Prediction of multi-sectoral longitudinal water withdrawals using hierarchical machine learning models

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November 23, 2022

Abstract

Accurate models of water withdrawal are crucial in anticipating the potential water use impacts of drought and climate change. Machine-learning methods are increasingly used in water withdrawal prediction due to their ability to model the complex, nonlinear relationship between water use and potential explanatory factors. However, most machine learning methods do not explicitly address the hierarchical nature of water use data, where multiple observations are typically available for multiple facilities, and these facilities can be grouped an organized in a variety of different ways. This work presents a novel approach for prediction of water withdrawals across multiple usage sectors using an ensemble of models fit at different hierarchical levels. A dataset of over 300,000 records of water withdrawal was used to fit models at the facility and sectoral grouping levels, as well as across facility clusters defined by temporal water use characteristics. Using repeated holdout cross validation, it demonstrates that ensemble predictions based on models learned from different data groupings improve withdrawal predictions for 63% of facilities relative to facility-level models. The relative improvement gained by ensemble modeling was greatest for facilities with fewer observations and higher variance, indicating its potential value in predicting withdrawal for facilities with relatively short data records or data quality issues. Inspection of the ensemble weights indicated that cluster level weights were often higher than sector level weights, pointing towards the value of learning from the behavior of facilities with similar water use patterns, even if they are in a different sector.

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2	learning models
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6	Key points:
7	1. A novel method for withdrawal prediction using hierarchical machine learning is presented.
8	2. Hierarchical ensemble models reduce predictive errors for a majority of facilities analyzed.
9	3. Ensemble models are most beneficial in facilities with high variance and fewer observations of
10	withdrawal.
11	
12	Keywords: Water management (6334), human impacts (4323), anthropogenic effects (4802,
13	4902), machine learning (0555)
14	
15	

16 Abstract:

Accurate models of water withdrawal are crucial in anticipating the potential water use impacts 17 18 of drought and climate change. Machine-learning methods are increasingly used in water 19 withdrawal prediction due to their ability to model the complex, nonlinear relationship between 20 water use and potential explanatory factors. However, most machine learning methods do not 21 explicitly address the hierarchical nature of water use data, where multiple observations are typically available for multiple facilities, and these facilities can be grouped an organized in a 22 variety of different ways. This work presents a novel approach for prediction of water 23 withdrawals across multiple usage sectors using an ensemble of models fit at different 24 25 hierarchical levels. A dataset of over 300,000 records of water withdrawal was used to fit models at the facility and sectoral grouping levels, as well as across facility clusters defined by temporal 26 water use characteristics. Using repeated holdout cross validation, it demonstrates that ensemble 27 predictions based on models learned from different data groupings improve withdrawal 28 29 predictions for 63% of facilities relative to facility-level models. The relative improvement gained by ensemble modeling was greatest for facilities with fewer observations and higher 30 31 variance, indicating its potential value in predicting withdrawal for facilities with relatively short 32 data records or data quality issues. Inspection of the ensemble weights indicated that cluster level weights were often higher than sector level weights, pointing towards the value of learning from 33 34 the behavior of facilities with similar water use patterns, even if they are in a different sector.

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38 **1. Introduction**

Sustainable water resources management requires accurate models, predictions, and 39 40 projections of water demand. Short-term water use forecasting can be crucial in drought management and utility operations, particularly given the climatic sensitivity of outdoor and 41 cooling water use. Longer-term projections of water use can help identify potential supply risks 42 43 under conditions of population growth (Vörösmarty et al., 2000) and climate change (Brown et al., 2013). Accurate models and projections of water demand are especially valuable in locations 44 where water management institutions have relatively limited control on water use; this is the case 45 in many Eastern states where large portions of withdrawal are not subject to permitting 46 requirements (Virginia Department of Environmental Quality, 2022). However, the factors that 47 govern water demand are highly complex and involve interactions between climatic and 48 environmental conditions, socio-economic factors, pricing, and institutional governance 49 structures. Given this complexity, it is unsurprising that many water use forecasts turn out to be 50 51 inaccurate in hindsight (Perrone et al., 2015).

Recognizing this need, numerous studies have used statistical regression models to 52 53 identify the environmental, socioeconomic, and institutional factors associated with greater volumes of water use. For instance, multiple studies have demonstrated the relationship between 54 55 climatic conditions, land use, and household water use in specific municipalities (e.g., Balling et 56 al., 2008; L. House-Peters et al., 2010; Lee et al., 2015; Mini et al., 2014). Several studies have leveraged broad-scale water use data to characterize drivers of broad geographic variability in 57 per-capita municipal water use efficiency and trends (Chinnasamy et al., 2021; 58 59 Sankarasubramanian et al., 2017; Worland et al., 2018) and irrigation withdrawals (Das et al.,

60 2018).

