

# Salt transport regimes caused by tidal and subtidal processes in narrow estuaries

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## Abstract

Tidally averaged transport of salt in estuaries is controlled by various subtidal and tidal processes. In this study we show the relative importance of various subtidal and tidal transport processes in a width-averaged sense. This is done for a large range of forcing and geometric parameters, which describe well-mixed to salt wedge estuaries. To this end we develop a width-averaged process-based model aimed at conducting and analysing a large number of experiments ( $\sim 40,000$ ). We find that the salt transport is dominated by one of seven salt transport balances, or regimes. Four of these regimes are dominated by subtidal processes, while the other three are dominated by tidal processes. Which regime occurs in a part of an estuary depends on four dimensionless parameters, representing local geometry and forcing conditions. One of the regimes features salt import by correlations between the depth-averaged tidal velocity and salinity. While this mechanism was previously only associated with along-channel geometric variations, we find it can also be a dominant mechanism in a significant part of the parameter space due to river-induced tidal asymmetry, independent of river geometry. We apply our classification to a case study of part of the Dutch Rhine delta and compare to decomposition results of a fully realistic three-dimensional model. We find the estuary features two regimes, with import dominated by subtidal shear transport in the seaward part of the estuary and by depth-averaged tidal correlations in the landward part of the estuary.

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## Key Points:

- Using a systematic model analysis, we identified 7 mechanistic regimes that can dominate the salt balance in narrow estuaries.
- We developed a model-based classification of the 7 regimes as a function of four dimensionless parameters.
- We identified a new regime, where salt import is dominated by correlations in the depth-averaged tidal flow and salinity.

## Abstract

Tidally averaged transport of salt in estuaries is controlled by various subtidal and tidal processes. In this study we show the relative importance of various subtidal and tidal transport processes in a width-averaged sense. This is done for a large range of forcing and geometric parameters, which describe well-mixed to salt wedge estuaries. To this end we develop a width-averaged process-based model aimed at conducting and analysing a large number of experiments ( $\sim 40,000$ ). We find that the salt transport is dominated by one of seven salt transport balances, or *regimes*. Four of these regimes are dominated by subtidal processes, while the other three are dominated by tidal processes. Which regime occurs in a part of an estuary depends on four dimensionless parameters, representing local geometry and forcing conditions. One of the regimes features salt import by correlations between the depth-averaged tidal velocity and salinity. While this mechanism was previously only associated with along-channel geometric variations, we find it can also be a dominant mechanism in a significant part of the parameter space due to river-induced tidal asymmetry, independent of river geometry. We apply our classification to a case study of part of the Dutch Rhine delta and compare to decomposition results of a fully realistic three-dimensional model. We find the estuary features two regimes, with import dominated by subtidal shear transport in the seaward part of the estuary and by depth-averaged tidal correlations in the landward part of the estuary.

## Plain Language Summary

The distribution of salt water in an estuary is controlled by the balance of various processes related to tidal flows and subtidal processes. We investigate which processes are most important in determining this salt distribution for a wide range of parameter configurations including depth, river discharge and tidal flow, thereby representing many types of estuaries found around the World. To this end, we develop a width-averaged model that is specifically suitable for investigating a large number of configurations ( $\sim 40,000$ ). We find that different processes are important for different parameter configurations, resulting in seven possible balances of processes, or regimes. The regime found in each part of an estuary is determined by four dimensionless parameters. We apply our classification to part of the Dutch Rhine delta and show that this estuary features two regimes depending on the location along the estuary.

## 1 Introduction

The distribution of salt in an estuary is continuously adapting to the balance between the various processes that import and export salt. On a tidally-averaged timescale, these transport processes are usually categorised as being a consequence of covariance between subtidal flows and subtidal salinity (hereafter 'subtidal processes') or as the covariance of tidally varying flow and salinity (hereafter 'tidal processes'). Analysis of salt transport balances in various estuaries has shown that either subtidal or tidal processes can be dominant depending on the estuary, the location in the estuary, and on forcing parameters, including river discharge and tidal amplitude (e.g. Fischer, 1972; Bowen & Geyer, 2003; Aristizábal & Chant, 2013; Wang et al., 2015; Bellafiore et al., 2021). The various subtidal and tidal transport processes each have a different sensitivity to changing forcing parameters (e.g. tides and river discharge) and geometry (depth or width). Hence, to understand how the salt intrusion and stratification in an estuary change with changing parameters, it is important to understand which mechanisms dominate the salt transport. However, there is currently no comprehensive theory that describes the relative importance of subtidal and tidal processes as a function of the estuarine parameters, such as depth, width, river run-off or tides for general estuaries. This work contributes to such theory focussing on width-averaged processes.

Most studies that describe the sensitivity of salt importing processes to changes in parameters have focussed on either subtidal or tidal processes in a certain type of estuary. For example, focussing on the width-averaged dynamics, the subtidal models of Hansen and Rattray (1965), Chatwin (1976), and MacCready (2004) apply to partially mixed estuaries and demonstrate the importance of subtidal transport due to gravitational circulation. The two layer models of e.g. Schijf and Schönfeld (1953), Armi (1986) apply to salt wedge estuaries and show that subtidal momentum advection and gravitational

circulation are both important. Concerning tidal width-averaged processes, Bowden (1965), Chatwin (1975), Larsen (1977), and Wei et al. (2016) focus on well-mixed estuaries and discuss the transport due to the correlation between vertical shear of the tidal velocity and tidally varying salinity. Other tidal salt transport mechanisms include those due to the interaction between the main channel and shallow flanks or side-embayments, called 'tidal trapping' (Okubo, 1973; MacVean & Stacey, 2011), temporally varying mixing in tidally varying salt wedge estuaries (Geyer & Farmer, 1989), and rapid changes in bathymetry and geometry (Geyer & Ralston, 2015). We refer to Fischer et al. (1979) and Geyer and Ralston (2011) for reviews of these and other processes.

The aim of this study is to construct a classification of salt transport balances, or *regimes*, which shows the relative importance of width-averaged subtidal and tidal salt transport processes across a large range of parameters. These include forcing parameters (i.e. related to river discharge and tide) and geometric parameters (i.e. depth and width) representative of narrow estuaries ranging from well-mixed to salt wedge. We further focus on single channel estuaries without side-embayments and only gradually varying bathymetry. Further important assumptions include temporally and spatially uniform mixing parameters and negligible surface elevation (i.e. rigid lid). The classification is derived from a systematic analysis of the results of a model that solves for the conservation laws for water motion and salinity. This work thereby extends that of Dijkstra and Schuttelaars (2021) (*hereafter DS21*), who constructed such a classification focussing only on width-averaged subtidal processes.

The forcing and geometric parameters usually vary strongly along an estuary and in time. Still, regions within an estuary and periods of time can typically be distinguished in which the tidal amplitude, river discharge, depth and width are fairly constant. This allows for developing a classification that is representative of the local dynamics, i.e. dynamics in a part of an estuary during a certain period of time. We model this by assuming an infinitely long estuarine domain with uniform forcing and geometric parameters representing these local dynamics. The dominant transport processes in this domain are then assumed to be representative for the local dynamics. Resolving an entire estuary like this has the advantage over a local model (e.g. a 1D-vertical model such as Lange and Burchard (2019)) that the response of the along-channel salinity gradient to the forcing and geometry is fully resolved, not prescribed. We will apply this approach and verify our results by classifying the dominant salt transport balances along the Rotterdam Waterway-Nieuwe Maas River (RWW, part of the Rhine Delta, Netherlands) by comparing our classification against results of a three-dimensional simulation model.

In Section 2 we first introduce the model equations, the dedicated solution procedure and the way the salt transport contributions are decomposed. Next, in section 3 we present our classification, describe the identified regimes and show the dependence of salt intrusion on the forcing and geometric parameters in each regime. The discussion in Section 4 presents the application to the RWW, a comparison of our classification to previous results, and an overview of processes that are not explicitly resolved. Finally, Section 5 summarises the main findings.

## 2 Model

### 2.1 Model equations

We develop a width-averaged model for water motion and salinity including both tidal and subtidal dynamics. As mentioned in the introduction, all forcing and geometric parameters are assumed constant in the along-channel direction. This also implies that the tidal velocity amplitude and phase are constant along the entire domain, so that the model does not resolve tidal propagation. This is done so that we can unambiguously speak of the effect caused by a tide of specific velocity amplitude and phase independent of estuary depth, length or friction. The model is forced by a constant river discharge and constant depth-integrated  $M_2$  tidal velocity. Overtides ( $M_4$ ,  $M_6$ , etc.) are resolved but assumed to have no depth-integrated contribution.

We assume that the (tidal) surface variations are small compared to the depth (rigid lid assumption). Hence the model domain is described by Cartesian coordinates with the vertical coordinate  $z$  from the flat bed  $z = -H$  to a fixed surface level at  $z = 0$  and the along-channel coordinate  $x$  from

the mouth at  $x = 0$  to the upstream boundary at  $x = L$ . The length  $L$  is chosen much larger than the salt intrusion limit, such that it does not affect the results. Note here that our assumption of an along-channel constant tidal velocity amplitude and phase also means that the estuary length does not affect the tides.

The water motion is described by the width-averaged continuity and momentum equations. Effects of turbulence are parametrised by a constant (in time and space) vertical eddy viscosity  $A_\nu$ . We assume that  $A_\nu$  is an independent parameter, meaning that the results do not rely on a specific choice of turbulence model to determine  $A_\nu$  but require explicit input of  $A_\nu$  (e.g. from a simple closure, observations, or simulation models). We further assume hydrostatic pressure and include density effects in the pressure terms only (i.e. Boussinesq approximation). The density  $\rho$  is related linearly to salinity as  $\rho = \rho_0(1 + \beta s)$ , where  $\rho_0$  is a reference density of  $1000 \text{ kg/m}^3$ ,  $\beta = 7.6 \cdot 10^{-4} \text{ psu}^{-1}$  and  $s$  is salinity. Under these assumptions, the model equations read as

$$u_x + w_z = 0, \quad (1a)$$

$$u_t + uu_x + ww_z = -g\zeta_x + g\beta \int_z^0 s_x dz' + A_\nu u_{zz}. \quad (1b)$$

Here,  $u, w$  denote the width-averaged horizontal and vertical velocity components. The width-averaged surface gradient  $\zeta_x$  strictly needs to be interpreted as a source of pressure due to the rigid lid assumption, whereas its effect on the surface elevation is ignored. Furthermore,  $g$  is the acceleration of gravity. Subscripts  $t, x$ , and  $z$  denote derivatives with respect to time and space. The boundary conditions are given by

$$A_\nu u_z = 0 \quad \text{at } z = 0, \quad (1c)$$

$$A_\nu u_z = s_f u \quad \text{at } z = -H, \quad (1d)$$

$$w = 0 \quad \text{at } z = 0, \quad (1e)$$

$$w = 0 \quad \text{at } z = -H, \quad (1f)$$

$$\frac{1}{H} \int_{-H}^0 u dz = -\mathcal{U}_r + \mathcal{U}_t \cos(\omega t) \quad \text{at } x = L, \quad (1g)$$

$$u_t - g\zeta_x + A_\nu u_{zz} = 0 \quad \text{at } x = L. \quad (1h)$$

Boundary condition (1c) represents a no-stress condition at the surface (i.e. no wind) and (1d) a partial slip condition at the bed, using a spatially and temporally constant friction coefficient  $s_f$  (in m/s). The third and fourth boundary conditions (1e)-(1f) prescribe no flux through the rigid surface and bed. Eq. (1g) describes the external forcing, consisting of a constant fresh water velocity  $\mathcal{U}_r \geq 0$  and a constant-amplitude  $M_2$  tidal velocity  $\mathcal{U}_t \cos(\omega t)$ , where  $\mathcal{U}_t \geq 0$  is the depth-averaged tidal velocity amplitude,  $\omega = 1.4 \cdot 10^{-4} \text{ s}^{-1}$  is the angular frequency of the  $M_2$  tide. Without loss of generality the phase is fixed at zero. Finally, Eq. (1h) describes that the velocity profile at  $x = L$  satisfies conditions of horizontally uniform (tidal) flow. Note that integration of the continuity equation (1a) over the cross-section and using boundary conditions (1e) and (1g) we find that the cross-sectionally averaged velocity equals  $-\mathcal{U}_r + \mathcal{U}_t \cos(\omega t)$  not only at  $x = L$  but in the entire estuary.

The salinity is described by the conservation equation

$$s_t + us_x + ws_z = K_\nu s_{zz} + K_h s_{xx}, \quad (2a)$$

where  $K_\nu$  and  $K_h$  are the vertical and horizontal eddy diffusivity. Like  $A_\nu$ , we assume that  $K_\nu$  and  $K_h$  are independent parameters, which are constant in space and time. We set the following boundary conditions:

$$K_\nu s_z = 0 \quad \text{at } z = 0, \quad (2b)$$

$$K_\nu s_z = 0 \quad \text{at } z = -H, \quad (2c)$$

$$s_t + us_x + ws_z = K_\nu s_{zz} \quad \text{at } x = 0, z \in (-H, 0], \quad (2d)$$

$$\max_t s = s_{\text{sea}} \quad \text{at } x = 0, z = -H, \quad (2e)$$

$$[s]_x = 0 \quad \text{at } x = 0, z = -H, \quad (2f)$$

$$s = 0 \quad \text{at } x = L, \quad (2g)$$

where  $[\cdot]$  means taking the tidal variation (i.e. removing the subtidal component). Conditions (2b)-(2c) are no-flux conditions at the surface and bed. The boundary conditions at  $x = 0$  are adapted from MacCready (2004) and DS21: Eq. (2d) describes that the salinity at  $x = 0$  satisfies the salt balance without horizontal diffusivity. Eq. (2e) fixes the maximum salinity that occurs during a tidal cycle at the bed at the seaward boundary to the sea/ocean salinity  $s_{\text{sea}}$ . Eq. (2f) requires that tidal variations of salinity should have a vanishing horizontal gradient for all time. Finally, Eq. (2g) describes a vanishing salinity at the upstream boundary.

The model contains three parameters related to turbulence in a water column:  $A_\nu$ ,  $K_\nu$  and  $s_f$ . We choose to relate these parameters to each other, so only  $A_\nu$  is considered an input variable to the model. The constant eddy viscosity and vertical eddy diffusivity are related as  $K_\nu = A_\nu/\sigma_\rho$  where, for simplicity, we choose a fixed Prandtl-Schmidt number  $\sigma_\rho = 2.2$  (e.g. Ralston et al., 2008). The eddy viscosity and friction parameter  $s_f$  are related to each other in terms of a dimensionless parameter  $R = \frac{A_\nu}{s_f H}$ . We assume a fixed value  $R = 0.5$  which is the best fit to typical boundary-layer solutions for barotropic flow in a  $k - \varepsilon$  model (Dijkstra, Brouwer, et al., 2017). The results of this study do not essentially depend on the choices of  $\sigma_\rho$  and  $R$ , see also Section 4.2.

## 2.2 Solution procedure

We look for dynamic equilibrium solutions of the model equations, i.e. solutions that are periodically varying with tidal and overtide frequencies but not on a subtidal timescale. The solution procedure is tailored to find this dynamic equilibrium in an efficient way, so that a large number of simulations of high numerical accuracy can be performed in a short time. This procedure extends that of DS21 by including the effects of tidal motion. Details of the solution procedure can be found in the supplemental material. Here we provide a summary of the essential steps.

First, we focus on the temporal and vertical structure of the flow and salinity. As the model is forced by an  $M_2$  tidal constituent and a constant river discharge, the solution consists only of a subtidal component, the  $M_2$  tide, and its overtides ( $M_4$ ,  $M_6$ , ...), which are created by nonlinear interactions. The temporal behaviour of the various physical quantities is approximated by a truncated series of  $q_{\text{max}}$  harmonic components plus a subtidal component. The vertical structure of the flow and salinity is approximated by a truncated eigenfunction expansion, using  $M + 1$  cosine functions. This approach has the advantage that the circulation and stratification can already be captured accurately by a small number of functions (i.e. here  $M = 12$ , see Table 1). The temporal structure of  $\zeta_x$  and the temporal-vertical structure of  $u$  and  $s$  are thus approximated as

$$u(x, z, t) = \sum_{m=0}^M \sum_{q=0}^{q_{\text{max}}} (\beta_{mq}^R(x) \cos(\omega_q t) + \beta_{mq}^I(x) \sin(\omega_q t)) \cos(\lambda_m z), \quad (3a)$$

$$s(x, z, t) = \sum_{m=0}^M \sum_{q=0}^{q_{\text{max}}} (\alpha_{mq}^R(x) \cos(\omega_q t) + \alpha_{mq}^I(x) \sin(\omega_q t)) \cos(\mu_m z), \quad (3b)$$

$$\zeta_x(x, t) = \sum_{q=0}^{q_{\text{max}}} \zeta_q^R(x) \cos(q\omega t) + \zeta_q^I(x) \sin(q\omega t). \quad (3c)$$

Here,  $\cos(\lambda_m z)$  and  $\cos(\mu_m z)$  are the eigenfunctions and  $\lambda_m$  and  $\mu_m$  are the eigenvalues chosen so that the eigenfunctions satisfy the vertical boundary conditions (1c)-(1d), (2b)-(2c) (see the supplemental material for the associated eigenvalue problem). Furthermore,  $\omega_q$  denotes the angular frequency of the  $q^{\text{th}}$  tidal component, with  $q = 0$  the subtidal component (i.e.  $\omega_0 = 0$ ). The vertical velocity  $w$  is obtained using the continuity equation (1a) and the expression for  $u$ .

We substitute the expansions for  $u$ ,  $s$ , and  $\zeta_x$  in the model equations, resulting in a set of  $(2M + 1)(2q_{\text{max}} + 1)$  equations for the unknown coefficient functions  $\alpha_{mq}^R(x)$ ,  $\alpha_{mq}^I(x)$ ,  $\beta_{mq}^R(x)$ ,  $\beta_{mq}^I(x)$ ,  $\zeta_q^R(x)$ ,  $\zeta_q^I(x)$ , which depend on the along-channel coordinate. This system of equations is discretised using a second-order accurate finite volume method on a grid with  $j_{\text{max}} + 1$  grid points. The spacing between the grid points is adapted in such a way that the highest resolution is located in the region with the largest along-channel salinity gradient. The total system of equations consists of  $(2M + 1)(2q_{\text{max}} + 1)$

$1)(j_{\max} + 1)$  nonlinear algebraic equations. This system is solved using a Newton-Raphson solver. For highly frictional situations ( $A_\nu$  large), the equations are nearly linear and this solver converges fast from any initial guess. When friction is weaker, the solutions are obtained using a continuation procedure, i.e. gradually decreasing  $A_\nu$  from one computation to the next and using the result of each computation as the initial guess for the next. The grid spacing is adapted based on the salinity gradient in this initial guess. This way, the solver converges within a few iterations.

