Hydrodynamics of non-equilibrium soil water retention

Itai Einav[1] and Mario Liu[2]

[1]University of Sydney
[2]Universitat Tubingen

November 28, 2022

Abstract

Water retention in soil exhibits diverse phenomena, including suction-saturation hysteresis, non-unique air entrapment at zero suction and negative suction under partial saturations. The constancy of suction after a long rest can be broken by relatively minor mechanical or hydraulic agitations such as low-amplitude wetting cycles – this fact is here being related to metastable states that differ from the true equilibrium. The complete suction-saturation relationships are thus being recovered using non-equilibrium Landau's hydrodynamic theory and Onsager's reciprocity principles. Equilibrium suction does not pertain to hysteresis, yet can be approached through small amplitude agitations over long duration. Conditions for rate independence are being described, while rate-dependency are also accommodated and illustrated. Finally, it is shown that the new non-equilibrium theory retains the rigorously derived equilibrium result of the effective stress of partially saturated soils.

Hydrodynamics of non-equilibrium soil water retention

Itai Einav[1], **Mario Liu**[2]

[1] av/M2v *2Mi`2 BM :2QK2+? MB+b M/ JBMBM; J i2`B Hb- a+?QQH Q7 *BpBH 1M;BM22`BM
av/M2v- kyye- av/M2v- mbi` HB
[2] h?2Q`2iBb+?2 S?vbBF- lMBp2`bBi i hC#BM;2M- dkyde hC#BM;2M- :2`K Mv

Key Points:

- i?Q‘Qm;?Hv ‘B;Q‘Qmb ?v/‘Q/vM KB+ i?2Q‘v 7Q‘ T ‘iB HHv b im‘ i2/
QT2/ i? i /BbiBM;mBb?2b K2i bi #H2 7‘QK i‘m2 2[mBHB#‘BmK bi i2b
- .2T ‘im‘2 7‘QK i‘m2 2[mBHB#‘BmK Bb +QMi‘QHH2/ #v ;‘ MmH ‘ i2KT2
MB}2b K2bQb+QTB+ KQiBQMb Q7 T ‘iB+H2 M/ ~mB/ BMi2‘7 +2b
- h?2 i?2Q‘v 2tTH BMb i?2 ‘B+? +QKTH2tBiv Q7 bQBH r i2‘ ‘2i2MiBQM-
?vbi2‘2bBb- T i?@/2T2M/2Mi B‘ 2Mi‘ TK2Mi M/ M2; iBp2 bm+iBQM

Corresponding author: Itai Einav, i t a i . e i n a v > s y d n e y . e d u . a u

Abstract

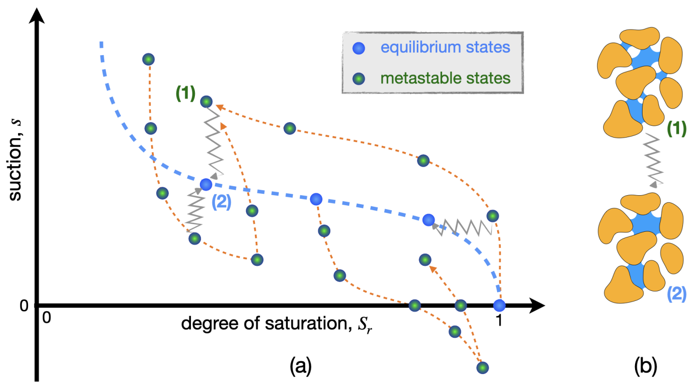
q i2‘ ‘2i2MiBQM BM bQBH 2t?B#Bib /Bp2‘b2 T?2MQK2M - BM+Hm/BM; bm+ bBb- MQM@mMB[m2 B‘ 2Mi‘ TK2Mi i x2‘Q bm+iBQM M/ M2; iBp2 bm+iBQ m‘ iBQMbX h?2 +QMbi M+v Q7 bm+iBQM 7i2‘ HQM; ‘2bi + M #2 #‘QF2M K2+? MB+ H Q‘ ?v/‘ mHB+ ;Bi iBQMb bm+? b HQr@ KTHBim/2 r2iiBM; +v #2BM; ‘2H i2/ iQ K2i bi #H2 bi i2b i? i /Bz2‘ 7‘QK i?2 i‘m2 2[mBHB#‘BmK bm+iBQM@b im‘ iBQM ‘2H iBQMb?BTb ‘2 i?mb #2BM; ‘2+Qp2‘2/ mbBM; M ?v/‘Q/vM KB+ i?2Q‘v M/ PMb ;2‘öb ‘2+BT‘Q+Biv T‘BM+BTH2bX 1[mBHB# T2‘i BM iQ ?vbi2‘2bBb- v2i + M #2 TT‘Q +?2/ i?‘Qm;? bK HH KTHBim/2 /m‘ iBQMX *QM/BiBQMb 7Q‘ ‘ i2 BM/2T2M/2M+2 ‘2 #2BM; /2b+‘B#2/- r?B ‘2 HbQ ++QKKQ/ i2/ M/ BHHmbi‘ i2/X 6BM HHv- Bi Bb b?QrM i? i i?2 M i?2Q‘v ‘2i BMb i?2 ‘B;Q‘QmbHv /2‘Bp2/ 2[mBHB#‘BmK ‘2bmHi Q7 i?2 2z2+ b im‘ i2/ bQBHbX

1 Introduction

h?2 +QMM2+iBQM Q7 B‘ M/ r i2‘ T‘2bbm‘2b iQ i?2B‘ /2MbBiB2b BM T bQBHb Bb MQ‘K HHv 2tT‘2bb2/ #v ‘2H iBM; bm+iBQM U B‘ KBMmb r i2‘ T‘ Q7 b im‘ iBQM M/ TQ‘QbBivX h?2b2 bQBH r i2‘ ‘2i2MiBQM ‘2H iBQMb?BT i2‘M iBp2Hv b+ TBHH ‘v T‘2bbm‘2@b im‘ iBQM ‘2H iBQMb- ‘2 +‘BiB+ H im‘2- ;2Qi2+?MQHQ;v M/ T2i‘QH2mK 2M;BM22‘BM;- b i?2v +QMi‘QH i‘ M kyRjc > bb MBx /2? 2i HX- kyykV M/ 2z2+iBp2 bi‘2bb2b UCB M; 2i HX- k kyRdVX 6‘QK T‘2pBQmb 2tT2‘BK2Mi H ‘2b2 ‘+? Q7 i?2b2 ‘2H iBQMb?BTb #22M B/2MiB}2/X JQbi ;2M2‘ HHv- mM/2‘ 2H2p i2/ bm+iBQMb i?2 KQmMi ‘2/m+2b U"‘QQFb- RNe8c Jm H2K- RNdeV- r?BH2 mM/2‘ +QMbi Mi bm+iBQ m‘ iBQM /2T2M/b QM i?2 TQ‘QbBiv U: HHBTQHB 2i HX- kyyjc bbQmHBM .‘vBM; 7‘QK 7mHH b im‘ iBQM Q++ bBQM HHv ‘2[mB‘2b H ‘;2 bm+iBQM iQ i?2 r i2‘ +QMi2Mi Q‘ b im‘ iBQM U6‘2/HmM/ sBM;- RNN9c S b? 2i HX- k KQMQiQMQmbHv 7‘QK /‘v bi i2b + MMQi ‘2; BM 7mHH b im‘ iBQM rBi?Qm H2iiBM; r i2‘ T‘2bbm‘2 ;2ib H ‘;2‘ i? M i?2 B‘ T‘2bbm‘2- K2 MBM; 2bb2Mi iBQM Q‘ + TBHH ‘v T‘2bbm‘2 UaFD 2p2H M/ 2i HX- kyyyc Hb?2‘B7 2i H *?2M 2i HX- kyRNVX lM/2‘ x2‘Q bm+iBQM i?2 /2;‘22 Q7 b im‘ iBQM Bb H #‘Q /Hv /2T2M/b QM i?2 K tBK Q7 T‘2pBQmbHv TTHB2/ bm+iBQM USQmH H r h vHQ‘- RNdec > KK2‘pQH/ 2i HX- RNN3VX *v+H2b Q7 r2iiBM; UBK# BM; U/‘ BM ;2V T‘2b2Mi ?vbi2‘2iB+ Ub+ MMBM;V ‘2bTQMb2 U> BM2b- RN kyRjc S b? 2i HX- kyRdVX q?BH2 i?2 2z2+iBp2 bi‘2bb Q7 bQBHb ? b #22 iQ bQBH r i2‘ ‘2i2MiBQM mM/2‘ i?2‘KQ/vM KB+ 2[mBHB#‘BmK +QM/BiBQ 7Q‘ HH T‘ +iB+ H Tm‘TQb2b i?2 BKT +i Q7 ?v/‘ mHB+ ?vbi2‘2bBb QM Bi b 2tT2‘BK2Mi HHv THQii2/ ; BMbi b im‘ iBQM UE? HBHB w ‘; ‘# b?B- kyR

AM i?2 r2HH@FMQrM rQ‘F Q7 U> bb MBx /2? :‘ v- RNNjV i?2‘KQ/vM iQ ‘iB+mH i2 MmK#2‘ Q7 mb27mH ‘2bi‘B+iBQMb QM i?2 ‘2H iBQMb?BT #2ir22M bm+iBQ b im‘ iBQMV ‘2H iBQMb?BTbX >Qr2p2‘- i?2‘2 Bb +m‘‘2MiHv MQ i?2‘KQ/vM T #H2 Q7 2tTH BMBM; HH i?2 #Qp2 HBbi2/ Q#b2‘p iBQMb mbBM; bBM; bBbi2Mi T?vbB+ H i‘2 iK2Mi i?2 Tm‘TQb2 Q7 i?Bb T T2‘ Bb iQ ‘2bQHp2 i BM; T‘2pBQmb 7Q‘KmH iBQM 7Q‘ i?2‘KQ/vM KB+ HHv 2[mBHB#‘ i2/ bQB kyRdc 1BM p GBm- kyR3- kykyV iQ MQM@2[mBHB#‘BmK +QM/BiBQMbX MQK2M Q7 ?v/‘ mHB+ ?vbi2‘2bBb- B‘ 2Mi‘ TK2Mi- T i?@/2T2M/2Mi b im iBQM M/ M2; iBp2 bm+iBQM rBHH HH #2 ‘2bQHp2/ mbBM; bBM;H2 2pQH iBQM /2pB iBQM 7‘QK r2HH /2}M2/ MQM@?vbi2‘2iB+ i‘m2 2[mBHB#‘BmK Qmi M22/BM; iQ i F2 Mv //BiBQM H bbmKTiBQMbX

h?2 +m‘‘2Mi /2‘Bp iBQM y7Q}HHQrb i?2i-H?qB}+m?HreH Q}Be;BM HHv 7Q‘KmH i2/ #v G M/ m UG M/ m GB7b?Bix- RN3y- RN3dV- M/ 2ti2M/2/ i ~mB/Biv UE? H iMBFQp- kyR3V M/ HB[mB/ +‘vbi Hb U/2 :2MM2b S‘Qbi-



6B;m'2 R, *QM+2Tim H /BbiBM+iBQM #2ir22M 2[mBHB#'BmK M/ K2i bi #
'2i2MiBQM b?QrBM; K2i bi #H2 bi i2b pBbBi2/ 7i2' KQMQiQMB+ /'vBM; Q
Q' M;2 HBM2bV M/ i'm2 2[mBHB#'BmK bi i2b U/ b?2/ #Hm2 HBM2V TT'Q
im/2 T2'im'# iBQMb Q7 bm+iBQM Q' b im' iBQM U;'2v xB;x ;bV- Q' 2p2M
/vM KB+b Q7 ~mB/ T i+?2b M/ BMi2'7 +2b i? i ++QKT Mv bm+? T2'im'# i

#2 2 bBHv H27i #2?BM/ mTQM bK HH ?v/' mHB+ Q' K2+? MB+ H pB#' iBQ
BM; iQ i?2b2 KBMBK t' 2 s QMM@ e2eMQi +im HHv BM 2[mBHB#'BmK Ub22 6B
PM i?2 Qi?2' ? M/- #v +QMiBMmQmbHv //BM; 7m'i?2' ?v/' mHB+ Q' K2+?
iQ i?2 bQBH- i?2 bi i2 + M ;' /m HHv TT'Q +? i?2 i'm2 2[mBHB#'BmK bi i
#2+ mb2 KB+'QK2+? MB+ HHv i?Bb + M mMHQ+F i?2 KQiBQM Q7 Qi?2'rB
T i+?2b Ub22 6B; R#VX 1tT2'BK2Mib i? i BKTQb2 bm+? pB#' iBQMb i #Q
' M 2i HX- kyyNV M/ M2; iBp2 U Hb?2'B7 2i HX- kyR8V bm+iBQMb '2 p
im'2- M/ bmTTQ'i i?2 T'2b2M+2 Q7 QMHv bBM;H2 i'm2 2[mBHB#'BmK b

S 'iB HHv b im' i2/ bQBHb '2 i?'22@T? b2 K2/B - r?2'2 2 +? T? b2 BM
i?2 Qi?2'bX ++Q'/BM; iQ i?2 T'QTQb2/ i?2Q'v- bK HH bQHB/ T 'iB+H2 pB
TH2- 2t+Bi2/ #v +QmbiB+ r p2bV rQmH/ + mb2 i?2 iQi H TQ'2 +QMbi M; i2K
AM+'2 b2/ pTHK2 Q7 bi'QM;2' KQiBQM Q7 HQ+ HBb2/ ~mB/ T i+?2bX AM
i?2b2 T i+?2b KB;' i2 iQ 2tTHQ'2 Qi?2' HQ+ iBQMb BM i?2 TQ'2 bT +2b- r
7 pQm' #Hv '2biX h?Bb KQiBQM Bb ++QKT MB2/ #v i?2 Hi2' iBQMb Q7 B
;2QK2i'B2b Ub22 6B; R#V- M/ i?mb +? M;2b iQ i?2 K2 bm'2/ bm+iBQMX
;Bi iBQM biQTb2HM t2b # +F iQ x2'Q- i?2 bvbi2K ii BM M2r K2i bi #H2 b
Q7 bm+iBQM- r?B+? K v TQbbB#Hv HB2 QM i?2 i'm2 2[mBHB#'BmK +m'p2
2p2M M22/BM; iQ 2tT2'B2M+2 Mv +? M;2 iQ i?2 ?v/' mHB+ ?BbiQ'v iQ Q#iBM
2tT2'BK2Mi BHHmbi' i2b r?v T 'iB+H2 KQiBQM b?QmH/ MQi #2 /Bb+QMM2
'2i2MiBQMX h?2 T'QTQb2/ ?v/'Q/vM KB+ i?2Q'v +QMbB/2'b i?Bb 7Q'K Q7
iBK2- b r2HH b '2+Qp2'BM; i?2 Qi?2' 7Q'K Q7 bQHB/@~mB/ +QmTHBM; #
M/ ~mB/ T'2bbm'2b BM bQBHbX

h?2 T T2' Bb Q'; MBb2/ b 7QHHQrbX AM a2+X k i?2 ?v/'Q/vM KB+ T'B
iB HHv b im' i2/ bQBHb rBHH #2 '2pB2r2/- #2BM; p HB/ 7Q' #Qi? 2[mBHB#
+QM/BiBQMbX " b2/ QM i?2b2 ;2M2' H T'BM+BTH2b- i?2 7QHHQrBM; a2+
pBQmbHv /2'Bp2/ bT2+B}+ '2bmHib Q7 T 'iB HHv b im' i2/ bQBHb mM/2' i
HB#'BmK c i?2 T'2b2Mi iBQM rBHH // M2r +QMi2ti M/ 2t KTH2b iQ bmTTQ
K BM T 'ib Q7 i?2 T T2'X AM i?2 7QHHQrBM; b2+iBQMb i?2 7Q'KmH iBQM
MQM@2[mBHB#'BmK +QM/BiBQMbX AM a2+X 9 i?2 i'2 iK2Mi rBHH 7Q+mb
+2bb2bc r?BH2 a2+X 8 rBHH 2tT M/ i?Bb 7Q'KmH iBQM iQ ' i2@/2T2M/2M
i?2 HBKBi Q7 bHQr /'vBM; M/ r2iiBM; T'2/B+ib bBKBH ' '2bmHib b HQM
HBKBi BM a2+X 9X "Qi? Q7 i?2b2 b2+iBQMb rBHH BM+Hm/2 MmK#2' Q7

