Simulation of Dynamo Action Generated by a Precession Driven Flow.

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Abstract

Since many years precession is regarded as an alternative flow driving mechanism that may account, e.g., for remarkable features of the ancient lunar magnetic field [Dwyer 2011; Noir 2013; Weiss 2014] or as a complementary power source for the geodynamo [Malkus 1968; Vanyo 1991]. Precessional forcing is also of great interest from the experimental point of view because it represents a natural forcing mechanism that allows an efficient driving of conducting fluid flows on the laboratory scale without making use of propellers or pumps. Within the project DRESDYN (DREsden Sodium facility for DYNamo and thermohydraulic studies) a dynamo experiment is under development at Helmholtz-Zentrum Dresden-Rossendorf (HZDR) in which a precession driven flow of liquid sodium with a magnetic Reynolds number of up to Rm=700 will be used to drive dynamo action. Our present study addresses preparative numerical simulations and flow measurements at a small model experiment running with water. The resulting flow pattern and amplitude provide the essential ingredients for kinematic dynamo models that are used to estimate whether the particular flow is able to drive a dynamo. In the strongly non-linear regime the flow essentially consists of standing inertial waves. Most remarkable feature is the occurrence of a resonant-like axisymmetric mode which emerges around a precession ratio of $\Omega p/\Omega c = 0.1$ on top of the directly forced re-circulation flow. The combination of this axisymmetric mode and the forced m=1 Kelvin mode is indeed capable of driving a dynamo at a critical magnetic Reynolds number of Rmc=430 which is well within the range achievable in the experiment. However, the occurrence of the axisymmetric mode slightly depends on the absolute rotation rate of the cylinder and future experiments are required to indicate whether it persists at the extremely large Re that will be obtained in the large scale sodium experiment.

NG21A-0142 - Simulation of Dynamo Action Generated by a Precession Driven Flow

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The precession dynamo experiment at HZDR

Motivated by the idea of a precession-driven flow as energy source for the early geodynamo or the ancient lunar magnetic field, a large precession dynamo experiment is under development at HZDR.

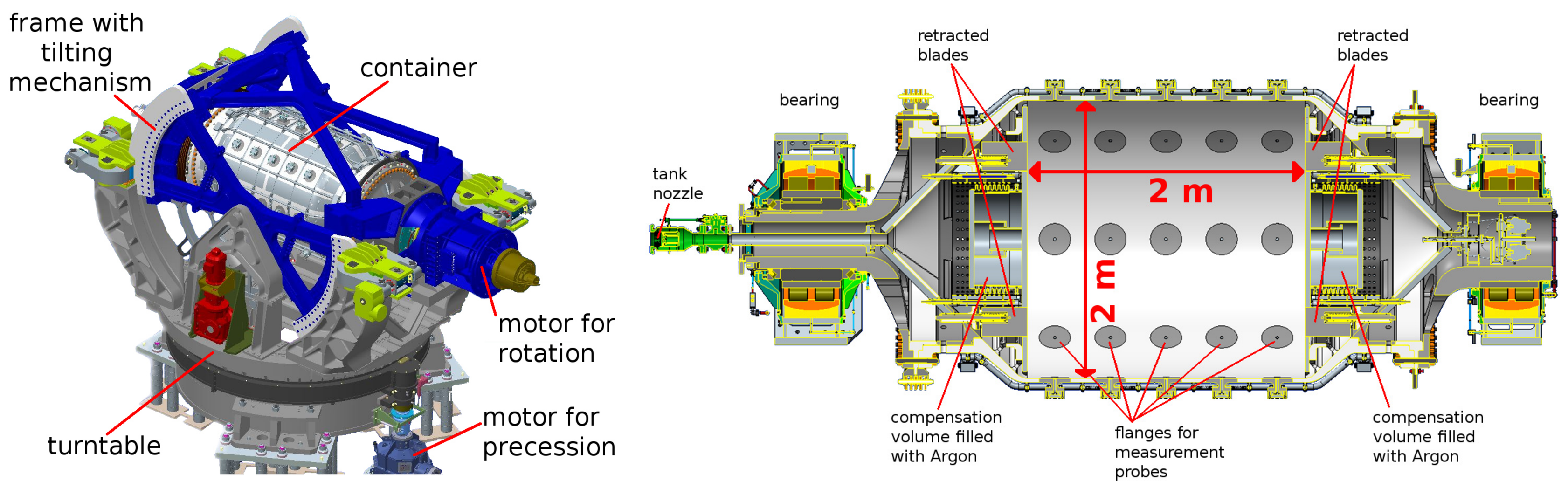

Fig. 1: Sketch of the facility (left) and the container (right). The cylinder has radius R=1 m and height H=2 m. The tilt between rotation axis and precession axis can be varied from 45° to 90°.

