# A Long-Lived Lunar Magnetic Field Powered by Convection in the Core and a Basal Magma Ocean

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1 A Long-Lived Lunar Magnetic Field Powered by Convection in the Core and a 2 Basal Magma Ocean 3 4 Saira S. Hamid<sup>12</sup>, Joseph G. O'Rourke<sup>2</sup>, Krista M. Soderlund<sup>3</sup> 5 6 ABSTRACT 7 8 An internally generated magnetic field once existed on the Moon. This field 9 reached high intensities (~10-100  $\mu T,$  perhaps intermittently) from ~4.3-3.6 10 Gyr ago and then weakened to  $\lesssim$  5  $\mu$ T before dissipating by ~1.9-0.8 Gyr ago. 11 While the Moon's metallic core could have generated a magnetic field via a 12 dynamo powered by vigorous convection, models of a core dynamo often fail to 13 explain the observed characteristics of the lunar magnetic field. In 14 particular, the core alone likely may not contain sufficient thermal, 15 chemical, or radiogenic energy to sustain the high-intensity fields for >100 16 Myr. A recent study by Scheinberg et al. suggested that a dynamo hosted in 17 electrically conductive, molten silicates in a basal magma ocean (BMO) may 18 have produced a strong early field. However, that study did not fully explore 19 the BMO's coupled evolution with the core. Here we show that a coupled BMO-20 core dynamo driven primarily by inner core growth can explain the timing and 21 staged decline of the lunar magnetic field. We compute the thermochemical 22 evolution of the lunar core with a 1-D, parameterized model tied to extant 23 simulations of mantle evolution and BMO solidification. Our models are most 24 sensitive to four parameters: the abundances of sulfur and potassium in the 25 core, the core's thermal conductivity, and the present-day heat flow across 26 the core-mantle boundary. Our models best match the Moon's magnetic history 27 if the bulk core contains ~6.5-8.5 wt% sulfur, in agreement with seismic 28 structure models.

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29	1. INTRODUCTION
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31	Paleomagnetic analyses of lunar meteorites and Apollo samples suggest that
32	a high-intensity magnetic field of ~10-100 $\mu T$ existed ~4.25-3.56 billion years
33	(Gyr) ago, followed by a weakened field of $\lesssim$ 5 $\mu$ T that persisted until ~1.9-
34	0.8 Gyr ago (e.g., Tikoo et al. 2014, Tikoo et al. 2017, Mighani et al. 2020,
35	Strauss et al. 2021, Wieczorek et al. 2022). Generation of an intrinsic
36	magnetic field via dynamo action requires vigorous motion of an electrically
37	conducting fluid such as the liquid portion of a metallic core (e.g., Bullard
38	1949, Elsasser 1950, Bullen 1954, Glatzmaier and Roberts 1995, Kageyama et
39	al. 1995). Various observations indicate that the Moon has a metallic core,
40	including seismic data from the Apollo missions (e.g., Garcia et al. 2011,
41	Weber et al. 2011), electromagnetic sounding (e.g., Hood et al. 1999, Shimizu
42	et al. 2013), and gravity data from the Gravity Recovery and Interior
43	Laboratory (GRAIL) mission (e.g., Williams et al. 2014), which are all
44	consistent with a core radius of $\sim 250-430$ km. Today, a solid inner core with
45	a radius up to $\sim\!250$ km may also exist (Williams et al. 2014, Weber et al.
46	2011).
47	Models of the thermal evolution of the lunar core have difficulty
48	reproducing the history of the lunar magnetic field (e.g., Evans et al. 2018,

49 Laneuville et al. 2014, Scheinberg et al. 2015). These models have two goals 50 that often seem incompatible: 1) sustaining a long-lived field (e.g., 51 multiple Gyr) and 2) sustaining an early strong field (i.e., >10  $\mu T,$  at least 52 for the first ~1 Gyr). With available energy sources internal to the core 53 (e.g., radiogenic, latent, and chemical energy, plus inner core precession), 54 the Moon can sustain a low-intensity field for long durations (e.g., 55 Laneuville et al. 2014; Scheinberg et al. 2015, Evans et al. 2018, Stys & 56 Dumberry 2020). However, Evans et al. (2018) showed that those energy sources 57 could only sustain a >10  $\mu T$  field for <50 Myr, assuming that the radius of the 58 core is  $\leq$ 380 km as favored by Weber et al. (2011) and Williams et al. (2014). 59 So, sustaining a >10  $\mu T$  field for ~1 Gyr is highly improbable without an 60 external mechanism, such as mechanical stirring between the solid mantle and 61 the liquid core from precession of the lunar spin axis (e.q., Dwyer et al. 62 2011; Meyer & Wisdom 2010; Cuk et al. 2019) and/or impact-induced changes in 63 the rotation rate of the solid mantle (e.g., Le Bars et al. 2011). Another 64 solution to this seeming paradox is to invoke intermittency during the high-65 intensity epoch. For example, a recent study proposed that foundering of 66 relatively cold material in the lunar mantle may have excited episodes of

67 rapid core cooling that lasted <1 Myr (Evans & Tikoo 2022). Finally, in this 68 study, we explore the idea that the core is not the only potential host for a 69 lunar dynamo as argued by Scheinberg et al. (2018).

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1.1. A Basal Magma Ocean

73 Almost any scenario for the formation of the Moon involves enough 74 energy to melt much of the newly formed Moon (e.g., Hartmann & Davis 1975, 75 Warren 1985, Elkins-Tanton et al. 2011, Canup 2012, Ćuk & Stewart 2012, 76 Nakajima & Stevenson 2014). The resulting magma ocean is often modeled as 77 solidifying in three primary stages (e.g., Elardo et al. 2011, Wieczorek et 78 al. 2006, Hess & Parmentier 1995, Hamid & O'Rourke 2022). As the lunar magma 79 ocean cooled, dense mafic cumulates (e.g., olivine and pyroxene) formed and 80 sank towards the bottom. Once most of the lunar magma ocean solidified, 81 anorthositic plagioclase with lower density began to crystallize, rising to 82 form the lunar crust. The final, highly evolved liquids, "ur-KREEP" (enriched 83 in uranium, thorium, potassium, rare earth elements, and phosphorus), 84 alongside ilmenite-rich cumulates, would be gravitationally unstable because 85 of their high densities. Some fraction of this ur-KREEP-ilmenite mixture 86 eventually sank to the base of the mantle, ponding as a layer above the core-87 mantle boundary (CMB). Radiogenic heat from elements present in this fallen 88 ur-KREEP layer, such as uranium, thorium, and potassium (with concentrations 89 up to ~12 times higher than the bulk mantle), could fully melt this layer 90 (e.g., Scheinberg et al. 2018). The result is a basal magma ocean (BMO) that 91 persists until convective heat loss into the overlying mantle causes 92 solidification. The nominal model of Scheinberg et al. (2018) had a 301-km 93 peak thickness BMO; less conservative models had BMO thicknesses up to 450 94 km.

95 Models are equivocal about the lifetime of a BMO. For example, a small 96 compositional density contrast between the BMO and the overlying mantle could 97 make the BMO short-lived (Stegman et al. 2003). In this scenario, thermal 98 expansion of the BMO can overcome the compositional density contrast between 99 the BMO and the overlying mantle, causing the BMO to buoyantly rise and re-100 mix with the mantle. Conversely, the persistence of interstitial fluid 101 trapped within the solidified cumulates could leave the BMO sequestered at 102 the CMB (Elkins-Tanton et al. 2011, Scheinberg et al. 2018). Indeed, 103 interpretations of geophysical data (Khan et al. 2014), seismic data (Weber 104 et al. 2011), and gravity data (Williams et al. 2014) have indicated that a

105 deep-seated zone of partial melt at the CMB may exist today. This partial 106 melt could be the last remnant of a once-thicker BMO.

107 The lunar BMO could have sustained a dynamo if it was vigorously 108 convecting and had an electrical conductivity,  $\sigma$ , of several thousand S/m 109 (Scheinberg et al. 2018). Such a BMO dynamo would have an advantage over the 110 core in terms of generating strong crustal fields because it is closer to the 111 surface (e.g., Ziegler & Stegman 2013). Magnetic fields attenuate rapidly 112 with distance, so a magnetic field generated in the BMO would appear stronger 113 at the surface than a magnetic field generated with the same strength in the 114 core (e.g., Scheinberg et al. 2018, Stevenson 1983, Christensen 2010). While 115 sufficiently high conductivity is a challenge for this hypothesis, thermal 116 coupling between the BMO and core can fortunately be explored regardless of 117 this uncertainty.

118 Our study is built on the whole-Moon models presented in Scheinberg et al. 119 (2018). That study focused on the thermal evolution of the solid mantle and 120 BMO to explain the early, strong (i.e., >10  $\mu$ T) lunar dynamo. Both the BMO and 121 the core were assumed to be well-mixed on the timescales of the overlying 122 solid mantle convection and to have an adiabatic temperature gradient, except 123 during the phase in which the magma ocean increases in temperature. That 124 study further tested the sensitivity of their model to the reference 125 viscosity in the solid mantle, the fraction of the KREEP layer that remained 126 near the surface, and the fraction of radioactive material concentrated in 127 the BMO. At the start of their simulations, the BMO exhibited a rapid 128 increase in heat flow from radiogenic heating, followed by a steady decline 129 to its solidus temperature. A detailed model of the core was not included 130 because the core is relatively small and does not strongly affect the thermal 131 evolution of the BMO and solid mantle. In this study, we do not directly 132 model the BMO-hosted dynamo, but rather focus on the core to test if models 133 of lunar evolution that feature a BMO as a boundary condition can explain 134 both the strong, early dynamo and the later dynamo that produced much weaker 135 fields (Figure 1).

