A Simple Multiscale Intermediate Coupled Stochastic Model for El Niño Diversity and Complexity

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Abstract

El Niño-Southern Oscillation (ENSO) is the most prominent interannual climate variability in the tropics and exhibits diverse features in spatiotemporal patterns. This paper develops a simple multiscale intermediate coupled stochastic model to capture the ENSO diversity and complexity. The model starts with a deterministic and linear coupled interannual atmosphere, ocean, and sea surface temperature (SST) system. It can generate two dominant linear solutions representing the eastern Pacific (EP) and the central Pacific (CP) El Niños, respectively. In addition to adopting a stochastic model for characterizing the intraseasonal wind bursts, another simple stochastic process is developed to describe the decadal variation of the background Walker circulation. The latter links the two dominant modes in a simple nonlinear fashion and advances the modulation of the strength and occurrence frequency of the EP and the CP events. Finally, cubic nonlinear damping is adopted to parameterize the relationship between subsurface temperatures and thermocline depth. The model succeeds in reproducing the spatiotemporal dynamical evolution of different types of ENSO events. It also accurately recovers the strongly non-Gaussian probability density function, the seasonal phase locking, the power spectrum, and the temporal autocorrelation function of the SST anomalies in all the three Niño regions (3, 3.4 and 4) across the equatorial Pacific. Furthermore, both the composites of the SST anomalies for various ENSO events and the strength-location bivariate distribution of equatorial Pacific SST maxima for the El Niño events from the model simulation highly resemble those from the observations.

A Simple Multiscale Intermediate Coupled Stochastic Model for El Niño Diversity and Complexity

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Key Points:

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12	•	The strength, occurrence frequency and spatiotemporal patterns of both the EP and
13		the CP events are realistically reproduced.
14	•	Strongly non-Gaussian statistics, seasonal phase locking and power spectrum are ac-
15		curately recovered in all Niño 3, 3.4 and 4 regions.
16	•	Both the composites of the SST anomalies and the strength-location bivariate distri-
17		bution of SST maxima highly resemble the observations.

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18 Abstract

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³⁸ Plain Language Summary

El Niño-Southern Oscillation (ENSO) is the most prominent interannual climate vari-39 ability in the tropics and exhibits diverse spatiotemporal characteristics. Developing a 40 dynamical model with intermediate complexity to simulate the ENSO diversity and com-41 plexity facilitates the understanding and predicting of the ENSO and the global climate. 42 To this end, a multiscale model is developed here. It starts with a coupled linear and deter-43 ministic interannual atmosphere, ocean, and sea surface temperature (SST) system. Then 44 suitable stochastic processes, nonlinearity, and seasonal synchronization are incorporated, 45 connecting the interannual components with the intraseasonal wind bursts and the decadal 46 variation of the background Walker circulation. The model can simulate different eastern 47 and central Pacific ENSO events with realistic spatiotemporal patterns, strength, and fre-48 quency. It also accurately recovers the strongly non-Gaussian probability density function, 49 the seasonal phase locking, the power spectrum, and the temporal autocorrelation function 50 of the SST anomalies in all the three Niño regions (3, 3.4 and 4) across the equatorial Pacific. 51 Furthermore, both the composites of the SST anomalies for various ENSO events and the 52 strength-location bivariate distribution of equatorial Pacific SST maxima for the El Niño 53 events from the model simulation highly resemble those from the observations. 54

55 **1** Introduction

El Niño-Southern Oscillation (ENSO) is the most prominent interannual variability in the tropics. It also affects the global climate, ecosystem, and socioeconomic development through atmospheric teleconnections (Ropelewski & Halpert, 1987; McPhaden et al., 2006). Therefore, understanding and predicting ENSO is a central problem with significant societal impacts. Bjerknes (1969) first suggested that ENSO is the product of tropical air-sea interaction. Since then, considerable achievements have been made in its simulation and prediction abilities (Latif et al., 1998; Neelin et al., 1998).

From the traditional point of view, El Niño is defined as the anomalous warm sea surface temperature (SST) in the equatorial eastern Pacific (EP) region. Zebiak and Cane (1987) developed the first coupled ocean-atmosphere model of intermediate complexity that successfully characterizes and predicts these EP warming events. Several deterministic and linear conceptual models were also proposed to explain the slow physics of ENSO. Among these models, the delayed oscillator (Schopf & Suarez, 1988) describes the delayed effects of oceanic wave reflection at the ocean's western boundary on the EP SST anomalies. In contrast, the recharge-discharge oscillator (F.-F. Jin, 1997a) combines SST dynamics and ocean adjustment dynamics into a coupled basinwide recharge oscillator that relies on the non-equilibrium between the zonal mean equatorial thermocline depth and wind stress.

With the continuously improved understanding of nature, the spatiotemporal diversity 73 and complexity of the ENSO have been progressively highlighted (Capotondi et al., 2015; 74 Timmermann et al., 2018). In particular, the observational data shows that the center of 75 anomalous SST is mainly located in the EP from 1980 to 2000. In contrast, it lies more 76 towards the central Pacific (CP) after 2000 (Ashok et al., 2007; Kao & Yu, 2009; Kim et al., 77 2012). See Panel (b) of Figure 1. The emergence of these different ENSO events is called the 78 El Niño diversity (Capotondi et al., 2015). It suggests the existence of at least two types of 79 El Niños, which are named the EP and the CP El Niños, when the peak of the SST anomaly 80 locates in the cold tongue and near the dateline region, respectively (Larkin & Harrison, 81 2005; Yu & Kao, 2007; Ashok et al., 2007; Kao & Yu, 2009; Kug et al., 2009). It is essential 82 to notice that the shift of the warming center can cause significant differences in the air-sea 83 coupling over the equatorial Pacific, which changes the way ENSO affects the global climate 84 and brings severe challenges to its prediction (D. Chen & Cane, 2008; E. K. Jin et al., 2008; 85 Barnston et al., 2012; Z.-Z. Hu et al., 2012; Zheng et al., 2014; Fang et al., 2015; Sohn et 86 al., 2016; Santoso et al., 2019). In addition to these two major categories, individual ENSO 87 events further exhibit diverse characteristics in spatial pattern, peak intensity, and temporal 88 evolution, known as the ENSO complexity (Timmermann et al., 2018). Thus, developing 89 effective dynamical models that capture the ENSO complexity is of practical importance, not 90 only for improving the understanding of the formation mechanisms of ENSO but advancing 91 the prediction of different ENSO events and the associated varying climatic impacts as well. 92

The main physical mechanisms of the EP and the CP El Niños are very different. Due 93 to the strong zonal asymmetry of the tropical Pacific air-sea system, the thermocline, a thin 94 layer that separates the upper warm water from the cold water in the lower layer, is deep 95 in the western Pacific (WP) while shallow in the EP region. Such a structure is consistent 96 with the easterly trade wind. As a result, the SST in the EP is more susceptible to the 97 oceanic vertical processes, i.e., the thermocline feedback. On the other hand, the background 98 mean state suggests that the CP is the region with the most significant zonal SST gradient. 99 Consequently, the anomalous zonal current can significantly affect the local SST variations, 100 which means the development of the CP type of ENSO is primarily influenced by the zonal 101 advective feedback (Kug et al., 2009, 2010; Ham & Kug, 2012; Kug et al., 2012; N. Chen 102 & Majda, 2016; Fang & Mu, 2018; Fang & Zheng, 2018). It is worthwhile to point out 103 that, since the state-of-the-art coupled general circulation models (CGCMs) in general have 104 difficulties in accurately describing the mean state of the tropical Pacific (e.g., the unrealistic 105 westward extension of the cold tongue), the simulations often contain biases in reproducing 106 the equatorial SST gradient and the relevant zonal advective feedback (Lin, 2007; Fang 107 & Zheng, 2018; Xie & Jin, 2018; Planton et al., 2021). Such an issue is one of the main 108 reasons that result in significant challenges for the CGCMs to simulate the El Niño diversity 109 correctly (Wittenberg, 2009; Ham & Kug, 2012; Capotondi et al., 2015). 110

In addition to the interannual state variables, it is essential to consider several other vital 111 112 variabilities belonging to different temporal scales in the modeling procedure to generate the ENSO complexity realistically. On the one hand, the intraseasonal atmospheric variability, 113 e.g., the westerly wind burst (WWB) (Harrison & Vecchi, 1997; Vecchi & Harrison, 2000; 114 Tziperman & Yu, 2007) and the Madden-Julian oscillation (MJO) (Hendon et al., 2007; 115 Puy et al., 2016), has been understood as one of the primary sources that lead to the ENSO 116 irregularity and extreme events. Specifically, WWBs can influence ENSO development by 117 stimulating eastward-propagating oceanic Kelvin waves, generating surface zonal currents, 118 and weakening evaporation. Many modeling works have been attempted to incorporate 119 semistochastic parameterization for the WWBs (Gebbie et al., 2007; Tziperman & Yu, 120