rotation rate	precession rate	nutation angle	Reynolds	magnetic Reynolds	aspect ratio	precession ratio
$f_c=\frac{\Omega_c}{2\pi}$	$f_p=\frac{\Omega_p}{2\pi}$	α	$\mathrm{Re}=\frac{\Omega_c R^2}{\nu}$	$\mathrm{Rm}=\frac{\Omega_c R^2}{\eta}$	$\Gamma=\frac{H}{R}$	$\mathrm{Po}=\frac{\Omega_p}{\Omega_c}$
$0\ldots10$ Hz	$0\ldots1$ Hz	$45°\ldots90°$	up to 10^8	up to 700	2	$0\ldots0.1$

- *natural* forcing allows efficient flow driving without propellers or pumps
- Gans' experiments in the 70's yield field amplification by a factor of 3
- precession dynamos are found in simulations around $\mathrm{Rm}\sim O(10^3)$

Hydrodynamic simulations & experiments

We conduct numerical simulations with *SEMTEX* (Spectral Element-Fourier method) and flow measurements in a down-scaled water experiment with Ultrasonic Doppler Velocimetry (UDV) (Fig. 2a)

$$\frac{\partial}{\partial t}\boldsymbol{u}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}=-\nabla P-2\boldsymbol{\Omega}_{\mathrm{p}}\times\boldsymbol{u}+\nu\nabla^2\boldsymbol{u} \quad \text{and} \quad \boldsymbol{u}_{\mathrm{bc}}=\boldsymbol{\Omega}_{\mathrm{c}}\times\boldsymbol{r}$$

