

Hydrodynamics of non-equilibrium soil water retention

Itai Einav¹ and Mario Liu²

¹University of Sydney

²Universitat Tübingen

November 28, 2022

Abstract

Water retention in soil exhibits diverse phenomena, including suction-saturation hysteresis, non-unique air entrapment at zero suction and negative suction under partial saturations. The constancy of suction after a long rest can be broken by relatively minor mechanical or hydraulic agitations such as low-amplitude wetting cycles – this fact is here being related to metastable states that differ from the true equilibrium. The complete suction-saturation relationships are thus being recovered using non-equilibrium Landau’s hydrodynamic theory and Onsager’s reciprocity principles. Equilibrium suction does not pertain to hysteresis, yet can be approached through small amplitude agitations over long duration. Conditions for rate independence are being described, while rate-dependency are also accommodated and illustrated. Finally, it is shown that the new non-equilibrium theory retains the rigorously derived equilibrium result of the effective stress of partially saturated soils.

Hydrodynamics of non-equilibrium soil water retention

Itai Einav¹, Mario Liu²

¹ $\frac{av}{M2v} * 2Mi^2 BM : 2QK2 + ? MB + b \frac{M}{JBMBM}; J i2^B Hb - a + ? QQH Q7 * BpBH 1M; BM22^B M$
 $\frac{av}{M2v} - kyye - \frac{av}{M2v} - mbi^H B$
² $h?2Q^2 iBb + ?2 S?vbBF - 1MBp2^b Bi i hC#BM; 2M - dkyde hC#BM; 2M - :2^K Mv$

Key Points:

- $i?Q'Qm; ?Hv^B; Q'Qmb ?v/'Q/vM KB + i?2Q'v 7Q'T^iB HHv b im' i2/QT2/i? i/BbiBM; mBb?2b K2i bi #H2 7'QK i'm2 2[mBHB#'BmK bi i2b$
- $.2T^im'2 7'QK i'm2 2[mBHB#'BmK Bb + QMi'QHH2/#v ;^MmH^i2KT2MB\}2b K2bQb + QTB + KQiBQM b Q7 T^iB + H2 M/ \sim mB/ BMi2'7 + 2b$
- $h?2 i?2Q'v 2tTH BMb i?2^B + ? + QKTH2tBiv Q7 bQBH r i2'^2i2MiBQM - ?vbi2'2bBb - T i?@/2T2M/2Mi B' 2Mi' TK2Mi M/ M2; iBp2 bm + iBQM$

Corresponding author: Itai Einav, itai.einav@sydney.edu.au

Abstract

q i2' '2i2MiBQM BM bQBH 2t?B#Bib /Bp2'b2 T?2MQK2M - BM+Hm/BM; bm
bBb- MQM@mMB[m2 B' 2Mi' TK2Mi i x2'Q bm+iBQM M/ M2; iBp2 bm+iBQ
m' iBQMbX h?2 +QMbi M+v Q7 bm+iBQM 7i2' HQM; '2bi + M #2 #'QF2M
K2+? MB+ H Q' ?v/' mHB+ ;Bi iBQMb bm+? b HQr@ KTHBim/2 r2iiBM; +v
#2BM; '2H i2/ iQ K2i bi #H2 bi i2bi? i /Bz2' 7'QK i?2 i'm2 2[mBHB#'BmK
bm+iBQM@b im' iBQM '2H iBQMb?BTb '2 i?mb #2BM; '2+Qp2'2/ mbBM; M
?v/'Q/vM KB+ i?2Q'v M/ PMb ;2'öb '2+BT'Q+Biv T'BM+BTH2bX 1[mBHB#
T2'i BM iQ ?vbi2'2bBb- v2i + M #2 TT'Q +?2/ i?'Qm;? bK HH KTHBim/2
/m' iBQM X *QM/BiBQM b 7Q' ' i2 BM/2T2M/2M+2 '2 #2BM; /2b+'B#2/- r?B
'2 HbQ ++QKKQ/ i2/ M/ BHHmbi' i2/X 6BM HHv- Bi Bb b?QrM i? i i?2 M
i?2Q'v '2i BMb i?2 'B; Q'QmbHv /2'Bp2/ 2[mBHB#'BmK '2bmHi Q7 i?2 2z2+
b im' i2/ bQBHbX

1 Introduction

h?2 +QMM2+iBQM Q7 B' M/ r i2' T'2bbm'2b iQ i?2B' /2MbBiB2b BM T
bQBHb Bb MQ'K HHv 2tT'2bb2/ #v '2H iBM; bm+iBQM U B' KBMmb r i2' T'
Q7 b im' iBQM M/ TQ'QbBivX h?2b2 bQBH r i2' '2i2MiBQM '2H iBQM b?BT
i2'M iBp2Hv b + TBHH 'v T'2bbm'2@b im' iBQM '2H iBQM b- '2+'BiB+ H
im'2- ;2Qi2+?MQHQ;v M/ T2i'QH2mK 2M;BM22'BM;- b i?2v +QM i'QH i' M
kyRjc > bb MBx /2? 2i HX- kyykV M/ 2z2+iBp2 bi'2bb2b UCB M; 2i HX- k
kyRdVX 6'QK T'2pBQmb 2tT2'BK2Mi H '2b2 '+? Q7 i?2b2 '2H iBQM b?BTb
#22M B/2MiB}2/X JQbi;2M2' HHv- mM/2' 2H2p i2/ bm+iBQM b i?2 KQmMi
'2/m+2b U''QQFb- RNe8c Jm H2K- RNdeV- r?BH2 mM/2' +QMbi Mi bm+iBQ
m' iBQM /2T2M/b QM i?2 TQ'QbBiv U: HHTQHB 2i HX- kyyjc bbQmHBM
'vBM; 7'QK 7mHH b im' iBQM Q++ bBQM HHv '2[mB'2b H '2; 2bm+iBQM iQ
i?2 r i2' +QM i2Mi Q' b im' iBQM U6'2/HmM/ sBM;- RNN9c S b? 2i HX- k
KQM QiQM QmbHv 7'QK /'v bi i2b + MMQi '2; BM 7mHH b im' iBQM rBi?Qm
H2iiBM; r i2' T'2bbm'2; 2ib H '2; i? M i?2 B' T'2bbm'2- K2 MBM; 2bb2Mi
iBQM Q' + TBHH 'v T'2bbm'2 UaFD 2p2H M/ 2i HX- kyyyc Hb?2'B7 2i HX
*?2M 2i HX- kyRNVX IM/2' x2'Q bm+iBQM i?2 /2; '22 Q7 b im' iBQM Bb H
#'Q /Hv /2T2M/b QM i?2 K tBK Q7 T'2pBQmbHv TTHB2/ bm+iBQM USQm
H r h vHQ'- RNdec > KK2'pQH/ 2i HX- RNN3VX *v+H2b Q7 r2iiBM; UBK#
BM; U/' BM ;2V T'2b2Mi ?vbi2'2iB+ Ub+ MMBM;V '2bTQM b2 U> BM2b- RN
kyRjc S b? 2i HX- kyRdVX q?BH2 i?2 2z2+iBp2 bi'2bb Q7 bQBHb ? b #22
iQ bQBH r i2' '2i2MiBQM mM/2' i?2'KQ/vM KB+ 2[mBHB#'BmK +QM/BiBQM
7Q' HH T' +iB+ H Tm'TQb2b i?2 BKT +i Q7 ?v/' mHB+ ?vbi2'2bBb QM Bi b
2tT2'BK2Mi HHv THQii2/ ; BMbi b im' iBQM UE? HBHB w ' ; '# b?B- kyF

AM i?2 r2HH@FMQRm rQ'F Q7 U> bb MBx /2? : ' v- RNNjV i?2'KQ/vM
iQ 'iB+mH i2 MmK#2' Q7 mb27mH '2bi'B+iBQM bTBMH Q'BT r2b2m22MiBQ
b im' iBQM V '2H iBQM b?BTbX >Qr2p2'- i?2'2 Bb +m''2MiHv MQ i?2'KQ/v
T #H2 Q7 2tTH BMBM; HH i?2 #Qp2 HBbi2/ Q#b2'p iBQM b mbBM; bBM;
bBbi2Mi T?vbB+ H i'2 iK2Mi i?2 Tm'TQb2 Q7 i?Bb T T2' Bb iQ '2bQH p2 i
BM; T'2pBQmb 7Q'KmH iBQM 7Q' i?2'KQ/vM KB+ HHv 2[mBHB#' i2/ bQE
kyRdc 1BM p GBm- kyR3- kykyV iQ MQM@2[mBHB#'BmK +QM/BiBQM bX
MQK2M Q7 ?v/' mHB+ ?vbi2'2bBb- B' 2Mi' TK2Mi- T i?@/2T2M/2Mi b im
iBQM M/ M2; iBp2 bm+iBQM rBHH HH #2 '2bQH p2/ mbBM; bBM;H2 2pQH
iBQM /2pB iBQM 7'QK r2HH /2}M2/ MQM@?vbi2'2iB+ i'm2 2[mBHB#'BmK
Qmi M22/BM; iQ i F2 Mv //BiBQM H bbmKTiBQM bX

h?2 +m''2Mi /2'Bp iBQ My7Q'HHQrba?2i-H?2B?m?Hre d Q}B; BM HHv
7Q'KmH i2/ #v G M/ m UG M/ m GB7b?Bix- RN3y- RN3dV- M/ 2ti2M/2/ iQ
~mB/Biv UE? H iMBFQp- kyR3V M/ HB[mB/ +?vbi Hb U/2 :2MM2b S'Qbi-

64 i?2 +QM+2Ti Q7 irQ@bi ;2 B''2p2'bB#Hv BM ?2i2'Q;2M2Qmb K2/B r b BM
 65 GBm- kyyNV- M/ /QTi2/ 7Q'; 'MmH 'K i2'B Hb UCB M; GBm- kyyN- ky
 66 kykRVX aBKBH 'Hv- UCB M; 2i HX- kyRdV ? p2 ?v/'Q/vM KB+ HHv /2'Bp
 67 Q7 T 'iB HHv b im' i2/ bQBHb BM i?2'KQ/vM KB+ 2[mBHB#'BmK- rBi?Qmi
 68 Qp2'r?2HKBM;Hv +QKTH2t b? T2b M/ /Bbi'B#miBQM Q7 B'@r i2'@bQHB
 69 i?2 7 +ii? i HH i?2b2 7 +iQ'b '2 +im HHv 2M+Q/2/ BM i?2 bQBH r i2' '2
 70 >2'2- i?2B' '2bQH p2/ '2H iBQM b?BT #2ir22M i?2 2z2+iBp2 bi'2bb M/ bQB
 71 b?QrM iQ #2 p HB/ 2p2M mM/2' MQM@2[mBHB#'BmK +QM/BiBQM bX

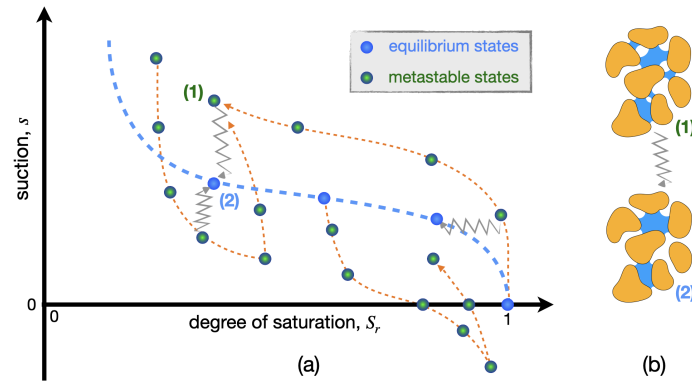
72 hQ ++QKKQ/ i2 7Q' MQM@2[mBHB#'BmK '2H iBQM b #2ir22M ~mB/ T'
 73 iB2b- i?2 +m''2Mi i'2 iK2Mi /QTib i?2 PMb ;2'öb '2+BT'Q+Biv '2H iBQM b
 74 .2 : 'QQi J xm'- kyRjV 7Q' i?2 +Q''2bTQM/BM; /BbbBT iBp2 KB+'Qb+QTB
 75 '2H2p M+2 Q7 i?2b2 '2H iBQM b ? b #22M MQi2/ 7Q' TQ'Qmb K2/B #v U"2
 76 2'b UGB 2i HX- RNN8c JQvM2 Jm' /- kyyec qBMFH2' 2i HX- kykyV- i?Qn
 77 mb2/ iQ 2tTH BM ?vbi2'2iB+ bQBH r i2' '2i2MiBQM BM bQBHX 6m'i?2'KQ'
 78 KQ/vM KB+ 7Q'KmH iBQM b Q7 T 'iB HHv b im' i2/ bQBHb- r2 /Q MQiM2;H
 79 Q'Qmb /BbiBM+iBQM UCB M; 2i HX- kyRdV #2ir22M i?2 2ti2'M HHv TTH
 80 iBQM M/ i?2 BMi'BMbB+ bm+iBQM i? i +im HHv /2p2HQTb BM i?2 K2/B X
 81 /2' K ;MBim/2 /Bz2'2Mi- v2i i?2v '2 bi'B+iHv +QMM2+i2/ i?2'KQ/vM KB+

82 MQi?2' F2v bi2T BM i?2 +m''2Mi ?v/'Q/vM KB+ /2'Bp iBQM Bb i?2 BM-
 83 i? M QM2 b2i Q7 i2KT2' im'2b M/ 2Mi'QTB2bX Hi?Qm;? BM r i2' '2b2 '+?
 84 K H i2KT2' im'2 ? b #22M +QM bB/2'2/ T'BQ' iQ i?Bb rQ'F- //BiBQM H i2K
 85 #22M ++QmMi2/ 7Q' BM +QM iBMMmK 7Q'KmH iBQM b Q7 ;' MmH ' M/ KQ
 86 GBm- kyyNc E K'BM "Qm+?#BM/2'- kyR9VX >2'2 r2 /QTi i?2 irQ bi ;2 B
 87 T'BM+BTH2 7Q' 2M2';v ~Qr #2ir22M i2KT2' im'2b Q7 /BbiBM+i b+ H2b Q7
 88 kyyNV- M/ B/2MiB7v irQ b2ib Q7 i2KT2' im'2b M/ 2Mi'QTB2bX h?2 }'bi b
 89 p2MiBQM H i?2'K H i2KT2' im'2M M/ 2Mi'QTB2bX h?2 }'bi b
 90 /QK M/ ~m+im iBM; KQiBQM Q7 2H2K2Mi 'v iQKBbiB+ T 'iB+H2bX h?2 b
 91 i?2 K2bQ@'2H i2/ i2KT2' im'2M M/ 2Mi'QTB2bX h?2 }'bi b
 92 7'22/QK M/ ~m+im iBM; KQiBQM Q7 K2bQb+QTB+ 72 im'2b i i?2 b+ H2 Q
 93 i?2 r i2'@ B'@T 'iB+H2 BMi2'7 +2b BM #2ir22MX h?2 b+ H2b Q7 i?2b2 K2
 94 +QKT ' #H2 M/ Qp2'r?2HKBM;Hv H ;'2' i? Mi? i Q7 iQKb- M/ i?mb i?2B
 95 '2 HmKT2/ rBi?BM i?2T'BM;H2 b2i Q7

96 h?2'K H 2z2+ib QM bQBH r i2' '2i2MiBQM ? p2 #22M '2TQ'i2/ 2tT2'BK
 97 K Mb . M2- RN3eVX h?2b2 +QmH/ #2 M im' HHv ++QmMi2/ 7Q' BM i?2 i?
 98 7Q' 7mim'2 +QM bB/2' iBQM b bBM+2 i?2v i2M/ iQ #2 bK HH BM T' +iB+ H '
 99 i?2 Qi?2' ? M/ - i?2 /2T2M/2M+2 Q7 bQBH r i2' '2i2MiBQM QM i?2 ~m+im i
 100 T 'iB+H2b M/ ~mB/ @bQHB/ BMi2'7 +2bX iB'Qm; #22M T'QMB@/2'2/ U
 101 Qmb i?2'KQ/vM KB+ +QM bB/2' iBQM bc ?2'2- i?Bb +QM+2Ti Bb #2BM; /2p
 102 iQ + Tim'2 '2H t iBQM iQr '/b i?2 H2 bi 2M2';v bi i2b Q7 i'm2 2[mBHB#'Br

103 h?2 bB;MB}+ M+2 Q7 i?2 ii'm2 2[mBHB#'BmKö7Q' bQBH r i2' '2i2MiBQ
 104 +H2 'Hv /2KQM bi' i2/- M/ i?2 +Q''2bTQM/BM; bi i2b '2 b?QrM iQ #2 mME
 105 K i2'B HX h?Bb Bb BM +QM i' bi iQ i?2 T'2pBQmb i?2'KQ/vM KB+ B/2 b #v
 106 M/ U> bb MBx /2? : 'v- RNNjV- r?B+? BMpQH p2/ irQ /Bz2'2Mi 2[mBHB#'B
 107 7Q' /'vBM; M/ r2iiBM;X >Qr2p2'- bm+? KQ/2Hb 7 BH iQ /2+B/2 r?2'2 iQ '2
 108 bB/BM; BM #2ir22M- mMH2bb #2BM; 7Q'+2/ iQ ' #Bi' 'BHv U"2HB 2p > b
 109 bm+? i?2 T'2pBQmb KQ/2Hb + MMQi 2tTH BM r?v i?2 bQBH + M/2p2HQT M
 110 M2; iBp2 + TBHH 'v T'2bbm'2bVX aBKBH 'Hv- i?2v + MMQi 2tTH BM i?2 M
 111 m H b im' iBQM- B' 2Mi' TK2Mi- M/ i?2 TT'Q +? Q7 bm+iBQM iQr '/b 7
 112 r2iiBM; @/'vBM; +v+H2b rBi? /BKBM bB?BM; KTHBim/2 UJm' H22i? ' M 2i

113 AM +QM i' bi- i?2 +m''2Mi 7Q'KmH iBQM Bb b?QrM iQ T'2/B+i HH Q7 i'
 114 7'QK }'bi T'BM+BTH2bX i MQM@2[mBHB#'BmK +QM/BiBQM b- i?2 '2i2MiE
 115 QMHv i b? HHQr 2M2';v KBMBK X h?2b2 KBMBK '2 ' i?2' T'2+ 'BQmb- #



6B;m'2 R, *QM+2Tim H /BbiBM+iBQM #2ir22M 2[mBHB#BmK M/ K2i bi #
'2i2MiBQM b?QrBM; K2i bi #H2 bi i2b pBbBi2/ 7i2' KQM QiQMB+ /'vBM; Q
Q' M; 2 HBM2bV M/ i'm2 2[mBHB#BmK bi i2b U/ b?2/ #Hm2 HBM2V TT'Q
im/2 T2'im' # iBQM b Q7 bm+iBQM Q' b im' iBQM U; '2v xB; x ;bV- Q' 2p2M
/vM KB+b Q7 ~mB/ T i+?2b M/ BMi2'7 +2b i? i ++QKT Mv bm+? T2'im' # i

116 #2 2 bBHv H27i #2?BM/ mTQM bK HH ?v/' mHB+ Q' K2+? MB+ H pB# iBQ
117 BM; iQ i?2b2 KBMBK e t' a QMHQ e2eM Qi +im HHv BM 2[mBHB#BmK Ub22 6B
118 PM i?2 Qi?2' ? M/- #v +QM iBMMQmbHv //BM; 7m'i?2' ?v/' mHB+ Q' K2+?
119 iQ i?2 bQBH- i?2 bi i2 + M ;' /m HHv TT'Q +? i?2 i'm2 2[mBHB#BmK bi i
120 #2+ mb2 KB+'QK2+? MB+ HHv i?Bb + M mMHQ+F i?2 KQiBQM Q7 Qi?2'rB
121 T i+?2b Ub22 6B; R#VX 1tT2'BK2Mib i? iBKTQb2 bm+? pB# iBQM b i #Q
122 ' M 2i HX- kyyNV M/ M2; iBp2 U Hb?2'B7 2i HX- kyR8V bm+iBQM b '2 p
123 im'2- M/ bmTTQ' i i?2 T'2b2M+2 Q7 QMHv bBM; H2 i'm2 2[mBHB#BmK b

