Mathematical Analysis of Memristor through Fractal-Fractional Differential Operator: A Numerical Study

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Abstract

The newly generalized energy storage component namely memristor is a fundamental circuit element so called universal charge-controlled mem-element is proposed for controlling the analysis and coexisting attractors. The governing differential equations of memristor are highly non-linear for mathematical relationships. The mathematical model of memristor is established in terms of newly defined fractal-fractional differential operators so called Atangana-Baleanu, Caputo-Fabrizio and Caputo fractal-fractional differential operator. A novel numerical approach is developed for the governing differential equations of memristor on the basis of Atangana-Baleanu, Caputo-Fabrizio and Caputo fractal-fractional differential operator. We discussed chaotic behavior of memristor under three criteria as (i) varying fractal order, we fixed fractional order, (ii) varying fractional order, we fixed fractal order and (ii) varying fractal and fractional orders simultaneously. Our investigated graphical illustrations and simulated results via MATLAB for the chaotic behaviors of memristor suggest that newly presented Atangana-Baleanu, Caputo-Fabrizio and Caputo fractal-fractional differential operator has generates significant results as compared with classical approach.

1. Introduction

A memristor is the fourth passive or circuit element that has the capability for remembering its state history in power-off modes due to its nonlinear nature and plasticity properties. There are four categories of memristor (i) ionic thin film memristor, (ii) spin memristor, (iii) molecular memristor and (iv) magnetic memristor; each types of memristor has its own significance as hysteresis under the application of charge is detected by ionic thin film memristor, degree of freedom in electron is relied by spin memristor, anomalous current-voltage is exhibited by molecular memristor and a bilayer-oxide films substrate is perceived by magnetic memristor respectively [1-2]. The first fabricated physical memristor was found by Strukov et al. [2] as a missing memristor so called fourth fundamental circuit element in 2008. Bao et al. [3] presented dimensionless mathematic model based on a fifth-order chaotic circuit with two memristors. They discussed stability analysis, dynamical analysis methods and the memristor initial states. They also described transient hyperchaos state transitions with excellent nonlinear dynamical phenomena. The two memristors connected in antiparallel has been observed by Buscarino et al. [4] when a sinusoidal input is applied. Their setting for two memristors was consisted of two capacitors, an inductor, one negative resistor, two memristors connected in antiparallel in which characterization of the four embedded circuit parameters was also analyzed within dynamical behaviors. Adhikari et al. [5] exhibited role of memristors on the basis of three conditions as (i) pinched hysteresis loop when frequency tends to infinity, (ii) critical frequency decreases monotonically when excitation frequency increases and (iii) bipolar periodic signal in the voltage-current is assumed to be periodic. The three-dimensional chaotic system has been modified by Li et al. [6] in terms of four-dimensional memristive system on the basis of dissipativity and symmetry. Their main focus was to investigate the complex dynamics includes as hyperchaos, limit cycles, chaos, torus and few others. Chen et al. [7] studied classical memristive chaotic circuit with a first-order memristive diode bridge in which theoretical and numerical investigation has been displayed for complex nonlinear phenomena coexisting attractors and bifurcation modes. Zhou et al. [8] perceived the effective role of hyperchaotic multi-wing attractor in a 4D memristive circuit within complicated dynamics. Here they presented interesting controller parameters for 4D memristive circuit includes Lyapunov exponents, phase portrait, bifurcation diagram and Poincaré maps. The dynamical illustrations can be continued on memristors, we include here few latest attempts subject to chaos analysis [9-11].

Although fractional calculus is a burning field of mathematics that studies the generalization of classical concepts in mathematics and engineering via differential and integral operators yet the fractal calculus is relatively a new science of differential and integral operators based on two parameters. The fractal-fractional differentiation consists two dimensions namely one for fractional order and other for fractal order. The main significance of the fractal-fractional differentiation is to describe fractal kinetics effectively in which the fractal time is replaced into the continuous time. The fractal-fractional differentiation provides the fractal dimension through which the model can capture preferential paths for capturing the flow in fractured aquifers. It plays an extremely effective role in the phenomena of hierarchical or porous media, for instance, fractal gradient of temperature in a fractal medium [12-15]. Very recently an African Professor Atangana presented his concept of fractal-fractional differentiation based on Mittage-Leffler, exponential decay and power-law memories in which he described that fractal-fractional differentiation attracts more non-local natural problems that display at the same time fractal behaviors [16]. Atangana and Qureshi [17] captured self-similarities in the chaotic attractors based on the basis of three numerical schemes for systems of nonlinear differential equations. Their investigated dynamical systems containing the general conditions for the existence and the uniqueness have been explored. Gomez-Aguilar [18] presented the Shinriki's oscillator model for the prediction of chaotic behaviors related to the fractal derivative in convolution with power-law, exponential decay law and the Mittag-Leffler function in which the Adams-Bashforth-Moulton scheme has been invoked for the numerical simulations at symmetric and asymmetric cases. Sania et al. [19] employed the concept of fractal-fractional operators presented in [16] for investigating the chaotic behaviors the Thomas cyclically symmetric attractor, the King Cobra attractor, Rossler attractor, the Langford attractor, the Shilnikov attractor. They claimed that new strange behaviors of the attractors have been which were impossible by fractional and classical differentiations. In short, the study can be continued for the charming and effective role of fractional calculus but we include here recent attempt therein [20-31]. Motivating by above discussion, our aim is to propose the controlling analysis and coexisting attractors provided by memristor through highly non-linear for mathematical relationships of governing differential equations. The mathematical model of memristor is established in terms of newly defined fractal-fractional differential operator so called Caputo-Fabrizio fractal-fractional differential operator. A novel numerical approach is developed for the governing differential equations of memristor on the basis Caputo-Fabrizio fractal-fractional differential operator. We discussed chaotic behavior of memristor under three criteria as (i) varying fractal order, we fixed fractional order, (ii) varying fractional order, we fixed fractal order and (ii) varying fractal and fractional orders simultaneously. Our investigated graphical illustrations and simulated results via MATLAB for the chaotic behaviors of memristor suggest that newly presented Caputo-Fabrizio fractal-fractional differential operator has generates significant results as compared with classical approach.

2. Fractal-Fractional Modeling of Memristor

The regulation of the flow of electrical current in a circuit and remembrance the amount of charge is rectified electrical component so called memristor. The main significance of memristor is that it retains memory without power. This electrical component is passive two-terminal and a nonlinear and can link magnetic flux and electric charge on the basis of non-volatile memory. It is the significant capability of memristor that it increases the flow of current in one direction based on resistance switching and it decreases the flow of current in the opposite direction based on resistance switching. Motivating from the features of memristor, we designed a new chaotic circuit is designed as shown in Fig. 1 which is based on a deformed structure of Chua's dual circuit as:

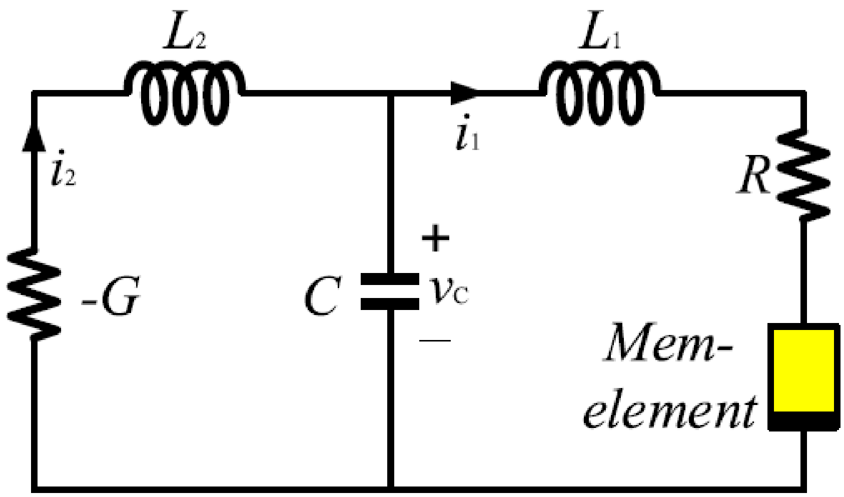

Fig. 1. A chaotic circuit

The chaotic circuit designed in Fig. 1 includes fewer dynamic components such as four state variables are i_1, i_2, v_C, and$q$ respectively and a capacitor, two inductors and emulator with a resistor and a negative conductance. The circuit depicted in Fig. 1 is followed by volt-ampere characteristic of elements and Kirchhoff's laws for generating the state equations containing nonlinear terms as written below:

$$\frac{\mathrm{dq}}{\mathrm{dt}} - i_1 = 0, \tag{1}$$

$$\frac{dv_c}{\mathrm{dt}} + \frac{i_1}{C} - \frac{i_2}{C} = 0, \tag{2}$$

$$\frac{di_2}{\mathrm{dt}} + \frac{v_c}{L_2} + \frac{i_2}{\mathrm{GL}_2} = 0, \tag{3}$$

$$\frac{di_1}{\mathrm{dt}} - \frac{v_c}{L_1} + \frac{v_m(q)}{L_1} + \frac{Ri_1}{L_1} = 0. \tag{4}$$

Here, the terminal voltage is symbolized by $v_m(q)$. For replacing the terminal voltage say $v_m(q)$ by applying the voltage across the memristor on the state equations (1-4) containing nonlinear terms; take place as

$$\frac{\mathrm{dq}}{\mathrm{dt}} - i_1 = 0, \tag{5}$$

$$\frac{dv_c}{\mathrm{dt}} + \frac{i_1}{C} - \frac{i_2}{C} = 0, \tag{6}$$

$$\frac{di_2}{\mathrm{dt}} + \frac{v_c}{L_2} + \frac{i_2}{\mathrm{GL}_2} = 0, \tag{7}$$

$$\frac{di_1}{\mathrm{dt}} - \frac{v_c}{L_1} + \frac{Ri_1}{L_1} + \frac{1}{L_1}\left(\frac{R_2\ R_0^2\ \mathrm{q}}{10C_1R_3R_1} - R_3^{-1}R_2R_0\right) = 0. \tag{8}$$

In order to bring the dynamic equation of the system, the time scale transformation is being carried out by keeping in mind the following transformations as

$$\begin{array}{c} a = L_2L_1^{-1}, b = L_2C^{-1}, c = R, k = G^{-1}, m = -R_3^{-1}R_2R_0, n = R_2R_0^2\left(C_1R_3R_1\right)^{-1} \\ u = L_2^{-1}q, z = i_2, t = d\tau L_2, x = i_1,\ y = v_c \end{array}, \tag{9}$$

