Modelling Viscosity of Volcanic Melts with Artificial Neural Networks

Dominic Langhammer^{1,1,1}, Gerd Steinle-Neumann^{1,1,1}, and Danilo Di Genova^{2,2,2}

¹Universität Bayreuth ²University of Bayreuth

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Abstract

Viscosity is of great importance in governing the dynamics of volcanoes, including their eruptive style. The viscosity of a volcanic melt is dominated by temperature and chemical composition, both oxides and water content. The changes in melt structure resulting from the interactions between the various chemical components are complex, and the construction of a physical viscosity model that depends on composition has not yet been achieved. We therefore train an Artificial Neural Networks (ANN) on a large database of measured compositions, including water, and viscosities that spans virtually the entire chemical space of terrestrial magmas, as well as some technical and extraterrestrial silicate melts. The ANN uses composition, temperature, a structural parameter reflecting melt polymerisation and the alkaline ratio as input parameters. It successfully reproduces and predicts measurements in the database with significantly higher accuracy than previous global models for volcanic melt viscosities. A calculator based on our ANN model is available at https://share.streamlit.io/domlang/visc_calc/main/final_script.py. Viscosity measurements are restricted to low and high viscosity range, which exclude typical eruptive temperature range. To overcome this limitation, we use the ANN to create a synthetic viscosity data in the high and low viscosity regime and fit these points using a physically motivated, temperature-dependent viscosity model. An Excel file to calculate viscosities using these parameters and the MYEGA equation is supplied in the Supporting Information.

Modelling Viscosity of Volcanic Melts with Artificial Neural Networks

D. Langhammer¹, D. Di Genova¹, G. Steinle-Neumann¹

 $^1\mathrm{Bayerisches}$ Geoinstitut, Universität Bayreuth, 95440 Bayreuth, Germany

5 Key Points:

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6	•	We train an Artificial Neural Network that calculates temperature- and composition-
7		dependent viscosity of volcanic melts
8	•	The Neural Network reproduces and predicts experimental viscosity significantly
9		better than previous global models
10	•	A synthetic data approach based on the Neural Network is combined with a phys-
11		ical model to predict viscosity at eruptive temperatures

Corresponding author: Dominic Langhammer, dominic.langhammer@uni-bayreuth.de

12 Abstract

Viscosity is of great importance in governing the dynamics of volcanoes, including their 13 eruptive style. The viscosity of a volcanic melt is dominated by temperature and chem-14 ical composition, both oxides and water content. The changes in melt structure result-15 ing from the interactions between the various chemical components are complex, and the 16 construction of a physical viscosity model that depends on composition has not yet been 17 achieved. We therefore train an Artificial Neural Networks (ANN) on a large database 18 of measured compositions, including water, and viscosities that spans virtually the en-19 tire chemical space of terrestrial magmas, as well as some technical and extraterrestrial 20 silicate melts. The ANN uses composition, temperature, a structural parameter reflect-21 ing melt polymerisation and the alkaline ratio as input parameters. It successfully re-22 produces and predicts measurements in the database with significantly higher accuracy 23 than previous global models for volcanic melt viscosities. A calculator based on our ANN 24 model is available at https://share.streamlit.io/domlang/visc_calc/main/final 25 _script.py. Viscosity measurements are restricted to low and high viscosity range, which 26 27 exclude typical eruptive temperatures. Without training data at such conditions, the ANN cannot reliably predict viscosities for this important temperature range. To overcome 28 this limitation, we use the ANN to create a synthetic viscosity data in the high and low 29 viscosity regime and fit these points using a physically motivated, temperature-dependent 30 31 viscosity model. An Excel file to calculate viscosities using these parameters and the MYEGA equation is supplied in the Supporting Information. 32

³³ Plain Language Summary

Magma viscosity is a key parameter that controls the style of a volcanic eruption. 34 whether it will be effusive or explosive. For this reason, any volcanic hazard mitigation 35 plan requires detailed knowledge of this property. Melt viscosity can vary by up to 15 36 orders of magnitude (a factor of a quadrillion) with temperature and composition. Un-37 fortunately, it is not possible to perform measurements over this range continuously in 38 the laboratory, but only in two distinct temperature regimes, termed high and low vis-39 cosity ranges. In order to obtain a model to predict how composition and temperature 40 control viscosity, we use machine learning and train an artificial neural network on a large 41 viscosity database. This allows us to calculate high- and low-temperature viscosity data 42 that we call synthetic. Since most magmas are erupted at temperatures between the high 43 and low-temperature ranges, we combine the synthetic data and a physically motivated 44 equation to describe the dependence of viscosity on temperature. This model can com-45 pute viscosities in the region without measurements, including typical eruption temper-46 atures of volcanoes. Our model serves the scientific community studying volcanic erup-47 tion mechanisms and its future prediction on a data driven basis. 48

