and the integral of the second term is extended to the entire length L of the optical path of each light beam. It appears therefore, in particular on the basis of items 3) and 4) above, that the actual value of the speed of light measured with respect to a stationary observer should be lower than the value predicted by the Galilean formula of speed composition, when that formula is evaluated using the peak value of the fluid speed inside the pipes and the nominal value of the refraction index of the stationary fluid, and this result is consistent with the outcome of the experiment. The actual amount of deviation would depend on the specific characteristics of the experimental setup being considered, in particular for what concerns its hydraulic characteristics and parameters.

Recent repetitions of the Fizeau experiment have highlighted that the effects due to turbulence could be the major contributor to the fringe shift observed as a result of the variation of the fluid flow rate and average velocity. In particular Lahaye *et al.* [9] explicitly mention that for low value of the fluid speed, $\bar{v} < 1$ m/s, it has not been possible to acquire any valid test point because of the difficulties in getting stable pictures on the digital sensor used to detect the fringes and their variation. Also in a similar work from Maers *et al.*[10] the experimental data of fringe shift versus flow velocity-difference have been measured only for water velocities in the range $0.5 < \bar{v} < 3.6$ m/s for which the flow is fully in the turbulent regime, having Reynolds number in the range $12.700 < Re < 91.400$.

The expression of the Reynolds number for the flow into circular pipes is:

$$Re = \frac{\rho D}{\mu}\,v = \frac{\nu}{D}\,\bar{v} \tag{56}$$

where ρ is the density of the fluid, μ and ν represent its dynamic and kinematic viscosity, D is the pipe diameter and $\bar{v}$ the macroscopic velocity of the fluid flow. According to the previously described assumption we could write the dependency of the equivalent refraction index from the Reynolds number as follows:

$$n^* = \begin{cases} n & if \quad Re < Re_L \\ n + \alpha(Re - Re_T) & if \quad Re > Re_T \end{cases} \tag{57}$$

where α is a constant to be determined, and Re_L and Re_T represent, respectively, the Reynolds numbers corresponding to the end of the laminar flow regime and to the onset of the turbulent one. Considering velocities of the fluid flow in the turbulent range, $\bar{v} > v_T$, it is therefore possible to put the expression of the equivalent index of refraction of the turbulent fluid in the form:

$$n^* = n(1+\delta) \qquad where \qquad \delta = \delta(Re) = \frac{\alpha(Re - Re_T)}{n} \ll 1 \tag{58}$$

The evaluation of the resultant speed of light into the moving turbulent flow using expression (55) would require calculating the integral of all the local axial velocities of the fluid along the optical path, but this in turn would require the precise knowledge of the flow field in each point into the pipes, which is not available. Due to the lack of detailed knowledge of the flow velocity

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field at every position inside the pipes, it will be assumed, as stated also in [10], that the velocities of the fluid are constant in the straight sections of the tubes through which the light beams travels, and have a radial profile typical of a turbulent regime. In this way the expression of the resultant speed of light becomes:

$$W = \frac{c}{n(1+\delta)} + \bar{v}; \tag{59}$$

Taking into account that $\delta \ll 1$, it is possible to expand the first term of (59) into powers of δ. Making then use of the definition of δ and truncating the expansion to first order it results:

$$W = \frac{c}{n}(1 - \delta + \delta^2 ...) + \bar{v} \simeq \frac{c}{n} - \alpha \frac{c}{n^2} \Delta Re + \bar{v} \tag{60}$$

where $\Delta Re = (Re - Re_T) = \frac{\nu}{D}(\bar{v} - v_T)$. Substituting this into the previous equation gives:

$$W = \frac{c}{n} + \bar{v} - \alpha \frac{c}{n^2} \frac{\nu}{D}(\bar{v} - v_T) \tag{61}$$

Putting now

$$\alpha = \frac{D}{\nu c} \tag{62}$$

the expression of the light speed with respect to the stationary observer finally becomes:

$$W = \frac{c}{n} + \bar{v}(1 - \frac{1}{n^2}) + \frac{1}{n^2} v_T \tag{63}$$

that corresponds to the expression obtained by Fizeau by taking into account that the term v_T/n^2 can be neglected, being much smaller, by several orders of magnitude, than the other constant term c/n.