2 General hydrodynamics for partially saturated soils

2.1 Densities

$$\varrho_\beta \equiv \frac{M_\beta}{V},\ \varrho \equiv \sum \varrho_\beta. \qquad \text{(URV)}$$

$$\hat{\varrho}_\beta \equiv \frac{M_\beta}{V_\beta},\ V \equiv \sum V_\beta. \qquad \text{(UkV)}$$

$$c_\beta \equiv \frac{M_\beta}{M} = \frac{\varrho_\beta}{\varrho}, \quad \phi_\beta \equiv \frac{V_\beta}{V} = \frac{\varrho_\beta}{\hat{\varrho}_\beta},$$
$$n \equiv \frac{V_V}{V} = \phi_{\mathrm{q}} + \phi\ , \quad e \equiv \frac{V_V}{V_S} = \frac{n}{1-n}, \quad S_r \equiv \frac{V_{\mathrm{q}}}{V_V} = \frac{\phi_{\mathrm{q}}}{n}, \qquad \text{(UjV)}$$

$$\sum \phi_\beta \equiv 1, \quad \sum c_\beta = 1. \qquad \text{(U9V)}$$

2.2 Balance laws

$$\partial_t U + \nabla_i E_i = \varrho v_i G_i, \qquad \text{(U8V)}$$

$$U \equiv U(g_i, \imath, \varepsilon_{ij}^2, \varrho_\beta, N_\zeta) = g_i^2/(2\varrho) + u(\imath, \varepsilon_{ij}^2, \varrho_\beta, N_\zeta), \qquad \text{(UeV)}$$

i?2B‘ QrM # H M+2 H rX "2HQr- i?2b2 2pQHmiBQM H rb ‘2 bT2+B}2/ 7Q‘ i #H2b- H2 pBM; i?2 MQM@2[mBHB#‘BmK p ‘B #H2b 7Q‘ a2+X 9X

h?2 KQK2MimK /2MbBiv Bb ;Bp2M #v Bib QrM +QMb2‘p iBQM H r,

$$\partial_t g_i + \nabla_j (g_i v_j + \sigma_{ij}) = \varrho G_i, \qquad \text{U d V}$$

r?2‘2 σ_{ij} Bb i?2 iQi H bi‘2bb i2MbQ‘X

h?2 i?2‘K H 2Mi‘QTv Bb ;Bp2M 7‘QK i?2 b2+QM/ H r Q7 i?2‘KQ/vM KB+

$$\partial_t \mathfrak{s} + \nabla_i (\mathfrak{s} v_i - F_i) = R/T \geq 0, \qquad \text{U 3 V}$$

r?2‘2 v_i M/ F_i ‘2 i?2 +QMp2+iBp2 M/ /BbbBT iBp2 2M2‘;v +m‘‘2Mib- ‘2bT2+ R M/ T ‘2 i?2 i?2‘K H 2Mi‘QTv T‘Q/m+iBQM M/ i2KT2‘ im‘2X

h?2 2H biB+ bi‘ BM i2MbQ‘ Bb ;Bp2M 7‘QK,

$$d_t \varepsilon_{ij}^2 + \Omega_{ik}\varepsilon_{kl}^2 - \varepsilon_{kj}^2 \Omega_{ik} = \dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^p, \quad d_t = \partial_t + v_k \nabla_k, \qquad \text{U N V}$$

r?2‘2 Ω_{ij} Bb i?2 MiB@bvKK2i‘B+ +QKTQM2Mi Q7 $\omega_{ij} = \frac{1}{2}(\nabla_j v_i - \nabla_i v_j)$ i?2 iQi H bvKK2i‘B+ bi‘ B/MBbiB- b7 Bb $\dot{\varepsilon}_{ij}^p$ i?2 +m‘‘2Mi FMQrM b i?2 TH biB+ bi‘ BM ‘ i2X JQ‘2 /2i BH QM i?2 bB;MB}+ M+2 Q7 i?Bb H r Bb ;Bp kyR3VX

6BM HHv- i?2 # H M+2 H r 7Q‘ i?2 T ‘iB H /2MbBiB2b Q7 i?2 i?‘22 bT2 Qp2‘ HH +QMb2‘p iBQM H r 7Q‘ i?2 iQi H T ‘iB H /2MbBiv ‘2 ;Bp2M 7‘QK,

$$\partial_t \varrho_\beta \equiv -\nabla_i (\varrho_\beta v_i^\beta) = -\nabla_i (\varrho_\beta v_i - J_i^\beta), \quad \partial_t \varrho \equiv -\nabla_i (\varrho v_i), \qquad \text{U R y V}$$

r?2‘2 v_i^β M/ J_i^β ‘2 i?2 p2HQ+Biv M/ /BbbBT iBp2 /2MbBiv+ β M/ 2Mi U~mtV Q7 T ‘iB+mH ‘- i?2 bmK Q7 i?2 bT2+B2b KQK2Mi Bb ;Bp2M #v i?2 iQi H KQK2 i?2 bQBH KBtim‘2

$$\varrho v_i = \sum \varrho_\beta v_i^\beta, \qquad \text{U R R V}$$

r?BH2 i? MFb iQ 1[X U R V $\partial_t \varrho = -\sum \partial_t \varrho_\beta$ bQ /#v +QK#BMBM; i?2 #Qp2,

$$\sum J_i^\beta = 0 \quad \Rightarrow \quad J_i^\beta = \varrho_\beta (v_i - v_i^\beta). \qquad \text{U R k V}$$

2.3 Hydrodynamic procedure – general results

6QHHQrBM; i?2 ?v/‘Q/vM KB+ T‘Q+2/m‘2 U1BM p GBm- kyR3V- #v +Q RkV i?2 i?2‘K H i2KT2‘ im‘2 Bb ‘2+Qp2‘2/ 7‘QK Bib +Q‘‘2bTQM/BM; 2Mi‘Q

$$T \equiv \frac{\partial u}{\partial \mathfrak{s}}, \qquad \text{U R j V}$$

r?BH2 i?2 2H biB+ bi‘2bb- i?2‘KQ/vM KB+ T‘2bbm‘2- M/ pBb+Qmb bi‘2bb

$$\sigma_{ij}^2 \equiv \frac{\partial u}{\partial \varepsilon_{ij}^2}, \quad P_T \equiv -\frac{\partial (u/\varrho)}{\partial (1/\varrho)}\bigg|_{\varepsilon_{ij}^2}, \quad \sigma_{ij}^{\cdot} = \sigma_{ij} - \sigma_{ij}^2 - P_T \delta_{ij}, \qquad \text{U R 9 V}$$

r?2‘2 σ_{ij} Bb i?2 E‘QM2+F2‘ /2Hi i2MbQ‘X aBKBH ‘Hv- Bi Bb TQbbB#H2 iQ /2 iB H +?2KB+ H TQi2MiB Hb 7Q‘ i?2 iQi H KBtim‘2- BM/BpB/m H bT2+B2b- 7‘QK i?2 +?2KB+ H TQi2MiB H Q7 i?2 bQHB/ T? b2,

$$\mu \equiv \frac{\partial u}{\partial \varrho}, \quad \mu_\beta \equiv \frac{\partial u}{\partial \varrho_\beta}, \quad X_\beta \equiv \mu_\beta - \mu_{\mathrm{a}}. \qquad \text{U R 8 V}$$

h? MFb iQ i?2 ?v/‘Q/vM KB+ T‘Q+2/m‘2 U1BM p GBm- kyR3V Bi Bb H /2‘Bp2 i?2 ‘ i2 Q7 i?2‘K H 2Mi‘QTv T‘Q/m+iBQM,

$$R = F_i \nabla_i T_{\mathrm{K}} + J_i \nabla_i X_{\mathrm{a}} + J_i^{\mathrm{q}} \nabla_i X_{\mathrm{q}} + \mathcal{D} \geq 0, \quad \mathcal{D} = \sigma_{ij}^{\cdot} \dot{\varepsilon}_{ij} + \sigma_{ij}^2 \dot{\varepsilon}_{ij}^p, \qquad \text{U R e V}$$

$$\sigma_{ij}^{2z} \equiv \sigma_{ij} - P_T\delta_{ij}, \quad P_T = -\frac{\partial(u/\varrho)}{\partial(1/\varrho)} = \mu\varrho - u. \qquad \text{UR dV}$$

3 Partially saturated soils under equilibrium

3.1 Equilibrium conditions

$$\min_{\{\hat{\varrho}_\beta, N_\zeta\}} u(\mathfrak{s}, \varepsilon_{ij}^2, \varrho_\beta, \hat{\varrho}_\beta, N_\zeta) = u_{2[}(\mathfrak{s}, \varepsilon_{ij}^2, \varrho_\beta) \quad (2[mBHB\#`BmK) \qquad \text{UR 3V}$$

3.2 Internal energy

$$u_{2[} \equiv u_2(\varepsilon_{ij}^2, \varrho_a) + f(\varrho_\beta). \qquad \text{URNV}$$

$p^2 \sim \varrho_{\mathsf{a}}^3(\varepsilon_v^2)^2$ M/ 2H biB+ $u_2 \sim \varrho_{\mathsf{a}}^3(\varepsilon_v^2)^3$ Ur?2 $p^2 = \frac{1}{3}\sigma_{ii}^2$ M/ $\varepsilon_v^2 = \varepsilon_{ii}^2$ VX h?mb- Hi?Qm;? Bi Bb TQbbB#H2 iQ + ''v HQ MQ P_T i?2 'QQm;? B#mi MQM2@7 T2M/2M+2 QM ϱ_{a}- i?Bb Bb 2MiB'2Hv M2;HB;B#H2 '2H f iBp2Mi Q i?2 +QMi'B#m KQbi T' +iB+ HHv $\varepsilon_v^2 \lesssim$ HX h?2'27Q'2- Bi Bb ++m'#iv2 BQM #QH? +1[X URdV M/ 1[bX UR9-R8-RjV bQ i? i,

$$\sigma_{ij}^{2\mathsf{z}} \equiv \sigma_{ij} - P_T\delta_{ij}, \quad P_T = -\frac{\partial(f/\varrho)}{\partial(1/\varrho)} = \mu\varrho - f, \qquad \text{UkyV}$$

M/

$$\sigma_{ij}^2 = \frac{\partial u_2}{\partial \varepsilon_{ij}^2}, \quad \mu = \frac{\partial f}{\partial \varrho}, \quad \mu_\beta = \frac{\partial f}{\partial \varrho_\beta}, \qquad \text{URV}$$

'2bT2+iBp2HvX b b?QrM #2HQr- f QM2 i?2 T 'MB2QM+b /Q7 MbBiB2b Bb +H2 '- i?r 2M #HBM; M ++m' i2 P_T bQMM/ $\sigma_{ij}^{2\mathsf{z}}$ 7BBQM 7Q'

3.3 Free energy and suctions

h?2 7'22 2M2';v Q7 i?2 i?'22 /QK BMb- r?B+? '2T'2b2Mib i?2B' pQHmK B#BHBiv- +QmH/ #2 + H+mH i2/ M HviB+ HHv UCB M; 2i HX- kyRdV,

$$f \equiv \sum \phi_\beta \hat{f}_\beta(\hat{\varrho}_\beta) = \sum \frac{\varrho_\beta}{\hat{\varrho}_\beta}\hat{f}_\beta(\hat{\varrho}_\beta), \quad \hat{P}_\beta \equiv -\frac{\partial(\hat{f}_\beta/\hat{\varrho}_\beta)}{\partial(1/\hat{\varrho}_\beta)} = \hat{\mu}_\beta\hat{\varrho}_\beta - f_\beta, \quad \hat{\mu}_\beta \equiv \frac{\partial \hat{f}_\beta}{\partial \hat{\varrho}_\beta}, \qquad \text{UkkV}$$

r?2'2 $\hat{P}_\beta$ - $\hat{P}_\beta$ M/ $\hat{\mu}_\beta$ '2 i?2 BMi'BMbB+ 7'22 2M2';v- T'2bbm'2 M/ +?2KB+ H TQi β 'BbBM; /m2 iQ TQbbB#H2 bK HH p 'B iBQMb BM i?2 $\hat{\varrho}_\beta$ XQH@bTQM/BM; BM i?Qm;? bK HH '2H iBp2 iQ p 'B iBQMb BM ϱ_β- i?2 TQbbB#H2 Mbb $\hat{\varrho}_\beta$ 7Q'p 'B #H2 Kmbi #2 HHQr2/ 7Q' iQ ++QKQ/ i2 T?2MQK2M '2H i2/ iQ i?2 /2p2HQTK2 bQHB/ BMi2'7 +2b mM/2' T 'iB H b im' iBQMbX AM T 'iB+mH '- mM/2' T 'i p 'B iBQMb T'Q/m+2 /Bz2'2Mi bB;Mb 7Q' i?2 BMi'BMbB+ T'2bbm'2b BM i? T'Q/m+BM; BMi'BMbB+ bm+iBQM,

$$\hat{s} \equiv \hat{P} \;\; - \hat{P}_{\mathsf{q}}, \qquad \text{UkjV}$$

r?B+? b?QmH/ MQi #2 +QM7mb2/ rBi? i?2 K2 bm'2/ bm+iBQM 7'QK i?2 /Bz B'uUV M/ r i2' u_{q} UV T'2bbm'2b i? i '2 #2BM; TTHB2/ Q' K2 bm'2/ 2ti2'M HH /2pB+2b Qmi Q7 i?2 bQBH,

$$s \equiv u \;\; - u_{\mathsf{q}}. \qquad \text{Uk9V}$$

Hi?Qm;? MQi #2BM; +QMbB/2'2/ T'BQ' iQ UCB M; 2i HX- kyRdV- KQb $u \neq \hat{P}$ $u_{\mathsf{q}} \neq \hat{P}_{\mathsf{q}}$ - M/ $s \neq \hat{s}$- bBM+2 HQM; #QmM/ 'B2b QMHv i?2 T 'iB H +?2KB+ H TQ M/ Qmi Q7 i?2 K2 bm'2K2Mi /2pB+2b M22/ iQ #2 2[m H mM/2' 2[mBHB#'B

6m'i?2'KQ'2- r?BH2 mM/2' 2[mBHB#'BmK i?2 T'2b2M+2 Q7 bm'7 +2 i2 ;2QK2i'B+ HHv +QKTHB+ i2/ BMi2'7 +2b K $\hat{P}_{\mathsf{q}} < 0$ + $u_{\mathsf{q}} < 0$ 2V +QKT'2bbBQM U +QKT'2bb2/ B' T $\hat{P}$ 2bbm'2b $u > 0$V- M/ i?mb TQbBiBp2 bm+ $\hat{s} > 0$ $s > 0$ iBQMb U V- Bi rBHH #2 b?QrM H i2' i? i Qz 2[mBHB#'BmK i?2b2 bm+iBQMb +QmH/ iBM; 7'QK p2'v /'v b KTH2bX