### 2.3 Controlling dimensionless parameters

The model contains seven independent variables  $H, \mathcal{U}_t, \mathcal{U}_r, \omega, s_{\text{sea}}, A_\nu$ , and  $K_h$ , two parameters that are assumed to be constant  $R = 0.5$ ,  $\sigma_\rho = 2.2$  (see above for motivation of these numbers), and two fundamental constants  $g$  and  $\beta$ . In order to reduce the number of variables, we rewrite the model equations to a dimensionless system and consider the dimensionless parameters controlling it. As derived in Section 2 of the supplemental information, this reduces the independent variables to the following four dimensionless parameters:

$F_r$	Estuarine Froude number	$\frac{\mathcal{U}_r}{c_I}$ ,
$F_t$	Tidal Froude number	$\frac{\mathcal{U}_t}{c_I}$ ,
$Ra$	Estuarine Rayleigh number	$\frac{c_I^2 H^2}{A_\nu K_h}$ ,
$St$	Stokes number	$\frac{A_\nu}{\omega H^2}$ ,

where  $c_I = \sqrt{\beta s_{\text{sea}} g H}$  is a measure for the internal wave speed of the fastest internal wave (e.g. MacCready, 2004). The estuarine Froude number  $F_r$  and Rayleigh number  $Ra$  are measures for the river-induced velocity and the reciprocal of mixing. These numbers also appear in subtidal models (see e.g. Hansen & Rattray, 1965; Guha & Lawrence, 2013, DS21). The tide additionally adds a tidal Froude number  $F_t$  (e.g. Geyer, 2010), which is the ratio of the tidal velocity amplitude and  $c_I$ , and the Stokes number (e.g. Souza, 2013), which is the ratio of a timescale for the water column to fully mix and the tidal timescale.

One of the important output parameters of the model is the salt intrusion length  $L_s$ , here defined as the distance from the mouth to the point where the bottom salinity equals 1 psu (see also DS21). The salt intrusion is made dimensionless by scaling. The length scale used for this scaling is not important; it simply determines the magnitude of the dimensionless length scale. Here we choose to use a dispersive length scale  $K_h/c_I$ , i.e.  $\mathcal{L}_s = \frac{L_s c_I}{K_h}$  (see also DS21). Being a dependent parameter, the dimensionless salt intrusion length  $\mathcal{L}_s$  may be expressed as some nonlinear function of the four independent parameters and the two chosen constants  $R$  and  $\sigma_\rho$ , i.e.

$$\mathcal{L}_s = f(F_r, F_t, Ra, St; \sigma_\rho, R). \quad (4)$$

### 2.4 Analysis of the salt transport

Transport of salt is analysed by considering the depth-averaged, time-averaged salinity equation in equilibrium (e.g. Hansen & Rattray, 1965). Using our assumptions of constant depth and prescribed tides, this is obtained by integrating the salinity equation (2a) over depth, time averaging and applying continuity (1a). This result reads as

$$T = \frac{1}{H} \left\langle \int_{-H}^0 u s - K_h s_x dz \right\rangle = 0, \quad (5)$$

where  $\langle \cdot \rangle$  denotes averaging over a tidal cycle.

To disentangle the various contributions to this transport balance we first follow the approach of Fischer (1976). That is, we write the velocity and salinity as a subtidal part,  $u_0, s_0$ , and tidally varying part,  $u_1, s_1$  containing all resolved tidal harmonics. Next, each of these components is decomposed into its depth-mean  $\bar{u}_i, \bar{s}_i$  ( $i = 0, 1$ ), and depth variation  $u'_i, s'_i$ , such that  $\bar{u}'_i = 0, \bar{s}'_i = 0$ . Thus the velocity and salinity are written as

$$u = \bar{u}_0 + \bar{u}_1 + u'_0 + u'_1, \quad (6a)$$

$$s = \bar{s}_0 + \bar{s}_1 + s'_0 + s'_1. \quad (6b)$$

The transport balance (5) can then be rewritten as

$$T = \underbrace{\bar{u}_0 \bar{s}_0}_{\text{subtidal depth-averaged transport}} + \underbrace{\langle \bar{u}_1 \bar{s}_1 \rangle}_{\text{tidally correlated depth-averaged transport}} + \underbrace{\overline{u'_0 s'_0}}_{\text{subtidal shear transport}} + \underbrace{\langle \overline{u'_1 s'_1} \rangle}_{\text{tidally correlated shear transport}} \underbrace{-K_h \bar{s}_{0,x}}_{\text{dispersive transport}} = 0. \quad (7)$$

Similar decompositions as in the above transport balance have been applied to analyse observations and models (e.g. Lerczak et al., 2006; Aristizábal & Chant, 2013; Bellafore et al., 2021), although in many instances the transport by tidal depth-averaged correlations  $\langle \bar{u}_1 \bar{s}_1 \rangle$  and tidal shear correlations  $\langle \overline{u'_1 s'_1} \rangle$  are merged and the dispersive transport is not considered.

To more explicitly relate the decomposition above to the underlying physical mechanisms, the transport balance is further decomposed. We distinguish contributions by the *river discharge* ( $u_{\text{riv}}, s_{\text{riv}}$ ), *barotropic tidal forcing* ( $u_{\text{tide}}, s_{\text{tide}}$ ), *baroclinic pressure* ( $u_{\text{gc}}, s_{\text{gc}}$ ), *momentum advection* ( $u_{\text{adv}}, s_{\text{adv}}$ ), and *horizontal dispersion* ( $s_{K_h}$ ). These contributions are determined from a decomposition of the momentum equation (1b), boundary condition (1g) and the salinity equation (2a) and is such that the sum of all contributions equals the total velocity and salinity. The decomposition is mathematically described by the following equations (terms in red mark the differences between the equations)

$$\begin{cases} u_{t,\text{riv}} - g\zeta_{x,\text{riv}} - A_\nu u_{zz,\text{riv}} = 0, \\ \frac{1}{H} \int_{-H}^0 u_{\text{riv}} dz = -\mathcal{U}_r, \end{cases} \quad \text{(river)} \quad (8a)$$

$$\begin{cases} u_{t,\text{tide}} - g\zeta_{x,\text{tide}} - A_\nu u_{zz,\text{tide}} = 0, \\ \frac{1}{H} \int_{-H}^0 u_{\text{tide}} dz = \mathcal{U}_t \cos(\omega t), \end{cases} \quad \text{(tide)} \quad (8b)$$

$$\begin{cases} u_{t,\text{gc}} - g\zeta_{x,\text{gc}} - A_\nu u_{zz,\text{gc}} = g\beta \int_z^0 s_x dz', \\ \frac{1}{H} \int_{-H}^0 u_{\text{gc}} dz = 0 \end{cases} \quad \text{at } x = L, \quad \text{(baroclinic pressure)} \quad (8c)$$

$$\begin{cases} u_{t,\text{adv}} - g\zeta_{x,\text{adv}} - A_\nu u_{zz,\text{adv}} = -(u u_x + w u_z), \\ \frac{1}{H} \int_{-H}^0 u_{\text{adv}} dz = 0 \end{cases} \quad \text{at } x = L, \quad \text{(momentum advection)} \quad (8d)$$

$$s_{t,\text{riv}} - K_\nu s_{zz,\text{riv}} = -(u_{\text{riv}} s_x + w_{\text{riv}} s_z), \quad \text{(river)} \quad (8e)$$

$$s_{t,\text{tide}} - K_\nu s_{zz,\text{tide}} = -(u_{\text{tide}} s_x + w_{\text{tide}} s_z), \quad \text{(tide)} \quad (8f)$$

$$s_{t,\text{gc}} - K_\nu s_{zz,\text{gc}} = -(u_{\text{gc}} s_x + w_{\text{gc}} s_z), \quad \text{(baroclinic pressure)} \quad (8g)$$

$$s_{t,\text{adv}} - K_\nu s_{zz,\text{adv}} = -(u_{\text{adv}} s_x + w_{\text{adv}} s_z), \quad \text{(momentum advection)} \quad (8h)$$

$$s_{t,K_h} - K_\nu s_{zz,K_h} = K_h s_{xx}, \quad \text{(horizontal dispersion)} \quad (8i)$$

These decompositions are computed in a post-processing step as Eq. (8c)-(8i) require the solution  $u$  and  $s$ . For the velocity, we observe that the depth-averaged parts of  $u_{\text{gc}}$  and  $u_{\text{adv}}$  are zero. The salinity components are determined up to some constant. Hence, this provides a decomposition of the salinity shear, while there is no decomposition of the depth-averaged salinity. We may thus summarise the obtained components as

$\bar{u}_0$	$\bar{u}_{0,\text{riv}}$	$\bar{s}_0$	no decomposition
$\bar{u}_1$	$\bar{u}_{1,\text{tide}}$	$\bar{s}_1$	no decomposition
$u'_0$	$u'_{0,\text{riv}}, u'_{0,\text{gc}}, u'_{0,\text{adv}}$	$s'_0$	$s'_{0,\text{riv}}, s'_{0,\text{tide}}, s'_{0,\text{gc}}, s'_{0,\text{adv}}, s'_{0,\text{Kh}}$
$u'_1$	$u'_{1,\text{tide}}, u'_{1,\text{gc}}, u'_{1,\text{adv}}$	$s'_1$	$s'_{1,\text{riv}}, s'_{1,\text{tide}}, s'_{1,\text{gc}}, s'_{1,\text{adv}}, s'_{1,\text{Kh}}$

Taking all correlations of these decomposed velocity and salinity contributions in the transport equation (5) results in a total of 33 physical mechanisms that transport salt. This number of contributions may seem too much to aid intuitive interpretation. However, it will turn out in the next section that only a few of these physical mechanisms are sufficient to understand the dominant balance and resulting salinity distribution. Hence, this decomposition does offer a useful framework for interpreting the results.



	Parameter	Meaning	Value
Physical	$\mathcal{U}_r$	Cross-sectionally averaged river velocity	$5 \cdot 10^{-4} - 2.2$ m/s
	$\mathcal{U}_t$	Cross-sectionally averaged tidal velocity	$0 - 1$ m/s
	$K_h$	Horizontal dispersion parameter	$25 - 500$ m <sup>2</sup> /s
	$A_\nu$	Vertical eddy viscosity	$3.5 \cdot 10^{-5} - 3$ m <sup>2</sup> /s
	$H$	Depth	20 m
	$\omega$	$M_2$ angular frequency	$1.4 \cdot 10^{-4}$ s <sup>-1</sup>
	$s_{\text{sea}}$	Maximum seaward salinity	30 psu
	$\sigma_\rho$	Prandtl-Schmidt number $A_\nu/K_\nu$	2.2
	$R$	viscosity-bottom friction ratio	0.5
Numerical	$M$	Number of spectral components in the vertical	12
	$q_{\text{max}}$	Number of harmonic components	3
	$j_{\text{max}}$	Number of adaptively spaced grid cells	200

**Table 1.** Default parameter and numerical settings, including ranges of parameters used for the sensitivity study.

## 2.5 Model experiments

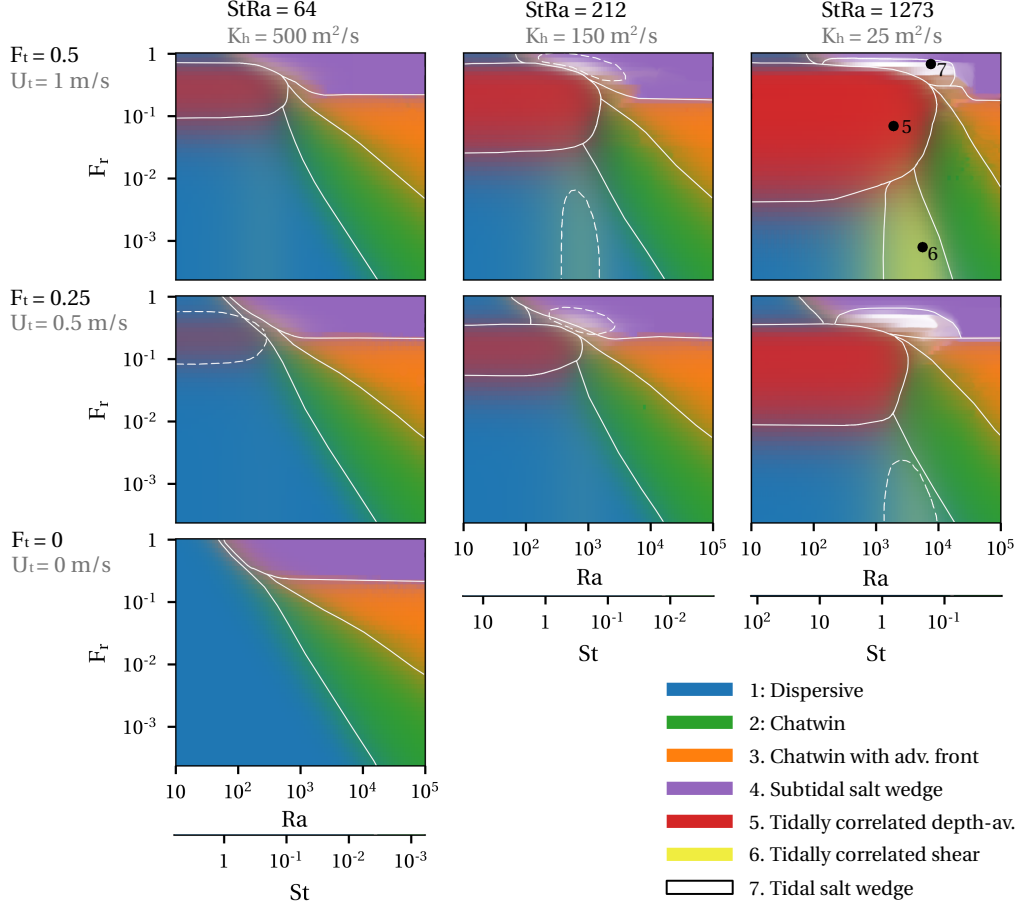
In order to gain insight into the entire parameter space, the model is run for a large number of different settings of the four dimensionless parameters  $F_r$ ,  $F_t$ ,  $Ra$ , and  $StRa$ . This is done by varying the corresponding dimension-carrying model input parameters  $\mathcal{U}_r$ ,  $\mathcal{U}_t$ ,  $A_\nu$ , and  $K_h$ . The ranges of these parameters as well as the values of the other parameters are listed in Table 1. The experiments are carried out using a continuation approach (cf. Section 2.2). i.e. by keeping values of  $\mathcal{U}_r$ ,  $\mathcal{U}_t$ , and  $K_h$  constant and gradually decreasing  $A_\nu$  and repeating this for other values of  $\mathcal{U}_r$ ,  $\mathcal{U}_t$ , and  $K_h$ . Over 40,000 model experiments have been carried out to sample the complete parameter space.

## 3 Results

We show the identified regimes in Section 3.1 and the corresponding stratification and salt intrusion in Section 3.2. This is then used to choose several specific parameter settings for which we analyse the water motion, salinity, and transport in more detail in Sections 3.3-3.5.

### 3.1 Regimes in parameter space

We systematically examined the salt transport processes in each of the model experiments. From this analysis, we identified 7 distinct balances of salt transport mechanisms, called *regimes*. The precise algorithmic procedure for defining these regimes is provided in Section 3 of the supplemental information. The occurrence of the 7 regimes in the parameter space is visualised in Fig. 1. Each panel in the figure shows regimes indicated by different colors as a function of  $F_r$  (vertical axis) and  $Ra$ ,  $St$ . Note that  $Ra$  and  $St$  change simultaneously on the horizontal axis, which corresponds to changing  $A_\nu$ . The various panels represents different combinations of  $F_t$  and the product  $StRa = \frac{c_T^2}{\omega K_h}$ . Moving from the lower panels up,  $F_t$  increases (i.e. tidal velocity increases). Moving from the left panels to the right,  $StRa$  increases (i.e.  $K_h$  decreases). In each model experiment, between 80 and 100 % (in most experiments well over 90 %) of the salt transport is described by the processes in either one regime or a combination of these regimes. The white lines in the figure separate regions where the mechanisms from one regime are more important than those in each of the other regimes. Close to these white lines, the colors in the figure blend continuously into each other. Here, one regime still dominates but 1 to 3 other regimes also play a role in the salt balance. White dashed lines are added in some of the panels where certain regimes never occur as most important, but do have a share in the salt balance of more than 20 %.



**Figure 1.** The seven regimes (colors) as a function of the four dimensionless parameters  $F_t$ ,  $F_r$ ,  $Ra$  and  $St$ . Each panel has a different constant value of  $F_t$  and  $StRa$ , while the axes in each panel represent varying  $F_r$  (vertical axis) and  $Ra$  and  $St$  (horizontal axis, corresponding to changing  $A_\nu$ ). White lines indicate the transitions between regimes, white dashed lines indicate a balance that is not dominant but contributes at least 20% to the total salt transport balance. The black dots in the upper-right panel mark the cases that are studied in Sections 3.3-3.5.

Of the seven regimes, the first four regimes were already identified in the subtidal study of DS21. We provide a short summary of these regimes here and refer to DS21 for an elaborate discussion. The four subtidal regimes are

1. **Dispersive regime.** Governed by a balance between salt transport due to background dispersion (through  $K_h$ ) and river-induced flushing. The background dispersion parametrises various mechanisms including lateral variations of the flow and salinity and effects of temporally varying eddy viscosity.
2. **Chatwin regime.** Regime as described by Chatwin (1976), which is a balance between the transport due to subtidal shear by gravitational circulation  $\langle u'_{0,gc} s'_{0,gc} \rangle$  and river-induced flushing.
3. **Chatwin regime with advection dominated front.** This is a transition regime that is characterised by satisfying the Chatwin regime when considering regions close to the mouth. When considering the regions near the salt intrusion limit, transport due to subtidal shear by momen-

tum advection becomes dominant, i.e.  $\langle \overline{u'_{0,adv} s'_{0,gc}} \rangle$ ,  $\langle \overline{u'_{0,gc} s'_{0,adv}} \rangle$ , and  $\langle \overline{u'_{0,adv} s'_{0,adv}} \rangle$ . This leads to the formation of a front-like salinity profile locally.

4. **Subtidal salt wedge regime.** Transport due to subtidal shear by momentum advection dominates in the entire estuary. The solution for the salinity resembles a stationary salt wedge and is well described by two-layer theory (e.g. Schijf & Schönfeld, 1953).

Naturally, these four regimes are the only regimes occurring when there are no tides (i.e. bottom panel of Fig. 1). The subtidal regimes also take a prominent place in the parameter space when tides are included. Besides these four dominantly subtidal regimes, we identify three regimes where tidal processes dominate. These are summarised here and discussed in more detail in the next sections.

5. **Tidally correlated depth-averaged transport regime.** A balance between transport due to tidally correlated depth-averaged transport  $\langle \overline{u_1 \bar{s}_1} \rangle$  and river induced flushing. In literature, tidally correlated depth-averaged transport has only been associated with the interchange of salt between a main channel and side-embayments and shallows (i.e. 'tidal trapping') (Okubo, 1973; Fischer et al., 1979; MacVean & Stacey, 2011) or other geometric features, such as shoals or river junctions (Dronkers & van de Kreeke, 1986). However, none of these geometric features are explicitly included in this study. We will show that presence of both a tidal flow and significant river flow can also cause a strong tidally correlated depth-averaged transport, independent of variations in bathymetry or geometry.
6. **Tidally correlated shear transport regime.** Balance between transport due to tidally correlated shear transport specifically by the term  $\langle \overline{u'_{1,tide} s'_{1,tide}} \rangle$  and river induced flushing. Here  $u'_{1,tide}$  is the vertical shear in the tidal velocity and  $s'_{1,tide}$  is the vertical salinity shear induced by the tidal velocity shear. This regime has been discussed from a theoretical perspective by several authors, including Bowden (1965), Chatwin (1975), Larsen (1977), Watson (1983), Ou et al. (2000), and Wei et al. (2016).
7. **Tidal salt wedge regime.** The net transports by subtidal processes and tidal correlations related to momentum advection dominate in the entire estuary. Hence, besides the mechanisms in regime 4, we find an important role for the transport contributions  $\langle \overline{u'_{1,adv} s'_{1,gc}} \rangle$ ,  $\langle \overline{u'_{1,gc} s'_{1,adv}} \rangle$ , and  $\langle \overline{u'_{1,adv} s'_{1,adv}} \rangle$ .