2007; Gebbie & Tziperman, 2009; Levine et al., 2016; Thual et al., 2016, 2017) and suggest 121 that the coupled feedbacks between the interannual SST and the intraseasonal WWBs is 122 sufficient to transfer a damped system to a semi-regular self-sustained oscillator. Likewise, 123 in light of an intermediate coupled model (ICM), Lian et al. (2014) found that the WWBs 124 are responsible for the existence of the irregularity and intensity of El Niño. The associated 125 specific characteristics depend on the timing of the WWBs relative to the phase of the 126 recharge-discharge cycle. On the other hand, ENSO is also modulated by the decadal 127 variation of the background mean state. Notably, McPhaden et al. (2011) and Xiang et al. 128 (2013) revealed the changes in the equatorial Pacific around the 2000s, i.e., a La Niña-like 129 background state with enhanced trade winds and a more tilted thermocline, is in favor of 130 the occurrence of more frequent CP El Niño events. This is consistent with the findings 131 in Capotondi and Sardeshmukh (2015), which highlighted the importance of a La Niña-132 like initial/background state based on results from a linear inverse model. Power et al. 133 (2021) also emphasizes the role of decadal variability in affecting the equatorial Pacific. In 134 addition, by extending the original recharge oscillator into a three-region (i.e., WP, CP, 135 and EP) conceptual model that contains a set of 6 stochastic ordinary differential equations 136 and includes both the thermocline and zonal advective feedbacks, N. Chen et al. (2022) 137 demonstrated that the decadal variability plays a crucial role in modulating the occurrence 138 of the CP and EP El Niños. 139

The conceptual model in N. Chen et al. (2022) captures many desirable large-scale fea-140 tures of the ENSO complexity. It thus provides an essential theoretical basis for developing 141 a more sophisticated dynamical model, namely an ICM, that aims to reproduce detailed 142 spatiotemporal patterns of the ENSO complexity realistically. Unlike the conceptual mod-143 els, the ICMs have the unique advantage of incorporating more elaborate underlying physics 144 and spatially-extended dynamics into the model development that facilitate the understand-145 ing and prediction of nature. The ICMs also serve as a bridge that connects the low-order 146 conceptual models and the more complicated CGCMs with a relatively low computational 147 cost. 148

To this end, a stochastic ICM for the ENSO complexity is developed in this paper. 149 The dynamical core of this new stochastic ICM is deterministic and linear, which involves 150 a coupled interannual atmosphere, ocean, and SST system that drives the essential ENSO 151 dynamics in a simple fashion. Here, the latent heating proportional to the SST is depleted 152 from the ocean and forces an atmospheric circulation. In turn, the resulting zonal wind 153 stress forces ocean dynamics that provide feedback to the SST through the thermocline 154 depth anomalies and the ocean zonal advection. The coupled linear and deterministic inter-155 annual starting model can generate two dominant linear solutions representing the EP and 156 the CP El Niños, respectively, which are essential for simulating the ENSO complexity. The 157 interannual components are coupled with the intraseasonal and the decadal variabilities, 158 described by suitable stochastic processes. The former is the main contributor to the ENSO 159 irregularity and extreme events. At the same time, the latter links the two dominant modes 160 in a simple nonlinear fashion and advances the modulation of the strength and occurrence 161 frequency of the EP and the CP events. Seasonal synchronization is further incorporated 162 into the model, facilitating the ENSO events to tend to peak in boreal winter. Finally, cubic 163 nonlinear damping is adopted to parameterize the relationship between subsurface temper-164 atures and thermocline depth. See Panel (a) of Figure 1 for a schematic illustration of the 165 model structure and the key components. Note that the originally pioneering Zebiak and 166 Cane (1987) ICM was not designed to characterize the ENSO complexity. A recently devel-167 oped revised version captures certain diversity features of the ENSO (Geng & Jin, 2022). 168 Nevertheless, the new simple ICM to be developed in this paper differs significantly from 169 the Zebiak and Cane (1987) model and its revised version. The new model highlights the 170 interactions between variabilities at different time scales, where only a minimum nonlinear-171 ity is adopted to maintain the model in a simple fashion. The model also exploits suitable 172 stochastic processes to effectively characterize the dynamical properties and accurately re-173 produce the non-Gaussian statistics of the ENSO complexity in different Niño regions across 174



Figure 1. Panel (a): A schematic illustration of the multiscale model developed here. Panel (b): The observational SST anomaly from 1980 to 2020 (unit: $^{\circ}$ C). It is based on the GODAS dataset (Behringer & Xue, 2004) and is computed by averaging over 5°S to 5°N followed by removing the monthly mean climatology of the entire period.

the equatorial Pacific. The latter is particularly crucial to simulate various ENSO events
realistically. It is an essential prerequisite for the unbiased statistical forecast of the ENSO
complexity as well (Majda & Chen, 2018; Fang & Chen, 2022).

The rest of the paper is organized as follows. Section 2 presents the details of the simple 178 stochastic ICM, including the deterministic and linear interannual components, the stochas-179 tic intraseasonal parameterization, the stochastic decadal process, the seasonal synchroniza-180 tion, and the nonlinearly coupled multiscale system. Section 3 contains the observational 181 datasets and the definitions of different types of ENSO events. The model simulations are 182 presented in Section 4 and are compared with the observations. In addition to showing 183 the spatiotemporal patterns of different ENSO events, the skill of reproducing several key 184 statistics in different Niño regions and the composite analysis are also highlighted in this 185 section. Finally, Section 5 contains the conclusions and discussion. 186

¹⁸⁷ 2 The Simple Stochastic ICM

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2.1 The starting deterministic and linear interannual model

The starting interannual model is a deterministic and linear coupled atmosphere-ocean SST system:

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Atmosphere:

$$-yv - \partial_x \theta = 0$$

$$yu - \partial_y \theta = 0$$

$$- (\partial_x u + \partial_y v) = E_a / (1 - \overline{Q})$$
(1)

Ocean:

$$\partial_t U - c_1 Y V + c_1 \partial_x H = c_1 \tau_x$$

$$Y U + \partial_Y H = 0$$

$$\partial_t H + c_1 (\partial_x U + \partial_Y V) = 0$$
(2)

SST:

$$\partial_t T = -c_1 \zeta E_q + c_1 \eta_1 H + c_1 \eta_2 U. \tag{3}$$

The coupled system (1)–(3) consists of a non-dissipative Matsuno–Gill type atmosphere 191 model (Matsuno, 1966; Gill, 1980), a simple shallow-water ocean model (Vallis, 2016) and 192 an SST budget equation (F.-F. Jin, 1997b). Here, the state variables u and v are the zonal 193 and meridional wind speeds, θ is the potential temperature, U and V are the zonal and 194 meridional ocean currents, H is the thermocline depth, and T is the SST. All of them are 195 anomalies. For the coordinate variables, t is the interannual time coordinate, x is the zonal 196 coordinate, while y and Y are the meridional coordinates for the atmosphere and ocean 197 components, respectively. The reason for adopting two distinct meridional axes is that the 198 atmosphere and ocean deformation radii are different. In these equations, $E_q = \alpha_q T$ is 199 the latent heat with \overline{Q} a constant representing the background vertical moisture gradient 200 (Majda & Stechmann, 2009), $\tau_x = \gamma u$ is the wind stress, ζ is the latent heating exchange 201 coefficient, η_1 and η_2 are the strengths of the thermocline and zonal advective feedback, 202 respectively. Here, η_1 is stronger in the EP due to the shallower thermocline, while η_2 is 203 stronger in the CP because of the more significant zonal gradient of the background SST 204 in that region. The constant c_1 is related to the ratio between the ocean and atmosphere 205 phase speeds. The atmosphere extends over the entire equatorial belt $0 \le x \le L_A$ with 206 periodic boundary conditions, namely $u(0, y, t) = u(L_A, y, t)$, and similar for other atmo-207 spheric variables. The Pacific Ocean extends over $0 \le x \le L_O$ with reflection boundary 208 conditions $\int_{-\infty}^{\infty} U(0, Y, t) \, dY = 0$ and $U(L_O, Y, t) = 0$ (Cane et al., 1981; F.-F. Jin, 1997b). 209 The detailed parameter values are listed in Appendix. 210

The above model retains a few essential ingredients that couple the interannual atmosphere, ocean, and SST components and drive the critical ENSO dynamics in a simple fashion. Specifically, the latent heating E_q proportional to the SST T is removed from the ocean and forces an atmospheric circulation. The resulting zonal wind stress τ_x , in turn, forces ocean dynamics that provide feedback to the SST through the thermocline depth anomalies H and the zonal current U. See the dashed box in Panel (a) of Figure 1 that depicts the interannual components.