3.4 Energy minimisation, and equilibrium suctions and effective stress

b T'2b2MiHv bi i2/ BM 1[X Ukk fV/ 2T2M/b QM i?2 p 'B;v/2MbBiB2b i?2 i?'22 T 'iB H QM2b '2[mB'2/ 7Q' 1[X URNV- b r2HH b i?'22 MQM@'2[mB' $\hat{\varrho}_\beta$X h?2'27Q'2- bQHpBM; 7Q' i?2 2z2+iBp2 bi'2bb BM 1[X UkyV rQmH/ /2T2 i'BMbB+ /2MbBiB2bX hQ i?Bb 2M/ Bi Bb TQbbB#H2 iQ }'bi bQHp2 2[mBHB#' BM; i?2 2M2';v iQr '/b Bib ;HQ# H KBMBKmK UCB M; 2i HX- kyRdV- #27Q 2[mBHB#'BmK b+2M 'BQb r?2'2 i?2 bi i2b + M ;2i bim+F BM HQ+ H KBMBK

b T 'i Q7 i?2 2M2';v KBMBKBb iBQM- Bi Bb mM/2'biQQ/ i? i mM/2' ;B iB H /2MbBiB2b ϱ_β i?2 BMi'BMbB+ /2MbBiB2b $\hat{\varrho}_\beta$ r?B+? /2T2M/ iQ KBMBKBb2 i?2 2'2;vX q?2M i?2 2M2';v Bb KBMBKBb2/- r?BH2 i FBM; 1[X U9V b i?2 QMHv

$$P_T \equiv u \quad -\chi s, \quad \chi \equiv \chi(\varrho_\beta), \tag{3}$$

$$\chi_{2[} \equiv \chi_{2[}(\varrho_\beta) = \frac{\varrho}{\bar{\varrho}} + \frac{\varrho_S}{\bar{\varrho}_S}\left[\frac{\bar{\varrho}\ (\partial \hat{s}_{2[}/\partial \varrho\ ) - \bar{\varrho}_S(\partial \hat{s}_{2[}/\partial \varrho_S)}{\bar{\varrho}\ (\partial \hat{s}_{2[}/\partial \varrho\ ) - \bar{\varrho}_{\mathrm{q}}(\partial \hat{s}_{2[}/\partial \varrho_{\mathrm{q}})}\right], \tag{4}$$

$$s_{2[} \equiv s_{2[}(\varrho_\beta) = \frac{\varrho_{\mathrm{q}}}{\bar{\varrho}_{\mathrm{q}} K_{\mathrm{q}}}\left[\bar{\varrho}\ \frac{\partial \hat{s}_{2[}}{\partial \varrho} - \bar{\varrho}_{\mathrm{q}}\frac{\partial \hat{s}_{2[}}{\partial \varrho_{\mathrm{q}}}\right]\hat{s}_{2[}, \tag{5}$$

$$\chi_{2[} \equiv \chi_{2[}(S_r, e) = S_r - e\frac{\partial \hat{s}_{2[}/\partial e}{\partial \hat{s}_{2[}/\partial S_r}, \tag{6}$$

$$s_{2[} \equiv s_{2[}(S_r, e) = -\left[\frac{S_r}{K_{\mathrm{q}}}\frac{\partial \hat{s}_{2[}}{\partial S_r}\right]\hat{s}_{2[}, \tag{7}$$

3.5 Illustration of equilibrium states

h?2 M HviB+ r v iQ mb2 i?2 '2H iBQMb?BTb BM 1[bX Uk3-kNV Bb iQ i' Q'/BM 'v /Bz2'2MiB H 2[m iBQM UP 1V- BQ2#M2 bFHHQ2M 7QK'TB'B+ H H r 7Q' $s_{2[} = s_{2[}(S_r, e)$- Bi Bb TQbbB#H2 iQ BMi2;' i2 = ?2 bQHmiBQM M BMBiB H bm+iBQMH2bb #QmM/ 'v $\hat{s}(M, e) \neq 0$ X 7Q' HH2b? 2iQ' bm+? M@ HviB+ bQHmiBQMb Bb iQ BKTQb2 ?vT2'#QHB+ '2H iBQMb?BT #2ir22M UCB M; 2i HX- kyRdc 1BM p GBm- kyR3- kykyV X M Hi2'M iBp2 2t KTH i?2 irQ bi2Tb #2HQr- r?B+? vB2H/ '2H iBQMb?BTb r?Qb2 K i?2K iB+ H 7Q r? i KQ'2 p2'b iBH2 M/ 2H2; Mi i? M i?2 T'2pBQmbHv T'QTQb2/ ?vT2'#QH

U Vb }'bi bi2T- +QMbB/2' '2i2MiBQM 7mM+iBQMb BM- bTB'2/ #v i?'22@ λV AMp2'b2 q2B#mHH 7mM+iBQMX q?2'2 i?Bb 7mM+iBQM r b Q'B;BM /Bbi'B#mi2/ bi'2M;i? BM bQHB/ K i2'B Hb- ?2'2 Bi Bb / Ti2/ iQ '2T'2b /2M+2 Q7 bm+iBQM QM b im' iBQM,

$$s_{2[} = ce^{-\lambda}(-\ln S_r)^{\alpha}, \qquad (28)$$

r?2'2 α Bb +QMbi Mi +QMi'QHHBM; i?2 /2T2M/2M+2 Q7 bQBH r i2' '2i2 S_r c r?BH2 $c e^{-\lambda}$ +QMi'QHb Bib /2T2M/2M+2 QM pQB/ ' iBQ Ur?B+? +QmH/ #2 7'QK 2KTB'B+ H '2H iBQMb 7Q' B' 2Mi'v p Hm2bVX SmiiBM; i?Bb T'Q M/ bQHpBM; 7Q' $\hat{s}(1, m) = 0$ M/ bm#bQMi #QmM/ 'v +QM/BiBQM- r2 }M/,

$$\hat{s}_{2[} = \sqrt{\tfrac{2K_{\mathrm{q}}}{1+\alpha}(-\ln S_r)s_{2[}} \qquad (29)$$

r?B+? mTQM /Bz2'2MiB iBQM mbBM; 1[X Uk3V ;Bp2b,

$$\chi_{2[} = S_r\left(1 + \tfrac{\lambda}{1+\alpha}\ln S_r\right). \qquad (30)$$

U#V q?BH2 i?2 bm+iBQMb BM i?2 T'QTQb2/ '2H iBQMb #Qp2 /Bp2';2 i x2 bQBHb 2t?B#Bi /Bp2';BM; #2? pBQm' i }MBi2 bK HH p Hm2 Q7 '2bB S_{r0} U b M //BiBQM H 7Qm'i? T ' K2i2'VX hQ i? i 2ti2Mi- Bi Bb TQbbB#H #Qp2 '2H iBQMb #v }'bi bm#bi S_r BM 7Q' $s_{2[}$ M/ $\hat{s}_{2[}$ BM 1[X UjRV rBi? M 2z2+iBp2 b im@ ' iBQM $S_r^* = (S_r - S_{r0})/(1 - S_{r0})$- bQ i? i 7'QK 1[bX Uk3-kNV,

$$s_{2[} = ce^{-\lambda}\left(\tfrac{S_r}{S_r - S_{r0}}\right)(-\ln S_r^*)^{\alpha}, \qquad (31)$$

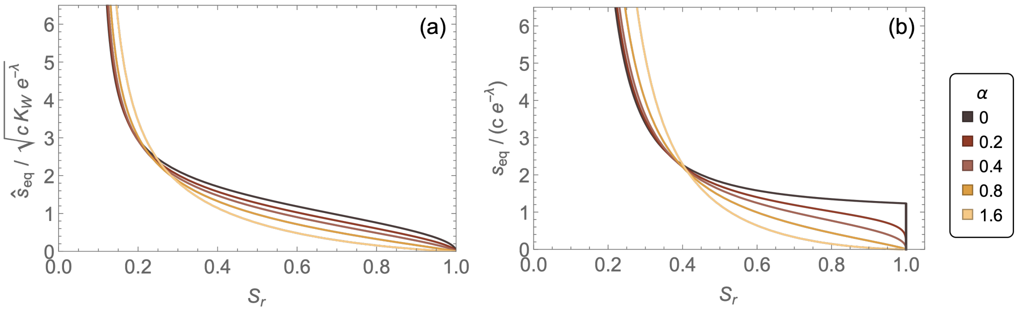
$$\hat{s}_{2[} = \sqrt{\tfrac{2K_{\mathrm{q}}}{1+\alpha}(-\ln S_r^*)s_{2[}} \qquad (32)$$

$$\chi_{2[} = S_r\left(1 + \tfrac{\lambda(S_r - S_{r0})}{(1+\alpha)S_r}\ln S_r^*\right), \qquad (33)$$

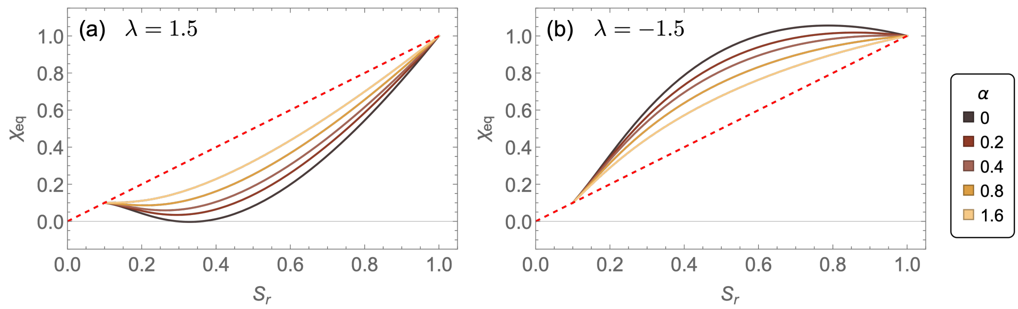
r?B+? vB2H/b i?2 b K2 '2bmHib b 1[bX Ujy@jkV 7Q' $S_{r0} = 0$X

hQ BHHmbi' i2 i?2b2 7mM+iBQMb- 6B;X k b?Qrb i?2 /2T2M/2M+2 Q7 i? bm'2/ bm+iBQMb QM i?2 b?T2 T ' K2i2'b α M/ λX h?2 2tTQM2Mi α ? p2 #22M MQM@/BK2MbBQM H 2t+Hm/2 i?2 2z2+ib Q7 i?2 T'QT2'iB2b Q7 c M/ K_{q} r?B+? /Q MQi +? M;2 i?2 [m H@ Bi iBp2 b? T2 M/ K2'2Hv b+ H2 i?2 p Hm2 Q7 i?2 2z2+iBp2 B' 2Mi'v p Hm 2z2+iBp2 B' 2Mi'v p Hm2 '2T'2b2Mib i?2 '2[mB'2/ bm+iBQM iQ bi ' i /'vBM 7mHH b im' iBQMVX L2p2'i?2H2bb- i?2 T ' K2i2'b λ M/ α 7Q' i?2 2z2+iBp2 bi'2bb "Bb?QT öb +Q2{+B2Mi $\chi_{2[}$- r?B+? Bb MQi 7Q' MQ $\lambda = -1$ QM 6B;X j M/ 6B;X j#- '2@ bT2+iBp2Hv- 7Q' p 'BQmb p Hm2b Q7 α M/ λ MQ i?2 [m HBi iBp2 /2T2M/2M+2 Q7 $\chi_{2[}$ QM S_rX

*QKT 'BM; 6B;X k M/ 6B;X k#- Bi Bb +H2 ' i? i i?2 2[mBHB#'BmK BMi bm'2/ bm+iBQMb '2 MQi bBKBH ' #Qi? [m HBi iBp2Hv M/ BM p Hm2b UbB /Bz2'V- b /Bb+mbb2/ #v U1BM p GBm- kykyVX b α BM+'2 b2b- i?2 b2 B /m+2 b? 'T2' 'Bb2 i ?B;? b im' iBQMb 7QHHQr2/ #v ~ ii2' 'Bb2 i BMi2' ' iBQMb- i' MbBiBQMBM; 7'QK ivTB+ H +H v iQ ivTB+ H b M/ '2i2MiBQM +



6B;m'2 k, h?2 /2T2M/2M+2 mM/2' i?2 i'm2 i?2'KQ/vM KB+ 2[mBHB#'BmK
M/ i?2 U#V K2 bm'2/ bm+iBQMb QM i?2 /2;'22 Q7 b im' iBQM 7Q' /Bz2'2M
' K2i2 α 'mbBM; 1[bX Uj9-jjV- '2bT2+iBp2HvX h?2b2 bm+iBQMb ? p2 #22M M
MQM@/BK2MbBQM H MmK#2'b QM i?2 v@ t2bX h?2 $S_y = 0.1$ bB/m H b im' iBQM



6B;m'2 j, h?2 /2T2M/2M+2 Q7 i?2 χ"BbQ TQM i?2 /2;'22 Q7 b im' iBQM
S_r 7Q' /Bz2'2Mi b? T2 αT M/K2i2' TQbBiBp2 U#V M2; iBp2- +Q'@
'2bTQM/BM; iQ 1[X Uj8VX b BM 6B;X k- i?2 '2bB/m H b im' iBQM T ' K2i2
'2/ HBM2b b?Qr i?2 $\chi = S_r$ bQHmiBQM- r?B+? i?2 +m''2Mi KQ/2H '2T'Q/m+2b

T 'iB+mH 'Hv +H2 ' 7'QK i?2 2[mBHB#'BmK K2 bm'2/ bm+iBQM 6B;X k#- r
'2+Qp2' i'm2 B' 2Mi'v p $S_r = 1$X bBMM ?2 Q i?2' ? M/ 6B;X j -# BHHmbi' i2
i? i TQbBiBp2 M/ M2; iBp2 p Hm2b Q7 λ ?Hm2b #2HQr M/ χ_2 #QpZ2HB M2-
'2bT2+iBp2HvX "Qi? b+2M 'BQb ? p2 #22M '2TQ'i2/ BM i?2 HBi2' im'2 UC
RNekc E? HBHB w '; '# b?B-ky RHyV2b ?#Q'B2 i?B;? b im' iBQMb- M/
?BMiBM; iQr '/b p Hm2b i? i K v ;Q #2HQr y b HQM; b i?2 bQBH '2K BMb

Ai b?QmH/ HbQ #2 K2MiBQM2/ i? i BM bm+?2MK'2BMmb2K"2mM Hb MQ7-
RNekc E? HBHB w '; '# b?B- ky RBbV Mi?2M QK/ HHBv#2BM; T'2p2Mi2/ 7'Q
+? M;BM; /m'BM; i?2 i2bib @ m+?MT2b2M2 BM HQi BM+Hm/2/ BM /' rBM; 6B;X
r?B+? iQ Q#2 +QMbi MiX >Qr2p2'- mbBM; 1[X Uj8V i?2 Q#2 i?M Q7 i?2
i?2 HBM2b +QmH/ #2 +QMbB/2'2/- M/ i?2 HBM2 h?2 b?2 i?2 iM?2
Q7 i?Bb /BbiQ'iBQM rQmH/ /2T2M/ QM i?2 K M'BM; Q7 i?2 b'B iBQMb Q
M/ i?2 H2p2H Q7 b2M bBiBpBiv iQ i?2 ++Q'/BM; iQ 1[X Uj8VX