The parameters involved in equation (9) for implementing the memristor are specified as$R0\ = 1k\Omega, R1\ = 500\Omega, R2\ = 1k\Omega, R3\ = 1\ k\Omega,\ C1\ = 100\ nF$. Invoking equation (9) among equations (5-8), we arrive at the simplified system of evolutionary differential equations containing nonlinear terms:

$$\begin{array}{c} \frac{\mathrm{dx}}{\mathrm{dt}} - a\left(nux + mx + cx - y\right) = 0, \\ \frac{\mathrm{dy}}{\mathrm{dt}} + bx - bz = 0, \\ \frac{\mathrm{dz}}{\mathrm{dt}} + y - kz = 0, \\ \frac{\mathrm{du}}{\mathrm{dt}} - x = 0, \end{array}, \tag{10}$$

subject to the initial conditions,

$$x\left(0\right) = y\left(0\right) = z\left(0\right) = u\left(0\right) = 0.01, \tag{11}$$

The chaotic phenomena and the phase portraits can be obtained by specification of embedded parameters in equation (10), as$a = 2,\ b = 1,\ c = 0.2,\ k = 0.92,\ m = -0.002,\ n = 0.04$. Developing the system of evolutionary differential equation containing nonlinear terms say (10) in terms of the new idea of fractal-fractional differential operator, we transferred governing nonlinear differential equation (10) of memristor as:

$$\begin{array}{c} \mathfrak{D}_t^{\alpha,\ \ \beta}x(t) - a\left(nux + mx + cx - y\right) = 0, \\ \mathfrak{D}_t^{\alpha,\ \ \beta}y(t) + bx - bz = 0, \\ \mathfrak{D}_t^{\alpha,\ \ \beta}z(t) + y - kz = 0, \\ \mathfrak{D}_t^{\alpha,\ \ \beta}u(t) - x = 0, \end{array}, \tag{12}$$

Here, $\mathfrak{D}_t^{\alpha,\ \ \beta}x(t)$,$\mathfrak{D}_t^{\alpha,\ \ \beta}y(t)$,$\mathfrak{D}_t^{\alpha,\ \ \beta}z(t)$ and$\mathfrak{D}_t^{\alpha,\ \ \beta}u(t)$ represent the fractal-fractional differential operators.

3. Fractal-Fractional Integrals and Differential Operators

Caputo Fractal-Fractional differential and integral operators[16]

$$\mathfrak{D}_t^{\alpha,\ \ \beta}\}\left(t\right) = \left(n-\alpha\right)^{-1}\frac{d}{dt^{\beta}}\int_0^t\left(t-s\right)^{n-\alpha-1}\}\left(s\right)\ ds, \qquad n-1 < \alpha, \beta \leq n \in N, \tag{13}$$

$$\mathcal{I}_t^{\alpha}\}\left(t\right) = \frac{\beta}{\Gamma(\alpha)}\int_0^t\left(t-s\right)^{\alpha-1}s^{\beta-1}\}\left(s\right)ds. \tag{14}$$

Caputo-Fabrizio Fractal-Fractional differential and integral operators [16]

$$\mathfrak{D}_t^{\alpha,\ \beta}\}(t) = M(\alpha)(1-\alpha)^{-1}\frac{d}{dt^{\beta}}\int_0^t \exp\left\{\frac{-\alpha(t-s)}{1-\alpha}\right\}\}(s)\ ds. \tag{15}$$

$$\mathcal{I}_t^{\alpha,\beta}\}(t) = \alpha\beta M(\alpha)\int_0^t s^{\alpha-1}\}(s)\,ds + \frac{\}(t)\,s^{\beta-1}(1-\alpha)\alpha}{M(\alpha)}. \tag{16}$$

Atangana-Baleanu Fractal-Fractional differential and integral operators [16]

$$\mathfrak{D}_t^{\alpha,\ \beta}\}(t) = AB(\alpha)(1-\alpha)^{-1}\frac{d}{dt^{\beta}}\int_0^t \mathbf{E}_{\alpha}\left\{\frac{-\alpha(t-s)^{\alpha}}{1-\alpha}\right\}\}(s)\ ds. \tag{17}$$

$$\mathcal{I}_t^{\alpha,\beta}\}(t) = \frac{}{\mathrm{AB}(\alpha)}\int_0^t s^{\beta-1}(t-s)^{\alpha-1}\}(s)\,ds + \frac{\}(t)\,t^{\beta-1}(1-\alpha)\alpha}{AB(\alpha)}. \tag{18}$$

3. Development of Numerical Scheme for Fractal-Fractional Models

3.1 Numerical Scheme for Caputo Fractal-Fractional Model

The Adams-Bashforth-Moulton method is a linear multi-step integration method. Though this numerical approach so called the Adams-Bashforth-Moulton method, one can solve the system of evolutionary differential equations containing nonlinear terms say (12) based on the new idea of fractal-fractional differential operator. In order to bring the fractal-fractionalized the system of evolutionary differential equations (12), we converted the system of evolutionary differential equations (12) of memristor in terms of Caputo fractal-fractional differential operator as defined

$$\begin{aligned}
&\mathfrak{D}_t^{\xi_1,\ \eta_1}x(t) - a(nux + mx + cx - y) = 0,\\
&\mathfrak{D}_t^{\xi_1,\ \eta_1}y(t) + bx - bz = 0,\\
&\mathfrak{D}_t^{\xi_1,\ \eta_1}z(t) + y - kz = 0,\\
&\mathfrak{D}_t^{\xi_1,\ \eta_1}u(t) - x = 0,
\end{aligned}\quad , \tag{19}$$

we set the structure of equation (19) for the numerical method, equation (19) takes the following expression as

$$\begin{aligned}
&\mathfrak{D}_t^{\xi_1,\ \eta_1}x(t) = \xi_1 t^{\xi_1-1}\}_1(x,y,z,u,t),\\
&\mathfrak{D}_t^{\xi_1,\ \eta_1}y(t) = \xi_1 t^{\xi_1-1}g_2(x,y,z,u,t),\\
&\mathfrak{D}_t^{\xi_1,\ \eta_1}z(t) = \xi_1 t^{\xi_1-1}g_3(x,y,z,u,t),\\
&\mathfrak{D}_t^{\xi_1,\ \eta_1}u(t) = \xi_1 t^{\xi_1-1}g_4(x,y,z,u,t),
\end{aligned} \tag{20}$$

Implementing equation (14) (Caputo- integral in terms of fractal-fractional sense) on equation (20), we arrive at

$$\begin{aligned} x(t) &= x(0) + \tfrac{\xi_1}{\Gamma(\eta_1)} \int_0^t \Lambda^{\xi_1-1} (t-\Lambda)^{\eta_1-1} \}_1 (x,y,z,u,\Lambda)\, d\Lambda, \\ y(t) &= y(0) + \tfrac{\xi_1}{\Gamma(\eta_1)} \int_0^t \Lambda^{\xi_1-1} (t-\Lambda)^{\eta_1-1} \}_2 (x,y,z,u,\Lambda)\, d\Lambda, \\ z(t) &= z(0) + \tfrac{\xi_1}{\Gamma(\eta_1)} \int_0^t \Lambda^{\xi_1-1} (t-\Lambda)^{\eta_1-1} \}_3 (x,y,z,u,\Lambda)\, d\Lambda, \\ u(t) &= u(0) + \tfrac{\xi_1}{\Gamma(\eta_1)} \int_0^t \Lambda^{\xi_1-1} (t-\Lambda)^{\eta_1-1} \}_4 (x,y,z,u,\Lambda)\, d\Lambda, \end{aligned} \tag{21}$$

By the setting equation (21) at t_{n+1}, we obtained the numerical scheme as

$$\begin{aligned} x^{n+1}(t) &= x_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \int_0^{t_{n+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_1 (x,y,z,u,\Lambda)\, d\Lambda, \\ y^{n+1}(t) &= y_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \int_0^{t_{n+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_2 (x,y,z,u,\Lambda)\, d\Lambda, \\ z^{n+1}(t) &= z_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \int_0^{t_{n+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_3 (x,y,z,u,\Lambda)\, d\Lambda, \\ u^{n+1}(t) &= u_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \int_0^{t_{n+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_4 (x,y,z,u,\Lambda)\, d\Lambda, \end{aligned} \tag{22}$$

The simplified form of equation (22) can be expressed for approximation within the interval $[t_j, t_{j+1}]$ in the compact form as

$$\begin{aligned} x_{n+1}(t) &= x_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_1 (x,y,z,u,\Lambda)\, d\Lambda, \\ y_{n+1}(t) &= y_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_2 (x,y,z,u,\Lambda)\, d\Lambda, \\ z_{n+1}(t) &= z_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_3 (x,y,z,u,\Lambda)\, d\Lambda, \\ u_{n+1}(t) &= u_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} \}_4 (x,y,z,u,\Lambda)\, d\Lambda, \end{aligned} \tag{23}$$

Applying the elementary procedure of integration and the Lagrange polynomial piece-wise interpolation on the expressions say$\Lambda^{\eta_1-1}\}_1(x,y,z,u,\Lambda), \Lambda^{\eta_1-1}\}_2(x,y,z,u,\Lambda), \Lambda^{\eta_1-1}\}_3(x,y,z,u,\Lambda)and\lambda^{\eta_1-1}\}_4(x,y,z,u,\Lambda)$involved in equation (23), as defined below

$$\begin{aligned} P_j(\Lambda) &= \tfrac{\Lambda-t_{j-1}}{t_j-t_{j-1}} t_j^{\eta_1-1} \}_1 (x_j,y_j,z_j,u_j,\Lambda_j) - \tfrac{\Lambda-t_j}{t_j-t_{j-1}} t_{j-1}^{\eta_1-1} \}_1 (x_{j-1},y_{j-1},z_{j-1},u_{j-1},\Lambda_{j-1}), \\ Q_j(\Lambda) &= \tfrac{\Lambda-t_{j-1}}{t_j-t_{j-1}} t_j^{\eta_1-1} \}_2 (x_j,y_j,z_j,u_j,\Lambda_j) - \tfrac{\Lambda-t_j}{t_j-t_{j-1}} t_{j-1}^{\eta_1-1} \}_2 (x_{j-1},y_{j-1},z_{j-1},u_{j-1},\Lambda_{j-1}), \\ R_j(\Lambda) &= \tfrac{\Lambda-t_{j-1}}{t_j-t_{j-1}} t_j^{\eta_1-1} \}_3 (x_j,y_j,z_j,u_j,\Lambda_j) - \tfrac{\Lambda-t_j}{t_j-t_{j-1}} t_{j-1}^{\eta_1-1} \}_3 (x_{j-1},y_{j-1},z_{j-1},u_{j-1},\Lambda_{j-1}), \\ S_j(\Lambda) &= \tfrac{\Lambda-t_{j-1}}{t_j-t_{j-1}} t_j^{\eta_1-1} \}_4 (x_j,y_j,z_j,u_j,\Lambda_j) - \tfrac{\Lambda-t_j}{t_j-t_{j-1}} t_{j-1}^{\eta_1-1} \}_4 (x_{j-1},y_{j-1},z_{j-1},u_{j-1},\Lambda_{j-1}), \end{aligned} \tag{24}$$