The above derivation shows that it is possible to provide an interpretation based on the Galilean vector sum of velocities of the experimental results obtained by Fizeau, without the need to invoke any space-time distortion. The proposed approach is based on the hypothesis that the index of refraction of the fluid is altered by the turbulent flow regime. This hypothesis could be verified by further experimental investigations of the optical properties of fluids under turbulent flow regimes, or by a theoretical analysis that would however require to have a very detailed model representing the complex velocity field of the fluid under such flow regime.

5. A test case for the velocity composition rule

In this section a test case is proposed to investigate the validity of the Galilean velocity vector addition rule versus the relativistic one that derives from the Lorentz transformation. The test is based on the analysis of the phenomenon of stellar aberration, i.e. on the observed variation of the position of the celestial objects as a function of the motion of the observer and of its velocity, motion that coincide with that of the Earth along its orbit in the case of a terrestrial telescope. Since the two formulas for the composition of the velocity of the light with the velocity of the observer are different, the expected variation of the position of the star evaluated by means of the relativistic rule is different from that obtained with the classical vector sum rule, and the amount of the difference depends on the value of the ratio of the speed of the observer with respect to the speed of light. Being the orbital velocity of the Earth about 10^4 times smaller than c, such differences are very small and their analysis therefore requires very accurate measurements of the observed position of the celestial objects in order to resolve the differences between the two cases.

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Let us consider the light coming from a very far celestial source, such that the corresponding wavefront can be considered planar over the entire area of the Earth's orbit. For an observer at rest into the center of mass of the Solar system the position of this source is fully characterized by two angles which can be expressed as the in-plane azimuth angle and the out-of-plane elevation angle with respect to the plane of the Earth's orbit.

Let $\mathbf{V}$ be the velocity vector describing the motion of an observer that is moving into the ecliptical plane. Let $\mathbf{c}$ be the vector defining the velocity of propagation of the incoming light with respect to the stationary frame, and let us consider a moving reference frame having its x axis aligned with the direction of the velocity vector $\mathbf{V}$ of the observer and the y axis lying into the plane formed by the direction of the incoming light and $\mathbf{V}$. The resultant vector $\mathbf{c}'$ that defines the apparent position of the light source for the moving observer, will also lie into the xy plane according both to the Galilean vector-sum rule and to the relativistic velocity-composition rule. However, the observed variation of the angle of incidence, i.e. the amount of *aberration*, is different in the two cases. It can be calculated by applying the two velocity composition rules and focusing the analysis on the x and y components of the vectors.

Let us define, in the reference frame of the Sun, the direction of the light source by the angle θ that the incoming light vector makes with the direction of the velocity of the observer. Let v be the speed of the observer, which is assumed to be directed along the positive direction of the x axis of the observer's reference frame, and $\beta = v/c$ the ratio of the observer speed with respect to the speed of light. Let us indicate with θ' the aberrated direction of the source as seen by the moving observer. The relationship between the true angle θ and the observed, aberrated, angle θ' is given by the following two trigonometric expressions (where the pedices G and R, respectively, stands for Galilean and Relativistic), which are derived from the classical vector sum and from the relativistic velocity composition rule :

$$sin(\theta_G - \theta') = \beta\, sin(\theta') \tag{64}$$

and

$$sin(\theta_R - \theta') = \frac{\beta\, sin(\theta')}{1 - \beta\, cos(\theta')} + \frac{\sqrt{1-\beta^2} - 1}{1 - \beta\, cos(\theta')}\ \frac{sin(2\theta')}{2} \tag{65}$$

