

# A Spatiotemporal-Aware Climate Model Ensembling Method for Improving Precipitation Predictability

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## Abstract

Multimodel ensembling has been widely used to improve climate model predictions, and the improvement strongly depends on the ensembling scheme. In this work, we propose a Bayesian neural network (BNN) ensembling method, which combines climate models within a Bayesian model averaging framework, to improve the predictive capability of model ensembles. Our proposed BNN approach calculates spatiotemporally varying model weights and biases by leveraging individual models' simulation skill, calibrates the ensemble prediction against observations by considering observation data uncertainty, and quantifies epistemic uncertainty when extrapolating to new conditions. More importantly, the BNN method provides interpretability about which climate model contributes more to the ensemble prediction at which locations and times. Thus, beyond its predictive capability, the method also brings insights and understanding of the models to guide further model and data development. In this study, we apply the BNN weighting scheme to an ensemble of CMIP6 climate models for monthly precipitation prediction over the conterminous United States. In both synthetic and real case studies, we demonstrate that BNN produces predictions of monthly precipitation with higher accuracy than three baseline ensembling methods. BNN can correctly assign a larger weight to the regions and seasons where the individual model fits the observation better. Moreover, its offered interpretability is consistent with our understanding of localized climate model performance. Additionally, BNN shows an increasing uncertainty when the prediction is farther away from the period with constrained data, which appropriately reflects our predictive confidence and trustworthiness of the models in the changing climate.

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## Key Points:

- We develop a spatiotemporal-aware weighting scheme using Bayesian neural networks for improving model ensemble predictions
- The method calculates model skill-consistent weights, provides interpretability, and quantifies uncertainty
- We demonstrate the method's superior performance over three baseline ensembling methods in predicting precipitation in CONUS

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**Abstract**

Multimodel ensembling has been widely used to improve climate model predictions, and the improvement strongly depends on the ensembling scheme. In this work, we propose a Bayesian neural network (BNN) ensembling method, which combines climate models within a Bayesian model averaging framework, to improve the predictive capability of model ensembles. Our proposed BNN approach calculates spatiotemporally varying model weights and biases by leveraging individual models' simulation skill, calibrates the ensemble prediction against observations by considering observation data uncertainty, and quantifies epistemic uncertainty when extrapolating to new conditions. More importantly, the BNN method provides interpretability about which climate model contributes more to the ensemble prediction at which locations and times. Thus, beyond its predictive capability, the method also brings insights and understanding of the models to guide further model and data development. In this study, we apply the BNN weighting scheme to an ensemble of CMIP6 climate models for monthly precipitation prediction over the conterminous United States. In both synthetic and real case studies, we demonstrate that BNN produces predictions of monthly precipitation with higher accuracy than three baseline ensembling methods. BNN can correctly assign a larger weight to the regions and seasons where the individual model fits the observation better. Moreover, its offered interpretability is consistent with our understanding of localized climate model performance. Additionally, BNN shows an increasing uncertainty when the prediction is farther away from the period with constrained data, which appropriately reflects our predictive confidence and trustworthiness of the models in the changing climate.

**Plain Language Summary**

Precipitation is one of the key climatic factors affecting fluxes of water, energy, and biogeochemical cycles. Global climate models (GCMs) are usually used for improving precipitation prediction and advancing understanding of precipitation's responses to climate change. A large set of GCMs are available and they show large uncertainties in physical process representations, varying prediction skills at different locations and times, and are usually not constrained by observations. Here, we propose a Bayesian neural network ensembling method to address these challenges and thus improve precipitation predictability by providing accurate and uncertainty-aware predictions.

**1 Introduction**

Precipitation is one of the key climatic factors affecting fluxes of water, energy, and biogeochemical cycles. It has been observed that climate change non-uniformly shifts regional and seasonal distributions of the precipitation, where dry regions/seasons get drier and wet regions/seasons get wetter (Stegall & Kunkel, 2019). This shift of precipitation patterns significantly affects natural ecosystem health and human society development (E. Martin, 2018; Greve et al., 2014). For instance, in humid regions, the heavy precipitation can increase flood and landslide risks, degrade water quality for human consumption, and disrupt regional ecosystem balance. In arid regions, the decreased precipitation can exacerbate droughts, which leads to water shortages, agricultural production loss, and energy supply risks. Therefore, improving our ability to accurately predict current and future patterns in precipitation is vital for assessing the vulnerability of ecosystems, preparing for extreme precipitation events, and concurrently enhancing water resources management (Konapala et al., 2020).

Global climate models (GCMs) have been used for improving precipitation prediction and advancing understanding of precipitation's responses to climate change (Weigel et al., 2021; Demory et al., 2020). One of the most inclusive sets of GCMs is from the Coupled Model Intercomparison Project (CMIP), initialized by the Working Group on Coupled Modeling under the organization of the World Climate Research Program (Eyring

64 et al., 2016; Taylor et al., 2012). CMIP is now in its sixth phase. CMIP6 consists of about  
65 100 GCMs produced by 49 different modeling groups/institutes (Zelazowski et al., 2018).  
66 These GCMs have large uncertainties in physical process representations, show varying  
67 prediction skills at different locations and times, and are usually not constrained by ob-  
68 servations (Eyring et al., 2019). Each of these aforementioned factors affects the accu-  
69 rate prediction of precipitation at regional scales. One strategy that can improve the pre-  
70 cipitation prediction is a comprehensive multi-model ensembling approach that lever-  
71 ages each individual model’s spatiotemporally varying predictive skill, integrates obser-  
72 vations to reduce prediction bias, and quantifies predictive uncertainty using a formal  
73 calibration and uncertainty quantification (UQ) framework (Que et al., 2020; Fothering-  
74 ham et al., 2015).

75 Several multi-model ensembling methods have been developed. Some approaches  
76 assume model independence and model democracy, in which each model is weighted equally.  
77 Although studies have demonstrated that under certain conditions equal-weight model  
78 averaging could produce better prediction performance than the individual models (Gleckler  
79 et al., 2008; Knutti et al., 2010; Pincus et al., 2008), the assumption on model indepen-  
80 dence and democracy is not true. Many GCMs in CMIP share components or are vari-  
81 ants of other models in the ensemble, and these models have large inconsistency in their  
82 skills at a given location and time (Alexander & Easterbrook, 2015; Abramowitz & Bishop,  
83 2015; Sanderson et al., 2015; Bishop & Abramowitz, 2013). Even an individual model  
84 shows considerably inconsistent skills in different locations and at different times. By re-  
85 cognizing the distinct capabilities among the models, some studies assigned unequal weights  
86 to individual ensemble members (Amos et al., 2020; Brunner et al., 2019; Wenzel et al.,  
87 2016; Karpechko et al., 2013; Räisänen et al., 2010). One of the most frequently adopted  
88 ensemble weighting schemes was proposed by Sanderson et al. (2015). It calculates model  
89 weights by balancing the model skill and model uniqueness; the coefficient controlling  
90 the balance is determined subjectively, and its value could significantly impact the en-  
91 semble results (Knutti et al., 2017; Sanderson et al., 2017).

92 Although some weighted average methods have been proposed, the unequal weights  
93 assigned to the individual models are mostly uncalibrated against the observations, an  
94 uniform weight is assigned to a model across the space and time, and the same weight  
95 is applied for future projections without UQ. Since the model skill varies at regional and  
96 seasonal scales, the spatiotemporally uniform weight does not fully leverage each indi-  
97 vidual model’s capability, resulting in the loss of information and possibly large biases  
98 in predicting the distribution of precipitation (G. M. Martin et al., 2017; Kumar et al.,  
99 2014). Stegall and Kunkel (2019) discovered that assigning unequal but spatiotempo-  
100 rally uniform weights to individual models can improve the mean prediction of the pre-  
101 cipitation, but the estimated regional precipitation distribution still had a large incon-  
102 sistency with the observations. Additionally, the model weights need to be calibrated against  
103 observations in each grid cell at each time step to reasonably reflect the individual model’s  
104 spatiotemporally varying skill in fitting the observed data and produce observationally  
105 constrained ensemble predictions. Studies have shown that many models contributing  
106 to CMIP yielded large discrepancies compared with observations, and these model bi-  
107 ases should be reduced by calibration before being used for prediction (Ukkola et al., 2020;  
108 Lorenz et al., 2018; Mueller & Seneviratne, 2014). Finally, UQ is required for the ensem-  
109 ble prediction to avoid overconfidence—especially when we project the precipitation in  
110 the future changing climate.

111 In this work, we propose a Bayesian neural network (BNN) ensembling method to  
112 improve precipitation predictability by providing accurate and uncertainty-aware pre-  
113 dictions. The BNN ensembling approach combines GCMs within a Bayesian model av-  
114 eraging framework. It calculates spatiotemporally varying model weights and biases, cal-  
115 ibrates the weights and biases against observations, and accounts for the varying qual-  
116 ity of the observed data. Additionally, the BNN method quantifies epistemic uncertainty

117 when extrapolating the prediction to new conditions. More importantly, BNN also pro-  
 118 vides interpretability about each individual model’s contribution to the ensemble pre-  
 119 diction in different regions and at different times.

120 The proposed BNN ensembling scheme overcomes the limitations of existing meth-  
 121 ods by leveraging the power of machine learning (ML) in data analytics and predictive  
 122 analytics. ML techniques have been applied for predicting precipitation (Jose et al., 2022;  
 123 Heinze-Deml et al., 2021; Li et al., 2021; Ahmed et al., 2020). Most of these applications  
 124 used ML methods either as a surrogate model of an individual GCM to reduce compu-  
 125 tational costs in simulation or as a data-driven, black-box regression model to simulate  
 126 the precipitation directly. The former application considers only a single GCM, and the  
 127 latter regression model simulation lacks mechanical interpretation and process under-  
 128 standing. Here, we use ML techniques in the context of multiple model analysis to cal-  
 129 culate the model weights of an ensemble of GCMs. The proposed BNN weighting strat-  
 130 egy sufficiently leverages each individual GCM’s diverse performance in heterogeneous  
 131 geography and different seasons by calculating spatiotemporally varying model weights  
 132 and biases. By fusing diverse GCMs, the BNN ensembling embeds our best physical knowl-  
 133 edge; and by constraining the ensemble prediction with the observations, BNN enables  
 134 accurate predictions that match the historical data. Additionally, the BNN method quan-  
 135 tifies both aleatoric uncertainty from the data noise and epistemic uncertainty when pro-  
 136 jecting to the unknown future. Furthermore, besides providing high-quality ensemble pre-  
 137 dictions with UQ, our method also brings insights and understanding of the climate model  
 138 performance to guide further model development and prioritize data collection.