124 S 'iB HHv b im' i2/ bQBHb '2 i?'22@T? b2 K2/B - r?2'2 2 +? T? b2 BM
125 i?2 Qi?2'bX ++Q'/BM; iQ i?2 T'QTQb2/ i?2Q'v- bK HH bQHB/ T 'iB+H2 pB
126 TH2- 2t+Bi2/ #v +QmbiB+ r p2bV rQmH/ + mb2 i2iQ22bH2Q' 22Xi2/ i2KT2'
127 AM+'2 b2/ pT' H2 Q7 bi'QM; 2' KQiBQM Q7 HQ+ HBb2/ ~mB/ T i+?2bX AM
128 i?2b2 T i+?2b KB; ' i2 iQ 2tTHQ'2 Qi?2' HQ+ iBQM b BM i?2 TQ'2 bT +2b- r
129 7 pQm' #Hv '2biX h?Bb KQiBQM Bb ++QKT MB2/ #v i?2 Hi2' iBQM b Q7 B
130 ;2QK2i'B2b Ub22 6B; R#V- M/ i?mb +? M; 2b iQ i?2 K2 bm'2/ bm+iBQM X
131 ;Bi iBQM bi Q7 b2Hm2b # +F iQ x2'Q- i?2 bvbi2K ii BM M2r K2i bi #H2 b
132 Q7 bm+iBQM- r?B+? K v TQbbB#Hv HB2 QM i?2 i'm2 2[mBHB#BmK +m'p2
133 2p2M M22/BM; iQ 2tT2'B2M+2 Mv +? M; 2 iQ K? B7Bp' 2B Q7H2im' QnB; QM
134 2tT2'BK2Mi BHHmbi' i2b r?v T 'iB+H2 KQiBQM b?QmH/ MQi #2 /Bb+QMM2
135 '2i2MiBQM X h?2 T'QTQb2/ ?v/'Q/vM KB+ i?2Q'v +QmbB/2'b i?Bb 7Q'K Q7
136 iBK2- b r2HH b '2+Qp2'BM; i?2 Qi?2' 7Q'K Q7 bQHB/ @~mB/ +QmTHBM; #
137 M/ ~mB/ T'2bbm'2b BM bQBHbX

138 h?2 T T2' Bb Q'; MBb2/ b 7QHHQrbX AM a2+X ki?2 ?v/'Q/vM KB+ T'B
139 iB HHv b im' i2/ bQBHb rBHH #2 '2pB2r2/- #2BM; p HB/ 7Q' #Qi? 2[mBHB
140 +QM/BiBQM bX " b2/ QM i?2b2 ;2M2' H T'BM+BTH2b- i?2 7QHHQrBM; a2+
141 pBQmbHv /2'Bp2/ bT2+B}+ '2bmHib Q7 T 'iB HHv b im' i2/ bQBHb mM/2' i
142 HB#BmKc i?2 T'2b2Mi iBQM rBHH // M2r +QM i2ti M/ 2t KTH2b iQ bmTTQ
143 K BM T 'ib Q7 i?2 T T2'X AM i?2 7QHHQrBM; b2+iBQM b i?2 7Q'KmH iBQM
144 MQM@2[mBHB#BmK +QM/BiBQM bX AM a2+X 9 i?2 i'2 iK2MirBHH 7Q+mb
145 +2bb2bc r?BH2 a2+X 8 rBHH 2tT M/ i?Bb 7Q'KmH iBQM iQ ' i2@/2T2M/2M
146 i?2 HBKBi Q7 bHQr /'vBM; M/ r2iiBM; T'2/B+ib bBKbH ' '2bmHib b HQM
147 HBKBi BM a2+X 9X "Qi? Q7 i?2b2 b2+iBQM b rBHH BM+Hm/2 MmK#2' Q7

148 $bB;MB\}+M+2Q7i?2M2ri?2Q'vX6BMHHv-BMi?2+QM+Hm/BM;a2+Xe i$
 149 $T T2'rBHH\#2bmKK'Bb2/-7QHHQr2/\#vMmK\#2'Q7bm;;2biBQM b7Q'7$
 150 $M/bim/B2bX$

151 2 General hydrodynamics for partially saturated soils

152 2.1 Densities

$S'iBHHvbim'i2/bQBHb+QMiBMi?'22/QK=BMb/2MQi2/\#vMBM/$
 $2+?bT2+B2bbm\{+B2MiHv/BbT2'b2/bm+?i?iBi+QmH/\#2bi iBbiB+HH$
 $KQ/vMKB+b iQ?QH/BMBiXh?2bQBH?b r?2T2'2b2MbT2p2pQ HmK2$
 $?bKmbc b'2bmHi-i?'22i?2'KQ/vMKB+T'iB H/2MbBiB2b+M\#2B/$
 $//BiBQM;Bp2bi?2Qp2'HH T'iB H/2MbBiv,$

$$\varrho_{\beta} \equiv \frac{M_{\beta}}{V}, \varrho \equiv \sum \varrho_{\beta}. \quad \text{URV}$$

$aBKBH'Hv-BiBbTQbbB\#H2iQ/2\}M2+Q''2bTQM/BM;B Mi'BMbB+/2M$
 $pQHmK2b?B+?//mTiQ;Bp2i?2iQiH pQHmK2,$

$$\hat{\varrho}_{\beta} \equiv \frac{M_{\beta}}{V_{\beta}}, V \equiv \sum V_{\beta}. \quad \text{Ukv}$$

$lbBM;i?2b2/2MbBiB2b p'B2ivQ7+QKKQMhvmb2//BK2MbBQMh2b$
 $\}M2/,$

$$c_{\beta} \equiv \frac{M_{\beta}}{M} = \frac{\varrho_{\beta}}{\varrho}, \quad \phi_{\beta} \equiv \frac{V_{\beta}}{V} = \frac{\varrho_{\beta}}{\hat{\varrho}_{\beta}},$$

$$n \equiv \frac{V_V}{V} = \phi_q + \phi, \quad e \equiv \frac{V_V}{V_S} = \frac{n}{1-n}, \quad S_r \equiv \frac{V_q}{V_V} = \frac{\phi_q}{n}, \quad \text{Ujv}$$

153 $\#2BM;i?2+QM+2Mi'iBQM bM/pQHmK27'+iBQM bQ7i?2bT2+B2b-i?2$
 154 $i?2/2;'22Q7bim'iBQM-'2bT2+V iBp2Bbci?2BpQ2$
 155 $F2vTQBMi?2'2Bbi?iHHi?2\#Qp2'iBQb'22MiB'2Hv;Bp2M7'QKi?2$
 156 $T'iB HbM/i?'22B Mi'BMbB+bVX$

$6BMHHv-mbBM;bBKTH2//BiBQM b$

$$\sum \phi_{\beta} \equiv 1, \quad \sum c_{\beta} = 1. \quad \text{U9V}$$

157 2.2 Balance laws

$++Q'/BM;iQi?2\}'biHrQ7i?2'KQ/vMKB+bHHK2/BKmbi+QM b2'p$
 $Bb;Bp2M\#vi?2\#H M+2Q7i?2+QM b2;p2/2M2';v/2MbBivU$

$$\partial_t U + \nabla_i E_i = \varrho v_i G_i, \quad \text{U8V}$$

$r?2'2iBbi?22M2'pG_i-Bb i?2;'pBi iBQM H7Qb+Bb2M2Biv+MM@$
 $i'B+p2HQ+BivXh?2+QM b2'p2/2M2';vBbvi2+Hv2/FiMqBim pbH?22i'2biU$
 $BMi2'MH2M2';vV2MbBivU$

$$U \equiv U(g_i, \vartheta, \varepsilon_{ij}^2, \varrho_{\beta}, N_{\zeta}) = g_i^2/(2\varrho) + u(\vartheta, \varepsilon_{ij}^2, \varrho_{\beta}, N_{\zeta}), \quad \text{UeV}$$

158 $r?2'2iBbi?22M2'pG_i-Bb i?2;'pBi iBQM H7Qb+Bb2M2Biv+MM@$
 159 $i'B+p2HQ+BivXh?2+QM b2'p2/2M2';vBbvi2+Hv2/FiMqBim pbH?22i'2biU$
 160 $BMi2'MH2M2';vV2MbBivU$
 161 $\#Qi?2[mBHB\#BmK M/MQM@2[mBHB\#BmK+QM/BiB/QmbXPMi?2Qi?2'$
 162 $MQi2bHbBi iQ'7@iQ@#2@bT2+B}2/MQM@2[mBHB\#BmKbi i2p'B\#H2b-$
 163 $Bb?mM/2'2[mBHB\#BmKXHHi?2b22[mBHB\#BmK M/MQM@2[mBHB\#B$

164 i?2B' QrM # H M+2 H rX "2HQr- i?2b2 2pQHmiBQM H rb '2 bT2+B}2/ 7Q' i
165 #H2b- H2 pBM; i?2 MQM@2[mBHB#'BmK p 'B #H2b 7Q' a2+X 9X

h?2 KQK2MimK /2MbBiv Bb ;Bp2M #v Bib QrM +QM b2'p iBQM H r,

$$\partial_t g_i + \nabla_j (g_i v_j + \sigma_{ij}) = \varrho G_i, \quad \text{UdV}$$

166 r?2'2_{ij} Bb i?2 iQi H bi'2bb i2MbQ'X

h?2 i?2'K H 2Mi'QTv Bb ;Bp2M 7'QK i?2 b2+QM/ H r Q7 i?2'KQ/vM KB+

$$\partial_t s + \nabla_i (s v_i - F_i) = R/T \geq 0, \quad \text{U3V}$$

167 r?2'2_i M*F_i* '2 i?2 +QM p2+iBp2 M/ /BbbbBT iBp2 2M2';v +m''2Mib- '2bT2+
168 R M*II* '2 i?2 i?2'K H 2Mi'QTv T'Q/m+iBQM M/ i2KT2' im'2X

h?2 2H biB+ bi' BM i2MbQ' Bb ;Bp2M 7'QK,

$$d_t \varepsilon_{ij}^2 + \Omega_{ik} \varepsilon_{kl}^2 - \varepsilon_{kj}^2 \Omega_{ik} = \dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^p, \quad d_t = \partial_t + v_k \nabla_k, \quad \text{UNV}$$

169 r?2'2_{ij} Bb i?2 MiB@bvKK2i'B+ +QKTQM2Mi Q_j'=i?2'2(∇_i2H Q+Biv ;' /B2Mi-
170 ∇_jv_i) i?2 iQi H bvKK2i'B+ bi' B_{ij}MBbi2i b?2BH2BT iBp2 +m''2Mi FMQrM bi?2
171 TH biB+ bi' BM ' i2X JQ'2 /2i' BH QM i?2 bB;MB}+ M+2 Q7 i?2Bb H r Bb ;Bp
172 kyR3VX

6BM HHv- i?2 # H M+2 H r 7Q' i?2 T 'iB H /2MbBiB2b Q7 i?2 i?'22 bT2
Qp2' HH +QM b2'p iBQM H r 7Q' i?2 iQi H T 'iB H /2MbBiv '2 ;Bp2M 7'QK,

$$\partial_t \varrho_\beta \equiv -\nabla_i (\varrho_\beta v_i^\beta) = -\nabla_i (\varrho_\beta v_i - J_i^\beta), \quad \partial_t \varrho \equiv -\nabla_i (\varrho v_i), \quad \text{URyV}$$

r?2'2_i^β M*II_i^β* '2 i?2 p2HQ+Biv M/ /BbbbBT iBp2 /2MbBi_{ij}X+AM2Mi U~mtV Q7
T 'iB+mH '- i?2 bmK Q7 i?2 bT2+B2b KQK2Mi Bb ;Bp2M #v i?2 iQi H KQK2
i?2 bQBH KBtim'2

$$\varrho v_i = \sum \varrho_\beta v_i^\beta, \quad \text{URRV}$$

r?BH2 i? MFb iQ 1[X URV r2∑BibQp2M /#v +QK#BMBM; i?2 #Qp2,

$$\sum J_i^\beta = 0 \Rightarrow J_i^\beta = \varrho_\beta (v_i - v_i^\beta). \quad \text{URkV}$$

173 2.3 Hydrodynamic procedure – general results

6QHHQrBM; i?2 ?v/'Q/vM KB+ T'Q+2/m'2 U1BM p GBm- kyR3V- #v +Q
RkV i?2 i?2'K H i2KT2' im'2 Bb '2+Qp2'2/ 7'QK Bib +Q''2bTQM/BM; 2Mi'Q

$$T \equiv \frac{\partial u}{\partial s}, \quad \text{URjV}$$

r?BH2 i?2 2H biB+ bi'2bb- i?2'KQ/vM KB+ T'2bbm'2- M/ pBb+Qmb bi'2bb

$$\sigma_{ij}^2 \equiv \frac{\partial u}{\partial \varepsilon_{ij}^2}, \quad P_T \equiv -\frac{\partial (u/\varrho)}{\partial (1/\varrho)} \Big|_{\varepsilon_{ij}^2}, \quad \sigma_{ij} = \sigma_{ij} - \sigma_{ij}^2 - P_T \delta_{ij}, \quad \text{UR9V}$$

r?2'2_j Bb i?2 E'QM2+F2' /2Hi i2MbQ'X aBKbH 'Hv- Bi Bb TQbbbB#H2 iQ /2
iB H +?2KB+ H TQi2MiB Hb 7Q' i?2 iQi H KBtim'2- BM/BpB/m H bT2+B2b-
7'QK i?2 +?2KB+ H TQi2MiB H Q7 i?2 bQHB/ T? b2,

$$\mu \equiv \frac{\partial u}{\partial \varrho}, \quad \mu_\beta \equiv \frac{\partial u}{\partial \varrho_\beta}, \quad X_\beta \equiv \mu_\beta - \mu_{\mathfrak{a}} \quad \text{UR8V}$$

h? MFb iQ i?2 ?v/'Q/vM KB+ T'Q+2/m'2 U1BM p GBm- kyR3V Bi Bb H
/2'Bp2 i?2 ' i2 Q7 i?2'K H 2Mi'QTv T'Q/m+iBQM,

$$R = F_i \nabla_i T_K + J_i \nabla_i X + J_i^{\mathfrak{q}} \nabla_i X_{\mathfrak{q}} + \mathcal{D} \geq 0, \quad \mathcal{D} = \sigma_{ij} \dot{\varepsilon}_{ij} + \sigma_{ij}^2 \dot{\varepsilon}_{ij}^p, \quad \text{UReV}$$

174 $r?2'2BbB/2MiB\}2/ \quad b i?2K2+?MB+H/BbbbBTiBQMx$

$q?BH2i?2;2M2'H+QM/BiBQM7Q';m'Mi22BM;i?2TQbBiBp2M2bbC$
 $i'QTvT'Q/mr+KBQMQHHQr7'QKPMb;2'öb'2+BT'Q+H'2HiBQMb?BTb-iQ$
 $BiBb+H2\sigma_{ij}^2i?2\epsilon_{ij}X^*BiB+Hbi i2i2biBM;BMbQBHK2+?MB+b'2;2M2'HH$
 $/2'p2'vHQrbi'BM'i2b-BMr?B+?+b2i?2pBb+Qmb0X'2bbT'+iB+HHv$
 $aQ\#vB/2MiB7vBM;i?22HbiB+bi'2bbb2z2+iBp2Ub\#Qi?+QK2biQ'2T$
 $'B2/QmiQMhv\#vi?2bQBHBf2H2iQMV-i?2?v/'Q/vMKB+T'Q+2/m'2'2p$
 $2'H'2bmHi7Q'i?22z2+iBp2bi'2bbQ7bQBhbU\#Qi?7mHHvbim'i2/M$

$$\sigma_{ij}^{2z} \equiv \sigma_{ij} - P_T \delta_{ij}, \quad P_T = -\frac{\partial(u/\varrho)}{\partial(1/\varrho)} = \mu\varrho - u. \quad \text{URdV}$$

175 $aBMp2/2T2M/bQM i?2/2iMBQB/QM i?22tTHB+Bi bQHmiBQM7Q' i?22$
 176 $iBp2bi'2XbAKTQ'iMiHv- b/Bb+mbb2/BMi?27QHHQrBM;b2+iBQM i?2;2$
 177 $Q7QM i?2p'BQmb/2MbBiB2bBb+H2'-i?mb2M\#pBM/+m'i2bQHmiB$
 178 $\sigma_{ij}^{2z}X$

179 3 Partially saturated soils under equilibrium

180 $h?2+m'i'2Mi b2+iBQM'2pB2rbi?27mM/K2MiH'2bmHib7'QKi?2i?2'$
 181 $iBQM7Q'2[mBHB\#i2/bQBHBvbi2Kb\#vUCB M;2iHX-kyRdV-r?B+?'2$
 182 $/'QK2+?MB+H'2HiBQMb?BTb\#2ir22Mi?22z2+iBp2bi'2bbQ7T'iBHH$
 183 $BMi'BMbB+M/2ti2'MHHvK2bm'2/fTTHB2/bm+iBQM p Hm2bXh?BbBb$
 184 $7Q'2T'Q+22/BM;iQ//i'2bbi?2KQ'2;2M2'H+b2Q7bm+?'2HiBQMb?B$
 185 $+QM/BiBQMbx$

186 3.1 Equilibrium conditions

$h'm2i?2'KQ/vMxKB+e b@}2ibu'272'iQ+QM/BiBQMbr?2'2HHi?2/BbbbB$
 $iBp2\sim mt2bBMi?2bvbi2KpMBb?M/HHi?2MQM@2[mBHB\#BmKbi i2p$
 $pHm2-bQi?ii?2B'+QM'i'B\#miBQM iQ i?22M2';vpMBb?2bxIM/2'bm+?$
 $i2'MH2M2T;T'Q+?2bBibmMB[m2KBMBKmkpHm2,$

$$\min_{\{\hat{\varrho}_\beta, N_\zeta\}} u(\mathcal{A}, \varepsilon_{ij}^2, \varrho_\beta, \hat{\varrho}_\beta, N_\zeta) = u_2(\mathcal{A}, \varepsilon_{ij}^2, \varrho_\beta) \quad (2[mBHB\#BmK]) \quad \text{UR3V}$$

187 $r?B+?Bb7QmM/\#vKBMBKBbBM;i?2Bmi2'MH2M2';\hat{\varrho}_\beta QCBM2i?'22Bmi$
 188 $2iHX-kyRdV-b r2HHbHHi?2TQbbB\#H2p\#Hm2bQ2[mBHB\#BmK\#2@$
 189 $bi i2p'B\#H2bBMi?27QHHQrBM;-i?Bb2[mBHB\#BmKbi i2Bb}'bi/2b+'B$
 190 $BM2[-M/H\{i2'2bT2+B\}+MQM@2[mBHB\#BmKbi i2p'B\#H2b rBHH\#2/2$
 191 $HQr/2T'im'2b7'QKi?2i'm22[mBHB\#BmK+QM/BiBQMbx iMQM@2[mB$
 192 $rQmH/\#2b?QrMmbBM;i?2M2ri?2Q'vi?iT'iBHHvbim'i2/bQBhb+Q$
 193 $K2ibi\#H2bi i2bi?i/QMQib iBb7vi?2i'm22[mBHB\#BmK+QM/BiBQM$

194 3.2 Internal energy

$L2;H2+iBM;i?2'KH2z2H2V?QQmi?2?v/'QK2+?MB+H'2HiBQMb-i?2$
 $2[mBHB\#BmKpHm2Q7i?2Bmi2'MH2M2';vQ7T'iBHHvbim'i2/bQB$
 $\#vbmKKBM;mti?22HbiB+bi'BM2M2';MQ7?i?2'2Q7HMQ7;2H2iQM$
 $i?2i?'22/QKBmb,$

$$u_2[\equiv u_2(\varepsilon_{ij}^2, \varrho_a) + f(\varrho_\beta). \quad \text{URNV}$$

$h?2'2HiBQM\#2ir22Mi?2T'iB H/2MbBiB2pb'Bmbl\#XRVK2Mbi?i$
 $h?2'27Q'2a bBMf+2T2M/QQM2v\#Qi?+QM'i'B\#mi2iQ i?2i?2'KQ/vMKB+T'2$
 $bmP2X>Qr2p2'-rp2K2bm'2K2Mib'2p2H2HbiB+\#mHFKQ/mHBBM bQ$
 $(\varrho^3\varrho^2)^{1/2}U_m\#BM1BMp-kyRRc oB;;BMB iFBMbQM-RNN8V-K2MBM;2$

$$p^2 \sim \varrho_a^3 (\varepsilon_v^2)^2 M / 2 H b i B + u_2 M^2 \varrho_a^3 (\varepsilon_v^2)^3 U r ? 2 p^2 = \frac{1}{3} \sigma_{ii}^2 M \varepsilon_v^2 = \varepsilon_{ii}^2 V X h ? m b -$$

$$H i ? Q m ; ? B i B b T Q b b B \# H 2 i Q + ' ' v H i Q M Q i ? 2 ' Q Q M ? ' B \# m Q B Q M 2 Q$$

$$T 2 M / 2 M + 2 B i M B a M i ? B b B b 2 M i B ' 2 H v M 2 ; H B ; B \# H 2 ' 2 H f i B B 2 M i Q i ? 2 + Q M i ' B \# r$$

$$K Q b i T ' + i B v^2 \lesssim H H h ? 2 ' 2 7 Q ' 2 - B i B b ++ m ' \# i 2 B Q M ' \# Q H ? 1 1 X U R d V$$

$$M / 1 [b X U R 9 - R 8 - R j V b Q i ? i ,$$

$$\sigma_{ij}^{2z} \equiv \sigma_{ij} - P_T \delta_{ij}, \quad P_T = - \frac{\partial(f/\varrho)}{\partial(1/\varrho)} = \mu \varrho - f, \quad U k y V$$