Because the factors that influence water use tend to be highly complex and nonlinear, 61 there has been increasing interest in the use of machine learning to model the relationship 62 between water use and potential explanatory factors. For instance, Bolorinos et al. (2020) used 63 random forests to model bi-monthly water demand for municipal customers and identify 64 drought-induced changes in water consumption. Lamb et al. (2021) demonstrated how boosted 65 66 regression trees could be used to model the relationship between groundwater pumping and numerical and categorical explanatory variables that can better represented governance 67 differences. Recognizing that a large number of machine learning methods exist and may differ 68 in terms of their predictive capabilities, Wongso et al. (2020) compared the ability of four 69 different machine learning methods to predict state-level per-capita water use. When compared 70 with linear regression approaches, ML models are often able to achieve lower predictive errors 71 72 (Bolorinos et al., 2020; Wongso et al., 2020), pointing towards their potential value in water use modeling. 73

74 Across this body of research, one factor that is not always explicitly considered is the way in which the structure of water use data influences model predictions and inferences. Water 75 76 use data is inherently hierarchical, with multiple potential options for grouping and categorizing 77 observations. For instance, water use datasets often include observations through time for multiple customers or water users. These water users in turn can be grouped or classified based 78 79 on geographic location, water use sector, or institutional governance structures. Depending on their structure, regression approaches may be capturing different drivers of variability that lead to 80 81 different management implications. Models of cross-sectional variability across different users 82 and locations can assist in targeting conservation measures (Deoreo & Mayer, 2012; Suero et al., 83 2012), whereas models of temporal variability can lead to more accurate predictions of water use

84	under different policy and drought conditions (Hester & Larson, 2016). Recognizing this,
85	longitudinal or mixed effects regression has become a standard statistical approach in modeling
86	water use (Baerenklau et al., 2014; L. A. House-Peters & Chang, 2011; Polebitski & Palmer,
87	2010; J. Shortridge & DiCarlo, 2020). Longitudinal regression, and other forms of hierarchical
88	regression more generally, allow model parameters to differ across groups in the data, while still
89	constraining those parameter values based on population-level characteristics. This provides a
90	middle ground between pooled regression models, where all observations are grouped together
91	and described via a single set of model parameters, and unpooled regression where a unique
92	model is fit for each group in the data (Gelman & Hill, 2007).
93	Although hierarchical data structures have received less focus in the machine learning
94	methodologies, recent research has begun to develop new approaches that account for grouped
95	data structures. For example, the mixed effects random forest (MERF) approach models
96	individual predictions through time as an additive function of a random forest model of
97	population-level mean behavior processes and individual-level random effects (Hajjem et al.,
98	2014). This approach was later extended to account for high-dimensional data that includes a
99	large number of predictor variables relative to observations (Capitaine et al., 2021). Several
100	studies have proposed methods that integrate regression and classification trees within a mixed
101	modeling framework to address subgroups and hiearchies that exist in clinical trial data
102	(Fokkema et al., 2018; Fokkema et al., 2021; Seibold et al., 2019). Other methods leverage
103	ensemble learning, where predictions from multiple models are aggregated into a single
104	prediction. For instance, Eygi Erdogan et al. (2021) create an ensemble of support vector
105	machine models trained on different time intervals in a panel dataset. Ensemble learning, in
106	which multiple models are independently fit to a dataset and combined into a single prediction,

has been found to generally reduce model variance which results in more accurate predictions on
new data (James et al., 2021; Kuncheva, 2014).

109 The objective of this research was to develop and assess a novel approach for prediction 110 of water withdrawals across multiple usage sectors using an ensemble of machine learning models fit at different hierarchical levels. This work leverages 29 years of monthly withdrawal 111 112 data from approximately 2,500 water using facilities across Virginia. Models were fit at different grouping levels, ranging from single-facility models to sector-wide models, and used multiple 113 climatic and socioeconomic variables as predictor variables. A cluster analysis was also 114 conducted to identify clusters of facilities with similar temporal patterns of water withdrawal, 115 116 even if they were not in the same usage sector, with cluster-level models fit to observations within these groups. Grouping level models were then combined into a weighted ensemble 117 prediction using quadratic programming. The predictive accuracy of all models was evaluated 118 through a repeated holdout cross validation approach, and compared to a null model where 119 120 facility-level withdrawal was based on long-term averages. In addition to assessing predictive accuracy, the relative weights applied to different grouping level models were used to evaluate 121 the relative value of different grouping levels within the ensemble approach. Finally, the facility-122 123 level characteristics associated with improved ensemble predictions were identified to better 124 understand the conditions in which ensemble modeling provides the most value.