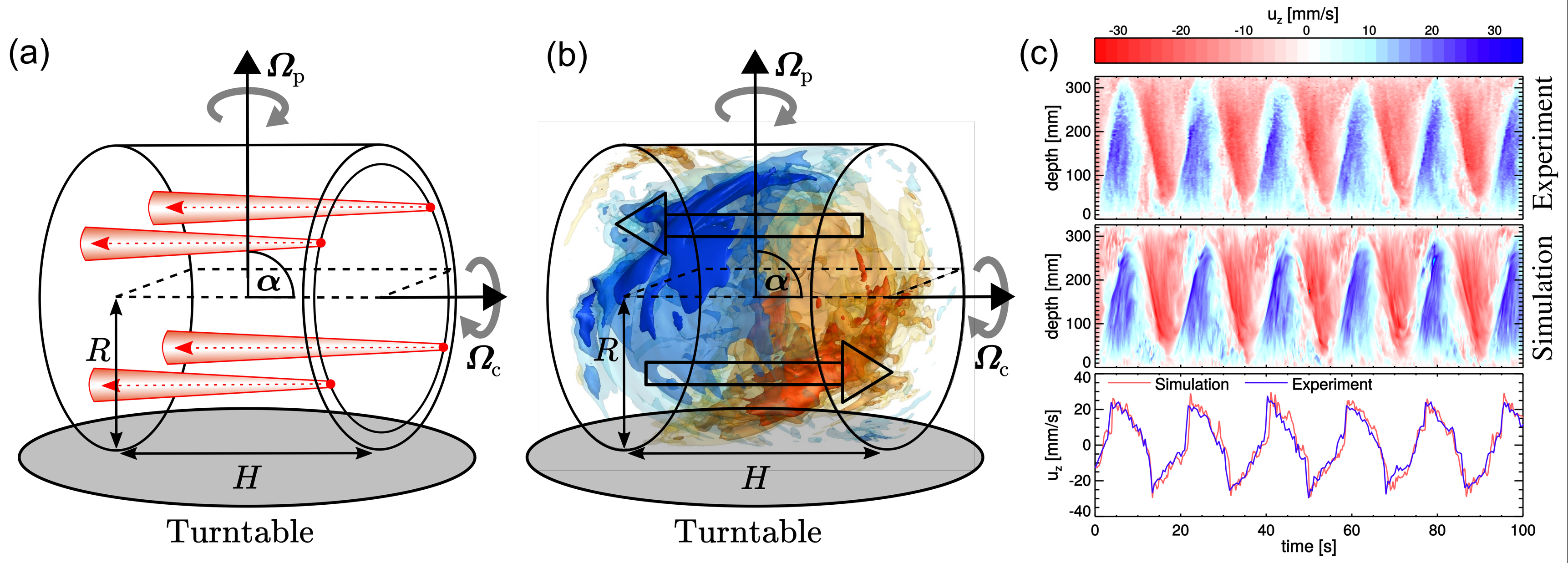

Fig. 2: (a) Set-up of the experiment with location of UDV probes. Nutation angle is fixed at $\alpha=90°$. (b) Isosurfaces of axial velocity from simulations that illustrate the recirculation flow (arrows), the concentration in the vicinity of the lateral walls, and the tilt with respect to the symmetry axis. (c) Direct comparison of UDV measurements (top) and simulations (center) at $\mathrm{Re}=10^4$ and $\mathrm{Po}=0.10$

- excellent agreement between simulation and experiment at Re=10^4 which is the upper limit for simulations and lower limit for motor (Fig. 2c)
- flow determined by directly forced flow $(m=1)$ overlapped by higher azimuthal modes (essentially $m=2$) that represent standing inertial waves due to nonlinear self-interactions (Fig. 2b and 2c)

Inertial waves & amplitudes

Quantitative analysis by decomposing u_z in axial modes $\propto\sin(\pi zk/H)$ with axial wavenumber k and Fourier transformation in azimuth and time

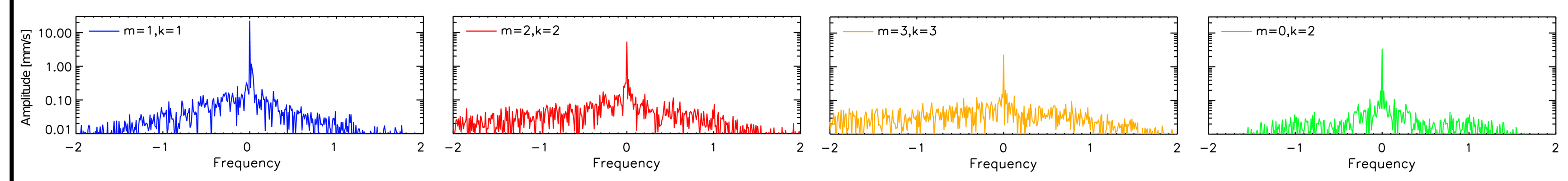

Fig. 3: Spectra of the directly forced mode (blue), its first harmonics (red & orange) and the axisymmetric flow (green) from simulations in the precession frame with $\mathrm{Re}=10^4$ and $\mathrm{Po}=0.10$.

- laminar flow consisting of large scale inertial modes (standing waves in precession frame) with only weak time-dependent contributions