Invoking equation (24) in to (23), we arrive at

$$\begin{aligned} x_{n+1}(t) &= x_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} P_j(\Lambda)\, d\Lambda, \\ y_{n+1}(t) &= y_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} Q_j(\Lambda)\, d\Lambda, \\ z_{n+1}(t) &= z_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} R_j(\Lambda)\, d\Lambda, \\ u_{n+1}(t) &= u_0 + \tfrac{\eta_1}{\Gamma(\xi_1)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \Lambda^{\eta_1-1} (t_{n+1}-\Lambda)^{\xi_1-1} S_j(\Lambda)\, d\Lambda, \end{aligned} \tag{25}$$

we investigated the numerical scheme for Caputo fractal-fractional operator as

$$
\begin{aligned}
x_{n+1} &= x_0 + \frac{\eta_1(\eta_1 t)^{\xi_1}}{\Gamma(\xi_1+2)} \sum_{j=0}^{n} \Big[t_j^{\eta_1-1} \}_1 \left(x_j, y_j, z_j, u_j, t_j\right) \ \Big\{ (n+1-j)^{\xi_1} \left(n-j+2+\xi_1\right) - (n-j)^{\xi_1} \\
&\quad \times \ \left(n-j+2+2\xi_1\right) \Big\} - t_{j-1}^{\eta_1-1} \}_1 \left(x_{j-1}, y_{j-1}, z_{j-1}, u_{j-1}, t_{j-1}\right) (n+1-j)^{\xi_1+1} - (n-j)^{\xi_1} \ \left(n-j+1+\xi_1\right) \Big], \\
y_{n+1} &= y_0 + \frac{\eta_1(\eta_1 t)^{\xi_1}}{\Gamma(\xi_1+2)} \sum_{j=0}^{n} \Big[t_j^{\eta_1-1} \}_2 \left(x_j, y_j, z_j, u_j, t_j\right) \ \Big\{ (n+1-j)^{\xi_1} \left(n-j+2+\xi_1\right) - (n-j)^{\xi_1} \\
&\quad \times \ \left(n-j+2+2\xi_1\right) \Big\} - t_{j-1}^{\eta_1-1} \}_2 \left(x_{j-1}, y_{j-1}, z_{j-1}, u_{j-1}, t_{j-1}\right) (n+1-j)^{\xi_1+1} - (n-j)^{\xi_1} \ \left(n-j+1+\xi_1\right) \Big], \\
z_{n+1} &= z_0 + \frac{\eta_1(\eta_1 t)^{\xi_1}}{\Gamma(\xi_1+2)} \sum_{j=0}^{n} \Big[t_j^{\eta_1-1} \}_3 \left(x_j, y_j, z_j, u_j, t_j\right) \ \Big\{ (n+1-j)^{\xi_1} \left(n-j+2+\xi_1\right) - (n-j)^{\xi_1} \\
&\quad \times \ \left(n-j+2+2\xi_1\right) \Big\} - t_{j-1}^{\eta_1-1} \}_3 \left(x_{j-1}, y_{j-1}, z_{j-1}, u_{j-1}, t_{j-1}\right) (n+1-j)^{\xi_1+1} - (n-j)^{\xi_1} \ \left(n-j+1+\xi_1\right) \Big], \\
u_{n+1} &= u_0 + \frac{\eta_1(\eta_1 t)^{\xi_1}}{\Gamma(\xi_1+2)} \sum_{j=0}^{n} \Big[t_j^{\eta_1-1} \}_4 \left(x_j, y_j, z_j, u_j, t_j\right) \ \Big\{ (n+1-j)^{\xi_1} \left(n-j+2+\xi_1\right) - (n-j)^{\xi_1} \\
&\quad \times \ \left(n-j+2+2\xi_1\right) \Big\} - t_{j-1}^{\eta_1-1} \}_4 \left(x_{j-1}, y_{j-1}, z_{j-1}, u_{j-1}, t_{j-1}\right) (n+1-j)^{\xi_1+1} - (n-j)^{\xi_1} \ \left(n-j+1+\xi_1\right) \Big].
\end{aligned}
\tag{26}
$$

3.2 Numerical Scheme for Caputo-Fabrizio Fractal-Fractional Model

In order to bring the fractal-fractionalized the system of evolutionary differential equations (12), we converted the system of evolutionary differential equations (12) of memristor in terms of Caputo-Fabrizio fractal-fractional differential operator as defined

$$
\begin{aligned}
\mathfrak{D}_t^{\xi_2, \ \ \eta_2} x(t) &= \xi_2 t^{\xi_2-1} \}_1 (x, y, z, u, t), \\
\mathfrak{D}_t^{\xi_2, \ \ \eta_2} y(t) &= \xi_2 t^{\xi_2-1} g_2 (x, y, z, u, t), \\
\mathfrak{D}_t^{\xi_2, \ \ \eta_2} z(t) &= \xi_2 t^{\xi_2-1} g_3 (x, y, z, u, t), \\
\mathfrak{D}_t^{\xi_2, \ \ \eta_2} u(t) &= \xi_2 t^{\xi_2-1} g_4 (x, y, z, u, t),
\end{aligned}
\tag{27}
$$

Implementing equation (16) (Caputo-Fabrizio integral in terms of fractal-fractional sense) on equation (27), we arrive at

$$
\begin{aligned}
x(t) &= x(0) + \frac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_1 (x, y, z, u, t) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^t \Lambda^{\eta_2-1} \}_1 (x, y, z, u, \Lambda)\, d\Lambda, \\
y(t) &= y(0) + \frac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_2 (x, y, z, u, t) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^t \Lambda^{\eta_2-1} \}_2 (x, y, z, u, \Lambda)\, d\Lambda, \\
z(t) &= z(0) + \frac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_3 (x, y, z, u, t) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^t \Lambda^{\eta_2-1} \}_3 (x, y, z, u, \Lambda)\, d\Lambda, \\
u(t) &= u(0) + \frac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_4 (x, y, z, u, t) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^t \Lambda^{\eta_2-1} \}_4 (x, y, z, u, \Lambda)\, d\Lambda,
\end{aligned}
\tag{28}
$$

By the setting at t_{n+1} in equation (28), we obtained the numerical scheme as

$$
\begin{aligned}
x_{n+1}(t) &= x_0 + \frac{\eta_2 t_n^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_1 (x^n, y^n, z^n, u^n, t_n) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^{t_{n+1}} \Lambda^{\eta_2-1} \}_1 (x, y, z, u, \Lambda)\, d\Lambda, \\
y_{n+1}(t) &= y_0 + \frac{\eta_2 t_n^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_2 (x^n, y^n, z^n, u^n, t_n) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^{t_{n+1}} \Lambda^{\eta_2-1} \}_2 (x, y, z, u, \Lambda)\, d\Lambda, \\
z_{n+1}(t) &= z_0 + \frac{\eta_2 t_n^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_3 (x^n, y^n, z^n, u^n, t_n) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^{t_{n+1}} \Lambda^{\eta_2-1} \}_3 (x, y, z, u, \Lambda)\, d\Lambda, \\
u_{n+1}(t) &= u_0 + \frac{\eta_2 t_n^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_4 (x^n, y^n, z^n, u^n, t_n) + \frac{\xi_2\eta_2}{M(\xi_2)} \int_0^{t_{n+1}} \Lambda^{\eta_2-1} \}_4 (x, y, z, u, \Lambda)\, d\Lambda,
\end{aligned}
\tag{29}
$$

The simplified form of equation (29) can be expressed by taking the difference between the consecutive terms as

$$
\begin{aligned}
x_{n+1}(t) = x_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_1\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \\
\times \}_1\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) + \tfrac{\xi_2\eta_2}{M(\xi_2)} \int_{t_n}^{t_{n+1}} \lambda^{\eta_2-1} \}_1(x, y, z, u, \Lambda)\, d\Lambda, \\
y_{n+1}(t) = y_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_2\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \\
\times \}_2\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) + \tfrac{\xi_2\eta_2}{M(\xi_2)} \int_{t_n}^{t_{n+1}} \lambda^{\eta_2-1} \}_2(x, y, z, u, \Lambda)\, d\Lambda, \\
z_{n+1}(t) = z_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_3\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \\
\times \}_3\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) + \tfrac{\xi_2\eta_2}{M(\xi_2)} \int_{t_n}^{t_{n+1}} \lambda^{\eta_2-1} \}_3(x, y, z, u, \Lambda)\, d\Lambda, \\
u_{n+1}(t) = u_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_4\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \\
\times \}_4\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) + \tfrac{\xi_2\eta_2}{M(\xi_2)} \int_{t_n}^{t_{n+1}} \lambda^{\eta_2-1} \}_4(x, y, z, u, \Lambda)\, d\Lambda,
\end{aligned} \tag{30}
$$

Applying the elementary procedure of integration and the Lagrange polynomial piece-wise interpolation on equation (30), we investigated the suitable expressions as

$$
\begin{aligned}
x_{n+1}(t) = x_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_1\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_1\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \\
+ \tfrac{\xi_2\eta_2}{M(\xi_2)} \left\{ \tfrac{3h}{2} t_n^{\eta_2-1} \}_1\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{h}{2} t_{n-1}^{\eta_2-1} \}_1\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \right\}, \\
y_{n+1}(t) = y_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_2\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_2\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \\
+ \tfrac{\xi_2\eta_2}{M(\xi_2)} \left\{ \tfrac{3h}{2} t_n^{\eta_2-1} \}_2\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{h}{2} t_{n-1}^{\eta_2-1} \}_2\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \right\}, \\
z_{n+1}(t) = z_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_3\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_3\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \\
+ \tfrac{\xi_2\eta_2}{M(\xi_2)} \left\{ \tfrac{3h}{2} t_n^{\eta_2-1} \}_3\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{h}{2} t_{n-1}^{\eta_2-1} \}_3\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \right\}, \\
u_{n+1}(t) = u_n + \tfrac{\eta_2 t^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_4\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{\eta_2 t_{n-1}^{\eta_2-1}(1-\xi_2)}{M(\xi_2)} \}_4\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \\
+ \tfrac{\xi_2\eta_2}{M(\xi_2)} \left\{ \tfrac{3h}{2} t_n^{\eta_2-1} \}_4\left(x^n, y^n, z^n, u^n, t_n\right) - \tfrac{h}{2} t_{n-1}^{\eta_2-1} \}_4\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right) \right\},
\end{aligned} \tag{31}
$$