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140 Figure 1: We study three stages in the coupled evolution of the lunar BMO and 141 core. (Left) Convection in the BMO with the potential to produce an early, high 142 intensity dynamo ~4.25-3.56 Gyr ago while the core was fully liquid. Dashed 143 arrows indicate that in limited scenarios, thermal convection in the core may 144 have occurred in tandem with the BMO-hosted dynamo. (Middle) Compositional 145 convection in the core produced a late, low intensity dynamo until ~1.9-0.8 Gyr 146 ago once the inner core started growing and the BMO began to solidify. (Right) 147 The internal field ceased ~1 Gyr ago once the BMO solidified sufficiently, the 148 inner core grew too large, and convection ceased in the liquid outer core. 149

#### 2. METHODS

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#### 2.1. Structure of the Metallic Core

154 We assume that the lunar core is an iron alloy that starts fully liquid 155 with no chemical or thermal stratification. To build our models, we assume 156 that sulfur is the major light element in the core, given its siderophile 157 behavior and cosmochemical abundance (e.g., Pommier et al. 2018, Cameron 158 1973). Our models also include trace amounts of potassium as a source of 159 radiogenic heating. Other studies have speculated about the possible roles of 160 other light elements in the lunar core, including carbon (e.g., Dasgupta et 161 al. 2009), silicon (e.g., Berrada et al. 2020), and phosphorous (e.g., Yin et 162 al. 2019). However, the complexities of a core with multiple light elements 163 are beyond the scope of this study.

164 A 1-D, parameterized description of the structure of the core is the 165 foundation of our models. As described in Appendix A, we used hydrostatic 166 equilibrium and equations of state detailed in Khan et al. (2017) to 167 calculate the radial profiles of density, pressure, temperature, and 168 gravitational acceleration within the core. Our fiducial structural model 169 assumes that the core contains 6 wt% sulfur and has a central pressure and 170 temperature of 5.15 GPa and 1800 K, respectively, to match the core 171 parameters described in Scheinberg et al. (2018). The radius of the core is 172 then 350 km, which is also the same as in Scheinberg et al. (2018) and in 173 agreement with available observational constraints. However, Scheinberg et 174 al. (2018) used an average density for the core appropriate to a composition

175 of pure iron, which would increase the total mass of the core by ~20%. 176 Fortunately, most of the structural parameters that are key to our 177 thermodynamic calculations (e.g.,  $K_0$ ,  $K_1$ ,  $L_\rho$ , and  $A_\rho$  in Table D1) are not 178 sensitive to the bulk composition of the core. Sulfur is most important to 179 the thermal evolution of the lunar core via its effect on the bulk liquidus. 180 Using a fixed sulfur content to calculate other parameters (e.g.,  $\rho_0$ ,  $P_0$ , and 181  $M_{\rm C}$ ) should only introduce inaccuracies that are smaller than the observational 182 uncertainties.

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#### 2.2. Energetics of the Metallic Core

186 The overlying BMO controls the evolution of the core. In the models of 187 Scheinberg et al. (2018), the BMO is initially set to 1700 K at 4.2 Ga, heats 188 up for ~200 Myr due to radiogenic heating, and subsequently cools until it 189 reaches the initial temperature when the models are stopped. We start 190 tracking the evolution of the core at the time when the BMO starts cooling 191 again. At that time, we assume the core is fully molten and has an adiabatic 192 temperature gradient throughout. We set that "initial" temperature at the top 193 of the core equal to that at the bottom of the BMO. From the results of 194 Scheinberg et al. (2018), we know the total heat flow across the core-mantle 195 boundary  $(Q_{CMB})$  over time:

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#### $Q_{CMB} = Q_B - Q_{S_{BMO}} - H_{BMO} (1)$

197 Here,  $Q_{\text{B}}$  is the heat flow outward from the BMO into the solid mantle,  $Q_{\text{SrMO}}$  is 198 heat associated with secular cooling, and  $H_{\text{BMO}}$  is the radiogenic heating in the 199 BMO. In order to model the magnetic history of the Moon until present day 200 (i.e. after the BMO model has stopped), we further assume that  $\mathcal{Q}_{CMB}$  changes 201 linearly to a specified present day value, which could be the same or (much) 202 less than the value of  $Q_{CMB}$  when the BMO solidifies. With the boundary 203 condition provided by the BMO model, we then use a well-established method, 204 developed to study Earth's core (e.g., Labrosse 2015), to model the 205 thermodynamic evolution of the lunar core once it starts cooling again. 206 First, we can calculate the global heat budget of the core:

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#### $Q_{CMB} = Q_L + Q_G + Q_R + Q_S (2)$

Here,  $Q_s$  represents the secular cooling of the core and is proportional to the core's specific heat. We assume that trace amounts of potassium produce radiogenic heating  $(Q_R)$ . The remaining two terms are only relevant once the inner core nucleates: energy from latent heat  $(Q_L)$  and gravitational energy 212 from the exclusion of light elements into the outer core  $(Q_G)$  that are 213 released as the inner core grows.

Given the total heat flow, we solve for the rate of change in the CMB temperature. As shown in Appendix B, most of the terms on the right side of equation (2) are products of  $dT_{CMB}/dt$  and a term ( $\tilde{Q}$ ) that depends only on the thermodynamic properties of the core and its structural parameters. Each of those terms is calculable using polynomial functions. We can thus rearrange equation (2):

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$$\frac{dT_{CMB}}{dt} = \frac{Q_{CMB} - Q_R}{\widetilde{Q_S} + \widetilde{Q_G} + \widetilde{Q_L}}$$
(3)

221 The growth rate of the inner core is directly proportional to  $dT_{CMB}/dt$  also 222 (see Appendix B). Because equation (2) does not include any secular cooling 223 of the inner core, we are implicitly assuming that the inner core is 224 perfectly insulating (i.e., with zero thermal conductivity). We could also 225 model a conductive inner core with infinite thermal conductivity, but the 226 associated heat flow is a minor contribution to the global heat budget if the 227 inner core extends to only <75% of the core radius, as expected at present 228 day. Technically, equations (2) and (3) are only valid if the liquid portion 229 of the core is convective and thus maintaining a nearly adiabatic thermal 230 profile. This assumption is not valid at present day since thermal 231 stratification probably exists since the core heat flux was likely lower than 232 the heat flux that can be conducted along the adiabat for most of the Moon's 233 evolution (e.g., Laneuville et al. 2014).

234 Our models use a liquidus for the core that depends on the bulk 235 composition. We adapted Equation 29 from Buono & Walker (2011), in which the 236 Fe-FeS liquidus is fit to a polynomial that is fourth-order in both pressure 237 and sulfur content. Our model uses an approximation of the liquidus that is 238 first-order in both pressure and sulfur content. Specifically, we estimated 239 the approximate pressure derivative  $(dT_L/dP)$  based on the difference in the 240 liquidus temperatures at 5.15 GPa at the center of the core versus 4.43 GPa 241 at the CMB for 6 wt% sulfur. We found the approximate compositional 242 derivative  $(dT_L/dc)$  based on the difference in liquidus temperatures for 0 vs. 243 25 wt% sulfur at 5 GPa (Table D1). 244

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2.3. Strength of a Core-Hosted Dynamo

247 Vigorous convection in the core can produce a dynamo through the 248 conversion of kinetic to magnetic energy. In general, there are two types of

249 power sources for convection in the core. First, the buoyancy of light 250 elements released from inner core solidification can drive compositional 251 convection. Second, thermal buoyancy from secular cooling of the core, 252 freezing of the inner core, and/or radiogenic heating can power thermal 253 convection. For thermal convection to occur from secular cooling alone,  $Q_{CMB}$ 254 must exceed the adiabatic heat flow  $(Q_{AD})$ , which equals the product of the 255 thermal conductivity of the core and the adiabatic temperature gradient (see 256 Appendix B). Once the inner core nucleates, the critical heat flow above 257 which convection occurs is lowered.

We combined the energy and entropy budgets for the core to calculate the total dissipation available to power a dynamo (e.g., Labrosse 2015):

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$$\Phi_{CMB} = \Phi_L + \Phi_G + \Phi_R + \Phi_S - \Phi_K. (4)$$

261 Here,  $\Phi_{\rm L}$ ,  $\Phi_{\rm G}$ ,  $\Phi_{\rm R}$ , and  $\Phi_{\rm S}$  are the dissipation terms associated with  $Q_{\rm L}$ ,  $Q_{\rm G}$ ,  $Q_{\rm R}$ , 262 and  $Q_{s}$ , respectively. The last term ( $\Phi_{K}$ ) corresponds to the entropy sink 263 associated with thermal conduction in the core. Appendix B contains the 264 polynomial expressions for each dissipation term, which, like the energy 265 terms, depend on the thermophysical properties of the core and its overall 266 cooling rate. Critically, we assume a dynamo exists if the dissipation is 267 positive (i.e., if  $\Phi_{ ext{CMB}}$  > 0 W). This criterion yields similar predictions as 268 another often-used criterion, which is that the magnetic Reynolds number 269 (defined below) exceeds a critical value of 50-100 (e.g., Roberts 2007). 270 Several scaling laws are available to convert the dissipation (in Watts) 271 into the strength of the magnetic field at the equatorial surface of the Moon 272 (in Teslas). First, we use a scaling law based on core energetics (see 273 Appendix B) to calculate the total dipole moment  $(D_M)$  of the Moon (units of A 274  $m^2$ ). In this case, assuming the lunar magnetic field is dipolar, the surface 275 field strength at the magnetic equator is

 $B = \frac{\mu_0 D_M}{4\pi R_M^3}, (5)$ 

277 where  $R_{\rm M}$  is the radius of the Moon and  $\mu_0$  is the permeability of free space. 278 Additionally, we estimate the magnetic field intensity using three scaling 279 laws that relate the associated convective power to the anticipated 280 convective velocities (e.g., Christensen 2010). These scaling laws use 281 different force balances to calculate the strength of the magnetic field in 282 the core ( $B_c$ ). First, mixing length theory (ML) assumes a balance between 283 inertial and buoyancy forces:

284 
$$B_{ML} = \left[2c\mu_0(\rho_0 R_C^2 \Phi_{CMB}^2)^{\frac{3}{2}}\right]^{\frac{1}{2}}, (6)$$

where  $c \sim 0.63$  is a constant of proportionality,  $\rho_0$  is the central density in the core, and  $R_c$  is the radius of the core. Second, assuming a balance of Coriolis, inertial, and gravitational (Archimedes) (CIA) forces yields:

288 
$$B_{CIA} = \left[2c\mu_0(\rho_0^2 R_c^4 \Omega \Phi_{CMB}^3)^{\frac{1}{5}}\right]^{\frac{1}{2}}, (7)$$