In Fig. 1 we see a prominent place for the tidally correlated depth-averaged transport regime (regime 5, red color). The tidally correlated shear transport regime (regime 6, yellow) is only dominant in a part of the parameter space in the top-right panel (large tide, small  $K_h$ ). In two other panels, this regime does make contribution of more than 20 % in part of the parameter space (indicated by the dashed white lines). Finally, the tidal salt wedge regime only dominates a part of the parameter space in the right column (small  $K_h$ ) and makes a contribution of over 20 % in the middle column.

### 3.2 Stratification and salt intrusion

In this section we investigate the characteristics of the regimes in terms of stratification and dimensionless salt intrusion length. Here, the dimensionless salt intrusion length is a global quantity, assuming that the entire estuary can be characterised by the same value of the (dimensionless) parameters. Fig. 2 presents the maximum (in time and space) top-to-bottom stratification, tidally averaged dimensionless salt intrusion length  $\langle \mathcal{L}_s \rangle$ , and the ratio of the tidal variation of  $L_s$  and the subtidal  $L_s$ . This is done for the case without tides (corresponding to the bottom-left panel in Fig. 1) and with  $\mathcal{U}_t = 1$  m/s and  $StRa = 1273$  (corresponding to the top-right panel in Fig. 1). The white lines again indicate the boundaries between the regimes and were copied from the respective panels in Fig. 1. Concentrating on the first row, which plots stratification, the case without tides shows that stratification increases with increasing  $F_r$  and increasing  $Ra$  (i.e. decreasing  $St$  or decreasing mixing). As concluded by DS21, stratification is not uniquely related to the regime, except for regime 4, which is always strongly stratified. Adding tides (top-right panel in Fig. 2) we again do not find a unique relation between stratification and regime. The main difference with the case without tide is found for intermediate  $F_r$  and small  $Ra$ . Comparison to Fig. 1 shows this part of the parameter space is dominated by the tidally correlated depth-averaged transport regime (regime 5). In most of this regime,

the stratification is small; a few psu. However, substantial stratification may occur in the transition zone of this regime to the Chatwin and salt wedge regimes.

The second row in Fig. 2) shows the dimensionless salt intrusion length  $\mathcal{L}_s$ . In both the subtidal and tidal case,  $\mathcal{L}_s$  increases with decreasing  $F_r$ . For small  $Ra$  (large mixing), estuaries are in regime 1 or 5 and  $\mathcal{L}_s$  is insensitive to changing  $Ra$ . For larger  $Ra$ , the regimes change and  $\mathcal{L}_s$  increases for increasing  $Ra$ . The value of  $\mathcal{L}_s$  shows little relation to the regimes 1, 2, 3, and 6. In the tidally correlated depth-averaged transport regime (regime 5),  $\mathcal{L}_s$  is fairly independent of both  $F_r$  and  $Ra$ . In the salt wedge regimes 4 and 7, the salt intrusion length is always relatively small. This does not imply that estuaries with a salt distribution looking like a salt wedge necessarily have a small salt intrusion length. Salt wedge-like salt distributions occur in a broad transition region from regimes 3 and 5 to regimes 4 and 7 and hence can also feature larger salt intrusion length. In contrast, our definition of the salt wedge regimes is very strict; requiring the corresponding transport mechanisms are dominant in the entire estuary. Comparing the subtidal and tidal cases, the addition of tides mainly leads to a difference in salt intrusion in regime 5. This will be explained in more detail in Section 3.3.

The third row in Fig. 2 shows the ratio of the tidal excursion of the salt intrusion and the subtidal salt intrusion. As the tidal variation of the salt intrusion scales with the tidal excursion length, this ratio is small when the subtidal salt intrusion greatly exceeds the tidal excursion length. From the figure, it is clear that this is characteristic for the subtidal dispersive and Chatwin regimes (regime 1, 2, and 3) as well as the tidally correlated shear transport regime (regime 6). On the other hand, the tidal variation of the salt intrusion length and subtidal salt intrusion length are of the same order of magnitude in the subtidal and tidal salt wedge regimes (regime 4 and 7) and the tidally correlated depth-averaged transport regime (regime 5). This implies that the salinity is almost entirely flushed from the estuary during part of the tide in the cases where these regimes dominate the entire estuary.

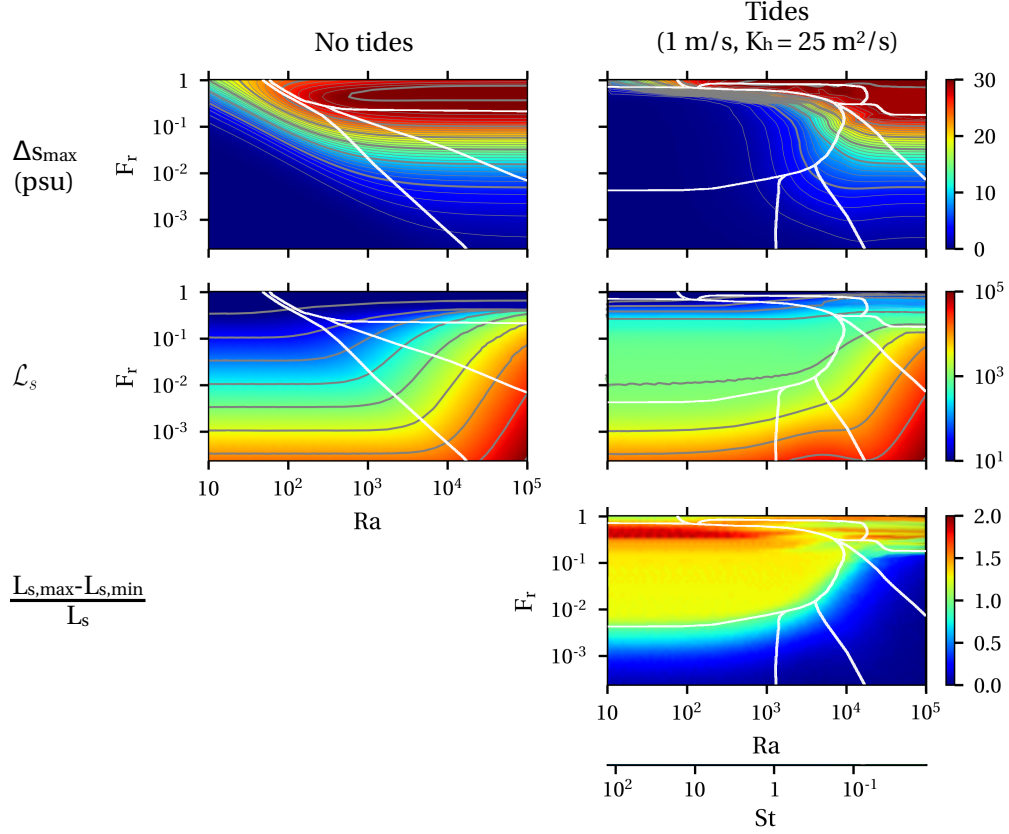
### 3.3 Tidally correlated depth-averaged transport regime

The first tide-dominated regime we discuss is the tidally correlated depth-averaged transport regime. This regime can be described by the following simplified form of the Fischer decomposition (Eq. (7)):

$$\underbrace{\bar{u}_{0,\text{riv}}\bar{s}_0}_{\text{transport by mean flow}} + \underbrace{\langle \bar{u}_{1,\text{tide}}\bar{s}_1 \rangle}_{\text{tidal mean transport}} = 0. \quad (9)$$

Fig. 3 shows an example of a case from the tidally correlated depth-averaged transport regime using  $F_r = 7 \cdot 10^{-2}$ ,  $F_t = 0.5$ ,  $Ra = 1940$ , and  $StRa = 1273$  (marked by 5 in Fig. 1). The top panels show the surface velocity and bottom salinity as a function of  $x$  and  $t$ , which are representative of the entire water column as the salinity is well-mixed and there is little exchange flow. The grey dashed lines indicate the time of slack water. The surface velocity shows an almost perfectly symmetric tide, affected only slightly by the river discharge (visible as slack water occurs at  $t/T_{\text{tide}} < 0.25$  and  $t/T_{\text{tide}} > 0.75$ ). Subtidal salinity decays in the along-channel direction but there is strong temporal variation over the tide, so that the salt is flushed out of the estuary during part of the tidal cycle. Maximum salt intrusion coincides with high water slack, while minimum salt intrusion occurs before low water slack. In terms of tidal components, the phase difference between the  $M_2$  tidal salinity and tidal velocity is approximately 80 degrees. The higher harmonics,  $M_4$  and  $M_6$ , of the salinity signal are the reason that maximum salt intrusion still occurs at high water slack. The bottom panel in the figure shows the most important contributions to the salt transport per  $\text{m}^2$  cross-sectional area and divided by the depth- and time-averaged salinity. Here, TM0, TM1 denote subtidal and tidal covariance, respectively, and the terms following it denote the contributions to  $u$  and  $s$  (see Section 2.4). The subtidal salt import is entirely governed by tidally correlated depth-averaged transport (red line), balanced by river-induced export (blue line), confirming Eq. (9) is a good approximation.

Many theoretical analyses of tidal salt transport conclude that tidally-correlated depth-averaged transport is (approximately) zero in along-channel uniform estuaries with a tide-dominated current (e.g. Chatwin, 1975). This is because  $\bar{u}$  and  $\bar{s}$  are 90 degrees out of phase in that case. Okubo (1973) and MacVean and Stacey (2011) show that tidally-correlated depth-averaged transport can be caused



**Figure 2.** Maximum subtidal top-bottom stratification (in psu; upper panels), dimensionless salt intrusion (middle panels) and relative tidal salt intrusion (lower panel) for the case without tides (left) and with 1 m/s tides (right). The white lines indicate the transitions between regimes from Fig. 1.

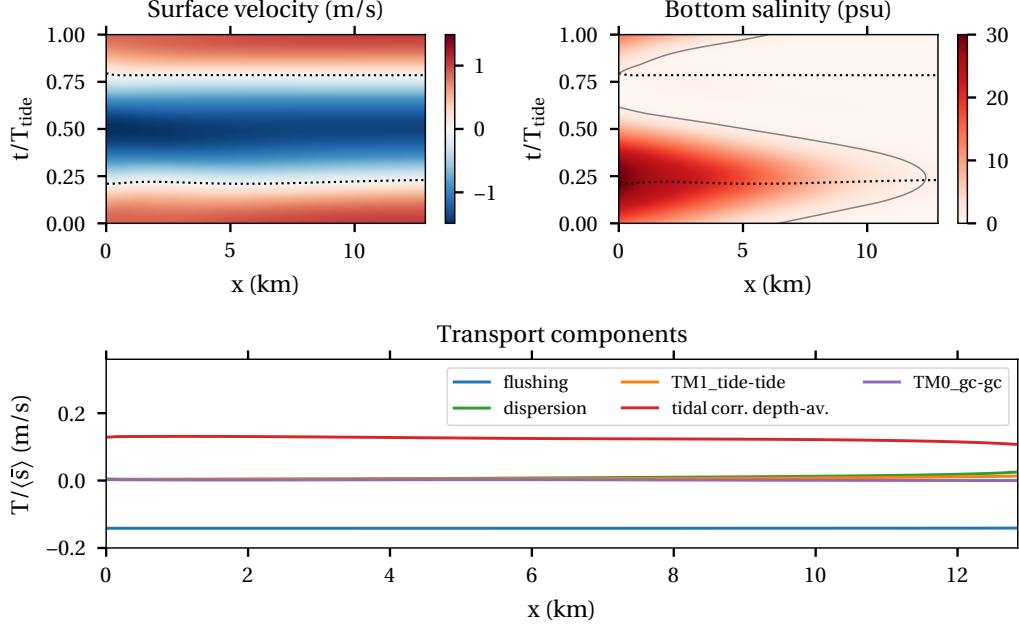
by tidal trapping: the interaction between the main channel and shallower sides or side-embayments, which change the phase difference between the depth-averaged tidal velocity and salinity (also see Fischer et al., 1979). Dronkers and van de Kreeke (1986) argue that also other geometric features, such as shoals, river branching and the transition to the open sea may be causes for such a phase difference. They furthermore use the term ‘non-local’ transport, as they show the tidal depth-averaged transport depends on the variations of  $\bar{u}$  and  $\bar{s}$  over the tidal excursion length. All above-mentioned authors assume that the river flow may be neglected compared to the tidal flow. As our case has along-channel uniform parameters, one would conclude from literature that tidally correlated depth-averaged transport should vanish.

To understand why we nevertheless find a big tidally correlated depth-averaged transport, we look closer at the analytical solution of the approximate transport balance Eq. (9). This is used together with a simplified depth-averaged balance for the  $M_2$  salinity component and our definitions  $\bar{u}_0 = \mathcal{U}_r$  and  $\bar{u}_1 = \mathcal{U}_t \cos(\omega t)$ . The equations considered are

$$-\mathcal{U}_r \bar{s}_{0,x} + \langle \mathcal{U}_t \cos(\omega t) \bar{s}_{1,x} \rangle = 0, \quad (10a)$$

$$\bar{s}_{1,t} - \mathcal{U}_r \bar{s}_{1,x} + \mathcal{U}_t \cos(\omega t) \bar{s}_{0,x} = 0, \quad (10b)$$

where  $\bar{s}_0$  still denotes the depth-averaged subtidal salinity and  $\bar{s}_1$  now specifically denotes the depth-averaged  $M_2$  component of the salinity. The derivation and solution procedure for this simplified system is presented in Appendix A. The resulting subtidal salinity follows an exponential profile  $\bar{s}_0 = s_{\text{sea}} e^{-\lambda x}$



**Figure 3.** Example representing the tidally correlated depth-averaged transport regime (see mark 5 in Fig. 1). Model results are shown for the surface velocity, bottom salinity and the tidally-averaged salt transport divided by the subtidal depth-averaged salinity. For the transport contrirbutions in the lower panel TM0, TM1 denote subtidal and tidal covariance, respectively, and the terms following it denote the contributions to  $u$  and  $s$  (see Section 2.4).

with a decay rate

$$\lambda = \frac{\omega}{\sqrt{\frac{1}{2}\mathcal{U}_t^2 - \mathcal{U}_r^2}}, \quad (\text{assuming } \mathcal{U}_r^2 < \frac{1}{2}\mathcal{U}_t^2) \quad (11a)$$

$$\approx \sqrt{2} \frac{\omega}{\mathcal{U}_t} \quad \text{if } \mathcal{U}_r \ll \mathcal{U}_t. \quad (11b)$$

For river velocities much smaller than the tidal amplitude, Eq. (11b), the salt intrusion length scales with the tidal excursion length. As we defined the salt intrusion length as the distance to the 1 psu line, we obtain  $L_s = \ln(s_{\text{sea}})/\lambda$ , or (assuming  $s_{\text{sea}} = 30$  psu)

$$L_s \approx 2.4 \frac{\mathcal{U}_t}{\omega}. \quad (12)$$

In terms of dimensionless numbers, the dimensionless salt intrusion length  $\mathcal{L}_s$  scales with  $F_t St Ra$ . As the subtidal salinity can be approximately be described by an exponential profile, it can also be considered as the solution from a 1D dispersion law (i.e. following  $D_{\text{tide-depth-averaged}} \bar{s}_{0,x} + \mathcal{U}_r \bar{s}_0 = 0$ ) with dispersion parameter:

$$D_{\text{tide-depth-averaged}} \sim \frac{\mathcal{U}_r \mathcal{U}_t}{\omega} \quad (\text{assuming } \mathcal{U}_r \ll \mathcal{U}_t). \quad (13)$$

In dimensionless numbers:  $\frac{D_{\text{tide-depth-averaged}}}{K_h} \sim F_r F_t St Ra$ .

The above reasoning requires no geometric features to create a phase difference between  $\bar{u}$  and  $\bar{s}$ , as suggested necessary by the literature. Also, the above reasoning does not depend on the seaward boundary condition, which was used only to set the value of  $\bar{s}_0$  at  $x = 0$ , so the effects of exchange between the estuary and the sea are not needed as well. The reason that the tidally correlated depth-averaged transport nevertheless occurs in this case, is the presence of both a tidal and river flow. The



non-zero river discharge results in a phase difference between velocity and salinity smaller than 90 degrees. The finding that river discharge can induce tidally correlated depth-averaged transport is new.

This finding also explains why the tidally correlated depth-averaged transport regime disappears for small river discharges. It is expected that a depth-averaged  $M_4$  overtide can play a similar role as river discharge leading to a significant tidal depth-averaged transport, so it is likely that this regime occurs in estuaries with a small river flow but strong overtides. Since we assumed a zero depth-averaged  $M_4$  velocity, this is not explicitly resolved here.

These analytical estimates explain our findings in Figs. 1-2. Firstly, the expression for the salt intrusion length agrees with the observation from Fig. 2 that the salt intrusion length in the tidally correlated depth-averaged transport regime is almost constant under varying  $F_r$  and  $Ra$ ,  $St$  if  $StRa$  and  $F_t$  are kept constant. Small variations of the salt intrusion length nevertheless occur, as the regimes gradually transition into each other. Secondly, we see from the figure that this regime disappears for high  $F_r$ , which is consistent with Eq. (11a) becoming invalid for  $\mathcal{U}_r > \frac{1}{\sqrt{2}}\mathcal{U}_t$  as the simplified balance of Eq. (9) has no solution in that case (i.e. import is insufficient to balance the large river-induced export). Thirdly, the dimensionless dispersion coefficient associated with the tidally correlated depth-averaged transport increases with  $\mathcal{F}_r$ ,  $\mathcal{F}_t$  and  $StRa$ , while the dimensionless dispersion coefficient in the dispersive regime (regime 1) is a constant. Hence, the current regime dominates over the dispersive regime for large  $F_r$ ,  $F_t$ , and  $StRa$ , as confirmed by Fig. 1.

Only accounting for the subtidal and  $M_2$  component is a coarse approximation. A much closer approximation is obtained when also taking into account the  $M_4$  salinity component. The analytical solution procedure for the resulting system of equations is shown in Section 4 of the supplemental information. With this addition, the subtidal salt intrusion no longer follows an exponential profile and is therefore not precisely described by a dispersion law. Otherwise, all conclusions drawn above remain valid, and the salt intrusion length-scale derived above still provides a good approximation of the salt intrusion length. Section 4 of the supplemental information also describes the effect of accounting for phase propagation of the tidal velocity  $\mathcal{U}_t$ , showing that this effect is typically negligible.