Calculating the simplification of equation (31), we investigated the numerical scheme for Caputo-Fabrizio fractal-fractional operator as

$$
\begin{aligned}
x_{n+1}(t) = x_n + \eta_2 t_n^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{3\xi_2 h}{2M(\xi_2)} \right) \}_1\left(x^n, y^n, z^n, u^n, t_n\right) - \eta_2 t_{n-1}^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{\xi_2 h}{2M(\xi_2)} \right) \\
\times \}_1\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right), \\
y_{n+1}(t) = y_n + \eta_2 t_n^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{3\xi_2 h}{2M(\xi_2)} \right) \}_2\left(x^n, y^n, z^n, u^n, t_n\right) - \eta_2 t_{n-1}^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{\xi_2 h}{2M(\xi_2)} \right) \\
\times \}_2\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right), \\
z_{n+1}(t) = z_n + \eta_2 t_n^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{3\xi_2 h}{2M(\xi_2)} \right) \}_3\left(x^n, y^n, z^n, u^n, t_n\right) - \eta_2 t_{n-1}^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{\xi_2 h}{2M(\xi_2)} \right) \\
\times \}_3\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right), \\
u_{n+1}(t) = u_n + \eta_2 t_n^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{3\xi_2 h}{2M(\xi_2)} \right) \}_4\left(x^n, y^n, z^n, u^n, t_n\right) - \eta_2 t_{n-1}^{\eta_2-1} \left(\tfrac{1-\xi_2}{M(\xi_2)} + \tfrac{\xi_2 h}{2M(\xi_2)} \right) \\
\times \}_4\left(x^{n-1}, y^{n-1}, z^{n-1}, u^{n-1}, t_{n-1}\right).
\end{aligned} \tag{32}
$$

3.3 Numerical Scheme for Atangana-Baleanu Fractal-Fractional Model

In order to bring the fractal-fractionalized the system of evolutionary differential equations (12), we converted the system of evolutionary differential equations (12) of memristor in terms of Atangana-Baleanu fractal-fractional differential operator as defined

$$\begin{aligned}
\mathfrak{D}_t^{\xi_3,\ \ \eta_3} x(t) &= \xi_3 t^{\xi_3 - 1} \}_1 (x, y, z, u, t), \\
\mathfrak{D}_t^{\xi_3,\ \ \eta_3} y(t) &= \xi_3 t^{\xi_3 - 1} g_2 (x, y, z, u, t), \\
\mathfrak{D}_t^{\xi_3,\ \ \eta_3} z(t) &= \xi_3 t^{\xi_3 - 1} g_3 (x, y, z, u, t), \\
\mathfrak{D}_t^{\xi_3,\ \ \eta_3} u(t) &= \xi_3 t^{\xi_3 - 1} g_4 (x, y, z, u, t),
\end{aligned} \tag{33}$$

Implementing equation (18) (Atangana-Baleanu integral in terms of fractal-fractional sense) on equation (33), we arrive at

$$\begin{aligned}
x(t) &= x(0) + \tfrac{\eta_3 t^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_1 (x, y, z, u, t) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^t \Lambda^{\eta_3 - 1} (t - \Lambda)^{\xi_3 - 1} \}_1 (x, y, z, u, \Lambda)\, d\Lambda, \\
y(t) &= y(0) + \tfrac{\eta_3 t^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_2 (x, y, z, u, t) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^t \Lambda^{\eta_3 - 1} (t - \Lambda)^{\xi_3 - 1} \}_2 (x, y, z, u, \Lambda)\, d\Lambda, \\
z(t) &= z(0) + \tfrac{\eta_3 t^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_3 (x, y, z, u, t) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^t \Lambda^{\eta_3 - 1} (t - \Lambda)^{\xi_3 - 1} \}_3 (x, y, z, u, \Lambda)\, d\Lambda, \\
u(t) &= u(0) + \tfrac{\eta_3 t^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_4 (x, y, z, u, t) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^t \Lambda^{\eta_3 - 1} (t - \Lambda)^{\xi_3 - 1} \}_4 (x, y, z, u, \Lambda)\, d\Lambda,
\end{aligned} \tag{34}$$

By the setting at t_{n+1} in equation (34), we obtained the numerical scheme as

$$\begin{aligned}
x_{n+1} &= x_0 + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_1 (x_n, y_n, z_n, u_n, t_n) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^{t_{n+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_1 (x, y, z, u, \Lambda)\, d\Lambda, \\
y_{n+1} &= y_0 + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_2 (x_n, y_n, z_n, u_n, t_n) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^{t_{n+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_2 (x, y, z, u, \Lambda)\, d\Lambda, \\
z_{n+1} &= z + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_3 (x_n, y_n, z_n, u_n, t_n) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^{t_{n+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_3 (x, y, z, u, \Lambda)\, d\Lambda, \\
u_{n+1} &= u_0 + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3)}{\mathrm{AB}(\xi_3)} \}_4 (x_n, y_n, z_n, u_n, t_n) + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \int_0^{t_{n+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_4 (x, y, z, u, \Lambda)\, d\Lambda,
\end{aligned} \tag{35}$$

The simplified form of equation (35) can be expressed for approximation within the interval $[t_j, t_{j+1}]$ in the compact form as

$$\begin{aligned}
x_{n+1} &= x_0 + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3) \}_1 (x_n, y_n, z_n, u_n, t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_1 (x, y, z, u, \Lambda)\, d\Lambda, \\
y_{n+1} &= y_0 + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3) \}_2 (x_n, y_n, z_n, u_n, t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_2 (x, y, z, u, \Lambda)\, d\Lambda, \\
z_{n+1} &= z_0 + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3) \}_3 (x_n, y_n, z_n, u_n, t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_3 (x, y, z, u, \Lambda)\, d\Lambda, \\
u_{n+1} &= u_0 + \tfrac{\eta_3 t_n^{\eta_3 - 1}(1-\xi_3) \}_4 (x_n, y_n, z_n, u_n, t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{\xi_3 \eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \Lambda^{\eta_3 - 1} (t_{n+1} - \Lambda)^{\xi_3 - 1} \\
&\qquad \times \}_4 (x, y, z, u, \Lambda)\, d\Lambda,
\end{aligned} \tag{36}$$

Calculating the simplification of equation (36), we investigated the numerical scheme for Atangana-Baleanu fractal-fractional operator as

$$
\begin{aligned}
x_{n+1} = x_0 + \tfrac{\eta_3 t_n^{\eta_3-1}(1-\xi_3)\}_1(x_n,y_n,z_n,u_n,t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{(\eta_3 t)^{\xi_3}\eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3+2)} \textstyle\sum_{j=0}^{n} \Big[t_j^{\eta_3-1}\}_1(x_j,y_j,z_j,u_j,t_j) \\
\times\Big\{(n+1-j)^{\xi_3}(n-j+2+\xi_3)-(n-j)^{\xi_3}(n-j+2+2\xi_3)\Big\} - t_{j-1}^{\eta_3-1}\}_1(x_{j-1},y_{j-1},z_{j-1},u_{j-1},t_{j-1}) \\
\times(n+1-j)^{\xi_3+1}-(n-j)^{\xi_3}(n-j+1+\xi_3)\Big], \\
y_{n+1} = y_0 + \tfrac{\eta_3 t_n^{\eta_3-1}(1-\xi_3)\}_2(x_n,y_n,z_n,u_n,t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{(\eta_3 t)^{\xi_3}\eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3+2)} \textstyle\sum_{j=0}^{n} \Big[t_j^{\eta_3-1}\}_2(x_j,y_j,z_j,u_j,t_j) \\
\times\Big\{(n+1-j)^{\xi_3}(n-j+2+\xi_3)-(n-j)^{\xi_3}(n-j+2+2\xi_3)\Big\} - t_{j-1}^{\eta_3-1}\}_2(x_{j-1},y_{j-1},z_{j-1},u_{j-1},t_{j-1}) \\
\times(n+1-j)^{\xi_3+1}-(n-j)^{\xi_3}(n-j+1+\xi_3)\Big], \\
z_{n+1} = z_0 + \tfrac{\eta_3 t_n^{\eta_3-1}(1-\xi_3)\}_3(x_n,y_n,z_n,u_n,t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{(\eta_3 t)^{\xi_3}\eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3+2)} \textstyle\sum_{j=0}^{n} \Big[t_j^{\eta_3-1}\}_3(x_j,y_j,z_j,u_j,t_j) \\
\times\Big\{(n+1-j)^{\xi_3}(n-j+2+\xi_3)-(n-j)^{\xi_3}(n-j+2+2\xi_3)\Big\} - t_{j-1}^{\eta_3-1}\}_3(x_{j-1},y_{j-1},z_{j-1},u_{j-1},t_{j-1}) \\
\times(n+1-j)^{\xi_3+1}-(n-j)^{\xi_3}(n-j+1+\xi_3)\Big], \\
u_{n+1} = u_0 + \tfrac{\eta_3 t_n^{\eta_3-1}(1-\xi_3)\}_4(x_n,y_n,z_n,u_n,t_n)}{\mathrm{AB}(\xi_3)} + \tfrac{(\eta_3 t)^{\xi_3}\eta_3}{\mathrm{AB}(\xi_3)\Gamma(\xi_3+2)} \textstyle\sum_{j=0}^{n} \Big[t_j^{\eta_3-1}\}_4(x_j,y_j,z_j,u_j,t_j) \\
\times\Big\{(n+1-j)^{\xi_3}(n-j+2+\xi_3)-(n-j)^{\xi_3}(n-j+2+2\xi_3)\Big\} - t_{j-1}^{\eta_3-1}\}_4(x_{j-1},y_{j-1},z_{j-1},u_{j-1},t_{j-1}) \\
\times(n+1-j)^{\xi_3+1}-(n-j)^{\xi_3}(n-j+1+\xi_3)\Big].
\end{aligned} \tag{37}
$$

4. Numerical Results

In this section, we present complex dynamics generated by fractal-fractional-order system of memristor with interesting characteristics based on the graphical illustration so called chaotic behavior say figures (2-10).The various chaotic attractors have been demonstrated by fractal-fractional-order system of memristor by the numerical simulations based on the three types of fractal-fractional operators namely Atangana-Baleanu, Caputo-Fabrizio and Caputo. From comparison point of view, the fractal-fractional mathematical operators have played their important roles in capturing some hidden chaotic behaviors that could not be revealed by non-fractional operators. It is observed from Figs. (2-10) that the differences and similarities within the behavior of the solution of the attractors have generated rich dynamics for fractal-fractional-order system of memristor. Figures from (2-10) are prepared by invoking the control parameters as$R0 = 1k\Omega, R1 = 500\Omega, R2 = 1k\Omega, R3 = 1\ k\Omega,\ C1 = 100\ nF$and $a = 2,\ b = 1,\ c = 0.2,\ k = 0.92,\ m = -0.002,\ n = 0.04$subject to initial conditions say$x(0) = y(0) = z(0) = u(0) = 0.01$.Fig. 2 is depicted for chaotic behaviors of memristor given by numerical scheme of Caputo fractal-fractional operator keeping (fractional parameter) $\xi_1 = 1$ and (fractal parameter) $\eta_1 = 0.98$at $t = 400$. Fig. 3 illustrates the chaotic behaviors of memristor given by numerical scheme of Caputo fractal-fractional operator keeping (fractional parameter) $\xi_1 = 0.99$ and (fractal parameter)$\eta_1 = 1$ at $t = 900$. In order to disclose the hidden phenomenon, we presented Fig. 4 for three dimensional chaotic behaviors of memristor given by numerical scheme of Caputo fractal-fractional operator keeping (fractional parameter) $\xi_1 = 0.99$ and (fractal parameter) $\eta_1 = 0.98$ at $t = 300$. Fig. 5-7 elucidates the chaotic behaviors of memristor given by numerical scheme of Caputo-Fabrizio fractal-fractional operator; in which we varied fractional parameter ($\xi_1 = 0.99$) and kept fractal parameter equal to one ($\eta_1 = 1$) while reciprocally, we kept fractional parameter equal to one ($\xi_1 = 1$) and varied fractal parameter. Such chaotic behaviors can be seen in in Fig. 5 and 6. Meanwhile, we varied fractional parameter as well as fractal parameter($\xi_1 = 0.99, , \eta_1 = 0.98$) in Fig. 7. The similar trend is employed in Figs. (8-10)

Which present chaotic behaviors of memristor given by numerical scheme of Atangana-Baleanu fractal-fractional operator.