289 where  $\Omega$  is the present-day angular velocity of the Moon, which may 290 underestimate the field strength since the Moon likely rotated faster in the 291 past. Third, the Magneto-Archimedes-Coriolis (MAC) scaling assumes a balance 292 between Lorentz, gravitational, and Coriolis forces:

293 
$$B_{MAC} = \left[2c\mu_0(\rho_0 R_c^2 \Omega \Phi_{CMB})^{\frac{1}{2}}\right]^{\frac{1}{2}}.(8)$$

294 With these three scaling laws, we calculate the surface field strength of the 295 dipole component as

$$B_S = \frac{1}{7} B_C \left(\frac{R_C}{R_M}\right)^3. (9)$$

297 The ratio of the Moon's core radius to the Moon's radius  $(R_{\rm M})$  accounts for the 298 fact that the dipole field at the surface is smaller than the dipole field at 299 the core (Scheinberg et al. 2018). The pre-factor of 1/7 assumes an Earth-300 like power spectrum for the magnetic field and accounts for the fact that not 301 all of the energy in the magnetic field is partitioned into the poloidal 302 components that can reach the surface (e.g., Christensen et al. 2009, 303 Scheinberg et al. 2018). Note that the core field is assumed to diffuse 304 across an electrically insulating mantle in this approach, thus neglecting 305 the contribution of the BMO. Because the BMO is argued to have a relatively 306 large electrical conductivity, our surface field strength calculations may be 307 considered as lower-bound estimates (discussed further in section 4.3). 308

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#### 2.4. Local Rossby Number

We further assess the dipolarity of the Moon's magnetic field, particularly whether a dipole-dominated or multipolar dynamo may be preferred. Although there are numerous hypotheses for what controls the breakdown of the dipole (e.g., Soderlund & Stanley 2020), we consider here the local Rossby number:

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$$Ro_l = \frac{U}{2\Omega l},\tag{10}$$

316 where  $\Omega = 2\pi/T$  is the angular velocity of the Moon, T is the rotation period 317 in seconds, l is the characteristic length scale of the flow, and U is the 318 characteristic fluid velocity. This dimensionless parameter measures the

319 relative importance of inertial to Coriolis forces at convective length 320 scales. Numerical models of planetary dynamos indicate that dipole-dominated 321 solutions tend to be found approximately when  $Ro_l < 0.1$  (i.e., when inertial 322 effects are relatively weak compared to rotation), with multipolar solutions 323 occurring for larger  $Ro_l$  values (e.g., Christensen & Aubert 2006).

324 In order to estimate this parameter, we assume a characteristic fluid 325 velocity and length scale following scaling law predictions as done for the 326 magnetic field strengths (e.g., Christensen 2010). The mixing length (ML) 327 scaling yields

328 
$$U_{ML} = \left(\frac{\Phi_{CMB}R_C}{\rho_0}\right)^{\frac{1}{3}}, \quad l_{ML} = R_C, \quad Ro_{l,ML} = \left(\frac{\Phi_{CMB}R_C}{\rho_0}\right)^{\frac{1}{3}} (2 \ \Omega \ R_C)^{-1}, \tag{11}$$

329 the Coriolis, inertial, and gravitational (Archimedes) (CIA) scaling yields

330 
$$U_{CIA} = \left(\frac{\Phi_{CMB}}{\rho_0}\right)^{\frac{2}{5}} \left(\frac{R_c}{\Omega}\right)^{\frac{1}{5}}, \quad l_{CIA} = \left(\frac{U_{CIA} R_c}{\Omega}\right)^{\frac{1}{2}}, \quad Ro_{l,CIA} = \left(\frac{\Phi_{CMB}}{\rho_0}\right)^{\frac{2}{5}} \left(\frac{R_c}{\Omega}\right)^{\frac{1}{5}} (4 \ \Omega \ U_{CIA} R_c)^{-\frac{1}{2}}, \quad (12)$$

331 and the Magneto-Archimedes-Coriolis (MAC) scaling yields

$$U_{MAC} = \left(\frac{\Phi_{CMB}}{\rho_0 \Omega}\right)^{\frac{1}{2}}, \quad l_{MAC} = R_C \qquad Ro_{l,MAC} = \left(\frac{\Phi_{CMB}}{\rho_0 \Omega}\right)^{\frac{1}{2}} (2 \ \Omega \ R_C)^{-1}. \tag{13}$$

Here,  $\phi = \Phi_{CMB}$  /  $V_{oc}$  is the volumetric thermodynamically available power over 333 334 the fluid core. We could also use these velocity scalings to confirm that the 335 magnetic Reynolds number, which relates the Ohmic diffusion timescale to the 336 convective timescale, exceeds the critical value of ~50 for magnetic field 337 generation to occur (e.g., Roberts 2007). With the definition

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 $Re_m = \mu_0 \sigma U l$ , (14)339 a flow velocity faster than  $\sim 0.1-1$  mm/s produces Rem > 50 if we assume the

340 length scale is equal to the core radius and the electrical conductivity is  $\sigma$ 341  $\sim 10^5$  to  $10^6$  S m<sup>-1</sup> (e.g., Berrada et al. 2020, Pommier et al. 2020).

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2.5. Model Parameters

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345 Our model ingests the BMO model outputs from Scheinberg et al. (2018) and 346 calculates the energy and dissipation budgets for the core to determine when 347 the core may host a dynamo (see Table D2). Following the nomenclature of 348 Scheinberg et al. (2018), naming of the BMO models corresponds to the 349 parameters chosen to describe the mantle and the initial solidification of 350 its magma ocean. For example, 'V19' indicates a reference mantle viscosity of 351 10<sup>19</sup> Pa s, 'K50' indicates that 50% of the KREEP layer remained trapped near 352 the surface, and 'p54' indicates that 54% of the internal radiogenic heating

353 occurs in the sunken KREEP material. We focus on the BMO models that generate 354 magnetic fields with lifetimes of <2.9 Gyr for consistency with the 355 paleomagnetic record (e.g., Mighani et al. 2020). We adopt the nominal BMO 356 case, V19K50p54, as the basis for our nominal model of the core as it assumes 357 moderate yet reasonable values for the mantle parameters. To test the 358 sensitivity of our models to the properties of the core, we scan across four 359 different parameters: the abundance of sulfur and potassium in the core, the 360 thermal conductivity of the core, and the present-day heat flux at the CMB. 361 As with other planets, the Moon's core is expected to be an alloy of iron 362 and light elements, such as sulfur (e.g., Steenstra et al. 2016). Properties

362 and fight elements, such as suffice (e.g., steenstrate et al. 2016). Properties 363 of the FeS system are relatively well known (e.g., Fei et al. 1997, 2000, 364 Chudinovskikh & Boehler 2007, Morard et al. 2007, 2008, Stewart et al. 2007, 365 Chen et al. 2008, Buono & Walker 2011, Pommier 2018) and concentrations of 366 sulfur in the lunar core are likely <6-8 wt% based on interpretations of 367 seismic data (e.g., Weber et al. 2011) and models of the lunar core (e.g., 368 Scheinberg et al. 2015, Laneuville et al. 2014). We vary the sulfur 369 abundance, [S], in the bulk core from 1-9 wt% in increments of 0.5 wt%.

370 Potassium is a potential heat source in planetary cores and soluble in 371 iron alloys at planetary conditions (Murthy et al. 2003, Lee et al., 2004). 372 However, the potassium content of the lunar core remains uncertain. Based on 373 previous studies (e.g., Laneuville et al. 2014, Scheinberg et al. 2015), we 374 test a lower limit of 0 ppm, which assumes a complete lack of radiogenic 375 heating in the lunar core. Although the lower pressures and temperatures in 376 the lunar interior might lead to lower amounts of potassium in the lunar core 377 (e.g., Steenstra et al. 2018), we use plausible concentrations of potassium 378 in Earth's core as an upper limit (e.g., Hirose et al. 2013). In our models, 379 we assume that potassium is incompatible in the inner core, meaning that the 380 outer core becomes enriched in potassium as the inner core grows. We vary the 381 bulk potassium abundance, [K], from 0-50 ppm in increments of 25 ppm. 382 The thermal conductivity,  $k_c$ , of iron alloys defines the adiabatic heat 383 flux of the core. We assume that the maximum plausible value of  $k_c$  is ~50 W m<sup>-1</sup> 384 K<sup>-1</sup>, cited from thermal conductivity experiments on Fe-FeS alloys in the lunar 385 pressure and temperature range (e.g., Pommier 2018). Small amounts of 386 impurities, such as sulfur, can cause a large reduction in the thermal 387 conductivity. We investigate  $k_c$  and [S] independently in our models to isolate 388 the effects of each parameter, but they are coupled in reality. A minimum 389 value of 10 W  $m^{-1}$  K<sup>-1</sup> is selected to represent relatively large impurities of 390 sulfur (e.g., Pommier 2018). Other proposed compositions for the lunar core,

391 such as Fe-Si alloys, have thermal conductivities that are intermediate 392 between these upper and lower bounds (Berrada et al. 2020). Overall, we vary 393  $k_c$  from 10-50 W m<sup>-1</sup> K<sup>-1</sup> in increments of 10 W m<sup>-1</sup> K<sup>-1</sup>.

394 The present-day heat flux at the CMB is highly uncertain and may have 395 been susceptible to higher heat fluxes out of the lower mantle from the 396 enrichment of water and other incompatible elements during solidification of 397 the lunar magma ocean (e.g., Elkins-Tanton & Grove 2011, Khan et al. 2014, 398 Evans et al. 2014, Weiss & Tikoo 2014, Dygert et al. 2017, Greenwood et al. 399 2018). To monitor how the core's temperature evolves given a certain heat 400 flow, we test a range of values using thermal evolution models as a guide 401 (e.g., Laneuville et al. 2014). After the BMO solidifies, we assume that  $Q_{CMB}$ 402 decreases linearly from the final BMO simulation output ( $\sim 0.90-3.70$  GW) to a 403 heat flux value specified at present. We therefore vary the present-day heat 404 flow,  $Q_c$ , from 0-2 GW in increments of 1 GW. While the lower limit of 0 GW may 405 represent an extreme scenario, we want to explore a full range of modeling 406 possibilities to account for multiple scenarios for the lunar solid mantle. 407 Furthermore, 1-D models for small planetary bodies typically indicate that 408 the heat flux varies slightly during most of the core's evolution (e.g., 409 Laneuville et al. 2014). We find that model outputs from simulations with a 410  $Q_{CMB}$  equal to the final BMO simulation output are similar to those from models 411 where the  $Q_{\text{CMB}}$  slightly decreases.