49 1 Introduction

The shear viscosity (η) of volcanic melts is of great importance for the transport 50 dynamics of magmas and the eruptive styles of volcanoes (Cassidy et al., 2018; Colucci 51 & Papale, 2021; Di Genova et al., 2017a; Dingwell, 1996; Gonnermann & Manga, 2007; 52 Papale, 1999), making η an important quantity for physical volcanology. Melt viscos-53 ity depends dominantly on chemical composition (x) and temperature (T). Previous work 54 has often used a T-dependent expression to fit data from η measurements to a specific 55 anhydrous composition. Common examples are the VFT (Fulcher, 1925; Tammann & 56 Hesse, 1926; Vogel, 1921) and MYEGA (Mauro et al., 2009) models. With a critical in-57 fluence of water on eruption dynamics (Gonnermann & Manga, 2013) and its strong con-58 trol on viscosity, the dependence of η on H₂O is often considered separately in models. 59 Such models are typically built by empirical modifications of the fitting parameters to 60 include H_2O dependence (e.g., Dingwell et al., 1998; Giordano et al., 2009; Langham-61

⁶² mer et al., 2021; Misiti et al., 2011; Robert et al., 2015; Vetere et al., 2006; Whitting-

ton, Hellwig, et al., 2009).



Figure 1. Total alkaline and silica content (Le Bas et al., 1986) for all data set in the database. Blue symbols indicate training/validation data sets and red symbols the test sets. Anhydrous data sets are shown by circles and hydrous by crosses. Values and references for the training/validation sets are given in Table 1 and for the test sets in Table 2.

In contrast to the individual melt fits, global models predict viscosity based on x, 64 T (Giordano et al., 2008; Hui & Zhang, 2007) and pressure P (Duan, 2014). These three 65 global models are based on empirical descriptions and are fitted on large data sets. The 66 complex and non-linear relation between chemical components, melt structure and vis-67 cosity prevents the use of a model approach based on physical principles. But advances 68 in machine learning algorithms, specifically artificial neural networks (ANN), provide an 69 alternative route to describe the composition-viscosity relation. Using a large database, 70 ANNs can find highly non-linear mappings between input and and output without prior 71 knowledge of the mathematical form of this connection (Aggarwal, 2018). Recently, Tandia 72 et al. (2019) have shown the capabilities of ANN to accurately fit and predict melt η for 73 a database of technical glasses. Based on this success, Cassar (2021) produced an ANN 74 architecture with a plethora of input parameters to predict η of silicate melts, the ANN 75 by Le Losq et al. (2021) predicts several physical outputs but is limited to compositions 76 in the $K_2O-Na_2O-Al_2O_3-SiO_2$ system. These studies use grey box approaches for the vis-77 cosity calculation, by predicting input parameters of certain viscosity equations (e.g., VFT) 78 or MYEGA) which are then used to compute η . This approach permits a physical in-79 terpretation of the predictions, within the constraints of the model. 80

However, with a scarcity of η data, this approach cannot be used for volcanic melts. 81 The problem of sparse data is even more pronounced when considering the effect of wa-82 ter on η (Duan, 2014; Hui & Zhang, 2007; Giordano et al., 2008). Therefore, we do not 83 follow the previous ANN studies with a grey box approach, but use a black box to utilise as many data as possible; this approach maps the input (composition x and T, where 85 we treat H_2O on equal footing with the oxide components) directly to viscosity. We train 86 an ANN using a database of viscosities for volcanologically relevant melts which we col-87 lected from the literature. We use 3482 data points from 153 data sets (not counting the 88 variable H₂O content) for melts covering virtually the entire compositional space of mag-89 mas on Earth and some analogues of extraterrestrial melts (Figure 1). We show that the 90 ANN is not only capable of fitting given data, but also of predicting η with high accu-91 racy for the viscosity ranges in which measurements are performed. 92

To generate models that inter- and extrapolate in a physically sound way, which 93 cannot be guaranteed using the black box approach, we combine the ANN with a "syn-94 thetic data" approach: For a composition of interest, we generate sets of $\eta -T$ values from 95 an ANN in two distinct η intervals in which viscosity measurements can be performed 96 either by concentric cylinder or micropenetration/parallel plate viscometry (low viscos-97 ity, L η , range: 10⁻³ Pa s < η < 10⁵ Pa s and high viscosity, H η , range: 10⁸ Pa s < 98 $\eta < 10^{13}$ Pa s). These isochemical η values are then fit using the MYEGA equation (Mauro 99 et al., 2009) for T dependence. Our approach combines the accuracy of the neural net-100 work trained on a large data set of η values and the physical basis of the MYEGA equa-101 tion. An application/calculator of our model is available at https://share.streamlit 102 .io/domlang/visc_calc/main/final_script.py (code and model can be found at https:// 103 github.com/DomLang/Visc_Calc). 104