For very small values of the observer speed, compared to the speed of light, the difference between the two angles θ and θ' is also very small, therefore it is possible to determine the solution of the above two expressions by approximating the sine function with its argument, $sin(\theta - \theta') \simeq (\theta - \theta')$. Expanding then, for $\beta \ll 1$, the second expression as a power series of β truncated to the terms of second order, gives:

$$\theta_G = \theta' + \beta\, sin(\theta') \tag{66}$$

and

$$\theta_R \simeq \theta' + \beta\, sin(\theta') + \frac{1}{4}\,\beta^2\, sin(2\theta') \tag{67}$$

The comparison of equations (66) and (67) shows that the reconstructed position of the light source calculated using the relativistic formula differs from the one obtained from the Galilean vector sum by a term which is quadratic into β. For a given value of the speed v of the observer, the amplitude of this term depends on the angle between the incident light and the direction of the velocity vector of the observer, being maximum when $|sin(2\theta')| = 1$, therefore when $\theta' = \pi/4 + k\pi$,

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and being null when the observer velocity is either parallel or forms a right angle with respect to the direction of the incident light.

Let us now consider the case of an observer moving around the Sun with constant angular velocity Ω on a circular orbit having radius R, and of a distant light source located into the same plane of this orbit and stationary with respect to the Sun, as shown in Figure 5. The vector of the observer velocity always lies into the plane of the orbit, therefore in this case the aberration of the incoming light produces, for such moving observer, an apparent motion of the source which is also always lying into the same plane of the orbit. For this orbiting observer the stationary light source thus shows an apparent oscillation of its position along an horizontal line parallel to the plane of the orbit and characterized by the same time period of the orbit.

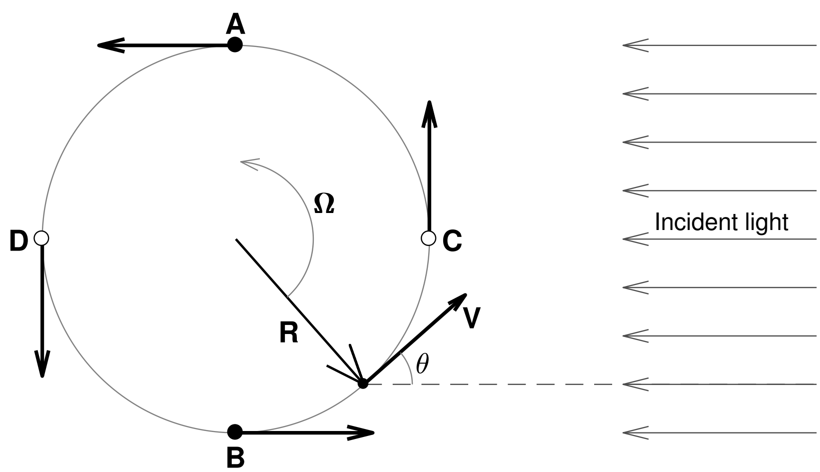

Figure 5: *Orbiting observer with a distant complanar light source*

It is possible to identify four notable locations along the orbit which are significant because of their peculiar properties with respect to the aberration of the source. In the two positions indicated in Figure 5 with **A** and **B** the velocity of the observer is parallel to the direction of the incident light, therefore when the moving observer is in these two points of the orbit there is no aberration of the incoming light and the observed position of the star coincides with the one observed into the stationary frame of the Sun. The position of the celestial object observed in these two points can therefore be taken as a reference position, since it requires no calculation in order to remove the aberration term.

Conversely, when the moving observer is in the two locations labeled **C** and **D**, its velocity vector is orthogonal to the direction of the incident light. In these two locations there is the maximum aberration of the apparent position of the star. However, the value of the aberration term is the same for both the classical and the relativistic rule. Therefore, the calculation of the un-aberrated position of the light source, by means of equations (66) or (67) leads to the same result for both the classical and the relativistic rule. In the particular case of a stationary source considered here, the position of the source calculated by the moving observer located in these two points results coincident with the position observed at locations **A**, **B**.