M /

$$\sigma_{ij}^2 = \frac{\partial u_2}{\partial \varepsilon_{ij}^2}, \quad \mu = \frac{\partial f}{\partial \varrho}, \quad \mu_\beta = \frac{\partial f}{\partial \varrho_\beta}, \quad U k r V$$

195

196

$$' 2 b T 2 + i B p 2 H v X \quad b b ? Q r M \# 2 H Q r f - Q M i ? 2 2 p M B 2 Q m + 2 / Q m b B i B 2 b B b + H 2 ' - i ? r$$

$$2 M \# H B M ; M ++ m ' i 2 p Q M H i ? B Q M 7 Q '$$

197

3.3 Free energy and suctions

$$h ? 2 7 ' 2 2 2 M 2 ' ; v Q 7 i ? 2 i ? ' 2 2 / Q K B M b - r ? B + ? ' 2 T ' 2 b 2 M i b i ? 2 B ' p Q H m K$$

$$B \# B H B i v - + Q m H / \# 2 + H + m H i 2 / M H v i B + H H v U C B M ; 2 i H X - k y R d V ,$$

$$f \equiv \sum \phi_\beta \hat{f}_\beta(\hat{\varrho}_\beta) = \sum \frac{\varrho_\beta}{\hat{\varrho}_\beta} \hat{f}_\beta(\hat{\varrho}_\beta), \quad \hat{P}_\beta \equiv - \frac{\partial(\hat{f}_\beta/\hat{\varrho}_\beta)}{\partial(1/\hat{\varrho}_\beta)} = \hat{\mu}_\beta \hat{\varrho}_\beta - f_\beta, \quad \hat{\mu}_\beta \equiv \frac{\partial \hat{f}_\beta}{\partial \hat{\varrho}_\beta}, \quad U k k V$$

$$r ? 2 ' \hat{P}_\beta - \hat{P}_\beta M \hat{\mu}_\beta ' 2 i ? 2 B M i ' B M b B + 7 ' 2 2 2 M 2 ' ; v - T ' 2 b b m ' 2 M / + ? 2 K B + H T Q i$$

$$\beta ' B b B M ; / m 2 i Q T Q b b B \# H 2 b K H H p ' B i B Q M b B M i ? 2 p X Q H @ b T Q M / B M ; B M$$

$$i ? Q m ; ? b K H H ' 2 H i B p 2 i Q p ' B i B Q M b i B M T Q b B B \# B 2 H B b B 2 p ' B \# H 2$$

$$K m b i \# 2 H H Q r 2 / 7 Q ' i Q ++ Q K Q / i 2 T ? 2 M Q K 2 M ' 2 H i 2 / i Q i ? 2 / 2 p 2 H Q T K 2$$

$$b Q H B / B M i 2 ' 7 + 2 b m M / 2 ' T ' i B H b i m ' i B Q M b X A M T ' i B + m H ' - m M / 2 ' T ' i$$

$$p ' B i B Q M b T ' Q / m + 2 / B z 2 ' 2 M i b B ; M b 7 Q ' i ? 2 B M i ' B M b B + T ' 2 b b m ' 2 b B M i ?$$

$$T ' Q / m + B M ; B M i ' B M b B + b m + i B Q M ,$$

$$\hat{s} \equiv \hat{P} - \hat{P}_q, \quad U k j V$$

$$r ? B + ? b ? Q m H / M Q i \# 2 + Q M 7 m b 2 / r B i ? i ? 2 K 2 b m ' 2 / b m + i B Q M 7 ' Q K i ? 2 / B z$$

$$B ' u U V M / r i 2_q M T ' 2 b b m ' 2 b i ? i ' 2 \# 2 B M ; T T H B 2 / Q ' K 2 b m ' 2 / 2 t i 2 ' M H H$$

$$/ 2 p B + 2 b Q m i Q 7 i ? 2 b Q B H ,$$

$$s \equiv u - u_q. \quad U k 9 V$$

198

199

200

$$H i ? Q m ; ? M Q i \# 2 B M ; + Q M b B / 2 ' 2 / T ' B Q ' i Q U C B u M \neq \hat{P}_i - H X - k y R d V - K Q b$$

$$u_q \neq \hat{P}_q - M s \neq \hat{s} - b B M + 2 H Q M ; \# Q m M / ' B 2 b Q M H v i ? 2 T ' i B H + ? 2 K B + H T Q$$

$$M / Q m i Q 7 i ? 2 K 2 b m ' 2 K 2 M i / 2 p B + 2 b M 2 2 / i Q \# 2 2 [m H m M / 2 ' 2 [m B H B \# ' B$$

201

202

203

204

205

$$6 m ' i ? 2 ' K Q ' 2 - r ? B H 2 m M / 2 ' 2 [m B H B \# ' B m K i ? 2 T ' 2 b 2 M + 2 Q 7 b m ' 7 + 2 i 2$$

$$; 2 Q K 2 i ' B + H H v + Q K T H B + i 2 / B M i 2 ' 7 + 2 b K 2_q M b u_d \leq 0 X + Q K T ' 2 b b B Q M U$$

$$+ Q K T ' 2 b b 2 / B ' T \hat{P} 2 b b m ' 2 b 0 U - M / i ? m b T Q b B i B p 2 0 b m + 0 B Q M b U$$

$$V - B i r B H H \# 2 b ? Q r M H i 2 ' i ? i Q z 2 [m B H B \# ' B m K i ? 2 b 2 b m + i B Q M b + Q m H /$$

$$i B M ; 7 ' Q K p 2 ' v / ' v b K T H 2 b X$$

206

3.4 Energy minimisation, and equilibrium suctions and effective stress

207

208

209

210

211

212

$$b T ' 2 b 2 M i H v b i i 2 / B M 1 [X U k k f V / 2 T 2 M / 2 2 Q M B ; v / 2 M b B i B 2 b i ? 2$$

$$i ? ' 2 2 T ' i B H Q M 2 b ' 2 [m B ' 2 / 7 Q ' 1 [X U R N V - b r 2 H H b i ? ' 2 2 M Q M @ ' 2 [m B '$$

$$\hat{\varrho}_\beta X h ? 2 ' 2 7 Q ' 2 - b Q H p B M ; 7 Q ' i ? 2 2 z 2 + i B p 2 b i ' 2 b b B M 1 [X U k y V r Q m H / / 2 T 2$$

$$i ' B M b B + / 2 M b B i B 2 b X h Q i ? B b 2 M / B i B b T Q b b B \# H 2 i Q \} ' b i b Q H p 2 2 [m B H B$$

$$B M ; i ? 2 2 M 2 ' ; v i Q r ' / b B i b ; H Q \# H K B M B K m K U C B M ; 2 i H X - k y R d V - \# 2 7 Q$$

$$2 [m B H B \# ' B m K b + 2 M ' B Q b r ? 2 ' 2 i ? 2 b i i 2 b + M ; 2 i b i m + F B M H Q + H K B M B$$

$$b T ' i Q 7 i ? 2 2 M 2 ' ; v K B M B K B b i B Q M - B i B b m M / 2 ' b i Q Q / i ? i m M / 2 ' ; B$$

$$i B H / 2 M b B i B 2 b B M i ' B M b B b / 2 M b B i B 2 b m b i / 2 T 2 M / 2 M i H v i Q K B M B K B b 2 i ? 2$$

$$2 ' ; v X q ? 2 M i ? 2 2 M 2 ' ; v B b K B M B K B b 2 / - r ? B H 2 i F B M ; 1 [X U 9 V b i ? 2 Q M H v$$

$$QM2 \} M/b \hat{P}_\beta = u_\beta = P_0 \quad b \quad bm+iBQM H2bb \quad HBK \quad B \quad Xr?Q'r2p2'-bm+?$$

i'BpB H bQHmiBQM B;MQ'2b i?2 T'2pBQmbHv K2MiBQM2/ 'QH2 Q7 BMi2
iB HHv b im' i2/ bQBH- bi?2 +im H '2 bQM 7Q' Q#b2'pBM; i?2 MQM@x2
i'BmBb+ bm+iBQM b BM i?2 }'bi TH +2X S'2pBQmb ii2KTib iQ i' +F i?2 +Q
bm+? BMi2'7 +2b BM bQBHb ? p2 Hr vb '2HB2/ QM Qp2'bBKTHB7vBM; b
#v UCB M; 2i HX- kyRdV KQ'2 T' ;K iB+- ++m' i2 M/ ;2M2' H TT'Q +?
i?2 K2 bm'2/ bsm b iBQM im HHv FMQrM [m MiBiv #v KBMBKBbBM; i?2 2M2'
i?2 BMi'BMbB+ b m m iBQM BQM H +QMbi' BMiX h?Bb +im HHv ? b M 2t +i
bQHmiBQM- #mi i?2 bQHmiBQM Bb 7 ' iQQ HQM; iQ #2 BHHmKBM iBM;X >
'Bp2 i T' +iB+ HHv i?2 b K2 '2bmHi bi?2 M HviB+ QM2 #v TT'QtBK iB
bm+iBQM H2bb HBKBi UCB M; 2i HX- kyRdV- r?B+? T'2b2Mib i?2 7QHHQr
'BmK bi i2b

$$P_T \equiv u - \chi s, \quad \chi \equiv \chi(\varrho_\beta), \quad U k 8 V$$

213

r?2'2 Bb i?2 +H bbbB+ H "Bb?QTöb +Q2{+B2Mi U"Bb?QT "HB;?i- RNejVX

6m'i?2' F22TBM; BM KBM/ i?2 Q' /2' Q7 K ;MBim/2bKQ7 #Qi? i?2 +QKT'
M/ BMi'BMbB+ b m m iBQM BQM H +QMbi' BMiX h?Bb +im HHv ? b M 2t +i
M2;HB;B#H2 p Hm2b- r2 }M/ i? i mM/2' 2[mBHB# 'BmK +QM/BiBQM b,

$$\chi_2[\equiv \chi_2(\varrho_\beta) = \frac{\varrho}{\bar{\varrho}} + \frac{\varrho_S}{\bar{\varrho}_S} \left[\frac{\bar{\varrho}}{\bar{\varrho}} \frac{(\partial \hat{s}_2 / \partial \varrho)}{(\partial \hat{s}_2 / \partial \varrho)} - \bar{\varrho}_S \frac{(\partial \hat{s}_2 / \partial \varrho_S)}{(\partial \hat{s}_2 / \partial \varrho_S)} \right], \quad U k e V$$

$$s_2[\equiv s_2(\varrho_\beta) = \frac{\varrho_q}{\bar{\varrho}_q K_q} \left[\bar{\varrho} \frac{\partial \hat{s}_2}{\partial \varrho} - \bar{\varrho}_q \frac{\partial \hat{s}_2}{\partial \varrho_q} \right] \hat{s}_2, \quad U k d V$$

214

r?2'2 i?2 bm#b+'BTi i2[öx-b M/2iQQB;?HB;?i i? i i?2 #Qp2 bQHmiBQM b

215

+Q''2bTQM/ iQ 2[mBHB# 'BmK p Hm2b- QrBM; iQ i?2 2M2';v KBMBKBb iBQ

216

i?Bb MQR b +QKT '2/ iQ i?2 Q'B;BM H 2tTQbBiBQM UCB M; 2i HX- kyRdV

217

BM; 2[mBHB# 'BmK rQmH/ #2 /BbiBM;mBb?2/ 7'QK MQM@2[mBHB# 'BmK +

218

r?BH2 b T 'i Q7 i?2 2M2';v KBMBKBb iBQ M 2?2 BmHiB B/biQ+p/2 MbBiB2b

219

iQ Q#i BM i?2 #Qp2 '2bmHi- i?2b2 p 'B iBQM b '2 +2'i BMHv bK HH M/ i'

220

p Hm2 BM i?2 #Qp2 +QmH/ #2 i?2 b m /22 2 QM+ 22K H 2i im'2 M/ iKQ@

221

bT?2'B+ B' T'2bbm'2X Ai Bb HbQ MQi2/ i? i i?2 r i2' +QKT'2bbbB#BHBiv

222

Bb FMQrM KiQ=22: S X

223

h?2 '2H iBQM b?BTb BM 1[bX Uke-kdV '2 KQbi;2M2' H M/ ++m' i2-

224

QM2 iQ bbmK2 /2T2M/2M+2-BM(ö)2 Q7QBKH/ K2 bm'2/ bQBH r i2' '2@

225

i2MiBQM BM i?2(ö) iK i+QmH/ #2 +QKT '2/ rBi? 2tT2'BK2Mi H K2 bm'2K2

226

Hi?Qm;? i?2 BMi'BMbB+ /2MbBiB2b r2'2 HHQr2/ iQ +? M;2 /m'BM; i?2 2M

227

iQ 'Bp2 iQ i?2 '2bmHib #Qp2- i?2b2 +? M;2b '2 2MiB'2Hv bK HH M/ i?n

228

;2M2' HHv pö' B2 #H22 mQ2 #Qp2 b +QMbi Mi BMi'BMbB+ /2MbBiB2b i? i+Q

229

bTQM/ iQ i?2B' FMQrM p Hm2b i R iK M/ 'QQK i2KT2' im'2X

Hi?Qm;? BM ;2M2' H i?2 2z2+ib Q7 i?2 BMi'BMbB+ B' /2MbBiv QM i?2
Q7 bQBHb Bb +QM bB/2'2/ BM i?2 #Qp2- KQbi 2KTb'B+ H bQBH r i2' '2i2M
iQ M2;H2+i i?Bb i?2Q'2iB+ H +QM i'B#miBQM X Ai B b iBm b mb27mH iQ M2;
i?2 #Qp2- M/ 7m'i?2' +QM bB/2' i?2 '2H iBQM b?BTb #2ir22M i?2 BM/2t T
iQ T'Q/m+2 KQ'2 +QKKQM /2T2M/2M+B2b M/ bBKTH2' bi'm+im'2bX PM2
#2 2t?B#Bi2/ BM i2'Kb Q7 i?2 /2S_r'2 M Q7QB i m'LBKQM p GBm- kykyV,

$$\chi_2[\equiv \chi_2(S_r, e) = S_r - e \frac{\partial \hat{s}_2 / \partial e}{\partial \hat{s}_2 / \partial S_r}, \quad U k 3 V$$

$$s_2[\equiv s_2(S_r, e) = - \left[\frac{S_r}{K_q} \frac{\partial \hat{s}_2}{\partial S_r} \right] \hat{s}_2, \quad U k n V$$

230

r?B+? '2[mB'2b QM2 iQ(ö) iK i+QmH/ vB2H/ K2 bm'2/ bQBH r i2' '2@

231

i2MiBQM BM i?2(ö) iK ;'22 #H2 rBi? i?2 2tT2'BK2MibX

3.5 Illustration of equilibrium states

$h?2 \text{ M HviB} + r \text{ v iQ mb} 2 i?2 '2H \text{ iBQMb?BTb BM } 1[bX \text{ Uk3-kNV Bb iQ i'}$
 $Q'/BM 'v /Bz2'2MiB \text{ H } 2[m \text{ iBQM U} \hat{s}_2]V;BQ2\#M2 \text{ bQM} Q2M2Q' \text{ TB'B+ H H r}$
 $7Q'2[=s_2(S_r, e) - BiBb \text{ TQbbB\#H2 iQ BMi2; ' } \hat{s}_2 \neq s_2(S_r, e)1m \text{ Q B Mi}$
 $M \text{ BMBiB H bm+iBQM H2bb \# Qm} (M/e)' \neq 0 \text{ K7mH2H2b Kt'Hi2BQM' bm+? M@}$
 $HviB+ bQHmiBQM b Bb iQ BKTQb2 \text{ ?vT2'\#QHb+ '2H iBQM b?BT \#2ir22M}$
 $UCB \text{ M; 2i HX- kyRdc } 1BM \text{ p GBm- kyR3- kykyVX M Hi2'M iBp2 2t KTH}$
 $i?2 \text{ irQ bi2Tb \#2HQR- r?B+? vB2H/ '2H iBQM b?BTb r?Qb2 K i?2K iB+ H } 7Q$
 $r? iKQ'2 \text{ p2'b iBH2 M/ } 2H2; Mi i? M i?2 T'2pBQmbHv T'QTQb2/ ?vT2'\#Q$

$U \text{ Vb } \}'bi \text{ bi2T- +QM bB/2' '2i2MiBQM } 7mM+iBQM b \text{ B-MbTB'2/ \#v i?'22@}$
 $\lambda V \text{ AMp2'b2 q2B\#mHH } 7mM+iBQM X \text{ q?2'2 i?Bb } 7mM+iBQM \text{ r b Q'B;BM}$
 $/Bbi'B\#mi2/ bi'2M; i? BM \text{ bQHB/ K i2'B Hb- ?2'2 BiBb / Ti2/ iQ '2T'2b}$
 $/2M+2 Q7 \text{ bm+iBQM QM b im' iBQM,}$

$$s_{2l} = ce^{-\lambda}(-\ln S_r)^\alpha, \quad UjyV$$

$r?2'2 \text{ Bb } +QM bi \text{ Mi +QM i'QH HBM; i?2 /2T2M/2M+2 Q7 bQBH r i2' '2i2}$
 $S_r \text{ c r?B+H2 +QM i'QH b Bi b /2T2M/2M+2 QM pQB/ ' iBQ Ur?B+? +QmH/ \#2}$
 $7'QK 2KTb'B+ H '2H iBQM b 7Q' B' 2Mi'v p Hm2bVX \text{ SmiiBM; i?Bb T'Q}$
 $M/ bQH pBM(1,m) = BM b \# QmM/ 'v +QM/BiBQM- r2 \}M/,$

$$\hat{s}_{2l} = \sqrt{\frac{2K_a}{1+\alpha}}(-\ln S_r)s_{2f} \quad UjRV$$

$r?B+? mTQM /Bz2'2MiB \text{ iBQM mbBM; } 1[X \text{ Uk3V ;Bp2b,}$

$$\chi_{2l} = S_r \left(1 + \frac{\lambda}{1+\alpha} \ln S_r\right). \quad UjkV$$

$U\#V?BH2 i?2 \text{ bm+iBQM b BM i?2 T'QTQb2/ '2H iBQM b \#Qp2 /Bp2'; 2 i \times 2}$
 $bQBHb 2t?B\#Bi /Bp2'; BM; \#2? pBQM' i \}MBi2 bKS, H p Hm2 Q7 '2bB$
 $U \text{ b M //BiBQM H } 7Qm' i? T ' K2i2'VX hQ i? i 2ti2Mi- BiBb \text{ TQbbB\#H}$
 $\#Qp2 '2H iBQM b \#v \}S_b B2T' H Q B M[X UjRV rBi? M 2z2+iBp2 b im@$
 $' iBQM = (S_r - S_{r0})/(1 - S_{r0}) - bQ i? i 7'QK 1[bX \text{ Uk3-kNV,}$

$$s_{2l} = ce^{-\lambda} \left(\frac{S_r}{S_r - S_{r0}} \right) (-\ln S_r^*)^\alpha, \quad UjjV$$

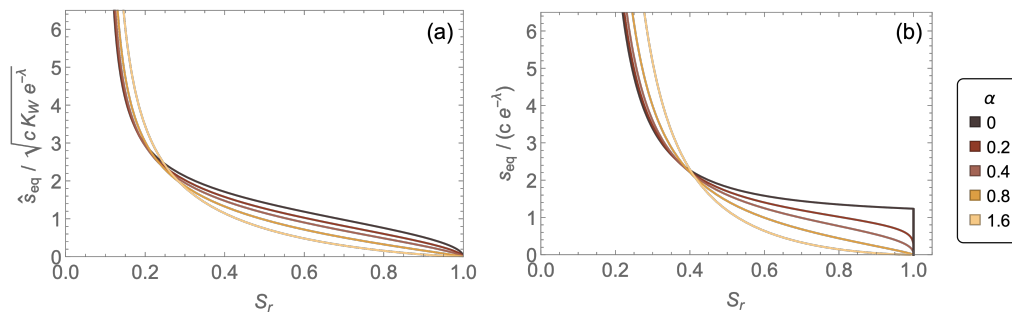
$$\hat{s}_{2l} = \sqrt{\frac{2K_a}{1+\alpha}}(-\ln S_r^*)s_{2f} \quad Uj9V$$

$$\chi_{2l} = S_r \left(1 + \frac{\lambda(S_r - S_{r0})}{(1+\alpha)S_r} \ln S_r^*\right), \quad Uj8V$$