5. Conclusion

This manuscript is investigated to present the fractal-fractional model based on highly non-linear for mathematical model of memristor in terms of fractal-fractional differential operator so called Atangana-Baleanu, Caputo-Fabrizio and Caputo fractal-fractional differential operator. The numerical solutions for mathematical model of memristor have extensively been discussed by means of Adams-Bashforth-Moulton method. With the help of numerical schemes of fractal-fractional differential operators, chaotic behavior of memristor under three criteria is discussed as (i) varying fractal order, we fixed fractional order, (ii) varying fractional order, we fixed fractal order and (ii) varying fractal and fractional orders simultaneously. Such analysis of attractors is simulated via MATLAB. At the end, chaotic behaviors of memristor suggest that newly presented Atangana-Baleanu, Caputo-Fabrizio and Caputo fractal-fractional differential operator has generates significant results as compared with classical approach.

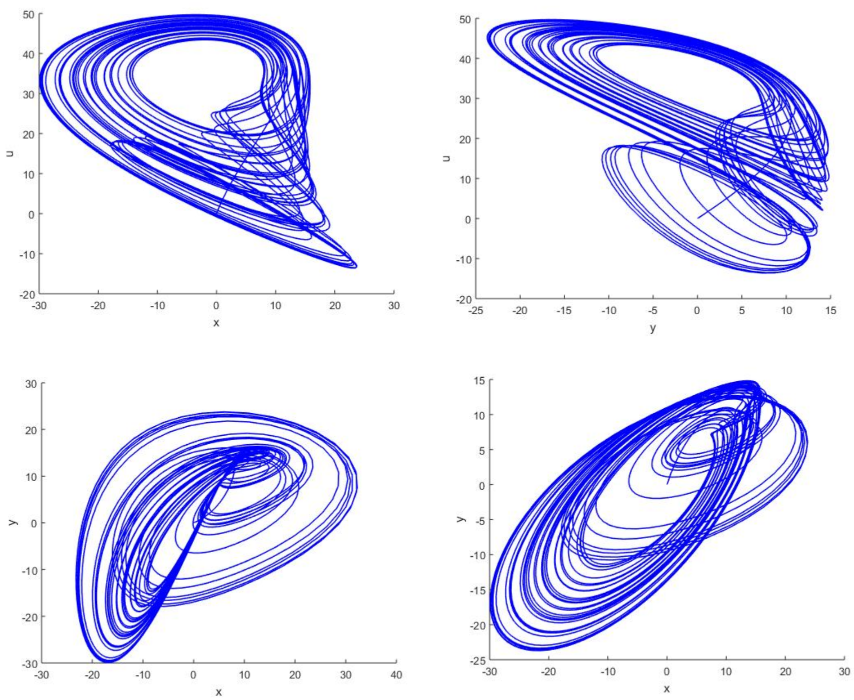

Fig.2. Chaotic Behaviors of memristor given by numerical scheme of Caputo fractal-fractional operator keeping (fractional parameter)$\xi_1 = 1$ and (fractal parameter) $\eta_1 = 0.98$ at$t = 400$.

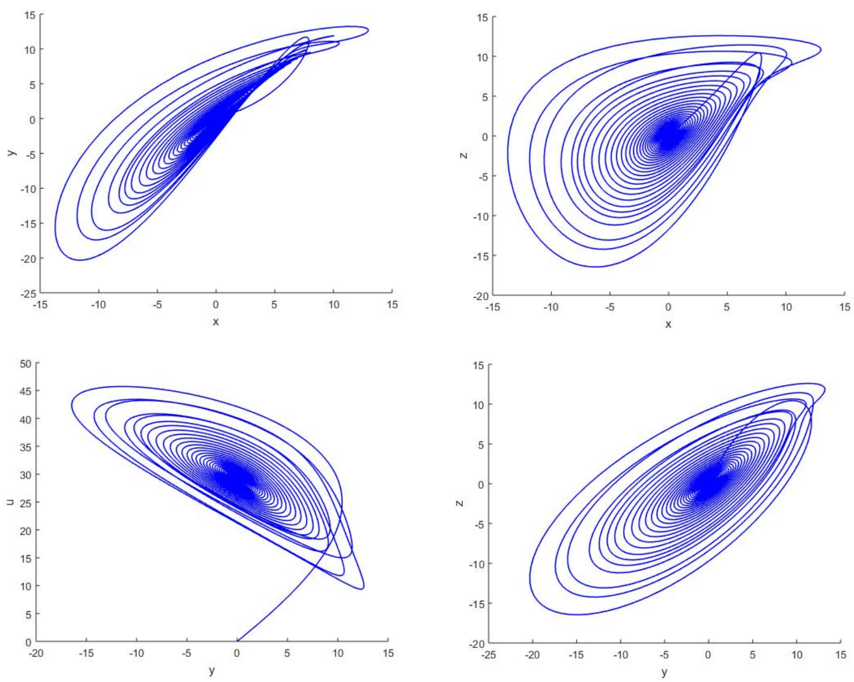

Fig.3. Chaotic Behaviors of memristor given by numerical scheme of Caputo fractal-fractional operator keeping (fractional parameter)$\xi_1 = 0.99$ and (fractal parameter) $\eta_1 = 1$ at$t = 900$.

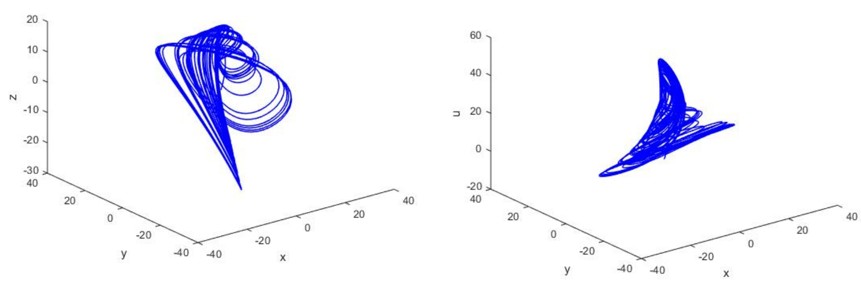

Fig.4. Chaotic Behaviors of memristor given by numerical scheme of Caputo fractal-fractional operator keeping (fractional parameter)$\xi_1 = 0.99$ and (fractal parameter) $\eta_1 = 0.98$ at$t = 300$.

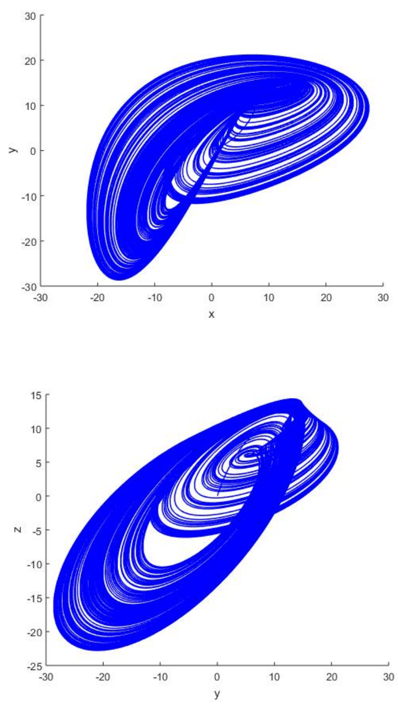

Fig.5. Chaotic Behaviors of memristor given by numerical scheme of Caputo-Fabrizio fractal-fractional operator keeping (fractional parameter) $\xi_1 = 1$ and (fractal parameter) $\eta_1 = 0.98$at $t = 1800$.

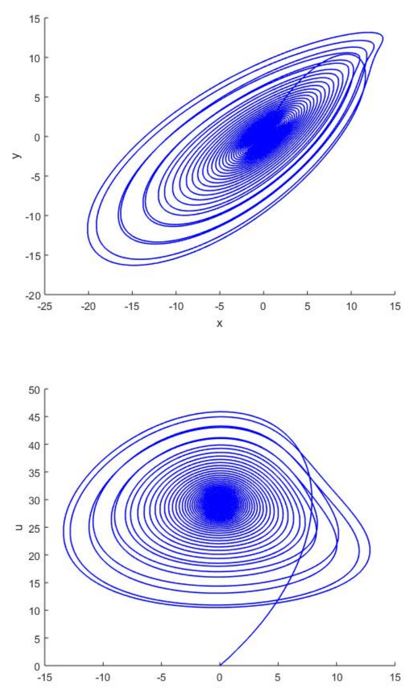

Fig.6. Chaotic Behaviors of memristor given by numerical scheme of Caputo-Fabrizio fractal-fractional operator keeping (fractional parameter) $\xi_1 = 0.99$ and (fractal parameter) $\eta_1 = 1$at $t = 1800$.

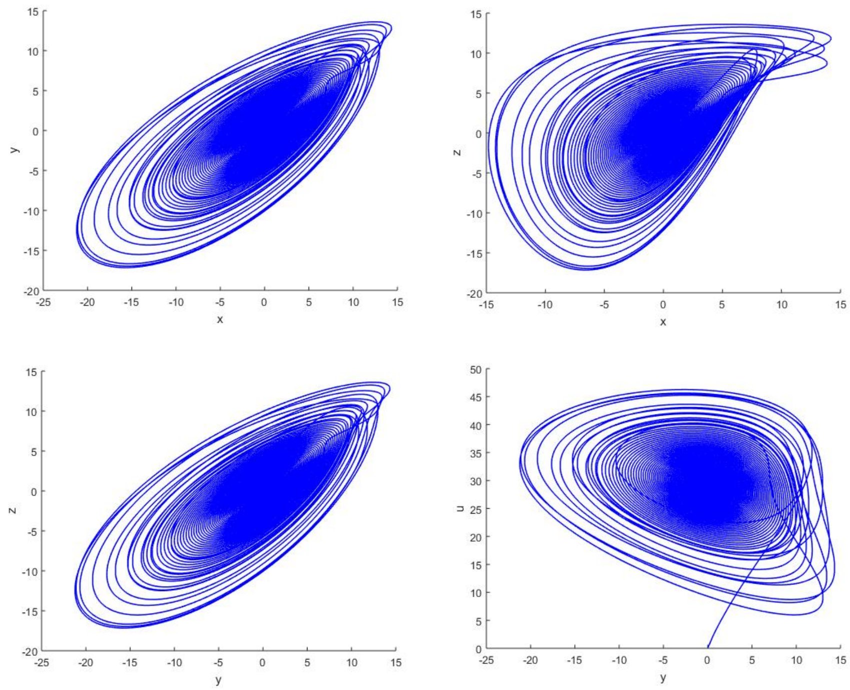

Fig.7. Chaotic Behaviors of memristor given by numerical scheme of Caputo-Fabrizio fractal-fractional operator keeping (fractional parameter) $\xi_1 = 0.99$ and (fractal parameter)$\eta_1 = 0.98$ at $t = 1800$.