4 Non-equilibrium soil water retention (rate-independence)

4.1 Non-equilibrium intrinsic suction and effective stress

lM/2‘ MQM@2[mBHB#‘BmK +QM/BiBQMb- i?2 BMi‘BMbB+ M/ K2 bm‘2/ pB i2 7‘QK i?2B‘ 2[mBHB#‘BmK p Hm2b- i?mb 2t?B#BiBM; /vM KB+ T?2MQ bBbX Ai Bb i?2‘27Q‘2 mb27mH iQ +QMbB/2‘ i?2 ıMQM@2[mBHB#‘BmK BM p ‘B #H2,

$$\xi \equiv \hat{s} - \hat{s}_{2[} \qquad \text{UjeV}$$

7Q‘ r?B+? M 2tTHB+Bi 2pQHmiBQM 2[m iBQM Bb ‘2[mB‘2/X _2bQHpBM; i K ‘v 7Q+mb Q7 i?2 +m‘‘2Mi /2‘Bp iBQMX hQ ‘2i BM i?2 2[mBHB#‘BmK ‘2br Qmb b2+iBQMb- i?2 2pQHmiBQM Q7 i?Bb M2r BM/2T2M/2Mi bi i2 p ‘B #H2 H t iQ x2‘Q- b$-\hat{s}_{2[}$ mM/2‘ i‘m2 2[mBHB#‘BmK ‘2bTQMb2 Mi Q7 HH T‘2@ pBQmb bi i2 p ‘B #H2b ϱ_β VX h?2 +Hm#‘2 ‘2H iBQMb BM 1[bX Uk3-kNV M/ mb@ BM; 1[X UjeV- mM/2‘ MQM@2[mBHB#‘BmK +QM/BiBQMb,

$$\chi(\varrho_\beta, \xi) = \chi_{2[}(\varrho_\beta), \quad s(\varrho_\beta, \xi) = \frac{\hat{s}}{\hat{s}_{2[}} s_{2[} = \left(1 + \frac{\xi}{\hat{s}_{2[}(\varrho_\beta)}\right) s_{2[}(\varrho_\beta). \qquad \text{UjdV}$$

b bm+? i?2 "Bb?QT bi‘2bb T ‘ K2i2‘ χ ‘2K BMb i?2 ‘KQ/vM KB+ T‘2bbm‘2 BM 1[X M/ i?2 2z2+iBp2 bi‘2bb BM 1[X UkyV ‘2 HH BMb2MbBiBp2 iQ i?2 MQM@2 #mi i?2 K2 bm‘2/ bm+iBQM s BMb2MbBiBp2- bBM+2 Bi /2T2M/b QM ξ Bb p HB/ 7Q‘ Mv bT2+B}+ 2tTQbBiBQM- BM+Hm/BM; r?2M i?2 2z2+i Q7 iBQM Bb B;MQ‘2/ M/ i?2 /2MbBiB2b ‘2 ‘2TH +2/ #v i?2 b im‘ iBQM M/ pQ

$$\chi(S_r, e, \xi) = \chi_{2[}(S_r, e), \quad s(S_r, e, \xi) = \left(1 + \frac{\xi}{\hat{s}_{2[}(S_r, e)}\right) s_{2[}(S_r, e), \qquad \text{Uj3V}$$

r?2‘2 i?2 2[mBHB#‘BmK p Hm2b ‘2 i F2M 7‘QK 1[bX US_r3-kdV- r?B+? Tm‘2 M/ eX

aBM+2 i?2 K2 bm‘2/ bm+iBQM /2T2M/b QM i?2 ξ-QMi‘B#mi BQM +QMb+iBQM /2 iQ i‘ +F i?2 /vM KB+b Q7 i?2 H ii2‘ p ‘B #H2X Aib ;2M2‘ H 2pQHmiBQM 2[Bi iQ #Qi? /p2+i HQM; i?2 # ‘v+2MiB+ pBHQ+Biv v_i M/ B‘‘2p2‘bB#Hv i?‘Qm;

$$(\partial_t + v_i \nabla_i)\xi + Z_\xi = 0, \qquad \text{UjNV}$$

r?2‘2 Z_ξ Bb Bib /BbbBT iBp2 ~mtX *m‘‘2MiHv mMFMQrM- i?Bb /BbbBT iBp2 bT2+B}2/ #v 7QHHQrBM; PMb ;2‘öb ‘2+BT‘Q+ H +QM/BiBQMb M/ CB M; p2‘bB#BHBiv T‘BM+BTH2 UCB M; GBm- kyyNVX

4.2 Non-equilibrium, meso-related entropy

AM i?2‘KQ/vM KB+b i?2 b2+QM/ H r BM 1[X U3V ‘272‘b iQ # H M+2 Q7 BM i2‘Kb Q7 i?2 /2;‘22b Q7 7‘22/QK Q7 2H2K2Mi ‘v KB+‘Qb+QTB+ T ‘iB+H 2p2‘- i?2 /2b+‘BTiBQM Q7 bQBHb 7m‘i?2‘ ‘2[mB‘2b iQ +QMbB/2‘ i?2 /2;‘22 b+QTB+ 72 im‘2b bm+? b bQBH T ‘iB+H2b M/ B‘@r i2‘ BMi2‘7 +B H +m‘ Q‘/2‘ Q7 K ;MBim/2b H ‘;2‘ i? M i?Qb2 iQKbX Ai ? b #22M b?QrM M/ +QM KQiBQM Q7 i?2b2 K2bQb+QTB+ 72 im‘2b +QMi‘QHb i?2 ‘?2QHQ;v M/ /vM /Bbi‘B#mi2/ ;2QK2i‘v Q7 B‘@r i2‘ BMi2‘7 +2b ? b HbQ #22M b?QrM 2tT2 2i HX- kyRjc 6mFmb?BK 2i HX- kykRV M/ +QKTmi iBQM HHv U: M 2i H bQBH r i2‘ ‘2i2MiBQM ?vbi2‘2bBb T?2MQK2M X h?2‘27Q‘2- BM //BiBQM # H M+2 Q7 i?2 i?2‘K H 2Mi‘QTv BM 1[X U3V- r2 7m‘i?2‘ +QMbB/2‘ i?2 # H 2Mi‘QTv,

$$\partial_t \mathfrak{s}_\mathsf{K} + \nabla_i(\mathfrak{s}_\mathsf{K} v_i - F_i^\mathsf{K}) = R_\mathsf{K}/T_\mathsf{K}, \qquad \text{U9yV}$$

r?2‘2 $\mathfrak{s}_\mathsf{K} v_i$ M/ F_i^K ‘2 i?2 K2bQ@‘2H i2/ +QMp2+iBp2 M/ /BbbBT iBp2 2M2‘;v + bT2+iBp2Hv- R_K r?BH2 T_K Bb i?2 K2bQ@‘2H i2/ ‘ i2b Q7 2Mi‘QTv T‘Q/m+iBQM M/ T2‘ im‘2X >2‘2- i?2 #Qp2 2[m iBQM H2ib mb +QMbB/2‘ ?v/‘ mHB+ HHv ‘2

T?2MQK2M bm+? b bQBH r i2' '2i2MiBQM ?vbi2'2bBbX >Qr2p2'- MQi2 i? 2[m iBQM +QmH/ M/ ? b #22M mb2/ iQ 2tTH BM K2+? MB+ HHv '2H i2/ MQK2M BM b M/- bm+? b bi'2bb@bi' BM ?vbi2'2bBb UCB M; GBm- kyyN LQiB+2 i? i mMHBF2 1[X U3V- R_K bi'B+i2/ 7'QK #2BM; M2; iBp2- v2i r2 i F2 $\ell \geq$ 0 M $R + R_K \geq 0$X h?2 KB+'Q@K2bQ b2T ' iBQM #2ir22M i?2 irQ 2Mi'QTB2b T2' im'2b ? b #22M 7m'i?2' /Bb+mbb2/ #v UGBm- kykRV BM i?2 +QMi2ti Q T?vbB+bX

4.3 Internal energy at non-equilibrium states

.m'BM; MQM@2[mBHB#'BmK +QM/BiBQMb i?2 BMi2'M H 2M2';v /2MbB QM #Qi? 2[mBHB#'BmK M/ MQM@2[mBHB#'BmK +QMi'B#miBQMb- i?2 H i?2 irQ M2r MQM@2[mBHB#'BmK bi i2 p 'B #H2b s_K M/ ξ K2MiBQM2/ i2/ 2Mi'Q bB+ bm+iBQM X 2pB/2Mi 2Q'2- M2;H2+iBM; i?2 2z2+ib Q7 s 'i?2 i?2'K H 2Mi'QT QM i?2 ?v/'QK2+? MB+ H '2bTQMb2 Q7 T 'iB HHv b im' i2/ bQBHb,

$$u \equiv u(\varrho_\beta, \varepsilon_{ij}^2, s, s_K, \xi) = u_{2[}(\varepsilon_{ij}^2, \varrho, \varrho_\beta) + u_{M2}(s_K, \xi), \quad \text{U9RV}$$

h?2 }'bi i2'K $u_{2[}$ + Tim'2b BMi2'M H 2M2';v i? i K BMi BMb /m'BM; 2[mBHB +QmH/ #2 i F2M /B'2+iHv 7'QK 1[X URNVX PM i?2 Qi?2' ? M/- u_{M2} i?2 b2+QM/ //BiBQM H BMi2'M H 2M2';v i MQM@2[mBHB#'BmK bi i2bX h?2 i?2'KQ/v i?2 }'bi i?'22 +QMb2'p2/ bi i2 p 'B #H2b ? p2 #22M /2}M2/ BM 1[bX UkRVX M KB+ +QMDm; i2b Q7 i?2 irQ '2K BMBM; BM/2T2M/2Mi bi i2 p 'B #H2b MQM@2[mBHB#'BmK T 'i Q7 i?2 BMi2'M H 2M2';v,

$$T_K = \frac{\partial u_{M2}}{\partial s_K}, \quad Y_\xi = \frac{\partial u_{M2}}{\partial \xi}, \quad \text{U9kV}$$

#2BM; i?2 K2bQ@'2H i2/ i2KT2' im'2 M/ ömM# H M+2/ BMi2'7 +B H i2tim

LQi2 i? i BM 1[X U9RV i?2 BMi2'M H 2M2';v 'BbBM; 7'QK 2[mBHB#'B bi i2 p 'B #H2b r b 2t+HmbBp2Hv /2+QmTH2/ BMiQ i?2B' +Q''2bTQM/BM; i?Bb Bb MQi M2+2bb 'BHv '2[mB'2/ 7Q' i?2 +m''2Mi 7Q'KmH iBQM- i?Bb b irQ '2 bQMb, URV i?2'2 '2 MQ +m''2Mi 2tT2'BK2Mib i? i rQmH/ bmTTQ'i i iBQM Q7 i?2 ++m' i2 7Q'K Q7 bm+? +QmTHBM;c UkV bm+? 7Q'K Q7 +QmT 2/Hv r2 F- bQ //BM; Bi rQmH/ QMHv +QKTHB+ i2 i?2 /2'Bp iBQM rBi?QmiB im H '2bmHi Q7 i?Bb T T2'X

++Q'/BM;Hv- bBM+2 r2 H'2 /v bT2+B}2/ i?2 2[mBHB#'BmK BMi2'M H r? i Bb H27i Bb iQ bT2+B7v i?2 MQM@2[mBHB#'BmK BMi2'M H 2M2';v BM bi i2 p 'B #H2bX ; BM- BM i?2 #b2M+2 Q7 2tT2'BK2Mib i? i rQmH/ ?2HT +i 7Q'K- i?2 bBKTH2bi 7Q'K Bb +QMbB/2'2/- r?B+? BMpQHp2b 7m'i?2' /2T2M/2Mi [m /' iB+ +QMi'B#miBQMb 7'QK i?2 irQ MQM@2[mBHB#'BmK b

$$u_{M2}(s_K, \xi) = \tfrac{1}{2}\omega s_K^2 + \tfrac{1}{2}\kappa \xi^2, \quad \text{U9jV}$$

bQ mbBM; 1[X U9kV,

$$T_K = \omega s_K, \quad Y_\xi = \kappa \xi, \quad \text{U99V}$$

r?2'2 $\omega \geq 0$ M $\kappa \geq 0$X h?2 [m /' iB+ 7Q'K Q7 i?2 TQi2MiB H #Qp2 +QmH/ #2 b i?2 HQr2bi Q'/2' i2'K r?B+? ? b KBMBKmK Q7 h vHQ' TT'QtBK iBQM iQ i'm2 2M2';v TQi2MiB HX b bm+?- Bi Bb +H2 ' i? i b i?2 irQ M2r MQM@2[m p 'B #H2b $s_K \to 0$ M $\xi \to 0$V- i?2 iQi H BMi2'M H 2M2';v TT'Q +?2b Bib KBM KmK 2[mBHB#'BmK p H$u \to u_{2[}$ - i?mb F22TBM; i?2 T'2pBQmb i'm2 2[mBHB#'BmK '2b

*QMbB/2'BM; ; BM i?2 [m2biBQM Q7 +QmTHBM; Q' MQi i?2 2[mBHB#' BMi2'M H 2M2';B2b BM 1[X U9RV- QM2 K v rBb? iQ +QMbB/2' i?2 +Q2{+B2 2' HHv /2T2M/ QM i?2 T 'iB H /2MbBivX >Qr2p2'- Bi +QmH/ #2 b?QrM i? i

4.4 Total entropy production under typical experimental conditions

$$R + R_{\mathrm{K}} \equiv F_i \nabla_i T + F_i^{\mathrm{K}} \nabla_i T_{\mathrm{K}} + J_i \nabla_i X + J_i^{\mathrm{q}} \nabla_i X_{\mathrm{q}} + Y_\xi Z_\xi + \mathcal{D} \geq 0. \qquad (98)$$

4.5 Parallel force decomposition of the total entropy production

$$\nabla_i T \sim 0, \quad \nabla_i T_{\mathrm{K}} \sim 0, \quad \mathcal{D} \sim 0, \quad J_i \ll J_i^{\mathrm{q}}, \qquad (9e)$$

$$R + R_{\mathrm{K}} = J_i^{\mathrm{q}} \nabla_i X_{\mathrm{q}} + Y_\xi Z_\xi \geq 0. \qquad (9d)$$

i?2 T?vbB+ H K2 bm'2 Q7 bHQrHv '2H tBM; ~m+im iBQMb- b /2b+'B#2/ 7 7 ' b i?2 K2+? MB+ H T 'i Q7 i?2 +QMbiBimiBp2 #2? pBQm' Bb +QM+2'M2 Q#i BM ' i2@BM/2T2M/2Mi 2t T'2bbBQMb 7Q' T_K +2H2 tB+r b M/ bQrM 7Q' 2t KTH2 7Q' /'v b M/ BM i2'Kb Q7 i?2 ;' MmH ' i2KT2' im'2 UGBm- kykRV- r?B+? Bb ?2'2 +QMbB/2' iQ T_K X

A''2bT2+iBp2 Q7 i?2b2 irQ Hi2'M iBp2 TQbbB#H2 /2+QKTQbBiBQMb- i?2 pB#' iBQM Q7 ~mB/@bQHB/ BMi2'7 +2b b?QmH/ /2+ v iQ 7m'i?2' pB#' i?2KX hQ i?Bb 2M/- R M/ R_K ?2'2 i 2'Kb b?QmH/ BM+Hm/2 bQm'+2 i2'Kb $\pm\gamma T_K^2$ U-/ bBMF i2'Kb U '2bT2+iBp2HvX ++Q'/BM;Hv- r?BH2 i?2 +QMbiBimiBp2 2[m iBQMb 7Q' i?2 bT2+B}2/ #2HQr bm+? i? i R_K ?2'2 i 2'Kb 2M/ Q7 2HKUVH2UM i'QTv T'Q/m+iBQMb rQmH/ #2 bi'B+iHv MQM@M2; iBp2- i?2 K2bQ@b+ H2 R_K BbM/2M2'B+ T'Q/m+iBQM 7'QK #2BM; M2; iBp2- M/ ??2'2 K v #2 bQ /m2 iQ i?2 +Q''2bTQM/BM; bBMF bmHi 7QHHQrb i?2 T'BM+BTH2 Q7 ıirQ@bi ;2 B''2p2'bB#BHBivö 7Q' i?2 T H ';2' iQ i?2 bK HH2' b+ H2- r?B+? r b Q'B;BM HHv T'QTQb2/ 7Q' /'v ;' M GBm- kyyNV- #mi Bb ?2'2 i F2M iQ '2~2+i bBKBH ' ?v/'Q/vM KB+b BM T bQBHbX

h FBM; HH Q7 i?2b2 BMiQ ++QmMi- i?2 Qp2' HH 2Mi'QTv T'Q/m+iBQ Bib K2bQ@'2H i2/ M/ i?2'K H +QMi'B#miBQMb- '2bT2+iBp2Hv,

$$R_K = J_i^{q}\,\nabla_i X_{qK} - \gamma T_K^2, \qquad \text{U 9 3 V}$$

$$R = J_i^{q}\,\nabla_i X_{q} + Y_\xi Z_\xi + \gamma T_K^2, \qquad \text{U 9 N V}$$

r?2'2 $\nabla_i X_{qK}$ M/ $\nabla_i X_{q}$ '2 i?2 K2bQ@'2H i2/ M/ i?2 i?2'K HHv@'2H i2/ U iQKB T 'ib Q7 $\nabla_i X_{q}$ - #2BM; i?2 +?2KB+ H TQi2MiB H ;' /B2Mi Q7 i?2 r i2' '2H iBp2 AM Q'/2' iQ '2i BM 1[X U9dV i?2B' bmKK iBQM '2[mB'2b,

$$\nabla_i X_{q} + \nabla_i X_{qK} \equiv \nabla_i X_{q}. \qquad \text{U 8 y V}$$

h?2 BMi2'T'2i iBQM Q7 i?2 #Qp2 Bb i? i i?2 ?QKQ;2MBb2/ ~mt2b BMi ;Q2b BMiQ KQpBM; K2bQ@b+QTB+ M/ iQKBbiB+ $J_i^{qK} = J_i^{q}$ 2b Q7 7'22/QK '2 J_i^{q} - v2i i?2 7Q'+2b 7'QK i?2 /Bz2'2M+2b BM +?2KB+ H TQi2MiB Hb BM i?2 72'2Mi M/ //iQ ;Bp2 i?2 iQi H ?QKQ;2MBb2/ TQi2MiB H $\nabla_i X_{q}$ X