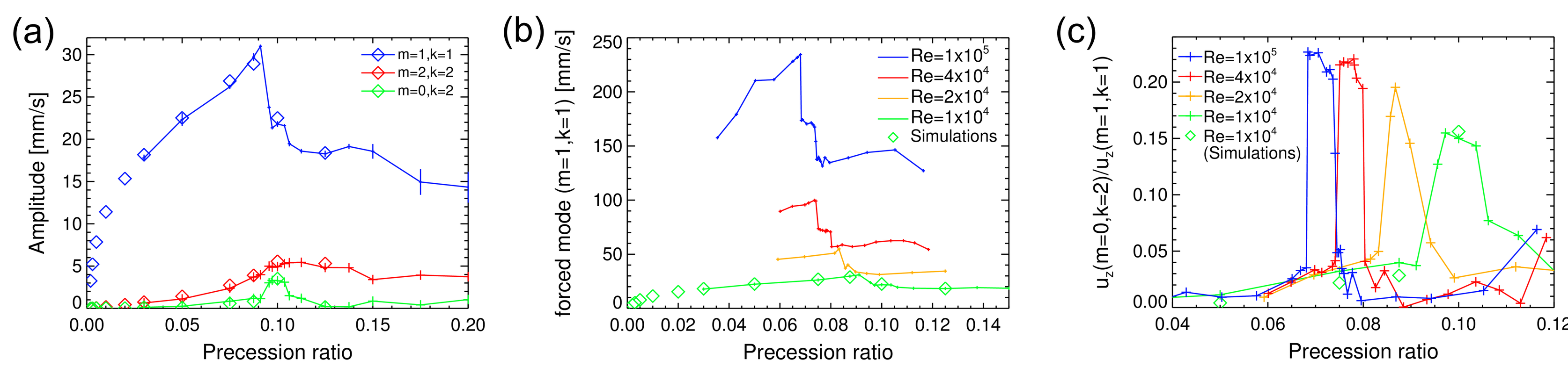

Fig. 4: (a) Amplitude of dominant modes for Re=10^4 and Po=0.1. (b) Amplitude of forced mode (m,k)=(1,1) for various Re. (c) Relative amplitude of the axisymmetric mode (m,k)=(0,2) for various Re. The width of the regime with axisymmetric mode (~0.006) is rather independent of Re.

- resonance-like emergence of axisymmetric mode in conjunction with reduction of amplitude of directly forced mode (m,k)=(1,1)

Structure of axisymmetric mode

- double roll structure in the meridional plane (similar to mean flow in VKS)
- geostrophic azimuthal mode opposite to solid body rotation

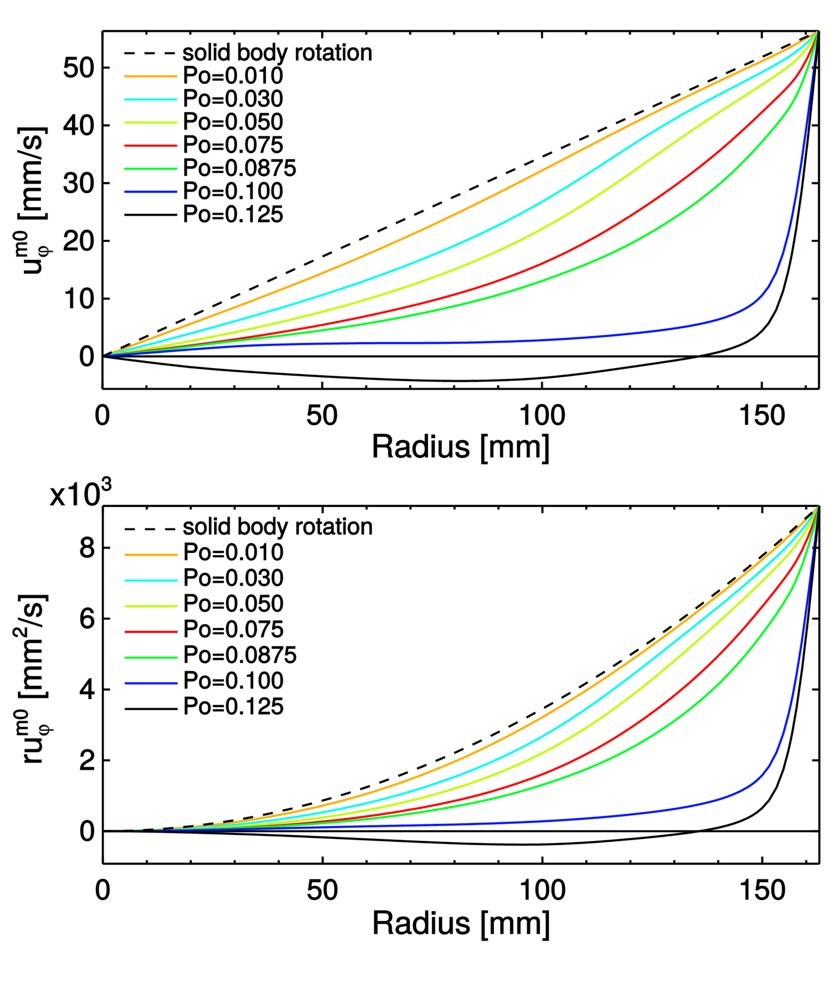

Fig. 5: Radial profiles of the time-averaged azimuthal velocity (top) and the corresponding angular momentum (bottom) in the equatorial plane from simulations at Re=10^4. The Rayleigh criteria is violated when the slope of ru_φ becomes negative.