139 We apply the BNN ensembling method for monthly precipitation prediction over  
 140 the conterminous United States (CONUS). We consider an ensemble of GCMs from CMIP6  
 141 and use the European Centre Reanalysis Data (ERA5) as “observations” for model cal-  
 142 ibration and performance evaluation. We perform both synthetic and real case studies  
 143 to verify, evaluate, and demonstrate the method’s capability with respect to prediction  
 144 accuracy, interpretability, and UQ. The main contributions of this effort are as follows.  
 145

- 146 • We propose a BNN ensembling approach for precipitation prediction by leveraging  
 147 individual GCM’s spatiotemporally varying skill and calibrating the model weights  
 148 and biases against the observations.
- 149 • We demonstrate the superior prediction performance of the proposed method in com-  
 150 parison with three widely used ensembling approaches on GCMs from CMIP6 and  
 151 additionally show that BNN can reasonably calculate the epistemic uncertainty in  
 152 extrapolation to avoid overconfident projections.
- 153 • We investigate the interpretability of the BNN method in terms of which GCMs con-  
 154 tribute more to the ensemble prediction at which locations and times and demonstrate  
 155 that the calculated spatiotemporally varying weights are consistent with the GCMs’  
 156 simulation skill.

## 157 **2 Methods and Data**

158 In this section, we introduce the BNN ensembling method and describe the climate  
 159 models and the precipitation data. Next, we briefly introduce three state-of-the-art en-  
 160 sembling schemes with which we compare the BNN for performance evaluation. Lastly,  
 161 we discuss some evaluation metrics.

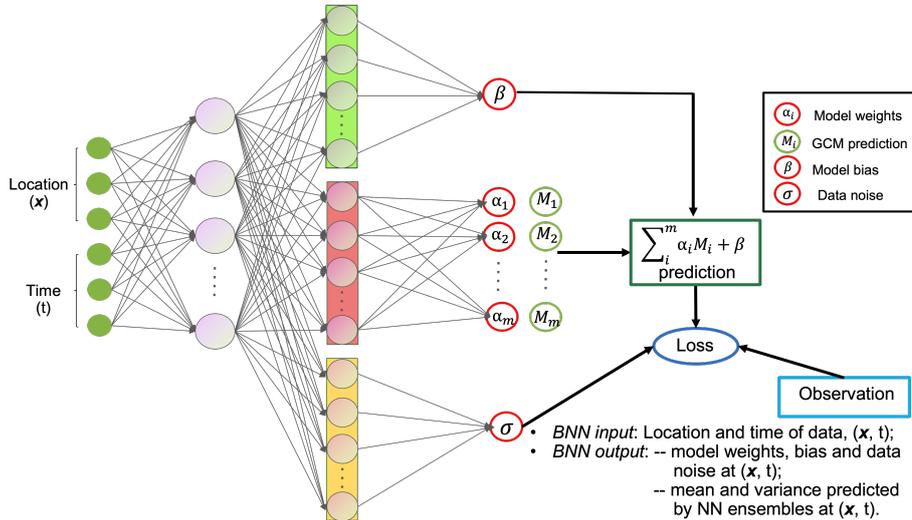
### 162 **2.1 Bayesian Neural Networks for Ensemble Model Predictions**

We assume that observations  $y(\mathbf{x}, t)$  at a given location  $\mathbf{x}$  and time  $t$  can be re-  
 presented as a sum over an ensemble of  $m$  GCM predictions  $M_i(\mathbf{x}, t)$  weighted by their

respective weights  $\alpha_i(\mathbf{x}, t)$ , a bias term  $\beta(\mathbf{x}, t)$ , and a data noise term  $\epsilon(\mathbf{x}, t)$ :

$$y(\mathbf{x}, t) = \sum_{i=1}^m \alpha_i(\mathbf{x}, t) M_i(\mathbf{x}, t) + \beta(\mathbf{x}, t) + \epsilon(\mathbf{x}, t). \quad (1)$$

163 The model weights are positive and their sum over the ensemble models is one:  $\alpha_i(\mathbf{x}, t) >$   
 164  $0$ , and  $\sum_{i=1}^m \alpha_i(\mathbf{x}, t) = 1$ . The model bias  $\beta(\mathbf{x}, t)$  represents the discrepancy of the weighted  
 165 ensemble model simulations from the observation. The data noise  $\epsilon(\mathbf{x}, t)$  considers the  
 166 observation quality varying across the location and time, which is assumed following a  
 167 Gaussian distribution with a zero mean and a heteroscedastic standard deviation  $\sigma(\mathbf{x}, t)$ .  
 168 The combination of the first two terms at the right-hand side of Eq. (1) forms the BNN  
 169 ensemble model prediction:  $\hat{y}(\mathbf{x}, t) = \sum_{i=1}^m \alpha_i(\mathbf{x}, t) M_i(\mathbf{x}, t) + \beta(\mathbf{x}, t)$ . This ensembling  
 170 scheme expresses the model weights as a function of location and time to leverage in-  
 171 dividual models' spatiotemporally varying simulation skills. The ensemble prediction ad-  
 172 ditionally considers a bias term that is also a function of space and time. Incorporat-  
 173 ing the bias term in ensembling is crucial, especially when all the individual models have  
 174 an over- or under-prediction. In this situation, the weighted ensemble model simulations  
 175  $\sum_{i=1}^m \alpha_i(\mathbf{x}, t) M_i(\mathbf{x}, t)$  would not perform better than the best-performing individual model,  
 176 no matter what their weights are. Incorporating the spatiotemporally varying model bias  
 177 into the ensemble prediction reflects the ensemble model deficiency.



**Figure 1.** Architecture of the proposed Bayesian neural networks (BNNs).

In implementation, BNN reads the data location ( $\mathbf{x}$ ) and time ( $t$ ) as inputs and estimates the model weights, biases, and data noises at the given  $(\mathbf{x}, t)$  by calibrating the ensemble prediction against the observations. As illustrated in Figure 1, BNN first uses a set of dense layers to extract common information of the model weights, biases, and data noises. Then, three sets of dense layers are designed to learn the information specific to each component. Next, BNN incorporates the multiple GCM predictions  $M_i(\mathbf{x}, t)$  and combines them with the estimated model weights, biases, and data noises in the loss function for optimization. The weights, biases, and noises are calibrated as probabilistic functions by specifying distributions over the parameters of the neural networks (NNs) (i.e., we perform the optimization in the Bayesian context). For computational efficiency, we train the BNN using the randomized maximum a posteriori (MAP) sampling (Pearce et al., 2018) instead of the computationally intractable full Bayesian inference, which may require Markov chain Monte Carlo simulation. The MAP sampling approach uses multiple NNs to quantify the ML model parameter uncertainty. Specifically, for the  $j$ -th net-

work, we draw a sample from the prior distribution over the network parameters (assumed Gaussian)  $\theta_{anc,j} \sim N(\mu_{prior}, \Sigma_{prior})$ , and compute the MAP estimate corresponding to a prior re-centered at  $\theta_{anc,j}$ . When we consider a dataset of  $N$  observations  $y_k$  where  $k = 1, \dots, N$  and specify the data likelihood by assuming a Gaussian noise with the heteroscedastic standard deviation of  $\sigma(\mathbf{x}_k, t_k)$ , the calculation of the MAP estimate is equivalent to minimize the following loss function for the  $j$ -th network:

$$Loss_j = \sum_{k=1}^N \frac{(y_k - \hat{y}_j(\mathbf{x}_k, t_k))^2}{\sigma_j^2(\mathbf{x}_k, t_k)} + \sum_{k=1}^N \log(\sigma_j^2(\mathbf{x}_k, t_k)) + \|\Sigma_{prior}^{-1/2}(\theta_j - \theta_{anc,j})\|_2^2. \quad (2)$$

178 After training, the output of an ensemble of  $n_e$  such networks is thus a mixture of  $n_e$   
 179 Gaussians,  $N(\hat{y}_j(\mathbf{x}_k, t_k), \sigma_j^2(\mathbf{x}_k, t_k))$ . Then, the mean prediction of these networks  $\frac{1}{n_e} \sum_j \hat{y}_j$   
 180 is the BNN prediction result. The variance  $\frac{1}{n_e} \sum_j \sigma_j^2 + \frac{1}{n_e} \sum_j \hat{y}_j^2 - (\frac{1}{n_e} \sum_j \hat{y}_j)^2$  quanti-  
 181 fies predictive uncertainty where the first term quantifies aleatoric data uncertainty, and  
 182 the combination of the second and third terms quantifies the epistemic uncertainty de-  
 183 scribing the model’s ignorance about the conditions outside the observational records.

184 Attributed to this special NN design and Bayesian training, the ensembling strat-  
 185 egy of BNN not only calculates spatiotemporally varying model weights and biases, but  
 186 it also calibrates the weights and biases against observations to fully leverage each in-  
 187 dividual model’s simulation capability, allowing for more accurate and observationally  
 188 constrained predictions. Furthermore, we trained the BNN using the computationally  
 189 efficient randomized MAP sampling, which enables rapid quantification of the aleatoric  
 190 and epistemic uncertainty. Last but not the least, a key strength of this BNN approach  
 191 is the models interpretability, which can explain which models perform well in which lo-  
 192 cations at which times. This interpretability extends the usage of ML techniques beyond  
 193 its predictive capabilities to bring insight and understanding to the climate models.