4.6 Reciprocal conditions

++Q'/BM; iQ 1[bX U3-9dV i?2 MQM@M2; iBp2M2bb Q7 i?2 #Q R M/ R_K 2MbmB'2b i?2 Bb}2/X h?2 MQM@M2; iBp2M2bb Q7 i?2 i?2 $R \geq 0$ M/ 2KT?bBv 7'QK i?2 BM; PMb ;2'öb '2+BT'Q+ H +QM/BiBQMb 7Q' ;2M2' HHv +QmTH2/ B''2p2'b BM;Hv- 7Q' i?2 +m''2Mi T'Q#H2K

$$\begin{pmatrix} Z_\xi \\ \nabla_i X_{q} \end{pmatrix} = \begin{pmatrix} r^{\xi\xi} & r_j^{\xi q} \\ r_i^{q\xi} & r_{ij}^{qq} \end{pmatrix} \cdot \begin{pmatrix} Y_\xi \\ J_j^{q} \end{pmatrix}, \qquad \text{U 8 R V}$$

r?2'2 r2 ? p2 BMi'Q/m+2/ BM i?2 K i'Bt i?2 ;2M2' HBb2/ '2bBbiBpBiv +Q2{ Q7 i?2 i' MbTQ'i +Q2{+B2Mib BM UPMb ;2'- RNjRVVX ++Q'/BM; iQ U* bB bBbiBpBiv +Q2{+B2Mib Kmbi b iBb7v,

$$r_i^{q\xi} = -r_i^{\xi q}, \quad r_{ij}^{qq} = r_{ji}^{qq}, \qquad \text{U 8 k V}$$

r?2'2 $r^{\xi\xi} > 0$ - $r_{ij}^{qq} > 0$ 7Q' $i = j$ - M/ $\det(r_{ij}^{qq}) > 0$ X

6m'i?2'KQ'2- Bi Bb Hr vb TQbbB#H2 iQ /2}M2

$$r_i^{\xi q} = r^{\xi q}\,\hat{e}_i, \quad J_i^{q} = J_{q}\,\hat{e}_i, \qquad \text{U 8 j V}$$

r?2'2 $r^{\mathsf{q}\,\xi}$ M/ J_{q} /2MQi2 i?2 K ;MBim/2b Q7 $J_i^{\xi\mathsf{q}}$ M/ J_i^{q}- '2bT2+iBp2Hv- rBi? $\hat{e}_i$ i?2 mMBi p2+iQ' HQM; r?B+? J_i^{q} T2'7Q'K2- M/ i?2 2tT'2bbBQM +QMbi'mi2/ BM i2'Kb Q7 $\hat{e}_i$ X h?2M- bBM+2 1- i?2 }'bi PMb ;2' '2H iBQM BM 1[X U8RV '2/m+2b iQ,

$$Z_\xi = r^{\xi\xi} Y_\xi + r^{\xi\,\mathsf{q}} J_{\mathsf{q}} \,. \tag{U89V}$$

6BM HHv- ;Bp2M i?2 TQbBiBp2 /B ;QM H +Q2{+B2MIb 1[X U9NV- U8R-8k- Bi Bb +H2 ' i? i $R \geq 0$ M/ $R_{\mathsf{K}} \geq 0$ r2 Kmbi i?mb 7m'i?2' '2[mB'2

$$\nabla_i X_{\mathsf{qK}} = \eta J_i^{\mathsf{q}} \,, \tag{U88V}$$

rBi? $\eta \geq 0$ X

4.7 Water flux through finite experimental volumes

aBM+2 $\sum_i J_i^{\beta} = 0$ M/ $J_i \ll J_i^{\mathsf{q}}$ 7'QK 1[bX URk-9eV- r2 H p2 $J_i^{\mathsf{q}} \approx -J_i^{\mathsf{q}}$ X / i? i lbBM; i?2 +QMb2'p iBQM Q7 bQHB/ /2MbBiv BM 1[X URyV- Bi 7QHHQrb i? i $\partial_t \varrho_{\mathsf{a}} = -\nabla_i(\varrho_{\mathsf{a}} v_i + J_i^{\mathsf{q}})$ X >Qr2p2'- /m'BM; 2tT2'BK2Mi H r i2' '2i2MiBQM i2bib i?2 K bb Bb F2Ti +QMbi Mi- bQ r2 ? p2 $\varrho_{\mathsf{q}}\varrho_{\mathsf{a}} \approx 0$ M/ $J_i^{\mathsf{q}} = -\varrho_{\mathsf{a}} v_i$ X *QK#BMBM; i?2 H ii2' '2H @ iBQM rBi? i?2 +QMb2'p iBQM Q7 r i2' /2MbBiv BM 1[X URyV- r2 ? p2,

$$\partial_t \varrho_{\mathsf{q}} = \left(1 + \frac{\varrho_{\mathsf{q}}}{\varrho_{\mathsf{a}}}\right) \nabla_i J_i^{\mathsf{q}} \,. \tag{U8eV}$$

h?2 #Qp2 2[m iBQM '2H i2b i?2 ' i2 Q7 +? M;2 Q7 i?2 T 'iB H r i2' /2MbBiv- r i2' ~mt i?'Qm;? i?2 pQHmK2 Q7 M BM}MBi2bBK H '2T'2b2Mi iBp2 pQHmK2 2H2K2MiX h?2 H bi 2[m iBQM +QmH/ #2 +QMp2'i2/ iQ +Q''2bTQM/ iQ V +QMi BMBM; i?2 pQHmK2 i2' '2i2MiBQM b KTH2b i?'Qm;? BMi2;' iBQMX lbBM; $\int \nabla_i J_i^{\mathsf{q}} dV =$ $\oint J_i^{\mathsf{q}} dS_i = J_{\mathsf{q}} A$ U? pBM; $dS_i = \hat{e}_i dA$- rBi? A #2BM; i?2 bm'7 +2 '2 i?'Qm;? r?B+? i?2 r i2' ~Qrb BMiQ VX- i?2'27Q'2- r2 }M/ i? i

$$J_{\mathsf{q}} = \left(\frac{\varrho_{\mathsf{a}}}{\varrho_{\mathsf{a}} + \varrho_{\mathsf{q}}}\right) l \partial_t \varrho_{\mathsf{q}} \,, \tag{U8dV}$$

r?2'2 $l = \frac{V}{A}$ Bb i?2 ivTB+ H 2tT2'BK2Mi H /BK2MbBQMX h?2'27Q'2- i?2 ~mt Q7 i?2 2tT2'BK2Mi H b KTH2 b+ H2b TT'QtBK i2Hv rBi? i?2 ' i2 Q7 r i2' /2MbBiv BK2Mi H /BK2MbBQM X $J_{\mathsf{q}} \sim l\partial_t \varrho_{\mathsf{q}}$ lbBM; i?2 /2}MBiBQMb BM 1[X UjV- Bi +QmH/ HbQ T'2bb2/ BM i2'Kb Q7 ' i2 Q7 i?2 b im' iBQM M/ pQB/ ' iBQ,

$$J_{\mathsf{q}} = \mathcal{C}\left(\partial_t S_r + \frac{S_r}{e(1+e)}\partial_t e\right), \quad \mathcal{C} \equiv \mathcal{C}(S_r, e) = l\left(\frac{e}{1+e}\right)\left(\frac{\bar{\varrho}_{\mathsf{a}}\bar{\varrho}_{\mathsf{q}}}{\bar{\varrho}_{\mathsf{a}} + \bar{\varrho}_{\mathsf{q}} S_r e}\right). \tag{U83V}$$

h?2 #Qp2 Bb ;2M2' HX >Qr2p2'- BM KQbi 2tT2'BK2Mi H K2 bm'2K2Mi '2i2MiBQM T'QT2'iB2b i?2 pQB/ ' iBQ e Bb bQK2rB+Q MbB Mi- r?BH2 i?2 BMi'BMbB+ /2MbBiB2b +? M;2 7'QK i?2B' '272'2M+2,

$$J_{\mathsf{q}} = \mathcal{C}\partial_t S_r \tag{U8NV}$$

bm+? i? i i?2 r i2' ~mt BMiQ i?2 b KTH2 '2H i2b /B'2+iHv iQ i?2 ' i2 Q7 b i

4.8 Non-equilibrium soil water retention

*QK#BMBM; 1[bX U9y-99V rBi? 1[X U9j-8yV- i?2 2tT'2bbBQM 7Q' $\partial_t s \approx 0$ 7Q' 2 BK2Mi H +QM/BiBQMb- M/ bbmKBM; ?QKQ;2M2Qmb i2bBiBM; +QM/BiBQM /B2Mib +QmH/ #2 B;MQ'2/,

$$\partial_t T_{\mathsf{K}} = \frac{\omega R_{\mathsf{K}}}{T_{\mathsf{K}}} = \frac{\omega\gamma(T_{\mathsf{K}0}^2 - T_{\mathsf{K}}^2)}{T_{\mathsf{K}}}, \tag{UeyV}$$

r?2'2 mbBM; 1[X U8jV,

$$T_{\mathsf{K}0} = \sqrt{\frac{\eta}{\gamma}}|J_{\mathsf{q}}|, \qquad \text{UeRV}$$

Bb i?2 bi iBQM 'v T_{K} HQ/ $\partial_t T_{\mathsf{K}} = 0$- Q7B+? '2T'2b2Mib i?2 p Hm2 Q7 i?2 K2bQ@ '2H i2/ i2KT2' im'2 i i?2 ' i2 BM/2T2M/2M+2 HBKBi Q7 p2'v bHQr r2iiBM; /BiBQMbX h?2 KQ'2 ;2M2' H + b2 Q7 ' i2@/2T2M/2Mi T'Q+2bb2b Bb /Bb+

L2ti- /QTiBM; 1[X U89V HQM; rBi? 1[X UjNV 7Q' ?QKQ;2M2Qmb i2biB

$$\partial_t \xi + r^{\xi\xi} Y_\xi + r^{\xi\,\mathsf{q}} J_{\mathsf{q}} = 0. \qquad \text{UekV}$$

q?2M MQ r i2' Bb //2/ Q' bm#i' +i2/ 7'QK i?2 2tT2'BK2Mi H b KTH2- r $J_{\mathsf{q}} = 0$- r2 2tT2+i i?2 BMi'BMbB+ bmi ξ iQ ++Q'/BM;Hv '2Hti iQ Bib 2[mBHB#'BmK QMH r?2M i?2 K2bQ@b+QTB+ BMi2'7 +2b '2 #2BM; bm{+B2MiHv ;Bi i2/X am+ 2tT2+i2/Hv 2M #H2 iQ mMD K B'@r i2' BMi2'7 +2b i? i K v Qi?2'rBb2 #2 2[mBHB#'B HQM; T 'iB+H2 'Qm;?M2bb M/ T i+?2b Q7 ~mB/bX AM i?2 ?v i?2 2z2+ib Q7 i?Qb2 ;Bi iBQMb QM i?2 2M2';v H M/b+ T2 '2 +QMbB/2'2/ '2H i2/ 2MiiQTv ϑ_{K} Q7 i2KT2' im'2 T_{K} '2b +QM+2'Tim HBb2/ #v 6B;X 8X h FBM; i?2b2 B BMiQ ++QmMi- r2 '2[mB'2,

$$r^{\xi\xi} = r T_{\mathsf{K}}, \qquad \text{UejV}$$

rBi? $r \geq 0$- r?BH2 ;Bp2M i?2 bi iBQM 'v $T_{\mathsf{K}} = T_{\mathsf{K}0}$ bQHmiBQM 1[X UeRV,

$$r^{\xi\xi} = r\sqrt{\frac{\eta}{\gamma}}|J_{\mathsf{q}}|. \qquad \text{Ue9V}$$

_2+ HHBM; i? i $Y_\xi = k\xi$ 7'QK 1[X U99V- b r2HH b 1[X U8NV,

$$\partial_t \xi = -rk\sqrt{\frac{\eta}{\gamma}}|J_{\mathsf{q}}|\xi - r^{\xi\,\mathsf{q}} J_{\mathsf{q}}, \quad J_{\mathsf{q}} = \mathcal{C}\partial_t S_r. \qquad \text{Ue8V}$$

h?2 #Qp2 2[m iBQM '2T'2b2Mib i?2 K BM '2bmHi Q7 i?Bb T T2' ?v/ '2bQHp2/ 2[m iBQM 7Q' bQBH r i2' '2i2MiBQM i MQM@2[mBHB#'BmK +Q BMi'BMbB+ bm+iBQM ξ 2tT'2bb2/ bQ i? i Bi '2iB+ HHv iQ +v+HB+ H r2iiBM;@/'vBM iQ S_rX LQiB+2 HbQ i? i i?2 +Q2{+B2Mib $\{r, r_{\xi\,\mathsf{q}}, \eta, \gamma\}$ BM 1[X Ue8V K v HH /2T2M/ QM i?2 p '@ BQMb T 'iB H ϱ_β 2M/BM; QM i?2 B''2bT2+iBp2 iQ +v+H2 M/ i?2 bQ@+ HH2/ ξ i?2 2[mBHB#'BmK bm+iBQM rQmH/ BM ;2M2' H /2T2M/ QM i?Qb2 /2MbB QM S_r M/ eVX 6BM HHv- M 2p2M bBKTH2' 7Q'K Q7 i?Bb ?v/'Q/vM KB+ 2[m iB p2HQT2/ #v MQiB+BM; i? i 1[X Ue8V Bb 2MiB'2Hv ' i2 BM/2T2M/2Mi- bQ K iBp2b BM i?2 #Qp2 2[m iBQM +QmH/ #2 bm#biBimi2/ #v }MBi2 BM+'2K2 BXX- $\partial_t \xi \to d\xi$ dxV,

$$\boxed{d\xi = -\mathcal{A}|dS_r|\xi - \mathcal{B}dS_r} \qquad \text{UeeV}$$

r?2'2 $\mathcal{A}$ M/ $\mathcal{B}$ '2 2z2+iBp2 MQM@M2; iBp2 +Q2{+B2Mib 7Q' i?2 BMi'BMbB+ b r?B+? '2 ;Bp2M #v

$$\mathcal{A} \equiv \mathcal{A}(S_r, e) = rk\sqrt{\frac{\eta}{\gamma}}\mathcal{C}, \quad \mathcal{B} \equiv \mathcal{B}(S_r, e) = r^{\xi\,\mathsf{q}}\mathcal{C}. \qquad \text{UedV}$$

h?2 #Qp2 2[m iBQM 7Q' ξ +QmH/- b r2 rBb? iQ ?B;?HB;?i Bi b i?2 K BM '2bm i?Bb T T2'X q?BH2 i?2 /2'Bp iBQM 7QHHQrb +H2 ' v2i bm#iH2 ?v/'Q/vM K M H '2bmHi Bb 2ti'2K2Hv bBKTH2 M/ b?QmH/ i?mb Qz2' bQm'+2 Q7 BMb TB'B+ H BMMQp iBQMb BM ?v/'QHQ;v M/ bQBH K2+? MB+bX :Bp2M i?2 2[mBHB#'BmK BMBiB H +QM/BiBQMb $\xi = \xi_{\mathsf{eq}} + \xi_{2[}$ BM 1[X UjeV- +QKTH2i2 KQ/2H 7Q' bQBH r i2 '2i2MiBQM $S_r = S(S_r, e)$ +QmH/ i?2M #2 Q#i BM2/ mbBM; 1[X UjdV- ;Bp2M i?2 '2H i 7Q' i?2 2[mBHB#'BmK bm+iBQM ξ_{eq} M/ $\xi_{2[}$ +QMbiBimiBp2 '2H iBQMb iBb7vBM; 1[X UkNVVX