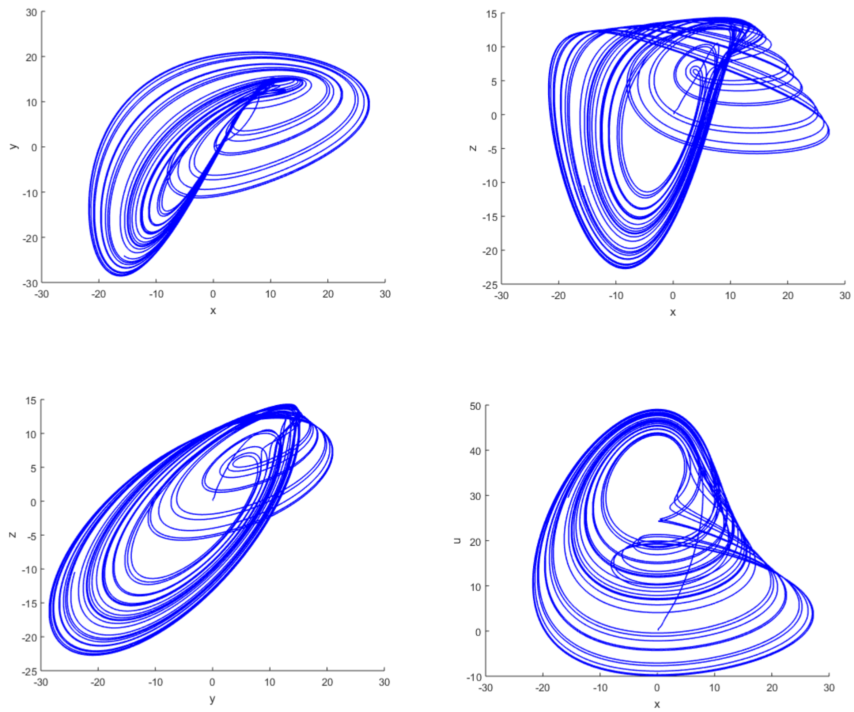

Fig.8. Chaotic Behaviors of memristor given by numerical scheme of Atangana-Baleanu fractal-fractional operator keeping (fractional parameter) $\xi_1 = 1$ and (fractal parameter) $\eta_1 = 0.99$at $t = 300$.

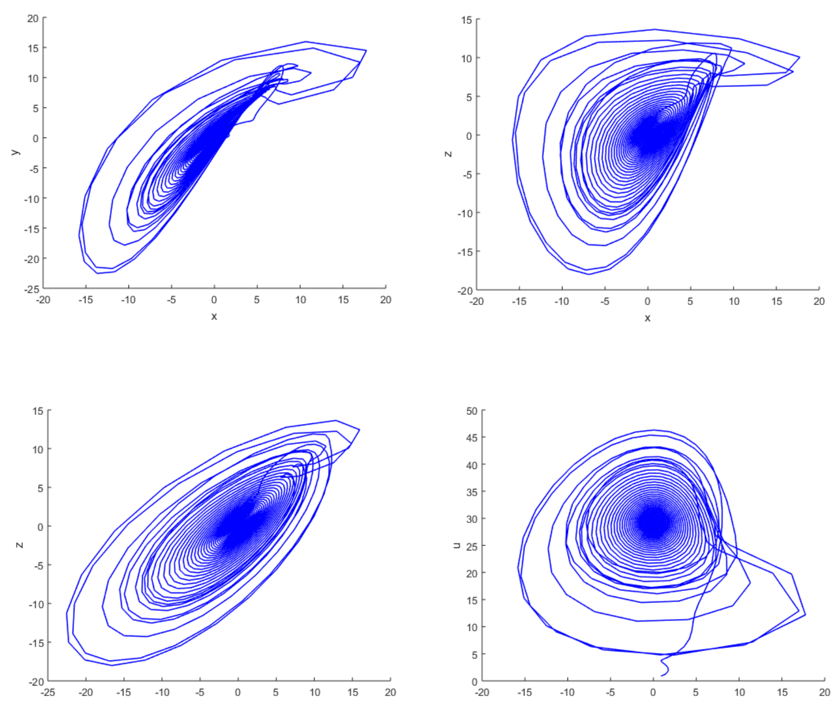

Fig.9. Chaotic Behaviors of memristor given by numerical scheme of Atangana-Baleanu fractal-fractional operator keeping (fractional parameter) $\xi_1 = 0.99$ and (fractal parameter) $\eta_1 = 1$at $t = 1800$.

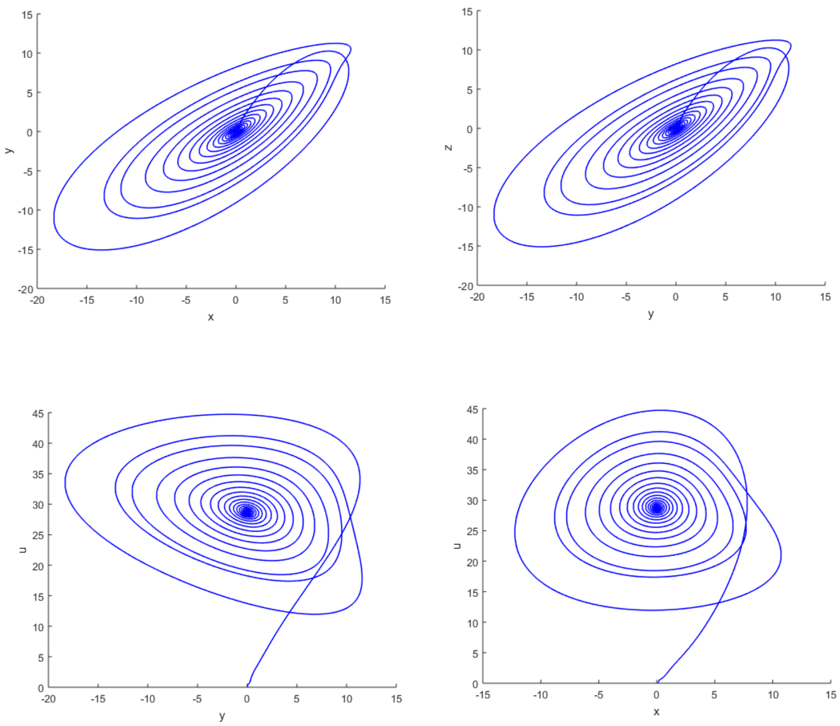

Fig.10. Chaotic Behaviors of memristor given by numerical scheme of Atangana-Baleanu fractal-fractional operator keeping (fractional parameter) $\xi_1 = 0.99$ and (fractal parameter)$\eta_1 = 0.99$ at $t = 1800$.

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Conflict of interest

The authors declare no conflict of interest.

References

[1] Chua L O, Memristor-The missing circuit element, IEEE Trans. Circuit Th., 18, 507-519 (1971).

[2] Strukov D B, Snider G S, Stewart, D. R. & Williams, R. S., The missing memristor found, Nature, 459 (2008).

[3] Bao B C, Shi G D, Xu J P, Liu Z, Pan S H, Dynamics analysis of chaotic circuit with two memristors, Sci. China Tech. Sci. 54, 2180-2187 (2011).

[4] Buscarino A, Fortuna L, Frasca M, Gambuzza L V, A chaotic circuit based on Hewlett-Packard memristor, Chaos, 22, 023136 (2012).

[5] Adhikari SP, Sah MP, Kim H, Chua LO, Three fingerprints of memristor, IEEE Trans. Circuits Syst, I(60), 3008-3021 (2013).

[6] Li Q D, Zeng HZ, Li J, Hyperchaos in a 4D memristive circuit with infinitely many stable equilibria, Nonlinear Dyn, 79, 2295-2308 (2015).

[7] Chen M, Li M Y, Yu Q, Bao B C, Xu Q, Wang J, Dynamics of self-excited attractors and hidden attractors in generalized memristor-based Chua's circuit, Nonlinear Dyn., 81, 215-226 (2015).

[8] Zhou L,Wang C H, Zho L L, Generating hyperchaotic multi-wing attractor in a 4D memristive circuit, Nonlinear Dyn., 85, 2653-2663 (2016).

[9] Yang N N, Xu C, Wu C J, Jia R, Modeling and analysis of a fractional-order generalized memristor-based chaotic system and circuit implementation, Int J Bifurcat Chaos, 27 (13) (2017), Article 1750199, DOI: 10.1142/S0218127417501991.

[10] Sánchez-López C, Carbajal-Gómez V H, Carrasco-Aguilar M A and Carro-Pérez I, Fractional-order memristor emulator circuits Complexity, 2018, ID 2806976 https://doi.org/10.1155/2018/2806976.

[11] Lin Q, Wen C, Fajie W, Ji L, A non-local structural derivative model for memristor, Chaos, Solitons & Fractals, 126, 169-177 (2019).

[12] H Cheng, The Casimir effect for parallel plates in the space time with a fractal extra compactified dimension, Int J Theor Phys, 52 (2013), 3229-3237.

[13] J H He, A tutorial review on fractal space time and fractional calculus, Int J Theor Phys, 53 (2014), 3698-3718.

[14] F. Brouers, T.J. Al-MusawiBrouers-Sotolongo fractal kinetics versus fractional derivative kinetics: a new strategy to analyze the pollutants sorption kinetics in porous materials, J Hazard Mater, 350 (2018) 162-168.

[15] M. Pan, L. Zheng, F. Liu, A spatial-fractional thermal transport model for nanofluid in porous media, Appl Math Model, 53 (2018), 622-634.

[16] Atangana A, Fractal-fractional differentiation and integration: Connecting fractal calculus and fractional calculus to predict complex system, Chaos, Solitons & Fractals, 102, (2017), 396-406.

[17] Atangana A, Qureshi S, Modeling attractors of chaotic dynamical systems with fractal-fractional operators, Chaos, Solitons & Fractals, 123, (2019), 320-337.

[18] Gomez-Aguilar J F, Chaos and multiple attractors in a fractal-fractional Shinriki's oscillator model, Physica A (2019), doi: https://doi.org/10.1016/j.physa.2019.122918.