¹⁰⁵ 2 Artificial Neural Networks

Artificial neural networks broadly refer to algorithms for pattern recognition, and 106 here we make use of a simple architecture called a dense feed-forward multilayer percep-107 tron (Aggarwal, 2018). It consists of three layer types: input, hidden and output (Fig-108 ure 2). The layers contain so-called neurons, each storing a single numerical value. Each 109 neuron of a layer is connected to every neuron in the following layer, hence dense, and 110 every connection has a weight associated with it. The neuron values are propagated along 111 these connections, and the weights are optimised during the learning process, analogous 112 to variable parameters when fitting an equation to d ata. The neurons of the input layer 113 store the input data (Section 3.1) which are propagated through all neurons of the hid-114 den layers until the output (η) is calculated. The output is compared to the correspond-115 ing measurement using a loss function (Abadi et al., 2015), for which we use the mean 116 squared error (MSE). The loss function is minimised, going backwards through the net-117 work tuning the weights. Furthermore, non-linearity is added using so-called activation 118 functions (Aggarwal, 2018) which can be applied to the values of each neuron before they 119 are propagated further through the network. This way the hidden layers become a non-120 linear mapping between input and output. 121



Figure 2. Sketch of a feed-forward multilayer perceptron. Circles are neurons storing single values. Each line connecting the neurons has a weight (regression parameter) and activation function associated with it. The black dots indicate that the number of neurons can hypothetically be arbitrary. The number of hidden layers can vary; here, we illustrate two layers used for our artificial neural network.

122 **3 Database**

We compile a database of 3482 tuples (T, x, η) for relevant melts from the litera-123 ture. Of these, 2538 are data from measurements of only anhydrous samples, while 193 124 data sets with the remaining 942 data contain H_2O . The data span a large chemical do-125 main in the total alkaline – silica (TAS) diagram (Figure 1). The database is separated 126 into a training/validation (Table 1) and a test set (Table 2) with 3194 and 288 data points, 127 respectively. The training/validation set contains 144 rock types which yield a total of 128 320 compositions, counting each H_2O content as a unique composition. Of these, 142 are 129 anhydrous and 178 hydrous. The test set contains 15 rock types and again taking into 130 account differing H_2O contents, 29 unique measured compositions. Of these, 14 are an-131 hydrous and 15 hydrous data sets. The test sets are chosen such that they probe a rea-132 sonable chemical domain to check the reliability of the predictions made by the ANN. 133



Figure 3. Histograms indicating the data distribution for one choice of the training, validation and test sets in terms of SiO_2 content (left column) and SM structural parameter (right column).



Figure 4. Viscosity $(\log \eta)$ values from the data sets used here as a function of inverse temperature (10000/T). Data shown in blue are used in the training/validation process (Table 1), red crosses mark the test set (Table 2).

134	Silica content and the empirical parameter termed structure modifier content (SM,
135	in mol%) (Giordano & Dingwell, 2003a) for one choice of the training and validation sets
136	(Section 3.2) and for the test set are shown in Figure 3. The distributions for training
137	and validation sets are very similar which indicates that the validation set tests the in-
138	terpolation, not extrapolation, capabilities of a trained ANN. The SiO_2 content of data
139	ranges from $\sim 37 \text{ wt\%}$ ($\sim 40 \text{ mol\%}$) to $\sim 80 \text{ wt\%}$ ($\sim 85 \text{ mol\%}$) and the H ₂ O content
140	from 0 wt% to ~ 6 wt% (~ 16 mol%). Measured η for L η range from ~ 10 ⁻¹ to ~ 10 ⁵ Pa s
141	and for H η from ~ 10 ⁸ to ~ 10 ¹⁴ Pa s. T spans approximately 1180 to 2020 K and 590
142	to 1270 K in the L η and H η range, respectively (Figure 4).

Table 1. Data sets used in the training and validation process, listed with decreasing SiO_2 content. Oxide compositions and H_2O are given in wt%. The H_2O content is given as the range found in the respective reference. TA = $Na_2O + K_2O$ states the total alkali content (wt%). The first column gives the reference from which the data are taken, the last column indicates the sample name used in the respective publication. Detailed oxide compositions of the melts listed here are given in Table S1 in the Supporting Information as an Excel file.