125 **2. Methods**

126 2.1 Data Sources and Processing

127 2.1.1 Withdrawal Data

128	This analysis used long-term records water withdrawal provided by the Virginia
129	Department of Environmental Quality (VDEQ). All water users in Virginia who withdrawal
130	more than 10,000 gallons per day (non-agricultural users) or 1 million gallons (MG) in any
131	single month for crop irrigation are required to report monthly water withdrawal to VDEQ. This
132	dataset includes 313,321 nonzero monthly withdrawal records between 1990 and 2018 from
133	2,579 water using facilities across eight water use sectors (Table 1). County location is available
134	for all facilities within the dataset. Note that agriculture refers to livestock and agricultural
135	processing operations, rather than crop irrigation. Additional details on withdrawal data are
136	presented in Shortridge and DiCarlo (2020). However, many of these facilities only have short-
137	term records of water withdrawal or a majority of months with zero reported withdrawals. To
138	ensure that all facilities had sufficient data available for model training, weighting, and
139	validation, only facilities with at least 36 nonzero withdrawal observations were retained for
140	inclusion in the analysis.

	All Data		Retained for Analysis		
	Facilities	Observations	Facilities	Observations	Total water use
Sector	(n)	(n)	(n)	(n)	(MG/month)
Agriculture (Ag)	155	7,032	36	5,914	129
Aquaculture (Aq)	14	2,978	12	2,913	866
Commercial (Com)	463	55,573	292	52,721	740
Industrial (Ind)	211	37,464	154	36,968	16,000
Irrigation (Irr)	727	23,036	187	18,157	1,310
Mining (Min)	91	14,394	70	14,260	1,320
Municipal (Mun)	894	166,747	735	164,718	24,900
Thermoelectric (Thm)	24	6,097	23	6,073	201,000
Total	2,579	313,321	1,509	301,724	246,000

 Table 1: Summary of withdrawal data used in model development

Water withdrawal volumes across different users, even within a single usage sector, often vary across several orders of magnitude. Many water users also exhibit significant seasonal water user patterns as well. To address this variability, all water withdrawal records were converted to water withdrawal anomalies as in Equation 1:

147

148
$$W_{AN_{f,t}} = \frac{W_{O_{f,t}} - W_{O_{f,m}}}{sd(W_{O_{f,m}})}$$
(1)

149

where $W_AN_{f,t}$ is the withdrawal anomaly in facility *f* at time period *t*; $W_O_{f,t}$ is the observed withdrawal in facility *f* at time *t*, $\overline{W_O}_{f,m}$ is the average withdrawal in facility *f* for month *m*; and $sd(W_O_{f,m})$ is the standard deviation of withdrawal in facility *f* during month *m*. Note that estimating anomalies thus requires at least two years of data for each facility.

154 2.1.2 Predictor Variables

A total of thirteen socio-economic and four climatic predictor variables were included as 155 potential predictors of water withdrawal. Socio-economic variables are summarized in Table 2. 156 These variables represented a variety of population, economic, and land-use characteristics that 157 158 have been shown to have relationships with water withdrawals in previous research (e.g., Sankarasubramanian et al., 2017; Shortridge & DiCarlo, 2020; Worland et al., 2018). Socio-159 economic predictor variables were obtained from the U.S. Census (US Census Bureau, 2022), 160 U.S. Bureau of Economic Analysis (US Bureau of Economic Analysis, 2022), USDA National 161 Agricultural Statistics Service (US Department of Agriculture, 2017), the USGS FORecasting 162 SCEnarios of Land-use Change (FORE-SCE) model (Sohl et al., 2007), and the U.S. Energy 163

Information Administration (US Energy Information Administration, 2022). Predictor variables
that exhibited trends through time were linearly detrended prior to inclusion in the model (see
Table 2).

Additionally, three climatic predictor variables were included to account for widespread 167 evidence of the relationship between weather and water withdrawals (e.g., Brown et al., 2013; 168 169 House-Peters & Chang, 2011; Lee et al., 2015). This included maximum daily temperature, total precipitation, and daily precipitation variability as quantified by the Gini coefficient (Marston & 170 171 Ellis, 2019; Rajah et al., 2014). The Gini coefficient is a non-dimensional representation of the 172 inequality in a distribution ranging from 0 to 1, with higher values indicating more inequality. Recent research has demonstrated an increasing trend in Gini coefficient rainfall values in the 173 Eastern U.S., indicating that a larger portion of rainfall is occurring on a smaller number of days 174 (Marston & Ellis, 2019; Rajah et al., 2014); this increase in rainfall variability is also correlated 175 with greater water use (Shortridge & DiCarlo, 2020). Gridded daily values of maximum 176 177 temperature and precipitation from 1990 - 2018 were obtained from the PRISM dataset (Daly et al., 2008) and spatially aggregated to county-level daily values. These county level values were 178 then temporally aggregated to monthly estimate of average high temperature, total precipitation, 179 180 and precipitation Gini coefficient.