To facilitate the computation of the model solution, a meridional projection and trun-218 cation are applied to the coupled system, which is known to have meridional basis functions 219 in the form of parabolic cylinder functions (Majda, 2003; Thual et al., 2016). To develop 220 a simple ICM, only the leading basis function is kept for the atmosphere and the ocean, 221 denoted by $\phi_0(y)$ and $\psi_0(Y)$, respectively. Both $\phi_0(y)$ and $\psi_0(Y)$ have Gaussian profiles and 222 are centered at the equator, but the meridional span of $\phi_0(y)$ is more significant than that 223 of $\psi_0(Y)$. The details of these basis functions are included in Appendix. The meridional 224 truncations trigger atmosphere Kelvin, Rossby waves K_A, R_A , and ocean Kelvin, Rossby 225 waves K_O, R_O . Once the system (1)–(3) is projected to the leading meridional basis func-226 tions, the dependence of y and Y are eliminated. The resulting system is only a function of 227 t and x. It reads: 228

Atmosphere:

$$\partial_x K_A = -\chi_A E_q (2 - 2\bar{Q})^{-1} - \partial_x R_A / 3 = -\chi_A E_q (3 - 3\bar{Q})^{-1} (B.C.) \quad K_A(0,t) = K_A (L_A,t) (B.C.) \quad R_A(0,t) = R_A (L_A,t)$$
(4)

Ocean:

$$\partial_t K_O + c_1 \partial_x K_O = \chi_O c_1 \tau_x / 2$$

$$\partial_t R_O - (c_1 / 3) \partial_x R_O = -\chi_O c_1 \tau_x / 3$$

(B.C.) $K_O(0,t) = r_W R_O(0,t)$
(B.C.) $R_O(L_O,t) = r_E K_O(L_O,t)$
(5)

SST:

$$\partial_t T = -c_1 \zeta E_q + c_1 \eta_1 (K_O + R_O) + c_1 \eta_2 (K_O - R_O), \tag{6}$$

where r_W and r_E are the reflection coefficients associated with the ocean reflection boundary conditions (B.C.). The constants χ_A and χ_O are the meridional projection coefficients with $\chi_A = \int_{-\infty}^{\infty} \phi_0(y)\phi_0(y/\sqrt{c})dy$ and $\chi_O = \int_{-\infty}^{\infty} \psi_0(Y)\psi_0(\sqrt{c}Y)dY$. Once these waves are solved, the physical variables can be reconstructed,

$$u = (K_A - R_A)\phi_0 + (R_A/\sqrt{2})\phi_2$$

$$\theta = -(K_A + R_A)\phi_0 - (R_A/\sqrt{2})\phi_2$$

$$U = (K_O - R_O)\psi_0 + (R_O/\sqrt{2})\psi_2$$

$$H = (K_O + R_O)\psi_0 + (R_O/\sqrt{2})\psi_2$$
(7)

where ϕ_2 and ψ_2 are the third meridional bases of atmosphere and ocean, respectively. 229 Note that T, τ and E_q in (4)–(6) stand for the variables after the meridional projection. 230 Despite adopting the same notation, they differ from the original variables in (1)-(3). In 231 addition, the reflection coefficients r_W and r_E are calculated by using the boundary con-232 ditions: $\int_{-\infty}^{\infty} U(0, Y, t) dY = 0$ at the western boundary and $U(L_O, Y, t) = 0$ at the eastern 233 boundary. The former integrates U in (7) and gives $r_W = 0.5$. The latter implies U is zero 234 at different latitude points. In other words, it requires the projected velocity field to each 235 meridional basis function to be zero. Thus, the velocity projected to the leading basis ψ_0 236 leads to $K_O = R_O$ or $r_E = 1$. It is also worth remarking that the equations (4)–(6) are 237 projected only to the leading meridional basis function while the reconstruction in (7) also 238 includes the third one. This is due to the use of the so-called raising and lowering operators 239 in deriving the truncated equations, which connect the nearby meridional basis functions. 240 See Majda (2003); Biello and Majda (2006); Stechmann and Majda (2015) for more tech-241 nical details. It is worth remarking that the higher order meridional basis functions would 242 include off-equatorial contributions to the equatorial dynamics, which could be essential to 243 account for the negative feedback associated with off-equatorial Rossby waves (Kirtman, 244 1997; Capotondi et al., 2006) or the effect of off-equatorial influences, for example, the 245 north and south Pacific meridional modes (Chiang & Vimont, 2004; Zhang et al., 2014). 246 Nevertheless, this work aims to develop a simple ICM that characterizes explicit physics at 247 the leading order. The stochastic noise can effectively describe certain statistical feedback 248 from the off-equatorial regions. 249

After applying a spatial discretization in the x direction, the coupled system (4)–(6) is solved numerically via an upwind finite difference scheme. Since the coupled system is linear and deterministic, its final solution, after the numerical discretization is applied, can be written as a superposition of a set of non-interacting linear modes (the so-called linear solutions). Each linear solution is associated with one eigenmode of the system. In the numerical discretization here, the entire equatorial band is divided into N_A equidistance

grids, and there are N_O grid points in the Pacific ocean. In the simulations of this paper, 256 $N_A = 128$ and $N_O = 56$ are utilized. In other words, the distance between every two 257 grid points is 312.5km, as the entire equatorial band and the span of the Pacific ocean are 258 40,000km and 17,500km, respectively. With an appropriate choice of physical parameters 259 (see Appendix), all the eigenvalues have negative real parts, indicating the decaying nature of 260 these linear solutions. It is essential to highlight that although the eigenvalues of the strongly 261 decaying and fast oscillating small-scale modes may vary by changing the resolution in the 262 spatial discretization, the leading two eigenmodes with the slowest decaying rate remain 263 almost unchanged as long as the spatial discretization is not too coarse. The leading two 264 eigenmodes appear as a pair, and the associated eigenvalues are complex and conjugate, 265 where the associated oscillation frequency lies on the interannual time scale. Due to the 266 slowest decaying rate, the full solution of the coupled system (4)-(6) is dominated by these 267 eigenmodes (see Appendix for details). 268

Figure 2 shows the spatiotemporal evolution of the leading two eigenmodes. In this 269 figure, the decaying rate is manually set to zero to demonstrate the spatiotemporal pattern of 270 such linear solutions. By varying the strength of the zonal advective feedback coefficient η_2 , 271 the dominant eigenmodes can have distinct behavior (see Appendix for the exact parameter 272 values). Specifically, if the role of the zonal advection is weakened, then the leading linear 273 solution exhibits spatiotemporal patterns with the EP El Niño dominating. See Panel (a). 274 In such a situation, the thermocline feedback is the primary mechanism for generating the 275 EP events. In addition, the convergence center of the atmospheric wind lies in the eastern 276 Pacific. On the other hand, if the zonal advective feedback becomes stronger, then the CP 277 El Niño pattern becomes dominant. Correspondingly, the zonal ocean current leads to the 278 warming in the CP region, and the associated convergence center of the atmospheric wind 279 shifts westward. See Panel (b). In addition, the thermocline is deeper than average during 280 the development phase of EP events and steeper than average (La Niña-like) during the 281 development phase of CP events, as described in Capotondi and Sardeshmukh (2015). It 282 is worth highlighting that the occurrence frequency of the CP events (every 2.5 years) is 283 higher than that of the EP events (every 4.5 years) in these dominant linear solutions, which 284 is consistent with the observations. Note that these two linear solutions are the necessary 285 conditions and mechanisms for the model to capture the ENSO complexity. 286

It is worth mentioning that Fedorov and Philander (2001) suggested in their stability 287 analysis the EP-like and CP-like linear modes feature eastward and westward SST propa-288 gation, respectively. This is slightly different from the results shown here. In Fedorov and 289 Philander (2001), the mean state of the SST is calculated by a simple model with the spec-290 ified mean thermocline depth (H) and the temperature difference across the thermocline 291 (ΔT) . Note that this simple model is proper for the anomalous fields in the tropical Pacific 292 but could be too crude to depict the mean state since the latter is much more complicated 293 than the former. As a result, the reconstructed mean state of the SST is flat in the central 294 Pacific, i.e., with nearly zero zonal gradients. Since the zonal SST gradient of the mean 295 state directly determines the strength of the zonal advective feedback $(-uT_x)$, it is crucial 296 for developing the CP type of ENSO event. To this end, a more refined structure function 297 η_2 is adopted here to represent the strength of the zonal advective feedback (or the zonal 298 SST gradient of the mean state), which shows a large center in the central Pacific region. 299

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2.2 Simple stochastic models for the intraseasonal and decadal variabilities

As was seen in Section 2.1, the coupled system (4)–(6) can generate basic linear solutions that exhibit regular patterns of the EP and CP El Niño events in different situations. However, the irregularity and complexity of ENSO require extra mechanisms beyond the deterministic and linear dynamics. In particular, the ENSO variability is often triggered or inhibited by a broad range of random atmospheric disturbances in the tropics, such as the WWBs (Harrison & Vecchi, 1997; Vecchi & Harrison, 2000; Tziperman & Yu, 2007), the easterly wind bursts (EWBs) (S. Hu & Fedorov, 2016), as well as the convective envelope



Figure 2. Linear solutions of the coupled system (4)–(6) reconstructed utilizing the leading two eigenmodes, which have the slowest decaying rate. These two modes appear as a pair of complex conjugate and therefore the reconstructed spatiotemporal pattern is real-valued. Panel (a) shows the solution by multiplying a small number to the ocean zonal advective feedback η_2 coefficient to lower its role and thus gives a EP El Niño dominant mode. Panel (b) shows the solution of the system with a stronger zonal advective feedback and leads to a CP El Niño dominant mode. In both panels, the four columns present the hovmoller diagrams of the interannual atmosphere wind u (unit: m/s), the ocean current U (unit: m/s), the thermocline depth H (unit: m) and the SST T (unit: °C). The detailed parameter values corresponding to the results here are listed in Appendix. Note that, for the purpose of illustration, the decaying rate is manually set to be zero in demonstrating the spatiotemporal pattern of such linear solutions here.

of the MJO (Hendon et al., 2007). On the other hand, it has been shown that the EP and 308 CP events were alternatively prevalent every 10 to 20 years over the past century (Yu & 309 Kim, 2013; Dieppois et al., 2021). For example, the EP events were the dominant ones in 310 the 1980s and 1990s, while the CP El Niños more frequently occurred after 2000 (D. Chen 311 et al., 2015; Freund et al., 2019). These findings imply that the decadal variability plays a 312 crucial role in driving the transitions between the CP- and EP-dominant regimes. Thus, it 313 is also essential to incorporate the decadal effect into the coupled ENSO model to link the 314 different linear solutions. 315

To this end, two stochastic processes are developed and coupled to the starting interannual model (4)-(6). These two stochastic processes characterize the intraseasonal random wind bursts and the decadal variability, respectively. The former is a natural component that depicts random atmospheric disturbances. The latter describes the decadal variation of the background Walker circulation. It may be related to the climate change scenario and plays a vital role in modulating the strength and the occurrence frequency of the EP and the CP events (D. Chen et al., 2015; N. Chen et al., 2022).