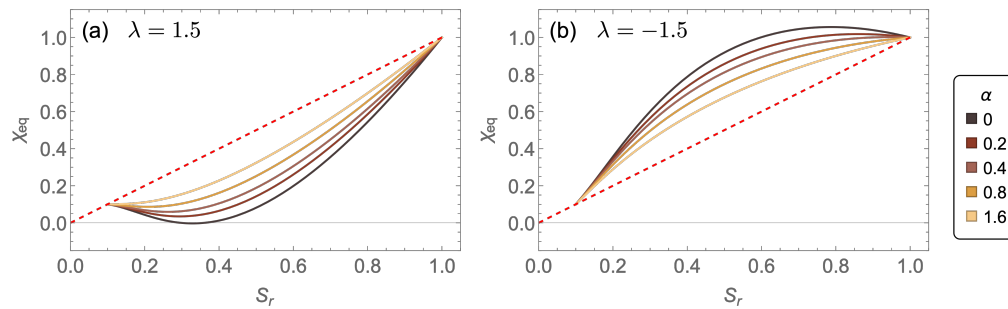
$r?B+? vB2H/b i?2 \text{ b K2r0-2bmHrB7Q1[bX Ujy@jkVX}$

$hQ \text{ BHHmbi' i2 i?2b2 } 7mM+iBQM b- 6B; X \text{ k b?Qrb i?2 /2T2M/2M+2 Q7 i?}$
 $bm'2/ bm+iBQM b QM i?2 bX \text{ T2T'2K2H2b ? p2 \#22M MQM@/BK2MbBQM H}$
 $2t+Hm/2 i?2 2z2+ib Q7 d? M p Q B/K2B2Q?B+? /Q MQi+? M; 2 i?2 [m H@}$
 $Bi iBp2 b? T2 \text{ M/ K2'2Hv b+ H2 i?2 p Hm2 Q7 i?2 2z2+iBp2 B' 2Mi'v p Hm}$
 $2z2+iBp2 B' 2Mi'v p Hm2 '2T'2b2Mib i?2 '2[mB'2/ bm+iBQM iQ bi 'i/'vBM}$
 $7mHH b im' iBQM VX L2p2' i?2H2H2b-bi?22T'2bK2H27Q' i?2 2z2+iBp2 bi'2bb$
 $"Bb?QTöb +Q2[+B2M2QrM \neq Q' M\Lambda = -1 \text{ QM } 6B; X j \text{ M/ } 6B; X j\#- '2@}$
 $bT2+iBp2Hv- 7Q' p 'BQ mQip M2b2Q7 i?2 [m HBi iBp2 /2T2M/2M+2 Q7$
 $Q M, X$

$*QKT 'BM; 6B; X \text{ k M/ } 6B; X \text{ k\#- BiBb +H2 ' i? i i?2 2[mBHB\#'BmK BMi}$
 $bm'2/ bm+iBQM b '2 MQi bBKBH ' \#Qi? [m HBi iBp2Hv \text{ M/ BM p Hm2b UbE}$
 $/Bz2'V- b /Bb+mbb2/ \#v U1BM \text{ p GBm- kykyVxbBM? QBMX Q TMQBOM+'2 b2 B}$
 $/m+2 \text{ b? 'T2' 'Bb2 i?B;? b im' iBQM b 7QHHQr2/ \#v ~ ii2' 'Bb2 iB Mi2'}$
 $' iBQM b- i' MbBiBQM BM; 7'QK ivTB+ H +H v iQ ivTB+ H b M/ '2i2MiBQM +$



6B;m'2 k, h?2 /2T2M/2M+2 mM/2' i?2 i'm2 i?2'KQ/vM KB+ 2[mBHB#'BmK
M/i?2 U#V K2 bm'2/ bm+iBQM b QM i?2 /2;'22 Q7 b im' iBQM 7Q' /Bz2'2M
' K2i2'mbBM; 1[bX Uj9-jjV- '2bT2+iBp2HvX h?2b2 bm+iBQM b ? p2 #22M M
MQM@/BK2MbBQM H MmK#2'b QM i?2 v@ t2bX h?2=2.2B/m H b im' iBQM



6B;m'2 j, h?2 /2T2M/2M+2 Q7 i?2 \$\chi_{eq}\$ B-b?Q1TQM i?22/2;'22 Q7 b im' iBQM
\$S_r\$ 7Q' /Bz2'2Mi b? T2\$\alpha\$T M/K2i2' TQb-B iBp2' U#V M2 \$\chi_{eq}\$ =B p.2- +Q'@
'2bTQM/BM; iQ 1[X Uj8VX b BM 6B;X k- i?2 '25B/m0.1Xb. im?2iBQM T ' K2i2
'2/ HBM2b b?Qr i?25. BQBH iBQM- r?B+? i?2 +m''2Mi KQ#2HX '2T'Q/m+2b rE

256 T 'iB+mH 'Hv +H2 ' 7'QK i?2 2[mBHB#'BmK K2 bm'2/ b2m+iBQM 6B;X k#- r
257 '2+Qp2' i'm2 B' 2Mi'v p\$H=12xbBM?2 Q i?2' ? M/ 6B;X j -# BHHmbi' i2
258 i? i TQbBiBp2 MA M2Hmb2p2B p2H m2b #2HQR M/\$\chi_{eq}\$#Q2HB M2-
259 '2bT2+iBp2HvX "Qi? b+2M 'BQb ? p2 #22M '2TQ'i2/ BM i?2 HBi2' im'2 UC
260 RNeKc E? HBHB w ' ; '# b?B-ky RNV2b?Qp2MR i?B;? b im' iBQM b- M/
261 ?B MiBM; iQr '/b p Hm2b i? i K v ;Q #2HQR y b HQM; b i?2 bQBH '2K BMb
262 Ai b?QmH/ HbQ #2 K2MiBQM2/ i? i BM b2C+2MK2BM mb2K'2mHb Q/-
263 RNeKc E? HBHB w ' ; '# b?B-ky RNV2b?Qp2MR i?B;? b im' iBQM b- M/
264 +? M;BM; /m'BM; i?2 i2bibX @m? 2T Qb2MiB MQi BM+Hm/2/ BM /' rBM; 6B;X
265 r?B+? iQ QF#2 +Qmbi MiX >Qr2p2'- mbBM; 1[XeUj8V iB2Q2m2 Qim Q7 i?2
266 i?2 HBM2b +QmH/ #2 +QmbB/2'2/- M/ \$\chi_{eq}\$#HBM2Xh?2b2Q2iM p2
267 Q7 i?Bb /BbiQ' iBQM rQmH/ /2T2M/ QM i?2 K pM Bi'BM2; Q72i?2bp b' B iBQM b Q
268 M/ i?2 H2p2H Q7 b2M b2B iBQM Q72Bi ++Q' /BM; iQ 1[X Uj8VX

280 $T?2MQK2M \text{ } bm+? \text{ } b \text{ } bQBHr \text{ } i2' \text{ } '2i2MiBQM \text{ } ?vbi2'2bBbX >Qr2p2' - MQi2 \text{ } i?$
 281 $2[m \text{ } iBQM + QmH/ \text{ } M/ \text{ } ? \text{ } b \text{ } \#22M \text{ } mb2/ \text{ } iQ \text{ } 2tTH \text{ } BM \text{ } K2+? \text{ } MB+ \text{ } HHv \text{ } '2H \text{ } i2/ \text{ } M$
 282 $MQK2M \text{ } BM \text{ } b \text{ } M/ - \text{ } bm+? \text{ } b \text{ } bi'2bb@bi' \text{ } BM \text{ } ?vbi2'2bBb \text{ } UCB \text{ } M; \text{ } GBm - \text{ } kyyN$
 283 $LQiB+2 \text{ } i? \text{ } i \text{ } mMH \text{ } B \text{ } F \text{ } 2B \text{ } H \text{ } X \text{ } M \text{ } Q \text{ } 3V2bi' \text{ } B+i2/ \text{ } 7'QK \text{ } \#2BM; \text{ } M \text{ } \geq \text{ } iBp2 - \text{ } v2i \text{ } r2 \text{ } i \text{ } F2$
 284 $0 \text{ } M \text{ } R + \text{ } R_K \geq 0 \text{ } X \text{ } h?2 \text{ } KB+ \text{ } 'Q@K2bQ \text{ } b2T \text{ } ' \text{ } iBQM \text{ } \#2ir22M \text{ } i?2 \text{ } irQ \text{ } 2Mi' \text{ } QTB2b$
 285 $T2' \text{ } im'2b \text{ } ? \text{ } b \text{ } \#22M \text{ } 7m'i?2' \text{ } /Bb+mbb2/ \text{ } \#v \text{ } UGBm - \text{ } kykRV \text{ } BM \text{ } i?2 \text{ } +QMi2ti \text{ } Q$
 286 $T?vbB+bX$

287 4.3 Internal energy at non-equilibrium states

$.m'BM; MQM@2[mBHB\# 'BmK + QM/BiBQM b \text{ } i?2 \text{ } BMi2' \text{ } M \text{ } H \text{ } 2M2'; v \text{ } /2MbB$
 $QM \text{ } \#Qi? \text{ } 2[mBHB\# 'BmK \text{ } M/ \text{ } MQM@2[mBHB\# 'BmK + QMi' \text{ } B \text{ } \#miBQM b - \text{ } i?2 \text{ } H$
 $i?2 \text{ } irQ \text{ } M2r \text{ } MQM@2[mBHB\# 'BmK \text{ } bi \text{ } i2 \text{ } p \text{ } 'B \text{ } \#H2b \text{ } Mi?2' \text{ } K \text{ } BMi \text{ } \#M \text{ } i2/ \text{ } 2Mi' \text{ } Q$
 $bB+ \text{ } bm+iBQM \text{ } K2 \text{ } p \text{ } B2i' \text{ } B \text{ } Q \text{ } i'2 - \text{ } M2; H2+iBM; i?2 \text{ } 2z2+ib \text{ } Q7Q' \text{ } v2 \text{ } i?2' \text{ } K \text{ } H \text{ } 2Mi' \text{ } Q$
 $QM \text{ } i?2 \text{ } ?v/ \text{ } 'QK2+? \text{ } MB+ \text{ } H \text{ } '2bTQM b2 \text{ } Q7 \text{ } T \text{ } 'iB \text{ } HHv \text{ } b \text{ } im' \text{ } i2/ \text{ } bQBHb,$

$$u \equiv u(\varrho_\beta, \varepsilon_{ij}^2, \mathcal{J}, \mathcal{J}_K, \xi) = u_2(\varepsilon_{ij}^2, \varrho, \varrho_\beta) + u_M(\mathcal{J}_K, \xi), \quad U9RV$$

$h?2 \text{ } \}'bi \text{ } i2' \text{ } K+ \text{ } Tim'2b \text{ } BMi2' \text{ } M \text{ } H \text{ } 2M2'; v \text{ } i? \text{ } i \text{ } K \text{ } BMi \text{ } BMb \text{ } /m'BM; 2[mBHB$
 $+QmH/ \text{ } \#2 \text{ } i \text{ } F2M \text{ } /B'2+iHv \text{ } 7'QK \text{ } 1[X \text{ } URNVX \text{ } PM \text{ } i?2 \text{ } M \text{ } Q+? \text{ } Tim'2b - \text{ } i?2 \text{ } b2+QM/$
 $//BiBQM \text{ } H \text{ } BMi2' \text{ } M \text{ } H \text{ } 2M2'; v \text{ } i \text{ } MQM@2[mBHB\# 'BmK \text{ } bi \text{ } i2bX \text{ } h?2 \text{ } i?2' \text{ } KQ/v$
 $i?2 \text{ } \}'bi \text{ } i?2'22 + QMb2' \text{ } p2/ \text{ } bi \text{ } i2 \text{ } p \text{ } 'B \text{ } \#H2b \text{ } ? \text{ } p2 \text{ } \#22M \text{ } /2\}M2/ \text{ } BM \text{ } 1[bX \text{ } UKRVX$
 $M \text{ } KB+ \text{ } +QMDm; i2b \text{ } Q7 \text{ } i?2 \text{ } irQ \text{ } '2K \text{ } BMBM; BM/2T2M/2Mi \text{ } bi \text{ } i2 \text{ } p \text{ } 'B \text{ } \#H2b \text{ } '$
 $MQM@2[mBHB\# 'BmK \text{ } T \text{ } 'i \text{ } Q7 \text{ } i?2 \text{ } BMi2' \text{ } M \text{ } H \text{ } 2M2'; v,$

$$T_K = \frac{\partial u_M}{\partial \mathcal{J}_K}, \quad Y_\xi = \frac{\partial u_M}{\partial \xi}, \quad U9kV$$

288 $\#2BM; i?2 \text{ } K2bQ@'2H \text{ } i2/ \text{ } i2KT2' \text{ } im'2 \text{ } M/ \text{ } \ddot{om}M\# \text{ } H \text{ } M+2/ \text{ } BMi2' \text{ } 7 \text{ } +B \text{ } H \text{ } i2tim$
 289 $LQi2 \text{ } i? \text{ } i \text{ } BM \text{ } 1[X \text{ } U9RV \text{ } i?2 \text{ } BMi2' \text{ } M \text{ } H \text{ } 2M2'; v \text{ } 'BbBM; 7'QK \text{ } 2[mBHB\# 'B$
 290 $bi \text{ } i2 \text{ } p \text{ } 'B \text{ } \#H2b \text{ } r \text{ } b \text{ } 2t+HmbBp2Hv \text{ } /2+QmTH2/ \text{ } BMiQ \text{ } i?2B' \text{ } +Q' \text{ } '2bTQM/BM;$
 291 $i?Bb \text{ } Bb \text{ } MQi \text{ } M2+2bb \text{ } 'BHv \text{ } '2[mB'2/ \text{ } 7Q' \text{ } i?2 \text{ } +m' \text{ } '2Mi \text{ } 7Q' \text{ } KmH \text{ } iBQM - \text{ } i?Bb \text{ } b$
 292 $irQ \text{ } '2 \text{ } bQM b, \text{ } URV \text{ } i?2'2 \text{ } '2 \text{ } MQ +m' \text{ } '2Mi \text{ } 2tT2' \text{ } BK2Mib \text{ } i? \text{ } irQmH/ \text{ } bmTTQ' \text{ } i$
 293 $iBQM \text{ } Q7 \text{ } i?2 \text{ } ++m' \text{ } i2 \text{ } 7Q' \text{ } K \text{ } Q7 \text{ } bm+? \text{ } +QmTHBM; c \text{ } UkV \text{ } bm+? \text{ } 7Q' \text{ } K \text{ } Q7 \text{ } +QmT$
 294 $2/Hv \text{ } r2 \text{ } F - \text{ } bQ \text{ } //BM; Bi \text{ } rQMh/ \text{ } QMHv \text{ } +QKTHB+ \text{ } i2 \text{ } i?2 \text{ } /2' \text{ } Bp \text{ } iBQM \text{ } rBi? \text{ } Qmi$
 295 $im \text{ } H \text{ } '2bmHi \text{ } Q7 \text{ } i?Bb \text{ } T \text{ } T2' \text{ } X$

$++Q' /BM; Hv - \text{ } bBM+2 \text{ } r2 \text{ } H'2 \text{ } /v \text{ } bT2+B\}2/ \text{ } i?2 \text{ } 2[mBHB\# 'BmK \text{ } BMi2' \text{ } M \text{ } H$
 $r? \text{ } i \text{ } Bb \text{ } H27i \text{ } Bb \text{ } iQ \text{ } bT2+B7v \text{ } i?2 \text{ } MQM@2[mBHB\# 'BmK \text{ } BMi2' \text{ } M \text{ } H \text{ } 2M2'; v \text{ } BM$
 $bi \text{ } i2 \text{ } p \text{ } 'B \text{ } \#H2bX \text{ } ; \text{ } BM - \text{ } BMi?2 \text{ } \#b2M+2 \text{ } Q7 \text{ } 2tT2' \text{ } BK2Mib \text{ } i? \text{ } irQmH/ \text{ } ?2HT$
 $+i \text{ } 7Q' \text{ } K - \text{ } i?2 \text{ } bBKTH2bi \text{ } 7Q' \text{ } K \text{ } Bb \text{ } +QMbB/2'2/ - \text{ } r?B+? \text{ } BMPQH p2b \text{ } 7m'i?2' \text{ } /2$
 $/2T2M/2Mi \text{ } [m \text{ } /' \text{ } iB+ \text{ } +QMi' \text{ } B \text{ } \#miBQM b \text{ } 7'QK \text{ } i?2 \text{ } irQ \text{ } MQM@2[mBHB\# 'BmK \text{ } b$

$$u_M(\mathcal{J}_K, \xi) = \frac{1}{2}\omega \mathcal{J}_K^2 + \frac{1}{2}\kappa \xi^2, \quad U9jV$$

$bQ \text{ } mbBM; 1[X \text{ } U9kV,$

$$T_K = \omega \mathcal{J}_K, \quad Y_\xi = \kappa \xi, \quad U99V$$

296 $r?2' \text{ } \mathcal{Q} \geq 0 \text{ } M \text{ } k \geq 0 \text{ } X \text{ } h?2 \text{ } [m \text{ } /' \text{ } iB+ \text{ } 7Q' \text{ } K \text{ } Q7 \text{ } i?2 \text{ } TQi2MiB \text{ } H \text{ } \#Qp2 +QmH/ \text{ } \#2 \text{ } b$
 297 $i?2 \text{ } HQR2bi \text{ } Q' /2' \text{ } i2' \text{ } K \text{ } r?B+? \text{ } ? \text{ } b \text{ } KBMBKmK \text{ } Q7 \text{ } h \text{ } vHQ' \text{ } TT' \text{ } QtBK \text{ } iBQM \text{ } iQ$
 298 $i' \text{ } m2 \text{ } 2M2'; v \text{ } TQi2MiB \text{ } HX \text{ } b \text{ } bm+? - \text{ } Bi \text{ } Bb \text{ } +H2 \text{ } 'i? \text{ } i \text{ } bi?2 \text{ } irQ \text{ } M2r \text{ } MQM@2[r$
 299 $p \text{ } 'B \text{ } \#H2b \text{ } p_K \text{ } MB \text{ } \text{ } ? \text{ } M \text{ } \xi \rightarrow 0 \text{ } V - \text{ } i?2 \text{ } iQi \text{ } H \text{ } BMi2' \text{ } M \text{ } H \text{ } 2M2'; v \text{ } TT' \text{ } Q \text{ } +?2b \text{ } Bib \text{ } KBM$
 300 $KmK \text{ } 2[mBHB\# 'BmK \text{ } Q7 - \text{ } H \text{ } 2m2b \text{ } F22TBM; i?2 \text{ } T'2pBQmb \text{ } i' \text{ } m2 \text{ } 2[mBHB\# 'BmK \text{ } '2b$

301 $*QM bB/2'BM; \text{ } ; \text{ } BM \text{ } i?2 \text{ } [m2biBQM \text{ } Q7 +QmTHBM; Q' \text{ } MQi \text{ } i?2 \text{ } 2[mBHB\# '$
 302 $BMi2' \text{ } M \text{ } H \text{ } 2M2'; B2b \text{ } BM \text{ } 1[X \text{ } U9RV - \text{ } QM2 \text{ } K \text{ } v \text{ } rBb \text{ } ? \text{ } M \text{ } Q +Q \text{ } M2bB@2' \text{ } i?2 \text{ } +Q2\{+B2$
 303 $2' \text{ } HHv \text{ } /2T2M/ \text{ } QM \text{ } i?2 \text{ } T \text{ } 'iB \text{ } H \text{ } /2MbBivX >Qr2p2' - \text{ } Bi +QmH/ \text{ } \#2 \text{ } b? \text{ } QrM \text{ } i? \text{ } i$