Reference	SiO_2	TA	$\rm H_2O$	Name in source	Reference	SiO_2	ТА	H_2O	Name in source
(Hofmeister et al., 2014)	80.25	3.82		Moldavite	(Giordano et al., 2006)	60.71	4.42		MST
(Giordano et al., 2006)	79.43	3.91		MDV	(Giordano et al., 2000)	60.46	15.21	0.00 - 3.75	
(Le Losq & Neuville, 2013)	78.92	7.13		NAK83.8.0	(Giordano et al., 2009)	58.90	14.57		Mercato 1500
(Di Genova et al., 2017a)	78.87	6.29		F	(Giordano et al., 2009)	58.84	14.61	0.00 - 4.24	Mercato 1600
(Hess et al., 1995; Dingwell et al., 1996)	78.60	8.80	0.00 - 3.35	HPG8	(Whittington et al., 2001)	58.82	16.75	0.00 - 4.27	Phonolite
(Di Genova et al., 2017a)	77.86	6.39		G	(Giordano et al., 2009)	58.80	14.77		Mercato 1400
(Le Losq & Neuville, 2013)	77.82	8.99		NAK83.8.2	(Liebske et al., 2003)	58.69	4.87	0.00 - 1.96	Andesite
(Di Genova et al., 2017a)	77.63	6.06		Α	(Vetere et al., 2006)	57.95	5.19	2.73	MD25
(Di Genova et al., 2017a)	77.56	7.15		C	(Vetere et al., 2006)	57.95	5.19	5.60	MD12
(Di Genova et al., 2017a)	77.28	6.32		B	(Robert et al., 2019)	57.32	15.27		Jd100
(Di Genova et al., 2017a)	77.25	7.58		H	(Robert et al., 2019)	57.12	17.92		Jd625
(Di Genova et al., 2017a)	76.83	7.22		D	(Sehlke & Whittington, 2015)	57.1	0.60		NVP
(Le Losq & Neuville, 2013)	76.81	10.00		NAK83.8.4	(Liebske et al., 2003)	56.65	4.79		Unzen-A
(Stabile et al., 2016)	76.62	9.71	0.00.0.24	Ebu-N-red	(Liebske et al., 2003)	56.65	4.79		Unzen-3
(Romine & Whittington, 2015)	70.03	8.00	0.00-0.34	NGA EL N	(Liebske et al., 2003)	50.00	4.79		Unzen-2
(Stabile et al., 2016) (Di Camana et al., 2017a)	76.94	6.89		EDU-IN I	(Liebske et al., 2003) (Liebske at al. 2002)	00.00 EC CE	4.79		Unzen-4
(La Laga & Nauvilla, 2012)	76.10	10.75		I NAVO2 0 C	(Langhamman et al. 2003)	50.05	4.79	0.00.1.50	Unzen-5
(Cete et al. 2005)	76.02	7.09	0.00.0 58	NAR03.0.0	(Dahart et al. 2010)	55.00	9.08	0.00-1.59	Lat-DSC
(Goto et al., 2005) (Stabile et al., 2016)	75.02	10.10	0.00-0.38	Fbu B rod	(Robert et al., 2019) (Robert et al., 2010)	55.86	10.70		J075 Id275
(Here et al. 1005)	75.60	8 50		5Mg	(Robert et al. 2010)	54.95	19.10		Id50
(Stabile et al. 2016)	75.30	11.28		Ebu-C	(Robert et al. 2019)	54.50	21.64		Ido
(Di Genova et al. 2017a)	75.33	8 21		E	(Robert et al. 2013)	54.42	4 82	0.00-3.76	sha
(Stabile et al. 2016)	75.30	10.49		Ebu-B	(Giordano et al. 2009)	53.90	13.01	0.00 0.10	Pompei TR
(Stabile et al., 2016)	75.15	11.79		Ebu-C-red	(Giordano et al., 2006)	53.53	5.09		MRP
(Le Losa & Neuville 2013)	75.11	11 70		NAK83 8 8	(Bomano et al. 2003)	53.52	12.57	0.00-3.32	V 1631 W
(Hess et al., 1995)	74.60	13.60		5K	(Vetere et al., 2007)	53.47	8.38	0.00-4.75	Vul
(Friedman et al., 1963)	74.16	8.50	0.00-1.25		(Sehlke & Whittington, 2015)	53.3	0.35		IcP-HCT
(Hess et al., 1995)	74.10	13.40	0.000	5Na	(Romano et al., 2003)	53.14	13.05	3.07	V_1631_G
(Hess et al., 1995)	74.10	8.40		5Ca	(Robert et al., 2019)	53.12	20.64		Jd25
(Di Genova et al., 2017a)	73.75	6.72		J	(Robert, 2014)	53.08	4.76	0.00 - 2.92	fu18
(Whittington et al., 2004)	73.61	8.86	0.00 - 3.41	DK89	(Hofmeister et al., 2016)	53.08	4.76		Bas-and
(Stabile et al., 2016)	73.40	20.77		NFS-red	(Sehlke & Whittington, 2016)	53.02	3.23		CHW
(Hofmeister et al., 2014)	72.99	14.64		vase	(Sehlke & Whittington, 2016)	52.19	1.82		KREEPe
(Hofmeister et al., 2014)	72.91	15.71		1960	(Robert et al., 2015)	51.46	3.99	0.00 - 3.02	sb
(Hofmeister et al., 2014)	72.59	16.85		1895	(Sehlke & Whittington, 2016)	51.28	1.92		SHG