[19] Sania Q, Abdon A, Asif A S, Strange chaotic attractors under fractal-fractional operators using newly proposed numerical methods, Eur. Phys. J. Plus (2019) 134: 523, DOI 10.1140/epjp/i2019-13003-7.

[20] Kashif A A, Gomez-Aguilar J F, A comparison of heat and mass transfer on a Walter's-B fluid via Caputo-Fabrizio versus Atangana-Baleanu fractional derivatives using the Fox-H function, Eur. Phys. J. Plus (2019) 134, 101, DOI 10.1140/epjp/i2019-12507-4.

[21] Abro K A, Muhammad N M, Gomez-Aguilar J F, Functional application of Fourier sine transform in radiating gas flow with non-singular and non-local kernel, Journal of the Brazilian Society of Mechanical Sciences and Engineering (2019) 41:400 https://doi.org/10.1007/s40430-019-1899-0.

[22] Kanno R. Representation of random walk in fractal space-time, Physica A, 1998, 248, 165-75.

[23] Kashif A A, Ilyas K, Jose F G-A, Thermal effects of magnetohydrodynamic micropolar fluid embedded in porous medium with Fourier sine transform technique, Journal of the Brazilian Society of Mechanical Sciences and Engineering, 41, (2019) 174-181. https://doi.org/10.1007/s40430-019-1671-5.

[24] Abro K A, Ali A M, Anwer A M, Functionality of Circuit via Modern Fractional Differentiations, Analog Integrated Circuits and Signal Processing: An International Journal, 99(1) 11-21, (2019). https://doi.org/10.1007/s10470-018-1371-6.

[25] Chen W, Sun H G , Zhang X, Korosak D, Anomalous diffusion modeling by fractal and fractional derivatives, Comput Math Appl. 2010, 59(5), 1754-8.

[26] Ambreen S, Kashif, A A, Muhammad A S, Thermodynamics of magnetohydrodynamic Brinkman fluid in porous medium: Applications to thermal science, Journal of Thermal Analysis and Calorimetry (2018), DOI: 10.1007/s10973-018-7897-0

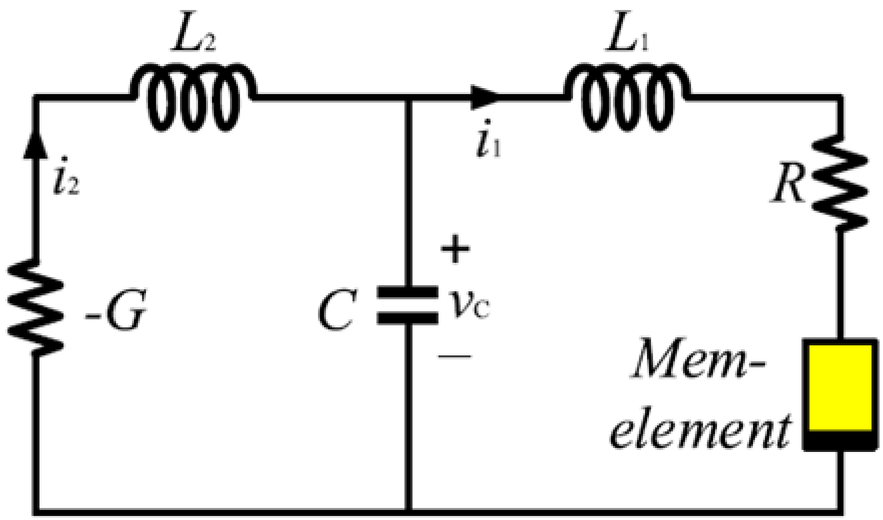
[27] Kashif A A, Ilyas K, Gomez-Aguilar J F, A mathematical analysis of a circular pipe in rate type fluid via Hankel transform, Eur. Phys. J. Plus (2018) 133: 397, DOI 10.1140/epjp/i2018-12186-7.

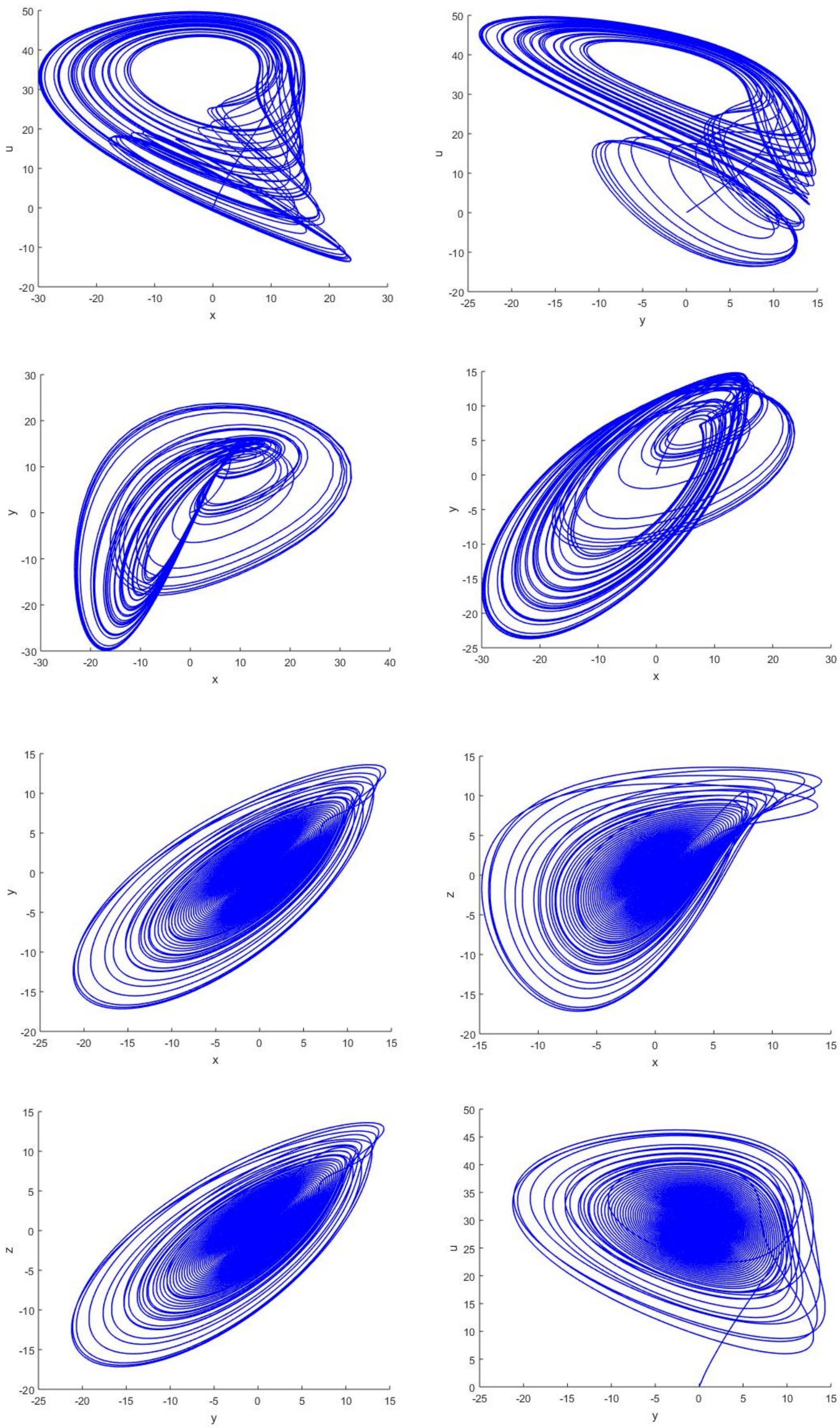
[28] Abro K A, Ali D C, Irfan A A, Ilyas K, Dual thermal analysis of magnetohydrodynamic flow of nanofluids via modern approaches of Caputo–Fabrizio and Atangana–Baleanu fractional derivatives embedded in porous medium, Journal of Thermal Analysis and Calorimetry, (2018) 1-11. https://doi.org/10.1007/s10973-018-7302-z.

[29] J. Singh, D. Kumar, D. Baleanu, S. Rathore. On the local fractional wave equation in fractal strings. Mathematical Methods in the Applied Sciences, 42(5), (2019), 1588-1595.

[30] Kashif Ali Abro, Anwar Ahmed Memon, Muhammad Aslam Uqaili, A comparative mathematical analysis of RL and RC electrical circuits via Atangana-Baleanu and Caputo-Fabrizio fractional derivatives, Eur. Phys. J. Plus, (2018) (2018) 133: 113, DOI 10.1140/epjp/i2018-11953-8.

[31] Abro K A, Ilyas K, Kottakkaran S N, Novel technique of Atangana and Baleanu for heat dissipation in transmission line of electrical circuit, Chaos, Solitons & Fractals, 129, 40-45, (2019), https://doi.org/10.1016/j.chaos.2019.08.001