aBM+2 i?2 }'bi i2'K BM 1[X UeeV H-BM 2 +Hv 2 p+BM B i BQM@ i'QHb i?2 '2H t iBQM ' i2 Q7 i?2 bvbi2K # +F iQ 2[mBHB#'BmK bi i2bX h?2 MQi /2T2M/ QM i?mb i?2 +Q2 B+B2Mi +QMi'QHb i?2 /'B7i ' i2 r v 7'QK i?2 2[mBHB#'BmK bi i2X h?2 +iBQMb Q7 i?2b2 irQ i2'Kb +QKT2i2- bQ QMHv B7 i? b2Mi- rQmH/ i?2 bm+iBQM /2pB iBQM '2H t iQ x2'Q- i?mb H2iiBM; i?2 bQ 'Bp2 iQ Bib 2[mBHB#'BmK bi i2X hQ 7m'i?2' mM/2'bi M/ i?2 'QH2 Q7 i?Bb i?2 bi iBQM 'v + b2 r?2'2 i?2b2 i2'Kb # H M+2,

$$\xi_{\mathsf{a}} = -\frac{\mathcal{B}}{\mathcal{A}}\frac{dS_r}{|dS_r|} = \mp\frac{\mathcal{B}}{\mathcal{A}} \quad (7\mathrm{Q}' \ d\xi = 0), \qquad \text{Ue3V}$$

r?B+? /Q2b MQi +im HHv /2T2M/ QM i?2 bvbi2K X/ BQM Bb B2Q'BM; +QMbi MiX $\mathcal{A}$ M/ $\mathcal{B}$- QM+2 TT'Q'B i2 BMb QM Bib bi iBQXM PMv ip2 Hmi2 2' ? M/- +QMb $\mathcal{B}/\mathcal{A}$ '2 T2M/2Mi QM S_r - i?2 BMi'BMbB+ bm+iBQM H/2pB iBQM /'B7i 7Q' ξ_{a} XK L2p2'i?2H2bb- i?2 bi iBQM 'v '2H iBQMb BM 1[X Ue3V rBHH #2 mb27mH BM/B+ iBQM iQ i?2 #QmM/ 'B2b Q7 i?2 /'vBM; M/ r2iiBM; T'Q+ 'Q#mbi M/ bBKTH2 QTiBQM 7Q' $\mathcal{B}$ i?2 /'B7i 2Q' S_r 2{2+i BM i?2 '2H t iBQM +Q2 B+B2Mi +QMbi Mi,

$$\mathcal{A} = a, \quad \mathcal{B} = -b\frac{\partial \hat{s}_{2[}}{\partial S_r}, \qquad \text{UeNV}$$

r?2'2 $\{a, b\} \geq 0$ '2 MQM@M2; iBp2 +QMbi Mib X $\hat{s}_{2[} = \hat{s}_{2[}(S_r, x)$ Bb i?2 T2M/2Mi QM bQ /Q2b $\mathcal{B}$ M/ i?mb i?2 T'2/B+i2/ bQBH r i2' '2i2MiBQMX P M i?2 b HbQ p 'v rBi Qi?2' ? M/- '2+ HH i? i 1[X UeeV B;MQ'2b i?2 TQbbB#BHBiv Q7 ?vbi2'2iB #Hi2 $\neq 0$- BM+Hm/BM; mMH2bb S_r X +?Q mH ?mim' 2 tT2'BK2Mib 2tTHQ'2 M/ '2 TQ'i bm+? T?2MQK2MQM- QM2 K $\mathcal{N}_{\mathsf{q}}$ 'TQH 1[X U8 V2Q'i? Bib KQ'2 ;2M@ 2' H 7Q'K BM 1[X U83VX LQi2 i? i i?2 2Mi'QTv $d\mathcal{E} = 0$ -Q'2 } Mi$= 2\frac{\partial \hat{s}_{2[}}{\partial S_r} dS_r$ - M/ bQ 1[X UeeV vB2H/b T'Q+2bb $d\xi = -a\,dS_r|\xi| + b\,d\hat{s}_{2[}$ X 6Q' $a = 0$ BMi2;' iBQM ;Bp2b $\xi \approx$ $b\hat{s}_{2[}$- b?QrBM; i? i i?2 2ti2Mi Q7 BMi'BMbB+ bm+iBQM /2pB iBQM 7'QK Bib #'Q /Hv /2T2M/ QM i?2 2[mBHB#'BmK bm+iBQM H2p2H Bib2H7X h?Bb b+ b HHv '2~2+i2/ 7'QK 2tT2'BK2Mi H '2i2MiBQM i2bib- i?mb T'QpB/BM; i?2 ?BM/ 1[X UeNVX

4.9 Illustration of non-equilibrium results

am#b2+iBQM jX8 BHHmbi' i2b i?2 T' +iB+ H bB;MB}+ M+2 Q7 i?2 2[m M KB+ i?2Q'v #v UCB M; 2i HX- kyRdV #v 2KTHQvBM; i?2 T?2MQK2MQHQ j8VX h?Bb KQ/2H HHQrb 7Q' b? 'T2' 'Bb2 Q7 $S_r = 1$ m+?2M KQ'2 i?2 pB+BM T '2/ rBi? i?2B' H i2bi T?2MQK2MQHQ;B+ H KQ/2H BM U1BM p GBm- ky Bb MQi i?2 K BM '2bmHi Q7 i?Bb T T2'X h?2 +im H BK M/ '2bmHi '2 iQ Q7 bQBH r i2' '2i2MiBQM BM MQM@2[mBHB#'BmK +QM/BiBQMbX 6Q' i? i?Bb K BM '2bmHi #v /QTiBM; i?2 b K2 2[mBHB#'BmK 1[bX Ujj@j8V b i?2 MQM@2[mBHB#'BmK 2[m iBQMb BM 1[X Uje-j3-eeVX ++Q'/BM;Hv- i?2 Q [mB'2/ 7Q' + Tim'BM; MQM@2[mBHB#'BmK bm+iBQMb '2 a M/ bX

hQ i?Bb 2M/- 6B;X 9 T'2b2Mib i?2 2z2+ib Q7 i?2b2 T ' K2i2'b QM i?2 M BMi'BMbB+ bm+iBQM U $\xi = \xi/\sqrt{\rho K_w e^{\lambda}}$ QM U 'QrV- BMi'BMbB+ bm+iBQM U $\hat{s}^* = \hat{s}/\sqrt{\rho K_w e^{\lambda}}$ QM U KB//H2 'QrV- M/ K2 bm'2/ bm+iBQM U $s^* = s/(\rho e^{\lambda})$ QM U iQK 'QrV- 7Q' a M/ b QMb p Hm2bX h?2b2 };m'2b +Q''2bTQM/ iQ bm+iBQM@+QMi'QHH2/ T'QiQ+QH 1 M/ $b = 0$- i?2M /'vBM; +QMiBMm2b mMiBH $s^* = 20 - 70$ QKH?B+? r2 TTHB2/ Ryy bK HH BM@ +'2K2Mi H +v+H2b QM 7 bm+iBQM HQQT r2/ #v 7m'i?2' /'vBM; i@BHH iBM; iBHH M2; iBp2 bm+iBQM Q7 H /'vBM; $s^* = 0$ X iQ

LQiB+2 i? i /m'BM; +QMiBMmQmb /' BM ;2 M/ BK#B#BiBQM i?2 bi i2 T ' HH2H iQ i?2 bi iBQM 'v bQHmiBQMb b?QrM #v i?2 / b?2/ Q' M;2 HBM2 KQbi BKTQ'i MiHv- Mv /2pB iBQM 7'QK i?2 / b?2/ #Hm2 2[mBHB#'BmK +m iQ MQM@2[mBHB#'BmK bi i2bX Ai Bb b?QrM i? i BM Q'/2' iQ '2; BM i?2 2[

6B;m`2 9, h?2 2z2+ib Q7 Mi K2ii[X UeNV QM i?2 /vM KB+b Q7 i?2 BMi`B bm+iBQM /2pB+2 [X Ue eV- i?2 MQM@2[mBHB#`BmK 1[X UjNV- i?2 MQM@2[mBHB#`BmK 1[X Ue9V- HH ; BMbi bi#BHBiv bQHmiBQMb `2 ;Bp2M #v 1[X Ue3V M/ 2[mBHB#`BmK bQHmiBQMb `2 b?QrM 1[bX Uj9-jjV 7Q` i?2 irQ +Q``2bTQM/BM; bm+iBQMbX

+ M // bm{+B2Mi r2iiBM;@/`vBM; MQBb2- r?B+? + mb2b /`B7iBM; 7`QK i?
iQ i?2 2[mBHB#`BmK QM2- Q` 2t2KTH2- B;MQ`BM; i?2 Ryy bK HH +v+H2b-
`2bi Q7 i?2 +m`p2 `2p2 Hb i?2 T`2pBQmbHv 2tTH BM2/ #2? pBQ`X h?2 Hv /m##
HX- kyR8c "2`BQxFBM Jm H2K- kyR3VV, ö}`bi /` BM ;2 +m`p2ö U HbQ F
/`vBM; +m`p2öV- ö}`bi BK#B#BiBQMö U HbQ FMQrM b öT`BK `v r2iiBM; +
iBQM- M/ i?2 7QHHQrBM; öb2+QM/ `v /` BM ;2ö U HbQ FMQrM b öb2+QM
i?2 }M H H2; Q7 Qm` i2biBM; T`QiQ+QH- r?B+? /Q2b MQi ;Q `2im`M iQ 7m
1 is=0X h?Bb BM #BHBiv iQ `2; BM 7mHH b im` iBQM 7i2` bm+? /`vBM;@
+H2 ? b #22M ii`B#mi2/ iQ ö B` 2Mi` TK2Miö M Q#pBQmb bvMQMvK 7Q
#mi r2 `2 MQi r `2 Q7 +H2 ` 7mM/ K2Mi H 2tTH M iBQMb iQ i?Bb /2p2H
T`QTQbBiBQMb iQ mb2 `#Bi` `v +m`p2@}iiBM; T`QiQ+QHbX h?Bb ; T Bb
i? MFb iQ i?2 MQM@2[mBHB#`BmK i?2`KQ/vM KB+ T?vbB+b- b r2 b? HH
i?2 M2ti bm#b2+iBQM ; BMbi 2tT2`BK2MibX

6BM HHv- Bi Bb mb27mH iQ +QKT `2 i?2 KB//H2 M/ `B;?i +QHmKMb B
a Mb +Q2{+B2Mib ? p2 #22M [m /`mTH2/ i?Bb +? M;2 +QmH/ `Bb2 7Q` 2
K i2`B H #v [m /`mTHBM; i?2 2tT2`BK2MiH B/2MiB}+ iBQM [X U83V-
r?B+? #2? p2b Q/2T2M/bHBM2 `Hv QM Bi U1[bX e3VX aBM+2 i?2 ` iBQ
b=a Bb B/2MiB+ H 7Q` #Qi? +QHmKMb- i?2 bi iBQM `v HBM2b QM i?2 +Q``2
B/2MiB+ H b r2HHX q2 i?mb /Q MQi Q#b2`p2 Km+? /Bz2`2M+2b r?2M i?2 `
i?2 bi iBQM `v HBM2bX PM i?2 Qi?2` ? M/- i?2 ` i2b Q7 TT`Q +?BM; iQ i?
bB;MB}+ MiHv- 7 +i i? i +QmH/ #2 i2bi2/ 2tT2`BK2Mi HHv BM i?2 7mim`2
r i2` `2i2MiBQM i2bib 7Q` B/2MiB+ H K i2`B Hb v2i /BbiBM+iBp2Hv /Bz2`

5.2 Drying-wetting cycles with diminishing amplitude

$$S_r(t) = 1 - a + a\cos(\pi \tfrac{t}{t_1}) e^{-\frac{t}{t_2}}, \quad t \geq 0 \tag{U3yV}$$

$$d_t S_r(t) = -\tfrac{a}{t_1 t_2} \left(t_1 \cos(\pi \tfrac{t}{t_1}) + \pi t_2 \sin(\pi \tfrac{t}{t_1}) \right) . \tag{U3RV}$$

6 Conclusions

6B;m‘2 RR, .vM KB+ /‘vBM;@r2iiBM; +v+H2b rBi? /BKBMBb?BM; KTHBim
iBK2b U V b im‘ iBQM Qp2‘ iBK2c U#V K2KQ‘v 7mM+iBQM Qp2‘ iBK2c M/
r i2‘ ‘2i2MiBQM +m‘p2bX

RXi i?2 HBKBi Q7 i?2‘KQ/vM KB+ 2[mBHB#‘BmK- i?2 /2‘Bp iBQM ;Bp2b
7mHHv +QMbBbi2Mi rBi? i?2 ‘B;Q‘Qmb }M/BM;b Q7 UCB M; 2i HX- ky
iBQM + ‘27mHHv /BbiBM;mBb?2/ i?2 2ti2‘M HHv K2 bm‘2/f TTHB2/ br
+ TBHH ‘v T‘2bbm‘2V 7‘QK i?2 BMi‘BMbB+ bm+iBQM i?‘Qm;? +?2KB+
Qmi i?2 bQBH- M/ ‘2H i2/ i?2b2 iQ i?2 2z2+iBp2 bi‘2bb Q7 i?2 bQBHX
kXhQ ++QKKQ/ i2 2K2‘;2Mi 72 im‘2b i MQM@2[mBHB#‘BmK +QM/BiBQ
Q‘v /BbiBM;mBb?2/ i?2 i‘m2 i?2‘KQ/vM KB+ 2[mBHB#‘BmK 7‘QK K2i b
r b ‘;m2/ M/ b?QrM i? imMHBF2 2[mBHB#‘BmK bi i2b- K2i bi #H2 bi
mMB[m2 M/ + M #2 2 bBHv /Bbim‘#2/ #v 2ti2‘M H T2‘im‘# iBQMbX T
im‘# iBQMb + M H2 /i?2 bvbi2K iQr ‘/b Bib mMB[m2 2[mBHB#‘BmK bi
jXhrQ MQM@2[mBHB#‘BmK BMi2‘M H p ‘B #H2b M/ i?2B‘ 2pQHmiBQM 2
/m+2/ M/ /2p2HQT2/, U V i?2 BMi‘BMbB+ bm+iBQM BM UCB M; 2
/2pB iBQM 7‘QK i?2 2[mBHB#‘ i2/ BMi‘BMbB+ bm+iBQM BM UCB M; 2
i?2 K2bQ@‘2H i2/ i?2K-Tr2?Bim?2‘ +Fb /2;‘22b Q7 7‘22/QK i i?2 b+ H2 Q
;‘ BMb M/ TQ‘2bX
9Xh?2 ‘ i2ξQr7b ‘B;Q‘QmbHv /2‘Bp2/ # b2/ QM PMb ;2‘öb ‘2+BT‘Q+Biv T‘B
/BbbBT iBp2 T‘Q+2bb2b i i?2 KBTK Qbb+‘2H+QXp2?2/‘mi2BQM7; i?2
UCB M; GBm-kyyNVöb T‘BM+BTH2 Q7 irQ@bi ;2 B‘‘2p2‘bB#BHBiv- r
iQ ‘2~2+i~m+im iBM;;‘ BM KQiBQMb-#mi?2‘2 7m‘i?2‘ i F2M iQ /2b+
Q7 ~m+im iBM; KQiBQMb Q7 ~mB/T i+?2b M/ BMi2‘7 +2bX
8Xi i?2 bi iBQM ‘v THBiK2BiMQ7 i?2Q‘v T‘2/B+ib ‘ i2@BM/2T2M/2M+2-r?2
QM+2 /‘vBM; Q‘ r2iiBM; biQT i Mv ;Bp2M bm+iBQM@b im‘ iBQM TQB
K BMb K2i bi #H2X h?2 K2i bi #BHBiv ?B;?HB;?ib i? i i?2b2 bi i2b ‘
+ ‘BQmb iQ bK HH T2‘im‘# iBQMb #v r2iiBM;@/‘vBM; +v+H2b Q‘ ;‘ BM
eXq?2 MK /Q2b MQi? p2 2MQm;?iBK2 iQ ‘2H t iQ Bib bi iBQM ‘v HBKBi-
b?Qrb ‘ i2@/2T2M/2M+2- b i?2 KQiBQM Q7 ~mB/T i+?2b M/ BMi2‘7
bm+iBQM Bb /vM KB+X
dXq?2‘2 i?2 2[mBHB#‘BmK M/ MQM@2[mBHB#‘BmK bi iBQM ‘v p Hm2b
BMi‘BMbB+ bm+iBQMb /Q MQi/2T2M/ QM 2tT2‘BK2Mi H b KTH2 /BK2I
‘2p2 Hb i? i i?2 ‘ i2 Q7 i?2 TT‘Q +? Q7 i?2 MQM@2[mBHB#‘BmK bm+
‘v p Hm2 b?QmH/ /2T2M/ QM BiX