412 Astute readers will realize that our modeling approach makes the 413 cooling rate of the core seem artificially smooth over time after the BMO 414 solidifies. While the BMO exists, we use  $Q_{CMB}$  from the 3-D solid mantle models 415 of Scheinberg et al. (2018), which contain realistic time-variability and 416 fluctuations. Once the BMO has presumably solidified, our parameterized model 417 is effectively 1-D and uses a simplified approach for  $Q_{CMB}$  to capture the 418 average field strength and lifetime of the core dynamo. In reality, some 419 smaller-scale temporal variations in  $Q_{CMB}$  should be expected and the very last 420 time step is not necessarily representative of the end of the time series.

421 We ran a total of ~800 simulations to test the sensitivity of the core 422 model to [S], [K],  $k_c$ , and  $Q_c$  using BMO model outputs from Scheinberg et al. 423 (2018) as boundary conditions.

3. RESULTS

- 424
- 425
- 426



428 Figure 2: Results of the nominal core model with  $k_c=40$  W m<sup>-1</sup> K<sup>-1</sup>,  $Q_c=0$  GW, 429 [S]=7.5 wt%, and [K] = 25 ppm coupled to the nominal BMO model 430 (V19K50p54). All models began at 4.2 billion years before the present day. 431 (a) Temperature at the core-mantle boundary (CMB), at the center of the 432 core or near the inner core boundary (ICB), and the average temperature of 433 the core. (b) Inner core radius with respect to time. (c) Sulfur abundance 434 in the outer core with respect to time. (d) Heat budget given by latent 435 heat,  $Q_L$ , radiogenic heating,  $Q_R$ , gravitational energy  $Q_G$ , adiabatic heat 436 flow in the core,  $Q_{AD}$ , heat flow across the core-mantle boundary,  $Q_{CMB}$ , and 437 secular cooling,  $Q_S$ .

438

439 Our nominal values for the core parameters are [S] = 7.5 wt, [K] = 25440 ppm,  $k_c = 40 \text{ W} \text{ m}^{-1} \text{ K}^{-1}$ , and  $Q_c = 0 \text{ GW}$  for the V19K50p54 BMO boundary condition 441 (Table 1). Figure 2 details the outputs of our nominal model for the core 442 coupled to the nominal BMO model (i.e., V19K50p54). The temperature at the 443 CMB begins at ~1760 K and quickly spikes to ~1940 K due to radiogenic heating 444 in the BMO (Fig. 2a). The BMO then begins solidifying as radiogenic heating 445 declines over time, followed by the core cooling in tandem with the BMO. Once 446 the BMO solidifies, an inner core forms at ~2.2 Gyr as relatively pure iron 447 crystallizes from the inside out (Fig. 2b), expelling sulfur into the outer 448 core (Fig. 2c). Our models assume that the lunar core always contains sub-449 eutectic amounts of sulfur. We verified that this assumption is consistent 450 with our results, which track the sulfur content of the outer core over time

451 (e.g., Figure 2c). The liquidus temperature of the outer core is lowered as 452 it is progressively enriched in sulfur. The result is a molten outer core and 453 a growing inner core. The heat flow is always less than that transported by 454 thermal conduction along the core adiabat,  $Q_{AD}$ . After the inner core 455 nucleates, most extracted heat from the core arises from the release of  $Q_G$  and 456  $Q_L$  (Fig. 2d). The release of  $Q_G$  is nonzero, but small compared to  $Q_L$ . Following 457 the release of  $Q_G$  and  $Q_L$ , there is a reduction in the core cooling rate due to 458 these heat sources acting as a buffer to secular cooling. We note that the  $\ensuremath{\mathsf{Q}_{\text{CMB}}}$ 459 is much lower than the heat flow across the upper boundary of the BMO ( $Q_B$  = 460 ~100 GW at 2.6 Ga) in Scheinberg et al. (2018) because  $Q_B$  includes radiogenic 461 heating in the BMO and the heat associated with cooling the BMO itself.

Abundant sulfur influences the core's ability to drive a magnetic field by lowering its solidus temperature and controlling the onset of inner core crystallization (discussed further in section 3.2.1). The nominal model produces an inner core radius of 250 km at present day (Fig. 2b) and is consistent with core radii derived from calculated models of lunar gravity data (Williams et al. 2014) and reanalyzed Apollo seismic data (Weber et al.



468 2011).

469 Figure 3: a) Surface field intensities of the nominal core model where core 470 convection is driven by inner core growth relatively late in the Moon's 471 history. The buoyancy flux (BF), mixing length (ML), Coriolis, inertial, 472 gravitational (Archimedes) (CIA), and Magneto-Archimedes-Coriolis (MAC) 473 scaling laws are used to estimate surface field intensities of the dipole 474 component. Surface field intensities are compared to the nominal BMO magnetic 475 field intensity assuming the ML scaling law. (b) The dissipation budget of 476 the nominal core model includes the entropy sink associated with thermal 477 conductivity,  $\Phi_{\scriptscriptstyle K}$ , the dissipation associated with secular cooling,  $\Phi_{\scriptscriptstyle S}$ , latent 478 heat,  $\pmb{\Phi}_{\text{L}}$ , gravitational energy,  $\pmb{\Phi}_{\text{G}}$ , radiogenic heating,  $\pmb{\Phi}_{\text{R}}$ , and the 479 dissipation available for a dynamo,  $\Phi_{\rm CMB}$ . (c) If  $k_{\rm C}$  is lowered to 30 W m<sup>-1</sup> K<sup>-1</sup>, 480 purely thermal convection occurs intermittently between ~0.7 and 2 Gyr. Those 481 resultant surface fields are several times weaker than the BMO-hosted field. 482 (d) Dissipation budget associated with a lower  $k_c$  of 30 W m<sup>-1</sup> K<sup>-1</sup>. 483

484 The lunar BMO suppresses convection in the core by lowering its cooling 485 rate. The core produces a dynamo that begins near the cessation of the 486 nominal BMO-hosted dynamo and ends ~1 Ga, consistent with the lower estimate 487 on the cessation of the lunar dynamo derived from radiometric dating of 488 Apollo 15 samples (e.g., Mighani et al. 2020) (Fig. 3a). The relatively weak 489 surface magnetic field strength of  $\lesssim$  2.55  $\mu$ T is also consistent with 490 paleomagnetic data and intensities from previous models of the lunar core 491 dynamo (e.g., Laneuville et al. 2014, Tikoo et al. 2014, Tikoo et al. 2017, 492 Evans et al. 2018, Mighani et al. 2020).

493 We next consider different BMO conditions for our core model. Table 1 494 presents the nominal core input parameters for each BMO boundary condition 495 used in this study. BMOs with a smaller fraction of KREEP that remained near 496 the surface (i.e., V19K25p54 and V18K00p100 in Table 1) have greater initial 497 thicknesses and tend to require lower sulfur abundances (6.5-7 wt%) in the 498 bulk core to initiate dynamo action during the observed timing of the low-499 intensity epoch. Because a BMO with a greater thickness will have a longer 500 lifetime (e.g., Scheinberg et al. 2018), the core will begin crystallizing at 501 a later time when the BMO eventually solidifies. Conversely, models with 502 shallower BMOs (i.e., 301 km) mostly require higher sulfur abundances in the 503 core (7-8.5 wt%) to achieve a core dynamo during the same period. BMO 504 boundary conditions with greater lifetimes additionally suppress inner core 505 growth for longer periods, resulting in smaller inner core radii at present 506 day. Furthermore, models that contain shallower BMOs match the estimated 507 timing of the lunar dynamo if balanced by less radiogenic heating in the core 508 (i.e.,  $\leq$  25 ppm of potassium). In general, BMO boundary conditions typically 509 require the core to have a higher thermal conductivity (i.e.,  $\geq 30 \text{ W m}^{-1} \text{ K}^{-1}$ ) 510 to match the estimated timing of the lunar dynamo.

#### Nominal Core Model Inputs

BMO Boundary Condition	V19K50p54	V19K50p36	V19K50p27	V19K25p54	V18.5K50p54	V18K00p100
BMO lifetime <sup>1</sup> (Gyr)	2.6	2.0	1.6	2.9	1.2	2.1
BMO thickness <sup>1</sup> (km)	301	301	301	383	301	450
[S] (wt%)	7.5	7.0	8.5	7.0	8.5	6.5
[K] (ppm)	25	0	0	50	0	50
Qc (GW)	0	0	0	0	0	0
k <sub>C</sub> (W m <sup>-1</sup> K <sup>-1</sup> )	40	10	30	40	30	30
Table 1: Nominal	core param	eters for ea	ach BMO bou	undary condit	ion used in	this study.

 $^1\mathrm{Values}$  from Scheinberg et al. (2018), Table 1.