Virginia exhibits several spatial patterns of variability in long-term climate conditions, most notably higher temperatures in the southern inland portion of the state, lower temperatures in the Appalachian and Blue Ridge Mountains, and a rain shadow along the western border of the state. To account for this cross-sectional variability as well as seasonal differences, all monthly climate data were transformed to monthly anomaly values prior to inclusion within the model (Shortridge et al., 2016): 187

$$AN_{c,t} = \frac{O_{c,t} - \bar{O}_{c,m}}{sd(O_{c,m})}$$
(2)

189

188

where O_{c,t} is the observed value of a climate variable (e.g., total precipitation) in county c and 190 time period t; $\bar{O}_{c,m}$ is the monthly normal (long-term mean from 1990-2018) observation of that 191 variable in that county; and $sd(O_{c,m})$ is the standard deviation of all observations in that month. 192 193 In this way, the anomaly value can represent how weather conditions compared to long-term averages for that specific month and location. For instance, an anomaly temperature value of 0.0 194 195 would indicate that the temperature in that month and county was exactly equal to the long-term 196 mean temperature in that month and county, while a value of 1.0 would indicate that the temperature was one standard deviation higher than the long-term mean. 197

Explanatory Variable	Description and Rationale (temporal and geographic resolution in parentheses)	Source	Sector models
Population	County-level population (detrended); included to account for higher municipal water withdrawals as population grows (annual, county)	US Census	Mun
Per-capita personal income	Percent change (from previous year) in personal income per capita; Spatially-explicit representation of general economic conditions at the county level (annual, county)	US Bureau of	All
Manufacturing GDP	Virginia GDP in current dollars from manufacturing (detrended); Spatially-general representation of manufacturing sector economic strength (annual, state)	Economic Analysis	Com, Ind, Min
Agricultural GDP	Virginia GDP in current dollars from agriculture (detrended); Spatially-general representation of agricultural sector economic strength (annual, state)		Ag, Aq, Com, Irr
USDA Price Ratio	Ratio of USDA Prices Received Index to Prices Paid Index (detrended); Spatially-general representation of economic conditions for agricultural producers (monthly, national)	USDA NASS	Ag, Aq, Irr
Agricultural Land Cover	Percentage of county land use classified as cultivated cropland, hay or pasture; Account for expansion and decline of agricultural land cover over study period (annual, county)	USGS FORE-	Ag, Aq, Irr
Developed land cover density	Population per square kilometer of developed area; Included to account for differing development densities (annual, county)	model	Mun
Energy Prices	Virginia total energy prices in dollars per million BTU (detrended); Spatially-general representation of energy prices (annual, state)		All
Electricity Sales	Virginia retail electricity sales in millions of kWh (detrended); Spatially-general representation of energy consumption (annual, state)	US Energy Information	Thm
Energy Production	Virginia primary energy production in trillion BTU (detrended); Spatially-general representation of energy production (annual, state)	Administration	Min, Thm

 Table 2: Summary of socio-economic predictor variables

198 2.2 Modeling Approach

199 The predictor variables described above were used to estimate monthly water withdrawal 200 at the facility level using an ensemble of models fit across different grouping levels. Grouping 201 level refers to the specificity of data included to fit the model and included a facility level, sector level, and two cluster level groupings. At each level, multiple model formulations (parametric 202 203 linear models, semi-parametric non-linear models, and nonparametric machine learning models) were tested and the best model in terms of out-of-sample predictive error minimization was 204 205 retained. These models were combined into a multi-level ensemble model that predicted 206 withdrawal as a weighted average of predictions from the different grouping level models. Model performance was quantified via a repeated cross validation approach where the data were 207 208 randomly partitioned at each iteration into distinct training, weighting, and testing datasets. An overview of this process is shown in Figure 1, and additional details are presented in the 209 210 following sections.



211

212 Figure 1: Diagram overview of modeling approach used during each iteration of cross

validation

214

Model name	Description and Rationale
Facility Grouping	Separate model fit to each facility in the dataset. Captures facility
Level	level water use behavior, but not generalizable to other facilities.
Sector Grouping	Model fit using data from all facilities within each water use sector.
Level	Captures general water use behavior across multiple facilities at the
	expense of accuracy at individual facility level.
Large Cluster	Model fit using data from all facilities within each large cluster
Grouping Level	(k=3). Clusters are defined based on temporal water use patterns, and
	thus contain facilities with similar withdrawal patterns even if they
	are different water use sectors.
Small Cluster	Model fit using data from all facilities within each small cluster
Grouping Level	(k=8). Same as large clusters, but with facilities partitioned into
	smaller groups with less in-group variability in temporal withdrawal
	patterns.
Ensemble	Withdrawal predictions are a weighted average of the four grouping
	level models above.
Null	Withdrawal predictions are equal to the long-term average
	withdrawal in each month for each facility. Included as a baseline for
	comparison.
Tab	le 3: Summary of models and rationale for inclusion