First, with the stochastic wind bursts, the wind stress τ_x now contains two components $\tau_x = \gamma(u + u_p)$, where u remains the same as the atmospheric circulation in (2) while u_p is the contribution from the stochastic wind bursts, which is assumed to have the following structure,

$$u_p(x, y, t) = a_p(t)s_p(x)\phi_0(y),$$

where $\phi_0(y)$ is again the leading meridional basis while $s_p(x)$ is a fixed spatial structure localized in the western Pacific because most of the observed wind bursts are active there (see Appendix). The time series $a_p(t)$ describes the wind burst amplitude and is governed by a simple one-dimensional real-valued stochastic process (Gardiner, 2009)

$$\frac{\mathrm{d}a_p}{\mathrm{d}t} = -d_p a_p + \sigma_p(T_C) \dot{W}_p,\tag{8}$$

where d_p is the damping term chosen such that the decorrelation time of the wind is about 323 one month. In (8), W_p is a white noise source while $\sigma_p(T_C)$ is its strength. When a_p 324 is positive and negative, it represents the WWB and EWB, respectively. It is important 325 to highlight that the noise strength $\sigma_p(T_C)$ is state-dependent (the so-called multiplicative 326 noise), as a function of the interannual SST from (6) averaged over the western-central 327 Pacific, namely Niño 4 region. In the absence of seasonal cycle and decadal influence, 328 $\sigma_p(T_C) = 1.6(\tanh(T_C)+1)$. The reason for choosing such a state-dependent noise coefficient 329 is that wind burst activity is usually more active with warmer SST in the western-central 330 Pacific due to the strengthening or eastward extension of the warm pool (Vecchi & Harrison, 331 2000; Hendon et al., 2007), which is modeled here in a simple parameterized fashion. This also implies that the level of stochastic forcing is larger during El Niño than La Niña events 333 (Capotondi et al., 2018). The choice of the hyperbolic tangent function guarantees the 334 bounded wind strength even with a very strong SST, which is more realistic than using a 335 linear function. Note that the enhanced SST only increases the amplitude of the wind bursts. 336 In contrast, the individual wind burst event generated from the stochastic process in (8) does 337 not prefer westerly or easterly. This allows an equal chance to create both the WWB and 338 the EWB as individual events consistent with the observations. Due to the state-dependent 339 noise coefficient, the modeling procedure here indicates that the intraseasonal wind bursts 340 not only affect the interannual variability but are also modulated by the latter. 341

Next, the decadal variability is driven by another simple stochastic process,

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\lambda(I-m) + \sigma_I(I)\dot{W}_I,\tag{9}$$

where the damping λ is set to be 5 years⁻¹ representing the decadal time scale. Similar to 342 (8), $\sigma_I(I)$ and W_I here are the state-dependent noise strength and the white noise source. 343 The reason for adopting a state-dependent noise coefficient, which is a function of I itself, 344 is to allow the distribution of I to be non-Gaussian. In particular, the trade wind in the 345 lower level of the Walker circulation in the decadal time scale is easterly, which means 346 the sign of I should stay the same throughout time, and thus the distribution of I is 347 non-Gaussian. This feature can be easily incorporated into the process of I with the state-348 dependent noise coefficient (Averina & Artemiev, 1988; Q. Yang et al., 2021). Based on the 349 limited observational data and the theory of inferring the least unbiased maximum entropy 350 solution for a distribution, a uniform distribution between [0,1] is adopted for I in this 351 work. Here, a larger I corresponds to a stronger easterly trade wind. The details of the 352 maximum entropy solution and the way to determine $\sigma_I(I)$ are included in the Appendix. 353 Note that the decadal variability I also stands for the zonal SST difference between the 354 WP and CP regions that directly determines the strength of the zonal advective feedback. 355 It is the primary interaction between decadal and interannual variabilities in the coupled 356 system. In fact, in Kang et al. (2020), a Walker circulation strength index is defined as 357 the sea level pressure difference over the CP/EP region ($160^{\circ}W-80^{\circ}W$, $5^{\circ}S-5^{\circ}N$) and the 358 Indian Ocean/WP region (80°E-160°E, 5°S-5°N). The monthly zonal SST gradient between 359 the WP and CP region is highly correlated with this Walker circulation strength index 360 (correlation coefficient being around 0.85), suggesting significant air-sea interaction over the 361 equatorial Pacific. Since the latter is more directly related to the zonal advective feedback 362 strength over the CP region, the decadal variable mainly illustrates such a feature. 363

³⁶⁴ 2.3 Seasonal synchronization

Seasonal phase locking is one of the remarkable features of ENSO, which manifests in the tendency of ENSO events to peak during boreal winter and is mainly related to the pronounced seasonal cycle of the mean state (Tziperman et al., 1997; Stein et al., 2014). Seasonal synchronization is incorporated into the multiscale coupled model developed above through two simple parameterizations.

First, the climatological SST in the central-eastern Pacific warms in spring and cools in 370 boreal fall. This is partly because of the seasonal motion of the Intertropical Convergence 371 Zone (ITCZ), which also modulates the strength of the upwelling and horizontal advection 372 processes that influence the evolution of the SST anomalies (Mitchell & Wallace, 1992). 373 Since the cool (warm) SSTs corresponds to the decreased (increased) convective activity 374 and upper cloud cover, a time-dependent damping term is incorporated into the system 375 to describe such a seasonal variation (Thual et al., 2017). It mimics the cloud radiative 376 feedback. Specifically, two sinusoidal functions are utilized for parameterizing the otherwise 377 constant α_q , which appears as $E_q = \alpha_q T$ in (6). One sinusoidal function has a period of 378 one year that naturally describes the seasonal cycle. The other sinusoidal function has a 379 period of half a year that represents a semiannual contribution to the seasonally modulated 380 variance, as was suggested by Stein et al. (2014). 381

Second, the increased wind burst activity in the western Pacific during the boreal winter as a direct response to the increased atmospheric intraseasonal variability, such as the MJO, is another primary contributor to the seasonal synchronization (Hendon et al., 2007; Seiki & Takayabu, 2007). Therefore, a sinusoidal function with a period of one year is utilized for parameterizing the seasonal variation of the wind burst strength coefficient σ_p in (8).

2.4 The nonlinearly coupled multiscale system

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The coupled model developed so far is a linear model, despite the state-dependent noise. However, the linear nature of the model is insufficient in characterizing some of the key observed dynamical and statistical features of the ENSO complexity.

From the dynamical point of view, at least two major nonlinearities are expected to be added to the starting linear model. First, the decadal variability determines the strength of the zonal advective feedback. Therefore, it is natural to treat the modulation of the decadal variability on the ENSO dynamics as nonlinear, where the decadal variability plays the role of a multiplicative factor of the zonal advection coefficient. In other words, the decadal variability I is incorporated into the SST budget equation (6) and appears in front of the zonal advection coefficient,

$$\partial_t T = -c_1 \zeta E_q + c_1 \eta_1 (K_O + R_O) + c_1 I \eta_2 (K_O - R_O) + c_1 \eta_2 c_2, \tag{10}$$

³⁹¹ such that a quadratic nonlinearity is introduced from $I(K_O - R_O)$ as I and K_O, R_O are ³⁹² both state variables (recall from (7) that K_O and R_O are the linear combination of H and ³⁹³ U). This nonlinearity represents the mechanism that strengthening the Walker circulation ³⁹⁴ in the decadal time scale will trigger more CP events. It is crucial in simulating the correct ³⁹⁵ occurrence frequencies of both the CP and the EP El Niños. One additional small constant ³⁹⁶ c_2 is added to (10), which guarantees all the variables have climatology with zero mean since ³⁹⁷ otherwise, the nonlinearity can cause a slight shift of the mean state.

Another nonlinearity incorporated here is the damping coefficient in the SST equation. 398 Recall that $E_q = \alpha_q T$ and therefore $-c_1 \zeta \alpha_q$ is the damping coefficient. Here α_q is parame-399 terized by a nonlinear quadratic function of the CP SST, and the spatial structure of such 400 a nonlinear function is concentrated in the CP area. In addition, the symmetric axis of 401 this quadratic function has a negative value, which means a stronger damping is imposed 402 when the CP SST is positive. This effectively gives a cubic damping in the CP. The reason 403 for introducing this nonlinearity is twofold. On the one hand, the relationship between the 404 subsurface temperatures and the thermocline depth is more complicated in the CP region 405 (Zhao et al., 2021). At the same time, only a simple shallow water model is utilized here. 406 Thus, such nonlinear damping is introduced to parameterize the additional relationship be-407 yond the capability of the shallow water model. On the other hand, it is justified from a 408 simple statistical analysis of the observational data that a linear relationship between the 409

damping and SST anomaly in the CP region is broken. In contrast, a cubic nonlinearity fits the data in a more accurate fashion (N. Chen et al., 2022). The nonzero symmetric axis in parameterizing α_q is also crucial for recovering the correct non-Gaussian statistics of the SST in the CP region.