5<u>12</u> 513 514

	Nominal Core Model Outputs				
BMO Boundary Condition	V19K50p54	V19K50p36	V19K50p27	V19K25p54	V18.5K50p54
Present-day inner core radius (km)	250	257	231	226	262
Compositional	0.07 (ML) 0.13 (CTA)	0.16	0.05	0.03	0.07
convection B <sub>max</sub> (µT)	0.36 (BF)	0.77	0.27	0.20	0.40
Thermal	2.55 (MAC) 3×10 <sup>-6</sup> (ML)	4.0	0.002	3×10 <sup>-4</sup>	0.001
convection	3×10 <sup>-7</sup> (CIA) 0.001(BF)	3×10 <sup>-4</sup>	0.001	9.3×10 <sup>-5</sup> 0.02	2×10 <sup>-4</sup>
$B_{max}$ ( $\mu$ T)	0.003 (MAC)	0.30	0.31	0.08	0.23
Combined $B_{max}$ ( $\mu T$ )	0.07 (ML) 0.13 (CIA) 0.36 (BF)	0.16 0.45 0.85	0.052 0.091 0.34	0.03 0.04 0.22	0.071 0.13 0.46
., ,	2.55 (MAC)	4.3	2.58	1.78	2.83

	0.07 (ML)	0.16	0.052	0.03	0.071	0.042	
Combined $B_{max}$	0.13 (CIA)	0.45	0.091	0.04	0.13	0.081	
(µT)	0.36 (BF)	0.85	0.34	0.22	0.46	0.34	
•	2.55 (MAC)	4.3	2.58	1.78	2.83	2.28	
Peak Local Rossby Number	0.02 (CIA) 0.003 (ML) 3×10 <sup>-4</sup> (MAC)	0.03 0.004 4×10 <sup>-4</sup>	0.02 0.003 2×10 <sup>-4</sup>	0.02 0.003 2×10 <sup>-4</sup>	0.02 0.003 2×10 <sup>-4</sup>	0.03 0.003 3×10 <sup>-4</sup>	
Compositional	1.14(ML)	1.83	1.90	0.66	2.23	1.42	
convection	1.14(CIA)	1.75	1.86	0.66	2.23	1.42	
duration	1.06(BF)	1.75	1.78	0.58	2.22	1.34	
(Gyr)	1.12(MAC)	1.89	1.95	0.71	2.33	1.49	
Thermal	0.05(ML)	0.87	0.08	0.12	0.85	1.65	
convection	0.05(CIA)	0.87	0.08	0.12	0.85	1.65	
duration	0.05(BF)	0.87	0.08	0.12	0.85	1.65	
(Gyr)	0.05(MAC)	0.87	0.08	0.12	0.85	1.65	
Tifotimo of	1.19(ML)	2.70	1.98	0.78	3.08	3.07	
LITECIME OI	1.19(CIA)	2.62	1.94	0.78	3.08	3.07	
core-nosted	1.11(BF)	2.62	1.86	0.7	3.07	2.99	
aynamo (Gyr)	1.17(MAC)	2.76	2.03	0.83	3.18	3.14	

Table 2: Compositional and thermal convection in the core sustains low intensity magnetic fields following the cessation of a BMO-hosted dynamo.  $B_{max}$  is the peak magnetic field intensity at the surface according to the ML, CIA, BF, and MAC magnetic field scaling laws, respectively, assuming that the mantle is electrically insulating. Thermal convection  $B_{max}$  corresponds to the ML, CIA, BF and MAC scalings, respectively. The combined  $B_{max}$  is the sum of surface fields generation from thermal and compositional convection. The peak local Rossby number corresponds to the CIA, ML, and MAC scaling laws, respectively. The thermal convection duration corresponds to the ML, CIA, BF, and MAC scalings, respectively.

516

V18K00p100

241

0.04 0.08 0.28 2.0 0.002 0.001 0.06 0.28



#### 3.2. Sensitivity Tests

Figure 4: The sensitivities of the nominal core model to core parameters  $k_c$ , [K], [S], and  $Q_c$ for the nominal V19K50p54 BMO model. (a) The surface magnetic field intensity is most sensitive to  $k_{\rm c}$  and [S] and less sensitive to [K] and  $Q_c$ . (b) Our choice of [S] controls the predicted timing of inner core growth and thus, а compositionally-driven core shaded dynamo. The region represents inner core radii that are probably inconsistent with gravity lunar data (e.g., Williams et al. 2014). The (C) duration of is the dynamo predicted to increase with increasing  $Q_c$  and decreasing  $k_c$ . High [S] tends to delay the onset of inner core crystallization and result in a shorter field shaded The region duration. represents durations that are likely inconsistent with constraints on the end of the lunar dynamo (e.g., Mighani et al. 2020). The magnetic field intensity and the duration of the core dynamo are given by the MAC scaling law.

## 3.2.1. Influence of Sulfur in the Core

An inverse relationship exists between the sulfur content and the solidus temperature of the core. As the sulfur content increases, the solidus temperature of the Fe-S system decreases, delaying core

563 solidification until lower temperatures are reached. Therefore, the timing of 564 inner core growth, and thus, the start time of compositional convection in 565 our models depends on the sulfur content of the bulk core (Fig. 4a). The 566 sulfur concentration is viable when the end of the core-hosted dynamo matches 567 the lower estimate on the cessation of the lunar dynamo at ~1 Gyr (e.g., 568 Mighani et al. 2020). Initial sulfur abundances of 1-6.5 wt% result in inner 569 core nucleation at higher temperatures, causing the core to solidify rapidly 570 early in its history (Fig. 4b). Sulfur abundances from 7-8.5 wt% result in 571 the inner core nucleating near the cessation of the BMO-hosted dynamo. 572 Increasing the bulk sulfur content to >8.5 wt% further delays inner core 573 growth and generally results in temporal gaps between the BMO-hosted and 574 core-hosted dynamo, a complete lack of core dynamo action, or contradictions 575 with timing estimates derived from paleomagnetic data (Fig. 4c). However, if 576 the BMO model assumes a lower solid mantle viscosity (i.e., V18.5K50p54), 577 then convective heat transfer is more efficient and results in shorter BMO 578 lifetimes (Scheinberg et al. 2018). As a result, the inner core begins 579 crystallizing earlier and a bulk sulfur content of up to 12 wt% can produce 580 results consistent with the lower estimate on the cessation of the lunar 581 dynamo (e.g., Mighani et al. 2020). The trends outlined in Fig. 4 that arise 582 from variations in  $k_c$ , [K], [S], and  $Q_c$  continue under all other BMO boundary 583 conditions.

584

585 586 3.2.2. Influence of the Core's Heat Budget and Thermal Conductivity

587 The duration and intensity of the core dynamo are also sensitive to  $k_c$ , 588  $Q_c$ , and [K] (Figure 4). A potassium abundance of 50 ppm in the core 589 contributes thermal energy to the dynamo but suppresses growth of the inner 590 core, which can decrease the predicted intensity of the magnetic field 591 overall. Decreasing [K] has a minimal effect on the field intensity because 592 radiogenic heating is nearly equivalent to secular cooling in the dissipation 593 budget. In contrast, increasing the total heat flow to 1-2 GW increases the 594 duration and strength of the core-hosted dynamo, unless a low sulfur 595 abundance leads to rapid core solidification. Furthermore, the duration and 596 intensity of the field generally increases with decreasing thermal 597 conductivity values. We find that purely thermal convection typically occurs 598 before the onset of inner core crystallization if the thermal conductivity is 599 low (i.e., 10-30 W m<sup>-1</sup> K<sup>-1</sup> as in Fig. 3c). As thermal conductivity decreases, 600 the super-adiabatic heat flow increases, leading to a stronger, more long-601 lived dynamo. Thermal convection-driven dynamos typically occur 602 simultaneously with BMO-hosted dynamos as the core is still hot and fully

 $\begin{array}{ll} 603 & \mbox{molten. Compared to the abundance of sulfur in the bulk core, our simulations} \\ 604 & \mbox{reveal that small variations in parameters such as $k_c$, $Q_c$, and [K] play an $605 & \mbox{overall negligible role in the onset of a compositionally-driven dynamo, $606 & \mbox{whereas a thermal convection-driven dynamo is largely dictated by $k_c$.} \\ 607 & \mbox{607} \end{array}$ 

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609

3.2.3. Strength and Timing of the Core Dynamo

610 Depending on the BMO boundary condition, inner core crystallization can 611 produce fields ~0.7-2.3 Gyr in duration, with peak magnetic fields of 0.16, 612 0.45, 0.77, and 4  $\mu$ T, for the ML, CIA, BF, and MAC scaling laws respectively 613 (Table 2). A general issue arises in the case of the CIA, ML, and BF scalings 614 laws in which the strengths are not sufficiently strong enough to reproduce 615 the first period of decline to ~4-7  $\mu T$  by 3.19 Ga (e.g., Strauss et al. 2021) 616 or the second period of decline to ~5  $\pm$  2  $\mu T$  by ~1-2 Ga (e.g., Tikoo et al. 617 2017, Mighani et al. 2020). However, intensities ranging from  $\sim 1.7-4$   $\mu T$  can be 618 achieved under all BMO boundary conditions if the MAC scaling law is assumed. 619 In particular, an intensity of 4  $\mu$ T is achieved if the BMO boundary condition 620 contains a lower fraction of radioactive material concentrated in the BMO 621 (i.e., V19K50p36). However, this magnetic field weakens to a maximum of  $4\,\mu\text{T}$ 622  $\sim 2$  Gyr after accretion, which is  $\sim 0.7$  Gyr later than what is observed in the 623 lunar paleomagnetic record (Strauss et al. 2021).

624 Surface magnetic fields are weaker if they are driven by thermal convection 625 rather than by inner core crystallization. The peak surface magnetic field driven 626 by thermal convection in the nominal core model is  $3 \times 10^{-6}$ ,  $3 \times 10^{-7}$ , 0.001, and 0.003 627  $\mu T$  for the ML, CIA, BF, and MAC scaling laws, respectively (Table 2). For all BMO 628 boundary conditions, thermal convection in the core is initiated ~3.7 Gyr ago 629 (albeit briefly in some models; e.g., Fig. 3a). Furthermore, depending on the BMO 630 boundary condition, thermal convection can persist intermittently for up to ~1.7 631 Gyr, resulting in an overlap with the BMO-hosted field (e.g., Fig. 3c). Thermal 632 convection produces intensities that are consistent with previous modeled 633 estimates of the core (e.g., Laneuville et al. 2014, Evans et al. 2018, 634 Scheinberg et al. 2015), but inconsistent with paleomagnetic analyses 635 constraining the initial and final decline of the lunar dynamo (e.g., Tikoo et 636 al. 2017, Mighani et al. 2020, Strauss et al. 2021). Furthermore, these results 637 are consistent with a low-intensity epoch that persisted from ~1.9-0.8 Ga (e.g., 638 Mighani et al. 2020, Tikoo et al. 2017, Tikoo et al. 2014, Strauss et al. 2021).

639 An uneven heat flow across the CMB may make the magnetic field 640 intermittent because dynamos can be sensitive to slight variations in heat 641 flow (Scheinberg et al. 2015). As an artifact of our modeling approach, early 642 magnetic fields produced via thermal convection are discontinuous due to 643 fluctuations in the  $Q_{CMB}$  from mantle dynamics. In some cases, thermal 644 convection generates fields that are predicted to drop to zero multiple times 645 before rising again from inner core crystallization. The duration of these 646 gaps in the magnetic field are much longer than the magnetic diffusion time 647 (Appendix C). Using the nominal models but with core conductivity lowered to 648  $k_c = 30 \text{ W} \text{ m}^{-1} \text{ K}^{-1}$  as an example case (i.e., Fig 3c), gaps in thermal convection 649 on average last ~140 Myr, whereas the magnetic diffusion time is only a few 650 hundred years. Alternatively, dynamos induced by thermal convection can 651 transition directly into those induced by inner core crystallization, 652 compounding the resultant fields.