For any other point of the orbit, the un-aberrated position of the source calculated by means of the classical rule will be different from the one obtained from the relativistic formula, and

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the maximum difference between the two results will occur when the moving observer is at the midpoints between **A**,**B** and **C**,**D**, i.e. at an azimuth angle along the orbit of $\psi = \pi/4 + k\pi/2$. Assuming a stationary source, since the angle between the light direction and the velocity of the observer is $\theta = \Omega t$, one of the two computed results, either the Galilean or the Relativistic one, will contain an harmonic oscillation of the horizontal position of the celestial object, having amplitude equal to $\beta^2/4$, and with period equal to one half the period of the observer's orbit. Such peculiar behaviour of the reconstructed position of the distant light source, characterized by a twice per revolution oscillation, that constitutes its specific signature, represents an artifact of the calculated solution, artifact which is due to the inconsistency of the analytical formula used with respect to the actual rule followed by the physical phenomenon.

Let us now consider the case of a terrestrial observer and let's approximate the Earth's orbit with a circle of radius $R = 150x10^6\,km$, and period T equal to one year. In this case the orbital speed is constant and its value is $v \simeq 30\,km/s$, which gives $\beta \simeq 10^{-4}$.

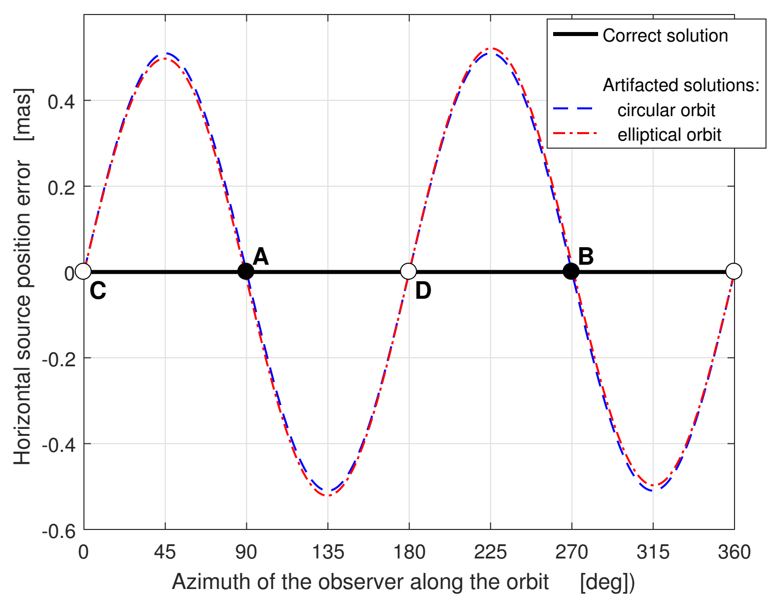

Figure 6: *Comparison of the un-aberrated position of the light source calculated by means of the two different velocity composition rules for an Earth based observer*

With these values of the orbital parameters the two resulting curves of the calculated horizontal position of the source, obtained from equations (66) or (67), are shown in Figure 6. In this figure, also the resulting artifacted solution calculated taking into account the elliptical shape of the Earth's orbit is presented. Due to the small eccentricity of the actual orbit, the deviations of these results from the reference case of a circular trajectory are very small, as shown in the graph that has been calculated considering a celestial object aligned to the major axis of the ecliptic.

The values of the un-aberrated position of the source corresponding to the four notable orbital locations **A**,**B** and **C**,**D** are indicated in Figure 6 with the same markers used in the previous Figure 5. Both the correct and the artifacted curves pass through points **A** and **B**, since for these locations there is no aberration at all and the value of the horizontal position of the celestial object is given directly by the observed position. Both curves also give the same results for locations **C**

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and **D** where the velocity of the observer is orthogonal to the incoming light direction. [5]

The above described artifact, characterized by its twice per revolution frequency content, must be present in either the classical or the relativistic computed results, and has the same specific signature characteristics for any observed stationary source lying into the orbital plane, with almost the same amplitude of oscillation and with the same frequency content, independently from the specific celestial object or the specific region of the electromagnetic spectrum being observed.