(Whittington, Bouhifd, & Richet, 2009)	72.31	9.49		HP96	(Giordano & Dingwell, 2003a)	51.20	11.65		Ves_W_tot
(Hofmeister et al., 2014)	72.19	3.62		Indoch	(Sehlke & Whittington, 2016)	50.65	0.35		KREEP
(Hess et al., 1995)	72.10	16.70		10Na	(Whittington et al., 2000)	50.56	10.05	0.00 - 2.27	Tephrite
(Di Genova et al., 2017a)	71.22	6.17		L	(Hofmeister et al., 2016)	50.40	2.79		P-MPRB
(Hess et al., 1995)	71.20	18.20		10K	(Hofmeister et al., 2016)	50.40	2.79		MORB
(Hess et al., 1995)	71.20	8.20		10Mg	(Misiti et al., 2009; Giordano et al., 2006)	50.17	4.63	0.00-4.16	SPZ, STB
(Hess et al., 1995)	70.60	8.20		10Ca	(Sehlke & Whittington, 2016)	50.06	3.51		Mu-Fe
(Langhammer et al., 2021)	70.50	9.90	0.00 - 3.55	Rhy14-DSC	(Sehlke & Whittington, 2016)	49.95	0.87		Lme
(Di Genova et al., 2013)	69.21	10.87	0.00 - 3.55	PS-GM	(Al-Mukadam et al., 2020)	49.90	0.00		Di
(Stabile et al., 2016)	69.14	24.61		NFS	(Sehlke & Whittington, 2016)	49.65	0.57		KOM
(Giordano et al., 2006)	68.80	10.19		CL_OF	(Robert et al., 2015)	49.40	3.70	0.00 - 2.7	fu06
(Le Losq & Neuville, 2013)	68.71	11.81		NAK75.12.0	(Giordano & Dingwell, 2003a)	49.20	9.20		Ves_G_tot
(Hofmeister et al., 2016)	68.41	7.02		Rhyo-dac	(Giordano et al., 2006)	49.07	4.83		STR
(Le Losq & Neuville, 2013)	68.28	12.49		NAK75.12.2	(Langhammer et al., 2021)	48.95	5.57	0.00-2.4	Bas1-DSC
(Le Losq & Neuville, 2013)	07.48	13.01		NAK75.12.5	(Giordano et al., 2009)	48.74	10.83		Pollena GM
(Le Losq & Neuville, 2013)	66.96	14.53		NAK75.12.7 NAK75.10.6	(Giordano et al., 2009)	48.05	11.00		Pollena TR
(Stabile et al. 2016)	66.26	14.20		KES	(Ciordano et al. 2009)	47.99	1.55	0.00.4.45	1006CM
(Ciandana & Diamall 2002a)	66.00	6 00	0.00.1.08	UNZ	(Hofmainter et al. 2005)	47.04	4.01	0.00=4.45	OID1
(Giordano & Dingweii, 2005a) (Le Lesa fr Neuville, 2012)	65.75	15.80	0.00-1.98	NAK75 19 10	(Ciordano & Dingwoll 2002b)	47.40	4.01	0 00 2 21	FTN
(Alidibirov et al. 1007)	65.28	5.62		NAR75.12.10	(Sobling & Whittington, 2016)	46.06	0.58	0.00-2.31	FUC
(Le Losa & Neuville 2013)	64.90	16.40		NAK75 12 12	(Morrison et al. 2019)	46.91	1.78		ANOR
(Whittington et al. 2001)	64.45	10.40	0.00-4.92	Trachute	(Sehlke & Whittington 2016)	46.60	2.24		NAK
(Stabile et al. 2016)	64.44	27.56	0.00-4.02	KFS-red	(Morrison et al. 2019)	45.00	4.05		ISC-1a
(Hess et al. 1995)	64.30	26.20		20K	(Giordano et al. 2006)	45.76	3.72		SLP
(Hofmeister et al. 2016)	64.04	6.05		Dac-and	(Morrison et al. 2020)	44 76	7 19		NYI-1948
(Giordano et al. 2004)	63.88	12.49	0.00=3.86	MNV	(Whittington et al. 2000)	43.57	8 55	0.00-2.06	NIO
(Hess et al., 1995)	63.20	7.50	0.00 0.00	20Mg	(Sehlke & Whittington, 2016)	42.16	0.52	0.00 2.00	LM
(Whittington Hellwig et al. 2009)	63.12	6.25	0.00-5.04	BBD	(Bobert et al. 2019)	40.51	21.63		Ne100
(Hess et al., 1995)	62.90	7.40		20Ca	(Robert et al., 2019)	40.33	25.55		Ne625
(Richet et al., 1996)	62.40	4.45	0.00 - 3.46	Andesite	(Morrison et al., 2020)	39.61	10.61		NYI-1977
(Hess et al., 1995)	62.40	26.90		20Na	(Robert et al., 2019)	39.41	24.03		Ne75
(Romano et al., 2003)	61.26	12.62	0.00-3.78	AMS_B1	(Robert et al., 2019)	39.34	27.09		Ne375
(Neuville et al., 1993)	61.17	5.24		Andesite	(Sehlke & Whittington, 2016)	39.13	11.05		NYI
(Hellwig, 2006)	61.05	5.81	0.00 - 4.94	Dacite	(Robert et al., 2019)	37.87	26.18		Ne50
(Giordano et al., 2004)	60.74	11.60	0.00-3.41	IGC	(Robert et al., 2019)	37.28	27.51		Ne25