#### 3.3. Local Rossby Number

In order to make initial predictions for the magnetic field morphologies in our models, we estimate the local Rossby number as a proxy for whether the core dynamos would be dipole-dominated or multipolar, as for example has been done previously for Ganymede's dynamo (Rückriemen et al. 2015). The CIA scaling law predicts higher values of the local Rossby number  $(Ro_1 \sim 10^{-2})$  relative to the ML  $(Ro_1 \sim 10^{-3})$  and MAC  $(Ro_1 \sim 10^{-4})$  scaling laws

![](_page_22_Figure_3.jpeg)

since inertia plays a larger role in the force balance (Christensen & Aubert 2006). However, for the nominal core model, all scaling laws predict that the local Rossby number is below the threshold value of ~0.1 throughout the lifetime of the core dynamo, suggesting a prevailing

### 674 dipole-dominated magnetic field (Table 2 and Figure 5).

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653 654

Figure 5: Predictions of the local Rossby number for the nominal core model estimated from CIA, ML, and MAC scaling laws.
4. DISCUSSION
In this study, we demonstrated that a BMO dynamo could naturally
dovetail with a core dynamo. Future studies should further explore this
hypothesis by addressing the following important issues.
4.1 Other Medes of Crystallization in the Core
4.1. Other Modes of Crystallization in the core
Future studies could model more complex modes of crystallization in the
lunar core. To recap, we made two relevant assumptions. First, we assumed
that the core always contains sub-eutectic amounts of sulfur, which most of
our models indeed predict (section 3.3). Second, we assumed that the core
solidifies from the center outwards. We set the liquidus temperature to
increase faster than the adiabatic temperature with pressure (e.g., with
gradients of 30 K/GPa versus ~23-25 K/GPa, respectively).
Future studies could relax these two assumptions, which would produce
more complicated behavior in models (e.g., Hauck et al. 2006). First, FeS
rather than Fe could crystallize from the outer core as it cooled if the
sulfur content were super-eutectic. Being sulfur-rich compared to the
residual liquid, solid FeS would float to the top of the liquid rather than
sink to form an inner core like solid Fe. Second, solidification could occur
at the top or middle of the outer core, rather than at its bottom. For
example, "iron snow" could occur in metallic cores if the liquidus crosses
the adiabat above the base of the outer core. This process could help drive a
dynamo as the solidified iron sinks and remelts in the warmer fluid below,

713 leading to compositional convection (e.g., Williams 2009, Breuer et al. 714 2015). Whether the Moon's core entered an FeS crystallization or Fe snow 715 regime at any time remains an ongoing question.

Scientists might make more realistic models of the thermal evolution of sulfur-rich cores if they include these processes. Such models require detailed phase diagrams for the Fe-FeS system. The neglect of Fe snow and FeS crystallization in our models does not change our takeaway message, however, that the presence of a basal magma ocean overlying the core may influence the timing and intensity of the core dynamo. Our models may interface with future, more detailed descriptions of Fe snow and FeS in the core.

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4.2. Morphology of the Lunar Dynamo

726 The geometry and paleo-orientation of the Moon's magnetic field remains 727 largely uncertain. Estimates of paleoinclinations from five Apollo samples 728 suggest the existence of a dipolar field and a paleopole located at  $\sim 75^{\circ}N$ 729 between 3.8 and 3.3 billion years ago (e.g., Cournède et al. 2012). These 730 findings are possibly best explained with a paleofield geometry close to the 731 present-day rotation axis of the Moon. Assumptions of the paleopole were made 732 based on the location of Apollo samples: samples collected from the northern 733 hemisphere were given a positive declination while samples collected from the 734 southern hemisphere were given a negative declination. However, the sign of 735 the inclination remains largely unknown and more data is required to confirm 736 interpretations made from lunar samples. Studies of Apollo 17 mare basalts 737 estimated an inclination of  $\sim 34^{\circ}$  based on the layering of its parent boulder 738 (Nichols et al. 2021). This inclination is consistent with, but does not 739 require, a dipole in the center of the Moon aligned along its rotation axis.

740 Conversely, Olson & Christensen (2006) hypothesized that the Moon's 741 magnetic field may have been multipolar rather than dipole-dominated. The 742 critical difference between our studies is the amplitude of buoyancy flux in 743 the core. Their study assumed that the average buoyancy flux associated with 744 convection in the lunar core was 0.3 times the terrestrial value. That is, 745  $F_{Moon} = 0.3 F_{Earth}$ , where  $F = \alpha g Q / (\rho C_n)$  with thermal expansivity  $\alpha$ , gravitational 746 acceleration g, convective heat flux Q, density  $\rho$ , and specific heat capacity 747  $C_p$ . This assumption was based on the idea that tidal dissipation could add 748 several TW of power to the ancient lunar core (e.g., Williams et al. 2001). 749 This larger heat flow leads to larger estimates of the local Rossby number 750 (e.g.,  $Ro_1 \sim 2$ ), which would shift the lunar dynamo into a multipolar regime.

751 In contrast, our models do not include tidal heating in the lunar core. So, 752 the total power available for convection is only several GW in our models, as 753 shown in Figure 2d.

754 It is also possible that the directional magnetization of lunar rocks 755 does not record a long-term orientation of the lunar magnetic field since 756 differential rotations between the mantle and core would cause a core dipole 757 field to drift across the lunar surface (e.g., Cuk et al. 2019). Relative 758 motions of the core and mantle or misalignment between the lunar dynamo and 759 spin axis may further explain the great variability in the inferred 760 orientation of the lunar dynamo from proposed paleopole locations (e.g., 761 Oliveira & Wieczorek 2017, Nayak et al., 2017). Future missions sampling the 762 lunar bedrock along varying latitudes will allow for more precise geometric 763 determinations of the Moon's magnetic field.

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765 766 4.3. Electromagnetic Core - Mantle Coupling

767 The effects of an electrically conducting lower mantle on the core 768 dynamo is not considered in our study. This limitation is significant for 769 several reasons. First, as noted in Section 2.3, the relatively large 770 conductivity of the BMO, especially when it is fully liquid, will likely 771 cause our estimates of surface magnetic field strengths to be artificially 772 small compared to if this conductivity were taken into account. Our estimates 773 for the core field strength assume that the entire mantle, including the BMO, 774 is electrically insulating such that the core-generated magnetic field 775 becomes a potential field that diffuses upward through the mantle. Given the 776 anticipated higher conductivity of metalliferous silicate melts compared to 777 solid mantle rocks (e.g., Scheinberg et al. 2018), the top of the dynamo 778 region may effectively be the top of the BMO, rather than the top of the 779 core, even if the BMO is subcritical for dynamo action.

780 Second, fluid flows within the BMO may also modulate the core field 781 itself (e.g., Gómez-Pérez et al. 2010). Conversely, if the BMO fluid is 782 stably-stratified, its presence may still filter out small-scale components 783 of the core field that rapidly vary via the magnetic skin effect (e.g., 784 Christensen 2006). Third, the BMO may have resulted in larger magnetic 785 coupling between the core-mantle in the past, relevant to studies of the 786 Moon's rotational dynamics over time (e.g., Dumberry & Wieczorek 2016). 787 Further work, such as numerical dynamo modeling, is needed to better 788 understand the full degree of coupling between the BMO and core of the Moon.

#### 4.4. Thermal Stratification in the Core

791 The effects of thermal stratification in the lunar core are not 792 considered in this study. The inclusion of thermal stratification can have 793 several effects on the heat flux at the CMB. Studies of Mercury's core (e.g., 794 Knibbe and Westrenen 2018, Knibbe and van Hoolst 2021) found that thermal 795 stratification can lead to an increased inner core size, higher temperatures, 796 and a larger heat flux at the CMB, which together results in an early start to 797 the magnetic field. Subsequent heat released upon core solidification would 798 enable slow core growth and an active magnetic field until present day. 799 Future work could apply these models of Mercury to the Moon.

800 4.5. The Early Evolution of the Moon

801 Thermal stratification is probably inevitable at present day, but could 802 also exist early in the Moon's history. In this study, we assumed that the 803 core was initially fully molten and had an adiabatic temperature gradient. If 804 radiogenic heating in the BMO ever made the bottom of the BMO hotter than the 805 top of the core, then heat would move from the BMO into the core, which would 806 cause thermal stratification at the top of the core that may delay the start 807 of a core-hosted dynamo. However, the Moon could have formed with "superheat" 808 (such that the core was initially hotter than the BMO) (e.g., Evans et al. 809 2018), in which case the core could deliver heat to the BMO even while the 810 BMO heats up radiologically. Neither our study nor Scheinberg et al. (2018) 811 modeled these two, countervailing possibilities in detail. Further work is 812 thus needed to better understand the formation and early evolution of the 813 Moon.

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#### 5. CONCLUSIONS

816 Our model for the coupled evolution of a basal magma ocean and the core 817 places estimates on the abundance of sulfur in the core (i.e., 6.5-8.5 wt% in 818 Table 1) and can explain the timing and relative intensity of the lunar 819 magnetic field consistent with other models of the lunar core (e.g., 820 Laneuville et al. 2014, Evans et al. 2018, Scheinberg et al. 2015). The basal 821 magma ocean does not need to be electrically conductive to explain the 822 results presented here, even if it was required to explain the results of

823 Scheinberg et al. (2018). While that may mean the early, intense lunar dynamo 824 remains unexplained, we find that the predicted timing of the lunar dynamo in 825 our models is most consistent with observational constraints of the long-826 lived low-intensity period when moderate abundances of sulfur and potassium 827 are assumed in the core, the core's thermal conductivity is high, and if the 828 present-day CMB heat flow is assumed to be low (or even zero). Excessively 829 high values of  $Q_{\text{CMB}}$  at present day (i.e., 1-2 GW) tends to increase the 830 duration of the magnetic fields longer than is consistent with timing 831 constraints on the end of the lunar dynamo (e.g., Mighani et al. 2020). 832 Modeled intensities are most consistent with paleomagnetic analyses 833 constraining the initial and final decline of the lunar dynamo (e.g., Tikoo 834 et al. 2017, Mighani et al. 2020, Strauss et al. 2021) when the BMO boundary 835 condition is assumed to have less radiogenic heating concentrated in the BMO 836 or when the MAC scaling is assumed. Other scaling laws (i.e., CIA, ML, and 837 BF) predict that magnetic field intensities would be  $\sim 1-2$  orders of magnitude 838 weaker at the surface than inferred from paleomagnetic data (although recall 839 that our intensities may be higher if electrical conductivity of the BMO is 840 taken into account).