194 To enable the BNN to produce physically consistent results, we encoded our do-  
 195 main knowledge into the network design and network training. First, in terms of net-  
 196 work design, we chose tanh activations for the hidden layers in Figure 1 because their  
 197 mean output is zero-centered, which stabilizes the training. Furthermore, the tanh ac-  
 198 tivations result in a predictably flat extrapolation outside the training set, which ensures  
 199 a realistic estimation of the model bias and data noise. For the set of dense layers in sim-  
 200 ulating the model weights, we use a softmax layer at the end to ensure that the model  
 201 weights sum to unity. Additionally, in terms of network training, we first transform lat-  
 202 itude, longitude, and time of each data point to a 6 dimensional space-time input. In a  
 203 climate model, we usually use latitude and longitude to represent a location and use a  
 204 scalar of  $t$  to represent the time (no matter what the unit is). However, directly inputting  
 205 the three numbers—latitude ( $lat$ ), longitude ( $lon$ ), and time ( $t$ )—to the BNN would be  
 206 problematic because the model weights, biases, and data noises generated by such a net-  
 207 work would be discontinuous and would not respect seasonality. To address this prob-  
 208 lem, we first represent the location input  $\mathbf{x}$  by its Euclidean coordinate  $[\cos(lat)\sin(lon),$   
 209  $\cos(lat)\cos(lon), \sin(lat)]$  and warp the time input  $t$  onto a 3D helix  $[\cos(2\pi t/T), \sin(2\pi t/T),$   
 210  $t]$ , where  $T$  is the time scale of the climate model simulation (here  $T = 1$  month). This  
 211 transformation of the time variable makes the network generate model weights and bi-  
 212 ases with both a strong monthly periodicity and a slow variation over the year, which  
 213 is more consistent with reality. Next, we rescale each column of space-time inputs to the  
 214 range  $[-a, a]$  to appropriately represent the varying frequency of the model weights and  
 215 biases across the space and time. A larger value of  $a$  results in a higher changing frequency.  
 216 In this study, the spatial coordinates are scaled into the  $[-2, 2]$  range, and the tempo-  
 217 ral coordinates are scaled into the  $[-1, 1]$  range. The network complexity (e.g., the num-  
 218 ber of layers and the number of nodes in each layer) and the number of networks for Bayesian  
 219 training are problem specific, depending on the GCM resolution, the model ensemble size,  
 220 and affordable computing resources. Generally speaking, a large number of complex NNs  
 221 is needed for an ensemble analysis of many high-resolution GCMs to calculate the spa-  
 222 tiotemporally varying weights and quantify the uncertainty, which meanwhile requires

223 a high computational cost. In this study, we use an NN structure in which each set of  
 224 dense layers in Figure 1 has a single hidden layer with 100 nodes, and we use 50 such  
 225 NNs for UQ.

## 226 2.2 Precipitation Data and Models

227 We apply the BNN ensembling method for precipitation prediction based on the  
 228 GCMs from CMIP6. The simulated precipitation data from the CMIP6-GCMs are down-  
 229 loaded from the Earth System Grid Federation (ESGF) archives (<https://esgf-node.llnl.gov/search/cmip6>).  
 230 We consider monthly data from 53 GCMs during the period of 1980–2014, and our anal-  
 231 yses focus on the CONUS area. The details of these models are listed in Table 1. We  
 232 use the European Centre for Medium-Range Weather Forecasts (ERA5) reanalysis data  
 233 from the same periods and regions as the reference or “observations” for model calibra-  
 234 tion and performance evaluation (Muñoz-Sabater et al., 2021). The original ERA5 data  
 235 are at 33 km horizontal grid spacing and the hourly scale. We aggregate the data to the  
 236 monthly scale to be consistent with the GCMs simulation data. Both the simulation and  
 237 reference data are remapped to a common 1° latitude–longitude grid using the bilinear  
 238 interpolation method.

## 239 2.3 Three Widely Used Ensembling Schemes

240 In this section, we introduce three state-of-the-art ensembling schemes, which serve  
 241 as baselines to evaluate the BNN’s prediction performance. The simple average method  
 242 is straightforward and normally used for multiple model analysis. The weighted aver-  
 243 age (Knutti et al., 2017) and spatially weighted average methods (Amos et al., 2020) have  
 244 an increasing application because of their consideration of model skills and model inde-  
 245 pendence and their good prediction performance. In the following, we briefly describe  
 246 these three methods where the symbols are consistent with those in Section 2.1.

**Simple Average** The simple average method performs weighted averaging by as-  
 signing individual models with equal weights. The ensemble prediction is calculated as

$$\hat{y}(\mathbf{x}, t) = \frac{1}{m} \sum_{i=1}^m M_i(\mathbf{x}, t). \quad (3)$$

**Weighted Average** The weighted average method was introduced by Knutti et  
 al. (2017), who used model ensembles to project the future sea ice change in the Arct-  
 ic. This weighted average considered model skill and model independence in calculat-  
 ing the weights. For an ensemble of  $m$  models, the weight  $w_i$  for model  $i$  is calculated  
 as

$$w_i = \exp\left(-\frac{D_i^2}{\sigma_D^2}\right) / \left(1 + \sum_{j \neq i}^m \exp\left(-\frac{S_{ij}^2}{\sigma_S^2}\right)\right), \quad (4)$$

247 where  $D_i^2$  represents the discrepancy between the model  $i$  and the observation, and  $S_{ij}^2$   
 248 describes the difference of the model  $i$  from the model  $j$ . Here, the model outputs and  
 249 observations in calculation of  $D_i^2$  and  $S_{ij}^2$  are an averaged value over space and time. This  
 250 method uses  $D_i^2$  and  $S_{ij}^2$  to consider model skill and model uniqueness. It also introduces  
 251 two constants,  $\sigma_D$  and  $\sigma_S$ , to control the influence of the model skill and uniqueness on  
 252 the weights calculation and, consequently, on the ensemble predictions. For example, when  
 253  $\sigma_D$  is assigned a small value, only a small number of models obtain weights, whereas when  
 254  $\sigma_D$  is assigned a large value, this weighted average converges to the simple average with  
 255 equal weights. Although the values of  $\sigma_D$  and  $\sigma_S$  significantly affect the ensemble pre-  
 256 dictions, it is unknown how to assign an appropriate value for a specific problem; cur-  
 257 rently, the values are determined in a heuristic way. Additionally, although this weight-  
 258 ing method considers model skill and independence, it does not consider the model’s spa-  
 259 tiotemporally varying skill and assigns a uniform weight across space and time.

**Table 1.** The 53 GCMs from 28 institutes in CMIP6 are considered in this study. The 28 models in bold from each institute are used for ensembling in the real case application in Section 3.3.

Country	Research institute	Model name
Australia	Commonwealth Scientific and Industrial Research Organization	<b>ACCESS-ESM1-5</b> ACCESS-CM2
Canada	Canadian Centre for Climate Modelling and Analysis	<b>CanESM5</b> CanESM5-CanOE
China	Beijing Climate Center Chinese Academy of Meteorological Sciences Chinese Academy of Sciences The State Key Laboratory of Numerical Modeling for LASG Nanjing University Research Center for Environmental Changes	<b>BCC-ESM1</b> BCC-CSM2-MR <b>CAMS-CSM1-0</b> <b>CAS-ESM2-0</b> <b>FGOALS-g3</b> FGOALS-f3-L <b>NESM3</b> <b>TaiESM1</b>
France	The First Institute of Oceanography, SOA Institut Pierre Simon Laplace Centre National de Recherches Meteorologiques	<b>FIO-ESM-2-0</b> <b>IPSL-CM6A-LR</b> <b>CNRM-CM6-1</b> CNRM-CM6-1-HR CNRM-ESM2-1
Germany	The Alfred Wegener Institute Helmholtz Centre for Polar and Marine Research Max Planck Institute for Meteorology	<b>AWI-ESM-1-1-LR</b> AWI-CM-1-1-MR <b>MPI-ESM1-2-LR</b> MPI-ESM-1-2-HAM MPI-ESM1-2-HR
Japan	The University of Tokyo, National Institute for Environmental Studies, and Japan Agency for Marine-Earth Science and Technology Meteorological Research Institute	<b>MIROC-ES2L</b> MIROC6 <b>MRI-ESM2-0</b>
Italy	Fondazione Centro Euro-Mediterraneo sui Cambiamenti Climatici	<b>CMCC-CM2-HR4</b> CMCC-CM2-SR5
Korea	Korea Meteorological Administration Seoul National University	<b>KACE-1-0-G</b> <b>SAM0-UNICON</b>
Netherlands /Ireland	EC-EARTH consortium published at Irish Centre for High-End Computing	<b>EC-Earth3-Veg-LR</b> EC-Earth3-Veg EC-Earth3
Norway	Bjerknes Centre for Climate Research, Norwegian Meteorological Institute	<b>NorESM2-MM</b> NorESM2-LM NorCPM1
Russia	Institute of Numerical Mathematics	<b>INM-CM4-8</b> INM-CM5-0
UK	Met Office Hadley Center	<b>HadGEM3-GC31-LL</b> HadGEM3-GC31-MM <b>UKESM1-0-LL</b>
USA	Natural Environment Research Council National Center for Atmospheric Research  Geophysical Fluid Dynamics Laboratory University of Arizona NASA/GISS (Goddard Institute for Space Studies) Department of Energy	<b>CESM2-WACCM-FV2</b> CESM2 CESM2-FV2 CESM2-WACCM <b>GFDL-CM4</b> GFDL-ESM4 <b>MCM-UA-1-0</b> <b>GISS-E2-1-G</b> GISS-E2-1-G-CC GISS-E2-1-H <b>E3SM-1-0</b> E3SM-1-1 E3SM-1-1-ECA

**Spatially Weighted Average** Recognizing that the calculation of  $D_i^2$  and  $S_{ij}^2$  in Eq. (4) did not consider the difference in space and time, Amos et al. (2020) proposed a spatially weighted average method that calculates  $D_i^2$  and  $S_{ij}^2$  as a function of location  $\mathbf{x}$  and time  $t$ . Specifically, for an ensemble of  $m$  models, the spatially weighed average is defined by

$$w_i = \exp\left(-\frac{D_i^2(\mathbf{x}, t)}{n\sigma_D^2}\right) / \left(1 + \sum_{j \neq i}^m \exp\left(-\frac{S_{ij}^2(\mathbf{x}, t)}{n\sigma_S^2}\right)\right), \quad (5)$$

where  $n$  is the number of data in calculating  $D_i^2(\mathbf{x}, t)$  and  $S_{ij}^2(\mathbf{x}, t)$ . Although this method considers model–observation discrepancy and model–model difference across space and time in computing the model weights, it still assigns a uniform weight  $w_i$  to an individual model  $i$ .

## 2.4 Evaluation Metrics of Prediction Performance

We used several statistics and visualization tools to evaluate the prediction performance. For assessing the overall performance, we used root mean square error (RMSE), density plots, and box plots. A better performing ensembling method would have a smaller RMSE value, a closer density/box plot to that of the reference. To evaluate the performance in each grid cell, we present the prediction error across the simulation domain. Additionally, we evaluate the BNN’s spatiotemporal-aware weighting scheme by plotting the weights over the spatial domain, in specific regions, and along the simulation time.