When the celestial object being analyzed does not lie into the orbital plane there will be a contribution due to aberration also in the out-of-plane position of the source. Considerations similar to those discussed for an in-plane source apply also to this more general case: the vertical component of the calculated position of the source will contain a twice per revolution spurious term in either the classical or the relativistic results. The amplitude of the artifacted vertical component is null when the celestial object is located in the orbital plane, it then increases with the out-of-plane elevation of the source, reaching a maximum for an elevation angle of $\pi/4$, for which the term $\beta^2 sin(2\theta')$ is maximum. For elevations greater than $\pi/4$ the amplitude of the vertical spurious term will then decrease again and will become zero for circumpolar objects, for which also the in-plane component vanishes.

The presence of a twice per revolution frequency term into the computed results of the unaberrated position of stationary celestial objects is therefore a general characteristics, a specific signature, that allows to identify the incorrect velocity composition rule between the two that have been analyzed.

6. Conclusions

In the previous sections it has been shown that simultaneity of events can be assessed in a unique and consistent way by means of a general criterion does not necessarily require the use of light signals. Thanks to this criterion two events that are simultaneous for one observer result simultaneous also for another observer that is moving with respect to the first one. This shows that the concept of simultaneity is independent from the state of motion of the observer and from the specific clock synchronization signal that has been selected. Such absolute nature of simultaneity allows to introduce a definition of time which is common for all observers.

The above considerations have led to an alternative physical interpretation of the Lorentz transformation of coordinates and suggest that some interferometric experiments on light propagation can be explained without invoking the space-time deformation assumed by the Theory of Relativity, and by applying, instead, the Ritz emission theory[6] which assumes that light is always emitted with the same relative speed, equal to c in vacuum, with respect to its source.

Finally, a test case to discriminate between the Relativistic and the Galilean velocity composition rules has been proposed. The test is based on the analysis of the aberration of the light coming from celestial objects as perceived by an orbiting observer, and on the different results obtained by using the two alternative velocity composition formulas to remove the aberration term from the observed position of the various light sources of the sky. In order to be applied to measured data, this comparison requires that the observed position of the sources is determined with high accuracy, since the differences that have to be investigated are of the order of milli-arcseconds, a level of accuracy that should be achievable by the most advanced ground telescopes or space-based astrometric instruments.

[5] In the general case of a non stationary source, the corresponding computed value of the horizontal position of the object evaluated at **C** and **D** could differ from the one corresponding to the reference locations **A** and **B**.

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Should the outcome of the test be in favour of the classical Galilean velocity vector sum, this could constitute a further supporting element to reconsider the validity of Ritz emission theory in place of the Special Theory of Relativity. Despite having radical differences in their fundamental assumptions, the two theories share some important aspects that marked a sharp distinction from the approach previously adopted for the analysis of electromagnetic phenomena and for classical mechanics. Regarding the propagation of light both theories negate the existence of the aether, whilst for what concerns mechanics and the dynamics of motion of bodies, in both theories the interactions between non-coincident physical entities are not instantaneous as it was assumed in the Newtonian approach. Because of the assumption of instantaneous action at distance, the equations of motion of classical Newtonian mechanics contain, as stated by L. Landau[11], *"a certain degree of imprecision"*. The removal of the hypothesis of instantaneous action at distance, which is inherent into the action-reaction principle when applied to physical entities having a non-null geometrical separation between them, allows both theories to provide the correct predictions of the precession of the motion of the perihelion of Mercury. It may be possible, therefore, that also other experimental observations that have been considered as being in agreement with the Theory of Relativity could find an alternative interpretation not based on the concept of space-time deformation.

7. Aknowledgments

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APPENDIX: Application to a symmetrical physical system

Let us consider a circular water basin of radius R, with a flat bottom surface such that the water has constant depth across the entire area of the basin. Let us suppose to have, at the center of the basin, a vertical thin rod in contact with the water surface. By oscillating vertically this rod it is possible to generate waves that propagate from the center to the edge of the basin, traveling with the same speed in all directions by virtue of the intrinsic symmetry of the physical system, as shown in Figure 7. At every instant of time the wavefronts of these waves describe concentric circles and, thanks to the symmetry properties of the system, every wavefront reach simultaneously all the points of the external perimeter of the basin, as it is also known by experience. Indicating with w the value of the radial speed of propagation of the waves, the time elapsed from the start of a given wavefront from the center of the basin to its arrival at the border is given by $\Delta t = R/w$. Because of the symmetry of the system this value is the same for every direction and for every point of arrival on the external circumference.