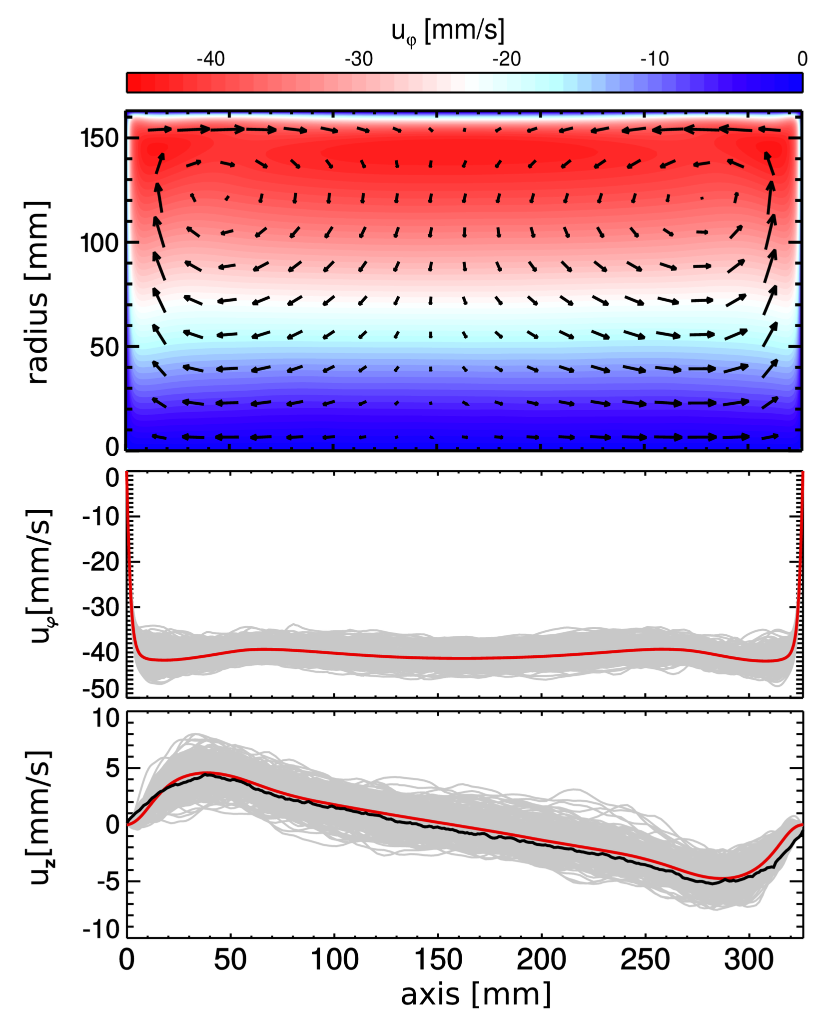

Fig. 6: Structure of the time-averaged axisymmetric velocity field from simulations at $\mathrm{Re}=10^4$ and $\mathrm{Po}=0.10$. The black curve in the bottom panel shows the profile obtained in the water experiment.

- strong axisymmetric mode within narrow regime of precession ratios
- goes along with suppression of solid body rotation
- violation of Rayleigh's instability criteria for rotating flows $\frac{d}{dr}(u_\varphi r)<0$

Kinematic dynamos

The time-averaged velocity fields obtained from hydrodynamic simulations constitute the basis for kinematic dynamo models. The magnetic induction equation is solved numerically using pseudo-vacuum boundary conditions.

$$\frac{\partial}{\partial t}\boldsymbol{B}=\nabla\times(\boldsymbol{u}\times\boldsymbol{B}-\eta\nabla\times\boldsymbol{B})$$

- results show dynamo action at $\mathrm{Rm}^{\mathrm{crit}}=430$ when applying the flow field from Re=10^4 and $\mathrm{Po}=0.1$
- essential for the dynamo is the combination of axisymmetric flow $(m=0)$ and the directly forced mode $(m=1)$
- flow fields obtained at other values of Po exhibit dynamo action only around $\mathrm{Rm}^{\mathrm{crit}}\approx2000\ldots5000$ (far above values achievable in the planned experiment)