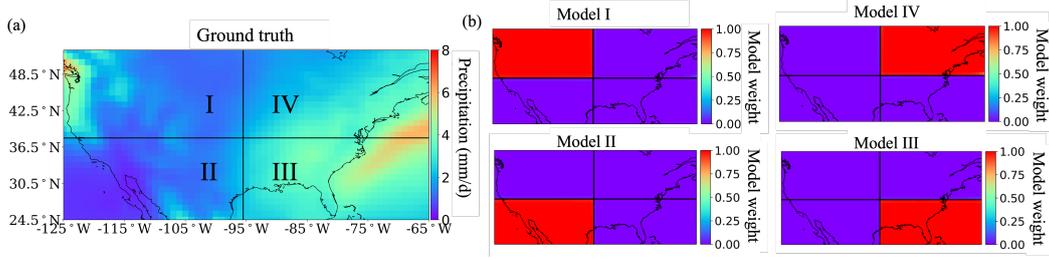
## 3 Results and Discussions

To validate and evaluate our proposed BNN ensembling scheme, we applied it to three case studies and compare its prediction performance and weight calculation with the three state-of-the-art methods introduced in Section 2.3. First, we designed a simple numerical experiment in which we know the ground truth to evaluate whether the BNN can accurately calculate the model weights reflecting the individual model’s spatiotemporally varying skill. Secondly, we designed a synthetic study where the “observations” come from one of the CMIP6 GCMs to further validate the BNN’s capability. In the last real case study, we applied the BNN for ensemble precipitation prediction using 28 CMIP6 GCMs from different institutes and use the ERA5 reanalysis data for calibration and evaluation. We analyze the results from three aspects: prediction performance, interpretability, and UQ.

### 3.1 A Simple Numerical Experiment

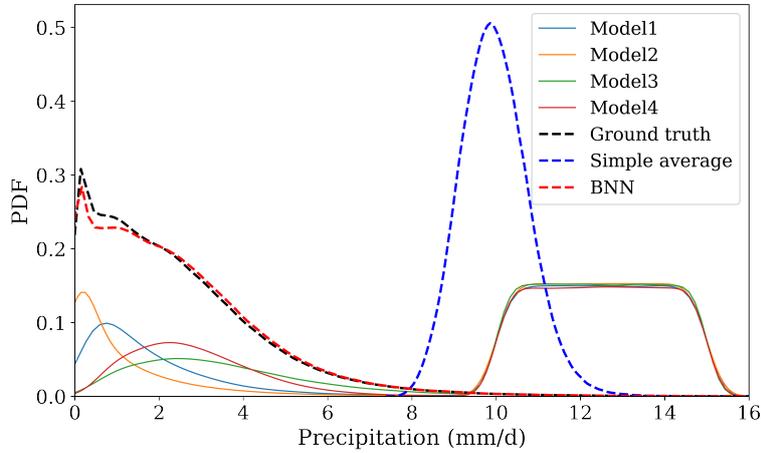
In the simple numerical experiment, we used 35 years of monthly ERA5 reanalysis data over CONUS in 1980–2014 as the ground truth, based on which we designed four individual models for ensemble analysis. Figure 2(a) shows the averaged ERA5 precipitation data over the 35 years. We divided the simulation domain into four equal regions: region I, II, III, and IV. Model  $i$  has the ground truth data in region  $i$  (where  $i$  represents I, II, III, and IV) and has random noises in the other three regions. We generated the random noise from the uniform distribution in the  $[10, 15]$  range, which is beyond the ground truth, having the maximum average value of  $8 \text{ mm/d}$ . We trained the BNN using the first 20 years of data and evaluated its performance on the remaining 15 years.

After training, the BNN calculates the model weights for each grid cell at each month. Figure 2(b) summarizes its averaged model weights over the 15 years of the unseen test period for the four individual models in the entire domain. We observed that the BNN successfully recovered the expected model weights; it assigns weights of 1.0 to the regions



**Figure 2.** (a) The ERA5 precipitation data used as a ground truth in the numerical experiment, in which we divide the domain into four regions to design four individual models for the ensemble analysis; (b) The BNN ensembling scheme accurately assigns the weight of 1.0 to the regions where the model is accurate and assign the weight of 0.0 to the regions where the model produces random noise.

300 where the individual model is accurate and weights of 0.0 to those regions where the model  
 301 produces random noises. Because our BNN reasonably leverages each individual model's  
 302 prediction skill by accurately calculating the spatially varying weights, its ensemble  
 303 predictions have a great agreement with the ground truth. As shown in Figure 3, the  
 304 probability density function (PDF) of the BNN prediction for the out-of-sample test period  
 305 closely overlaps with the PDF of the ground truth. In contrast, the prediction from the  
 306 simple average differs dramatically from the truth by assigning equal weights to the  
 307 models and uniform weights to the entire domain. This numerical example validates this BNN's  
 308 capability in successfully capturing individual model's spatiotemporally varying skill and  
 309 demonstrates its competence in accurate ensemble predictions.

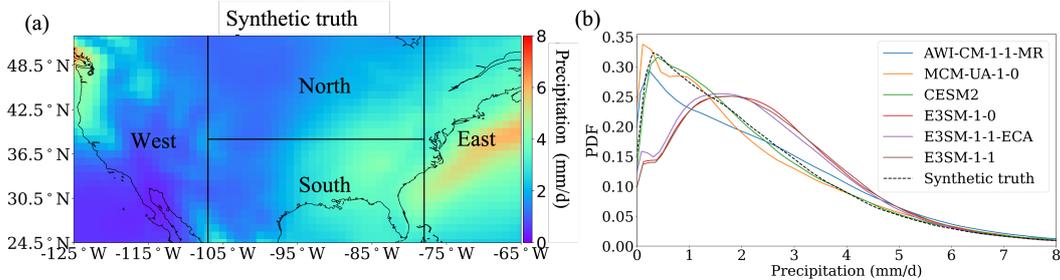


**Figure 3.** Probability density functions (PDFs) of the precipitation over the entire domain in the out-of-sample test period estimated by the simple average and BNN ensembling approaches, along with the data from the four individual models and the ground truth in the same period.

### 310 3.2 A Synthetic Study

311 In this second synthetic case study, we purposely selected seven CMIP6 GCMs from  
 312 Table 1 to investigate the BNN's capability. Those seven GCMs are the Alfred Wegener  
 313 Institute Climate Model (AWI-CM-1-1-MR), Manabe Climate Model v1.0 - University

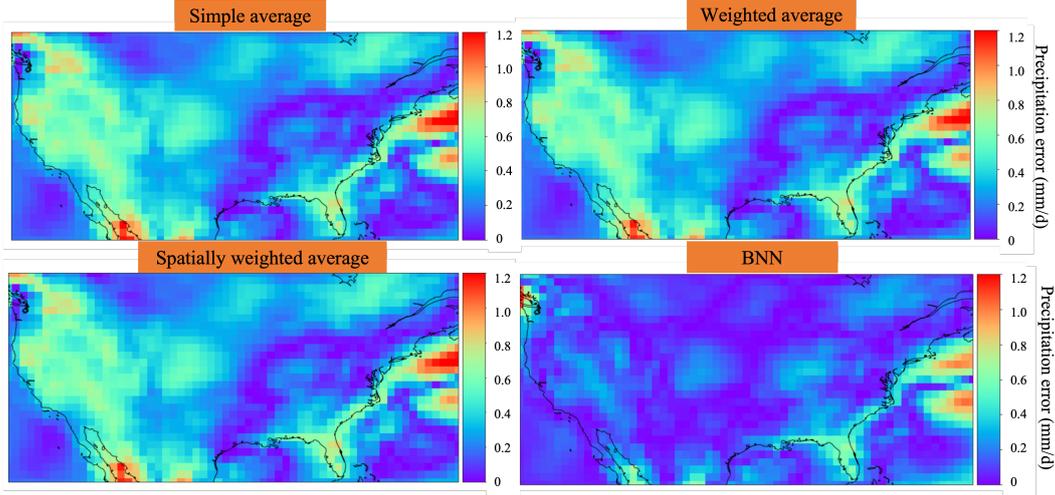
314 of Arizona (MCM-UA-1-0), Community Earth System Model Version 2 (CESM2, CESM2-  
 315 WACCM), and Energy Exascale Earth System Model (E3SM-1-0, E3SM-1-1, E3SM-1-  
 316 1-ECA). We chose the simulation data of model CESM2-WACCM as the synthetic truth  
 317 to calibrate the BNN in the training period and evaluate the BNN’s ensemble predic-  
 318 tion in the test period. Figure 4(a) shows the precipitation data of model CESM2-WACCM  
 319 in CONUS averaged over the 35 years, and Figure 4(b) summarizes the PDFs of the pre-  
 320 cipitation from the synthetic truth and the six models for ensemble analysis. We can see  
 321 that models AWI-CM-1-1-MR, MCM-UA-1-0, and CESM2 produce close predictions to  
 322 the synthetic truth, and the three E3SM models show similar performance, all perform-  
 323 ing differently from the other four models. In this selection of the individual models and  
 324 the synthetic truth, we expect that a good-performing ensembling scheme should assign  
 325 a large weight to those three models, AWI-CM-1-1-MR, MCM-UA-1-0, and CESM2, which  
 326 produce similar precipitation simulations with the synthetic truth, and assign a small  
 327 weight to the three E3SM models which have a relatively large discrepancy from the “truth”.  
 328 To further investigate the BNN’s capability in generating reasonable spatiotemporally  
 329 varying weights, we divided the simulation domain into four regions—North, East, South,  
 330 and West (Figure 4(a))—to evaluate whether its regional weights reflect the individual  
 331 model’s simulation skill locally. We used the first 20 years of data for training and the  
 332 last 15 years for out-of-sample testing.



**Figure 4.** (a) Precipitation data of the synthetic truth averaged over 35 years; (b) The PDFs of the precipitation data from the synthetic truth and the six GCMs for ensemble analysis in the synthetic case study.