$rQmH/QMHv // M mMM2+2bb 'v M2; B; B#B2H+QMi' B2/mB BQM2i QQM @$
 $i2ti Q7 2H biB+Biv BM i?2 T ' ; ' T? i? i 7QHHQrb 1[X URNVX h?2'27Q'2- i$
 $HQM; rQmH/ MQi Hi2' i?2 K BM '2bmHi Q7 i?Bb T T2'- BM i?2 7Q'K Q7 i?$
 $/ 'Q/vM KB+ 2[m iBQM Q7 MQM @2[mBHB# 'BmK bQBH r i2' '2i2MiBQMX$

$*QK\#BMBM; i?2 MQM @2[mBHB# 'BmK 1[bX UjN-9y-9kV rBi? i?2 T'2pBQ$
 $RkV- M/ 7QHHQrBM; i?2 b K2 ?v/ 'Q/vM KB+ T'Q+2/m'2- i?2 iQi H 2Mi'QT$
 $bmKb mT \#Qi? i?2'K H M/ K2bQ @ '2H i2/ +QMi' B\#miBQM b Bb ;Bp2M \#v,$

$$R + R_K \equiv F_i \nabla_i T + F_i^K \nabla_i T_K + J_i \nabla_i X + J_i^q \nabla_i X_q + Y_\xi Z_\xi + \mathcal{D} \geq 0. \quad U98V$$

4.4 Total entropy production under typical experimental conditions

$h?2 iQi H 2Mi'QTv T'Q/m+iBQM BM 1[X U98V Bb KQbi ;2M2' H- \#2BM;$
 $M/ 2tT2'BK2Mi H +QM/BiBQM b HBF2X h?Bb 2Mi'QTv T'Q/m+iBQM Bb H$
 $2p Mi iQ Mv K i2' B H TQBMi BM i?2 bT +2- b Bi /2T2M/b QM \#Qi? HQ+ H$
 $; ' /B2MibX AM ivTB+ H 2tT2'BK2Mi H bQBH r i2' +QM/BiBQM b- i?2 QMHv$
 $7'QK i?2 \sim mB/ \sim mt2bX IM/2' i?2b2 +QM/BiBQM b- r2 + M M2;H2+i i?2 i?2'K$
 $i2KT2' im'2 ; ' /B2Mib- br2HH bi?2 K2+? MB+ H /BbbBT iBQM- r?B+? B$
 $M2;HB;B\#H2 p2HQ+Biv ; ' /B2MibX aBKbH 'Hv- Bi Bb mb27mHiQ bi 'i \#v 2$
 $HB;B\#H2 K2+? MB+ H /BbbBT iBQM- bBM+2 ivTB+ H 2tT2'BK2Mi H b2imT$
 $i2' '2i2MiBQM T'QT2'iB2b /Q MQi BmpQH p2 /Bb+2'MB\#H2 K2+? MB+ H \#Q$
 $2ti2'M H +QmbiB+ 2t+Bi iBQMX h?2 BKT +i Q7 bm+? K2+? MB+ H K2+? M$
 $'2i2MiBQM ? b MQi \#22M 2tTHQ'2/ 2tT2'BK2Mi HHv BM i?2 HBi2' im'2- \#r$
 $Q'2iB+ HHv H i2' BM a2+X 9XRRX aBKbH 'Hv- r?BH2 \sim mB/ \sim mt2b '2 \#2B$
 $/2i2'KBM2 i?2 bQBH r i2' '2i2MiBQM '2H iBQM b?BTb- i?2 B' \sim mt Bb M2;H$
 $r i2' \sim mt- \#2+ mb2 i?2 /2MbBiv Q7 r i2' Bb i?'22 Q' /2'b H ;'2' i? Mi? i Q7$
 $BM;Hv- mM/2' 2tT2'BK2Mi H +QM/BiBQM b r2 + M MQ'K HHv mb2,$

$$\nabla_i T \sim 0, \quad \nabla_i T_K \sim 0, \quad \mathcal{D} \sim 0, \quad J_i \ll J_i^q, \quad U9eV$$

$bQ i? i$

$$R + R_K = J_i^q \nabla_i X_q + Y_\xi Z_\xi \geq 0. \quad U9dV$$

4.5 Parallel force decomposition of the total entropy production

$AM Q' /2' iQ + H+mH i2 i?2 ' i2b Q7 2Mi'QTB2b BM 1[bX U3-9yV Bi Bb M$
 $i?2 +QMi' B\#miBQM b iQ7 i?2 B' +mK mH Bp2 BM i?2 X U9dVX 1Mi'QTv$
 $T'Q/m+iBQM B2+ iBp2Hv 'Bb2b \#v /2T 'iBM; K2i bi \#BHBiv r?2M BMi2'7$
 $HQM; bQBH @T 'iB+H2 'Qm;?M2bb TQBMib '2 mMHQ+F2/- r?B+? pB\# i2$
 $BM i?2 pB+BMBivX h?2Y_\xi Z_\xi Q b2ii F2 M2iQ +QMi' B\#mi2 /B'2+iHv M/ 2t+HmbB$
 $iQ i?2 i?2'K H 2Mi'QTv RT'QBM +2BQMBM; Bi +QMi' B\#mi2 /B'2+iHv$
 $K2i bi \#BHBiv /2p2HQT b b22M 2tT2'BK2Mi HHvX PM i?2 Qi?2' ? M/- r2 +$
 $bB\#H2 /2+QKTQb BiBQM b i2QK i?2 \sim mt /2+QKTQb BiBQM ör?2'2 i?2 \sim mt i2'K$
 $J_i^q Bb /2+QKTQb2/ BMiQ iQKBbiB+ M/ K2bQ @ '2H i2/ T 'ib- M/ i7Q'+2 /$
 $r?2'2 i?2 +?2KB+ H \nabla_i Q_i +Bb2'K F2M BMiQ irQX h?2 mb2 Q7 T ' HH2H \sim mtf$
 $/2+QKTQb BiBQM b Bb p2'v +QKKQM BM /2bB;MBM; bT'BM; @bHB/2' bvb i2$
 $Ai Bb HbQ mb2/ BM ?v/ ' mHB+ iQ + H+mH i2 2z2+iBp2 T2'K2 \#BHBiv 7Q'$
 $' i2/ bQHB/ +QKTQM2MibX 1 +? Q7 i?2b2 /2+QKTQb BiBQM 7Q'K mH iBQM$
 $Bb ? ' /iQ /BbiBM; mBb? r?B+? Q7 i?2b2 Bb KQ'2 HBF2Hv rBi?Qmi 7m'i?2'$

$_2K 'F \#Hv- i?2 K BM '2bmHi Q7 i?Bb T T2' '2K Bmb B/2MiB+ H i i?2$
 $BM; mbBM; 2Bi?2' Q7 i?2b2 KQ/2Hb- v2i i?2 7Q'+2 /2+QKTQb BiBQM TT'Q$
 $BM/2T2M/2Mi '2bmHi KQ'2 /B'2+iHvX h?2'27Q'2- i?2 T ' HH2H 7Q'+2 7Q'$
 $\#2HQR- r?BH2 7Q' +QKTH2i2M2bb i?2 Hi2'M iBp2 T ' HH2H \sim mt 7Q'KmH$
 $X b T 'i Q7 i?2 Hi2'M iBp2 7Q'KmH iBQM bm\#iH2 ' i2@/2T2M/2M+2 7$
 $iBp2 Q7 i?2 K2bQ @ '2H i2/ i2'K F2M BMiQ irQX h?2 mb2 Q7 T ' HH2H \sim mtf$
 $/2+QKTQb BiBQM b Bb p2'v +QKKQM BM /2bB;MBM; bT'BM; @bHB/2' bvb i2$
 $Ai Bb HbQ mb2/ BM ?v/ ' mHB+ iQ + H+mH i2 2z2+iBp2 T2'K2 \#BHBiv 7Q'$
 $' i2/ bQHB/ +QKTQM2MibX 1 +? Q7 i?2b2 /2+QKTQb BiBQM 7Q'K mH iBQM$
 $Bb ? ' /iQ /BbiBM; mBb? r?B+? Q7 i?2b2 Bb KQ'2 HBF2Hv rBi?Qmi 7m'i?2'$

331 i?2 T?vbB+ H K2 bm'2 Q7 bHQrHv '2H tBM; ~m+im iBQMb- b /2b+'B#2/ 7
 332 7 ' b i?2 K2+? MB+ H T 'i Q7 i?2 +QMbiBimiBp2 #2? pBQm' Bb +QM+2'M2
 333 Q#i BM ' i2@BM/2T2M/2M+2H2 p2M- r?2bQrM 7Q' 2t KTH2 7Q' /'v b M/ BM
 334 i2'Kb Q7 i?2 ;' MmH ' i2KT2' im'2 UGBm- kykRV- r?B+? Bb ?2'2 +QMbB/2'
 335 iQ_KX

336 A''2bT2+iBp2 Q7 i?2b2 irQ Hi2'M iBp2 TQbbB#H2 /2+QKTQbBiBQMb
 337 i?2 pB#' iBQM Q7 ~mB/@bQHB/ BMi2'7 +2b b?QmH/ /2+ v iQ 7m'i?2' pB#'
 338 i?2KX hQ i?Bb 2H2M_{Ri}?2'Kb b?QmH/ BM+Hm/2 bQm_{++2'K}W- bBMFi2'Kb U
 339 '2bT2+iBp2HvX ++Q'/BM;Hv- r?BH2 i?2 +QMbiBimiBp2 2[m iBQMb 7Q' i?
 340 bT2+B}2/ #2HQR bm+? i? i H-Qi?M?M iQ?2H2UMi'QTv T'Q/m+iBQMb
 341 rQmH/ #2 bi'B+iHv MQM@M2; iBp2- i?2 K2bQ@V2BbiM/QM2'QTB+T2Q/m+iB
 342 7'QK #2BM; M2; iBp2- M/ ?2'2 K v #2 bQ /m2 iQ i?2 +Q''2bTQM/BM; bBMF
 343 bmHi 7QHHRb i?2 T'BM+BTH2 Q7 iirQ@bi ;2 B''2p2'bB#BHBivö7Q' i?2 T
 344 H ;'2' iQ i?2 bK HH2' b+ H2- r?B+? r b Q'B;BM HHv T'QTQb2/ 7Q' /'v ;' M
 345 Gbm- kyyNV- #mi Bb ?2'2 i F2M iQ '2~2+i bBKBH ' ?v/'Q/vm KB+b BM T
 346 bQBHbX

h FBM; HH Q7 i?2b2 BMiQ ++QmMi- i?2 Qp2' HH 2Mi'QTv T'Q/m+iBQ
 Bib K2bQ@'2H i2/ M/ i?2'K H +QM i'B#miBQMb- '2bT2+iBp2Hv,

$$\begin{aligned} R_K &= J_i^q \nabla_i X_{qK} - \gamma T_K^2, & U93V \\ R &= J_i^q \nabla_i X_q + Y_\xi Z_\xi + \gamma T_K^2, & U9NV \end{aligned}$$

r?2'Z_iX_{qK} M_{∇i}X_q '2 i?2 K2bQ@'2H i2/ M/ i?2 i?2'K HHv@'2H i2/ U iQKE
 T 'ib Q_iX_q - #2BM; i?2 +?2KB+ H TQi2MiB H ;' /B2Mi Q7 i?2 r i2' '2H iBp2
 AM Q'/2' iQ '2i BM 1[X U9dV i?2B' bmKK iBQM '2[mB'2b,

$$\nabla_i X_q + \nabla_i X_{qK} \equiv \nabla_i X_q. \quad U8yV$$

347 h?2 BMi2'T'2i iBQM Q7 i?2 #Qp2 Bb i? i i?2 ?QKQ;2MBb2/ ~mt2b BMi
 348 ;Q2b BMiQ KQpBM; K2bQ@b+QTB+ M/ iQKBbiB_i^{qK}/2 J_i^q22b Q7 7'22/QK '2
 349 J_i^q - v2i i?2 7Q'+2b 7'QK i?2 /Bz2'2M+2b BM +?2KB+ H TQi2MiB Hb BM i?2
 350 72'2Mi M/ // iQ ;Bp2 i?2 iQi H ?QKQ_{i2q}MBb2/ TQi2MiB H

4.6 Reciprocal conditions

351 ++Q'/BM; iQ 1[bX U3-9dV i?2 MQM@M2R+iBp2M2bb2Q7 #@i?
 352 Bb}2/X h?2 MQM@M2; iBp2M2bb Q7 i?2 i?2'K) B2M2'QTw TQq/ @+iBQM U
 353 BM; PMb ;2'öb '2+BT'Q+ H +QM/BiBQMb 7Q' ;2M2' HHv +QmTH2/ B''2p2'b
 354 BM;Hv- 7Q' i?2 +m''2Mi T'Q#H2K
 355

$$\begin{pmatrix} Z_\xi \\ \nabla_i X_q \end{pmatrix} = \begin{pmatrix} r_j^{\xi\xi} & r_j^{\xi q} \\ r_i^{q\xi} & r_{ij}^{qq} \end{pmatrix} \cdot \begin{pmatrix} Y_\xi \\ J_j^q \end{pmatrix}, \quad U8RV$$

r?2'2 r2 ? p2 BMi'Q/m+2/ BM i?2 K i'Bt i?2 ;2M2' HBb2/ '2bBbiBpBiv +Q2
 Q7 i?2 i' MbTQ'i +Q2{+B2Mib BM UPMb ;2'- RNjRVVX ++Q'/BM; iQ U* bB
 bBbiBpBiv +Q2{+B2Mib Kmbi b iBb7v,

$$r_i^{q\xi} = -r_i^{\xi q}, \quad r_{ij}^{qq} = r_{ji}^{qq}, \quad U8kV$$

356 r?2'2^ξ > 0 -r_{ij}^{qq} > 0 7Q' = j - Mdet(r_{ij}^{qq}) > 0 X

6m'i?2'KQ'2- Bi Bb Hr vb TQbbB#H2 iQ /2}M2

$$r_i^{\xi q} = r^{\xi q} \hat{e}_i, \quad J_i^q = J_q \hat{e}_i, \quad U8jV$$

$$r?2'2^q \xi \quad M J_q / 2 M Q i 2 i?2 K ; M B_i^{\xi q} / M J_i^q - 7'2 b T 2 + i B p 2 H v B b r i? B B M \\ b i?2 m M B i p 2 + i Q' \quad H Q M ; r? B_i^q? T?2 7 r 2 i 2 M + B t H H v H B 2 i X h?2 M - b B M + 2 \\ 1 - i?2 \} ' b i P M b ; 2' ' 2 H i B Q M B M 1 [X U 8 R V ' 2 / m + 2 b i Q,$$

$$Z_{\xi} = r^{\xi \xi} Y_{\xi} + r^{\xi q} J_q. \quad U 8 9 V$$

$$6 B M \quad H H v - ; B p 2 M i?2 T Q b B i B p 2 / B ; Q M \quad H + Q 2 \{ + B 2 B M b 1 B M U 9 N X U 8 R - 8 K \\ B i B b + H 2 R \geq i? X i h Q 2 M b m R \geq 0 r 2 K m b i i? m b 7 m' i?2' ' 2 [m B' 2$$

$$\nabla_i X_{qK} = \eta J_i^q, \quad U 8 8 V$$

$$r B i \eta \geq 0 X$$

4.7 Water flux through finite experimental volumes

$$a B M \sum_i 2_i^{\beta} = 0 \quad M J_i \ll J_i^q \quad 7' Q K 1 [b X U R k - 9 e V - r 2 J_i^q H b Q J M / i? i \\ l b B M ; i?2 + Q M b 2' p \quad i B Q M Q 7 b Q H B / 2 M b B i v B M 1 [X U R y V - B i 7 Q H H Q r b i? \\ \partial_t \varrho_a = - \nabla_i (\varrho_a v_i + J_i^q) X > Q r 2 p 2' - / m' B M ; 2 t T 2' B K 2 M i \quad H r i 2' ' 2 i 2 M i B Q M i 2 b i b i?2 \\ K \quad b b B b F 2 T i + Q M b i 0 M i \quad B Q r 2 J_i^q \neq - \varrho_a v_i X * Q K \# B M B M ; i?2 H i i 2' ' 2 H @ \\ i B Q M r B i? i?2 + Q M b 2' p \quad i B Q M Q 7 r i 2' / 2 M b B i v B M 1 [X U R y V - r 2 ? p 2,$$

$$\partial_t \varrho_q = \left(1 + \frac{\varrho_q}{\varrho_a} \right) \nabla_i J_i^q. \quad U 8 e V$$

$$h?2 \quad \# Q p 2 2 [m \quad i B Q M ' 2 H i 2 b i?2' \quad i 2 Q 7 +? M ; 2 Q 7 i?2 T ' i B \quad H r i 2' / 2 M \\ r i 2' \sim m t i? ' Q m ; ? i?2 p Q H m K 2 Q 7 \quad M B M \} M B i 2 b B K \quad H ' 2 T ' 2 b 2 M i \quad i B p 2 p Q H m \\ h?2 H \quad b i 2 [m \quad i B Q M + Q m H / \# 2 + Q M p 2' i 2 / i Q + Q 7 M 2 i B / 2' B K \} 2 M B i 2 i p Q H m K 2 \\ i 2' ' 2 i 2 M i B Q M b \quad K T H 2 b i? ' Q m ; ? B M i 2 ; ' \quad i B Q M X \quad l b B M ; d V?2 / B p 2' ; 2 M + 2 i?2 Q \\ \oint J_i^q d S_i = J_q A \quad U? p B M ; \hat{e}_i d A - r B i A \# 2 B M ; i?2 b m' 7 + 2' ' 2 i? ' Q m ; ? r? B + ? \\ i?2 r i 2' \sim Q r b B M i Q?2' 2 7 Q' 2 - r 2 \} M / i? i$$

$$J_q = \left(\frac{\varrho_a}{\varrho_a + \varrho_q} \right) l \partial_t \varrho_q, \quad U 8 d V$$

$$r?2'2 = \frac{V}{A} \quad B b i?2 i v T B + \quad H 2 t T 2' B K 2 M i \quad H / B K 2 M b B Q M X \quad h?2' 2 7 Q' 2 - i?2 \sim m t \\ i?2 2 t T 2' B K 2 M i \quad H b \quad K T H 2 b + \quad H 2 b \quad T T' Q t B K i 2 H v r B i? i?2' \quad i 2 Q 7 r i 2' / 2 M \\ B K 2 M i \quad H / B K 2 M b B Q M X \quad l b B M ; i?2 / 2 \} M B i B Q M b B M 1 [X U j V - B i + Q m H / \quad H b C \\ T' 2 b b 2 / B M i 2' K b Q 7' \quad i 2 Q 7 i?2 b \quad i m' \quad i B Q M \quad M / p Q B / ' \quad i B Q,$$

$$J_q = C \left(\partial_t S_r + \frac{S_r}{e(1+e)} \partial_t e \right), \quad C \equiv C(S_r, e) = l \left(\frac{e}{1+e} \right) \left(\frac{\bar{\varrho}_a \bar{\varrho}_q}{\bar{\varrho}_a + \bar{\varrho}_q S_r e} \right). \quad U 8 3 V$$

$$h?2 \quad \# Q p 2 B b ; 2 M 2' \quad H X > Q r 2 p 2' - B M \quad K Q b i 2 t T 2' B K 2 M i \quad H K 2 \quad b m' 2 K 2 M i \\ ' 2 i 2 M i B Q M T' Q T 2' i B 2 b i B 2 p Q B M / K 2 i B + Q M b i \quad M i - r? B H 2 i?2 B M i' B M b B + / 2 M b \\ +? M ; 2 7' Q K i?2 B' ' 2 7 2' 2 M + 2,$$

$$J_q = C \partial_t S_r \quad U 8 N V$$

$$b m + ? i? \quad i i?2 r i 2' \sim m t B M i Q i?2 b \quad K T H 2 ' 2 H i 2 b / B' 2 + i H v i Q i?2' \quad i 2 Q 7 b i$$

4.8 Non-equilibrium soil water retention

$$* Q K \# B M B M ; 1 [b X U 9 y - 9 9 V \quad r B i? 1 [b X U 9 y - 9 9 V Q i i P B M t T 2' 2 \\ B K 2 M i \quad H + Q M / B i B Q M b - \quad M / \quad b b m K B M ; ? Q K Q ; 2 M 2 Q m b i 2 b i B M ; + Q M / B i B Q M \\ / B 2 M i b + Q m H / \# 2 B ; M Q' 2 /,$$

$$\partial_t T_K = \frac{\omega R_K}{T_K} = \frac{\omega \gamma (T_{K0}^2 - T_K^2)}{T_K}, \quad U e y V$$