216 2.2.1 Facility Grouping and Clustering

The water withdrawal data used in this study can be grouped at multiple different levels, 217 218 some of which are hierarchical. Each water using facility has multiple observations of water use through time. Facilities are often categorized by water use sector, under the assumption that two 219 220 facilities in the same water use sector will exhibit similar water use behavior. For this study, 221 predictive models were fit at four different levels of facility grouping: facility level, sectoral 222 level, small cluster level, and large cluster level (Table 3). At the finest level, facility level 223 grouping entailed fitting a distinct model for each facility in the dataset. This allows for the model to be highly tailored to the water use characteristics of that facility. However, it can result 224 225 in a model that is less generalizable to new data, particularly in instances where a facility does 226 not have many observations to draw from (Gelman & Hill, 2007). The next level of grouping was the sectoral level, where a single model was fit to all facilities within that sector. This 227

provides a representation of generalized water use patterns in a given sector, such as the higher
irrigation withdrawals that are observed during periods of high temperature and low rainfall
(Shortridge & DiCarlo, 2020). This provides a model of how sectoral water withdrawals relate in
general with predictor variables, but will likely result in less accurate predictions for a single
facility.

One limitation with sectoral grouping is that facilities in a single sector might actually exhibit very different pattern of water use, particularly in the industrial and commercial sectors (Attaallah, 2018; McCarthy et al., 2022). Thus, the small and large cluster grouping levels were determined based on the results of a hierarchical cluster analysis (Everitt et al., 2011) that identified coherent facility groupings based on five water use characteristics calculated for each facility:

• Mean withdrawal volume (MG/month), log transformed. 239 240 • Coefficient of variation: standard deviation of withdrawal divided by mean. • Seasonality: the lowest three-month mean withdrawal divided by the highest three-month 241 242 mean withdrawal, where lower values indicate greater seasonality in withdrawal volume. • Autocorrelation: maximum degree of autocorrelation observed at any time lag. 243 Number of Observations: the number of nonzero withdrawal observations available. 244 Figures summarizing observed values for the five characteristics for all facilities are included as 245 supplementary material. To determine the optimal number of clusters, facilities were divided into 246 $k \in \{1, 15\}$ hierarchical clusters based on Euclidian distance using the FactoExtra package in R 247 248 (Kassambara & Mundt, 2020). Gap statistic estimates for each value of k (included in the supplementary material) exhibited non-monotonic behavior indicating well defined clusters at k 249

= 3 and k = 8, suggesting that there were three large clusters of facilities that could be further 250 divided into eight smaller subclusters (Tibshirani et al., 2001). An analysis of correspondence 251 252 between cluster assignment and sector indicated that while certain clusters largely corresponded to a single sector, the majority did not. This suggests that there are certain patterns of water use 253 behavior that cannot be explained simply by sectoral classifications, echoing previous research 254 255 that demonstrates significant variability across water use behavior across facilities that are generally grouped into a single sectoral classification (Attaallah, 2018; McCarthy et al., 2022). 256 257 Thus, models were also fit at the large (k = 3) and small (k = 8) cluster levels, where data from 258 all facilities within a single cluster were combined into a single model. Additional details and results of the cluster analysis are included in the supplementary material. 259

260 2.2.2 Regression and Machine Learning Models

For each of the grouping levels described above, multiple regression and machine 261 learning approaches were compared to identify the most effective predictor of water 262 withdrawals. The general formulation used in modeling withdrawal anomalies is shown in 263 Equation 3, where W_AN_f is a vector of anomaly withdrawal predictions of length t in facility f, 264 265 where t is the number of months of observations available in the training dataset. These were estimated using a generalized function of a *m* x *t* matrix of *m* predictor variables across *t* months 266 267 X_c , plus an error term ε . Note that because predictor variables were available at the county rather 268 than facility level, facility withdrawal is estimated as a function of predictor variables for its county location. 269

270
$$W_A N_f = f(X_c) + \varepsilon$$
(3)

Three different forms for the functional relationship f were tested at each grouping level. The 271 first was a gaussian linear regression (GLM) model. The second was a semi-parameteric 272 Gaussian generalized additive model (GAM), where smoothing functions are applied to the 273 predictor variables to capture non-linear relationships between the predictor and response 274 variables without *a*- *priori* assumptions about the form of that relationship (Hastie & Tibshirani, 275 276 1986). GAM models were fit using the mgcv package in R (Wood, 2011). The final model form was a nonparametric random forest (RF) model, where predictions from multiple rule-based 277 278 regression trees are combined into a single prediction (Breiman, 2001). RF models were fit using 279 the randomForest package in R (Liaw & Wiener, 2002). All model predictions were then converted from anomaly values back to a vector of withdrawal predictions as in Equation 4 prior 280 to estimating model error: 281

282
$$W_P_f = W_A N_f x \, sd(W_O_{f,m}) + \overline{W_O}_{f,m}$$
(4)