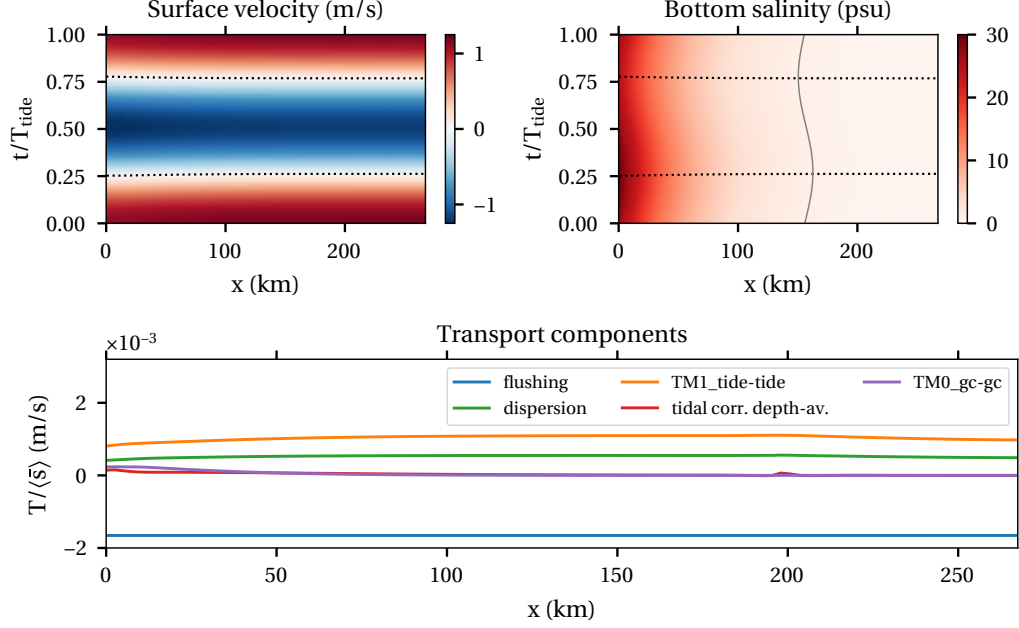
### 3.4 Tidally correlated shear transport regime

In the tidally correlated shear transport regime, salt is imported by the covariance of tidal velocity shear and tidal salinity shear and flushed out by the river discharge. Hence the balance reduces to approximately

$$\underbrace{\bar{u}_{0,\text{riv}}\bar{s}_0}_{\text{transport by mean flow}} + \underbrace{\langle u'_{1,\text{tide}}s'_{1,\text{tide}} \rangle}_{\text{tidal shear transport}} = 0. \quad (14)$$

An example of the tidal shear dispersion regime is shown in Fig. 4 using the parameter settings  $F_r = 8 \cdot 10^{-4}$ ,  $F_t = 0.5$ ,  $Ra = 5600$ , and  $StRa = 1300$  (see 6 in Fig. 1). As in the previous regime, the exchange flow is small and hence the surface velocity (top-left panel) shows an  $M_2$  tide modulated by a small river discharge. The bottom salinity (top-right panel) is representative of the salinity in the water column as top-to-bottom stratification is smaller than 1 psu. It shows a dominantly subtidal salt intrusion with a small tidal modulation, where maximum salt intrusion occurs close to high water slack. The bottom panel shows two important processes for salt import: the tidally correlated shear transport (TM1.tide-tide) and dispersion, with the tidal shear transport about twice more important than dispersion in this case. This case is thus a combination of regime 1 and 6, where regime 6 is most important.

The tidally correlated shear transport mechanism was first discussed by Bowden (1965) for the limit of large Stokes numbers and was generalised by e.g. Holley et al. (1970), Chatwin (1975), Larsen (1977), Fischer et al. (1979), and Watson (1983) (see Chatwin and Allen (1985) for a review). Ou et al. (2000) discussed this mechanism on a sloping bed. McCarthy (1993), Wei et al. (2016) provided the



**Figure 4.** Example representing the tidally correlated shear transport regime (see mark 6 in Fig. 1).

first analytical description of the above balance (14) appended with horizontal dispersive transport, but tidally correlated shear transport was never dominant in their case studies.

All the above authors have demonstrated that the tidally correlated shear transport can be approximated closely by a dispersion process. This dispersion process can be written as  $\langle u'_{1,\text{tide}} s'_{1,\text{tide}} \rangle = D_{\text{tide-tide}} \bar{s}_{x,0}$ , where

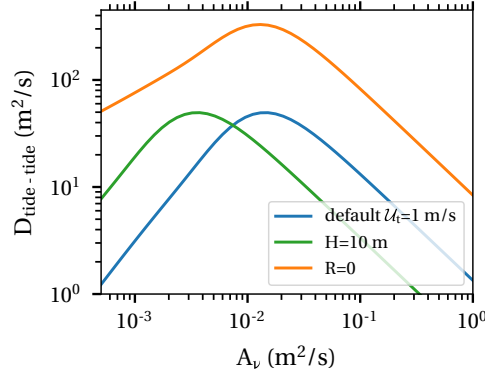
$$D_{\text{tide-tide}} = \frac{\mathcal{U}_t^2 H^2}{A_\nu} F(St; \sigma_\rho, R). \quad (15)$$

where  $F$  is some nonlinear function  $F$  that depends on the Stokes number  $St = \frac{A_\nu}{\omega H^2}$ ,  $\sigma_\rho$ , and  $R$ . Wei et al. (2016) provide a closed-form expression for  $F$  for the case  $\sigma_\rho = 1$ . Fig. 5 shows  $D_{\text{tide-tide}}$  as function of  $A_\nu$  for the parameter settings of the example in Fig. 4 and two variations, with  $H = 10$  m and  $R = 0$  (i.e. representing a no-slip bottom boundary) (c.f. Fischer et al. (1979, Fig. 7.4)). For the default settings (blue line),  $D_{\text{tide-tide}}$  varies over more than one order of magnitude with its maximum occurring for intermediate values of  $A_\nu$ . We may understand this physically as follows: for large values of  $A_\nu$  (i.e. large  $St$ ) the system is too strongly mixed to create substantial salinity shear, while for small  $A_\nu$  (i.e. small  $St$ ) the mixing is so small that the salinity shear cannot adjust to the tidal motion within the tidal period. As a result, the maximum tidally correlated shear transport occurs at intermediate  $A_\nu$  (intermediate  $St$ ). As this transport contribution scales with  $St$ , reducing  $H$  by a factor 2 (green line), means we find the maximum transport at a 4 times smaller  $A_\nu$ , such that the maximum occurs for the same value of  $St$ . When changing the viscosity-friction ratio  $R$  to represent a no-slip condition (orange line), the tidally correlated shear transport increases significantly by over an order of magnitude.

### 3.5 Tidal salt wedge regime

The tidal salt wedge regime is exemplified in Fig. 6, showing results using  $F_r = 0.73$ ,  $F_t = 0.5$ ,  $Ra = 7600$ , and  $StRa = 1300$  (see 7 in Fig. 1). Here, and everywhere where this regime dominates,  $F_r \geq F_t$ , so that the depth-averaged velocity is always negative. The bottom velocity (top-left panel) however is positive during the flood tide due to the strong estuarine circulation. Salt intrusion correspondingly





**Figure 5.** Equivalent dispersion parameter  $D_{\text{tide-tide}}$  (Eq. (15)) for the tidally correlated shear transport as a function of the eddy viscosity  $A_v$  for the default case (blue), a twice shallower case (green) and the default case with viscosity-friction ratio  $R = 0$ , representing a no-slip bottom boundary (orange).

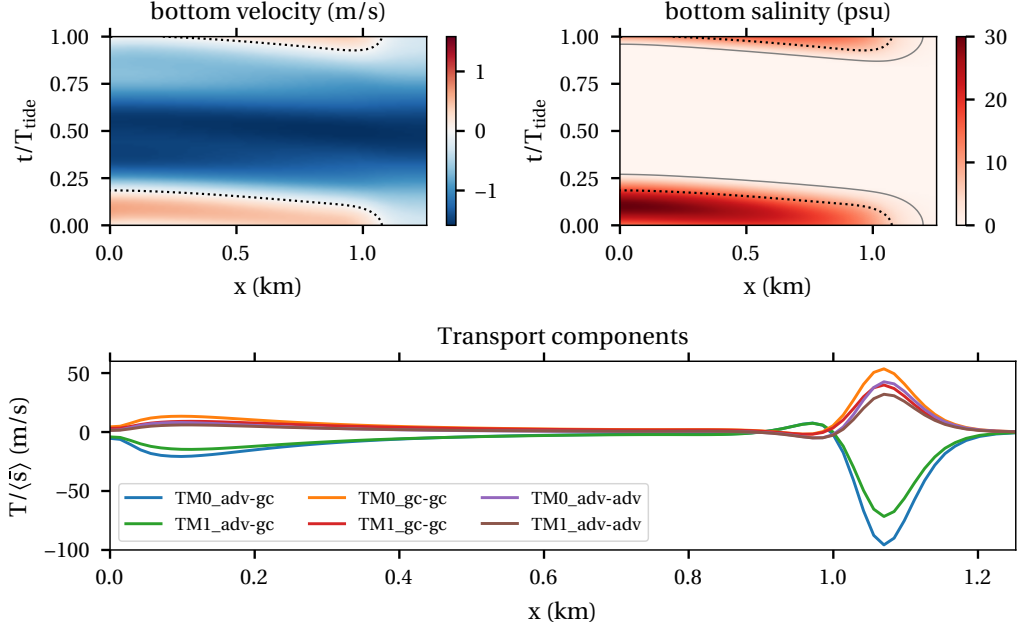
is greatest during the flood tide ( $0 < t/T_{\text{tide}} < 0.25$  and  $0.75 < t/T_{\text{tide}} < 1$ ), with the maximum intrusion occurring almost at peak flood ( $t/T_{\text{tide}} = 0$ ). Salt is flushed out of the estuary during the ebb tide, where it is important to note that salt water is assumed to remain close to the estuary mouth with our boundary conditions. Nevertheless, salt water only returns when the tide is close to peak flood. The bottom panel of Fig. 6 shows the governing transport processes are related to the baroclinic pressure and momentum advection. Throughout the estuary subtidal processes (indicated by TM0) are as important as the tidal correlations (indicated by TM1).

Analytical or otherwise highly simplified analysis of the tidal salt wedge seems restricted. Jay and Smith (1990) analyse a tidal two-layer model, but assume  $F_r \ll F_t$  and neglect momentum advection at the leading order. Given our results, this is clearly not the appropriate scaling for this regime. Geyer and Farmer (1989) and Geyer and Ralston (2011) offer an alternative approach by studying the tidal salt wedge assuming the flow and salinity are quasi-stationary at each moment during the tide. This assumption seems a reasonable, yet coarse approximation for the case studied in Fig. 6: the maximum salt intrusion occurs at peak flood consistent with a quasi-stationary view, but salt intrusion is not symmetric around peak flood, indicating salinity cannot fully adjust to instantaneous flow conditions. The main result of quasi-stationary theory states that a salt wedge cannot persist if the internal flow becomes critical, i.e. when the velocity in one layer exceeds the internal wave speed  $c_I$ . The exact time at which the flow becomes internally critical depends on the precise definition used for  $c_I$  and the layer velocity. Estimating these quantities for our case study indeed shows internally critical flow during most of the ebb phase. Geyer and Farmer (1989) also describe intense mixing in the process of breaking down the salt wedge. This requires mixing parameters that vary in time and space, which is not explicitly accounted for in our model. So while this model captures some of the important characteristics of a salt wedge, likely not all important physical processes are described (also see Section 4.3).

## 4 Discussion

### 4.1 Case study: the Rotterdam waterway

Below we first apply our classification to the Rotterdam waterway and compare the regimes found with transports in a simulation model. Next, we provide some general guidelines and lessons-learned for application to other estuaries.



**Figure 6.** Example representing the tidal salt wedge regime (see mark 7 in Fig. 1). Model results are shown for the near-bed velocity, bottom salinity and the tidally-averaged salt transport divided by the subtidal depth-averaged salinity.

### *Application to the Rotterdam waterway*

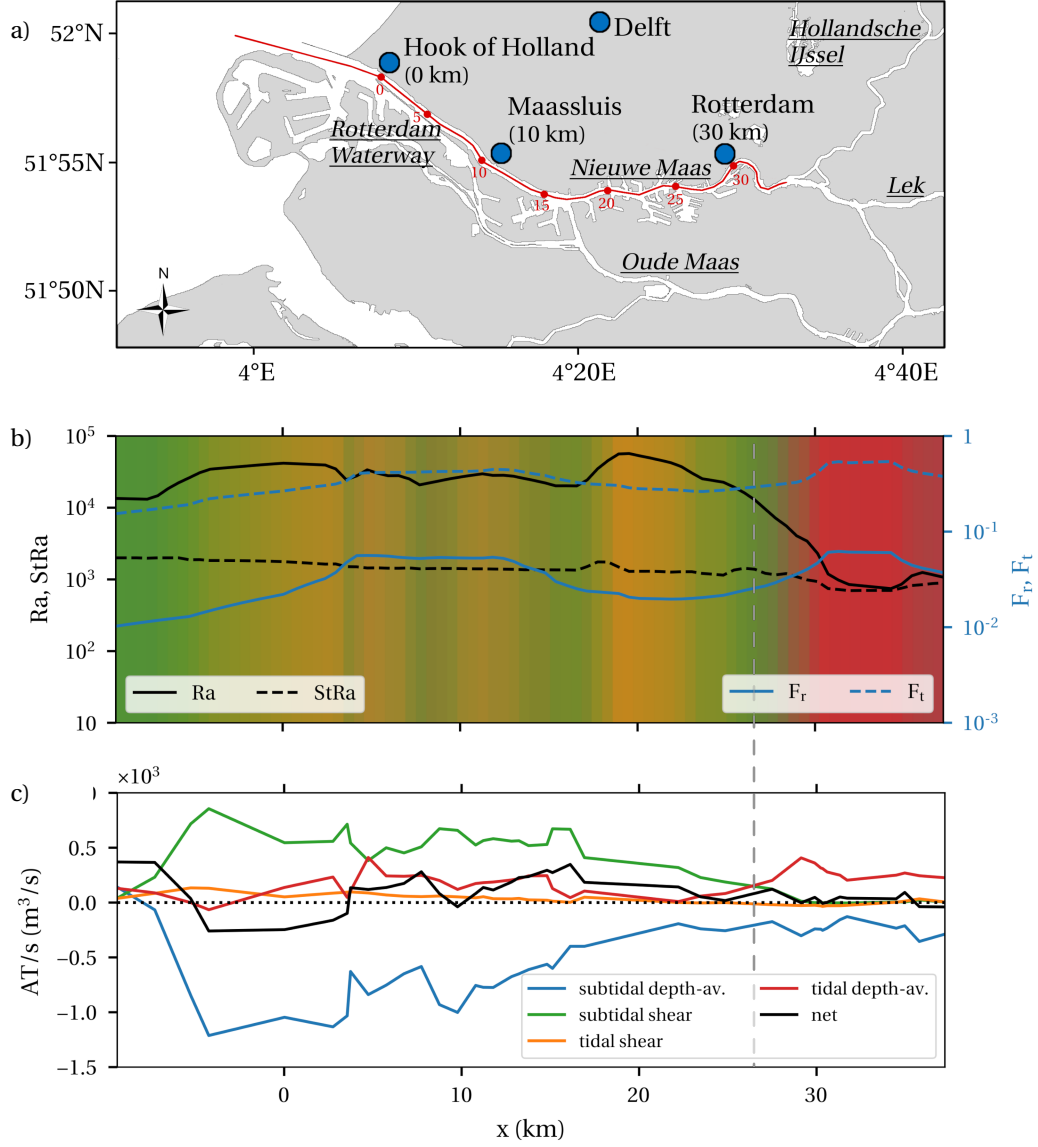
We apply our classification to determine the regimes in the Rotterdam Waterway-Nieuwe Maas estuary (RWW), which is a partially-mixed to salt wedge estuary in the Dutch Rhine delta. We verify our results by comparing our classification against results of the 3D hydrostatic OSR model (abbreviation from Dutch meaning 'operational flow model Rotterdam') of the lower Rhine-Meuse delta (Netherlands), developed and maintained by the Port of Rotterdam (see <https://www.portofrotterdam.com/en/operational-information/hydrometeo-data> for the operational results and W. M. Kranenburg et al. (2015), for a description and model evaluation). The OSR model is built within SIMONA, an established modelling suite that has been in use as official software for the Dutch Directorate-General for Public Works and Water Management since the early 1990s (De Goede, 2020). We focus on the part of the delta indicated by the red line in Fig. 7a following the Rotterdam Waterway-Nieuwe Maas (RWW) from the North Sea, approximately 10 km downstream of Hook of Holland (km 0) to the junction with the Hollandse IJssel (km 37). This part of the delta is a fairly narrow, almost prismatic and highly engineered estuary in the Rhine-Meuse delta, which forms the entrance channel to the Port of Rotterdam. We consider one spring-neap cycle in the period from August 17 to 31, 2003. During this time, the upstream discharge of the Rhine at the Dutch border at Lobith was low: 900-1000 m<sup>3</sup>/s.

To establish a classification for the RWW we first determined the values of the dimensionless numbers  $F_r$ ,  $F_t$ ,  $Ra$ , and  $St$  as a function of the along-channel coordinate using the OSR model input and output (spring-neap averaged  $H$ ,  $U_t$ ,  $U_r$ ) from 41 locations along the thalweg. Additionally, we used  $K_h = 20$  m<sup>2</sup>/s and checked that the regimes obtained are not sensitive to this choice (see below for more information). The eddy viscosity was estimated based on a reconstruction that used the velocity and salinity from the OSR model in a damping function-type closure as in Pacanowski and Philander (1981), Wei et al. (2021). See the supplemental information for details. The resulting values of the dimensionless numbers are plotted in Fig. 7b. For each along-channel position, we look up the regime from our classification for the local combination of the dimensionless parameters. In other words,

the local parameters are translated into an infinitely long estuary with uniform parameters, the salt transport is computed and the resulting dominant processes are assumed to also be dominant locally in the estuary (see also Section 1). We have recomputed the classification as in Fig. 1 using  $\sigma_\rho = 0.7$  as this is the value used in the OSR model, instead of  $\sigma_\rho = 2.2$ ; this results only in minor quantitative changes while the classification diagram of Fig. 1 is qualitatively the same.

The regime as a function of along-channel distance is indicated using the colors in Fig. 7b, where colors match those in Fig. 1. Between the mouth and km 26 we alternately find regimes 2 (green) and 3 (orange), which are both Chatwin regimes where subtidal shear due to gravitational circulation balances river-induced export. Between km 26-30 we find a transition to regime 5 (red), the tidally correlated depth-averaged regime, which is dominant in the upstream part of the estuary.

We compare our classification to a direct decomposition of the OSR model results made by W. M. Kranenburg and Van der Kaaij (2019) and W. M. Kranenburg et al. (2022), decomposing into the four contributions given in Eq. (7), i.e. subtidal and tidally correlated depth-averaged and shear transports (excluding dispersive transport). The result of their decomposition is shown in Fig. 7c. Note the net transport (black line) is not completely zero, meaning that the spring-neap averaged transports are not entirely in dynamic equilibrium. They find the transport by the subtidal shear contribution is dominant for  $x < 26.5$  km and transport by the tidally correlated depth-averaged transport dominates for  $x > 26.5$  km. This transition point is indicated by the grey-dotted line. Even though the three-dimensional OSR model explicitly accounts for many more processes related to turbulence and lateral correlations, as well as tidal trapping in the many harbour basins in the RWW, the results of the decomposition match our classification in 7b. Only the exact transition point is shifted between the two methods, as the red color appears only a few km ahead of the 26.5 km point (grey dotted line) in Fig. 7b. This case study demonstrates how our classification may be used for the classification of real estuaries and can qualitatively identify not only a regime, but also the transition of one regime into another within one estuary. The method can further be used to classify estuaries under various forcing conditions (e.g. high and low discharge) or compare different estuaries to each other.