$r?2'2 \text{ mbBM}; 1[X \text{ U}8jV,$

$$T_{K0} = \sqrt{\frac{\eta}{\gamma}} |J_q|, \quad \text{UeRV}$$

361 B b i?2 bi iBQM 'v b Q HQT2BM-Q7B+? '2T'2b2Mib i?2 p Hm2 Q7 i?2 K2bQ@
362 '2H i2/i2KT2' im'2 ' i i?2 ' i2 BM/2T2M/2M+2 HBKBi Q7 p2'v bHQR r2iiBM;
363 /BiBQMbX h?2 KQ'2 ;2M2' H + b2 Q7 ' i2 @/2T2M/2Mi T'Q+2bb2b Bb /Bb+i

$L2ti- /QTiBM; 1[X \text{ U}89V \text{ HQM}; rBi? 1[X \text{ U}jNV 7Q' ?QKQ; 2M2Qmb i2bi$

$$\partial_t \xi + r^{\xi \xi} Y_\xi + r^{\xi q} J_q = 0. \quad \text{UekV}$$

q?2M MQ r i2' Bb //2/ Q' bm#i' +i2/ 7'QK i?2 2tT2/ B K2Mi H b KTH2- r
0- r2 2tT2+ii?2 BMi'BMbB+ b m i B Q M v / i Q r B' / i B Q M i' m2 2[mBHB# 'BmK QMH
r?2M i?2 K2bQ@b+QTB+ BMi2'7 +2b '2 #2BM; bm{+B2MiHv ;Bi i2/X am+
2tT2+i2/Hv 2M #H2 iQ mMD K B' @r i2' BMi2'7 +2b i? i K v Qi?2'rBb2 #2
2[mBHB# 'B HQM; T 'iB+H2 'Qm;?M2bb M/ T i+?2b Q7 ~mB/bX AM i?2?v
i?2 2z2+ib Q7 i?Qb2 ;Bi iBQM b QM i?2 2M2';v H M/b+ T2 '2 +QM bB/2'2/
'2H i2/ 2Mi Q T v i2KT2' T m'2b +QM+2Tim HBb2/ #v 6B;X 8X h FBM; i?2b2 B
BMiQ ++QmMi- r2 '2[mB'2,

$$r^{\xi \xi} = rT_K, \quad \text{UejV}$$

$rBi? \geq 0 - r?BH2 ; Bp2M i?2 bi iBQM \text{ } \sqrt{T_{K0}} B m i B Q M \text{ } Q7V,$

$$r^{\xi \xi} = r \sqrt{\frac{\eta}{\gamma}} |J_q|. \quad \text{Ue9V}$$

364

$_{-2+} \text{ HHBM} \xi; i 2 \xi i7'QK 1[X \text{ U}99V- \text{ b } r2HH \text{ b } 1[X \text{ U}8NV ,$

$$\partial_t \xi = -rk \sqrt{\frac{\eta}{\gamma}} |J_q| \xi - r^{\xi q} J_q, \quad J_q = C \partial_t S_r. \quad \text{Ue8V}$$

h?2 #Qp2 2[m iBQM '2T'2b2Mib i?2 K BM '2bmHi Q7 i?Bb T T2' ?v/
'2bQH p2/ 2[m iBQM 7Q' bQBH r i2' '2i2MiBQM i MQM @2[mBHB# 'BmK +Q
BMi'BMbB+ bm+i B Q M T Q M b / b B Q M i2'2iB+ HHv iQ +v+HB+ H r2iiBM; @/'vBM
i Q_r X LQiB+2 HbQ i? i{+,Q2, {r B 2BMb1[X Ue8V K v HH /2T2M/ QM i?2 p '@
BQmb T 'iB H Q b 2M M B i B 2i B '2bT2+iBp2 iQ +v+H 2M /i?2m2p Q7
MQM @2[mBHB# 'BmK bm+iBQM rQM H/ BM ;2M2' H /2T2M/ QM i?Qb2 /2Mb
QM. M b V X 6BM HHv- M 2p2M bBKTH2' 7Q'K Q7 i?Bb ?v/'Q/vM KB+ 2[m i
p2HQT2/ #v MQiB+BM; i? i 1[X Ue8V Bb 2MiB'2Hv ' i2 BM/2T2M/2Mi- bQ K
iBp2b BM i?2 #Qp2 2[m iBQM +QmH/ #2 bm#bi B K m i 2b B B i? }MBi2 BM+'2K2
dx V,

$$\boxed{d\xi = -\mathcal{A} |dS_r| \xi - \mathcal{B} dS_r} \quad \text{UeeV}$$

$r?2'2 \text{ MB '2 2z2+iBp2 MQM @M2; iBp2 +Q2\{+B2Mib 7Q' i?2 BMi'BMbB+ b}$
 $r?B+? '2 ;Bp2M \#v$

$$\mathcal{A} \equiv \mathcal{A}(S_r, e) = rk \sqrt{\frac{\eta}{\gamma}} \mathcal{C}, \quad \mathcal{B} \equiv \mathcal{B}(S_r, e) = r^{\xi q} \mathcal{C}. \quad \text{UedV}$$

365 h?2 #Qp2 2[m i B Q M # Q Q 2/- b r2 rBb? iQ ?B;?HB;?i Bi b i?2 K BM '2bm
366 i?Bb T T2'X q?BH2 i?2 /2' Bp iBQM 7QH H Q r b +H2 ' v2i bm#iH2 ?v/'Q/vM K
367 M H '2bmHi Bb 2ti'2K2Hv bBKTH2 M/ b?QmH/ i?mb Qz2' bQm'+2 Q7 BMb
368 TB'B+ H BMP2biB; iBQM b BM ?v/'QH Q;v M/ bQBH K2+? MB+bX :Bp2M i?2
369 2[mBHB# 'BmK BMi3 B M b B z l B M +1 B Q M j e V- +QKTH2i2 KQ/2H 7Q' bQBH r i2
370 '2i2MiB Q M (S_r, e) +QmH/ i?2M #2 Q#i BM2/ mbBM; 1[X UjdV- ;Bp2M i?2 '2H i
371 7Q' i?2 2[mBHB# 'B m K M b m +U B Q M b i B b 7vBM; 1[X UKNVVX

aBM+2i?2}'bi i2'K BM 1[X UeeV H-BiM2 +Q2 b+BMBirBQM @
i'QHbi?2'2H t iBQM ' i2 Q7 i?2 bvbi2K # +F iQ 2[mBHB#'BmK bi i2bX h?
MQi /2T2M / QMi?mb i?2 +Q2 BHB BM+QMi'QHbi?2 /'B7i' i2 r v 7'QKi?2 2[
HB#'BmK bi i2X h?2 +iBQM b Q7 i?2b2 irQ i2'Kb +QKT2i2- bQ QMHv B7 i?
b2Mi- rQmH/ i?2 bm+iBQM /2pB iBQM '2H tiQ x2'Q- i?mb H2iiBM; i?2 bQE
'Bp2 iQ Bib 2[mBHB#'BmK bi i2X hQ 7m'i?2' mM/2'bi M/ i?2 'QH2 Q7 i?Bb
i?2 bi iBQM 'v + b2 r?2'2 i?2b2 i2'Kb # H M+2,

$$\xi_a = -\frac{B}{A} \frac{dS_r}{|dS_r|} = \mp \frac{B}{A} \quad (7Q\xi = 0), \quad \text{Ue3V}$$

r?B+? /Q2b MQi +im HCHV/2iT2m/ QMi?2 bvbi2KX/BQ2MBB2QBM;
+QMbiMMB- QM+2 TT'Q'2R2BMB QM Bib bi iBQMPMip2H2i2'2'
? M/- +QMbiB/L2'BM2M/2M; QMi- i?2 BMi'BMbB+ bm+iBQM/2pB iBQM
/'B7i 7ξQKL2p2'i?2H2bb- i?2 bi iBQM 'v '2H iBQM b BM 1[X Ue3V rBHH #2
mb27mH BM/B+ iBQM iQ i?2 #QmM/ 'B2b Q7 i?2 /'vBM; M/ r2iiBM; T'Q+
'Q#mbi M/ bBKTH2 QT iBQM 7QBi/Q2bB2iT2Q2(QMB2M2BH2 i?2
'2H t iBQM +Q2B bF2Mii +QMbi Mi,

$$A = a, \quad B = -b \frac{\partial \hat{s}_2}{\partial S_r}, \quad \text{UeNV}$$

372 r?2'2a,b} ≥ 0 '2 MQM@M2; iBp2 +QMiBiS(M(S,X) KBW+2T2Mε QM
373 bQ /Q2b M/ i?mb i?2 T'2/B+i2/ bQBH r i2' '2i2MiBQM Pmip2b HbQ p 'v rBi
374 Qi?2' ? M/- '2+ HH i? i 1[X UeeV B; MQ'2b i?2 TQbbB#BHBiv Q7 ?vbi2'2iB
375 #H2 ≠ 0- BM+Hm/BM; mM\$2X +Q2Q biHM7mim'2 2tT2'BK2Mib 2tTHQ'2 M/ '2(
376 TQ'i bm+? T?2MQK2MQM- QM2 KΛi '2TQ-K 42X?28mV2rQ i? Bib KQ'2 ;2M@
377 2' H 7Q'K BM 1[X U83VX LQ i2 =Hb-Qr2?} MmM≠2 $\frac{\partial \hat{s}_2}{\partial S_r} dS_r$ - M/ bQ
378 1[X UeeV vB2H/b TQ'Q+Bs_r|ξ2Hvξ₂X 6Q≡0 BMi2;' iBQM~;Bp2b
379 bξ₂[- b?QrBM; i? i i?2 2ti2Mi Q7 BMi'BMbB+ bm+iBQM /2pB iBQM 7'QK Bib
380 #'Q /Hv /2T2M/ QM i?2 2[mBHB#'BmK bm+iBQM H2p2H Bib2H7X h?Bb b+
381 b HHv '2~2+i2/ 7'QK 2tT2'BK2Mi H '2i2MiBQM i2bib- i?mb T'QpB/BM; i?2
382 ?BM/ 1[X UeNVX

4.9 Illustration of non-equilibrium results

383
384 am#b2+iBQM jX8 BHHmbi' i2bi?2 T' +iB+ H bB;MB}+ M+2 Q7 i?2 2[mf
385 M KB+ i?2Q'v #v UCB M; 2i HX- kyRdV #v 2KTHQvBM; i?2 T?2MQK2MQHQ
386 j8VX h?Bb KQ/2H HHQrb 7Q' b? 'T2' 'Bb2 Q7 bmm+2BQM QBiQ i?2 pB+BME
387 T '2/ rBi? i?2B' H i2bi T?2MQK2MQHQ;B+ H KQ/2H BM U1BM p GBm- kyl
388 Bb MQi i?2 K BM '2bmHi Q7 i?Bb T T2'X h?2 +im H BK M/ '2bmHi '2 iQ
389 Q7 bQBH r i2' '2i2MiBQM BM MQM@2[mBHB#'BmK +QM/BiBQM bX 6Q' i? i
390 i?Bb K BM '2bmHi #v /QTiBM; i?2 b K2 2[mBHB#'BmK 1[bX Ujj@j8V b i?2
391 MQM@2[mBHB#'BmK 2[m iBQM b BM 1[X Uje-j3-eeVX ++Q'/BM;Hv- i?2 QI
392 [mB'2/ 7Q' + Tim'BM; MQM@2[mBHB#'BmK bm+iBQM b '2

393 hQ i?Bb 2M/- 6B;X 9 T'2b2Mib i?2 2z2+ib Q7 i?2b2 T ' K2i2'b QM i?2 M
394 BMi'BMbB+ bm+iBQM/2pB eiBQim U 'QrV- BMi'BMbB+ /bmk_{te} iBQM U
395 KB//H2 'QrV- M/ K2 bms*2≠ b/(m+ iBQM MQiQK 'QrV- 7Q' MBQmb
396 p Hm2bX h?2b2 };m'2b +Q' '2bTQM/ iQ bm+iBQM@+QM i=QHH2/ T'QiQ+QI
397 1 Mξ = 0- i?2M /'vBM; +QM iBQM 2b'QBHrP B+? r2 TTHB2/ Ryy bK HH BM@
398 + '2K2Mi H +v+H2b Q7 bmm+2BQM HQ7r2/ #v 7m'i?2'2'≠50BM2i @BH
399 iBM; iBHH M2; iBp2=b+20-iBQM/ Q7H /'vBM;s#≠0ξ iQ

400 LQiB+2 i? i /m'BM; +QM iBMmQmb /' BM ;2 M/ BK#B#BiBQM i?2 bi i2 B
401 T ' HH2H iQ i?2 bi iBQM 'v bQHmiBQM b b?QrM #v i?2 / b?2/ Q' M;2 HBM2
402 KQbi BKTQ'i MiHv- Mv /2pB iBQM 7'QK i?2 / b?2/ #Hm2 2[mBHB#'BmK +r
403 iQ MQM@2[mBHB#'BmK bi i2bX Ai Bb b?QrM i? i BM Q'/2' iQ '2; BM i?2 2[

6B;m`2 9, h?2 2z2+ib Qa7 M b BM i2[ X UeNV QM i?2 /vM KB+b Q7 i?2 BMi`B
bm+iBQM /2pBMiB QMUeeV- i?2 MQM@2[mBHB#n B BM;B1M X` B MbV+ bm+iBQM
M/ i?2 MQM@2[mBHB#n B BM; 1rX+ iBjQM- HH ; \$ M bāibi BrQ`MiBQM
bQHmiBQM b `2 ;Bp2M #v 1[X Ue3V M/ 2[mBHB#`B BMK mb B2M; `2 b?QrM b
1[bX Uj9-jjV 7Q` i?2 irQ +Q``2bTQM/BM; bm+iBQM bX

9y9 + M //bm{+B2Mir2iiBM;@/`vBM;MQBb2- r?B+? + mb2b /`B7iBM; 7`QK i?
9y8 iQ i?2 2[mBHB#`BmK QM26Q?2t2KTH2- B;MQ`BM; i?2 Ryy bK HH +v+H2b-
9ye `2bi Q7 i?2 +m`p2 `2p2 Hb i?2 T`2pBQmbHv 2X2M2QK2MQH2;B7+2HHv /m#
9yd HX- kyR8c "2`BQxFBM Jm H2K- kyR3VV, ö}`bi /` BM ;2 +m`p2ö U HbQ F
9y3 /`vBM; +m`p2öV- ö}`bi BK#B#BiBQMö U HbQ FMQrM b öT`BK `v r2iiBM; +
9yN iBQM- M/ i?2 7QHHQrBM; öb2+QM/ `v /` BM ;2ö U HbQ FMQrM b öb2+QM
9Ry i?2 }M H H2; Q7 Qm`i2biBM; T`QiQ+QH- r?B+? /Q2B MQi; Q`2im`M iQ 7m
9RR 1 is=0X h?Bb BM #BHBiv iQ`2; BM 7mHH b im` iBQM 7i2`bm+? /`vBM;@
9Rk +H2 ? b #22M ii`B#mi2/ iQ ö B`2Mi` TK2Miö M Q#pBQmb bvMQMvK 7Q
9Rj #mir2 `2 MQi r`2 Q7 +H2 `7mM/ K2Mi H 2tTH M iBQMbiQ i?Bb /2p2H
9R9 T`QTQbBiBQMbiQ mb2 `#Bi` `v +m`p2@}iiBM; T`QiQ+QHbX h?Bb ; T Bb
9R8 i? MFbiQ i?2 MQM@2[mBHB#`BmK i?2`KQ/vM KB+ T?vbB+b- br2 b? HH
9Re i?2 M2ti bm#b2+iBQM ; BMbi 2tT2`BK2MibX

9Rd 6BM HHv- Bi Bb mb27mHiQ +QKT `2 i?2 KB//H2 M/`B;?i+QHmKMb B
9R3 a Mb +Q2{+B2Mib ? p2 #22M [m /`mTH2/ i?Bb +? M;2 +QmH/ `Bb2 7Q` 2
9RN K i2`B H #v [m /`mTHBM; i?2 2tT2f B K2B b B b B22MBB2Q MK U83V-
9ky r?BH2 #Qia MB b/2T2M/b HBM2 `Hv QM Bi U1[bX B A3 VX aBM+2 i?2 `iBQ
9kR b=aBb B/2MiB+ H 7Q` #Qi? +QHmKMb- i?2 bi iBQM `v HBM2b QM i?2 +Q`2
9kk B/2MiB+ H br2HHX q2 i?mb /Q MQi Q#b2`p2 Km+? /Bz2`2M+2b r?2M i?2 `
9kj i?2 bi iBQM `v HBM2bX PM i?2 Qi?2` ? M/- i?2 `i2b Q7 TT`Q +?BM; iQ i?
9k9 bB;MB}+ MiHv- 7 +ii? i+QmH/ #2 i2bi2/ 2tT2`BK2Mi HHv BM i?2 7mim`2
9k8 r i2` `2i2MiBQM i2bib 7Q` B/2MiB+ H K i2`B Hb v2i /BbiBM+iBp2Hv /Bz2`

6B;m'2 Ry, _ i2@/2T2M/2Mi'2bTQMb2b Q7 MQM@/BK2MbBQM HBb2/ bmmM/2' /Bz2'2Mi+QMbi Mid_tS_r2b Q7</0vBM;iBM; 7'QK 7mHH b im' iBQM rBi?'2H t2/ bT_Ki2 0X S ' K2i2'2b,0.5-a = 10 Mb = 2X HbQ b?QrM BM / b?2/ HBM2b '2 i?2 ' i2@BM/2T2M/2Mi K2i bi #H2 bi i2b M/i?2 bi i2b +Q''2bT i?2'KQ/vM KB+ 2[mBHB#'BmKX U ii BMBM;i?2 i'm2 i?2'KQ/vM KB+ 2[mB im'# iBQM b- b /2b+'B#2/ BM a2+X 9XV

585

5.2 Drying-wetting cycles with diminishing amplitude

b b2+QM/ M/ bHB;?iHv KQ'2 2H #Q' i2/ 2t KTH2 +QMbB/2' +QM i'QH r2iiBM; +v+H2b rBi? /BKBMbB?BM; KTHBim/2b p MBb?X ivTB+ H 2t KTH

$$S_r(t) = 1 - a + a \cos(\pi \frac{t}{t_1}) e^{-\frac{t}{t_2}}, \quad 7Q' r?B+? \quad U3yV$$

$$d_t S_r(t) = -\frac{a}{t_1 t_2} t_1 \cos(\pi \frac{t}{t_1}) + \pi t_2 \sin(\pi \frac{t}{t_1}) \cdot \quad U3RV$$

586

587

588

589

r?2'2'2T'2b2Mib i?2 /m' iBQM Q7 2 +? /'vBM; Q' in2T B+MH H2K-2 M/ /m'BM; r?B+? i?2 /'vBM; Q' r2iiBM; KTHBim/2b p MBb?X ivTB+ H 2t KTH iBQM rBi? iBKBb b?QrM QM 6B;X RR t2 Q2t i?2HQB2; QB i? Bib /2'Bp @ iBp2 b / b?2/ #H +F HBM2 QM 6B;X RR X

590

591

592

593

594

595

596

597

598

599

h?2 7Q'K Q7 +QmH/ #2 BMi2;' i2/ M H vBM+1 [XHV d8bV BM;/ vB2H/b +HQb2/@7Q'K 2tT'2bbbBQMX h?Bb bQHmiBQM Bb iQQ HQM; iQ #2 BHHm '2T'2b2Mi iBQM Bb mb27mH M/ ;Bp2M BM 6B;X RR X aQ Mi?222 /Bz2'2Mi iv i?2 ' i2 BM/2T2M/2Mi BHBK Bi2Q7 Wb22 1[X UddVV- Bi Bb mb27mH iQ +QKT i?2b2 BM 6B;X RR# Qp2' iBK2X 6Q' HQM; pt₁'vBM/2T2 Qi?2b'2b@ Bi? /2T2M/2Mi '2bT Q+QMp Q'2b iQ Bib ' i2@BM/2T2M/2Mi HBK BiX Ai Bb i?2M bm#biBimi2 i?2 (t)MBHBK 1[X UdeV M/ BMi2;'xi2 M/mi R2 MB '2H Qp2' i?2 MQM@2[mBHB#'BmK BMi'BMbB+ M/ K2 bm'2/ bm+iBQM bX h?2 +Q''2b BM 6B;X RR+- ; BM ?B;?HB;?iBM; +QM p2';2M+2 iQ i?2 ' i2@BM/2T2M/2M bHQr T'Q+2bb2bX