283

For each grouping level model, the GLM, GAM, and RF models were compared in terms of their mean absolute error across the weighting dataset, with the lowest error model retained. Following this process, each facility had four sets of withdrawal predictions generated by models fit at the facility, sector, small cluster, and large cluster grouping level. These predictions were then combined into a weighted ensemble prediction as follows:

$$W_P Ens_f = w_f \cdot W_P_{f_all}$$
(5)

Where w_f is a facility-specific vector of weights summing to 1.0, and $W_P_{f_all}$ is an $n \times 4$ matrix of predictions from the four different grouping level models (facility, sector, large cluster, and small cluster) for the n weighting data observations for facility f. The resulting $W_P_Ens_f$ is thus a vector of *n* predictions obtained from a weighted average of individual grouping level model
predictions. The values of the weights *w_f* for each facility were estimated using quadratic
programming problem (Goldfarb & Idnani, 1983) implemented via the quadprog package in R
(Turlach et al., 2019) of the form:

297
$$Minimize (W_O_f - w_f \cdot W_P_{f_all})^2$$
(6)

298 Subject to

299
$$\sum_{i=1}^{4} w_i = 1.0$$

300
$$w_i \ge 0.0 \ \forall \ i \in [1,4]$$
 (7)

301

The four grouping level models, as well as the ensemble model based on the weights calculatedin equations 6-7, were then used to generate withdrawal predictions for the testing dataset.

304

305 **2.3 Model Evaluation**

To evaluate models in terms of their out-of-sample predictive accuracy, a 100-fold crossvalidation approach was used (Hastie et al., 2009). At each iteration, the data were partitioned into three groups, with approximately 60% assigned to model training, 20% to model weighting, and 20% to model testing. The training data were used to fit three models of different functional forms (GLM, GAM, and RF) for each grouping level described above. These models were then used to predict withdrawal in the weighting dataset, with a single functional form selected for each grouping level based on root mean absolute error (RMAE). The predictions from the

selected models were then used to determines the weights used in the ensemble model. Finally, 313 the grouping level and ensemble models were used to predict withdrawals in the testing dataset. 314 These predictions were compared to a null model where each prediction of monthly water use 315 was equal to the long-term monthly mean value for that facility. Thus, the null model captured 316 seasonal variation for each facility but did not include climatic or socio-economic factors that 317 318 could induce variation beyond seasonal patterns. Mean absolute error (MAE) across the testing dataset was calculated for each model and facility by averaging absolute differences between 319 observed and predicted withdrawal in each observation *n* across each holdout iteration *h* 320 321 (Equation 8). Because absolute errors tend to scale with withdrawal volume, relative mean absolute error (RMAE), where MAE was presented as a fraction of mean facility withdrawal, 322 was also calculated to allow for comparison of error across facilities with different magnitudes of 323 withdrawal (Equation 9). 324

326
$$MAE_{f} = \frac{1}{H} \sum_{h=1}^{H} \frac{1}{N} \sum_{n=1}^{N} abs(W_{0,n} - W_{P_{f,n}})$$

325

327

329
$$RMAE_f = \frac{1}{H} \sum_{h=1}^{H} \frac{MAE_{f,h}}{W_{-}O_{f,h}}$$

328

To better understand the characteristics associated with facilities where ensemble modeling provided the most benefit relative to facility-specific models, the difference between RMAE from the facility grouping and ensemble models (Equation 10) was linearly regressed

(8)

(9)

against facility water use characteristics *C*. This included the logarithm of mean faculty
withdrawal, the number of nonzero observations, the coefficient of variation, autocorrelation,

seasonality, and a use sector, as defined in Section 2.2.1 above.

$$EnsImp_f = RMAE_{f_fac} - RMAE_{f_ens}$$
(10)

337

338
$$EnsImp_f = \alpha + \beta C_f + \varepsilon$$
(11)

339

340 3. Results

341 **3.1 Model Performance**

A summary of RMAE for testing dataset predictions across all 100 holdout cross validations is presented in Figure 2. For all model structures, relative error depended strongly on the sector assessed, with the highest predictive errors in agricultural and irrigation sectors. Variations in error across model structures were relatively small compared to differences across sectors and the variation in error across facilities within a sector (represented by the size of each box).



Figure 2: Relative percent errors across facilities in each water use sector. Boxplots show the
distribution of RMAE for all facilities in each sector averaged across cross validation iterations.