**Figure 7.** (a) Rotterdam Waterway - Nieuwe Maas river (RWW) in the Dutch Rhine delta. The red line indicates the along-channel section of the model domain that is represented in panels b and c. (b) Spring-neap averaged values of the dimensionless parameters (lines) along the RWW during low discharge conditions on 17-31 Aug, 2003 as estimated from the three-dimensional OSR model. Colors indicate the corresponding regime corresponding to colors in Fig. 1 with green and orange the Chatwin regimes (reg. 2 and 3) and red the tidally correlated depth-averaged transport regime (reg. 5). (c) Direct decomposition of the salt transport from the OSR model by W. M. Kranenburg and Van der Kaaij (2019) plotted as transport integrated over the cross-sectional area ( $A$ ) and divided by the mean salinity. The vertical dashed grey line indicate the location of the transition between regimes according to this decomposition.

### *Guidelines for application to other estuaries*

The current study is aimed at identifying the regimes and their occurrence in the parameter space, not as a foolproof classification tool. In applying the classification we found that it is especially difficult to determine the parameters  $K_h$  and  $A_\nu$ , which results in some degree of arbitrariness.

To select  $K_h$  one could start with a small value, so the estuary is not in or near regime 1. The exact value of  $K_h$  will then not affect the results, as dispersive transport is not important. Any small change in  $K_h$  will affect  $Ra$  and  $StRa$  in such a way that the final classification and salt intrusion length remains the same. If the observed and modelled salinity gradient or stratification are not similar, one could consider to increase  $K_h$  (possibly as a function of  $x$ ) so that dispersive transport becomes important and creates a better match between modelled and observed salinity.

The vertical eddy viscosity  $A_\nu$  can be determined to some extent on the basis of models or observations. While this worked in our application with a simple turbulence model, this is not infallible as  $A_\nu$  parametrises effects of vertical, lateral and temporal variations in mixing. The match between observed and modelled salinity gradients and stratification may be used here as well as a motivation to change  $A_\nu$ .

## 4.2 Comparison with other studies

Many of the regimes discussed in this study have been covered in previous studies focussing either on specific mechanisms or analysis of numerical models or observations. We look closer at the part of the parameter space covered by several of these studies in Table 2. Focussing first on general mechanistic studies, the regimes described by several studies is indicated by an x. The subtidal regimes 1, 2, 3, and 4 as well as the tidal correlations regimes 6 and 7 (to some extent) have been covered, although no study covers all of these regimes at once. This study for the first time describes the tidally correlated depth-averaged transport regime resulting from a river-induced asymmetry. Earlier studies only attributed this transport to geometric features of the estuary (see also Section 3.3).

Next concentrating on analysis methods for observations or complex numerical models, decomposition is usually done along the lines of Fischer (1976). A popular compact form of this decomposition used by e.g. Bowen and Geyer (2003), Lerczak et al. (2006) distinguishes between tidally correlated transport  $F_T$  and subtidal transport due to the exchange flow  $F_E$ . As shown in Table 2 dominance of either  $F_T$  or  $F_E$  may indicate dominance of multiple regimes. Dronkers and van de Kreeke (1986) chose a different compact form, distinguishing between tidally correlated depth-averaged transport (denoted by  $K_1$ , see Table 2) and all shear transport contributions ( $K_2$ ). Recently, W. M. Kranenburg and Van der Kaaij (2019) and Garcia et al. (2021) combined the above two approaches and distinguished between tidally correlated depth-averaged transport ( $F_1$ , see Table 2), subtidal shear transport ( $F_3$ ), and tidally correlated shear transport ( $F_4$ ). More elaborate decompositions have been made by e.g. Dyer (1974), Hughes and Rattray (1980), adding transport related to fluctuations of the surface level and lateral correlations, which in our approach is partly parametrised by  $K_h$ . Concluding, while no decomposition method is able to uniquely identify each individual physical process or regime, they do all distinguish collections of dominant processes or regimes. This can be used to estimate approximate parameter dependencies and is greatly helpful for understanding results of realistic models and observations. It also provides a useful tool for model reduction to operational one-dimensional models (W. M. Kranenburg et al., 2016).

## 4.3 Unresolved dynamics

Salt transport in real estuaries is greatly complex. In this study some of this complexity is parametrised, meaning that some of the salt transport is not explicitly accounted for. In this section we discuss how our assumptions relate to missing salt transport processes and how our assumptions affect the interpretation of the classification. Firstly, the model is width-averaged and hence the effect of any lateral variation in flow and salinity is parametrised in the dispersion parameter  $K_h$ . As Fischer et al. (1979) note, lateral correlations of velocity and salinity are dominant for the salt transport in quite a number of estuaries. Explicitly resolving the lateral dimension will likely result in a further specification of  $K_h$  and in new regimes that partly replace the dispersive regime 1.

Secondly, we assumed a tidally constant eddy viscosity. Most notably, this assumption does not account for the effect of tidal variations of stratification (or SIPS, Simpson et al. (1990)) on the eddy viscosity. Adding SIPS-induced tidal variations of the eddy viscosity affects the stratification, phasing

	Study	Regime						
		1	2	3	4	5	6	7
Mechanistic	Schijf and Schönfeld (1953)				x			
	Hansen and Rattray (1965)	x	x					
	Chatwin (1976)		x					
	Geyer and Farmer (1989)				x			x
	MacCready (2004)	x	x					
	Wei et al. (2016)	x					x	
	Dijkstra and Schuttelaars (2021)	x	x	x	x			
Decomposition	Dronkers and van de Kreeke (1986)		$K_2$	$K_2$	$K_2$	$K_1$	$K_2$	$K_2$
	Lerczak et al. (2006)		$F_E$	$F_E$	$F_E$	$F_T$	$F_T$	$F_T$
	Garcia et al. (2021)		$F_3$	$F_3$	$F_3$	$F_1$	$F_4$	$F_4$

**Table 2.** Selection of other studies presenting either detailed mechanistic description or decomposition methods for data or complex model results and the regimes that are described. Under ‘mechanistic’, an x indicates that this regime is described by the respective study. Under ‘decomposition’, the different terms correspond to the terms used in the respective studies (colors are used to emphasise the different terms), showing that different authors have made different groupings of regimes in their decompositions.

of the salinity relative to the velocity as well as the exchange flow through ESCO circulation (Dijkstra, Schuttelaars, & Burchard, 2017), which can cause a significant additional salt transport. This could impact our classification in several ways. For example, Geyer and MacCready (2014) discuss the existence of a SIPS-dominated regime. Furthermore, the tidal salt wedge regime is usually described in literature in terms of variations in mixing over the tidal cycle (Geyer & Farmer, 1989), so including tidal variations of the eddy viscosity is expected to be important for an accurate description of this regime.

We treated the eddy viscosity and vertical and horizontal eddy diffusivity as independent parameters in our study. In reality, these parameters are not independent but depend strongly and non-linearly on the flow and salinity through complicated turbulence closure models. This means that changes in e.g. tidal velocity in an estuary will not only affect the parameter  $F_t$  but simultaneously affect  $Ra$  and  $St$  through the turbulence closure. Three-dimensional models with advanced turbulence closures are therefore still necessary to determine how a change in estuarine parameters affects  $F_t$ ,  $F_r$ ,  $Ra$ , and  $St$  and hence the regimes in an estuary.

As discussed by e.g. Dronkers and van de Kreeke (1986), geometric elements (e.g. shallows, branches, side-basins) are potential causes for additional salt transport. Nevertheless, from our application in Section 4.1, it seems that the geometric elements in the RWW do not need to be considered to determine the dominant salt transport process.

Finally, when considering variations on short temporal scales (i.e. shorter than the adjustment time of the estuary, e.g. see C. Kranenburg (1986)) the dynamic equilibrium described in this study may not be attained. Furthermore, the combination of adjustment on short temporal scales over bathymetric variations on short spatial length scales in the estuary has been associated with features such as frontogenesis (Geyer & Ralston, 2015), which is also not described by this study.

## 5 Conclusions

We have investigated the relative importance of various subtidal and tidally correlated contributions to the tidally averaged estuarine salt balance. We focussed on a local description of the salt balance, so that all geometric and forcing parameters could be assumed along-channel uniform, while dynamically resolving the vertical and horizontal profiles of the salinity. This approach allowed us to

express the salinity and salt intrusion as a function of four dimensionless parameters describing the geometric and forcing conditions in an estuary. By varying these four dimensionless parameters over several orders of magnitude in over 40,000 model experiments, a large estuarine parameter space was systematically explored.

The main new finding of this work is that the entire parameter space features 7 balances of salt transport mechanisms, called regimes. Almost all essential transport processes in these regimes have been described individually in literature. However, the tidally correlated depth-averaged transport was previously only associated with tidal trapping or prominent geometric features. Here we demonstrated for the first time that this regime can also occur due to tidal asymmetry, in our case caused by a significant river discharge. Assuming a straight uniform prismatic channel, we found an analytical estimate for the typical salt intrusion length in the regime where this transport dominates, which scales with the tidal excursion length.

Although application of our classification to real estuaries is not the main goal, as a verification we compared our classification to transport contributions identified in a realistic simulation model of the Rotterdam Waterway-Nieuwe Maas (RWW). The classification qualitatively matches the results from the direct decomposition, also demonstrating the existence of the tidally correlated depth-averaged transport regime in the RWW upstream from km 26 during low discharge conditions. The classification elegantly shows how and why dominant physical mechanisms vary throughout a single estuary, vary between estuaries and vary as a function of variable forcing conditions.

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## Appendix A Analytical solution for the tidally correlated depth-averaged transport balance

The tidally correlated depth-averaged transport balance can be approximated analytically. In this section we show the simplest approximation that uses only information from the subtidal and  $M_2$  tidal components and which yields easy analytical length scale estimates. An extended approximation also using the  $M_4$  tide but otherwise following a similar derivation is presented in SI-Section 3.1. Furthermore, the reasoning is extended to include the effects of adding phase propagation of the tide in the limit  $\zeta/H \rightarrow 0$  in SI-Section 3.2.

For the present approximation we assume that the vertical structure of salinity and velocity are not important to the salt transport and hence use the depth-averaged balances for the subtidal salinity  $\bar{s}_0$  and  $M_2$  tidal salinity  $\bar{s}_1$ . Without loss of generality we assume that the depth-averaged tidal velocity may be written as  $\bar{u}_1 = \mathcal{U}_t \cos(\omega t)$ . Retaining only the terms essential in this regime, i.e.

$$-\mathcal{U}_r \bar{s}_{0,x} + \langle \mathcal{U}_t \bar{s}_{1,x} \rangle = 0, \quad (\text{A1a})$$

$$\bar{s}_{1,t} - \mathcal{U}_r \bar{s}_{1,x} + \mathcal{U}_t \bar{s}_{0,x} = 0. \quad (\text{A1b})$$

The  $M_2$  salinity is written as  $\bar{s}_1 = \Re((\bar{s}_1^R(x) + i\bar{s}_1^I(x))e^{i\omega t})$  with real functions  $\bar{s}_1^R(x)$  and  $\bar{s}_1^I(x)$ . Using this expression we obtain three equations: one for the subtidal salinity and two for the real and imaginary parts of the complex  $M_2$  salinity amplitude

$$-\mathcal{U}_r \bar{s}_{0,x} + \frac{1}{2} \mathcal{U}_t \bar{s}_{1,x}^R = 0, \quad (\text{A2a})$$

$$-\omega \bar{s}_1^I - \mathcal{U}_r \bar{s}_{1,x}^R + \mathcal{U}_t \bar{s}_{0,x} = 0, \quad (\text{A2b})$$

$$\omega \bar{s}_1^R - \mathcal{U}_r \bar{s}_{1,x}^I = 0. \quad (\text{A2c})$$



This system may be written in matrix-vector product form with a vector  $\underline{y} = [\bar{s}_0, \bar{s}_1^R, \bar{s}_1^I]^T$  as  $A\underline{y}_x + B\underline{y} = \underline{0}$  for some matrices  $A$  and  $B$ . After some rewriting, this can be cast in the form

$$\underline{y}_x = \begin{bmatrix} 0 & 0 & \frac{\omega \mathcal{U}_t}{\mathcal{U}_t^2 - 2\mathcal{U}_r^2} \\ 0 & 0 & \frac{2\omega \mathcal{U}_r}{\mathcal{U}_t^2 - 2\mathcal{U}_r^2} \\ 0 & \frac{\omega}{\mathcal{U}_r} & 0 \end{bmatrix} \underline{y} \quad (\text{A3})$$

Solutions consist of a linear combination of eigenvectors and  $e^{\lambda_i x}$  with eigenvalues  $\lambda_i$  ( $i = 1, 2, 3$ ). Eigenvalues of this matrix equation may be computed easily by hand. Assuming  $\mathcal{U}_r^2 < \frac{1}{2}\mathcal{U}_t^2$ , the eigenvalues are real and equal to 0,  $\frac{\omega}{\sqrt{\frac{1}{2}\mathcal{U}_t^2 - \mathcal{U}_r^2}}$ , and  $-\frac{\omega}{\sqrt{\frac{1}{2}\mathcal{U}_t^2 - \mathcal{U}_r^2}}$ . We may ignore non-negative eigenvalues as the corresponding eigenfunctions will not satisfy the condition  $s \rightarrow 0$  for  $x \rightarrow \infty$ . Hence the only relevant eigenvalue equals

$$\lambda = -\frac{\omega}{\sqrt{\frac{1}{2}\mathcal{U}_t^2 - \mathcal{U}_r^2}}. \quad (\text{A4})$$

And the solution reads as

$$\bar{s}_0 = \bar{s}_0(0)e^{\lambda x}, \quad (\text{A5a})$$

$$\bar{s}_1^R = \bar{s}_0(0)\frac{2\mathcal{U}_r}{\mathcal{U}_t}e^{\lambda x}, \quad (\text{A5b})$$

$$\bar{s}_1^I = -\bar{s}_0(0)\frac{\sqrt{2}\sqrt{\mathcal{U}_t^2 - 2\mathcal{U}_r^2}}{\mathcal{U}_t}e^{\lambda x}, \quad (\text{A5c})$$

where  $\bar{s}_0(0)$  is the subtidal salinity at  $x = 0$ . When  $\mathcal{U}_r^2 > \frac{1}{2}\mathcal{U}_t^2$ , eigenvalues are complex with zero real part, so that no solutions exist satisfying  $s \rightarrow 0$  for  $x \rightarrow \infty$ .

There are two apparent problems concerning this simple approximation. Firstly, Fig. 3 shows that the along-channel salinity profiles are not at all exponential as implied here. Secondly, this solution does not satisfy the boundary condition  $\bar{s}_{1,x} = 0$  at  $x = 0$ . These problems may be solved by including an  $M_4$  tidal component and retaining the horizontal dispersive terms in the equations. This is done in the SI-Section 3a. It has been checked that the length-scale derived above is a good estimate of the dominant length-scale of the solution presented in the SI.

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# Supporting Information for "Salt transport regimes caused by tidal and subtidal processes in narrow estuaries"

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## 1. Model development

### 1.1. Equations

We will investigate the momentum and continuity equations for the water motion

$$u_t - (A_\nu u_z)_z + uu_x + wu_z = -g\zeta_{\hat{x}} - g\beta \int_z^0 s_{\hat{x}} dz', \quad (1)$$

$$w_z + \frac{1}{B}(Bu)_{\hat{x}} = 0, \quad (2)$$

in Cartesian coordinates  $(\hat{x}, z)$  and time  $t$ . The model has boundary conditions

$$A_\nu u_z = 0 \quad (z = 0), \quad (3)$$

$$A_\nu u_z = s_f u \quad (z = -H), \quad (4)$$

$$w + uH_{\hat{x}} = 0 \quad (z = -H), \quad (5)$$

$$u_t - (A_\nu u_z)_z = -g\zeta_{\hat{x}} \quad (\hat{x} = L). \quad (6)$$

This means that partial slip and no-flux boundaries are prescribed at the bed and a no-stress boundary is prescribed at the surface. At the landward boundary, the flow satisfies a profile following from a balance between inertia, barotropic pressure, and mixing, thus eliminating advection and baroclinic pressure at this boundary.

The model is closed by prescribing the depth-averaged flow in the form of a river-induced and tidal discharge. As a result, the amplitude and phase of the depth-integrated tidal velocity are prescribed functions of  $x$ .

$$B \int_{-H}^0 u dz = Q_r + Q_t \cos(\omega t + \psi(x)). \quad (7)$$

The following equation for salinity is used

$$s_t - (K_\nu s_z)_z = -us_{\hat{x}} - ws_z + \frac{1}{B}(BK_h s_{\hat{x}})_{\hat{x}}, \quad (8)$$

with boundary conditions

$$K_\nu s_z - K_h s_{\hat{x}} \zeta_{\hat{x}} = 0 \quad (z = 0), \quad (9)$$

$$K_\nu s_z + K_h s_{\hat{x}} H_{\hat{x}} = 0 \quad (z = -H), \quad (10)$$

$$\max_t s = s_{\text{sea}} \quad (\hat{x} = 0, z = -H), \quad (11)$$

$$s_{\hat{x}} = 0 \quad (\text{only tidal } \hat{x} = 0, z = -H), \quad (12)$$

$$s_t - (K_\nu s_z)_z = -u s_{\hat{x}} - w s_z \quad (\hat{x} = 0, \sigma \in (-1, 0]), \quad (13)$$

$$s = 0 \quad (\hat{x} = L, \sigma \in [-1, 0]). \quad (14)$$

In words, at the seaward boundary, the salinity at the bed is prescribed, while the equation without diffusion should hold in the rest of the water column. At the landward boundary, the salinity is assumed to vanish.