3 Observational Data Sets and the Definitions of Different Types of the ENSO Events

416 **3.1 Data**

The monthly SST data is taken from the GODAS dataset (Behringer & Xue, 2004). Anomalies are calculated by removing the monthly mean climatology of the entire period. The Niño 4, Niño 3.4, and Niño 3 indices are the average SST anomalies over the zonal regions 160°E-150°W, 170°W-120°W and 150°W-90°W, respectively, together with a meridional average over 5°S-5°N.

3.2 Definitions of different types of the ENSO events

The definitions of different El Niño and La Niña events for studying the ENSO com-423 plexity follow those in Kug et al. (2009), which are based on the average SST anomalies 424 during boreal winter (December–January–February; DJF). When the EP is warmer than 425 the CP, and the EP SST is more significant than 0.5° C, it is classified as the EP El Niño. 426 Among this, based on the definitions used by Wang et al. (2019), an extreme El Niño event 427 corresponds to the situation that the maximum of EP SST anomaly from April to the fol-428 lowing March is more significant than 2.5° C. When the CP is warmer than the EP and 429 larger than 0.5° C, the event is defined as a CP El Niño. Finally, when either the CP or EP 430 SST anomaly is cooler than -0.5° C, it is defined as a La Niña event. 431

432 4 Model Simulation Results

The numerical solution of the model is calculated utilizing the forward Euler time 433 integration scheme with a time step of 0.5 days for the interannual variabilities. The Euler-434 Maruyama scheme is adopted to compute the stochastic processes of the decadal variability 435 and the intraseasonal wind bursts, with a numerical integration time step being 0.5 days 436 and 0.05 days, respectively. The monthly averaged model outputs for the interannual and 437 decadal variabilities are utilized in presenting the dynamical and statistical results. The 438 monthly averaged output has almost no difference from the direct model solution but is 439 adopted mainly to be consistent with the monthly averaged observational data. On the 440 other hand, the monthly average is not applied to the wind burst data. 441

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4.1 Model simulation of the ENSO complexity

Figure 3 shows a 50-year model simulation. With the random and nonlinear components 443 in the model, the resulting atmosphere-ocean-SST fields exhibit irregular spatiotemporal 444 patterns, mimicking the observed ENSO complexity (Panel (b) of Figure 1). To begin with, 445 the model simulation succeeds in reproducing both the realistic CP (e.g., 154, 157, 160) 446 and EP (e.g., 163, 175, 192) events as well as some mixed events (e.g., year 188). The two 447 different linear solutions presented in Figure 2 are now linked by the decadal variability, 448 which directly modulates the strength of the zonal advective feedback. In other words, the 449 decadal variability preconditions the model to have a preference towards either the EP or 450 the CP mode at each time instant, although the details of each single ENSO event are 451 still primarily affected by other stochastic and nonlinear effects. Next, the spatiotemporal 452 fields of the interannual atmosphere wind u, ocean current U, thermocline depth H, and the 453 SST T in Panels (a)–(d), as well as the wind bursts strength a_p in Panel (f), reveal distinct 454 formation mechanisms for the CP and EP El Niños. The EP El Niño, especially the extreme 455

EP El Niño, is triggered by the random wind bursts and the thermocline depth plays a vital 456 role in the event development. In contrast, the zonal advection is the dominant contributor 457 to the CP events. It is worth remarking that while the zonal advective feedback is the 458 dominant dynamical feedback in the CP region, the development of anomalous zonal currents 459 (for example, forced by anomalous winds) is also essential. A recent study (Capotondi & 460 Ricciardulli, 2021) links the occurrence of CP events to extratropical wind precursors related 461 to the North and South Pacific meridional modes. These off-equatorial wind anomalies can 462 give rise to heat content anomalies in the CP region (Anderson et al., 2013) or lead to 463 wind stress anomalies along the equator. The ICM developed here does not explicitly 464 take into account the off-equatorial effects. But the stochastic effects may play a role in 465 compensating for such effects. In addition to the response of the CP and EP events to 466 the zonal advective and thermocline feedbacks, the convergence center of the interannual 467 atmosphere wind locates in the CP and EP regions when these two types of events occur, 468 respectively. These causal relationships are consistent with observations, and the previous 469 findings (Kao & Yu, 2009; Kug et al., 2009; Xiang et al., 2013; Zheng et al., 2014; N. Chen 470 et al., 2018). Furthermore, as in the observations, the strength of the CP El Niños is overall 471 weaker than that of the EP ones (Zheng et al., 2014). Particularly, extreme El Niño events 472 are only observed in the EP region due to the anomalously intense wind bursts. It is also 473 noticed that the probability of generating CP events increases as the decadal variability 474 becomes stronger. This is again consistent with the observations, for example, the CP 475 events becoming more frequent as the strengthening of the Walker circulation in the 21st 476 century (McPhaden et al., 2011; Xiang et al., 2013). Nevertheless, regardless of the strength of the decadal variability, the model always allows both the CP and the EP events to be 478 triggered with a certain chance. Finally, the Niño indices shown in Panel (f) mimic reality, 479 where the Niño 4 index has a slightly larger value than Niño 3 at the CP El Niño phases. 480 In contrast, the Niño 3 index becomes much more significant than Niño 4 during extreme 481 EP events. 482

Figure 4 shows a simulation of the SST field for 200 consecutive years, accompanied by 483 the associated wind bursts and the decadal variability. To summarize the findings in these 484 figures, Table 1 lists examples of different ENSO events belonging to 9 refined categories 485 in such a long model simulation and are compared with observations. The results indicate 486 the ability of the model to reproduce the realistic ENSO complexity. First, the model can 487 simulate various EP El Niño events with different strengths. In addition to the moderate 488 EP El Niños, the extreme El Niño events, which appear as a result of the strong WWBs 489 generated from the intraseasonal model, are also reproduced by the model. It is worth highlighting that the so-called delayed super El Niño, as observed in 2014-2015 (S. Hu & 491 Fedorov, 2016; Capotondi et al., 2018; Thual et al., 2019; Xie & Fang, 2020), are realistically 492 simulated by the model, for example, during model years 905-906. The model succeeds in 493 recovering the associated peculiar westerly-easterly-westerly wind burst structure that is the crucial mechanism to trigger such an El Niño event. Here, the initial WWB tends to 495 start a strong El Niño, but the subsequent EWB kills the event and postpones it until the 496 following year, when another series of strong WWBs occur. Next, the model generates many 497 realistic CP El Niño events. In particular, both single-year (e.g., years 764 and 799) and 498 multi-year (e.g., years 760-761 and 899-900) CP El Niño events can be reproduced from the 499 model. The latter mimics the observed CP episodes, for example, during 2018-2020. In 500 addition to those events that belong to either the EP or the CP categories, the model also 501 creates some mixed CP-EP events (e.g., years 785 and 939), similar to the observed ones in 502 the early 1990s (e.g., the one that occurred in the year 1992). Finally, the La Niña events 503 from the model usually follow the El Niño ones as the consequence of the discharge phase. 504 Some La Niña events have cold SST in the CP region, while others have cold centers around 505 the EP area. The model can also simulate multi-year La Niña events. Namely, a La Niña 506 transitioning to another La Niña, such as the one that spans over the years 774-775 and 507 902-904, mimicking the observed events 1999-2000 and 1984-1986, respectively. It is also 508 worth pointing out that a few multi-year El Niño events (such as years 839-840, 918-919, 509



Figure 3. A 50-year model simulation of different fields. Panels (a)–(d): Hovmoller diagrams of the interannual atmospheric wind u (unit: m/s), the ocean current U (unit: m/s), the thermocline depth H (unit: m) and the SST T (unit: °C). The longitude ranges from 120°E (120) to 80°W (280). Panel (e): time series of the decadal variability I. Panel (f): time series of the intraseasonal random wind bursts a_p (unit: m/s). Panel (g): Niño 4, Niño 3.4 and Niño 3 indices.

Coarse category	Refined category	Model (yrs 750-950)	Observations (yrs 1980-2020)
EP El Niño			
	Moderate EP El Niño	792, 897	1987
	Super El Niño	752,862	1998
	Delayed super El Niño	918-919, 905-906	2014-2015
CP El Niño			
	Single-year CP El Niño	764,799	2005
	Multi-year CP El Niño:	760-761, 899-900	2018-2020
	CP La Niña	770,848	1989
Mixed events			
	Mixed EP-CP El Niño	785,939	1992
La Niña			
	Single-year La Niña	798,802	2006
	Multi-year La Niña	774-775, 902-904	1999-2000

Table 1. Examples of different ENSO events in the model simulation (from year 750 to year 950; showing in Figure 4) and observations (from year 1980 to 2020). Here, two examples from the model simulation and one example from observations are listed for each type of the ENSO events, respectively.

935-936) in the model simulation are not directly found in observations. The randomly
 generated wind bursts mainly cause them.