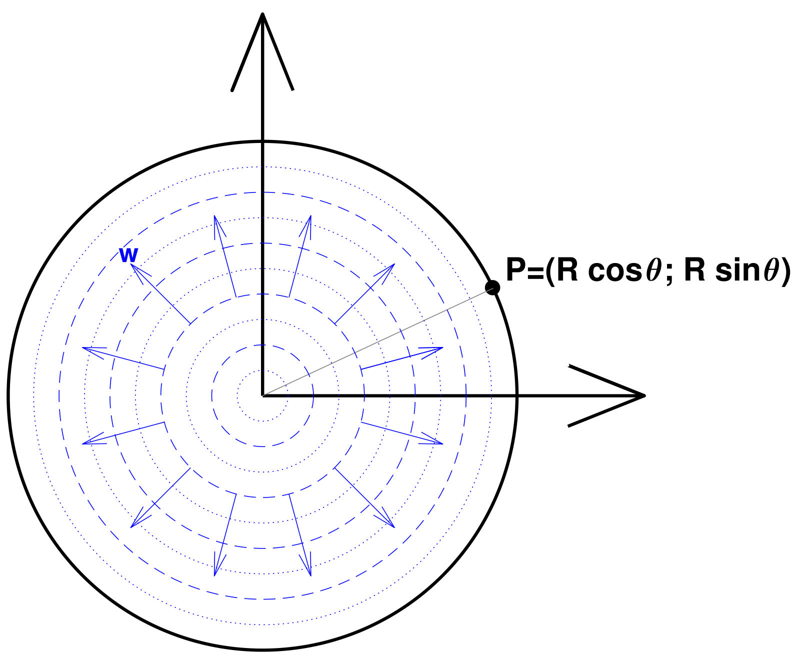

Figure 7: *Circular basin with constant depth water. The dashed and dotted blue circles indicate the position of the wave crests and troughs at a given instant of time.*

Let us now consider an observer K that is stationary with the basin, and let us put the origin of the corresponding reference frame at the center of the basin. For this reference frame the mathematical law that characterize the physical phenomenon being considered, i.e. the propagation of the wavefronts from the center of the basin, is given by:

$$v_x = w\,cos(\theta); \qquad v_y = w\,sin(\theta) \tag{68}$$

where $\theta = atg(y/x)$ and $(x; y)$ are the coordinates of the point. The magnitude of the speed is:

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{w^2\big(cos^2(\theta) + sin^2(\theta)\big)} = w \tag{69}$$

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for every value of θ, which reflects the isotropy of propagation of the waves for the observer K.

Let us now consider a second observer K', with its corresponding reference frame, that is moving horizontally with uniform speed V with respect to the observer K. Let us orient the two reference frames such that the x axis of both is directed parallel to V. We want now to transform the physical law of the system, given by eq. (68) for the observer K, into the moving frame K'. We will do this by applying both the Galilean transformations and the Lorentz ones, using the pedices G (Galilean) for the first and R (Relativistic) for the second, obtaining:

$$v'_{Gx} = w\,cos(\theta) - V; \qquad v'_{Gy} = w\,sin(\theta) \tag{70}$$

$$v'_{Rx} = \frac{w\,cos(\theta) - V}{1 - \frac{V\,w\,cos(\theta)}{c^2}}; \qquad v'_{Ry} = \frac{w\,sin(\theta)\sqrt{1-\frac{V^2}{c^2}}}{1 - \frac{V\,w\,cos(\theta)}{c^2}} \tag{71}$$