#### 1.1.1. Formal sigma transformation

In order to simplify the solution method, the width-averaged domain is transformed from  $(\hat{x}, z)$  coordinates to  $(x, \sigma)$  coordinates, where

$$x(\hat{x}, z) = \hat{x}, \quad (15)$$

$$\sigma(\hat{x}, z) = \frac{z}{H(x)}. \quad (16)$$

As a result,  $\sigma \in [-1, 0]$  (=bed to surface), regardless of the  $x$ -coordinate. The domain is thus rectangular. Using the transformation

$$\frac{\partial}{\partial \hat{x}} = \frac{\partial x}{\partial \hat{x}} \frac{\partial}{\partial x} + \frac{\partial \sigma}{\partial \hat{x}} \frac{\partial}{\partial \sigma} = \frac{\partial}{\partial x} - \sigma \frac{H_x}{H} \frac{\partial}{\partial \sigma}, \quad (17)$$

$$\frac{\partial}{\partial \hat{z}} = \frac{\partial x}{\partial \hat{z}} \frac{\partial}{\partial x} + \frac{\partial \sigma}{\partial \hat{z}} \frac{\partial}{\partial \sigma} = \frac{1}{H} \frac{\partial}{\partial \sigma}. \quad (18)$$

For the current study, we simplify the transformation by assuming that the terms involving  $H_x$  may be ignored. Furthermore, it is assumed that the surface slope does not affect the salinity profile. The equations for hydrodynamics (including the closure equation become

$$u_t - \frac{1}{H^2} A_\nu u_{\sigma\sigma} = -g\zeta_x - g\beta H \int_{\sigma}^0 s_x d\sigma', \quad (19)$$

$$\frac{1}{H} w_\sigma + \frac{1}{B} (Bu)_x = 0, \quad (20)$$

$$B \int_{-1}^0 u d\sigma = Q_r + Q_t \cos(\omega t + \psi(x)), \quad (21)$$

The boundary conditions read as

$$\frac{1}{H} A_\nu u_\sigma = 0 \quad (\sigma = 0), \quad (22)$$

$$\frac{1}{H} A_\nu u_\sigma = s_f u \quad (\sigma = -1), \quad (23)$$

$$w = 0 \quad (\sigma = -1), \quad (24)$$

$$u_t - \frac{1}{H^2} (A_\nu u_\sigma)_\sigma = -g\zeta_x \quad (x = L). \quad (25)$$

The transformed salinity equation reads

$$s_t - \frac{1}{H^2} (K_\nu s_\sigma)_\sigma + \frac{1}{H} w s_\sigma + u s_x - \frac{1}{B} (BK_h s_x)_x = 0, \quad (26)$$

with boundary conditions

$$\frac{1}{H} K_\nu s_\sigma = 0 \quad (\sigma = 0), \quad (27)$$

$$\frac{1}{H} K_\nu s_\sigma = 0 \quad (\sigma = -1), \quad (28)$$

$$\max_t s = s_{\text{sea}} \quad (x = 0, \sigma = -1), \quad (29)$$

$$s_x = 0 \quad (\text{only tidal } \hat{x} = 0, z = -H), \quad (30)$$

$$s_t - \frac{1}{H^2} (K_\nu s_\sigma)_\sigma = -u s_x - \frac{1}{H} w s_\sigma \quad (x = 0, \sigma \in (-1, 0]), \quad (31)$$

$$s = 0 \quad (x = L, \sigma \in [-1, 0]). \quad (32)$$

### 1.1.2. Turbulence model

We use the *KEFittedLead* model of iFlow with  $s_{f,0}$  as roughness parameter. Hence we use

$$A_\nu = \frac{1}{2} s_f H. \quad (33)$$

The eddy diffusivity is related to the eddy viscosity using a Prandtl-Schmidt number  $\sigma_\rho$  of 2.2, i.e.

$$Kv = \frac{A_\nu}{\sigma_\rho} \quad (34)$$

Throughout this memo we will use the relative roughness parameter  $R$ , which we define as

$$R = \frac{A_\nu}{s_f H}. \quad (35)$$

Using our turbulence closure,  $R$  is constant and has a value  $R = \frac{1}{2}$ .

## 1.2. Eigenfunction expansion

### 1.2.1. Horizontal velocity, salinity, and surface elevation

We will develop a numerical solution procedure for the fully non-linear equations. To this end, the velocity, salinity, and surface elevation are written as series expansions

$$u(x, \sigma, t) = \sum_{m=0}^{\infty} \sum_{q=0}^{\infty} (\beta_{mq}^R(x) \cos(q\omega t) + \beta_{mq}^I(x) \sin(q\omega t)) f_m(\sigma), \quad (36)$$

$$s(x, \sigma, t) = \sum_{m=0}^{\infty} \sum_{q=0}^{\infty} (\alpha_{mq}^R(x) \cos(q\omega t) + \alpha_{mq}^I(x) \sin(q\omega t)) g_m(\sigma), \quad (37)$$

$$\zeta(x, \sigma, t) = \sum_{q=0}^{\infty} (\zeta_q^R(x) \cos(q\omega t) + \zeta_q^I(x) \sin(q\omega t)). \quad (38)$$

Here,  $\beta^R$ ,  $\beta^I$ ,  $\alpha^R$ ,  $\alpha^I$ ,  $\zeta_q^R$ , and  $\zeta_q^I$  are real functions of  $x$  and the superscripts  $R$  and  $I$  indicate that these coefficients would be the real and imaginary part of complex variables  $\beta^R + i\beta^I$ ,  $\alpha^R + i\alpha^I$ , and  $\zeta_q^R + i\zeta_q^I$  when written in terms of complex exponentials. By definition  $\beta_{m,0}^I = 0$ ,  $\alpha_{m,0}^I = 0$ , and  $\zeta_0^I = 0 \forall m$ .

Many of the manipulations to these expansions below are similar for the cosine and sine components in time. Hence, we introduce a shorthand notation

$$u(x, \sigma, t) = \sum_{m=0}^{\infty} \sum_{q=-\infty}^{\infty} \beta_{mq}(x) f_m(\sigma) h_q(t), \quad (39)$$

$$s(x, \sigma, t) = \sum_{m=0}^{\infty} \sum_{q=-\infty}^{\infty} \alpha_{mq}(x) g_m(\sigma) h_q(t), \quad (40)$$

$$\zeta(x, t) = \sum_{q=-\infty}^{\infty} \zeta_q(x) h_q(t). \quad (41)$$

where

$$h_q(t) = \begin{cases} 1 & \text{if } q = 0, \\ \cos(q\omega t) & \text{if } q > 0, \\ \sin(q\omega t) & \text{if } q < 0. \end{cases} \quad (42)$$

Note that the sum over  $q$  is extended to negative integers. The sines are represented by negative  $q$ , while the cosines are represented by the positive  $q$ . These infinite sums over components in the vertical and in time are truncated as a finite number  $M$  and  $\pm q_{\max}$ , respectively.

The functions  $f$  and  $g$  are the eigenfunctions of the vertical mixing operator and read as

$$f_m = \cos(\lambda_m \sigma), \quad (43)$$

$$g_m = \cos(m\pi \sigma), \quad (44)$$

where the eigenvalues  $\lambda_m$  satisfy the equation

$$\lambda_m \tan(\lambda_m) = R^{-1}. \quad (45)$$

This implicit equation is solved using a numerical root-finding algorithm using that  $\lambda_m \in (m\pi - \frac{1}{2}\pi, m\pi + \frac{1}{2}\pi)$  for  $m = 0, 1, \dots$ . For the remainder of this memo we will assume  $\lambda_0 \neq 0$  (i.e.  $\lambda_0 > 0$ ). This is true as long as  $R < \infty$ , i.e.  $s_f H > 0$ .

### 1.2.2. Vertical velocity

The vertical velocity follows from the continuity equation and reads as

$$w(x, \sigma, t) = -H \sum_{m=0}^M \sum_{q=-Q}^Q \frac{1}{\lambda_m} \left( \beta_{mq,x}(x) + \frac{B_x}{B} \beta_{mq}(x) \right) (\sin(\lambda_m \sigma) + \sin(\lambda_m)) h_q(t). \quad (46)$$

### 1.2.3. Discharge

The river-induced and tidal discharge are additionally written as

$$Q_r + Q_t \cos(\omega t + \psi(x)) = \sum_{q=-1}^1 Q_q h_q(t), \quad (47)$$

where  $Q_0 = Q_r$ ,  $Q_{-1} = Q_t \cos(\psi(x))$ , and  $Q_1 = Q_t \sin(\psi(x))$ .

## 1.3. Galerkin forms

### 1.3.1. Water motion

**\***: Momentum equation The eigenfunction expansions are used to rewrite the momentum equation to

$$\begin{aligned} & \sum_{m=0}^M \sum_{n=0}^M \sum_{q=-q_{\max}}^{q_{\max}} \sum_{r=-q_{\max}}^{q_{\max}} \beta_{mq} \cos(\lambda_m \sigma) h_{q,t}(t) + \left( \frac{\lambda_m}{H} \right)^2 A_\nu \beta_{mq} \cos(\lambda_m \sigma) h_q(t) + \\ & \beta_{nr} \beta_{mq,x} \cos(\lambda_n \sigma) \cos(\lambda_m \sigma) h_q(t) h_r(t) + \frac{\lambda_n}{\lambda_m} \left( \beta_{mq,x} + \frac{B_x}{B} \beta_{mq} \right) \beta_{nr} \sin(\lambda_n \sigma) (\sin(\lambda_m \sigma) + \sin(\lambda_m)) h_q(t) h_r(t) \\ & = \sum_{q=-q_{\max}}^{q_{\max}} -g \zeta_{q,x} h_q(t) + \sum_{q=-q_{\max}}^{q_{\max}} g \beta H \left( \alpha_{0q,x} \sigma + \sum_{m=1}^M \frac{\alpha_{mq,x}}{m\pi} \sin(m\pi \sigma) \right) h_q(t). \end{aligned} \quad (48)$$

To derive this equation, it is used that

$$u_\sigma = \sum_{m=0}^M \sum_{q=-q_{\max}}^{q_{\max}} -\lambda_m \sin(\lambda_m \sigma) \beta_{mq} h_q(t), \quad (49)$$

$$\int_\sigma^0 s_x d\sigma' = \sum_{m=0}^M \sum_{q=-q_{\max}}^{q_{\max}} \int_\sigma^0 \alpha_{mq} \cos(m\pi \sigma') d\sigma' h_q(t), \quad (50)$$

$$= \sum_{q=-q_{\max}}^{q_{\max}} \left( -\alpha_{0q} \sigma + \sum_{m=1}^M \frac{\alpha_{mq}}{m\pi} \sin(m\pi \sigma') \Big|_\sigma^0 \right) h_q(t), \quad (51)$$

$$= \sum_{q=-q_{\max}}^{q_{\max}} - \left( \alpha_{0q} \sigma + \sum_{m=1}^M \frac{\alpha_{mq}}{m\pi} \sin(m\pi \sigma) \right) h_q(t). \quad (52)$$

To solve this equation we apply a Galerkin method to both the vertical and temporal dimension. Firstly focussing on the vertical dimension only, the equation and boundary conditions are multiplied by test functions of the type  $\cos(\lambda_k \sigma)$ ,  $k = 0, \dots, M$  and then integrated from  $\sigma = -1$  to 0. Using orthogonality of the cosines, the resulting equation for  $k = 0, \dots, M$  reads as (**using Einstein's summation convention**<sup>1</sup>)

$$\begin{aligned} & G_{1b,km} \beta_{mq} h_{q,t}(t) + \frac{A_\nu}{H} G_{1,km} \beta_{mq} h_q(t) + G_{2,kmn} \beta_{nr} \beta_{mq,x} h_q(t) h_r(t) + \\ & G_{3,kmn} \left( \beta_{mq,x} + \frac{B_x}{B} \beta_{mq} \right) \beta_{nr} h_q(t) h_r(t) = -g G_{4,k} \zeta_{q,x} h_q(t) + g \beta H G_{5,km} \alpha_{mq,x} h_q(t), \end{aligned} \quad (53)$$

where

$$G_{1,km} = \lambda_m^2 \int_{-1}^0 c_m c_k d\sigma, \quad (54)$$

$$G_{1b,km} = \int_{-1}^0 c_m c_k d\sigma, \quad (55)$$

$$G_{2,kmn} = \int_{-1}^0 c_n c_m c_k d\sigma, \quad (56)$$

$$G_{3,kmn} = \frac{\lambda_n}{\lambda_m} \int_{-1}^0 s_n (s_m + \sin(\lambda_m)) c_k d\sigma, \quad (\text{assuming } \lambda_m > 0) \quad (57)$$

$$G_{4,k} = \int_{-1}^0 c_k d\sigma, \quad (58)$$

$$G_{5,km} = \begin{cases} \int_{-1}^0 \sigma c_k d\sigma & \text{for } m = 0, \\ \frac{1}{m\pi} \int_{-1}^0 \bar{s}_m c_k d\sigma & \text{for } m > 0, \end{cases} \quad (59)$$

and,

$$c_k = \cos(\lambda_k \sigma), \quad (60)$$

$$s_k = \sin(\lambda_k \sigma), \quad (61)$$

$$\bar{c}_k = \cos(k\pi \sigma), \quad (62)$$

$$\bar{s}_k = \sin(k\pi \sigma). \quad (63)$$

Next, the Galerkin method is applied to the temporal dimension. To this end, the equation is multiplied by test functions  $h_s$  for  $s = -q_{\max}, \dots, q_{\max}$  and integrated from  $t = 0$  to 1. Again using orthogonality we find

$$\begin{aligned} & G_{1b,km} H_{2,sq} \beta_{mq} + \frac{A_\nu}{H} G_{1,km} H_{1,sq} \beta_{mq} + G_{2,kmn} H_{3,sqr} \beta_{nr} \beta_{mq,x} + \\ & G_{3,kmn} H_{3,sqr} \left( \beta_{mq,x} + \frac{B_x}{B} \beta_{mq} \right) \beta_{nr} = -g G_{4,k} H_{1,sq} \zeta_{q,x} + g \beta H G_{5,km} H_{1,sq} \alpha_{mq,x}, \end{aligned} \quad (64)$$

where

$$H_{1,sq} = \int_0^1 h_s(t) h_q(t) dt, \quad (65)$$

$$H_{2,sq} = \int_0^1 h_s(t) h_{q,t}(t) dt, \quad (66)$$

$$H_{3,sqr} = \int_0^1 h_s(t) h_q(t) h_r(t) dt. \quad (67)$$

**Boundary conditions:** The boundary conditions at the bed and surface are automatically satisfied by the choice of the eigenfunctions. The boundary condition at  $x = L$  rewrites to

$$G_{1b,km} H_{2,sq} \beta_{mq} + \frac{A_\nu}{H^2} G_{1,km} H_{1,sq} \beta_{mq} = -g G_{4,k} H_{1,sq} \zeta_{q,x}. \quad (68)$$

**\*: Closure equation** The closure equation is integrated in the  $x$  direction to find that the cross-sectionally integrated velocity equals the discharge everywhere in the domain. Using the eigenfunctions, this expression rewrites to

$$BH \int_{-1}^0 u d\sigma = BH \sum_{m=0}^M \sum_{q=-q_{\max}}^{q_{\max}} \beta_{mq} \int_0^1 \int_{-1}^0 \cos(\lambda_m \sigma) d\sigma h_q(t) dt, \quad (69)$$

$$= -BH \sum_{m=0}^M \sum_{q=-q_{\max}}^{q_{\max}} \frac{\beta_{mq}}{\lambda_m} \sin(\lambda_m) H_{1,q} = \sum_{q=-1}^1 H_{1,sq} Q_q dt. \quad (70)$$

For consistency of notation, this is written as (**using Einstein's notation**)

$$BHG_{9,m} H_{1,sq} \beta_{mq} = H_{1,sq} Q_q,$$



where

$$G_{9,m} = \frac{\sin(\lambda_m)}{\lambda_m}.$$

### 1.3.2. Salinity

**Salinity equation:** Substituting the series expression for  $u$ ,  $w$ , and  $s$  in the salinity equation, we obtain

$$\begin{aligned} \sum_{m=0}^M \sum_{n=0}^M \sum_{q=-q_{\max}}^{q_{\max}} \sum_{r=-q_{\max}}^{q_{\max}} \alpha_{mq} \cos(m\pi\sigma) h_{q,t}(t) + \frac{K_\nu}{H^2} (m\pi)^2 \alpha_{mq} \cos(m\pi\sigma) h_q(t) + \\ \alpha_{mq,x} \beta_{nr} \cos(m\pi\sigma) \cos(\lambda_n\sigma) h_q(t) h_r(t) + \\ \frac{m\pi}{B} \frac{(B\beta_{nr})_x}{\lambda_n} \alpha_{mq} \sin(m\pi\sigma) (\sin(\lambda_n\sigma) + \sin(\lambda_n)) h_q(t) h_r(t) = \\ \sum_{m=0}^M \sum_{q=-q_{\max}}^{q_{\max}} \frac{1}{B} (BK_h \alpha_{mq,x})_x \cos(m\pi\sigma) h_q(t), \end{aligned} \quad (71)$$

This is simplified using a Galerkin method with test functions  $\cos(k\pi\sigma)$  for  $k = 0, \dots, M$  and  $h_s(t)$  for  $s = -q_{\max}, \dots, q_{\max}$ , i.e. multiplying the equation with the test functions and integrating from  $\sigma = -1$  to 0 and  $t = 0$  to 1, we find (**using Einstein's notation**)

$$\begin{aligned} G_{6b,km} H_{2,sq} \alpha_{mq} + \frac{K_\nu}{H^2} G_{6,km} H_{1,sq} \alpha_{mq} + G_{7,kmn} H_{3,sqr} \alpha_{mq,x} \beta_{nr} + \frac{1}{B} G_{8,kmn} H_{3,sqr} (B\beta_{nr})_x \alpha_{mq} = \\ \frac{1}{B} G_{6b,km} H_{1,sq} (BK_h \alpha_{mq,x})_x, \end{aligned} \quad (72)$$

where

$$G_{6,km} = (m\pi)^2 \int_{-1}^0 \bar{c}_m \bar{c}_k d\sigma, \quad (73)$$

$$G_{6b,km} = \int_{-1}^0 \bar{c}_m \bar{c}_k d\sigma, \quad (74)$$

$$G_{7,kmn} = \int_{-1}^0 c_n \bar{c}_m \bar{c}_k d\sigma, \quad (75)$$

$$G_{8,kmn} = \frac{m\pi}{\lambda_n} \int_{-1}^0 (s_n + \sin(\lambda_n)) \bar{s}_m \bar{c}_k, \quad (\text{assuming } \lambda_n > 0). \quad (76)$$

**Boundary conditions:** The boundary condition at the seaward boundary for  $\sigma \in (-1, 0]$  is similar to the salinity equation but without the horizontal dispersion term. In Galerkin form this is applied for  $k = 1, \dots, M$  (i.e. all but the first element) and all  $q$ . It reads as (**using Einstein's notation**)

$$G_{6b,km} H_{2,sq} \alpha_{mq} + \frac{K_\nu}{H^2} G_{6,km} H_{1,sq} \alpha_{mq} + G_{7,kmn} H_{3,sqr} \alpha_{mq,x} \beta_{nr} + \frac{1}{B} G_{8,kmn} H_{3,sqr} (B\beta_{nr})_x \alpha_{mq} = 0. \quad (77)$$

The equation for  $k = 0$  for the subtidal component  $q = 0$  is replaced by the condition  $\max_t s = s_{\text{sea}}$  at  $x = 0$ ,  $\sigma = -1$ . Using the eigenfunction expansion this reads as

$$\cos(m\pi) h_q(\hat{t}) \alpha_{mq} = s_{\text{sea}}, \quad (78)$$

where  $\hat{t}$  the time such that  $s(0, -1, \hat{t}) = \max_t s(0, -1, t)$ . For the other temporal components, a Neumann condition is implemented, i.e.

$$\cos(m\pi) H_{1,sq} \alpha_{mq,x} = 0. \quad (79)$$

The landward boundary condition  $s = 0$  is rewritten to Galerkin form

$$G_{6b,mk} H_{1,sq} \alpha_{mq} = 0. \quad (80)$$

### 1.4. Newton-Raphson solution method

The Galerkin forms of the equations provide a set of  $(M+1)(2q_{\max}+1)$  non-linear differential equations for the horizontal velocity coefficients,  $(M+1)(2q_{\max}+1)$  non-linear differential equations for the salinity coefficients

and  $2q_{\max} + 1$  linear differential equations for  $\zeta_x$  coefficients. It is chosen to solve the entire set of  $(2(M + 1) + 1)(2q_{\max} + 1)$  equations together in terms of a state vector  $\hat{\mathbf{y}}$  defined as

$$\hat{\mathbf{y}} = [\hat{\alpha}, \hat{\beta}, \hat{\zeta}_{\mathbf{x}}]^t, \quad (81)$$

where

$$\hat{\alpha} = [\alpha_{0,-q_{\max}}, \dots, \alpha_{0,q_{\max}}, \dots, \alpha_{M,-q_{\max}}, \dots, \alpha_{M,q_{\max}}]^T, \quad (82)$$

$$\hat{\beta} = [\beta_{0,-q_{\max}}, \dots, \beta_{0,q_{\max}}, \dots, \beta_{M,-q_{\max}}, \dots, \beta_{M,q_{\max}}]^T, \quad (83)$$

$$\hat{\zeta}_{\mathbf{x}} = [\zeta_{-q_{\max},x}, \dots, \zeta_{q_{\max},x}]^T. \quad (84)$$

These vectors are functions of  $x$ . The equations are solved using a finite volume method on a grid in the along-channel direction. To this end, the solution is discretised to vectors  $\hat{\mathbf{y}}_j = \hat{\mathbf{y}}(x_j)$ , resulting in a total vector  $\mathbf{y}$  defined as

$$\mathbf{y} = [\hat{\mathbf{y}}_0, \hat{\mathbf{y}}_1, \dots, \hat{\mathbf{y}}_J]^T. \quad (85)$$

As the system of equations is non-linear, a Newton-Raphson method is used to solve the equations and denoted as  $\mathbf{y}^i$  with iteration number  $i = 0, 1, \dots$ .