Figure 3 presents a summary of the highest performing grouping level across facilities in 351 each water use sector, as well as all facilities in the dataset. Across all facilities, the model form 352 that most often resulted in lowest errors was the facility grouping, which had the best 353 performance for 33.1% of the analyzed facilities, followed by the ensemble model (21.0%). On 354 the other hand, the null model had the lowest errors in 15.9% of analyzed facilities. Facilities 355 across different sectors exhibited different patterns in terms of optimal model selection. For 356 instance, facility grouping models resulted in lowest errors in a large percentage of industrial 357 358 (46.1%), mining (42.9%) and thermoelectric facilities (52.2%). However, facility models resulted in lowest errors in only 16.7%, 26.0%, and 27.8% of aquaculture, commercial, and 359

- agricultural facilities, respectively. Sector-level models performed well for a relatively high
- percentage of irrigation facilities (lowest errors in 27.3% of facilities), but less well in the mining
- (7.1%), municipal (9.9%), and industrial (11.0%) sectors.
- 363



Highest Performing Models

364

365 366

Figure 3: Percentage of facilities where each model type resulted in the highest performance in terms of RMAE.

367

Figure 4 presents a summary of the percentage of facilities where predictions are improved through the use of a model ensemble relative to the other model forms. Across all facilities, the use of a model ensemble results in lower errors in 63% - 65% of facilities, depending on the model form with which it's compared. The ensemble process results in the broadest improvement for all non-facility models in the thermoelectric sector. For thermoelectric facilities, the ensemble model resulted in lower errors in anywhere from 78.3% to 95.7% of facilities when compared to the null, sector, and cluster grouping models. However, it only

resulted in improved performance in 39.1% of thermoelectric facilities when compared to 375 facility-specific models. Similar behavior is observed for industrial facilities, with the ensemble 376 approach reducing errors in 72.7% to 79.2% of facilities relative to the null, sector, and cluster 377 models, but only 50% of facilities relative to facility-specific models. Ensemble improvements 378 were most consistent in the municipal sectors, where errors were reduced by 61.5% to 68.3% of 379 380 facilities, regardless of the grouping level model with which the ensemble is compared. The results in Figures 3 and 4 collectively demonstrate that while the ensemble process is unlikely to 381 result in the optimum prediction for any single individual facility, it is likely to result in better 382 predictions relative to single grouping level models when applied to many facilities. In this 383 sense, its value is in providing a general approach that could be applied to many facilities across 384 a broad, heterogenous dataset, rather than being an optimum approach for a single facility. 385



386

387 388

Figure 4: Percentage of facilities where ensemble model reduced errors relative to null and grouping level models

390 3.2 Ensemble Model Structure

391 To better understand the relative influence that different individual grouping level models 392 play within the ensemble predictions, Figure 5 shows density plots of the average weights for each 393 grouping level model for all facilities in each sector. In all sectors, facility-level weights were generally higher than weights for other grouping levels models, with the highest facility-level 394 395 weights in the sectors where the facility models tended to perform best (industrial, mining, and thermoelectric). It is notable that the sector-level weights were generally no higher than the cluster 396 397 weights, and in several sectors (agriculture, aquaculture, and thermoelectric), the small cluster 398 weights were often higher than sector level weights. This demonstrates the value of learning from the behavior of facilities with similar water use patterns, even if they are in a different sector. 399



401 Figure 5: Density plot of mean ensemble model weights across facilities in each sector, averaged
 402 across cross validation iterations

	Estimate	Std. Error	n-value
Intercept	0.014	0.008	0.078
log(Water.Use.MGM)	-0.001	0.001	0.127
n.obs.nonzero	-2.85E-05	1.13E-05	0.012
Water.Use.COV	0.016	0.001	< 0.001
Water.Use.ACF.strength	-0.022	0.004	< 0.001
Water.Use.Seasonality	0.007	0.005	0.186
UseType (aquaculture)	0.003	0.013	0.839
UseType (commercial)	-0.001	0.007	0.832
UseType (industrial)	-0.005	0.008	0.466
UseType (irrigation)	-0.016	0.007	0.026
UseType (mining)	0.015	0.008	0.078
UseType (municipal)	0.004	0.007	0.603
UseType (thermoelectric)	-0.005	0.011	0.631

404

Table 4: Factors associated with improved ensemble performance relative to facility model performance. Bold rows indicate factors significantly associated with a

- 405relative to facility model performance. Bold rows indicate factors significantly associated with406difference in facility and ensemble performance (p < 0.05).
- 407

408 To better understand the facility water use characteristics associated with improved

409 ensemble model performance relative to facility-grouping models, the predictive improvement

410 from use of a model ensemble for each facility was regressed against facility water use

411 characteristics. The results of this regression are presented in Table 4. The ensemble model

tended to provide the most improvement relative to the facility-grouping model in facilities with

413 a lower number of observations, higher coefficient of variation, and less autocorrelation.