6.1 Perspectives

eXRXR 1tT2`BK2Mi H

T ‘i 7‘QK 2tTH BMBM; T‘2pBQmbHv ‘2TQ‘i2/ T?2MQK2M Q7 bQBH r i i?2 T‘QTQb2/ i?2Q‘v QT2Mb K Mv TQbbB#H2 p2Mm2b 7Q‘ 7mim‘2 ‘2b2 ‘+ 6Q‘ 2t KTH2- mb27mH 2tT2‘BK2Mi H +QMi‘B#miBQM +QmH/ #2 iQ 7m‘i?2 Q7 mMB[m2 2[mBHB#‘BmK BM T ‘iB HHv b im‘ i2/ bQBHbX hQ i?Bb 2M/- Q bK HH KTHBim/2 r2iiBM;@/‘vBM; +v+H2b 7‘QKS_pX‘ BQmb2/+v‘@2b Q7 b im‘ +H2b +QmH/ #2 /QM2 b bHQrHv b M22/2/ BM Q‘/2‘ iQ 2Mbm‘2 i?2 bvbi2K ‘ i2@BM/2T2M/2Mi ‘2bTQMb2- pBbBiBM; QMHv K2i bi #H2 bi i2bX lM/2‘ 7 ;2M2BiB2b BM 2tT2‘BK2Mi H b KTH2b KB;?i /2p2HQT- K2 MBM; i?2Q‘2iB- }2H/ Q7 K2bQ@‘2H i2T_kX>TQ2r2im2“2 i 2 +? TQBMi BM i?2 /QK BM i?2 i‘m2 HB#‘BmK ‘2i2MiBQM +m‘p2 Bb BM/2T2M/2MHQ7 i?2pK‘BMT_kBQm/2Q7 QMHv KQ/B}2b i?2 iBK2 M22/2/ iQ ‘‘Bp2 i i?Bb +m‘p2- MQi Bib 7Q‘KX

b2+QM/ 2tT2‘BK2Mi H TT‘Q +? iQ 2tTHQ‘2 i?2 +QM+2Ti Q7 mMB[m2 #2 iQ TTHv +QmbiB+ r p2bX *?QQbBM; bQmM/ r p2 i? iBb Km+? H ‘;2‘ T ‘iB+H2b- M2B;?#Qm‘BM; T ‘iB+H2b rQmH/ /2p2HQT bHB;?iHv /Bz2‘2Mi BM; iQ /BbbBT iBQM Q7 i?2 bQmM/ r p2- r?B+? ++Q‘/BM; iQ i?2 i?2Q‘v rB K2bQ@‘2H i2/ i2T_kT2r Bim2Qmi2p2M M22/BM; iQ +? S_pV2Xi?2Bbbim‘ iBQM Bb [mBi2 M HQ;Qmb iQ pBb+Qmb ?2 iBM; BM r i2‘X h?2 bK HH2‘ i?2 r p2 ‘2 i?2 p2HQ+Biv /Bz2‘2M+2b M/ /BbbBT iBQMX h?Bb T?vbB+b +? M;2b r Q7 r p2b rBi? r p2H2M;i? Km+? bK HH2‘ i? M i?2 ;‘ BMbX AM i?Bb + b2- ; i2b i?‘Qm;? ;‘ BM- i?2 /BbbBT iBQM QMHv ?2 ib mT i?2 i‘m2 i2KT2‘ im M/ K v MQi 2H_kXp i2

lbBM; 2Bi?2‘ Q7 i?2b2 T‘QTQb2/ 2tT2‘BK2Mi H T‘QiQ+QHb- r?2i?2‘ i? s +Q‘‘2bTQM/b iQ i?2 T‘BK ‘v /‘vBM; Q‘ r2iiBM; +m‘p2b- Q‘ Mv Qi?2‘ BMBi M H bi i2b b?QmH/ 2M/ mT QM mMB[m2 ‘2H S_p-QMQb?BiBMQir22M b im‘ iBQ bm+isBXQM

aBKBH ‘Hv- i?2 M2r i?2Q‘v QT2Mb M2r [m2biBQMb QM i?2 /2T2M/2M+ Q7 r i2‘ BM ;Bp2M bQBH QM i?2 /BK2MbBQM Q7 i?2 2tT2‘BK2Mi H bT2+B +mb b?QmH/ #2 QM BMbT2+iBM; i?2 bHQT2 Q7 i?2 ‘2i2MiBQM +m‘p2b mT b HbX 6m‘i?2‘KQ‘2- Bi 7QHHQr2/ 7‘QK i?2 s K?2vQHvbiQ /i?2vbHQT2biBb BM +QMbS_rMvi2imM/2‘ +v+HB+ Hep /2piBHQQMbBM + ‘27mH 2tT2‘BK2Mib i? i +Q BKTQb2 bm+? +QM/BiBQMb +QmH/ i?mb #2 BMbi‘m+iBp2 iQQX MQi?2‘ 2t T2‘BK2Mi H 2tTHQ‘ iBQM rQmH/ #2 iQ bim/v i?2 2z2+i Q7 T‘2pBQmb HQ / bi i2bX-;X+ +Q‘/BM; iQ i?2 i?2Q‘v- i?2 s Hb2Q2mHQ7 KQbi+HBQM +HQb2‘ iQ Bib 2[mBHB#‘BmS_rKri?2BH22H2iiBM; Bi‘2H t 7i2‘ 7 bi2‘ /‘vBM; UQ‘ r2iiBM;V i? M /‘vBM; UQ‘ r2iiBM;VX

eXRXk h?2Q`2iB+ H

h?2 T‘QTQb2/ i?2Q‘v HbQ QT2Mb M2r [m2biBQMb 7Q‘ 7m‘i?2‘ 7mM/ K M/ /2p2HQTK2MibX 6Q‘ 2t KTH2- r?BH2 BM i?2 +m‘‘2Mi T T2‘ i?2 ‘ i2@/2 iBQMb BMpQHp2/ +QMi‘QHH2/ b im‘ iBQM ‘ i2b- i?2b2 /Q MQi +im HHv ‘2 BM; +QM/BiBQMb BM i?2 T‘2pBQmb 2tT2‘BK2Mi H bim/B2b UhQTT 2i HX- RNdkc PmM; 2i HX- kyy8VX >2‘2- M2Bi?2‘ i?2 ‘ i2 Q7 b im‘ iBQM MQ‘ i?2 im HHv +QMi‘QHH2/- #mi ‘ i?2‘ K2 bm‘2/ i HQ+ H TQBMib BM bT +2X Ai B K2 bm‘2/ ‘ i2 Q7 b im‘ iBQM M/ BMi2;‘ i2 HQM; BiiQ bim/v i?2 T‘Q;‘2bbI Qp2‘ iBK2X Hi2‘M iBp2Hv- BirQmH/ #2 mb27mH iQ /QTi i?2 i?2Q‘v 7Q‘ b ‘v p Hm2 T‘Q#H2Kb- BMpQHpBM; M im‘ HHv ?vbi2‘2iB+ T2‘K2 #BHBiv- i?2 #QmM/ ‘v +QM/BiBQMb BM i?2b2 i2bib- r?BH2 2p Hm iBM; i?2 ‘ i2b Q7 iBQM iHQ+ H TQBMib BM bT +2 M/ b22 ?Qri?2v +QKT ‘2 rBi? i?Qb2 2tT

Appendix A Parallel flux decomposition of the total entropy production

$$R_K = J_i^{qK} r_i X_q \quad T_K^2; \qquad \text{U RV}$$

$$R = J_i^q r_i X_q + Y Z + T_K^2: \qquad \text{U kV}$$

$$J_i^q + J_i^{qK} \quad J_i^q; \qquad \text{U jV}$$

$$\begin{matrix} Z \\ r_i X_q \end{matrix} = \begin{matrix} r & r_j^{\,q} \\ r_i^q & r_{ij}^{qq} \end{matrix} \begin{matrix} Y \\ J_j^q \end{matrix}; \qquad \text{U 9V}$$

$$r_i X_q = J_i^{qK}: \qquad \text{U 8V}$$

6Q‘ +QMp2MB2M+2-‘2H2p Mi p2+iQ‘b +QmH/ #2 ‘2T‘2b2Mi2/ #v i?2B‘
MBim/2b iBK2b i?2 emHBQMp2 i?2QB‘ /B‘2+iBQMb,

$$J_i^q = J_q e_i; \quad J_i^{qK} = J_{qK} e_i; \quad J_i^q = J_q e_i: \qquad (U\ eV$$

b #27Q‘2-i?2 bi iBQM ‘v bQHmiBQM 7Q‘ i?2 K2bRQK@‘2H i2/i2KT2‘ im
0-r?B+? ?2‘2 RK2yMbi qK r iX q - bQ i? imbBM; i?2 #Qp2,

$$T_{K0} = \sqrt{-|J_{qK}|}: \qquad (U\ dV$$

AMb2‘ZBM;QK 1[X U 9V BMiQ i?2 2pQHmBBMQ1[X2UjNiB,QM 7Q‘

$$@ + r\ Y + r^q J_q = 0; \qquad (U\ 3V$$

715 bBMi+q2= r qei MAeiei = 1X

aBKBH ‘ ‘;mK2Mib 7QHHQr iQ mb2 1[X UejV-v2i iT?K2 M2r bi iBQM ‘v b
BM i?2 7QTK0 RQM 1[X U dV #Qp2 ;Bp2,

$$r = rT_K = r\sqrt{-|J_{qK}|}: \qquad (U\ NV$$

*QK#BMBM;i?2 b2+QM/PMb ;2‘‘2H iBQM BMr1iX[q UBM V-i?2 2tT‘2bbBC
1[X U 8V- M/i?2JiqK MB2M7Q[X U jV ;Bp2b,

$$J_q = \frac{J_q + r^q b}{+ r_{ij}^{qq}{}_{ij}}; \qquad (U\ RyV$$

r?2‘2j = eiej Bb i?2 E‘QM2+F2‘/2Hi X Ai i?mb 7QHHQrb i? i

$$@ = \sqrt{rb} - J_q \frac{J_q + \frac{r^q}{\sqrt{rb}}}{1 + \frac{r_{ij}^{qq}}{ij}} \quad r^q @\frac{J_q + \frac{r^q}{\sqrt{rb}}}{1 + \frac{r_{ij}^{qq}}{ij}}A: \qquad (U\ RRV$$

h?2 #Qp2‘2H iBQM b2‘p2bi?2 b K2 Tm‘TQb2 b 1[X Ue8V-r?B+?‘2br
T‘ HH2H 7Q‘+2 7Q‘KmH iBQM-v2i mMHBF2 #27Q‘2 Bi Bb MQi M2+2bb ‘B
2p2‘-mbBM;i?2 2tTJq2BbMB1QX 7QNV-‘i2@BM/2T2M/2M+2 + M ;BM #2‘2+
ii?2 HBK-BbQ7Q

$$@ = \sqrt{rb} - \frac{r_{ij}^{qq}}{r_{ij}^{qq}{}_{ij} +} Cj@S_rj \quad r^q \frac{}{r_{ij}^{qq}{}_{ij} +} \%_q C@S_r: \qquad (U\ RkV$$

bQi? ir2‘2i BMi?2K BM‘2bmHiQ7i?Bb T T2‘-b #Qt2/BM 1[X UeeV-r?
1[X UedV-i?2 +Q2{+B2Mib BM Bi ‘2 MQr ;Bp2M #v

$$A = \sqrt{rb} - \frac{r_{ij}^{qq}}{r_{ij}^{qq}{}_{ij} +} C; \quad B = r^q \frac{}{r_{ij}^{qq}{}_{ij} +} C. \qquad (U\ RjV$$

716

717 h?2‘27Q‘2-i?2 T ‘ HH2H ~mt 7Q‘KmH iBQM ‘2i BMb i?2 b K2 2pQHmiB
718 BM 1[X UeeV 7Q‘‘ i2 BM/2T2M/2Mi T‘Q+2bb2bX 6 +BM;rBi?i?2 b K2 2tT2
719 p Hm2b Q7 i?2 +AQ2MBBB2MiB?2 2pQHmiBQM 2Qm HBQ2K 7BQM bBKBH ‘-
720 v2ii?2 p Hm2b Q7 i?2 PMbfr;2‘q+Q2g+K2vM2bM2‘ HHv #2 /Bz2‘2MiX

721 6Q‘T?vbB+ H b KTH2b-mb2i?2 A:aL T2‘bBbi2Mi B/2MiB}2‘-b22i?2 A
722 a KTH2 LmK#2‘b 2i?2 TbBQMr;X ;mXQ`;fSm#HBb?@rBi?@ :lfSm#HBb?f mi?
723 @_2bQm`+2bf. i @ M/@aQ7ir `2@7Q`@ mi?Q`bOA:aL

724 +FMQrH2/;K2Mib
725 h?2 mi?Q‘b rQmH/HBF2iQ i? MF 7‘mBi7mH /Bb+mbbBQM rBi?:Bmb2TT
726 *?2M-6‘ M˘QBb:mBHH ‘/ M/.pB/_BH2vX A1 rQmH/HBF2iQ i? MF i?2 r
727 *QmM+BH 7Q‘ 7mM/BM;i?‘Qm;? ;‘ Mi MmK#2‘.SRNyRyj93dX