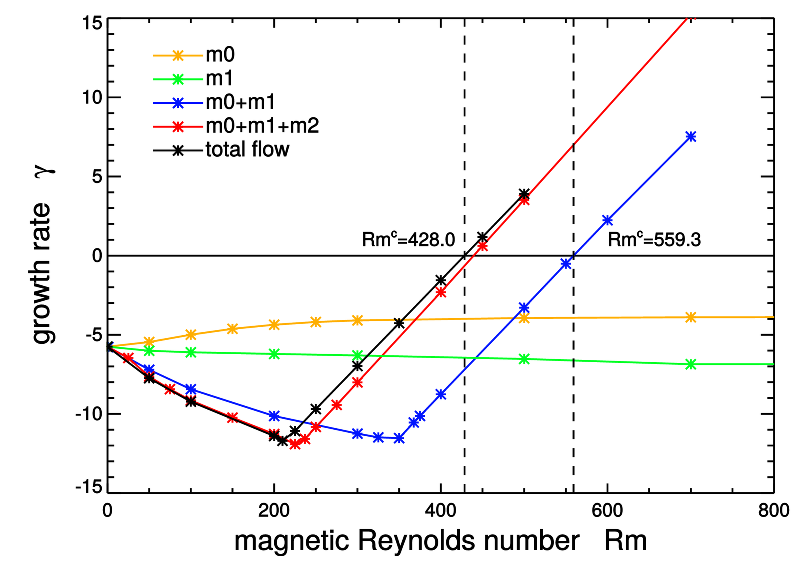

Fig. 7: Growth rates vs Rm for combinations of various azimuthal velocity modes obtained from simulations at $\mathrm{Re}=10^4$ and $\mathrm{Po}=0.1$

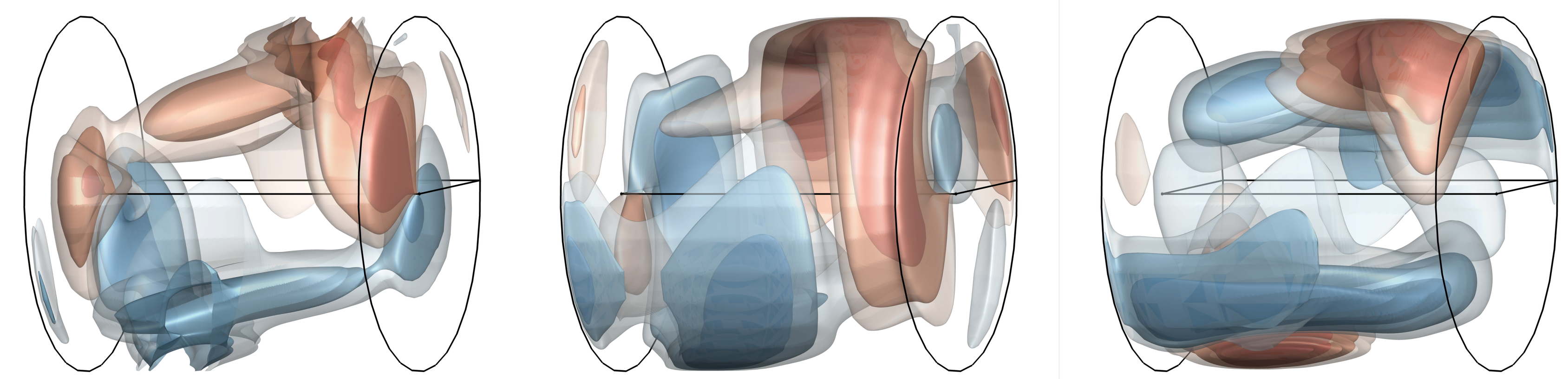

Fig. 8: Structure of the magnetic field at 12.5,25,50% of its maximum value. From left to right: B_r, B_φ, B_z. The field structure propagates around the cylinder axis.