600

6 Conclusions

601

602

603

604

605

606

h?Bb T T2' ? b T'2b2Mi2/i?2 /2p2HQTK2Mi Q7 ?v/'Q/vM KB+ i?2Q'v b im' i2/ bQBHb i MQM@2[mBHB#'BmK +QM/BiBQM bX h?2 i?2Q'v T'2/B+ Q'QmbHv M/ +QM bBbi2MiHv HH i?2 TBpQi H T?2MQK2M 2t?B#Bi2/ #v b H iBQM b?BTbX hQ i?Bb 2M/- MmK#2' Q7 BKTQ'i Mi i?2Q'2iB+ H bi2Tb ? /BbiBM;mBb? i?2 +m''2Mi rQ'F 7'QK T'2pBQmb i?2'KQ/vM KB+ 7Q'KmH iE im'2X

6B;m'2 RR, .vM KB+ /'vBM;@r2iiBM; +v+H2b rBi? /BKBMbB?BM; KTHBin
iBK2bU V b im' iBQM Qp2' iBK2c U#V K2KQ'v 7mM+iBQM Qp2' iBK2c M/
r i2' '2i2MiBQM +m'p2bX

607 R Xi i?2 HBKBi Q7 i?2'KQ/vM KB+ 2[mBHB#'BmK- i?2 /2'Bp iBQM ;Bp2b
608 7mHHv +QMbBbi2Mi rBi? i?2 'B;Q'Qmb }M/BM;b Q7 UCB M; 2i HX- ky
609 iBQM + '27mHHv /BbiBM;mBb?2/ i?2 2ti2'M HHv K2 bm'2/f TTHB2/ br
610 + TBHH 'v T'2bbm'2V 7'QK i?2 BMi'BMbB+ bm+iBQM i?'Qm;? +?2KB+
611 Qmi i?2 bQBH- M/ '2H i2/ i?2b2 iQ i?2 2z2+iBp2 bi'2bb Q7 i?2 bQBHX
612 kXhQ ++QKKQ/ i2 2K2';2Mi 72 im'2b i MQM@2[mBHB#'BmK +QM/BiBQ
613 Q'v /BbiBM;mBb?2/ i?2 i'm2 i?2'KQ/vM KB+ 2[mBHB#'BmK 7'QK K2i b
614 r b ' ;m2/ M/ b?QrMi? i mMHBf2 2[mBHB#'BmK bi i2b- K2i bi #H2 bi
615 mMB[m2 M/ + M #2 2 bBHv /Bbim'#2/ #v 2ti2'M H T2'im'# iBQMbX T
616 im'# iBQMb + M H2 / i?2 bvb i2K iQr '/b Bib mMB[m2 2[mBHB#'BmK bi
617 jXhrQ MQM@2[mBHB#'BmK Bmi2'M H p 'B #H2b M/ i?2B' 2pQHmiBQM 2
618 /m+2/ M/ /2p2HQT2/, U V i?2 BMi'BMbB+ bm+iBQM BM UCB M; 2
619 /2pB iBQM 7'QK i?2 2[mBHB#' i2/ BMi'BMbB+ bm+iBQM BM UCB M; 2
620 i?2 K2bQ@'2H i2/ i2'K-Tr2'Bi-m'2' +Fb /2;'22b Q7 7'22/QK i i?2 b+ H2 Q
621 ;' BMb M/ TQ'2bX
622 9Xh?2 ' i2ξQ7b 'B;Q'QmbHv /2'Bp2/ # b2/ QM PMb ;2'öb '2+BT'Q+Biv T'B
623 /BbbBT iBp2 T'Q+2bb2b i i?2 KBf_K Q bb+ '2+2Qp2?2/'mi2BQM; i?2
624 UCB M; GBm- kyyNVöb T'BM+BTH2 Q7 irQ@bi ;2 B''2p2'bB#BHBiv- r
625 iQ '2~2+i ~m+im iBM; ;' BM KQiBQMb- #mi ?2'2 7m'i?2' i F2MiQ /2b+
626 Q7 ~m+im iBM; KQiBQMb Q7 ~mB/ T i+?2b M/ Bmi2'7 +2bX
627 8Xi i?2 bi iBQM 'vT'KBiRBMQ7 i?2Q'v T'2/B+ib ' i2@BM/2T2M/2M+2- r?2
628 QM+2 /'vBM; Q' r2iiBM; biQT i Mv ;Bp2M bm+iBQM@b im' iBQM TQB
629 K BMb K2i bi #H2X h?2 K2i bi #BHBiv ?B;?HB;?ib i? i i?2b2 bi i2b '
630 + 'BQmb iQ bK HH T2'im'# iBQMb #v r2iiBM;@/'vBM; +v+H2b Q' ;' BM
631 eXq?2M_K /Q2b MQi ? p2 2MQm;? iBK2 iQ '2H tiQ Bib bi iBQM 'v HBKBi-
632 b?Qrb ' i2@/2T2M/2M+2- bi?2 KQiBQM Q7 ~mB/ T i+?2b M/ Bmi2'7
633 bm+iBQM Bb /vM KB+X
634 dXq?2'2 i?2 2[mBHB#'BmK M/ MQM@2[mBHB#'BmK bi iBQM 'v p Hm2b
635 BMi'BMbB+ bm+iBQMb /Q MQi /2T2M/ QM 2tT2'BK2Mi H b KTH2 /BK2I
636 '2p2 Hb i? i i?2 ' i2 Q7 i?2 TT'Q +? Q7 i?2 MQM@2[mBHB#'BmK bm+
637 'v p Hm2 b?QmH/ /2T2M/ QM BiX

6.1 Perspectives

eXRRR 1tT2`BK2Mi H

T 'i 7'QK 2tTH BMBM; T'2pBQmbHv '2TQ'i2/ T?2MQK2M Q7 bQBH r i
i?2 T'QTQb2/ i?2Q'v QT2Mb K Mv TQbbB#H2 p2Mm2b 7Q' 7mim'2 '2b2 '+
6Q' 2t KTH2- mb27mH 2tT2`BK2Mi H +QMi'B#miBQM +QmH/ #2 iQ 7m'i?2
Q7 mMB[m2 2[mBHB#`BmK BM T 'iB HHv b im' i2/ bQBHbX hQ i?Bb 2M/-Q
bK HH KTHBim/2 r2iiBM; @/'vBM; +v+H2b 7'QKSpX' BQ2mb2/2v'Q2b Q7 b im'
+H2b +QmH/ #2 /QM2 b bHQrHv b M22/2/ BM Q' /2' iQ 2Mbm'2 i?2 bvbi2K
' i2@BM/2T2M/2Mi '2bTQMb2- pBbBiBM; QMHv K2i bi #H2 bi i2bX IM/2' 7
;2M2BiB2b BM 2tT2`BK2Mi H b KTH2b KB; ?i /2p2HQT- K2 MBM; i?2Q'2iB-
}2H/ Q7 K2bQ@'2H i2T'XK>Qr2im2 "2 i2 +? TQBMiBM i?2 /QK BMi?2 i'm2
HB#`BmK '2i2MiBQM +m'p2 Bb BM/2T2M/2MhQ7 iH2p'BM BQM 2Q7
QMhv KQ/B}2b i?2 iBK2 M22/2/ iQ ''Bp2 ii?Bb +m'p2- MQi Bib 7Q'KX

b2+QM/ 2tT2`BK2Mi H TT'Q +? iQ 2tTHQ'2 i?2 +QM+2Ti Q7 mMB[m2
#2 iQ TTHv +QmbiB+ r p2bX *?QQbBM; bQmM/ r p2 i? iBb Km+? H ' ;2'
T 'iB+H2b- M2B; ?#Qm'BM; T 'iB+H2b rQmH/ /2p2HQT bHB; ?iHv /Bz2'2Mi
BM; iQ /BbbBT iBQM Q7 i?2 bQmM/ r p2- r?B+? ++Q' /BM; iQ i?2 i?2Q'v rE
K2bQ@'2H i2/ i2KkT0rB i?Qmi 2p2M M22/BM; iQ +? MYXiH2Bbim' iBQM
Bb [mBi2 M HQ; Qmb iQ pBb+Qmb ?2 iBM; BM r i2'X h?2 bK HH2' i?2 r p2
'2 i?2 p2HQ+Biv /Bz2'2M+2b M/ /BbbBT iBQM X h?Bb T?vbB+b +? M; 2b r
Q7 r p2b rBi? r p2H2M; i? Km+? bK HH2' i? M i?2 ;' BMbX AM i?Bb + b2-
; i2b i?'Qm; ? ;' BM- i?2 /BbbBT iBQM QMHv ?2 ib mTi?2 i'm2 i2KT2' im
M/ K v MQi 2H2p i2

IbBM; 2Bi?2' Q7 i?2b2 T'QTQb2/ 2tT2`BK2Mi H T'QiQ+QHb- r?2i?2' i?
s +Q''2bTQM/b iQ i?2 T'BK 'v /'vBM; Q' r2iiBM; +m'p2b- Q' Mv Qi?2' BMB
M H bi i2b b?QmH/ 2M/ mT QM mMB[m2 '2H sB-QmB?BetiMQ
bm+iBQM

aBKBH 'Hv- i?2 M2r i?2Q'v QT2Mb M2r [m2biBQMb QM i?2 /2T2M/2M+
Q7 r i2' BM ;Bp2M bQBH QM i?2 /BK2MbBQM Q7 i?2 2tT2`BK2Mi H bT2+B
+mb b?QmH/ #2 QM BMbT2+iBM; i?2 bHQT2 Q7 i?2 '2i2MiBQM +m'p2b mT
b HbX 6m'i?2'KQ'2- Bi 7QHhQr2/ 7'QK i?2 K?2QhbiQ /i2p2HQT2biBb BM
+QMbB, Wi2i mM/2' +v+HB+ Hep /B p2BQmBMM + '27mH 2tT2`BK2Mib i? i +Q
BKTQb2 bm+? +QM/BiBQMb +QmH/ i?mb #2 BMbi'm+iBp2 iQ QX MQi?2' 2t
T2`BK2Mi H 2tTHQ' iBQM rQmH/ #2 iQ bim/v i?2 2z2+i Q7 T'2pBQmb HQ /
bi i2bX; X++Q' /BM; iQ i?2 i?2Q'v- i?2 H2Q2HhQ K Qm+HB Q2M +HQb2' iQ Bib
2[mBHB#`BmK rB H22H2iiBM; Bi '2H t 7i2' 7 bi2' /'vBM; UQ' r2iiBM; V i? M
/'vBM; UQ' r2iiBM; VX

eXRXk h?2Q`2iB+ H

h?2 T'QTQb2/ i?2Q'v HbQ QT2Mb M2r [m2biBQMb 7Q' 7m'i?2' 7mM/ K
M/ /2p2HQTk2MibX 6Q' 2t KTH2- r?BH2 BM i?2 +m''2Mi T T2' i?2 ' i2@/2
iBQMb BMPQH p2/ +QMi'QHH2/ b im' iBQM ' i2b- i?2b2 /Q MQi +im HHv '2
BM; +QM/BiBQMb BM i?2 T'2pBQmb 2tT2`BK2Mi H bim/B2b UhQTT 2i HX-
RNdKc PmM; 2i HX- kyy8VX >2'2- M2Bi?2' i?2 ' i2 Q7 b im' iBQM MQ' i?2
im HHv +QMi'QHH2/- #mi ' i?2' K2 bm'2/ i HQ+ H TQBMib BM bT +2X Ai B
K2 bm'2/ ' i2 Q7 b im' iBQM M/ BMi2; ' i2 HQM; Bi iQ bim/v i?2 T'Q; '2bb
Qp2' iBK2X Hi2'M iBp2Hv- Bi rQmH/ #2 mb27mH iQ /QTi i?2 i?2Q'v 7Q' b
'v p Hm2 T'Q#H2Kb- BMPQH pBM; M im' HHv ?vbi2'2iB+ T2'K2 #BHBiv-
i?2 #QmM/ 'v +QM/BiBQMb BM i?2b2 i2bib- r?BH2 2p Hm iBM; i?2 ' i2b Q
iBQM i HQ+ H TQBMib BM bT +2 M/ b22 ?Qr i?2v +QKT '2 rBi? i?Qb2 2tT

687 6BM HHv- BM i?Bb T T2' i?2 2z2+i Q7 i?2 bT iB H ;' /B2Mi Q7 i?2 K2b
 688 im'r2T_K ? b #22M M2;H2+i2/ 7'QK i?2 2Mi' Q_T v i T_K Q_B M₁ i BQM 82'K
 689 M/ b bm+? 7'QK i?2 K2bQ @b+ H2 2Mi' QTv # H M +2 BM 1[X U9yVX ++Qm
 690 K2 M i? i i?2 ' i2 2[m iBQM Q7 i?2 K2bQ @k2b? Qm H2/KTm2' i 2n' BMpQH p2
 691 /BzmbBp2ri2T_K X JQ'2 bT2+B}+ HHv- i?Bb MQM @ H₂ v BMBzmbBp2 i2'K U
 692 i?2 2pQHmiBQM 2[m iBQM 7Q' i?2 K2bQ @k2b? Qm H2/KTm2' i 2n' BMpQH p2 iBK2 @
 693 b+ H2- r?B+? K v 2tTH BM i?2 /2p2HQT K2Mi Q7 ' i2 @/2T2M/2Mi HQ+ HB
 694 2tT2' BK2Mi H b KTH2b- b i?Qb2 '2 bi'QM;Hv '2H i2/ iQ i?2 bT22/ Q7 r2i
 695 im'BM; bm+? HQ+ HBb2/ 72 im'2b Bb i?2M TQbbB#H2 #v i'2 iBM; i?2 2tT2
 696 7mHH #QmM/ 'v p Hm2 T_K #22M; 2Bi Hm i2/ i Mv TQBMi BM i?2 2tT2' BK2
 697 i H /QK BMX M //BiBQM H /BzmbBp2 i2'K +QmH/ #2 //2/ iQ i?2 K bb +Qm
 698 bT2+B2b BM r v i? i rQmH/ MQi pBQH i2 i?2 Qp2' HH K bb +QMb2'p iBQM
 699 iQ #2 2p Hm i2/ 7Q' Mv TQBMi BM i?2 bT2+BK2M /QK BMX 6mim'2 '2b2 '-
 700 BMiQ i?2 BKTHB+ iBQM Q7 i?Bb i2'K- b Bi +QmH/ #'BM; /BzmbBp2 H2M;
 701 KQ/2Hb b Bi +QmH/ ?2HT i?2K '2bQH pBM; +QKTH2t T ii2'Mb BM T 'iB HH
 702 TQ'Qmb K2/B bm+? b rQ'K?QH2b M/ }M;2'BM; U>QKbv- RN3dc *m2iQ @6
 703 kyy3VX

704 *QM bB/2'BM; bQBH r i2' '2i2MiBQM bT2+BK2Mb b #QmM/ 'v p Hm2 T
 705 HHQr + Tim'BM; //BiBQM H 2K2';BM; ' i2 @/2T2M/2Mi T?2MQK2M - #2v
 706 a2+X 8X 6Q' 2t KTH2- 2 +? Q7 i?2 K2MiBQM2/ /BzmbBp2 i2'Kb BMi'Q/m+2
 707 /BiBQM iQ i?2 ivTB+ H ' i2 @/2T2M/2Mi T?2MQK2M - #2v
 708 /2T2M/ QM bQBH @bT2 +B₂ Q'22 Mm B2M iQ'7 +B H bBx2bV M/ z2+i K2 bm'2/
 709 2Mi'v p Hm2 M/fQ' ?v/' mHB+ +QM/m+iBpBivX h?2 '2H iBp2 TQbBiBQM Q
 710 HQ /BM; rBi? '2bT2+i iQ Mv Q7 i?2b2 K i2'B H iBK2b +QmH/ BMD2+i KQ'2
 711 bQMBM; iQ r?v /Bz2'2Mi 2tT2' BK2Mi H i2+?MB[m2b +QmH/ #2 7 pQm'2/ 7
 712 H₂ U-X7. bi BK#B#BiBQM #v tBb i' MbH iBQM 7Q' b M/- b QTTQb2/ iQ /'v
 713 p TQm' +QMi'QHH2/ i2+?MB[m2b 7Q' +H vVX

714 Appendix A Parallel flux decomposition of the total entropy production

b K2MiBQM2/ BM a2+X 9X8- r2 +QM bB/2'2/ irQ T Q_B B₂ H2 r vb iQ /2+
 i2'K B₂ Mi Q_R rBi?BM i?2 iQi H 2Mi'Q Rv r_K Q/7m1 iB Q_B MdvX h?2 7Q' @
 KmH iBQM BM a2+X 9X8 +QM bB/2'2/ iT ' rH X₂ Hr? Q++2r/ 2+QKT @bBiBQM ö
 b2M 7Q' bBKTHB+BivX >2'2- r2 7QH HQR i?2 bHB; ?iHv H₂ QM;2' öT ' HH2H ~
 BMiQ M iQ_K BbMB K2bQ @J2H T2'ib- ++Q'/BM;Hv,

$$\begin{aligned} R_K &= J_i^{qK} r_i X_q - T_K^2; & U_{RV} \\ R &= J_i^q r_i X_q + YZ + T_K^2; & U_{kV} \end{aligned}$$

rBi?

$$J_i^q + J_i^{qK} - J_i^q; \quad U_{jV}$$

bQ i? i #v bm R_K B₂ K₂ #Qp2- r2 '2i BM i?2 7Q'K Q7 1[X U9dVX AM Qi?2' rQ
 BM i?2 +QMi2ti Q7 Qm' i?2Q'v i?2 T ' HH2H ~mt /2+QKTQbBiBQM '272'b i
 ~m₂^w Bb #2BM; /2+QKTQb2/ BMiQ Bib iQKBbiB+ M/ K2bQb+QTB+ +QMi'B
 PMb ;2'öb '2+BT'Q+Biv +QM/BiBQM b 7Q' i?2Ri?2' K_B B₂ M₂ #Q_T v T'Q/m+iBQ

$$\frac{Z}{r_i X_q} = \frac{r}{r_i^q} \frac{r_j^q}{r_{ij}^{qq}} \frac{Y}{J_j^q}; \quad U_{9V}$$

r?2'2 i?2 +QM/BiBQM b QM i?2 ;2M2' HBb2/ '2bBbiBpBiv +Q2{+B2Mib '2K
 1[X U8kVX L2p2'i?2H2bb- MQiB+2 i? i i?2 K2 MBM; Q7 i?2b2 +Q2{+B2Mib
 i?2B' p Hm2b M22/ MQi #2 bBKbH 'X >Qr2p2'- 7Q' 2 b2 Q7 +QKT 'BbQM r
 i?2 +Q' '2bTQM/BM; bvK#QHbX HbQ MQiB+2 i? i mMHBF2 1[X U88V BM i?
 H iBQM- ?2'2- BM QR/2R_K Q₂ Mb m'2

$$r_i X_q = J_i^{qK}; \quad U_{8V}$$

$$6Q' + QMp2MB2M+2- '2H2p Mi p2+iQ'b +QmH/ \#2 '2T'2b2Mi2/ \#v i?2B' MBim/2b iBK2b i?2Qm MBQ M2+iQB' /B'2+iBQM b,$$

$$J_i^q = J_q \mathbf{e}_i; J_i^{qK} = J_{qK} \mathbf{e}_i; J_i^q = J_q \mathbf{e}_i: \quad U \text{ eV}$$

$$b \#27Q'2- i?2 bi iBQM 'v bQHmiBQM 7Q' i?2 K2bRk@'2H i2/ i2KT2' im 0- r?B+? ?2'2 Kky MqK r iXq - bQ i? i mbBM; i?2 \#Qp2,$$

$$T_{K0} = -jJ_{qKj}: \quad U \text{ dV}$$

$$AMb2'ZB M';QK 1[X U 9V BMiQ i?2 2pQHmBMQM X Uj NiBQM 7Q'$$

$$@ +r Y +r^q J_q = 0; \quad U \text{ 3V}$$

$$715 \quad bBM_i^q = r^q \mathbf{e}_i \quad M \mathbf{e}_i = 1 X$$

$$aBKBH ' ' ;mK2Mib 7QHHQr iQ mb2 1[X UejV- v2i \pi_k2 M2r bi iBQM 'v b BM i?2 7Q'k BM 1[X U dV \#Qp2 ;Bp2,$$

$$r = rT_K = r - jJ_{qKj}: \quad U \text{ NV}$$

$$*QK\#BMBM; i?2 b2+QM/ PMb ;2' '2H iBQM BMr1Xq UB MV- i?2 2tT'2bbB 1[X U 8V- M/ i?2JiQM BM Q[X U jV ;Bp2b,$$

$$J_q = \frac{J_q + r^q b}{+ r_{ij}^{qq} ij}; \quad U \text{ RyV}$$

$$r?2'q = \mathbf{e}_q Bb i?2 E'QM2+F2' /2Hi X Ai i?mb 7QHHQrb i? i$$

$$@ = r b^q - J_q \frac{J_q + \frac{r^q b}{1 + \frac{r_{ij}^{qq}}{ij}}}{r^q @ \frac{J_q + \frac{r^q b}{1 + \frac{r_{ij}^{qq}}{ij}}}{1 + \frac{r_{ij}^{qq}}{ij}}} A: \quad U \text{ RRV}$$

$$h?2 \#Qp2 '2H iBQM b2'p2b i?2 b K2 Tm'TQb2 b 1[X Ue8V- r?B+? '2br T ' HH2H 7Q'+2 7Q'KmH iBQM- v2i mMHBF2 \#27Q'2 Bi Bb MQi M2+2bb 'B 2p2'- mbBM; i?2 2tT'2BMBQM U QNV- ' i2@BM/2T2M/2M+2 + M ; BM \#2 '2+ i i?2 HBK' BbQ 7$$

$$@ = r b^q - \frac{r_{ij}^{qq} ij}{r_{ij}^{qq} ij +} C j @ S_j \quad r^q \frac{r_{ij}^{qq}}{r_{ij}^{qq} ij +} \%q C @ S_r: \quad U \text{ RkV}$$

$$bQ i? i r2 '2i BM i?2 K BM '2bmHi Q7 i?Bb T T2'- b \#Qt2/ BM 1[X UeeV- r? 1[X UedV- i?2 +Q2\{+B2Mib BM Bi '2 MQr ;Bp2M \#v$$

$$A = r b^q - \frac{r_{ij}^{qq} ij}{r_{ij}^{qq} ij +} C; \quad B = r^q \frac{r_{ij}^{qq}}{r_{ij}^{qq} ij +} C: \quad U \text{ RjV}$$