728 _ 2 7 2 ` 2 M + 2 b

729 H 2B - 1X - J ‘F b - "X - 1 B M p - A X U k y k R V X ? v / ‘ Q / v M K B + @ T H b i B + 7
730 K Q / 2 H H B M ; b M / m b B M ; K B M B K H ` b m 2 u i Q m 7 a T e ‘ m K T 2 i t 2 \ b e X g e H \ a n i H s
731 a n d p \ y s i H s o n U - T D R - y n 9 j e 3 i 3 d X s
732 H b ? 2 ‘ B 7 - L X - q v H H + 2 - X - G m - L X U k y R 8 V X J 2 b m ‘ B M ; i ? 2 b Q B H r
733 + m ‘ p 2 m M / 2 ‘ T Q b B i B p 2 M / M 2 ; i B p 2 K W i e B + t b m H + i n B i Q H M a ‘ e 2 ; ¦ B e K s 2 b X n X
734 ` n u } - ñ R U e 9 V - 9 9 k 9 8 R X
735 b b Q m H B M 2 - a X U k y y e V X J Q / 2 H B M ; i ? 2 ‘ 2 H i B Q M b ? B T # 2 i r 2 2 M b Q B H #
736 r i 2 ‘ ‘ 2 i 2 M i B Q M + a n d h p s 2 e X µ n n e U Ù k V - } 8 n 8 a 9 e 8 e j X
737 " 2 ‘ - C X U k k y R j V a X h i H s n T ¼ u i d s X i * n Q m m B ? 2 ‘ u * s Q h T e Q d ‘ i i B Q M X
738 " 2 H B 2 p - X u X - > b b M B x / 2 ? - a X J X U k y y R V X i ? 2 Q ‘ 2 i B + H K Q / 2 H Q 7
739 / v M K B + 2 z 2 + i b B M i ? 2 + T B H H ‘ v ‘ 2 H i B Q M 7 Q ‘ i r Q @ T ? b 2 ~ Q r B M T Q
740 ¦ } a n s q n } t i n p - V ñ T U j n V u - s 9 8 d e 8 R y X
741 " 2 ‘ B Q x F B M - X - J m H 2 K - u X U k y R 3 V X * Q K T ‘ i B p 2 M H v b B b Q 7 i ? 2
742 ‘ i B Q M ? v b i 2 ‘ 2 b B b T T ‘ Q + ? M / i ? 2 / Q K B M i ? 2 Q ‘ v Q 7 ? v b i 2 ‘ 2 b B b B M
743 T ‘ 2 / B + i B Q M Q 7 b + M M B M ; + m ‘ p 2 b M 8 / d B ‘ a 2 n M H i é s T K 2 n M ¯ X ã t e } | e]
744 s n u } ò H e - U k 8 j k e j X
745 " B b ? Q T - X q X - " H B ; ? i - : X U R N e j V X a Q K 2 b T 2 + i b Q 7 2 z 2 + i B p 2 b i ‘ 2 b b B
746 T ‘ i H v b i m ‘ i 2 W h e Q B i e h X \ o n U ù j X V u e R d d R N d X
747 " ‘ Q Q F b - _ X > X [U y R d N] e a 8 V X i H q } n q e } t X e * s Q H e T Q ‘ q / n Q } a n i u i 8 l h M e B d p 2 ‘ a @
748 b B i v X
749 * b B K B ‘ - > X " X : X U R N 9 8 V X P M Q M b ; 2 ‘ ö b T ‘ B M + B ¦ T e H 2 i Q e 7 ° K s B + ‘ Q b + Q T B + ‘
750 n T g n d e } n - p t U y k s @ i j W + s j 9 j X
751 * ? 2 M - S X - G m - L X - q 2 B - * X U k y R N V X : 2 M 2 ‘ H b + M M B M ; ? v b i 2 ‘ 2 b B b K Q /
752 ‘ 2 i 2 M i B Q M + ì n n u 2 } b n X a e n T W e n t e H \ n i H a e a n d W - e n e n v i } n n h e n t
753 o V U W R k V - y 9 y R N R R e X
754 * m 2 i Q @ 6 2 H ; m 2 ‘ Q b Q - G X - C m M 2 b - _ X U k y y 3 V X L Q M H Q + H B M i 2 ‘ 7 + 2
755 i 2 ‘ M 7 Q ‘ K i B Q M B M ; ‘ p B i v @ / ‘ B p 2 M m M b i m ‘ i 2 / p ~ \ Q y r s i ? ‘ Q m ; ? T Q ‘ Q m b
756 i H a e | e v i e o ° · U ò k e 9 t V t - e k ? 9 9 8 y 9 X
757 / 2 : 2 M M 2 b - S X : X - S ‘ Q b i G n X t U i R n U N u j N V X n X e H H a 2 n M / H Q s M S ‘ 2 b b - P t @
758 7 Q ‘ / X
759 . 2 : ‘ Q Q i - a X _ X - J x m ‘ - S X i n U h k] y e R j V u X i e i @ } i u h X t * e Q m ? h B m 2 d y n a h i H s
760 * Q ‘ T Q ‘ i B Q M X
761 1 B M p - A X - G B m - J X U k y R 3 V X > v / ‘ Q / v M K B + / 2 ‘ B p i B Q M Q 7 i ? 2 r Q ‘ F B M
762 T ‘ i B H H v b i m ‘ i 2 / n b u Q B m H a b e X n T t \ e g e H \ a n i H - s o a - n d p \ y s i H s n T
763 k y 8 k R d X
764 1 B M p - A X - G B m - J X U k y k y V X h ? 2 2 z 2 + i B p 2 b i ‘ 2 b b Q 7 m M b i m ‘ i 2 / b Q
765 M K B + + Q M M 2 + i B Q M b i Q B M i ‘ B M b B + - M i / e K ° 2 s b m m ? 2 h / b H m } + i n B s Q t M } b u X H] A M
766 t u } e s i n X } a n u U e T a T] X h j N a t e y V Y X a a e T ‘ s B M ; 2 ‘ X
767 6 ‘ 2 / H m M / - . X : X - s B M ; - X U R N N 9 V X 1 [m i B Q M b 7 Q ‘ i ? 2 b Q B H @ r i 2 ‘
768 + m ‘ p G 2 a X n a d i a n X e n t e H ¨ \ o h 9 i V + a 8 e k P a m 8 j u k } X n a e
769 6 m F m b ? B K - u X - > B ; Q - u X - J i b m b ? B K - h X - P i F 2 - u X U k y k R V X G B [m I
770 i ‘ B # m i B Q M i Q b ? 2 ‘ # 2 ? p B Q ‘ Q 7 m M b i m ‘ i 2 / b Q B H , K Q / 2 H B M ; M / T
771 K B + ‘ Q K 2 + ? M B + 8 b H K Q / M 2 e H X t - e R H R N X H a
772 : H H B T Q H B - . X U k y R k V X ? v b i 2 ‘ 2 i B + b Q B H @ r i 2 ‘ ‘ 2 i 2 M i B Q M K Q / 2 H +
773 p ‘ B i B Q M b Q 7 b m + i B Q M W e M / t p e Q H B V ¿ n Q i d B V Q X e y 8 e R e X
774 : H H B T Q H B - . X - q ? 2 2 H 2 ‘ - a X - E ‘ b i m M 2 M - J X U k y y j V X J Q / 2 H H B M ; i
775 / 2 ; ‘ 2 2 Q 7 b i m ‘ i B Q M B M / 2 7 Q ‘ K # H 2 W m Í M b t i e H ‘ \ U n 2 U / R k V Q - B H X
776 R y 8 R R k X
777 : M - u X - J ; ; B - 6 X - " m b + ‘ M 2 ‘ - : X - 1 B M p - A X U k y R j V X T ‘ i B + H 2 r i
778 K Q / 2 H 7 Q ‘ r i 2 ‘ ‘ 2 i 2 M i B Q M V Í 1 ? v b e i 2 ‘ \ 2 r b B k X U e 9 V e R 8 k e R e R X
779 > B M 2 b - q X " X U R N j y V X a i m / B 2 b B M i ? 2 T ? v b B + H T ‘ Q T 2 ‘ i B 2 b Q 7 b Q B
780 2 z 2 + i B M + T B H H ‘ v T ‘ Q T 2 ‘ i B 2 b - M / i ? 2 K Q / 2 b Q 7 K Q B b i m ‘ 2 / B b i ‘ B # m
781 i ? 2 ‘ 2 r B ¦ ? \ X e ` n u } n a e n T 8 X } - © H U u R e V t - u N] d a R e R - e H X i e n H e

> KK2`pQH/- qX GX- EMmib2M- ßX- Ap2`b2M- CX 1X- aFD¤p2H M/- aX JX
BHH `v T`2bbm`2 b+ MMBM; +m`p2b #v i?2 KB+`QmT QM2HK2K#` M2 i2+?
Q7 S2i`QH2mK a+B2M+2- kMU j1@M9V- Mk8 j BM3X
> bb MBx /2?- aX JX- *2HB - JX X- . ?H2- >X EX UkyykVX .vM KB+ 2
+ TBHH `v T`2bbm`2 b im` iBQM `2H iBQMb?BT M/ Bib BKT +ib QM m
o /Qb2 wQM2 CQm`M jH 8dX
> bb MBx /2?- aX JX- :` v- qX :X URNNjVX h?2`KQ/vM KB+ # bBb Q7 + T
bm`2 BM TQ`Qmb K2/B X Q m`+2b- kNU R y V-+?jj3N j9y8X
>2HKB;- _X- q2Bbb- X- qQ?HKmi?- "X AX UkyydVX .vM KB+ + TBHH `v
2`Q;2M2Qmb TQ`Qmb K2/B X M H :2QbV- k2 kd9X
>QKbv- :X JX URN3dVX oBb+Qmb }M;2`BM; BM TQ`Qmb K2/B X K2@
+? M- RRNURV- kdR jRRX
>QTK Mb- CX- . M2- CX URN3eVX h2KT2` im`2 /2T2M/2M+2 Q7 bQBH
+m`p2b H a+B2M+2 aQ+B2iv Q7 K2`B+ CQm`M H
C2MMBM;b- CX 1X "X- "m`H M/- CX "X URNekVX GBKBi iBQMb iQ i?
bi`2bb2b BM T `iHv b im`i2/ bQBHb X Rk8@R99X
CB M;- uX- 1BM p- AX- GBm- JX UkyRdVX i?2`KQ/vM KB+ i`2 iK2Mi Q
m` i2/ bQBHb `2p2 HBM; i?2 bi`m+im`2 Q7 +QmTH2/ 2bbMiB H+b
M/ S?vbB+b Q7 aQHB/b R9eX
CB M;- uX- GBm- JX UkyyNVX :` MmH :` bM HB /?Jv-+R/jVM KB+bX
RjNX
CB M;- uX- GBm- JX UkyR8VX TTHvBM; :a> iQ rB/2` M;2 Q7 2tT2`BK2
mH ` K2+? MB+b Q7 S?vbB+b- jRU9V- kdX1
E K`BM- EX- "Qm+?#BM/2`- 1X UkyR9VX hrQ@i2KT2` im`2 +QMiBMm
+? MB+b Q7 /27Q`KBM; KQ`Q?QmbbQ7 Hi2 2+? MB+b M/ S?vbB+b
Q7 aQHB/b- e N k33X
E? H iMBFQp- AX JX MUkyR9VX +iBQM iQ i?2 i?2Q`v Q7 bmT2`~mB/Biv
S`2bbX
E? HBHB- LX- w `; `# b?B- aX UkyRyVX AM~m2M+2 Q7 ?v/` mHB+ ?vb
bi`2bb BM mMb im`i2/ bQBHb- dkN dj9X
G M/ m- GX .X- GB7b?Bix- 1X aX iBbHB+ 3HXTXv bm i2`rQ`i?@
>2BM2K MMX
G M/ m- GX .X- GB7b?Bix- 1X `Q`i?X
GB- aX sX- S2M;` - .X "X- qQM;- SX@xX URNN8VX PMb ;2`öb `2+BT`Q+
?v/` mHB+ T2`K2 #BHBiv QM TQ`QmbX- 8d93X
GBm- JX UkykRVX h2KT2` im`2b BM S:GBM @T`Q+2bbV G2ii2`bV
Rj8UjV- jeyyjX
JQ``Qr- LX _X URNdyVX S?vbB+b M/ i?2`KQ/vM KB+b Q7 + TBHH `v
K2/B X KQiBQM BM 2K2`;2 +2 Q7 +QM- jk 8d3X
JQvM2- *X- Jm` /- JX X UkyyeVX irQ@b+ H2 KQ/2H 7Q` +QmTH2/ 2H
K2+? MB+ H T?2MQK2M M/ QMb ;2`öb `2+BT`Q+Biv `2H iBQMb BM 2
?QKQ;2MBx iBQM` MbTQ`i BM TQ`Qmb K2/B j3yX
Jm H2K- uX URNdeVX M2r KQ/2H 7Q` T`2/B+iBM; i?2 ?v/` mHB+ +QM/m
m` i2/ TQ`Qmb K2/B X Q m`+2b- R jk 8kkX
Jm` H22i? ` M- EX EX- GBm- *X- q2B- *X- EB##2v- hX *X- *?2M- GX Uk
2H biQTH iB+ 7` K2rQ`F 7Q` +QmTH2/ ?v/` mHB+ M/ K2+? MB+ H
mMb im` i2/ TQ`Qmb K2/B X KQ/2HHBM; BM :2QK2+? MB+b- jR 9 k9
PMb ;2`- GX URNjRVX _2+BT`Q+ H `2H iBQMb BM B``2p2`bB#H2 T`Q+2b
jdU9V- 9y8X
PmM;- PX- > bb MBx /2?- aX JX- "2xmBD2M- X Ukyy8VX hrQ@T? b2 ~Q
BM ;2Q+2Mi`B7m;2 M/ i?2 bB;MB}+ M+2 Q7 /vM KB+ + TBHH `v T`2
CQm`M H Q7 SQ`Qmb J2/B
S b? - X uX- E?Qb?;? H#- X- E? HBHB- LX UkyRdVX >vbi2`2iB+ KQ/2
iBQM Q7 r i2` `2i2MiBQM +m`p2 Q7 1BQ M22`BM; J2+? M@
B+b 9U dV- y9yRdyjX

3jd SQmHQp bbBHBb- X URNdyVX h?2 2z2+i Q7 i?2 2Mi` TT2/ B` QM i?2 ?v
TQ`Qmb #Q/v M/ QM Bib ?v/` mHB+ +QM/m+iBpBiv X RekX
3j3 _m#BM- JX "X- 1BM p- AX UkyRRVX H `;2 /27Q`K iBQM #`2 F ;2 KQ/2
K i2`B Hb BM+Hm/BM; TQ`QbBiv M/ BM2H biB+ M/ b2i iBQM H /27Q`K
M iBQM H CQm`M H Q7 1M;BM22`BM; aRB2M+2X
3jN aFD 2p2H M/- aX- aB[p2H M/- GX- EDQb pBF- X- h?QK b- qX- oB`MQpb
* TBHH `v T`2bbm`2 +Q``2H iBQM 7Q` K i2`B Hb bQ2 /2i2`KBM iBQM H
iBQM 1M;BM22`BM; edX
39y hQTT- :X- EHmi2- X- S2i2`b- .X URNedVX *QKT `BbQM Q7 r i2` +QM
?2 / / i Q#i BM2/ #v 2[mBHB#`BmK- bi2 /v@bi i2- M/ mMbi2 /v@bi i2
aQBH a+B2M+2 aQ+B2iv Q7 K2`B+ CQm`M H
39R o +? m/- :X- o m+HBM- JX- q FBH- JX URNdkVX bim/v Q7 i?2 mMB[m2M
KQBbim`2 +? ` +i2`BbiB+ /m`BM; /2bQ`TiBQM #v T`2bbm`2 +2HH @ BM ;2X
+B2iv Q7 K2`B+ CQm`M H 8jkX
39k o mM i- CX- * bBMB- 6X UkyRdVX T`Q+2/m`2 7Q` i?2 /B`2+i /2i2`KBM
T ` K2i2` 7`QK +? M;2b BM TQ`2 bBx2 /Bbi`B#miBQM- X
ejR ejeX
39j oB;;B MB- :X- iFBMbQM- CX URNN8VX AMi2`T`2i iBQM Q7 #2M/2` 2H2K
MB[m2b URV- R9N R89X
399 q M;- aX- hQFmM ; - hX EX- q M- CX- .QM;- qX- EBK- uX UkyReVX *
T`2bbm`2@b im` iBQM `2H iBQMb BM [m `ix M/ + `#QM i2 b M/b, GB
+Q``2H iBM; + TBHH `v M/ r2ii #BHBiv BM~m2M+2b QM B`- QBH- M
i` TTBM;X _2bQm`+2b- 8kU3V- eedR eeNyX
398 q `/H r- LX *X- h vHQ`- _X URNdeVX J2`+m`v + TBHH `v T`2bbm`2 +m
BMi2T`2i iBQM Q7 TQ`2 bi`m+im`2 M/ + TBHH `v #2?@pBQm` BM `2b2
H2iBM Q7 * M /B M S2i`QH2mK k2QHQ;vX
39e qBMFH2`- JX- :D2MM2bi /- JX X- "2/2 mt- .X- ED2Hbi`mT- aX- * #`BQHm
> Mb2M- X UkykyVX PMb ;2`@bvKK2i`v Q#2v2/ BM i?2`K H K2bQ
i2Kb, hrQ@T? b2 ~Qr BM TQ`Qmb K2/B - 3S2e yXB+b
39d w? Q- "X- J +JBMM- *X qX- axmH+x2rbFB- JX GX- L2m72H/- CX X- >mTT2
Cm M2b- _X UkyRjVX AMi2`7 +2 TBMMBM; Q7 BKKBb+B#H2 ;` pBiv
TQ`Qmb K2?Bb X+ H _2bU2kV- ykjyR8X
393 w?Qm- X@LX UkyRjVX +QMi +i M;H2@/2T2M/2Mi ?vbi2`2bBb KQ/2H
i2MiBQM #2?@pBQm` M/ :2Q9N- e@8kX
39N w?Qm- X@LX- a?2M;- .X- * `i2`- CX SX UkyRkVX JQ/2HHBM; i?2 2z2+i
QM bQBH@r i2` +? ` +i2`BbiB+b- 9N- e 3yX