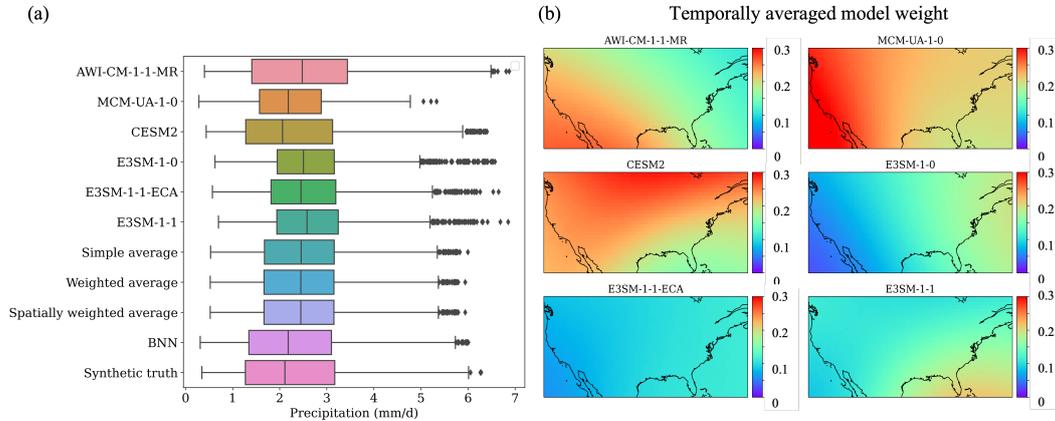
333 In the following, we analyze the ensemble prediction results. We first discuss the  
 334 ensemble prediction accuracy and compare the BNN performance with the three state-  
 335 of-the-art baselines. Next, we analyze the BNN’s weighting scheme in detail by looking  
 336 at its weights spatially and temporally and investigate the influence of the calculated model  
 337 biases on prediction performance. In the analysis, we additionally demonstrate the BNN’s  
 338 interpretability. Lastly, we explore the BNN’s capability in UQ.

339 Figure 5 shows the absolute prediction errors of the four ensembling approaches  
 340 averaged over the test period. The figure indicates that BNN produces more accurate  
 341 ensemble predictions than the other three methods by showing smaller prediction errors  
 342 in the simulation domain. Figure 6(a) summarizes the predictions of the six individual  
 343 models and the four ensembling methods in box plots. The box plots again demonstrate  
 344 that the ensemble predictions of BNN are closer to the synthetic truth, with similar median  
 345 and quantiles. On the other hand, the three baseline ensembling methods produce  
 346 quite similar results, all showing a relatively large difference from the synthetic truth.  
 347 In this case study, we fine-tuned the hyperparameters of  $\sigma_D$  and  $\sigma_S$  in the weighted average  
 348 and spatially weighted average methods and show here the best prediction results  
 349 we obtained after fine-tuning. However, the resulting ensemble predictions from these  
 350 two weighting schemes do not seem to bring much improvement from the simple average.  
 351 Their RMSEs are close to each other, with values of 1.44, 1.44 and 1.45 for simple



**Figure 5.** Absolute precipitation errors of the four ensembling methods averaged over the test period in the synthetic study.

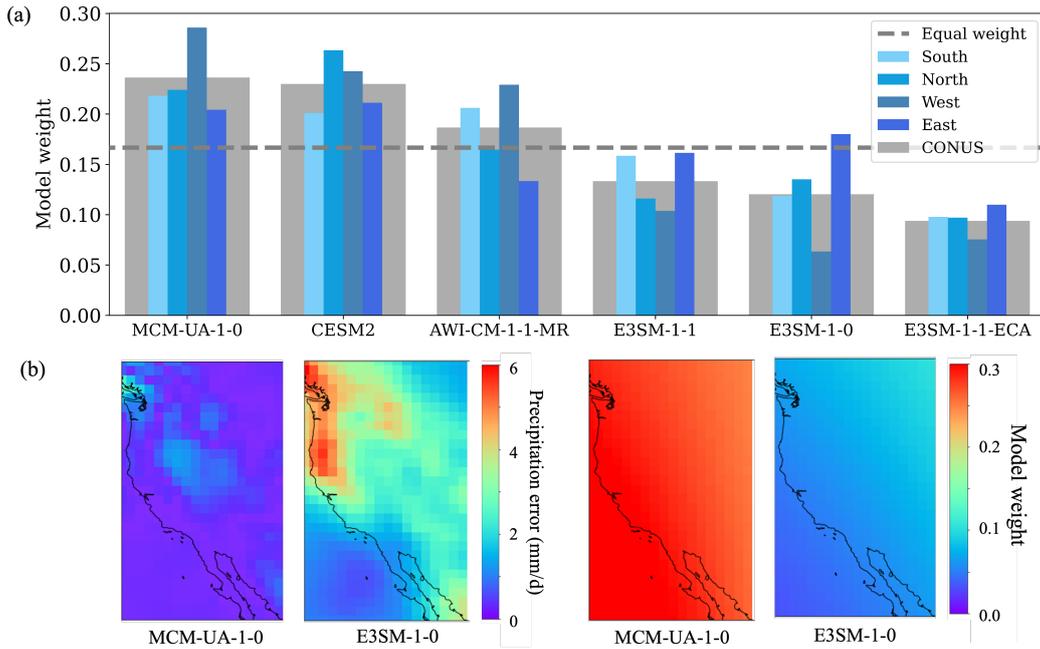
352 average, weighed average and spatially weighted average method, respectively. Due to  
 353 the computational costs, we do not perform hyperparameter tuning for the BNN in this  
 354 work. However, the current BNN architecture and the set of hyperparameters already  
 355 show a great improvement in prediction accuracy compared to the three ensembling base-  
 356 lines and the individual models. A higher improvement of BNN is expected after its hy-  
 357 perparameter tuning and architecture optimization.



**Figure 6.** (a) Boxplot of the precipitation data in the test period for the six GCMs and the four ensemble predictions in the synthetic study; (b) Temporally averaged model weights over the test period for the six GCMs.

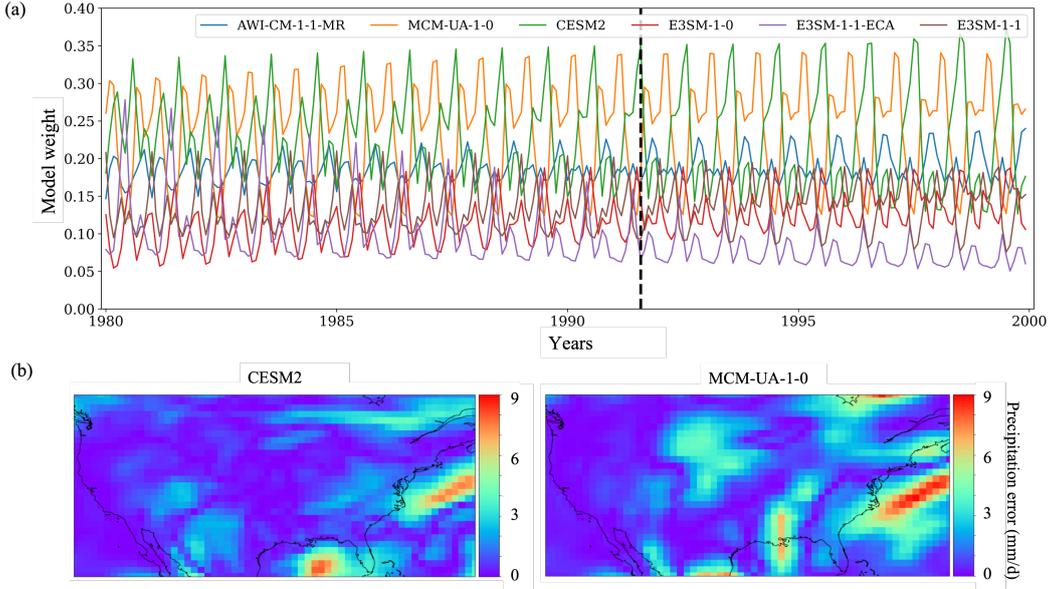
358 The superior prediction performance of our BNN is partially attributed to its spatio-  
 359 temporally varying weights. Figure 6(b) presents the temporally averaged weights over  
 360 the test period for the six individual GCMs in CONUS. We can see that the three top-  
 361 performing models—AWI-CM-1-1-MR, MCM-UA-1-0, and CESM2—receive higher weights  
 362 than the others overall, and the weights in each individual GCM vary spatially. We then  
 363 divided the simulation domain into four regions (see Figure 4(a)) to closely examine the

364 BNN’s spatial weighting and investigate whether its weighting aligns with the GCM’s  
 365 skill. Figure 7(a) summarizes the temporally averaged weights in the four regions and  
 366 the entire CONUS domain for the six individual models, and it also presents the equal  
 367 weights as a baseline. The figure indicates that although models MCM-UA-1-0 and CESM2  
 368 have the highest weights overall in CONUS, MCM-UA-1-0 contributes highly in the West,  
 369 and CESM2 is the dominant GCM in the North and East. This spatially varying weight  
 370 aligns well with each individual model’s spatially varying skill. Take the West region,  
 371 for example: Figure 7(b) indicates that model MCM-UA-1-0 performs better than E3SM-  
 372 1-0 with smaller prediction errors in the West, and the BNN also assigns a higher weight  
 373 to MCM-UA-1-0 in this region. This suggests that the BNN’s spatially varying weight-  
 374 ing reasonably reflects GCMs’ geographically heterogeneous prediction skill. Addition-  
 375 ally, we investigate the BNN’s temporally varying weights. Figure 8(a) plots the spatially  
 376 averaged weights of the 20 years for the six individual models. The figure indicates that  
 377 all the models present a seasonally changing weight, and no individual model performs  
 378 the best all the time. This suggests the importance of calculating temporally varying weights  
 379 in the ensembling. We picked a timestamp, August 1991, for a detailed analysis and present  
 380 the absolute prediction errors of model CESM2 and MCM-UA-1-0 at this specific time  
 381 in Figure 8(b). The figure indicates that model CESM2 predicts more accurate precip-  
 382 itation in August 1991 than MCM-UA-1-0 by producing smaller prediction errors. More-  
 383 over, the BNN accurately estimates the temporal-aware weights by assigning a larger value  
 384 to model CESM2 at this time step, which reasonably leverages the model’s seasonally  
 385 distinct skill.