414 **4. Discussion**

Comparing the relative accuracy of different model grouping levels can provide some
insights into the situations in which different model structures may provide optimal predictions.
Across all facilities, the facility-grouping level model most frequently resulted in the lowest
errors, but this still only occurred in 33.1% of facilities. Thus, there were numerous facilities

where a facility-specific model resulted in higher errors than other grouping levels. While the 419 ensemble model only resulted in error minimization in 21% of facilities, it resulted in error 420 reduction relative to grouping level models in over 60% of facilities. In this sense, its value is 421 likely highest in situations where a general modeling approach is needed to simulate withdrawals 422 across a heterogenous body of water users. The regression results in Table 4 indicate that this is 423 424 particularly true in facilities with fewer observations, greater variance, and less autocorrelation. This result mirrors previous discussions of pooled and unpooled regression models, where 425 426 unpooled models fit tend to be overfit and less generalizable when a small number of 427 observations are available (Gelman & Hill, 2007). High variance in water use observations could also potentially be indicative of data quality issues in self-reported water use data that have been 428 observed elsewhere (Chini & Stillwell, 2017; McCarthy et al., 2022; Zhang & Balay, 2014). The 429 ability to reduce the impact of these errors in predictive models is another potential benefit to the 430 ensemble modeling approach. 431

432 This work also provides some insights into the relative abilities and limitations in applying sector-based models for facility-level predictions. Sector grouping models performed 433 434 relatively well among agricultural and irrigation facilities, but less well in the industrial, mining, 435 and municipal sectors. This is possibly due to the high variability in water withdrawal practices in those sectors. In particular, the industrial sector contains facilities that have a wide range of 436 437 end uses for water, including cooling, incorporation into products, and landscape irrigation; this has been shown to lead to a high degree of variability across facilities in terms of water use and 438 439 consumption rates (Attaallah, 2018; McCarthy et al., 2022). It should also be noted that a single municipal water withdrawing facility in our dataset could potentially serve multiple water supply 440 utilities beyond the county in which the water is withdrawn. These water transfers present a 441

challenge for modeling and predicting withdrawal as water demand may be driven by conditionsin counties other than the location of the withdrawal.

444 Several areas of additional research could be envisioned to build on the results presented here. For instance, we grouped facilities based on temporal water usage characteristics and 445 sectoral classifications. However, water withdrawals could also depend on regulatory 446 447 governance, with different withdrawal patterns expected in more strict regulatory environments (such as groundwater management zones). Water source could also be used as a grouping level, 448 as surface water sources may experience more short-term fluctuations in water availability than 449 groundwater. Exploration of alternative grouping strategies could be a valuable area of additional 450 451 research. Similarly, this work used a global error metric (MAE) that aggregates predictive error across all observations equally to select the best performing model. However, as water supply 452 management is typically more concerned with periods of stress and potential water shortage, 453 454 other metrics that quantify model performance in terms of identifying periods of high withdrawal 455 may be of value. For instance, event detection metrics quantify the degree to which the model captures specific conditions of interest, such as values above a predefined threshold (Liemohn et 456 al., 2021). The concept of domain applicability can also be used to identify subdomains or 457 458 conditions (such as high or low withdrawal periods) in which different models perform optimally (Sutton et al., 2020). The exploration of different performance measures for model selection 459 would be a valuable area for further research. 460

461 **5.** Conclusions

462 This work presents a novel approach for prediction of longitudinal water withdrawals
463 across multiple usage sectors using an ensemble of machine learning models fit at different
464 hierarchical grouping levels. These grouping levels included facility and sectoral-level models,

as well as facility clusters determined based on temporal water use characteristics. Grouping 465 level models were also combined into an ensemble model that predicted withdrawal as a 466 467 weighted average of predictions from each individual grouping level model. For all model structures, relative error depended strongly on the sector assessed, with the highest predictive 468 errors in agricultural, irrigation, and municipal sectors. Across all facilities, the model form that 469 470 most often resulted in lowest errors was the facility grouping model (33.1%) followed by the ensemble model (21.0%). The use of an ensemble model resulted in more accurate predictions 471 472 relative to the facility model in 63% of facilities, and ensemble improvements were greatest for 473 facilities with relatively few records and high variance in withdrawal. This points to their potential value in predicting withdrawal for facilities with relatively short records of withdrawal 474 or data quality issues that could lead to highly variable withdrawal estimates. Inspection of the 475 weights used in the ensemble model indicated that small cluster weights were often higher than 476 477 sector level weights, pointing towards the value of learning from the behavior of facilities with 478 similar water use patterns, even if they are in a different sector. The ensemble modeling method presented here can thus provide a general approach for prediction of water withdrawals that can 479 480 be applied across heterogenous, multi-sector groupings of water users.

481 **6. Acknowledgements**

I would like to gratefully acknowledge the Virginia Department of Environmental
Quality for providing the data used in this project. All code and data used in this analysis are
available at: <u>https://osf.io/5pqvx/?view_only=a9b67a7867eb411585897076bf36a433</u>. This work
was financially supported through institutional funding provided by Virginia Tech; this support
is gratefully acknowledged.

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