716

$$717 \quad h?2'27Q'2- i?2 T ' HH2H \sim mt 7Q'KmH iBQM '2i BMb i?2 b K2 2pQHmiB \\ 718 \quad BM 1[X UeeV 7Q' ' i2 BM/2T2M/2Mi T'Q+2bb2bX 6 +BM; rBi? i?2 b K2 2tT \\ 719 \quad p Hm2b Q7 i?2 +Q2M BB2Mi i?2 2pQHmiBQM 2Qm HB Q2K 7BQM bBKBH '- \\ 720 \quad v2i i?2 p Hm2b Q7 i?2 PMbf; 2'q+Q'2 B+2M2M2' HHv \#2 /Bz2'2MiX$$

$$721 \quad 6Q' T?vbB+ H b KTH2b- mb2 i?2 A:aL T2'bBbi2Mi B/2MiB\}2'- b22 i?2 A \\ 722 \quad a KTH2 LmK\#2'b B2TbBQM rX ;mXQ` ;fSm\#HBb?@rBi?@ :lfSm\#HBb?f mi? \\ 723 \quad @_2bQm`+2bf. i @ M/@aQ7ir `2@7Q`@ mi?Q`bOA:aL$$

$$724 \quad +FMQrH2/;K2Mib$$

$$725 \quad h?2 mi?Q'b rQmH/ HBF2 iQ i? MF 7'mBi7mH /Bb+mbbBQM rBi? :Bmb2TT$$

$$726 \quad *?2M- 6' M^QBb :mBHH '/' M/ . pB/ _BH2vX A1 rQmH/ HBF2 iQ i? MF i?2 r$$

$$727 \quad *QmM+BH 7Q' 7mM/BM; i?'Qm;? ;' Mi MmK\#2' .SRNyRyj93dX$$

728 _272`2M+2b
 729 H 2B- 1X- J 'Fb- "X- 1BM p- AX UkykRVX ?v/'Q/vM KB+@TH biB+ 7
 730 KQ/2HHBM; b M/ mbBM; KBMBK H`br2ü Qñ aTe' nK2i2\teXgeH\aniHs
 731 and p\ys iHsonu-DR-y9p3i3Xs
 732 Hb?2'B7- LX- q vHH +2- X- Gm- LX UkyR8VX J2 bm'BM; i?2 bQBH r
 733 +m'p2 mM/2' TQbBiBp2 M/ M2; iBp2 KWie B+ b H+ iBQMa'el; Bk2bXnX
 734 `nu}-@B@V- 99k 98RX
 735 bbQmHBM2- aX UkyyeVX JQ/2HBM; i?2 '2H iBQM b?BT #2ir22M bQBH
 736 r i2' '2i2MiBQM a m hps2X . nneUU kV-}88ae8ejX
 737 "2 '- CX UkyRjVX iHs nT ¾u i d sXi nQmri B 2'u*Q hTeQd' iiB QMX
 738 "2HB 2p- XuX- > bb MBx /2?- aX JX UkyR3VX i?2Q'2iB+ H KQ/2H Q7
 739 /vM KB+ 2z2+ib BM i?2 + TBHH 'v '2H iBQM 7Q' irQ@T? b2 ~Qr BM TC
 740 S}ansqn}t in pV@jV- s9i de BRgX
 741 "2'BQxFBM- X- Jm H2K- uX UkyR3VX *QKT ' iBp2 M HvbBb Q7 i?2
 742 ' iBQM ?vbi2'2bBb TT'Q +? M/i?2 /QK BM i?2Q'v Q7 ?vbi2'2bBb BM
 743 T'2/B+iBQM Q7 b+ MMBM; +m'p2b M/d B'a2nMie f K2nMx te } |e]
 744 s nu } He-uk8j kejX
 745 "Bb?QT- X qX- "HB;?i- :X URNejVX aQK2 bT2+ib Q7 2z2+iBp2 bi'2bb B
 746 T 'iHv b im' i2WbQ B H B XonQjX u Rdd RNdX
 747 "'QQFb- _X >X URNp3VX iH q } nqe } tX e*Q H Q' q/Q} ani ui 8 lHMeBp2a@
 748 bBivX
 749 * bBKB'- >X "X :X URN98VX PM QMb ;2'öb T'BM+B T H2 iQe72KsB+'Qb+QTB+ '
 750 nT gnde } n-p Uyk@ijM-sj9jX
 751 *?2M- SX- Gm- LX- q2B- *X UkyRNVX :2M2' H b+ MMBM; ?vbi2'2bBb KQ/
 752 '2i2MiBQM +mhp2jBxae nT Went eH\ niHae and Wenenv i }nnhent
 753 oVWRkV- y9yRNRReX
 754 *m2iQ@62H; m2'QbQ- GX- Cm M2b- _X Ukyy3VX LQM HQ+ H BMi2'7 +2
 755 i2'M 7Q'K iBQM BM ;' pBiv@/'Bp2M mMb im' i2/p-Qrsi?'Qm;? TQ'Qmb
 756 iHae |ev i eo2¹Uk9Vt-ekp98y9X
 757 /2 :2MM2b- SX :X- S'QbiG C X tURnNujV Xxe HH a2M HQM S'2bb- Pt@
 758 7Q'/X
 759 .2 : 'QQi- aX _X- J xm'- SXi nUk j B j V u Xi e i @ } i u h X t * Q m h B 2 d y n a h i H s
 760 *Q'TQ' iBQMX
 761 1BM p- AX- GBm- JX UkyR3VX >v/'Q/vM KB+ /2'Bp iBQM Q7 i?2 rQ'F BM
 762 T 'iB HHv b im' i2hWQ B H B X nT t \e geH\aniH- a-Ad p\ys iHs nT -
 763 ky8 kRdX
 764 1BM p- AX- GBm- JX UkykyVX h?2 2z2+iBp2 bi'2bb Q7 mMb im' i2/ bQ
 765 M KB+ +QMM2+iBQM b iQ BMi'BMbB+ Mi/ek2sbmri 2H b H+ iBQm j u X J M
 766 t u } e s i n X } a n u l e T a T X j m t e g y X a e s B M ; 2 ' X
 767 6'2/HmM/- .X :X- sBM;- X URNN9VX 1[m iBQM b 7Q' i?2 bQBH@r i2'
 768 +m'p2X nadian Xente H C h 9 i V t a e k R a 8 j k } X n a e
 769 6mFmb?BK - uX- >B;Q- uX- J ibmb?BK - hX- Pi F2- uX UkykRVX GB[m
 770 i'B#miBQM iQ b?2 ' #2? pBQ' Q7 mMb im' i2/ bQBH, KQ/2HHBM; M/ T
 771 KB+'QK2+? MB 8 H K Q M Z B X t - F H R N X H a
 772 : HHBTQHB- .X UkyRkVX ?vbi2'2iB+ bQBH@r i2' '2i2MiBQM KQ/2H +
 773 p 'B iBQM b Q7 bm+iBQ M e M / t p Q B a n U i d B Q e y 8 e R e X
 774 : HHBTQHB- .X- q?22H2'- aX- E 'bimM2M- JX UkyyjVX JQ/2HHBM; i
 775 /2; '22 Q7 b im' iBQM BM /27Q'K #H2Wt M t e n H U d F b Q B H X
 776 Ry8 RRkX
 777 : M- uX- J ;;B- 6X- "mb+ 'M2' - :X- 1BM p- AX UkyRjVX T 'iB+H2 r
 778 KQ/2H 7Q' r i2' '2i2MiBQM p v i e i 2 ' 2 b B b Q u e 9 V e R k e R e R X
 779 > BM2b- qX "X URNjyVX aim/B2b BM i?2 T?vbB+ H T'QT2'iB2b Q7 bQB
 780 2z2+iBM + TBHH 'v T'QT2'iB2b- M/i?2 KQ/2b Q7 KQBbim'2 /Bbi'B#n
 781 i?2'2rB\$AXe `nu} nae nT 8X }-F H U R v t N j d a r e R e X i e n H e

d3k > KK2`pQH/- qX GX- EMmib2M- BX- Ap2`b2M- CX 1X- aFDp2H M/- aX JX
d3j BHH `v T`2bbm`2 b+ MMBM; +m`p2b #v i?2 KB+ C Qm QM2HK2K#` M2 i2+?
d39 Q7 S2i`QH2mK a+B2M+2- MW j10M9 B M22j` B B3X
d38 > bb MBx /2?- aX JX- *2HB - JX X- . ?H2- >X EX UkyykVX .vM KB+ 2
d3e + TBHH `v T`2bbm`2 b im` iBQM`2H iBQM b?BT M/ Bib BKT +ib QM n
d3d o /Qb2 wQM2 CQm VMjB 8dX
d33 > bb MBx /2?- aX JX- :` v- qX :X URNNjVX h?2`KQ/vM KB+ # bBb Q7 +
d3N bm`2 BM TQ`Qm b2K2/Bb Qm`+2b- k2B2yV`?jj3N j9y8X
dNy >2HKB;- _X- q2Bbb- X- qQ?HKmi?- "X AX UkyydVX .vM KB+ + TBHH `v
dNR 2`Q;2M2Qmb TQ`Qm b2K2/Bb Qm H :2Q B jB-2M+2 k d9X
dNk >QKbv- :X JX URN3dVX oBb+Qmb }M;2MBMm BM TQ`Qm b2K2/Bb Qm /XK2 @
dNj +? M B R M RV- kdR jRRX
dN9 >QTK Mb- CX- . M2- CX URN3eVX h2KT2` im`2 /2T2M/2M+2 Q7 bQBH
dN8 +m`p2b B H a+B2M+2 aQ+B2iv Q B yKj2`B8e kC Qm d`X M H
dNe C2MMBM;b- CX 1X "X- "m`H M/- CX "X URNekVX GBKBi iBQMbiQ i?
dNd bi`2bb2b BM T`iHv b iM Qi22/?MB B k2X Rk8@R99X
dN3 CB M;- uX- 1BM p- AX- GBm- JX UkyRdVX i?2`KQ/vM KB+ i`2 iK2Mi Q
dNN m` i2/ bQBHb`2p2 HBM; i?2 bi`m+ iM Q2m QM 2Hz Q7 iB?2J2i+2bM B +b
3yy M/ S?vbB+b Q R y Q H B /B 9eX
3yR CB M;- uX- GBm- JX UkyyNVX :` MmH :` bM Q H B /?v/ iR Q /vM KB+bX
3yk RjNX
3yj CB M;- uX- GBm- JX UkyR8VX TTHvBM; :a>iQ rB/2` M;2 Q7 2tT2`BK
3y9 mH `K2H B 2X1m`QT2 M S?vbB+jBUC Qm R M k dX1
3y8 E K`BM- EX- "Qm+?#BM/2`- 1X UkyR9VX hrQ@i2KT2` im`2 +QM iBMM
3ye +? MB+b Q7 /27Q`KBM; K C Qm Qm H b Q H B /2X2+? MB+b M/ S?vbB+b
3yd Q7 aQ H B /beN k33X
3y3 E? H iMBFQp- AX JX M B y R Q V X +iBQM iQ i?2 i?2Q v Q *bmT2`~mB/Biv
3yN S`2bbX
3Ry E? HBHB- LX- w `; `# b?B- aX UkyRyVX AM~m2M+2 Q7 ?v/` mHB+ ?vb
3RR bi`2bb BM mMb im`i22b Q B B y N 2V- dkN dj9X
3Rk G M/ m- GX .X- GB7b?Bix- 1X aX iB B N 3X V X v b B H B`rQ`i?@
3Rj >2BM2K MMX
3R9 G M/ m- GX .X- GB7b?Bix 6H M B X K 2R X 3M B i 2b rQ`i?X
3R8 GB- aX sX- S2M;` - .X "X- qQM;- SX@xX URNN8VX PMb ;2`ob`2+BT`Q+
3Re ?v/` mHB+ T2`K2 #BHBiv S?T Q B Q m b B B B X 1 8d93X
3Rd GBm- JX UkykRVX h2KT2` im`2b BM S`GBU l m M Q /T T M b B k B X G2ii2`bV
3R3 RjB jV- jeyyJX
3RN JQ``Qr- LX _X URNdYVX S?vbB+b M/i?2`KQ/vM KB+b Q7 + TBHH `v +
3ky K2/BAM/mbi`B H 1M;BM22`B M ;e v?2j k B B i Xv
3kR JQvM2- *X- Jm` /- JX X UkyyeVX irQ@b+ H2 KQ/2H 7Q` +QmTH2/ 2H
3kk K2+? MB+ H T?2MQK2M M/ QMb ;2`ob`2+BT`Q+Biv`2H iBQM b BM 2
3kj ?QKQ;2MBx iBQM` M b T Q B B X M TQ`Qm b pVK 2j B j3yX
3k9 Jm H2K- uX URNdeVX M2r KQ/2H 7Q` T`2/B+iBM; i?2 ?v/` mHB+ +QM/m
3k8 m` i2/ TQ`Qm b qKi2/B`2b Qm`+2b- R2M 2V`+8?Rj 8kkX
3ke Jm` H22i?` M- EX EX- GBm- *X- q2B- *X- EB##2v- hX *X- *?2M- GX UK
3kd 2H biQTH iB+ 7` K2rQ`F 7Q` +QmTHBM; ?v/` mHB+ M/ K2+? MB+ H #
3k3 mMb im` i2/AbM i2 H M X BQM H C Qm`M k B j Q 7 S d j B i N y B iv
3kN PMb ;2`- GX URNjRVX \_2+BT`Q+ H`2H iBQ M B B B H 2 p B B 2B #H2 T`Q+2b
3jy jdU9V- 9y8X
3jR PmM;- PX- > bb MBx /2?- aX JX- "2xmBD2M- X Ukyy8VX hrQ@T? b2~C
3jk BM ;2Q+2Mi`B7m;2 M/i?2 bB;MB}+ M+2 Q7 /vM KB+ + TBHH `v T`2H
3jj CQm`M H Q7 SQ`Qm b X 2/B
3j9 S b? - X uX- E?Qb?;? H#- X- E? HBHB- LX UkyRdVX >vbi2`2iB+ KQ/2
3j8 iBQM Q7 r i2``2i2MiBQM +m`p2 Q B i ?M p Q B Q 7 1 M Q B M 22`BM; J2+? M@
3je B+ B 9j dV- y9yRdyjyX

3jd SQmHQp bbBHBb- X URNdyVX h?2 2z2+i Q7 i?2 2Mi` TT2/ B` QMi?2 ?v
 3j3 TQ`Qmb #Q/v M/ QM Bib ?v/` naQBH+aQ B 2Mi+2 B B 9vXRekX
 3jN _m#BM- JX "X- 1BM p- AX UkyRRVX H `;2 /27Q`K iBQM #`2 F ;2 KQ/2
 39y K i2`B Hb BM+Hm/BM;TQ`QbBiv M/ BM2H biB+AMBi2i`@iBQM H /27Q`H
 39R M iBQM H CQm`M H Q7 1M 9BMR2V B MR a R B F2 Me+2X
 39k aFD 2p2H M/- aX- aB[p2H M/- GX- EDQb pBF- X- h?QK b- qX- oB`MQpb
 39j * TBHH `v T`2bbm`2 +Q``2H iBQM 7 QSK B 22/2@ pQ B 2 1 p` pQB@X
 399 iBQM 1M;BMj 22`BM;ey edX
 398 hQTT- :X- EHmi2- X- S2i2`b- .X URNedVX \*QKT `BbQM Q7 r i2`+QM
 39e ?2 // i Q#i BM2/ #v 2[mBHB#`BmK- bi2 /v@bi i2- M/ mMbi2 /v@bi i2
 39d aQBH a+B2M+2 aQ+B2iv Q7 Rk 2`B RkCj Q 9XM H
 393 o +? m/- :X- o m+HBM- JX- q FBH- JX URNdkVX bim/v Q7 i?2 mMB[m2M
 39N KQBbim`2 +? ` +i2`BbiB+ /m`BM; /2bQ`T iBQM #v+ B 2`MB-2 aHQ@BM ;2X
 38y +B2iv Q7 K2`B+-j eQM` m j R 8jkX
 38R o mM i- CX- * bBMB- 6X UkyRdVX T`Q+2/m`2 7Q` i?2 /B`2+i /2i2`KBM
 38k T `K2i2`7`QK +? M;2b BM TQ`2 bBx:2 Q B 1 +` B M 8 j B 2 Q M X
 38j ejR ejeX
 389 oB;;B MB- :X- iFBMbQM- CX URNN8VX AMi2`T`22 Q B 2 Q M @ 7 #2M/2` 2H2
 388 MB[-9 2URV- R9N R89X
 38e q M;- aX- hQFmM ; - hX EX- q M- CX- .QM;- qX- EBK- uX UkyReVX *
 38d T`2bbm`2@b im` iBQM`2H iBQM b BM [m`ix M/ + `#QM i2 b M/b, GB
 383 +Q``2H iBM;+ TBHH `v M/ r2ii #BHBiv BM~m2M+2b QM B`- QBH- M
 38N i` TTBM;j 2` _2bQm`+2b-2 2 3 V+2 edR eeNyX
 3ey q `/H r- LX \*X- h vHQ`- _X URNdeVX J2`+m`v + TBHH `v T`2bbm`2 +n
 3eR BMi2T`2i iBQM Q7 TQ`2 bi`m+im`2 M/ + TBHH `v m 12 @ pBQm` BM`2b2
 3ek H2iBM Q7 * M /B M S2i`Qk 12 k v K k 2 Q H k Q k X
 3ej qBMFH2`- JX- :D2MM2bi /- JX X- "2/2 mt- .X- ED2Hbi`mT- aX- * #`BQHm
 3e9 > Mb2M- X UkykyVX PMb ;2`@bvKK2i`v Q#2v2/ BM i?2`K H K2bQ
 3e8 i2Kb, hrQ@T? b2 ~Qr BM 6QQ Q i B 2 K 2 / B M 3 S e y k B + b
 3ee w? Q- "X- J +JBMM- *X qX- axmH+x2rbFB- JX GX- L2m72H/- CX X- >mTT2
 3ed Cm M2b- _X UkyRjVX AMi2`7 +2 TBMMBM; Q7 BKKBb+B#H2;` pBiv
 3e3 TQ`Qmb K 2 7 B b X + H _2 B 2 2 r v 1 y k j y R 8 X
 3eN w?Qm- X@LX UkyRjVX +QMi +i M;H2@/2T2M/2Mi?vbi2`2bBb KQ/2H
 3dy i2MiBQM #2? Q B Q mi 2 X b M / :2Q B 2 4 j e M S k + b
 3dR w?Qm- X@LX- a?2M;- .X- * `i2`- CX SX UkyRkVX JQ/2HHBM; i?2 2z2+i
 3dk QM bQBH@r i2`+? ` +i2`Bi2+B+M 8 j 2 b X e N e 3 y X