Table 2. Data sets used for testing the ANN, listed with decreasing SiO₂ content. Oxide compositions and H₂O are given in wt%. H₂O is given as the range found in the respective reference. TA = Na₂O + K₂O states the total alkali content (wt%). The first column gives the reference from which the data are taken, the last column indicates the sample name used in the respective publication. Detailed oxide compositions of the melts listed here are given in Table S2 in the Supporting Information as an Excel file.

Reference	SiO_2	ТА	H_2O	Name in source
(Hofmeister et al., 2014)	79.63	8.59		Haplogranite
(Webb, 2021)	67.02	11.81		h16b
(Webb, 2021)	63.28	11.46		h22b
(Hofmeister et al., 2016)	62.16	5.84		Dacite
(Webb, 2021)	61.81	9.24		h5a
(Misiti et al., 2006)	59.90	12.55	0.18 - 5.81	AMS
(Langhammer et al., 2021)	57.72	11.62	0.00 - 4.78	Tra3-DSC
(Misiti et al., 2011)	56.08	8.88	0.00 - 3.28	\mathbf{FR}
(Webb, 2021)	55.27	6.50		h34
(Sehlke & Whittington, 2015)	55.06	0.12		Enstatite Basalt
(Sehlke & Whittington, 2015)	55.02	6.47		NVP-Na
(Webb, 2021)	54.35	4.48		h10
(Hofmeister et al., 2016)	53.02	3.23		Dolerite
(Al-Mukadam et al., 2020)	49.90	0.00		Di, DSC derived viscosities
(Webb, 2021)	42.74	8.71		NIQ

3.1 General Data Preparation

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In the training/validation set (Table 1) we initially consider anhydrous and hydrous data sets separately. They are each shuffled and then split into a training and validation set, according to ratios and procedures given in more detail in Section 3.2. The anhydrous and hydrous sets are then combined to finally yield training and validation sets containing both anhydrous and hydrous compositions. Since the number of hydrous data points is relatively small, this procedure ensures that there are sufficient hydrous data in the training set.

The input parameters and corresponding viscosity measurements of the training 151 set are used for the regression during the learning process. After each learning cycle, the 152 validation set is used to predict $\log \eta$. As these data have not been used to train the neu-153 ral network, the validation set evaluates the ANN's ability to generalise, i.e., predict η . 154 The loss function calculated from the validation set (validation loss) is also used to avoid 155 overfitting. Overfitting is typically characterised as an increasing or constant validation 156 loss, while the training loss keeps decreasing. This indicates that the ANN's ability to 157 predict unknown values from input data remains constant or worsens despite an improv-158 ing fit. The improvement in the fit is explained by the ANN learning the data by heart. 159 This phenomenon can be mitigated using regularisation methods, such as dropout (Srivastava 160 et al., 2014) which we use here (Section 3.2). 161