where c is the speed of light in vacuum. Thus, when expressed into the moving frame K', the law of the phenomenon being observed has taken a new mathematical form, expressed respectively by eqs. (70) and (71), and the new form of this law now contains also the value of the relative speed V between the two reference frames. As it can be noted by calculating the magnitude of the transformed vector $\mathbf{v}'$, the new mathematical law does not correspond anymore to an isotropic speed of propagation of the wavefronts with respect to K', since the magnitude of both $\mathbf{v}'_{\mathbf{G}}$ and $\mathbf{v}'_{\mathbf{R}}$ is no longer the same for every direction considered, but becomes a function of θ. In fact, it results:

$$|\mathbf{v}'_{\mathbf{G}}|^2 = w^2 + V^2 - 2\,w\,V\,cos(\theta) \tag{72}$$

$$|\mathbf{v}'_{\mathbf{R}}|^2 = \frac{w^2 + V^2 - 2\,w\,V\,cos(\theta) - w^2V^2sin^2(\theta)/c^2}{1 - w\,V\,cos(\theta)/c^2} \tag{73}$$

This shows, consistently with what discussed in the introductory section, that a phenomenon which is characterized by an isotropic propagation speed for a stationary observer, does not appear isotropic to an observer which is translating with velocity V with respect to the first one, and this has been verified by applying both the Galilean transformations of coordinates and the relativistic ones expressed by the Lorentz transformation [6].

Let us now consider the two events that correspond to the start of a given wavefront from the center of the basin and to its arrival at a generic point P on the perimeter. Let us indicate with E_O the first event and E_P the second one. Setting the origin of time in correspondence of the first event, the in-plane and time coordinates of these two events into the reference frame K are:

$$\begin{aligned} E_O &= (x_O;\, y_O;\, t_O) &&= (0;\, 0;\, 0) \\ E_P &= (x_P;\, y_P;\, t_P) &&= \left(R\,cos(\theta);\, R\,sin(\theta);\, \tfrac{R}{w}\right) \end{aligned} \tag{74}$$

Using the Lorentz transformation it is possible to calculate, in the moving frame K', the primed time variable associated to the second event E_P:

$$t'_P = \gamma\,R\,\Big(\frac{1}{w} - \frac{V}{c^2}cos(\theta)\Big) \tag{75}$$

where $\gamma = 1/\sqrt{1 - V^2/c^2}$. Interpreting the variable t'_P as the time of the event for the moving observer would thus lead to conclude that the arrival of the wavefront at the perimeter of the basin

[6] As shown in section III, the only way to maintain the isotropy of propagation of the waves also for the moving observer would be that of using the transformations of coordinates defined by (6), with characteristic speed equal to the speed of propagation of the wave: $v_c = w$

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is no longer simultaneous for all points along the edge, since it would depend on the angular position of point P thru the $cos(\theta)$ function. But this conclusion is not in agreement with the experience, and it would constitute a violation of the symmetry of the physical system being considered, since it would mean that there are some points on the external circumference, some directions, for which the wavefront arrives earlier than other positions or directions, conclusion which is not in agreement with the symmetry characteristics of the water basin analyzed.

It is however still possible to properly assess the simultaneity of the wavefront arrival events into the moving reference frame K' by using the method that has been presented into Section III, based on the use of the space-time intervals. Let us in fact consider two events corresponding to the arrival of the wavefront at two different points A and B located on the edge of the circular basin. In the stationary frame K, these two events have coordinates:

$$\begin{aligned} E_A = \quad & (x_A;\, y_A;\, t_A) = \quad \Big(R\,cos(\alpha);\, R\,sin(\alpha);\, \tfrac{R}{w}\Big) \\ E_B = \quad & (x_B;\, y_B;\, t_B) = \quad \Big(R\,cos(\theta);\, R\,sin(\theta);\, \tfrac{R}{w}\Big) \end{aligned} \tag{76}$$

with α and θ representing the angular position of the two selected locations. We can calculate, in the stationary frame K, the relativistic intervals separating these two events from the event E_O that corresponds to the start of the wave from the center, obtaining:

$$s_{OA}^2 = c^2(t_A - t_O)^2 - (x_A - x_O)^2 - (y_A - y_O)^2 = \tag{77}$$

$$c^2\frac{R^2}{w^2} - R^2cos^2(\alpha) - R^2sin^2(\alpha) = R^2\Big(\frac{c^2}{w^2} - 1\Big) \tag{78}$$

and, similarly:

$$s_{OB}^2 = c^2\frac{R^2}{w^2} - R^2cos^2(\theta) - R^2sin^2(\theta) = R^2\Big(\frac{c^2}{w^2} - 1\Big) \tag{79}$$