**Figure 7.** (a) Temporally averaged model weights over the test period in CONUS and the four sub-regions (see Figure 4(a)) for the six GCMs in the synthetic study; (b) Prediction errors and model weights of model MCM-UA-1-0 and E3SM-1-0 in the West region.

386 By calculating the spatiotemporally varying weights that accurately reflect the in-  
 387 dividual model’s diverse skill across space and time, our BNN provides interpretability  
 388 about which model contributes more to the ensemble prediction in which region and at  
 389 which time. This scientific insight improves our understanding of each GCM’s predic-

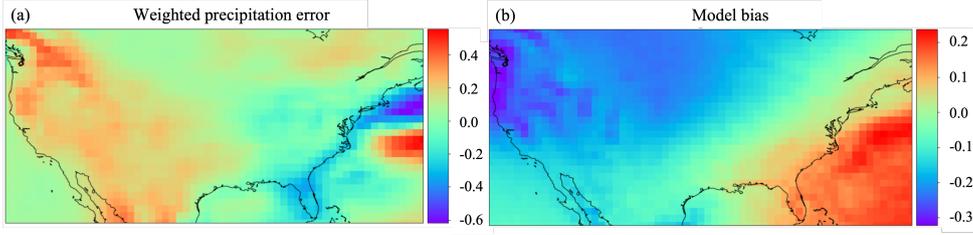


**Figure 8.** (a) Spatially averaged model weights over the simulation domain in the 20 year period for the six GCMs in the synthetic study; (b) Prediction errors of model CESM2 and MCM-UA-1-0 in the timestamp of August 1991 as highlighted in the black line of (a)

390 tive performance and help the model development by leveraging each model’s merits. For  
 391 example, BNN identified that the model MCM-UA-1-0 is more accurate in predicting  
 392 the precipitation in the West region of CONUS, and CESM2 performs better in the Sum-  
 393 mer season. Then we can go back to explore the mechanisms of the two models to in-  
 394 vestigate why they yield better performance in the specific region at the specific time.  
 395 On the other hand, we can also examine why a certain model performs poorly in a cer-  
 396 tain region at a certain time. Combining this comprehensive analysis, we can take ad-  
 397 vantage of each individual model’s strength to build a more powerful GCM for precipi-  
 398 tation prediction. And we can also explain that the BNN results in the superior ensem-  
 399 ble predictions because it assigns higher weights to the regions and times where the model  
 400 performs better. In essence, we are confident in that we are getting right answers for the  
 401 right reasons.

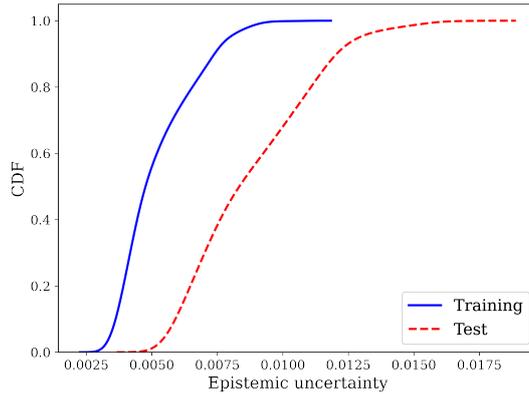
402 Besides the smart weighting scheme, the spatially varying bias term in the BNN  
 403 ensembling also plays an important role for accurate precipitation prediction. As shown  
 404 in Figure 9(a), which presents the weighted prediction errors of the six GCMs, the north-  
 405 west region has a relatively large positive prediction error. To compensate for the error  
 406 and make the ensemble prediction fit the calibration data well, the BNN estimates the  
 407 bias with a relatively large negative value in the region, as depicted in Figure 9(b). This  
 408 bias compensation scheme is particularly important when all the individual GCMs gener-  
 409 ate overestimation or underestimation, in which case the ensemble prediction will hardly  
 410 perform better than the best-performing individual GCMs despite the ensembling schemes.  
 411 In this situation, by introducing the bias term and calibrating its value against the data,  
 412 we can improve the ensemble predictions. Additionally, this bias term is a function of  
 413 space and time, so its calculation reflects the spatiotemporally varying model skill.

414 The BNN performs ensemble prediction in the Bayesian context, so it can quan-  
 415 tify the data uncertainty to consider the data noise and quantify the epistemic uncer-  
 416 tainty to consider the extrapolation error. Because the “observations” come from model



**Figure 9.** (a) Weighted precipitation errors (mm/d) of the six GCMs in the synthetic study; (b) The estimated bias (mm/d) ( $\beta$  in Figure 1) of BNN to compensate the weighted precipitation errors to enable a better ensemble prediction.

417 simulation data in this synthetic study, we do not have data noise. We focus more on  
 418 the epistemic uncertainty discussion. Figure 10 shows the cumulative density function  
 419 (CDF) of the epistemic uncertainty for the training and out-of-sample test data. The  
 420 figure indicates that the BNN can reasonably quantify the uncertainty, where the epis-  
 421 temic uncertainty of the test data in the extrapolation regime is greater than that of the  
 422 training data. This is highly desirable behavior and crucial in practice to prevent over-  
 423 confident projection in the future climate.



**Figure 10.** Epistemic uncertainty of the training and out-of-sample test data calculated by BNN in the synthetic study.

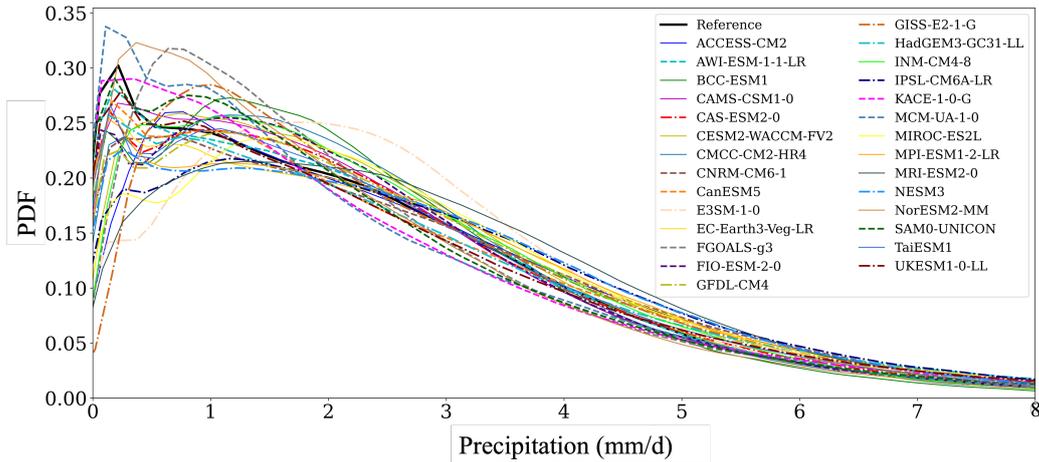
424 In this synthetic study, we demonstrate that our BNN produces superior predic-  
 425 tion performance compared to that of the three state-of-the-art ensembling methods. BNN  
 426 can accurately calculate the spatiotemporally varying model weights and biases, which  
 427 can be justified by the model’s prediction skill. This spatiotemporal-aware weighting scheme  
 428 meanwhile provides the interpretability of the BNN to help us understand which mod-  
 429 els contribute more to the ensemble prediction at which locations and times. Addition-  
 430 ally, we demonstrate that the BNN can reasonably quantify the epistemic uncertainty  
 431 by producing a larger uncertainty bound in the extrapolation regime to avoid overcon-  
 432 fident predictions.

### 433 3.3 A Real Case Application

434 After verification and validation of the BNN method, we applied it to a real case  
 435 study for precipitation prediction where the “observations” come from the ERA5 reanal-  
 436 ysis data. We considered the 53 GCMs from CMIP6 as the model set; these are described

437 in Section 2.2. The 53 GCMs are from 28 institutes. Given that the models from the same  
 438 institute have strong dependence/similarities, we first performed data screening by se-  
 439 lecting one model from one institute to roughly consider the model independence before  
 440 the ensemble analysis (Leduc et al., 2016; Ashfaq et al., 2022). For the models in the same  
 441 institute, we chose the one with the smallest RMSE compared to the ERA5 reference  
 442 data. The final selected 28 models are highlighted in bold in Table 1. Figure 11 shows  
 443 the PDFs of the precipitation data for the 28 GCMs and the ERA5 reference. As shown,  
 444 the 28 GCMs produce different precipitation simulations, and the major difference hap-  
 445 pens at the small precipitation values ( $\leq 2\text{mm/d}$ ). Some GCMs have close PDFs to the  
 446 “observations,” and some others deviate significantly from the reference.

447 We applied the BNN to the 28 GCMs for ensemble predictions and investigated  
 448 whether our model can leverage each individual model’s spatiotemporally varying skill  
 449 to produce an accurate prediction—and meanwhile reasonably quantify the predictive  
 450 uncertainty. We used the first 20 years of data for training and the remaining 15 years  
 451 for out-of-sample testing. In the training, we used the ERA5 data for model calibration;  
 452 in the testing, the ERA5 data were used as reference to evaluate the prediction perfor-  
 453 mance. In the following results discussion, we first evaluate the BNN’s prediction accu-  
 454 racy in comparison with the three baseline ensembling methods. Next, we analyze the  
 455 BNN’s model weights across the space and time and examine its interpretability. Lastly,  
 456 we present the UQ results and discuss the computational costs.



**Figure 11.** The PDFs of the precipitation data from the 28 GCMs for ensemble analysis and the reference data from the ERA5 reanalysis product.

457 Table 2 summarizes the RMSEs of the 15 years’ test data in the entire simulation  
 458 domain and the four regions for the four ensembling methods, where the four regions are  
 459 divided in the same manner as shown in Figure 4(a). The BNN produces the smallest  
 460 RMSEs in CONUS and in the West, North, and South regions, demonstrating the best  
 461 prediction performance. Additionally, the BNN produced consistently smaller predic-  
 462 tion errors than the simple average method, whereas in some cases, the advanced weighted  
 463 average and spatially weighted average even produced larger RMSEs than the simple av-  
 464 erage. Please note that the ensembling results of the weighted average and spatially weighted  
 465 average methods come from a fine-tuning of their hyperparameters, whereas for the BNN  
 466 approach, we did not perform a hyperparameter and architecture optimization. A fur-  
 467 ther improvement in the BNN prediction performance is expected after a better choice  
 468 of its hyperparameters and network architectures.