4.2 Comparison of the statistics between model simulations and observations

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In addition to the dynamical properties, the model statistics is another critical mea-514 surement for assessing its skill in reproducing realistic ENSO features. Since the focus is on 515 the ENSO complexity, it is essential to study various statistics that represent unique aspects 516 of the ENSO characteristics in different Niño regions across the equatorial Pacific. To this 517 end, four statistical quantities concerning the SST anomalies of the model simulation are 518 compared with those of the observations in Niño 4, Niño 3.4, and Niño 3 regions, respec-519 tively. They are 1) the probability density function (PDF), 2) the seasonal variance, 3) the 520 power spectrum, and 4) the autocorrelation function (ACF). Here, the statistics of the ob-521 servations are computed based on the observed SST between 1951 and 2020, which contains 522 70 years. On the other hand, a long simulation of 3500 years is utilized for computing the 523 model statistics. The total simulation is divided into 50 non-overlapping subperiods, each 524 having the same length as the observation. The statistics are then calculated for each of 525 these 50 subperiods, the difference among which reflects the uncertainty in calculating these 526 statistics. 527

Panel (a) of Figure 5 shows that the strong non-Gaussian statistics of the SST anoma-528 lies in all the three Niño regions are accurately recovered by the ICM developed here. In 529 particular, the PDF of the Niño 3 SST from observations is highly skewed towards the 530 positive direction with a one-sided fat tail. The tail corresponds to the occurrence of the 531 extreme EP El Niños events. The state-dependent noise in the wind burst process facilitates 532 the model to create such extreme events. Therefore, the model can accurately recover this 533 strong non-Gaussian PDF. In contrast, a negative skewness is found in Niño 4 SST from 534 observations. In addition, the kurtosis of the associated PDF is less than the standard Gaus-535 sian value, which is 3. These findings indicate the suppression of extreme El Niño events 536



Figure 4. Model simulation: Hovmoller diagram of SST T (unit: ^oC), time series of the decadal variability I and time series of the intraseasonal random wind bursts a_p (unit: m/s) from year 750 to year 950.

in the Niño 4 region. With the help of the cubic and non-centered damping in the CP 537 area, the model can accurately reproduce this skewed and light-tailed PDF. Similarly, the 538 model can produce the observed PDF of the Niño 3.4 SST, demonstrating a slight positive 539 skewness. It is worth highlighting that, despite the successes in recovering many dynami-540 cal features of ENSO, the CGCMs and many other dynamical models may not always be 541 skillful in capturing such highly non-Gaussian PDFs in all the three Niño regions. However, 542 recovering these statistics is one of the necessary conditions for reproducing the realistic 543 ENSO complexity. Panel (b) of Figure 5 reveals the model's capability in recovering the 544 observed seasonal synchronization of ENSO, which is represented by the monthly variance 545 of the SST in different Niño regions. The observed ENSO events usually favor starting in 546 spring and peaking in boreal winter. This will also be depicted in Figure 8 by the compos-547 ite analysis for the temporal evolutions of Niño 3.4 SST index for different ENSO events. 548 Overall, the model accurately captures these features, especially given that the model only 549 exploits simple sinusoidal functions for parameterizing the seasonal effects. 550

Next, Panel (a) of Figure 6 shows the power spectrums of the SST. It can be seen 551 that the significant signal of the power associated with the Niño 4 SST is between 2 and 4 552 years. The power decreases rapidly outside this window but another large power appears at 553 lower frequencies, consistent with the presence of a decadal component associated with CP 554 events (Sullivan et al., 2016). In contrast, the signal of the Niño 3 (and Niño 3.4) SST has a 555 broader range in the interannual time scale; that is, the power remains significant between 556 2 and 7 years. All of these features, except the low frequency in the Niño 4 region, are well 557 captured by the model simulations, which are the essential requirements for the model to 558 generate a similar degree of irregularity in oscillations as in observations. In addition, as shown in Panel (b), although the ACF associated with the model decays slightly faster than 560 the observations, the model can create very similar ACFs in different Niño 3.4 and Niño 561 4 regions as the observations. The results indicate that the model overall has a realistic 562 decaying rate and memory, which are consistent with nature in the equatorial Pacific and 563 are essential prerequisites for forecasting the ENSO complexity. 564

Finally, Figure 7 shows the scatter plot of the equatorial Pacific boreal winter (DJF) mean SST maxima for the El Niño events from the model simulation, which is compared with the observations. Each point displayed here is a function of the maximum SST and its



Figure 5. Comparison of the PDFs (Panel (a)) and the seasonal variances (Panel (b)) of SST anomalies between the model simulation and the observations in different Niño regions. The observations are based on the period of 1951-2020, which contains 70 years. Correspondingly, the model simulation has the length in total 3500 years and is divided into 50 equally long periods, each of which is 70 years. The blue shading area is the one standard deviation of the statistics computed from these 50 non-overlapped periods and the blue solid curve is the average value. The observational statistics is shown in red solid curve.



Figure 6. Similar to Figure 5 but for the spectrums (Panel (a)) and the autocorrelation functions (ACFs; Panel (b)).



Figure 7. Distribution of equatorial Pacific SST maxima for the El Niño events from the model simulation of 3500 years (blue) and the observations (red). For each of the qualified El Niño events, the winter-mean SST anomalies are averaged over the equatorial zone (from 5° S to 5° N), and then the Pacific zonal maximum is located. (a) Distribution of peak SST anomaly longitudes. (b) Scatter plot of the peak SST anomaly value v.s. the longitude at which it occurs. The blue (red) dots are for the model results (observations). (c) Distribution of peak SST anomaly values.

corresponding longitude. Panel (a) shows the PDF of the locations of these El Niño events. 568 It exhibits a bimodal distribution with two significant centers. One is near the dateline, 569 and another is in the eastern Pacific. These two peaks correspond to the CP and EP El 570 Niño events, respectively. The finding is consistent with those estimated from different 571 observational data sets by Dieppois et al. (2021). The distribution from the simple ICM 572 developed here is more accurate than those from many CGCM simulations. As was pointed 573 out by Capotondi et al. (2020), many CGCM simulations have biases; for example, the CP 574 events are often located too west compared with observations. Another desirable feature to 575 highlight is that, despite the bimodality, there remains a relatively large probability of the 576 event occurring in the region from the dateline to 120° W (240). This reveals that ENSO 577 diversity is not simply composed of events that belong to two separate categories. Instead, 578 there are many mixed EP-CP events. In fact, according to Panel (b), there are several 579 observed El Niño events (red dots) located in this region, indicating that the distribution 580 should be in the form of a continuum rather than two disjoint sets (Johnson, 2013; Capotondi 581 et al., 2015). However, this seems different in many CGCM results, as pointed out by 582 Capotondi et al. (2015). Next, in terms of strength, the events with CP SST anomaly peaks 583 are overall weaker than the corresponding EP ones. While the strongest events are always 584 located in the eastern Pacific, the EP events can exhibit a wide range of amplitudes. These 585 findings are consistent with physics and observations (F.-F. Jin et al., 2003). 586

El Niño			La N	iña		
	EP	CP	Extreme	Multi-year	Total	Multi-year
Obs	14	10	4	5	24	8
Model	18.2 ± 3.5	14.7 ± 3.5	3.1 ± 2.3	10.5 ± 1.6	26.1 ± 4.6	6.9 ± 2.6

Table 2. Occurrence frequency of different ENSO events per 70 years. The observations are based on the period of 1951-2020, which contains 70 years. For the model simulation, the mean value plus and minus one standard deviation computed from the 50 segments is shown for each case. Note that the counted number of the EP and the CP El Niños contains both single-year and multi-year events as well as the extreme events. Therefore, the total number of the El Niño events is simply the summation of the numbers in the first two columns (e.g., 24 in observations).

4.3 Composite analysis

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To provide a more quantitative assessment of the model performance on simulating 588 each type of the ENSO events, Table 2 summarizes the occurrence frequency of different 589 El Niño and La Niña events (as defined in Section 3.2) per 70 years. For the El Niño 590 events, although the occurrence frequency of both the CP and the EP El Niño events from 591 the model (18.2 and 14.7) is higher than that from the observations (14 and 10), the gap 592 in counting both types of the El Niños between the model and the observations is just 593 around one standard deviation of different model simulation segments, which is nevertheless 594 within a relatively reasonable range. Such a difference mainly comes from overestimating 595 the number of the multi-year El Niño events in the model. Except for this overestimation 596 issue, other statistics from the model simulation are similar to those from the observations. 597 First, the ratio between the numbers of the EP and the CP El Niño events relative to the 598 total number of events from the model simulation (55% v.s. 45%) is almost the same as 599 that in the observations (58% v.s. 42%), which indicates the skill of the model in capturing 600 the overall El Niño diversity. Second, four extreme El Niño events have occurred since 1951, 601 namely 1972–1973, 1982–1983, 1997–1998, and 2015–2016, while a comparable number of 602 3.1 ± 2.3 events is found in the model simulations. Third, the occurrence frequency for the 603 La Niña events (26.1 ± 4.6) from the model is very close to that in the observations (24). In 604 particular, the model and the observations share approximately the same numbers of single-605 year and multi-year La Niña events. It is worthwhile to remark that, as the classification 606 of El Niño events in observations is subject to the limited sample size and suffers from 607 uncertainties associated with varying datasets (Wiedermann et al., 2016; Capotondi et al., 608 2020), the perfect agreement of the occurrence frequency with observations should not be a 609 strict metric on evaluating the model performance. Therefore, it can be concluded that the 610 model is skillful in reproducing reasonably accurate numbers of different events. 611

Next, Figure 8 exhibits the composites of the DJF mean SST anomalies on the equa-612 torial Pacific for the EP El Niño, the CP El Niño, and the La Niña concerning the spatial 613 distribution. Here, all the ENSO events (i.e., during the entire 3500 model years) are com-614 pared with the observations instead of separating them into smaller segments (see Panels 615 a and b). It is seen that the composites computed from the model simulation are almost 616 identical to those from the observations in terms of both the spatial patterns and the am-617 plitudes. Specifically, the warming center locates in the EP and CP regions for the EP 618 and CP El Niños, respectively, although the simulated EP events are closer to the coastline 619 of South America. The cooling center of La Niña is located between the warming centers 620 of the EP and CP El Niño, which is also in accordance with the observations. Next, the 621 model succeeds in recovering the ENSO asymmetry. In other words, the amplitude of the 622 EP El Niño is overall stronger than those of the CP El Niño, and the La Niña (Hayashi et 623 al., 2020). It should be noticed that accurately reproducing the spatial distributions of the 624



Figure 8. Upper panels: composited winter (DJF) mean SST anomalies (lines) and the corresponding error bars (i.e., shaded by one standard deviation) over the equatorial Pacific for EP El Niño (black), CP El Niño (red) and La Niña (blue) events. (a) and (b) are for model and observation, respectively. Lower panels: composited temporal evolution of the Niño 3.4 index for different types of ENSO events. Year(0) and Year(+1) represent the developing and decaying years of each event, respectively.