As input parameters, we use mole fractions of SiO₂, TiO₂, Al₂O₃, FeO, Fe₂O₃, MnO, MgO, CaO, Na₂O, K₂O, P₂O₅, H₂O, temperature *T* in K, the SM parameter (Giordano & Dingwell, 2003a) and the alkali ratio $K_2O/(K_2O+Na_2O)$. Contrary to the global viscosity models by Hui and Zhang (2007) and Giordano et al. (2008) that consider the iron content in the melt as FeO only, we differentiate between FeO and Fe₂O₃. This distinction is important as Fe₂O₃ acts as a network former, leading to an increase in melt vis-

cosity, while FeO acts as a network modifier, and has the opposite effect on viscosity (Dingwell 168 & Virgo, 1987; Liebske et al., 2003; Bouhifd et al., 2004; Vetere et al., 2008; Chevrel et 169 al., 2013; Kolzenburg et al., 2018; Stabile et al., 2021). For samples with only the total 170 iron content FeO_{tot} reported, we distribute it evenly between FeO and Fe_2O_3 with a fac-171 tor of 1.11 to account for the higher molar weight of Fe_2O_3 (Langhammer et al., 2021). 172 SM reflects the effect of structural polymerisation on η , and the alkali ratio is known to 173 significantly affect the viscosity of SiO₂-rich systems (Di Genova et al., 2017a; Le Losq 174 & Neuville, 2013; Stabile et al., 2016). Cr_2O_3 is only used during the conversion from 175 wt% to mole fractions. It is omitted during the training as the vast majority of compo-176 sitions contain < 0.02 wt%. 177

To improve convergence and stability of the training process we scale the input data as follows (Montavon et al., 2012): (i) mole fractions of composition lie in the interval $\begin{bmatrix} 0,1 \end{bmatrix}$, (ii) T is normalised by dividing all values by the largest T within the data set (2023 K for Ne375, Table 1). (iii) We modify the definition of SM using mole fractions, also leading to values in the interval $\begin{bmatrix} 0,1 \end{bmatrix}$. (iv) Finally for every value of each input parameter x_i the z-score (Cassar, 2021) is calculated as input according to

$$z_{i,j} = \frac{x_{i,j} - \mu_i}{\sqrt{\sigma_i^2}} \tag{1}$$

$$\mu_i = \frac{1}{N} \sum_{j=1}^N x_{i,j} \tag{2}$$

$$\sigma_i^2 = \frac{1}{N} \sum_{j=1}^N (x_{i,j} - \mu_i)^2 .$$
(3)

Here $x_{i,j}$ and $z_{i,j}$ denote the *j*-th value of the *i*-th input parameter (e.g., SiO₂ content), *x* denoting the old and *z* the *z*-score used as input for the ANN. μ_i is the average and σ_i^2 is the variance of the input. The scaling achieves an average of 0 and variance of 1 for the new input z_i . Values for μ and σ^2 can be found in Table S3 in the Supporting Information.

189 3.

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3.2 Training Procedure

Critical parameters for training an ANN (hyperparameters) are the number of hid-190 den layers and neurons per layer which, through the associated weights, define the num-191 ber of adjustable parameters, the learning rate which defines the step size during train-192 ing, and the dropout value. The dropout randomly sets outputs of a layer to zero at a 193 probability given by the dropout value. This simulates various different ANN's during 194 a training session and reduces overfitting (Srivastava et al., 2014). As a measure for the 195 quality of the ANN we use the root-mean-square-error (RMSE), commonly used in other 196 publications. To search the space of hyperparameters, we use Bayesian optimisation as 197 a stochastic algorithm (Aggarwal, 2018; Snoek et al., 2012) to find architectures with low 198 validation errors. In this step, we perform the shuffling described above and combine 90%199 of the anhydrous and hydrous data sets each to the training set and the remaining 10%200 to the validation set. 201

In the Bayesian optimisation, we fix several hyperparameters: (i) We use two hidden layers. Several tests have shown little improvement when using three layers, and increasing the number of layers further increases the complexity which is not desired in our case given the scarcity of data. (ii) We apply the Adam optimiser (Kingma & Ba, 206 2014) as an algorithm, including the amsgrad flag (Reddi et al., 2019). (iii) For activation in all hidden layers we use the leaky ReLU function defined as

$$f(x) = \alpha x \text{ for } 0 > x, \tag{4}$$

$$f(x) = x \quad \text{for } 0 \le x . \tag{5}$$

We use the default value of $\alpha = 0.3$ (Abadi et al., 2015). In the in- and output layers identity is used as activation. (iv) As batch size we use the complete training set (full batch).

For other hyperparameters we vary the range: (i) The number of neurons in the hidden layers explored are 1 – 256 per layer, with final values of 256 for the first and 208 for the second layer. (ii) For the learning rate we use 0.057317962127906, after exploring 0.00001-0.5. (iii) From a range of 0.0-0.5, the chosen dropout value is 0.16569639948335368.