The system can be solved efficiently because the Galerkin form presented above results in a mode splitting between  $x$ ,  $z$ , and  $t$ . All  $z$  dependency is in the set  $G_i$  ( $i = 0, \dots, 9, 6b$ ) and all the time dependency is in the set  $H_i$  ( $i = 1, 2, 3$ ). These sets do not depend on  $x$  or on the unknowns  $\mathbf{y}$ . Hence, the expressions for  $G_i$ ,  $H_i$  only need to be computed once for all combinations of  $k$ ,  $m$ ,  $n$  and  $s$ ,  $q$ ,  $r$ . As these computations involve simple integrals of sines and cosines, the results are computed analytically. All  $x$ -dependency is in the Galerkin equations, which do not further depend on  $z$  and  $t$ . A minor exception is the seaward boundary condition, which is treated in more detail below.

#### 1.4.1. Newton-Raphson

The Newton-Raphson iteration for some non-linear equation of the form  $F(\mathbf{y}) = \mathbf{0}$  reads as

$$\mathbf{y}^{i+1} = \mathbf{y}^i - \left( \frac{\partial F(\mathbf{y}^i)}{\partial \mathbf{y}} \right)^{-1} F(\mathbf{y}^i) \quad (86)$$

Here,  $F$  consists of the three sets of equations (the momentum equation, salinity equation and closure equation) that we will call  $F_1$ ,  $F_2$ , and  $F_3$ . The equations are repeated below. The notation is changed slightly, writing derivatives of state variables are written using  $\Delta$  and  $\nabla$ :

$$F_1 = G_{6b,km} H_{2,sq} \alpha_{mq} + \frac{K_\nu}{H^2} G_{6,km} H_{1,sq} \alpha_{mq} + G_{7,kmn} H_{3,sqr} \Delta \alpha_{mq} \beta_{nr} + \frac{1}{B} G_{8,kmn} H_{3,sqr} (B_x \beta_{nr} + B \Delta \beta_{nr}) \alpha_{mq} - \frac{1}{B} G_{6b,km} H_{1,sq} ((BK_h)_x \Delta \alpha_{mq} + BK_h \nabla \alpha_{mq}), \quad (87)$$

$$F_2 = G_{1b,km} H_{2,sq} \beta_{mq} + \frac{A_\nu}{H^2} G_{1,km} H_{1,sq} \beta_{mq} + G_{2,kmn} H_{3,sqr} \beta_{nr} \Delta \beta_{mq} + G_{3,kmn} H_{3,sqr} \left( \Delta \beta_{mq} + \frac{B_x}{B} \beta_{mq} \right) \beta_{nr} + g G_{4,k} H_{1,sq} \zeta_{q,x} - g \beta H G_{5,km} H_{1,sq} \Delta \alpha_{mq}, \quad (88)$$

$$F_3 = B H G_{9,m} H_{1,sq} \beta_{mq} + H_{1,sq} Q_q, \quad (89)$$

with boundary conditions

$$\cos(m\pi) h_q(\hat{t}) \alpha_{mq} = s_{\text{sea}} \quad \text{at } j = 0, s = 0, k = 0, \quad (90)$$

$$\cos(m\pi) H_{1,sq} \Delta \alpha_{mq} = 0 \quad \text{at } j = 0, s \neq 0, k = 0, \quad (91)$$

$$G_{6b,km} H_{2,sq} \alpha_{mq} + \frac{K_\nu}{H^2} G_{6,km} H_{1,sq} \alpha_{mq} + G_{7,kmn} H_{3,sqr} \Delta \alpha_{mq} \beta_{nr} + \frac{1}{B} G_{8,kmn} H_{3,sqr} \Delta (B \beta_{nr}) \alpha_{mq} = 0 \quad \text{at } j = 0, k = 1, \dots, M, \quad (92)$$

$$G_{6b,km} H_{1,sq} \alpha_{mq} = 0 \quad \text{at } j = J, k = 0, \dots, M, \quad (93)$$

$$G_{1b,km} H_{2,sq} \beta_{mq} + \frac{A_\nu}{H^2} G_{1,km} H_{1,sq} \beta_{mq} = -g G_{4,k} H_{1,sq} \zeta_{q,x}. \quad \text{at } x = L, k = 0, \dots, M. \quad (94)$$

#### 1.4.2. Notation of $\mathbf{F}$ and Jacobian

The vector equation  $F$  can alternatively be written as

$$F(\hat{\mathbf{y}}) = M \hat{\mathbf{y}} - \hat{\mathbf{r}} = \begin{bmatrix} M_{1\hat{\alpha}} & M_{1\hat{\beta}} & M_{1\hat{\zeta}_{\mathbf{x}}} \\ M_{2\hat{\alpha}} & M_{2\hat{\beta}} & M_{2\hat{\zeta}_{\mathbf{x}}} \\ M_{3\hat{\alpha}} & M_{3\hat{\beta}} & M_{3\hat{\zeta}_{\mathbf{x}}} \end{bmatrix} \hat{\mathbf{y}} - \begin{bmatrix} \hat{\mathbf{r}}_1 \\ \hat{\mathbf{r}}_2 \\ \hat{\mathbf{r}}_3 \end{bmatrix}. \quad (95)$$

The Jacobian of this equation with respect to the state variable  $\hat{\mathbf{y}}$  can be written as

$$\frac{\partial F}{\partial \hat{\mathbf{y}}} = \begin{bmatrix} \frac{\partial F_1}{\partial \hat{\alpha}} & \frac{\partial F_1}{\partial \hat{\beta}} & \frac{\partial F_1}{\partial \hat{\zeta}_{\mathbf{x}}} \\ \frac{\partial F_2}{\partial \hat{\alpha}} & \frac{\partial F_2}{\partial \hat{\beta}} & \frac{\partial F_2}{\partial \hat{\zeta}_{\mathbf{x}}} \\ \frac{\partial F_3}{\partial \hat{\alpha}} & \frac{\partial F_3}{\partial \hat{\beta}} & \frac{\partial F_3}{\partial \hat{\zeta}_{\mathbf{x}}} \end{bmatrix} \quad (96)$$

The terms  $M_{\dots}$  and the partial derivatives can be easily derived from the equations when viewing the differential operators as linear operators that can be specified without knowing the variable to take the derivative of. We can then write:

$$\begin{aligned} \frac{\partial \Delta y}{\partial y} &= \Delta, \\ \frac{\partial \nabla y}{\partial y} &= \nabla. \end{aligned}$$

It follows that

$$\begin{aligned} M_{1\hat{\alpha}} &= G_{6b,km} H_{2,sq} + \frac{K_{\nu}}{H^2} G_{6,km} H_{1,sq} + G_{7,kmn} H_{3,sqr} (\beta_{nr} \Delta + B \Delta \beta_{nr}) \\ &\quad - \frac{1}{B} G_{6b,km} H_{1,sq} ((BK_h)_x \Delta + BK_h \nabla), \end{aligned} \quad (97)$$

$$M_{1\hat{\beta}} = \frac{1}{B} G_{8,kmn} H_{3,sqr} \alpha_{mq} (B_x + B \Delta), \quad (98)$$

$$M_{1\hat{\zeta}_{\mathbf{x}}} = \mathbf{0}, \quad (99)$$

$$\frac{\partial F_1}{\partial \hat{\alpha}} = M_{1\hat{\alpha}} + \frac{1}{B} G_{8,kmn} (B_x \beta_n + B \Delta \beta_n), \quad (100)$$

$$\frac{\partial F_1}{\partial \hat{\beta}} = M_{1\hat{\beta}} + G_{7,kmn} \Delta \alpha_m, \quad (101)$$

$$\frac{\partial F_1}{\partial \hat{\zeta}_{\mathbf{x}}} = \mathbf{0}. \quad (102)$$

$$M_{2\hat{\alpha}} = -g\beta H G_{5,km} H_{1,sq} \Delta, \quad (103)$$

$$M_{2\hat{\beta}} = G_{1b,km} H_{2,sq} + \frac{A_{\nu}}{H^2} G_{1,km} H_{1,sq} + G_{2,kmn} H_{3,sqr} \beta_{nr} \Delta + G_{3,kmn} H_{3,sqr} \left( \Delta \beta_{mq} + \frac{B_x}{B} \beta_{mq} \right), \quad (104)$$

$$M_{2\hat{\zeta}_{\mathbf{x}}} = g G_{4,k} H_{1,sq}, \quad (105)$$

$$\frac{\partial F_2}{\partial \hat{\alpha}} = M_{2\hat{\alpha}}, \quad (106)$$

$$\frac{\partial F_2}{\partial \hat{\beta}} = M_{2\hat{\beta}} + G_{2,kmn} H_{3,sqr} \Delta \beta_{mq} + G_{3,kmn} H_{3,sqr} \beta_{nr} \left( \Delta + \frac{B_x}{B} \right), \quad (107)$$

$$\frac{\partial F_2}{\partial \hat{\zeta}_x} = M_{2\hat{\zeta}_x}. \quad (108)$$

$$\mathbf{M}_{3\alpha} = \mathbf{0}, \quad (109)$$

$$\mathbf{M}_{3\beta} = B H G_{9,m}, \quad (110)$$

$$\mathbf{M}_{3\zeta_x} = \mathbf{0}, \quad (111)$$

$$\frac{\partial F_3}{\partial \alpha} = \mathbf{0}, \quad (112)$$

$$\frac{\partial F_3}{\partial \beta} = B H G_{9,m}, \quad (113)$$

$$\frac{\partial F_3}{\partial \zeta_x} = \mathbf{0}. \quad (114)$$

$$\hat{\mathbf{r}}_1 = \mathbf{0}, \quad (115)$$

$$\hat{\mathbf{r}}_2 = H_{1,sq} Q_q, \quad (116)$$

$$\hat{\mathbf{r}}_3 = \mathbf{0}. \quad (117)$$

#### 1.4.3. Boundary condition $\max_t s = s_{\text{sea}}$

The boundary conditions may be trivially written in a similar form. Only the boundary condition  $\max_t s = s_{\text{sea}}$  is non-linear and requires some more attention. To implement this condition in the Newton-Raphson solver, this condition is written as

$$F(\hat{\alpha}, \hat{t}(\hat{\alpha})) = 0,$$

where  $\hat{t}(\hat{\alpha})$  is the time such that  $\max_t(s(0, -1, t)) = s(0, -1, \hat{t})$ , which depends on  $\hat{\alpha}$ . Concretely, this function  $F$  is given by (using Einstein's notation)

$$F = \alpha_{mq} h_q(\hat{t}) \cos(m\pi) - s_{\text{sea}} \quad (118)$$

The time  $\hat{t}$  is the global maximum of  $F$ , which is a solution of  $g = F_{\hat{t}}(\alpha, \hat{t}) = 0$ . To find the global maximum,  $F$  is first computed as a discrete time series on a finite number of time instances. From this, an estimate of  $\hat{t}$  close to the global maximum is selected. This is then used as an initial estimate in an iterative solver solving for  $g = 0$ . We use a pre-implemented zero-finder in Scipy.

The Newton-Raphson solver requires the total derivative of  $F$  with respect to each element of the vector  $\hat{\alpha}$ :  $\alpha_{mq}$ , which reads as

$$\frac{dF}{d\alpha_{mq}} = \frac{\partial F}{\partial \alpha_{mq}} + \frac{\partial F}{\partial \hat{t}} \frac{\partial \hat{t}}{\partial \alpha_{mq}}. \quad (119)$$

To find  $\frac{\partial \hat{t}}{\partial \alpha_{mq}}$  note that

$$\frac{dg}{d\alpha_{mq}} = \frac{\partial g}{\partial \alpha_{mq}} + \frac{\partial g}{\partial \hat{t}} \frac{\partial \hat{t}}{\partial \alpha_{mq}} = 0. \quad (120)$$

This expression should equal zero as, by the definition of  $\hat{t}$ ,  $g = 0$  for all  $\alpha_{mq}$ . Re-arranging this expression yields

$$\frac{\partial \hat{t}}{\partial \alpha_{mq}} = - \left( \frac{\partial g}{\partial \hat{t}} \right)^{-1} \frac{\partial g}{\partial \alpha_{mq}}. \quad (121)$$

Hence, using

$$\frac{\partial F}{\partial \alpha_{mq}} = h_q(\hat{t}) \cos(m\pi), \quad (122)$$

$$\frac{\partial F}{\partial \hat{t}} = \alpha_{mq} h_{q,\hat{t}}(\hat{t}) \cos(m\pi), \quad (123)$$

$$\frac{\partial g}{\partial \alpha_{mq}} = h_{q,\hat{t}}(\hat{t}) \cos(m\pi), \quad (124)$$

$$\frac{\partial g}{\partial \hat{t}} = \alpha_{mq} h_{q,\hat{t}\hat{t}}(\hat{t}) \cos(m\pi), \quad (125)$$

it follows that

$$\frac{dF}{d\alpha_{mq}} = \cos(m\pi) \left( h_q(\hat{t}) - h_{q,\hat{t}}(\hat{t}) \frac{\alpha_{nr} h_{r,\hat{t}}(\hat{t}) \cos(n\pi)}{\alpha_{nr} h_{r,\hat{t}\hat{t}}(\hat{t}) \cos(n\pi)} \right). \quad (126)$$

This expression holds assuming  $\hat{t}$  changes gradually from one iteration to the next; sudden jumps are not captured accurately using this expression.

## 2. Dimensionless form

We may demonstrate formally that the model used here depends on 7 dimensionless numbers. To this end, we first repeat the model equations

$$u_x + w_z = 0, \quad (127a)$$

$$u_t + uu_x + wu_z = -g\zeta_x + A_\nu u_{zz} - g\beta \int_z^0 s_x dz', \quad (127b)$$

$$\frac{1}{H} \int_{-H}^0 u dz = -\mathcal{U}_r + \mathcal{U}_t \cos(\omega t), \quad (127c)$$

$$s_t + us_x + ws_z = K_\nu s_{zz} + K_h s_{xx}. \quad (127d)$$

We rewrite the model equations to dimensionless quantities, denoting dimensionless numbers using a  $*$  and defining the following quantities

$$\begin{aligned} x &= x^* L, & u &= u^* U, \\ z &= z^* H, & w &= u^* W, \\ \zeta &= \zeta^* A, & s &= s^* s_{\text{sea}}, \\ t &= t^* \omega^{-1}. \end{aligned}$$

Using these expressions, the equations are rewritten as

$$\frac{U}{L} u_{x^*}^* + \frac{W}{H} w_{z^*}^* = 0, \quad (128a)$$

$$U\omega u_{t^*}^* + \frac{UU}{L} u^* u_{x^*}^* + \frac{WU}{H} w^* u_{z^*}^* = -\frac{gA}{L} \zeta_{x^*}^* + \frac{A_\nu U}{H^2} u_{z^* z^*}^* - \frac{g\beta H s_{\text{sea}}}{L} \int_{z^*}^0 s_{x^*}^* dz', \quad (128b)$$

$$U \int_{-1}^0 u^* dz^* = -\mathcal{U}_r + \mathcal{U}_t \cos(t^*), \quad (128c)$$

$$s_{\text{sea}} \omega s_{t^*}^* + \frac{U s_{\text{sea}}}{L} u^* s_{x^*}^* + \frac{W s_{\text{sea}}}{H} w^* s_{z^*}^* = \frac{K_\nu s_{\text{sea}}}{H^2} s_{z^* z^*}^* + \frac{K_h s_{\text{sea}}}{L^2} s_{x^* x^*}^*. \quad (128d)$$

We see from Eq. (128a) that  $\frac{U}{L} = \frac{W}{H}$  and from Eq. (128c) that  $U$  is either  $\mathcal{U}_r$  or  $\mathcal{U}_t$ , depending on the frequency considered. Hence, we may say that  $\frac{U}{c}$  either scales with  $F_r$  or  $F_t$  depending on the frequency. Below we simplify the notation by writing only  $\frac{U}{c} = F_r$ . The remaining two expressions are rewritten using  $\frac{U}{L} = \frac{W}{H}$  and are divided by  $U\omega$  and  $s_{\text{sea}}\omega$ , respectively, to yield

$$u_{t^*}^* + \frac{U}{L\omega} u^* u_{x^*}^* + \frac{U}{L\omega} w^* u_{z^*}^* = -\frac{gA}{UL\omega} \zeta_{x^*}^* + \frac{A_\nu}{H^2\omega} u_{z^* z^*}^* - \frac{g\beta H s_{\text{sea}}}{UL\omega} \int_{z^*}^0 s_{x^*}^* dz', \quad (129a)$$

$$s_{t^*}^* + \frac{U}{L\omega} u^* s_{x^*}^* + \frac{U}{L\omega} w^* s_{z^*}^* = \frac{K_\nu}{H^2\omega} s_{z^* z^*}^* + \frac{K_h}{L^2\omega} s_{x^* x^*}^*. \quad (129b)$$

We will consider each of the remaining factors in these equations and rewrite them to our dimensionless numbers below:

$$\frac{U}{L\omega} = \frac{U}{c} \frac{c}{L\omega} = \frac{U}{c} \frac{c^2}{K_h \omega} \frac{K_h}{cL} = F_r St Ra \mathcal{L}_s^{-1}, \quad (130a)$$

$$\frac{gA}{UL\omega} = \frac{K_h}{Lc} \frac{c}{U} \frac{gA}{K_h \omega} = \frac{K_h}{Lc} \frac{c}{U} \frac{c^2}{K_h \omega} \frac{gA}{c^2} = \mathcal{L}_s^{-1} F_r^{-1} St Ra \frac{gA}{c^2} \quad (130b)$$

$$\frac{A_\nu}{H^2\omega} = St, \quad (130c)$$

$$\frac{g\beta H s_{\text{sea}}}{UL\omega} = \frac{c^2}{UL\omega} = \frac{c}{U} \frac{c}{L\omega} = F_r^{-1} St Ra \mathcal{L}_s^{-1}, \quad (130d)$$

$$\frac{K_\nu}{H^2\omega} = St \sigma_\rho^{-1}, \quad (130e)$$

$$\frac{K_h}{L^2\omega} = \frac{K_h^2}{L^2 c^2} \frac{c^2}{K_h \omega} = \mathcal{L}_s^{-2} St Ra, \quad (130f)$$

where  $F_r$  may be interchanged with  $F_t$  everywhere. The only remaining term is  $\frac{gA}{c^2}$ , which includes the scale  $A$  of  $\zeta$ . This term must be related to at least one of the other terms in the momentum equation, although a-priori it is unclear how. Hence, this term will not introduce any other dimensionless parameters.