841 Thermal convection can briefly exist with the BMO, but is generally 842 short-lived (Fig 3a) or intermittent (Fig 3c), generating magnetic field 843 intensities of up to ~0.3  $\mu T$  that persist for  $\lesssim 1.7$  Gyr (Table 2). Near 844 cessation of the lunar BMO dynamo, heat flows are too low for purely thermal 845 convection and later dynamo action requires inner core crystallization. 846 Magnetic fields generated from the onset of inner core crystallization can 847 reach intensities of up to ~4  $\mu T$  and can persist for  $\lesssim 2.3$  Gyr (Table 2). 848 Temporal gaps may arise between dynamos powered by different types of energy 849 in the core (i.e., thermal vs. compositional), which are neither confirmed 850 nor excluded by extant data. Temporal gaps in the magnetic field can lead to 851 complications in interpretations of the paleomagnetic record and may indicate 852 that a portion of Apollo samples with null paleointensities (e.g., Tarduno et 853 al. 2021) may not result from poor magnetic recording properties.

Estimates of the core sulfur abundance from our model can further translate into predictions of the radius of the inner core. These predictions can be verified with future missions, such as the Farside Seismic Suite (e.g., Panning et al. 2021), which will provide new constraints on the internal structure of the Moon, and the Lunar Geophysical Network (e.g., Weber et al. 2021), which aims to understand the size, state, and composition of the lunar core and the chemical and physical stratification of the mantle. 861 Together, these findings will help discriminate between hypotheses that seek 862 to explain the high-low intensity epoch. Research on the Moon's magnetic 863 history should remain fruitful for decades. 864 865 6. ACKNOWLEDGMENTS 866 This material is based upon work supported by the National Science Foundation 867 Graduate Research Fellowship under Grant No. 026257-001. We thank Aaron 868 Scheinberg for providing the complete outputs of published simulations. 869 870 7. REFERENCES 871 Berrada, M., Secco, R. A., Yong, W., & Littleton, J. A. H. (2020). Electrical 872 Resistivity Measurements of Fe-Si With Implications for the Early Lunar 873 Dynamo. Journal of Geophysical Research: Planets, 125(7), 1-15. 874 https://doi.org/10.1029/2020JE006380 875 Bland, M. T., Showman, A. P., & Tobie, G. (2008). The production of 876 Ganymede's magnetic field. Icarus, 198(2), 384-399. 877 https://doi.org/10.1016/j.icarus.2008.07.011 878 Blaske, C. H., & O'Rourke, J. G. (2021). Energetic Requirements for Dynamos 879 in the Metallic Cores of Super-Earth and Super-Venus Exoplanets. Journal of 880 Geophysical Research: Planets, 126(7), 1-19. 881 https://doi.org/10.1029/2020JE006739 882 Breuer, D., Rueckriemen, T., & Spohn, T. (2015). Iron snow, crystal floats, 883 and inner-core growth: modes of core solidification and implications for 884 dynamos in terrestrial planets and moons. Progress in Earth and Planetary 885 Science, 2(1). https://doi.org/10.1186/s40645-015-0069-y 886 Bullard, E. C. (1949). The magnetic field within the earth. Proceedings of 887 the Royal Society of London. Series A. Mathematical and Physical Sciences, 888 197(1051), 433-453. https://doi.org/10.1098/rspa.1949.0074 889 Bullen, K. E. (1954). On the Homogeneity, or Otherwise, of the Earth's Upper 890 Mantle. American Geophysical Union, 33(5). 891 https://doi.org/10.1029/TR035i005p00838. 892 Buono, A. S., & Walker, D. (2011). The Fe-rich liquidus in the Fe-FeS system 893 from 1bar to 10GPa. Geochimica et Cosmochimica Acta, 75(8), 2072-2087. 894 https://doi.org/10.1016/j.gca.2011.01.030 895 Cameron, A. G. W. (1973). Abundances of the Elements in the Solar System. 896 Space Science Reviews, 15, 121-146. 897 Canup, R. M. (2012). Forming a Moon with an Earth-like Composition via a 898 Giant Impact. Science, 338(6110), 1052-1055. 899 https://doi.org/10.1126/science.1226073.

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al. (2017), especially in their Appendix A, to calculate radial profiles of

1193
1194 APPENDIX A. RADIAL STRUCTURE OF THE LUNAR CORE
1195
1196 We approximated the lunar core as a mixture of liquid Fe and liquid Fe-10
1197 wt% S to make structure models. We followed the procedure detailed in Khan et

1198

1199 density, pressure, and temperature. We use the mass-weighted averages of the 1200 depth-dependent values of the Grüneisen parameter and the coefficient of 1201 thermal expansion. We then performed a least-squares fit to parameterize the 1202 radial density using a fourth-degree polynomial:

1203 
$$\rho(r) = \rho_0 \left[ 1 - \left(\frac{r}{L_\rho}\right)^2 - A_\rho \left(\frac{r}{L_\rho}\right)^4 \right], \qquad (A1)$$

1204 where  $\rho_0$  is density at the center of the core,  $L_\rho$  is a length scale, and  $A_\rho$  is 1205 a constant. The effective bulk modulus is then  $K_0 = 2\pi G (L_\rho \rho_0)^2/3$ , where G is 1206 the gravitational constant. The derivative of the effective bulk modulus is  $K_1$ 1207 = (10  $A_\rho$  + 13)/5. Finally, the adiabatic thermal gradient in the core is then 1208  $T_a(r) = T(0) [\rho(r)/\rho_0]^{\gamma}$ .

- 1209
- 1210 1211

#### APPENDIX B. ENERGETICS OF A DYNAMO IN THE LUNAR CORE

Section 2.2 describes the heat budget of the lunar core. For completeness, we list here the polynomial equations used to calculate the different terms. Analogous equations that were developed to model Earth's core can be found in Labrosse (2015), albeit with slightly different notation and additional complexities added to the analytic formulation, and in the Supporting Information for Blaske & O'Rourke (2021).

1218 In our models, the total heat flow across the core/mantle boundary can be 1219 partitioned into four different terms, each of which is proportional to the 1220 overall cooling rate of the core  $(dT_{CMB}/dt)$ . First, we have the heat flow 1221 associated with secular cooling of the fluid portion of the core. Before the 1222 inner core nucleates, we have

1223 
$$\tilde{Q}_{S} = -\frac{4}{3}\pi\rho_{0}C_{C}L_{\rho}^{3}f_{c}\left(\frac{R_{C}}{L_{\rho}},\gamma\right)\left[1-\left(\frac{R_{C}}{L_{\rho}}\right)^{2}-A_{\rho}\left(\frac{R_{C}}{L_{\rho}}\right)^{4}\right]^{-\gamma},\qquad(B1)$$

1224 where

1225 
$$f_c(x,\delta) = x^3 \left[ 1 - \frac{3}{5} (\delta+1) x^2 - \frac{3}{14} (\delta+1) (2A_\rho - \delta) x^4 \right].$$
(B2)

1226 After the inner core nucleates,

1227 
$$\tilde{Q}_{S} = -\frac{4}{3}\pi\rho_{0}C_{C}L_{\rho}^{3}\left[1 - \left(\frac{R_{I}}{L_{\rho}}\right)^{2} - A_{\rho}\left(\frac{R_{I}}{L_{\rho}}\right)^{4}\right]^{-\gamma}\left[\frac{dT_{L}}{dR_{I}} + \frac{2\gamma T_{L}(R_{I})\left(\frac{R_{I}}{L_{\rho}}\right)\left(1 + 2A_{\rho}\left(\frac{R_{I}}{L_{\rho}}\right)^{2}\right)}{1 - \left(\frac{R_{I}}{L_{\rho}}\right)^{2} - A_{\rho}\left(\frac{R_{I}}{L_{\rho}}\right)^{4}}\right]\left[f_{c}\left(\frac{R_{C}}{L_{\rho}},\gamma\right)\right]$$

1228 
$$-f_c\left(\frac{R_I}{L_{\rho}},\gamma\right)\left[\left(\frac{dR_I}{dT_c}\right),\qquad(B3)\right]$$

1229 where  $T_L(R_I)$  is the liquidus temperature evaluated at the inner core boundary 1230 given by

1231 
$$T_L(R_I) = T_L(0) - K_0 \left(\frac{dT_L}{dP}\right) \left(\frac{R_I}{L_{\rho}}\right)^2 + \frac{c_0}{f_c} \left(\frac{R_L}{L_{\rho}}, 0\right) \left(\frac{dT_L}{dc}\right) \left(\frac{R_I}{L_{\rho}}\right)^3. \tag{B4}$$

1232 Here  $c_0$  is the mass fraction of sulfur in the outer core, which increases as 1233 the inner core grows. Differentiating this equation yields the slope of the 1234 liquidus at the inner core boundary:

1235 
$$\frac{dT_L}{dR_I} = -2K_0 \left(\frac{dT_L}{dP}\right) \left(\frac{R_I}{L_\rho^2}\right) + \frac{3c_0}{f_c \left(\frac{R_c}{L_\rho}, 0\right)} \left(\frac{dT_L}{dc}\right) \left(\frac{R_I^2}{L_\rho^3}\right). \tag{B5}$$

1236 Following Nimmo (2015), we use this slope and the adiabatic thermal gradient 1237 to calculate the growth rate of the inner core

1238 
$$\frac{dR_I}{dT_C} = -\frac{1}{\left(\frac{dT_L}{dP} - \frac{dT_a}{dP}\right)_{R_I}} \left(\frac{T_L(R_I)}{T_{CMB}\rho(R_I)g(R_I)}\right). \tag{B6}$$