ENSO events and the ENSO asymmetry is still one of the main challenges for the stateof-the-art CGCMS (Planton et al., 2021). On the other hand, the model also realistically captures the composite results of temporal evolutions of the Niño 3.4 index for different types of ENSO events (see Panels c and d). Remarkably, they all show significant seasonal phase locking character, consistent with the discussion in Figure 5.

4.4 Sensitivity analysis

What remains is to study the role of each critical process in the coupled model, which facilitates understanding the model dynamics. To this end, several sensitivity tests are carried out in the following.

Let us begin by investigating the role of decadal variability. In the standard run, I is 634 driven by a simple stochastic process (9), and its value of the decadal variability time series 635 varies between 0 and 1. In the sensitivity tests, the model simulations with a fixed I of 636 either $I \equiv 0$ or $I \equiv 1$ are studied. In each trial, the strength of the wind bursts is slightly 637 tuned by multiplying a constant such that the variance of the SST remains the same as 638 the standard run, which allows a fair comparison of the occurrence frequency in different 639 scenarios. Note that the decadal variability may be related to climate change and climate 640 projection. Therefore, the interannual variability response due to the decadal variability 641 variation is of great interest. First, the decadal variability is set to be zero (i.e., $I \equiv 0$), 642 which corresponds to the situation with a weakened background Walker circulation between 643 1980 and 2000 but towards the more extreme case. In such a scenario, the model simulation 644 leads to an increase of the El Niño events (from 32.9 to 42.1 per 70 years) and a decrease of 645 the La Niña ones (from 26.1 to 19.4). More specifically, among the El Niño events, 65.8%646 events are the EP type while only 34.2% events remain as the CP El Niño. This means 647 the scenario with a weakened Walker circulation is more favorable for the EP than the CP 648

El Niño events, as the zonal advective feedback in this situation is reduced and the role 649 played by the thermocline feedback becomes dominant. The CP El Niño events can still 650 be generated because of the stochastic noise. Similar to the overall occurrence frequency, 651 more multi-year El Niño (from 10.5 to 15.9) and less multi-year La Niña (from 6.9 to 4.6) 652 is found in such a case. In addition, about 7.6 extreme El Niño events are produced per 70 653 years. This is nearly twice as many as that in the standard run and is in accordance with 654 the observations, where 3 out of the total four extreme El Niño events occurred from 1980 655 to 2000. Notably, such a result is consistent with the climate projection that an increased 656 frequency of extreme El Niño events will appear due to the greenhouse warming since a 657 projected surface warming over the EP is faster than that in the surrounding ocean waters 658 (Cai et al., 2014). Next, if the decadal variability is set to be $I \equiv 1$, then the model mimics 659 the situation when the Walker circulation and zonal thermocline slope are relatively strong, 660 similar to the period after 2000. In such a scenario, more CP El Niño events (from 10 to 661 20.6) are found in the model simulation as a natural consequence of the strengthened zonal 662 advection feedback. Similarly, less multi-year El Niño (from 10.5 to 9.6) and multi-year La 663 Niña (from 6.9 to 5.2) are generated (Iwakiri & Watanabe, 2022). In addition, there remain 664 only 1.3 extreme El Niño events per 70 years since the overall occurrence of the EP events 665 becomes lower. These findings further indicate that the difference between the positive and 666 negative phases of ENSO is weakened. As a result, the PDF of the EP SST becomes closer 667 to Gaussian, which is fundamentally different from the observed non-Gaussian PDF with a 668 fat tail. 669

The next analysis study is about the effect of the multiplicative noise $\sigma_p(T_C)$ in the stochastic wind burst process. The multiplicative noise is one of the main contributors to the asymmetry of the EP type of El Niño. If an additive noise (i.e., setting σ_p as a constant) is adopted, the PDF of the simulated EP SST becomes more symmetric and Gaussian. This is very different from the observations, where the amplitude of the extreme El Niño is typically larger than that of the strongest La Niña.

Finally, the nonlinear damping in the CP region is crucial to the ENSO dynamics and 676 statistics. According to the observations, the asymmetry concerning the SST PDF in the 677 CP is reversed compared with that in the EP. That is, the amplitude of the negative phase 678 of the CP SST is generally stronger than that of the positive one, which leads to a negative 679 skewness of the CP SST PDF. Such a negative skewness is accurately recovered with the 680 help of the nonlinear function of α_q with the non-zero symmetric axis. On the other hand, 681 the nonlinear damping in the CP region also plays a vital role in suppressing the amplitude 682 of the strong CP events since the damping becomes more significant as the amplitude of the 683 SST anomaly. As a result, the system favors the small and moderate SST anomalies, and a 684 reduced kurtosis appears for the CP SST PDF. 685

5 Conclusions and Discussion

This paper develops a simple multiscale stochastic ICM to capture the ENSO diversity 687 and complexity. The model highlights the interconnections between intraseasonal, interan-688 nual, and decadal variabilities. It also exploits suitable stochastic processes to facilitate the realistic simulation of the ENSO. The model succeeds in reproducing the spatiotemporal dy-690 namical evolution of different types of ENSO events. It also accurately recovers the strongly 691 non-Gaussian probability density function, the seasonal phase locking, the power spectrum, 692 and the temporal autocorrelation function of the SST anomalies in all the three Niño re-693 gions (3, 3.4 and 4) across the equatorial Pacific. Furthermore, both the composites of the 694 SST anomalies for various ENSO events and the strength-location bivariate distribution of 695 equatorial Pacific SST maxima for the El Niño events from the model simulation highly 696 resemble those from the observations. These desirable features of the model are essential 697 for realistically simulating different ENSO events. They are also the prerequisites for the 698 unbiased statistical forecast of the ENSO complexity. 699

Variable	Unit	Unit Value
x zonal axis	$[y]/\delta$	$15000 \mathrm{km}$
y atmospheric meridional axis	$\sqrt{c_A/\beta}$	$1500 \mathrm{km}$
Y oceanic meridional axis	$\sqrt{c_O/\beta}$	$330 \mathrm{km}$
t interannual time axis	[t]	34days
u zonal wind speed	δc_A	$5 \mathrm{ms}^{-1}$
v meridional wind speed	$\delta[u]$	$0.5 \mathrm{ms}^{-1}$
θ potential temperature	15δ	$1.5\mathrm{K}$
E_q latent heating	[heta]/[t]	$0.45 { m K. day^{-1}}$
U zonal current speed	$c_O \delta_O$	$0.25 { m ms}^{-1}$
V meridional current speed	$\delta \sqrt{c}[U]$	$0.56 {\rm cm s^{-1}}$
H thermocline depth	$H_O \delta_O$	$20.8 \mathrm{m}$
T sea surface temperature	[heta]	$1.5\mathrm{K}$
a_p wind burst amplitude	[u]	$5 \mathrm{ms}^{-1}$

Table A1. Model variables, definitions and units.

It is worthwhile to point out that the stochastic ICM developed here shares many com-700 mon features with the conceptual model in N. Chen et al. (2022). Both models include three 701 time scales, where the decadal variability modulates the solution that alternates between 702 the EP- and the CP-dominant regimes. At the same time, the intraseasonal wind bursts 703 trigger most of the irregularities and extreme events. The underlying principles of incorpo-704 rating stochastic wind bursts and the nonlinearity into both models also appear in a similar 705 fashion. Nevertheless, the ICM emphasizes more sophisticated physics and includes many 706 additional dynamical properties. It also involves spatially-extended structures, which allow 707 a better understanding and potentially an improved forecast of the spatiotemporal patterns. 708

A few important topics remain as future work. First, the intraseasonal model adopted 709 here is a one-dimensional simple stochastic process where the spatial structure is prescribed 710 and fixed. A more realistic intraseasonal model can be a spatially-extended (stochastic) 711 model for the wind bursts and the MJO; for example, one of the models in D. Yang and 712 Ingersoll (2013); Wang et al. (2016); Adames and Kim (2016); Thual et al. (2018). A 713 dynamical intraseasonal model allows the wind bursts and the MJO to have realistic spatial 714 propagation mechanisms from the Indian Ocean to the WP. Such a coupled model will 715 also help us understand the coupling between the MJO and the ENSO. Second, the model 716 developed here has a symmetric meridional structure because only the leading meridional 717 basis function is utilized in the meridionally truncated system. Yet, both the wind bursts 718 and the ENSO have certain meridionally asymmetric features, which could be essential to 719 account for the negative feedback associated with off-equatorial Rossby waves (Kirtman, 720 1997; Capotondi et al., 2006) or the effect of off-equatorial influences, for example, the 721 north and south Pacific meridional modes (Chiang & Vimont, 2004; Zhang et al., 2014). 722 Therefore, incorporating additional meridional basis functions into the model is a natural 723 extension of the current system. Third, the ICM developed here applies to the forecast of 724 different ENSO events. In particular, the ICM can be combined with the conceptual model 725 in N. Chen et al. (2022) (and possibly the coupled MJO-ENSO model to be developed as 726 well) for the multi-model data assimilation and forecast, which advances the understanding 727 of the role of each model and each component in improving the forecast of the ENSO 728 complexity. 729

⁷³⁰ Appendix A Variables and Parameters

The definitions and units of the model variables are listed in Table A1. The parameter values are summarized in Table A2.