Thus, the relativistic interval that separates the start of the wavefront from the center of the basin from its arrival on the perimeter is the same for all the locations on the external circumference, it being independent from the angular location of the point of arrival, it results therefore $s_{OA}^2 = s_{OB}^2$ for every couple of points on the external perimeter.

Let us now calculate the relativistic space-time coordinates of the two arrival events with respect to the moving frame K' by applying the Lorentz transformation, that gives:

$$E'_A = \Big(\gamma\, R\, \big(cos\alpha - \frac{V}{w}\big); \quad R\, sin\alpha; \quad \gamma\, R\, \big(\frac{1}{w} - \frac{V}{c^2}cos\alpha\big)\Big) \tag{80}$$

$$E'_B = \Big(\gamma\, R\, \big(cos\theta - \frac{V}{w}\big); \quad R\, sin\theta; \quad \gamma\, R\, \big(\frac{1}{w} - \frac{V}{c^2}cos\theta\big)\Big) \tag{81}$$

It is now possible to calculate the relativistic interval between E'_A and E'_O:

$$(s'_{OA})^2 = c^2(t'_A - t'_O)^2 - (x'_A - x'_O)^2 - (y'_A - y'_O)^2 = \gamma^2R^2\Big[c^2\Big(\frac{1}{w} - \frac{V}{c^2}cos\alpha\Big)^2 - \Big(cos\alpha - \frac{V}{w}\Big)^2 - \frac{1}{\gamma^2}sin^2\alpha\Big] \tag{82}$$

Inserting the expression of γ, expanding the various terms and taking into account that $1/\gamma^2 = (1 - V^2/c^2)$ finally gives:

$$(s'_{OA})^2 = \gamma^2R^2\Big(\frac{c^2}{w^2} + \frac{V^2}{c^2} - 1 - \frac{V^2}{w^2}\Big) = R^2\Big(\frac{c^2}{w^2} - 1\Big) \tag{83}$$

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and similarly:

$$(s'_{OB})^2 = \gamma^2 R^2 \left(\frac{c^2}{w^2} + \frac{V^2}{c^2} - 1 - \frac{V^2}{w^2} \right) = R^2 \left(\frac{c^2}{w^2} - 1 \right) \tag{84}$$

which shows that also in this case it results $(s'_{OA})^2 = (s'_{OB})^2$. We have therefore found that also for the moving observer K', the relativistic interval that separates the start of the wavefront from the center of the basin from its arrival on the perimeter, is the same for all the locations on the external circumference, it being independent from the angular location of the point of arrival. Therefore, on the basis of criterion (16) presented into Section III, the simultaneity of the arrival of the wavefronts on the edge of the basin is consistently maintained also for the moving observer and its reference frame, provided that the assessment is based on the criterion of equality of the relativistic intervals.

The same considerations and results would be found also if we consider a different kind of physical phenomenon, like for example an acoustic sonar pulse or a light flash emitted into the water at the center of the basin. Thanks to the symmetry properties of the system being considered, also in these cases the sound, or light, wavefronts will reach simultaneously all the points on the external circumference. Repeating the same previous analysis for this other physical phenomena requires just to replace the value of the characteristic radial speed of the surface water waves w with the speed of sound or with the speed of light into the medium, water in this case. In this way we will therefore find again that the propagation of the sound or light waves will not be isotropical for a moving observer K', and it will be possible to assess the simultaneity of the arrivals of the wavefront at the basin edge by using the criterion based on the equality of the intervals.

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