1239  $\,$  The growth of the inner core also releases latent heat

1240 
$$\tilde{Q}_L = 4\pi R_I^2 \rho(R_I) T_L(R_I) \Delta S_C \left(\frac{dR_I}{dT_C}\right), \tag{B7}$$

1241 where  $\Delta S_c = 200 \text{ J/K/kg}$  is the entropy of melting for the inner core (Nimmo 1242 2015). Next, we compute the gravitational energy related to the exclusion of 1243 sulfur from the inner core as it freezes:

1244 
$$\tilde{Q}_{G} = \frac{8\pi^{2}G\rho_{0}c_{0}\alpha_{I}R_{I}^{2}L_{p}^{2}}{f_{c}\left(\frac{R_{c}}{L_{\rho}},0\right)} - f_{\chi}\left(\frac{R_{I}}{L_{\rho}}\right) \left[\frac{dR_{I}}{dT_{c}}\right], \tag{B8}$$

1245 where  $\alpha_{I} = 2.3$  is the coefficient of compositional expansion for enriching the 1246 outer core in sulfur (Nimmo 2015). Here we leverage another useful function:

1247 
$$f_{\chi}(x) = x^3 \left\{ -\frac{1}{3} \left( \frac{R_I}{L_{\rho}} \right)^2 + \frac{1}{2} \left[ 1 + \left( \frac{R_I}{L_{\rho}} \right)^2 \right] x^2 - \frac{13}{70} x^4 \right\}.$$
(B9)

1248 Last and easiest, the radiogenic heat in the core is

1249 
$$Q_R = M_C H_K[K] \exp(-\lambda_K t), \qquad (B10)$$

1250 where  $\lambda_{K} = 1.76 \times 10^{-17} \text{ s}^{-1}$  and  $H_{K} = 4.2 \times 10^{-14} \text{ W/kg/ppm}$  are the decay constant 1251 and the heat production rate at t = 0 for potassium-40, respectively.

1252 The energy budget by itself does not reveal whether a dynamo may exist in 1253 the lunar core. We must compute the dissipation budgets, again following 1254 Labrosse (2015) and studies such as Blaske & O'Rourke (2021). First, we 1255 expand equation 3 in the main text as

1256 
$$\Phi_{CMB} = \left(\frac{T_D[T_L(R_I) - T_{CMB}]}{T_L(R_I)T_{CMB}}\right)Q_L + \left(\frac{T_D}{T_{CMB}}\right)Q_G + \left(\frac{T_D - T_{CMB}}{T_{CMB}}\right)Q_R + \left(\frac{T_D(T_S - T_{CMB})}{T_S T_{CMB}}\right)Q_S - \Phi_K.$$
(B11)

1257 Here we use the average temperature in the outer core:

1258 
$$T_D = \frac{T(R_I)}{\left[1 - \left(\frac{R_I}{L_\rho}\right)^2 - A_\rho \left(\frac{R_I}{L_\rho}\right)^4\right]^{\gamma}} \left[\frac{f_c \left(\frac{R_c}{L_\rho}, 0\right) - f_c \left(\frac{R_I}{L_\rho}, 0\right)}{f_c \left(\frac{R_c}{L_\rho}, -\gamma\right) - f_c \left(\frac{R_I}{L_\rho}, -\gamma\right)}\right], \tag{B12}$$

1259 The effective temperature associated with dissipation from secular cooling is 1260 almost identical to  $T_D$  but slightly hotter:

1261 
$$T_{S} = \frac{T(R_{I})}{\left[1 - \left(\frac{R_{I}}{L_{\rho}}\right)^{2} - A_{\rho} \left(\frac{R_{I}}{L_{\rho}}\right)^{4}\right]^{\gamma}} \left[\frac{f_{c} \left(\frac{R_{c}}{L_{\rho}}, \gamma\right) - f_{c} \left(\frac{R_{I}}{L_{\rho}}, \gamma\right)}{f_{c} \left(\frac{R_{c}}{L_{\rho}}, 0\right) - f_{c} \left(\frac{R_{I}}{L_{\rho}}, 0\right)}\right]. \tag{B13}$$

1262 Finally, we can calculate the dissipation sink associated with the thermal 1263 conductivity of the core fluid:

1264 
$$\Phi_{K} = 16\pi\gamma^{2}k_{C}L_{\rho}\left[f_{k}\left(\frac{R_{C}}{L_{\rho}}\right) - f_{k}\left(\frac{R_{I}}{L_{\rho}}\right)\right]T_{D},$$
(B14)

1265 where our last useful function is

1266 
$$f_k(x) = 0.2x^5 \left[ 1 + \frac{10}{7} \left( 1 + 2A_\rho \right) x^2 + \frac{5}{9} \left( 3 + 10A_\rho + 4A_\rho^2 \right) x^4 \right].$$
(B15)

1267 Note that we can then write the total adiabatic heat flow in terms of  $\Phi_{\text{K}}$ :

1268 
$$Q_{AD} = \left(\frac{T_S T_{CMB}}{T_D (T_S - T_{CMB})}\right) \Phi_K \tag{B16}$$

1269 which is an energy-based definition that is basically equivalent to the usual 1270 formula,  $Q_{AD} \sim 4\pi R_c^2 k_c (dT_a/dr)$ , derived from Fourier's law.

1271 1272

#### APPENDIX C. MAGNETIC DIFFUSION TIME

1273 We determine the time it takes for the field to decay after convection 1274 ceases following the procedure detailed in Stevenson (2003) to approximate 1275 the magnetic diffusion time:

$$\tau = \frac{R_c^2}{\pi^2 \lambda}.$$
 (C1)

1277 Here  $R_c$  is radius of the electrically conducting region (i.e., the core) and  $\lambda$ 1278 is magnetic diffusivity given by:

1279  $\lambda = \frac{1}{\mu_0 \sigma},\tag{C2}$ 

1280 where  $\mu_0$  is the permeability of free space and  $\sigma$  is the electrical 1281 conductivity. We assume  $\lambda \sim 1 \text{ m}^2/\text{s}$ , appropriate for terrestrial planets with a 1282 liquid iron alloy core (e.g., Schubert and Soderlund 2011), such that the 1283 magnetic field will diffuse across the core in  $\tau \sim 400$  years.

1284 1285

#### APPENDIX D: TABLES

Term	Description	Value
$\mu_0$	Permeability of free space	$1.257 \times 10^{-6} \text{ H} \cdot \text{m}^{-1}$
G	Gravitational constant	6.67 × 10 <sup>-11</sup> m <sup>3</sup> ·kg <sup>-1</sup> ·s <sup>-2</sup>
R	Universal gas constant	8.3145 J·K <sup>-1</sup> ·mol <sup>-1</sup>
$R_{M}$	Radius of the Moon	1737 km
R <sub>c</sub>	Radius of the core	350 km
Ω	Angular velocity of the Moon	2.66 × $10^{-6}$ rad·s <sup>-1</sup>
K <sub>0</sub>	Effective modulus	121.4 × 10 <sup>9</sup> Pa
K1	Effective derivative of effective modulus	5.7871
Aρ	Constant in density profile	1.59
ρo	Central density	6477 kg·m <sup>-3</sup>
P <sub>0</sub>	Central pressure	5.15 × 10° Pa
$M_{\mathbb{C}}$	Mass of the core	1.16 × 10 <sup>21</sup> kg
Vc	Volume of the core	$3.95 \times 10^{16} \text{ m}^3$
g	Gravitational acceleration near the core- mantle boundary	0.6311 m·s <sup>-2</sup>
γ	Grüneisen parameter for the core	1.65
Cc	Specific heat of the core	835 J·kg <sup>-1</sup> ·K <sup>-1</sup>
$\Delta S_{C}$	Entropy of melting for the inner core	200 J·K <sup>-1</sup> ·kg <sup>-1</sup>
$\alpha_{I}$	Coefficient of compositional expansion for enriching the outer core in sulfur	2.3
$\lambda_{ ext{K}}$	Average decay constant for potassium-40	$1.76 \times 10^{-17} \text{ s}^{-1}$
$\mathrm{H}_{\mathrm{K}}$	Heat production rate for potassium-40	4.2 × 10 <sup>-14</sup> W·kg <sup>-1</sup> ·ppm <sup>-</sup>
С	Constant of proportionality in equations 5-7	0.63
$dT_L/dc$	Compositional dependence of liquidus temperature	-2500 K
$dT_L/dP$	Pressure dependence of liquidus temperature	3 × 10 <sup>-8</sup> K·Pa <sup>-1</sup>

Table D2							
Definition	Definition of Model Inputs and Outputs						
Variable	Definition	Values					
Input para	meters						
[S]	Abundance of sulfur in the core <sup>a</sup>	1-6 wt%					
[K]	Abundance of potassium in the $core^b$	0-50 ppm					
k <sub>c</sub>	Thermal conductivity of the core $^{\circ}$	10-50 W m <sup>-1</sup> K <sup>-1</sup>					
Qc	Present-day heat flow across the core- mantle boundary <sup>d</sup>	0-2 GW					
Energy bud	get outputs of the core						
Q <sub>смв</sub>	Heat flow across the core-mantle boundary	GW					
QL	Latent heat from inner core nucleation	GW					
QG	Gravitational energy released from inner core nucleation	GW					
Q <sub>R</sub>	Radiogenic heating in the core	GW					
Qs	Secular cooling of the core	GW					
Entropy bu	dget outputs of the core						
$\Phi_{ ext{CMB}}$	Dissipation available to power a dynamo	MW					
$\Phi_{ ext{L}}$	Dissipation associated with latent heat	MW					
đ	Dissipation associated with	N /T.7					
$\Phi_{ extsf{G}}$	gravitational energy	MM					
$\Phi_{\mathbb{R}}$	Dissipation associated with radiogenic	MW					
₹ K	heating						
$\Phi_{ m s}$	Dissipation associated with secular	MW					
	cooling						
$\Phi_{ extsf{K}}$	Dissipation sink associated with thermal	MW					
257 1 . 7	conductivity						

1293 <sup>b</sup>Laneuville et al. 2014, Scheinberg et al. 2015, Hirose et al. 2013.

1294 °Pommier 2018.

1295 <sup>d</sup>Laneuville et al. 2014.