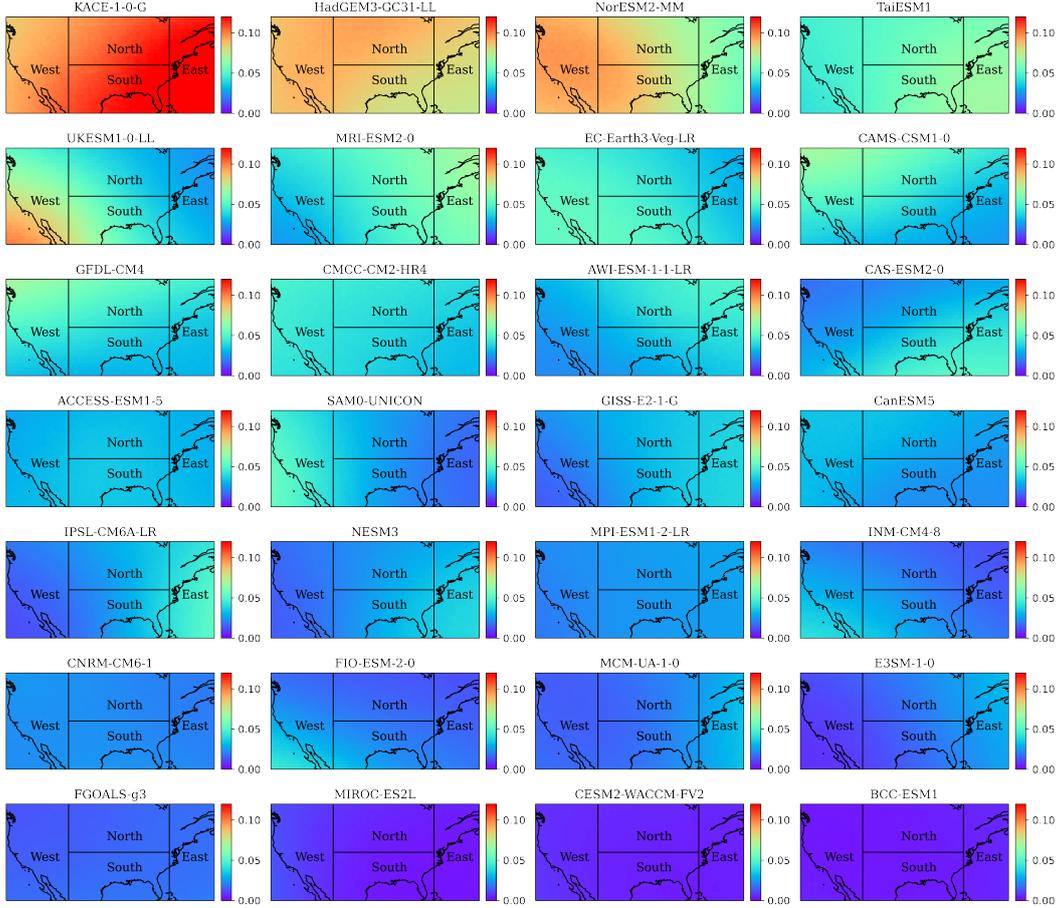
**Table 2.** The RMSEs of the 15 years' precipitation data (mm/d) in the test period at CONUS and the four sub-regions (Figure 4(a)) for the four ensembling methods.

	Simple average	Weighted average	Spatially weighted average	BNN
CONUS	1.48	1.48	1.51	1.45
West	0.71	0.68	0.67	0.57
North	0.25	0.27	0.28	0.23
South	0.50	0.48	0.49	0.43
East	0.46	0.44	0.43	0.44

469 The superior prediction performance of the BNN benefits from its spatiotemporal-  
470 aware weighting scheme. Figure 12 shows the temporally averaged model weights in CONUS  
471 for the 28 GCMs. We organized the models from the largest weights to the smallest weights  
472 in row-wise order. Each GCM presents geographically heterogeneous weights. Overall,  
473 model KACE-1-0-G, HadGEM3-GC31-LL, and NorESM2-MM in the top row gain the  
474 highest weights, and model MIROC-ES2L, CESM2-WACCM-FV2, and BCC-ESM1 on  
475 the bottom row obtain the lowest weights. However, the model's overall higher weight  
476 does not necessarily show a uniform higher weight across the domain at each grid cell.  
477 For example, in the second column of Figure 12, although model SAM0-UNICON has  
478 a smaller weight than MRI-ESM2-0 in most areas, it shows a higher weight in the West  
479 region.

480 Figure 13(a) summarizes the temporally averaged model weights in CONUS and  
481 the four regions for the 28 GCMs. We can clearly see that the models present region-  
482 ally varying weights. Ten models have a higher weight than the equal value (i.e., 1/28).  
483 For some models whose overall weights are below the equal weight, they could still show  
484 a higher weight in a certain region. For example, model SAM0-UNICON presents a larger  
485 weight in the West region than MRI-ESM2-0, although its overall weight in CONUS is  
486 smaller than the latter. This spatially varying weight assignment is consistent with the  
487 individual model's prediction skill. As shown in Figure 13(b), model SAM0-UNICON  
488 shows a smaller prediction error than MRI-ESM2-0 in the West, where the RMSEs of  
489 SAM0-UNICON and MRI-ESM2-0 are 0.7 and 1.2, respectively; we also observe a higher  
490 spatial weight of SAM0-UNICON in this region. Additionally, Figure 13(c) illustrates  
491 that model KACE-1-0-G and NorESM2-MM have similar weights in the West region,  
492 and these two models indeed show similar prediction performance: the RMSE of KACE-  
493 1-0-G is 0.65 close to that of the NorESM2-MM value of 0.63.

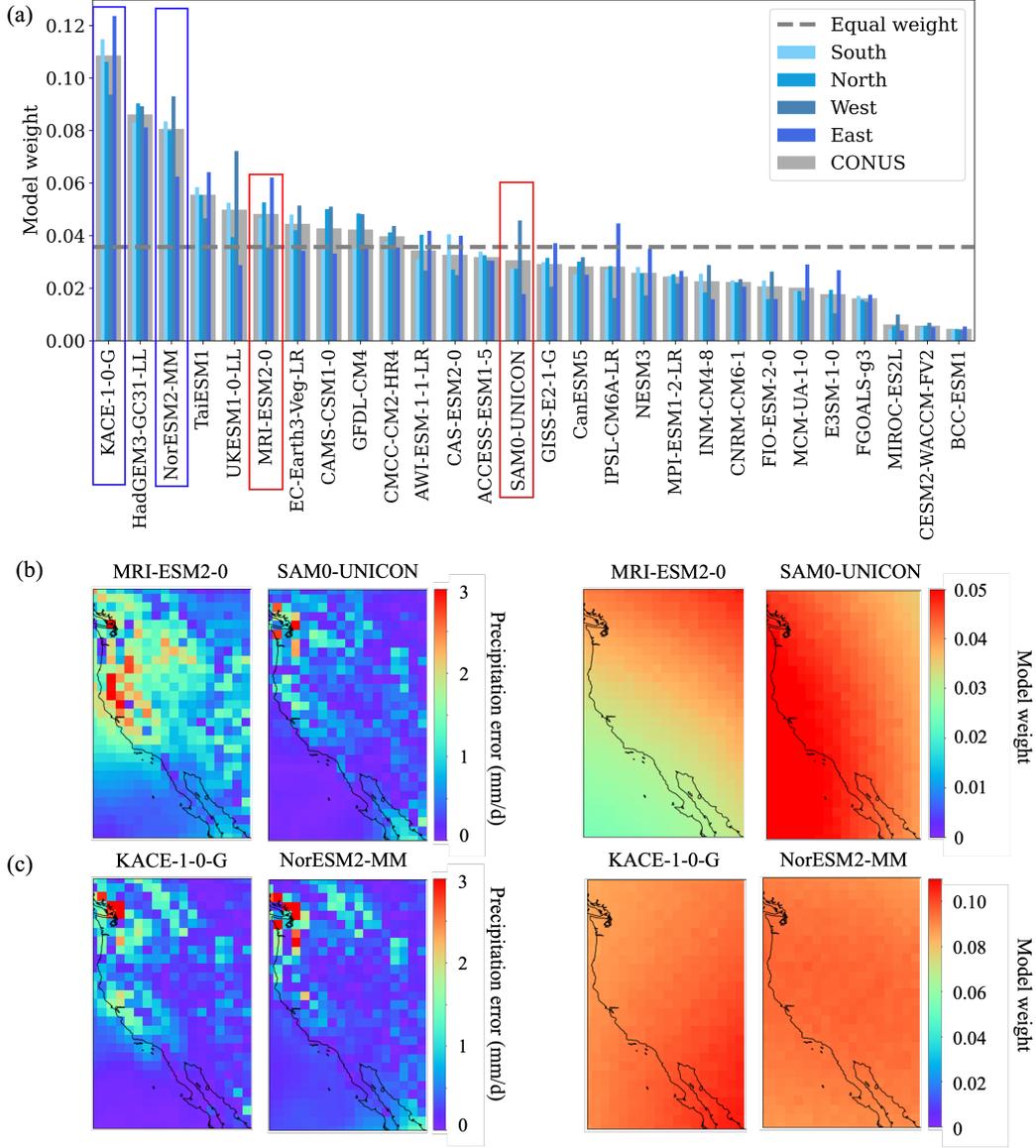
494 The BNN not only gives reasonable spatially aware weights that accurately reflect  
495 the individual model's spatially varying skill, but it also produces skill-consistent weights  
496 in the temporal dimension. To avoid a busy figure and for a better demonstration, Fig-  
497 ure 14(a) plots the spatially averaged model weights in the out-of-sample test period for  
498 three GCMs, which show the top prediction performance and have the highest spatial  
499 weights. All of the three models demonstrate a seasonally changing weight, and none of  
500 them obtain the highest weights all the time. The weights of model KACE-1-0-G show  
501 a decreasing annual trend regardless of the seasonality, the weights of model NorESM2-  
502 MM present an increasing annual trend, and there is no much annual change in the weights  
503 of model HadGEM3-GC31-LL. We picked two timestamps—at the beginning and at the  
504 end of the test period—to analyze the weights of model KACE-1-0-G and NorESM2-MM  
505 in detail. Figure 14(b) shows that KACE-1-0-G has a smaller prediction error than NorESM2-  
506 MM in February 2000, which justifies its higher model weights at this time. Addition-  
507 ally, the lower weight of KACE-1-0-G in July 2014 once again aligns with its relatively  
508 higher prediction errors at this time.