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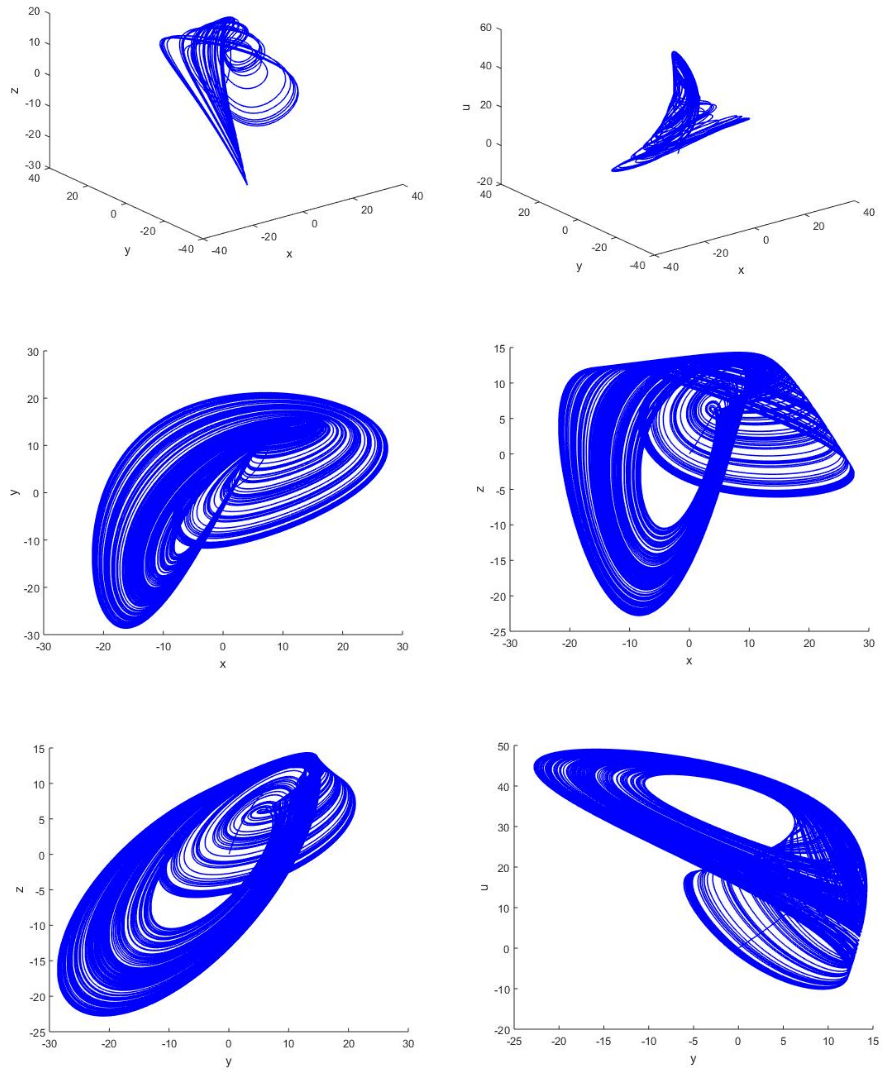
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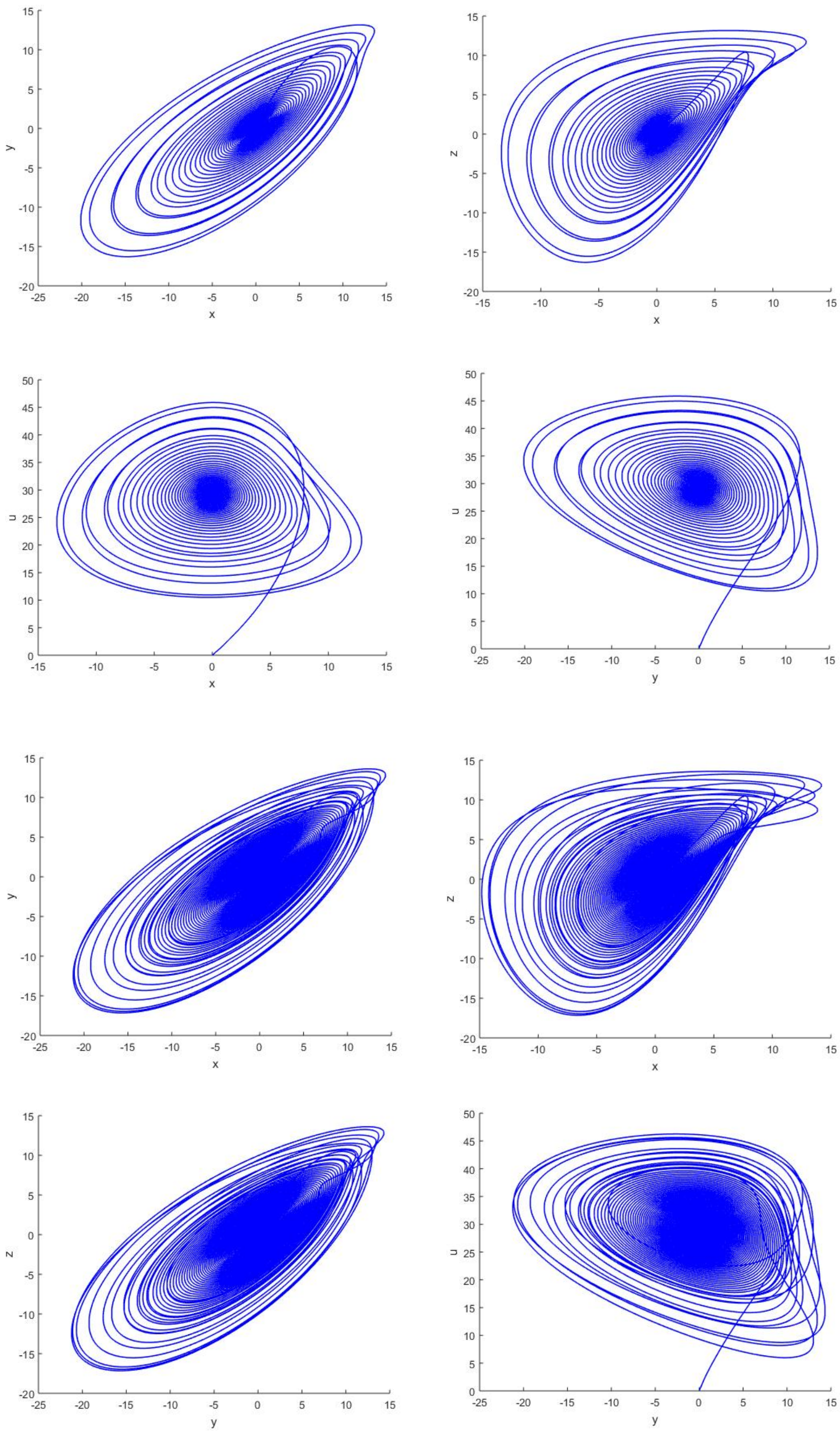
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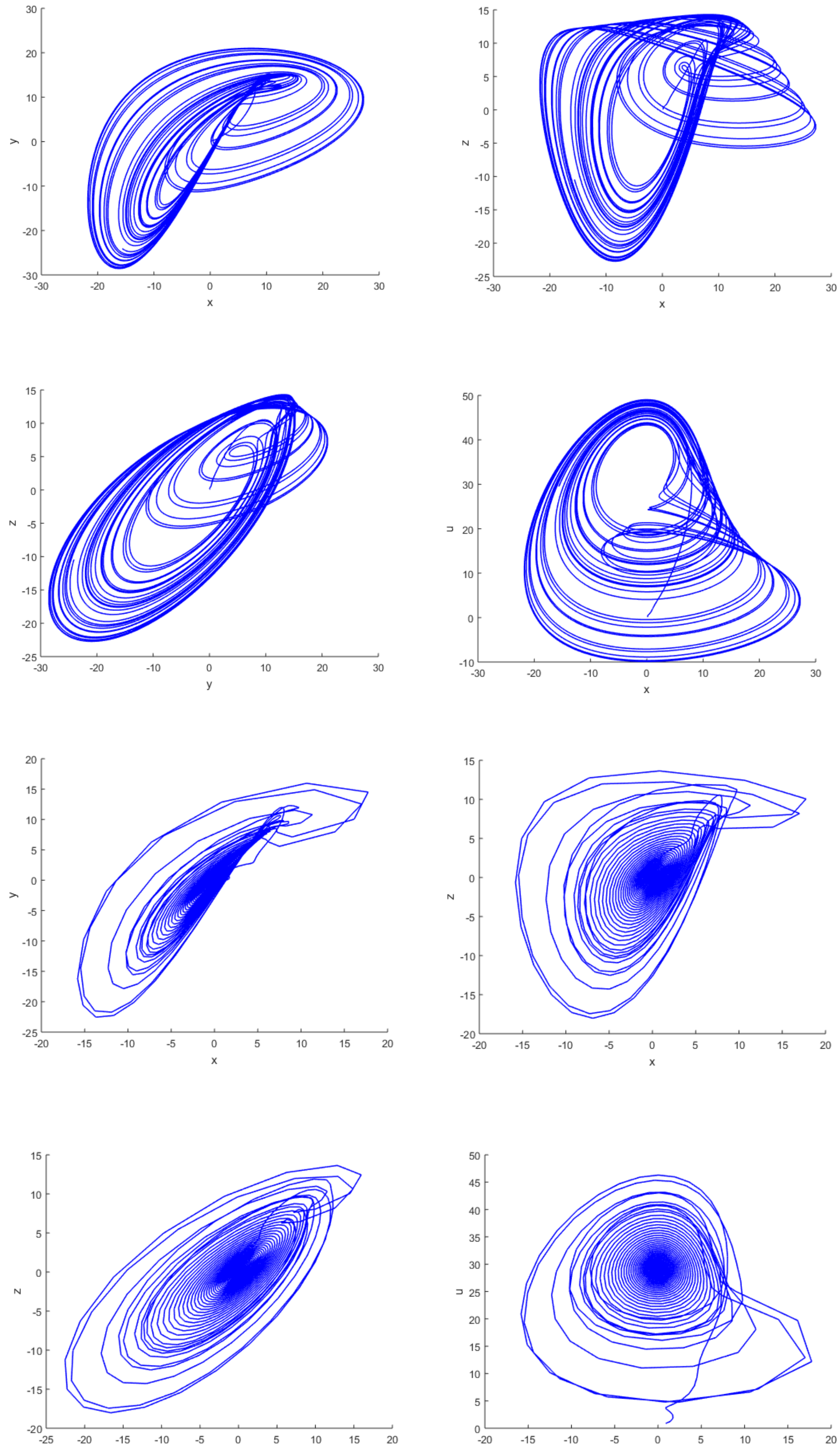
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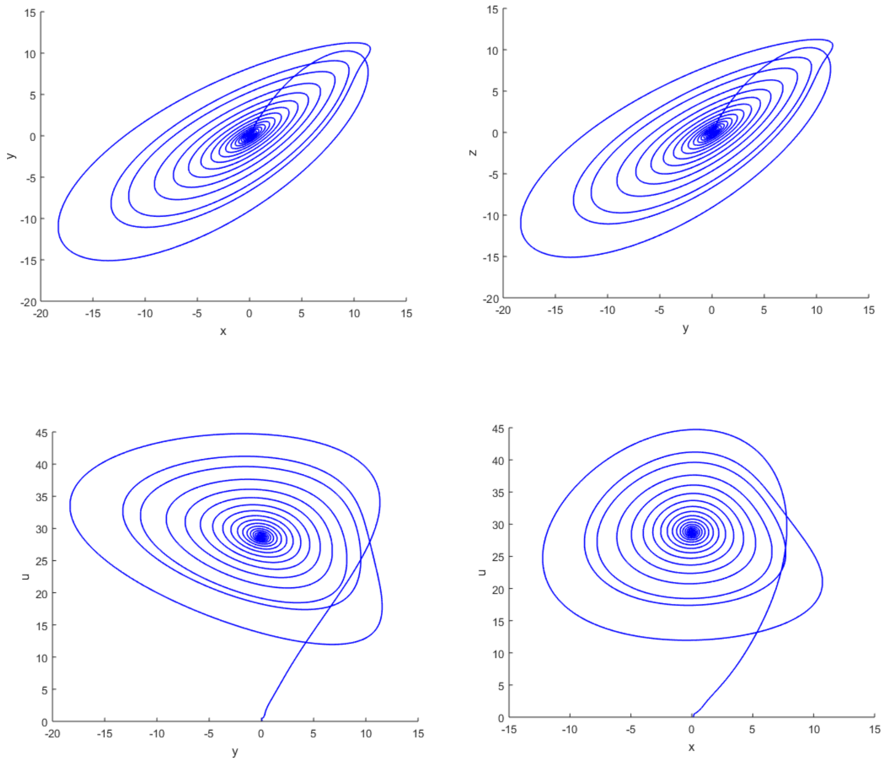

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