Parameter	Value
ϵ Froude number	0.4
δ long-wave scaling	0.1
δ_O arbitrary constant	0.1
c_A atmospheric phase speed	$50 \mathrm{ms}^{-1}$
c_O oceanic phase speed	$2.5 \mathrm{ms}^{-1}$
c ratio of oceanic/atmospheric phase speed	0.05
c_1 modified ratio of phase speed	0.15
β beta-plane parameter	$2.28 \ 10^{-11} \mathrm{m}^{-1} \mathrm{s}^{-1}$
g' reduced gravity	0.03ms^{-2}
H_O mean thermocline depth	$50\mathrm{m}$
$ \rho_O $ ocean density	1000 kg.m^{-3}
χ_A atmospheric meridional projection coefficient	0.31
χ_O oceanic meridional projection coefficient	1.38
L_A equatorial belt length	8/3
L_O equatorial Pacific length	1.2
\bar{Q} mean vertical moisture gradient	0.9
\bar{T} mean SST	16.6 (which is 25° C)
α_q latent heating factor	$\alpha_q = q_c q_e \exp(q_e \bar{T}) / \tau_q \times \beta_1(T) \times \beta_2(t)$
$\beta(T)$ state dependent component in α_q	$\beta_1(T) = 1.8 - \eta_2/3 + (0.2 + T_C + 0.4 \times \eta_2)^2/5$
$\beta(T)$ seasonal dependent component in α_q	$\beta_2(T) = 1 + 0.5 \sin(2\pi(t - 1/12)) + 0.1 \sin(2\pi t)\eta_2$
	$-0.0625\sin(4\pi(t-3/12))\eta_1$
q_c latent heating multiplier coefficient	7
q_e latent heating exponential coefficient	0.093
$ au_q$ latent heating adjustment rate	15
γ wind stress coefficient	6.53
r_W western boundary reflection coefficient	0.5
r_E eastern boundary reflection coefficient	
ζ latent heating exchange coefficient	8.7
c_2 mean correction coefficient	
η profile of thermocline feedback	$\eta(x) = 1.3 + (1.1 \times \tanh(7.5(x - L_O/3)))$
η_2 profile of zonal advective feedback	$\eta_2(x) = \max(0, 4\exp(-(x - L_O/(7/3))^2/0.1) \times 0.9)$
d_p wind burst damping	$1.12 \text{ (which is } 1 \text{mon}^{-1} \text{)}$
s_p wind burst zonal structure	$s_p(x) = \exp(-45(x - L_O/4)^2)$
$\sigma_p(T_C)$ wind burst noise coefficient	$\sigma_p(T_C) = 1.6(\tanh(T_C) + 1)(1 + 0.6\cos(2\pi t))(1 - 0.75I)$
λ damping of decadal variability	0.0186 (which is $5year^{-1}$)
m mean of I	0.5

 Table A2.
 Model parameter values.



Figure A1. Panels (a)–(d): Spatial structure functions of the meridional bases $\phi_0(y)$, $\phi_2(y)$, $\psi_0(y)$ and $\psi_2(y)$. Panels (e)–(g): Spatial structure functions of $s_p(x)$, $\eta_1(x)$ and $\eta_2(x)$.

As was stated in Section 2.1, different parabolic cylinder functions in the ocean and atmosphere were used in the coupled model. Their profiles are shown in Figure A1. The atmospheric parabolic cylinder functions read $\phi_0(y) = (\pi)^{-1/4} \exp(-y^2/2), \phi_2(y) = (4\pi)^{-1/4} (2y^2 - 1) \exp(-y^2/2)$. The ocean parabolic cylinder functions read $\psi_m(Y)$, which have the same profiles as the atmospheric ones but depend on the oceanic meridional axis Y.

To couple the ocean and atmosphere, projection coefficients are introduced, which read $\chi_A = \int_{-\infty}^{+\infty} \phi_0(y) \phi_0(y/\sqrt{c}) \, dy$ and $\chi_O = \int_{-\infty}^{+\infty} \psi_0(Y) \psi_0(Y/\sqrt{c}Y) \, dY$. The atmosphere uses a truncation of the Kelvin and first Rossby atmospheric equatorial waves of amplitude K_A and R_A . The ocean uses a truncation of zonal wind stress forcing to ψ_0 , $\tau_x = \tau_x \psi_0$. This is known to excite only the Kelvin and first Rossby oceanic waves of amplitude K_O and R_O . Similarly, for the SST model, a truncation ψ_0 , $T = T\psi_0$ is utilized. Then, the deterministic and linear part of the ENSO model truncated meridionally yields (4)–(6).

⁷⁴⁵ Appendix B Parameters for the Two Different Linear Solutions

The model starts with a deterministic and linear coupled interannual atmosphere, ocean, and SST system. Before the two stochastic processes on the other two time scales are further incorporated, confirming that the linear model can generate the basic solutions of the two types of ENSO events under different parameter settings is crucial. In addition, the potential difference should be as slight as possible for the physical interpretation.

In this model, the behavior of the leading mode is mainly determined by the relative 751 amplitude between the zonal advective feedback and the thermocline feedback, which is 752 consistent with the observational analyses. For this purpose, the only change of the setting 753 is the strength of the zonal advective feedback, i.e., η_2 . Specifically, for the linear solution 754 corresponding to the CP ENSO regime, $\eta_2(x) = \max(0, 4 \exp(-(x - L_O/(7/3))^2/0.05) \times 0.9)$ 755 is utilized (Panel (b) of Figure 2), while for that corresponding to the EP ENSO regime, 756 $\eta_2(x) = \max(0, 4 \exp(-(x - L_O/(7/3))^2/0.05) \times 0.9) \times 0.3$ is adopted (Panel (a) of Figure 757 2), which is 30% of the one in the CP ENSO regime. Figure B1 illustrates the spectrum of 758 the eigenmodes as a function of the frequency and the growth rate. The two panels show 759 the EP and CP El Niño dominant cases, corresponding to the two panels in Figure 2. 760

Appendix C Stochastic Process with Multiplicative Noise for the Decadal Variability

The decadal variability influences the occurrence frequency of the two types of El Niño and, thus, the ENSO complexity. A stochastic model is introduced for the decadal variability, which depicts the strength of the background Walker circulation and affects the related zonal



Figure B1. The spectrum of the eigenmodes as a function of the frequency (x-axis; unit: year⁻¹) and the growth rate (y-axis; unit: year⁻¹). Panels (a)–(b) here show the EP and CP El Niño dominant cases, corresponding to the two panels in Figure 2.

advective feedback. In the decadal model (9), a state-dependent (i.e., multiplicative) noise coefficient $\sigma_I(I)$ is adopted that allows I to be non-negative, which comes from the fact that the trade wind in the lower level of the Walker circulation in the decadal time scale is easterly. Here, as only limited data for the decadal variability is available, a uniform distribution function of I is adopted in the model. This is based on the fact that the uniform distribution is the maximum entropy solution for a function in the finite interval without additional information (Kapur & Kesavan, 1992; Branicki et al., 2013; Majda & Wang, 2006). The parameter m is the mean of I, which can be inferred directly from the data. The damping parameter λ can be determined by taking the inverse of the decorrelation time, which is defined as

$$\tau = \lim_{T \to \infty} \int_0^T ACF(t) \, \mathrm{d}t \qquad \text{with} \qquad ACF(t) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \frac{I(t+t')I(t')}{var(I)} \, \mathrm{d}t',$$

where T here is the time not SST. A sufficiently large T is used as a numerical approximation. Finally, the multiplicative noise coefficient $\sigma_I(I)$ is determined in the following way (Averina & Artemiev, 1988)

$$\sigma_I^2(I) = \frac{-2\lambda}{p(I)} \int_{-\infty}^{I} \left(s - \frac{m}{\lambda}\right) p(s) \,\mathrm{d}s.$$

763 Data availability statement

The monthly ocean temperature and current data were downloaded from the Na-

tional Centers for Environmental Prediction Global Ocean Data Assimilation System at

https://www.esrl.noaa.gov/psd/data/gridded/data.godas.html (Behringer & Xue, 2004). The

programs and data for the model can be obtained from this site https://doi.org/10.5281/zenodo.6797996

 $_{768}$ (N. Chen & Fang, 2022).

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Figure 1.



(a) Schematic Illustration of the Model Development

(b) Observed SST

Figure 2.

(a) EP El Nino dominant mode

(b) CP El Nino dominant mode





Figure 3.



Figure 4.



Figure 5.









(b) Seasonal variances



Nino 3



Figure 6.













Figure 7.



Figure 8.



Figure A1.



Figure B1.