**Figure 12.** Temporally averaged model weights over the 15 years of the test period in CONUS for the 28 GCMs in the real case application.

509 The BNN accurately calculates the model weights for each individual model in each  
 510 grid cell at each time step. Its weighting scheme sufficiently leverages models' predic-  
 511 tion skill and produces skill-consistent weights. This smart weighting not only improves  
 512 model prediction accuracy, but it also provides interpretability about each GCM's con-  
 513 tribution to the ensemble prediction. Please note that all the results and weights anal-  
 514 ysis presented in this real case application are based on the out-of-sample test data, so  
 515 when deploying the BNN method in practice for future projection where the ground truth  
 516 is unknown, its verified interpretable and skill-consistent weights increase our confidence  
 517 in the BNN's ensemble prediction. Certainly, when projecting to the future unknown con-  
 518 ditions, besides the point estimate, we are also interested in the predictive uncertainty.  
 519 BNN can reasonably quantify the epistemic uncertainty caused by the model ignorance  
 520 and data shortage. Figure 15 shows the CDFs of the epistemic uncertainty for the train-  
 521 ing and out-of-sample test data. The figure indicates that BNN produces a larger epis-  
 522 temic uncertainty of the test data than that of the training data, accurately reflecting  
 523 our lesser confidence in the unknown conditions and thus preventing overconfident ex-  
 524 trapolation.

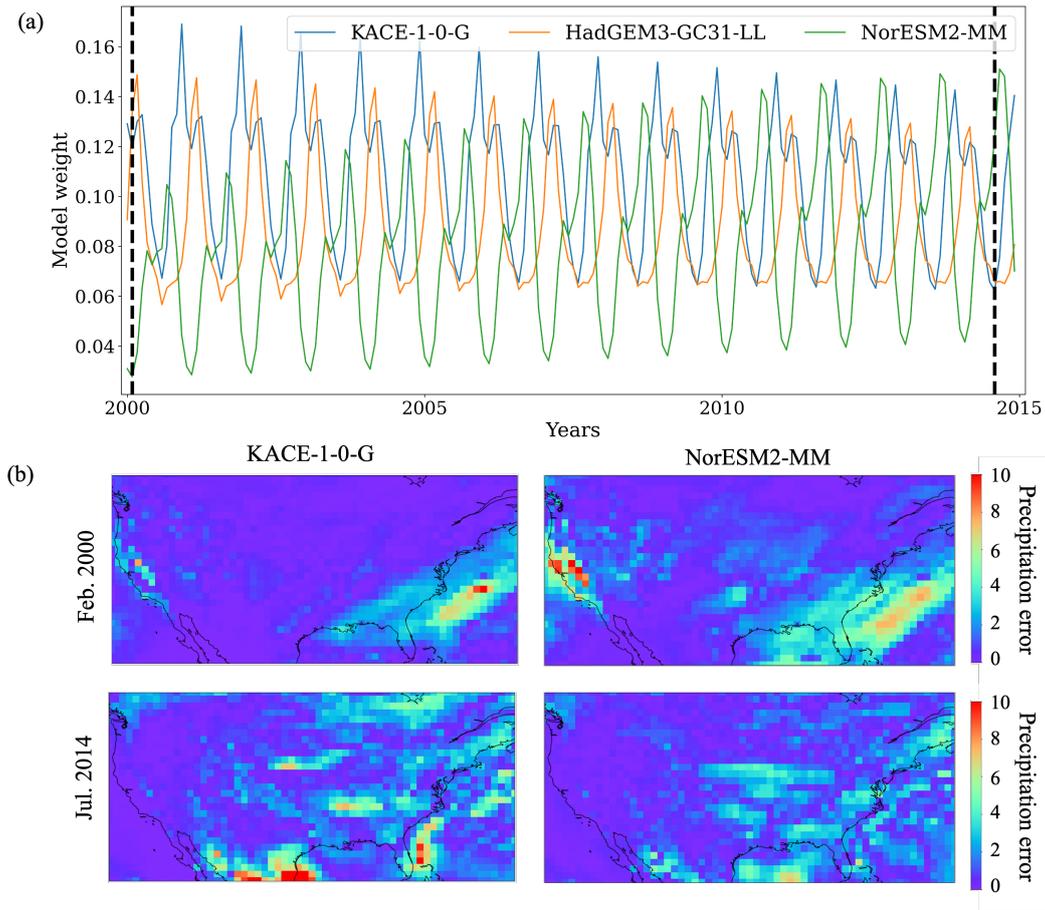
525 In this real case application, we successfully apply the BNN ensembling method  
 526 to 28 GCMs from CMIP6 for precipitation predictions in CONUS. We demonstrate BNN's  
 527 superior prediction performance regionally and locally in comparison to the three base-  
 528 line methods. We investigate BNN's spatiotemporal-aware weighting scheme, verify its



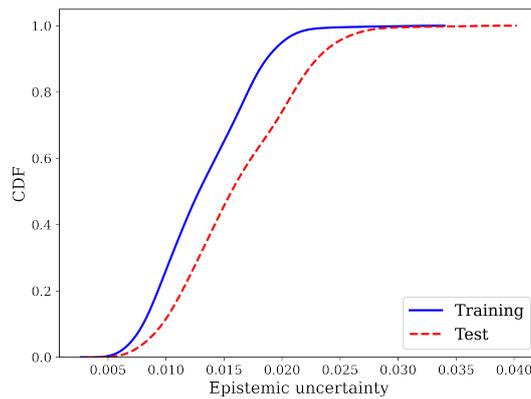
**Figure 13.** (a) Temporally averaged weights over the test period in CONUS and the four regions (see Figure 12) for the 28 GCMs considered in the real case application; (b) Prediction errors and model weights of model MRI-ESM2-0 and SAM0-UNICON in the West region; (c) Prediction errors and model weights of model KACE-1-0-G and NorESM2-MM in the West region.

529 weight’s consistency with the model prediction skill, and interpret the individual mod-  
 530 els’ contribution to the ensemble prediction spatially and temporally. Lastly, we analyze  
 531 the reasonableness of BNN’s UQ capability.

532 One possible limitation of the BNN ensembling scheme is the high computational  
 533 cost. In this work, all the training ends at 1000 epochs when the loss function shows marginal  
 534 decay. In the synthetic study in which six GCMs are considered, it takes about 35 min-  
 535 utes to train one NN and 29.17 hours to finish the training of 50 NNs in the BNN en-  
 536 sembling. For the real case application where 28 GCMs are analyzed, it takes about 40



**Figure 14.** (a) Spatially averaged weights in the 15 years of test period for the three top performing GCMs in the real case application; (b) Prediction errors (mm/d) of model KACE-1-0-G and NorESM2-MM in February 2000 and July 2014.



**Figure 15.** Epistemic uncertainty of the training and out-of-sample test data calculated by BNN in the real case application.

537 minutes to train one NN and 33.33 hours to train the 50 NNs. All the experiments were  
 538 performed on a 2.3 GHz Quad-Core Intel Core i7 CPU. Roughly speaking, the compu-

539 tational cost increases with increasing numbers of networks in BNN training and ensemble  
540 GCMs, as well as with the resolution of the GCMs; this is because the BNN calcu-  
541 lates weights at each time step in each grid cell. In spite of the relatively high compu-  
542 tational cost of the BNN compared to other ensembling schemes, the cost is affordable  
543 (e.g., within one or two days); more importantly, the BNN provides better prediction per-  
544 formance, interpretable ensembling results, and UQ.

## 545 **4 Conclusions and Future Work**

546 In this work, we propose a BNN ensembling method for multiple model analysis  
547 to enhance the predictive capability. The method improves prediction accuracy by learn-  
548 ing spatiotemporally varying model weights and biases based on the individual models’  
549 skill in simulating the observations across space and time. Additionally, the BNN method  
550 accounts for the varying quality of the observations by incorporating their aleatoric un-  
551 certainty and avoids overconfident extrapolating predictions by quantifying the epistemic  
552 uncertainty. More importantly, the method offers interpretability about which models  
553 contribute more to the ensemble prediction at which locations and seasons. This insight  
554 advances predictive understanding, guides process-based model development, and pri-  
555 oritizes data collection.

556 We apply the BNN ensembling method for precipitation prediction in CONUS based  
557 on the GCMs from CMIP6. In both synthetic and real case studies, we demonstrate that  
558 the BNN produces a better prediction performance than the three baseline ensembling  
559 approaches; it can correctly assign a higher weight to the regions and the seasons where  
560 the individual GCM fits the “observations” better; and it gives a reasonable bias value  
561 to compensate for the error of the weighted average to enable a better ensemble predic-  
562 tion than the individual models. Additionally, we verify that the proposed BNN’s inter-  
563 pretability is consistent with our prior knowledge in the synthetic design and with our  
564 understanding of localized GCM performance in the real case application. Finally, the  
565 BNN shows an increasing uncertainty when the prediction is farther away from the pe-  
566 riod with constrained data, which appropriately reflects our predictive confidence and  
567 the trustworthiness of the models in the changing climate. Although the BNN ensem-  
568 bling method produces high-quality, interpretable, and uncertainty-aware predictions at  
569 the expense of high computational costs in calculating the grid-specific and time-specific  
570 model weights and biases, the cost is affordable: for example, about 33 hours are spent  
571 in application of the 28 GCMs. More importantly, the provided high predictive accuracy  
572 and the insights of the model performance are significant. In the future, we will apply  
573 the BNN ensembling technique for other Earth system modeling problems, including pre-  
574 dictions of other response variables from the GCMs and problems in other disciplines  
575 such as hydrology and ecology.

## 576 **5 Data Availability Statement**

577 The simulated precipitation data used in this work are available from the CMIP6  
578 archive <https://esgf-node.llnl.gov/search/cmip6>. The Python scripts of our proposed BNN  
579 method can be found in <https://github.com/patrickfan/BNN>.

## 580 **6 Author Contributions**

581 MF implemented the numerical experiments, prepared the figures and analyzed the  
582 results. DL developed the algorithms, contributed to the research plan, and interpreted  
583 the results. DR processed the data and interpreted the results. EMP formulated the prob-  
584 lem and interpreted the results. All the four authors contributed to the manuscript writ-  
585 ing.

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