Looking closer at the boundary conditions, almost all boundary conditions contain only one term that equals zero and do not have to be considered further. The salinity boundary condition at  $x = 0$  contains multiple terms

but they are all terms that also appear in the salt equation and hence scale with the dimensionless parameters. Only the bottom boundary condition for the velocity remains. This reads as

$$A_\nu u_z - s_f u = 0. \quad (131)$$

In terms of dimensionless numbers this rewrites to

$$\frac{A_\nu}{s_f H} u_{z^*}^* - u^* = 0, \quad (132)$$

where we defined  $\frac{A_\nu}{s_f H} = R$ .

### 3. Algorithm for defining regimes

We have identified seven regimes with each their specific characteristics. A single model experiment is not necessarily in one regime, but may display mixed characteristics of two, three, or even four regimes. Hence, we assign each model experiment 7 scores, with score  $i$  representing to what extent the experiment displays characteristics of regime  $i$  ( $i = 1, \dots, 7$ ). The 7 scores are scaled to add up to 100%. The color plot in Fig. 1 in the main text is generated by assigning an rgb-color to each of the 7 regimes and defining the color of a model experiment as the score-weighted average of these colors in term of their rgb-values.

We will denote the transport contributions as  $T_{\text{Mi-contr1-contr2}}$ , where  $i = 0, 1$  and denotes whether we consider interactions between subtidal velocity and salinity ( $i = 0$ ) or tidal velocity and salinity ( $i = 1$ , representing all resolved tidal components) and 'contr1', 'contr2' are the contributions from velocity and salinity and v.v. Furthermore,  $T_{\text{dispersion}} = -K_h \bar{s}_x^0$  and  $T_{\text{tidal mean}}$  is the interaction between the tidal velocity and  $\bar{s}^1$ . We define the following auxiliary vectors

$$\text{ImportScore} = \frac{1}{\hat{T}_{\text{dispersion}} + \hat{T}_{\text{M0-gc-gc}} + \hat{T}_{\text{M1-tide-tide}} + \hat{T}_{\text{tidal mean}}} [\hat{T}_{\text{dispersion}}, \hat{T}_{\text{M0-gc-gc}}, \hat{T}_{\text{M0-tide-tide}}, \hat{T}_{\text{tidal mean}}]^T,$$

where  $\hat{T}$  indicates the transport averaged over the estuary up to the  $s = 1$  psu line (defined as  $x = L_s$ ). This vector measures the relative importance of four different import processes averaged over the estuary. Next we define

$$\begin{aligned} \text{FrontScore} = [1 - \text{ImportScore}_2 \frac{T_{\text{M0-adv-gc}}(x_f)}{T_{\text{dispersion}}(x_f) + T_{\text{tidal mean}}(x_f) + T_{\text{M0-adv-gc}}(x_f)}, \\ \text{ImportScore}_2 \frac{T_{\text{M0-adv-gc}}}{T_{\text{dispersion}}(x_f) + T_{\text{tidal mean}}(x_f) + T_{\text{M0-adv-gc}}(x_f)}]^T, \end{aligned}$$

which determines the relative importance of the gravitational-gravitational and advection-gravitational interaction at the salt intrusion front  $x = x_f$ , defined as the location where this advection-gravitational transport attains its maximum. We define a similar score measuring the relative importance of subtidal advection-gravitational interactions over the entire estuary

$$\text{AdvM0Score} = [1 - \text{ImportScore}_2 \min \left( \frac{\hat{T}_{\text{M0-adv-gc}}}{\hat{T}_{\text{M0-gc-gc}}} \right), \text{ImportScore}_2 \min \left( \frac{\hat{T}_{\text{M0-adv-gc}}}{\hat{T}_{\text{M0-gc-gc}}} \right)]^T.$$

Finally, we define a similar measure that distinguishes whether tidal advective processes are also important.

$$\text{AdvM1Score} = [1 - \text{AdvM0Score}_2 \min \left( \frac{\hat{T}_{\text{M1-adv-gc}}}{\hat{T}_{\text{M0-adv-gc}}} \right), \text{AdvM0Score}_2 \min \left( \frac{\hat{T}_{\text{M1-adv-gc}}}{\hat{T}_{\text{M0-adv-gc}}} \right)]^T.$$

The scores are then awarded as

$$\begin{aligned} \text{Score}_1 &= \text{ImportScore}_1 \text{FrontScore}_1 \text{AdvM0Score}_1 \text{AdvM1Score}_1, \\ \text{Score}_2 &= \text{ImportScore}_2 \text{FrontScore}_1 \text{AdvM0Score}_1 \text{AdvM1Score}_1, \\ \text{Score}_3 &= \text{FrontScore}_2 \text{AdvM0Score}_1 \text{AdvM1Score}_1, \\ \text{Score}_4 &= \text{AdvM0Score}_2 \text{AdvM1Score}_1, \\ \text{Score}_5 &= \text{ImportScore}_3 \text{FrontScore}_1 \text{AdvM0Score}_1 \text{AdvM1Score}_1, \\ \text{Score}_6 &= \text{ImportScore}_4 \text{FrontScore}_1 \text{AdvM0Score}_1 \text{AdvM1Score}_1, \\ \text{Score}_7 &= \text{AdvM1Score}_2. \end{aligned}$$

The sum of these scores is already normalised to 1 according to this definition.

#### 4. Analytical approximations to the tidal mean transport regime

##### 4.1. Approximation using $M_2$ and $M_4$ tides

There are two problems concerning the simple approximation of the tidal mean transport regime provided in the appendix of the main text. Firstly, the example of the tidal mean transport regime in the main text shows that the along-channel salinity profiles are not at all exponential, which is implied by the simple approximation. Secondly, the simpler solution does not satisfy the boundary condition  $\bar{s}_{1,x} = 0$  at  $x = 0$ . The first problem may be solved by including higher harmonics, especially the  $M_4$  salinity. The second problem may be solved by retaining the dispersive terms in the equation, allowing the formation of a boundary layer near  $x = 0$ . We assume the same form of the tidal salinity and write the tidal salinity in terms of a real and imaginary part of the complex amplitude, resulting in a system of five equations that reads as

$$\begin{aligned} -\mathcal{U}_r \bar{s}_{0,x} + \frac{1}{2} \mathcal{U}_t \bar{s}_{1,x}^R - K_h \bar{s}_{0,xx} &= 0, \\ -\omega \bar{s}_1^I - \mathcal{U}_r \bar{s}_{1,x}^R + \mathcal{U}_t \bar{s}_{0,x} + \frac{1}{2} \mathcal{U}_t \bar{s}_{2,x}^R - K_h \bar{s}_{1,xx}^R &= 0, \\ \omega \bar{s}_1^R - \mathcal{U}_r \bar{s}_{1,x}^I - \frac{1}{2} \mathcal{U}_t \bar{s}_{2,x}^I - K_h \bar{s}_{1,xx}^I &= 0, \\ -2\omega \bar{s}_2^I - \mathcal{U}_r \bar{s}_{2,x}^R + \frac{1}{2} \mathcal{U}_t \bar{s}_{1,x}^R - K_h \bar{s}_{2,xx}^R &= 0, \\ 2\omega \bar{s}_2^R - \mathcal{U}_r \bar{s}_{2,x}^I + \frac{1}{2} \mathcal{U}_t \bar{s}_{1,x}^I - K_h \bar{s}_{2,xx}^I &= 0. \end{aligned}$$

Next, we define  $\hat{\zeta}_i^R = \hat{s}_{i,x}^R$  and  $\hat{\zeta}_i^I = \hat{s}_{i,x}^I$  for  $i = 1, 2$ . Writing  $\underline{y} = [\hat{s}_1^R, \hat{s}_1^I, \hat{s}_2^R, \hat{s}_2^I, \hat{\zeta}_1^R, \hat{\zeta}_1^I, \hat{\zeta}_2^R, \hat{\zeta}_2^I, \bar{s}_{0,x}]^T$  and re-ordering the above equation yields

$$\begin{bmatrix} \hat{s}_1^R \\ \hat{s}_1^I \\ \hat{s}_2^R \\ \hat{s}_2^I \\ \hat{\zeta}_1^R \\ \hat{\zeta}_1^I \\ \hat{\zeta}_2^R \\ \hat{\zeta}_2^I \\ \bar{s}_{0,x} \end{bmatrix}_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\omega/K_h & 0 & 0 & -\mathcal{U}_r/K_h & 0 & \frac{1}{2} \hat{u}_1^R/K_h & 0 & \hat{u}_1^R/K_h \\ \omega/K_h & 0 & 0 & 0 & 0 & -\mathcal{U}_r/K_h & 0 & \frac{1}{2} \hat{u}_1^R/K_h & 0 \\ 0 & 0 & 0 & -2\omega/K_h & \frac{1}{2} \hat{u}_1^R/K_h & 0 & -\mathcal{U}_r/K_h & 0 & 0 \\ 0 & 0 & 2\omega/K_h & 0 & 0 & \frac{1}{2} \hat{u}_1^R/K_h & 0 & -\mathcal{U}_r/K_h & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \hat{u}_1^R/K_h & 0 & 0 & 0 & -\mathcal{U}_r/K_h \end{bmatrix} \begin{bmatrix} \hat{s}_1^R \\ \hat{s}_1^I \\ \hat{s}_2^R \\ \hat{s}_2^I \\ \hat{\zeta}_1^R \\ \hat{\zeta}_1^I \\ \hat{\zeta}_2^R \\ \hat{\zeta}_2^I \\ \bar{s}_{0,x} \end{bmatrix} \quad (133)$$

One of the eigenvalues of this system of equations is  $\lambda = -\frac{\mathcal{U}_r}{K_h}$ , i.e. the eigenvalue corresponding to the dispersive regime. Other eigenvalues follow as the roots of an eighth-order polynomial and need to be computed numerically. To satisfy the condition of vanishing salinity for  $x \rightarrow \infty$ , we only retain the eigenvalues with negative real part. Depending on the values of  $\mathcal{U}_r$ ,  $\mathcal{U}_t$  and  $K_h$ , these eigenvalues may be real or complex, and general simple expressions for the salt intrusion length cannot be derived. The full solution may however be found using a simple script computing the eigenvectors and imposing the boundary conditions at  $x = 0$ .

##### 4.2. Importance of phase in the tidal mean transport balance

It is possible to elegantly extend our simple analytical approximation of the tidal mean dispersion regime in the appendix in the main text for tides propagating at a finite constant speed. This derivation is valid in the limit  $\zeta/H \rightarrow 0$ , i.e. ignoring any effects of the moving surface on the salt transport. To incorporate the finite propagation speed, we assume the integrated tidal velocity is prescribed as

$$\bar{u}_1 = \mathcal{U}_t \cos(\omega t + \psi x).$$

where the propagation velocity of the tide equals  $\omega/\psi$ . We consider the simplified tidal mean dispersion balance of

$$-\mathcal{U}_r \bar{s}_{0,x} + \langle \mathcal{U}_t \cos(\omega t + \psi x) \bar{s}_{1,x} \rangle = 0, \quad (134)$$

$$\bar{s}_{1,t} - \mathcal{U}_r \bar{s}_{1,x} + \mathcal{U}_t \cos(\omega t + \psi x) \bar{s}_{0,x} = 0. \quad (135)$$

We next define the tidal salinity in terms of its real and imaginary parts, by which Eq. (135) rewrites to the following two equations

$$\begin{aligned} -\omega \bar{s}_1^I - \mathcal{U}_r \bar{s}_{1,x}^R &= -\mathcal{U}_t \cos(\psi x) \bar{s}_{0,x}, \\ \omega \bar{s}_1^R - \mathcal{U}_r \bar{s}_{1,x}^I &= -\mathcal{U}_t \sin(\psi x) \bar{s}_{0,x}. \end{aligned}$$

Defining  $\underline{y} = [\bar{s}_1^R, \bar{s}_1^I]^T$ , these equations can be written as

$$\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \underline{y} + \begin{bmatrix} -\mathcal{U}_r & 0 \\ 0 & -\mathcal{U}_r \end{bmatrix} \underline{y}_x = - \begin{bmatrix} \cos(\psi x) \\ \sin(\psi x) \end{bmatrix} \mathcal{U}_t \bar{s}_{0,x}. \quad (136)$$



We treat this system as an inhomogeneous system in  $y$ , i.e. assuming  $\bar{s}_{0,x}$  a known forcing term. We cannot regard  $\bar{s}_{0,x}$  as an additional unknown and solve the resulting system as we did for the case with constant phase, as this would yield a system of ODEs with variable coefficients with no easy analytical solution. Instead, the above system of equations is solved using variation of parameters. The solution to the homogeneous system of ODEs reads as

$$\underline{y} = \begin{bmatrix} e^{i\omega/\mathcal{U}_r} & e^{-i\omega/\mathcal{U}_r} \\ ie^{i\omega/\mathcal{U}_r} & -ie^{-i\omega/\mathcal{U}_r} \end{bmatrix} \underline{c} \quad (137)$$

for vector of unknown coefficients  $\underline{c}$ . Using variation of parameters, we find that the derivative of  $\underline{c}$  satisfies

$$\underline{c}' = \frac{\mathcal{U}_t}{2\mathcal{U}_r} \bar{s}_{0,x} \begin{bmatrix} e^{-i\left(-\frac{\omega}{\mathcal{U}_r}x + \psi x\right)} \\ e^{i\left(-\frac{\omega}{\mathcal{U}_r}x + \psi x\right)} \end{bmatrix}. \quad (138)$$

Next, since we are only interested in estimating a typical length scale of salt intrusion we seek for solutions of the form  $\bar{s}_0 = ke^{\lambda x}$ . We find

$$\underline{c} = k \frac{\lambda \mathcal{U}_t}{2\mathcal{U}_r} \begin{bmatrix} \left(\frac{i\omega}{\mathcal{U}_r} - i\psi + \lambda\right)^{-1} e^{-\left(\frac{-i\omega}{\mathcal{U}_r}x + i\psi - \lambda\right)x} \\ \left(-\frac{i\omega}{\mathcal{U}_r} + i\psi + \lambda\right)^{-1} e^{\left(\frac{-i\omega}{\mathcal{U}_r}x + i\psi + \lambda\right)x} \end{bmatrix} + \tilde{c}. \quad (139)$$

We choose to set the remaining constant  $\tilde{c}$  to zero as we are only interested in a length scale for salt intrusion. The solution for the tidal salinity is then found to be

$$\bar{s}_1 = k \frac{\lambda \mathcal{U}_t}{\mathcal{U}_r} \left(-\frac{i\omega}{\mathcal{U}_r} + i\psi + \lambda\right)^{-1} e^{(i\psi + \lambda)x}. \quad (140)$$

We can then write the reduced subtidal salinity balance as

$$-\mathcal{U}_r ke^{\lambda x} + \frac{1}{2}k \frac{\lambda \mathcal{U}_t^2}{\mathcal{U}_r} \operatorname{Re} \left( \left(-\frac{i\omega}{\mathcal{U}_r} + i\psi + \lambda\right)^{-1} \right) e^{\lambda x} = 0, \quad (141)$$

which is equivalent to

$$\mathcal{U}_r^2 - \frac{1}{2}\mathcal{U}_t^2 \frac{\lambda^2}{\lambda^2 + \left(-\frac{\omega}{\mathcal{U}_r} + \psi\right)^2} = 0.$$

Finally, this has solution

$$\lambda = \pm \frac{\omega - \mathcal{U}_r \psi}{\sqrt{\frac{1}{2}\mathcal{U}_t^2 - \mathcal{U}_r^2}}. \quad (142)$$

## 5. Reconstruction of eddy viscosity for the RWW case

The eddy viscosity is not part of the output of the OSR model and was therefore estimated. We estimate the eddy viscosity for the RWW case based on the model results of the velocity and salinity using the following function

$$A_\nu = c_v \langle U(H + \zeta) F(\overline{Ri}) \rangle, \quad (143)$$

where  $c_v$  is a calibration coefficient,  $\zeta$  is the water surface level,  $U$  is the depth-averaged velocity magnitude

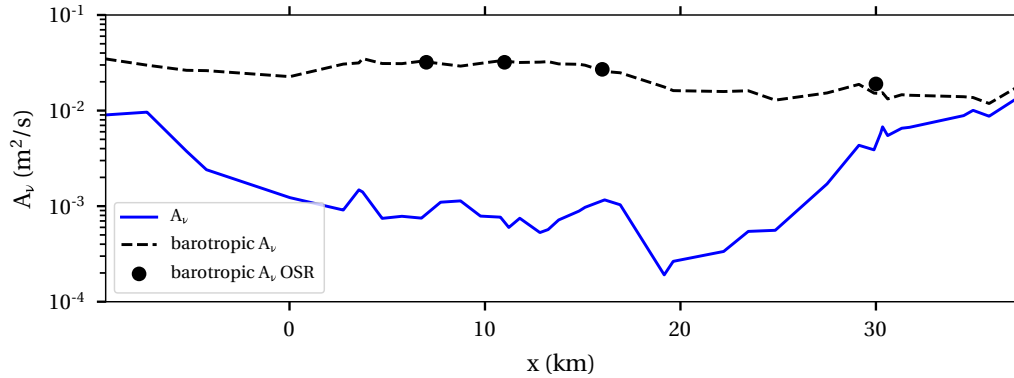
$$U = \sqrt{u^2 + v^2} \quad (144)$$

and  $F$  is a damping function dependent on the depth-averaged Richardson number  $\overline{Ri}$  based on the damping functions of Pacanowski and Philander (1981), see also Nunes Vaz and Simpson (1994), Basdurak, Valle-Levinson, and Cheng (2013), and Wei et al. (2021), i.e.

$$F = (1 + 5\overline{Ri})^{-2}, \quad (145)$$

$$Ri = -g\beta \frac{s_z}{u_z^2 + v_z^2}. \quad (146)$$

To determine the calibration coefficient  $c_v$  we used modelled eddy viscosity values for a barotropic simulation (i.e. a run without density effects on turbulence) of the same time period in the OSR model. The time-averaged depth-averaged barotropic eddy viscosity from the OSR model is shown by the black dots in Fig. S1. We calibrated Eq. (143) with  $F = 1$  to these black dots, resulting in  $c_v = 2.5 \cdot 10^{-3}$ , as shown by the black dashed line. The resulting eddy viscosity from Eq. (143) with  $F \neq 1$  is shown by the blue line.



**Figure S1.** Eddy viscosity from a barotropic run of the OSR model (i.e. without density effects) averaged over the period August 17 to 31, 2003 (black dots), calibration of Eq. (143) with  $F = 1$  (black dashed line), and resulting  $A_\nu$  from Eq. (143) with  $F \neq 1$ .

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