Conclusions for the planned sodium dynamo

Dynamos possible at Rm achievable in the planned experiment in a narrow regime characterized by presence of an axisymmetric mode

Scaling to planned sodium experiment:

- emergence of $m=0$ mode is connected with hysteresis found for transition into turbulent flow state at larger Re
- occurs at smaller Po when Re is increased
- measurements of power consumption and pressure point out asymptotic behavior with $\mathrm{Po}^{\mathrm{crit}}\approx0.06\ldots0.07$ for $\mathrm{Re}\gtrsim10^6$

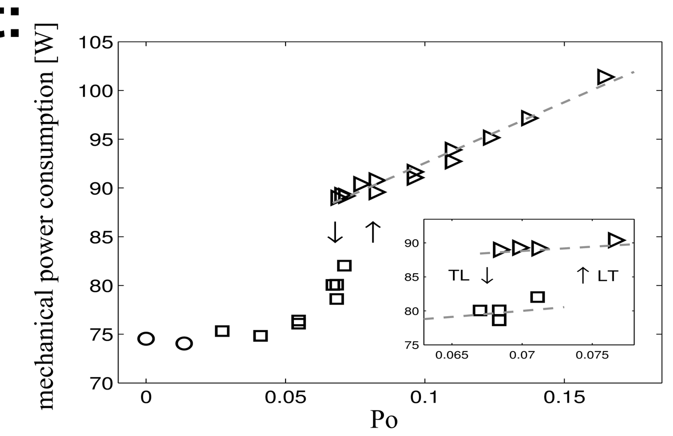

Fig. 9: Jump in electrical power consumption and hystereses at transition to turbulent state. Experiment Re=10^6

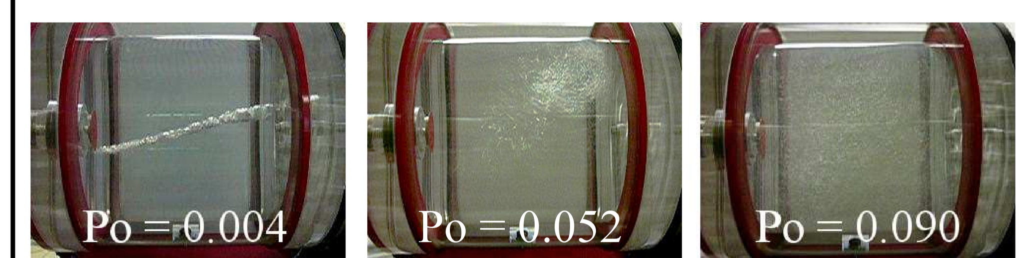

Fig. 10: Transition from laminar state (left) to a nonlinear fluid behavior (center) finally resulting in a turbulent flow state above a critical precession ratio (right). Re=2×10^6.

- $\mathrm{Rm}^{\mathrm{crit}}\approx430$ corresponds to $f_c\approx5\ldots6$ Hz
- $\mathrm{Po}^{\mathrm{crit}}\approx0.06$ corresponds to $f_p\approx0.35$ Hz
- global quantities (pressure, motor power, torque) for characterisation of flow state

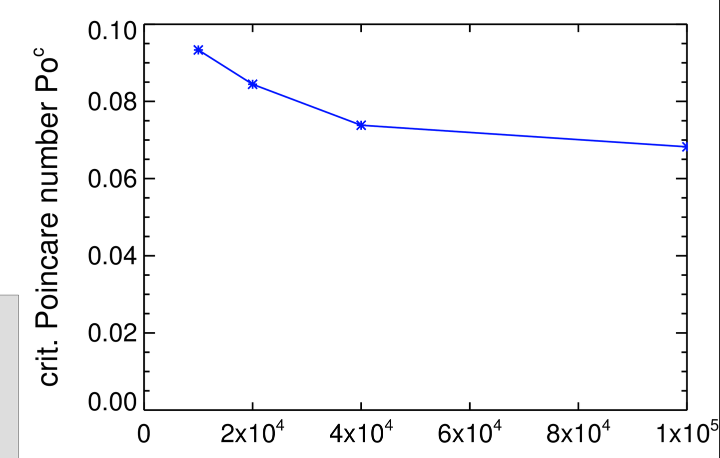

Fig. 11: Critical Po for appearance of axisymmetric mode from experiment

see Giesecke et al., arxiv: 1708.06314