Using the ANN hyperparameters chosen by the Bayesian optimisation, we apply 217 a 20-fold cross-validation process. All anhydrous and hydrous data sets in the training/ 218 validation database (Table 1) are shuffled separately and split into 20 sets each; one of 219 each is combined to create 20 sets that contain anhydrous and hydrous data (superset). 220 The ANN is trained 20 times, using 19 of the supersets for training and one as valida-221 tion. The validation superset is exchanged until each superset was used for validation 222 once. These architectures are used to predict all η values of the test set, and we present 223 and discuss results for the ANN with the lowest RMSE. 224

Neural networks are built using TensorFlow (Abadi et al., 2015) and the Bayesian
optimisation is performed with the KerasTuner (O'Malley et al., 2019), using Python
as programming language. Data is managed and prepared using the Pandas and NumPy
packages.

²²⁹ 4 Training Results and Evaluation

The SiO_2 distribution for the training and validation set belonging to the network 230 that displays the lowest RMSE when predicting the test set are very similar (Figure 3). 231 Their RMSE values are 0.09 and 0.12, respectively (Figure 5). Therefore we expect the 232 ANN to be trained to interpolate rather than extrapolate. This in turn leads us to rec-233 ommend the use of this model only within the, albeit extensive, chemical bounds given 234 by the training/validation data set (Figure 1). The models of Hui and Zhang (2007) and 235 Giordano et al. (2008) applied to the training/validation sets in our database produce 236 RMSE values of 1.36 and 1.18 respectively (Figure S1 in the Supporting Information). 237



Figure 5. Calculated versus measured viscosities of the training (top) and validation sets (bottom) for the ANN yielding the lowest RMSE when applied to the test sets. The black solid line indicates the one to one correspondence, and dashed lines ± 1 log-unit deviation from identity. RMSE values are 0.09 for the training and 0.12 for the validation set.



Figure 6. Comparison of measured and calculated viscosities for the artifical neural network (ANN, red crosses, RMSE = 0.45), the model by Giordano et al. (2008) (GRD08, blue diamonds, RMSE = 1.23) and Hui and Zhang (2007) (HZ07, green squares, RMSE = 0.89) for melts in the test sets. The one to one correspondence is shown by the solid black line, dashed lines indicate a ± 1 log-unit deviation from identity.

For the test sets we compare measured η to predictions of our ANN and the global models of Hui and Zhang (2007) and Giordano et al. (2008) in Figure 6. The ANN predictions show the lowest RMSE with 0.45, compared to 0.89 and 1.23 for the models by Hui and Zhang (2007) and Giordano et al. (2008), respectively.

Both literature models show significantly larger deviations than the ANN predic-242 tion for specific data sets. The model by Hui and Zhang (2007) overestimate viscosity 243 measurements for the enstatite and a Na₂O-rich basalt (NVP-Na) by Sehlke and Whit-244 tington (2015) and for the diopside (Di) determined from calorimetry experiments by 245 Al-Mukadam et al. (2020). The model by Giordano et al. (2008) underestimate viscosi-246 ties for these samples as well as for a haplogranite (Hofmeister et al., 2014). The two com-247 positions by Schlke and Whittington (2015) – models for basalts on Mercury – show a 248 high content in CaO and MgO as does the diopside by Al-Mukadam et al. (2020) (Ta-249 ble S2 in the Supporting Information), outside typical terrestrial compositions. It may 250 therefore not be surprising that the models by Hui and Zhang (2007) and Giordano et 251 al. (2008) fail to reliably predict their η , while the training/validation set of our ANN 252 contains two similar compositions from Sehlke and Whittington (2015) and further plan-253 etary tholeiitic melts (Sehlke & Whittington, 2016). 254

The high η for the fully polymerised haplogranite by Hofmeister et al. (2014) is not reproduced well by the model of Giordano et al. (2008) which also shows a poor fit for a similar model haplogranite (HPG8) by Hess et al. (1995).



Figure 7. Viscosity $(\log \eta)$ as a function of inverse temperature (1000/T) for some anhydrous compositions in the training/validation database (panels a and b) and the test set (panel c). Measurements are shown by filled circles, viscosity values predicted from the ANN by crosses, where bold crosses indicate the η range for the synthetic data approach. Solid (dashed) lines show MYEGA fits to the experimental (synthetic) data. Data references can be found in Tables 1 and 2. The horizontal lines indicate 10^{12} Pa s.

5 Synthetic Models

Eruptive temperatures of most magmas lie between the $H\eta$ and $L\eta$ range. At these 259 temperatures, volcanic melts tend to crystallise faster than the timescale of the viscos-260 ity measurement. Therefore an interpolation – or extrapolation if data only exist at $H\eta$ 261 - between these ranges is required to determine η . This is done by fitting η data using 262 models, such as the MYEGA and VFT equations (Figure 7), sometimes modified to in-263 clude a H₂O dependence. Contrary to MYEGA fits to experimental data, viscosities di-264 rectly determined from the ANN in the range 10⁵ Pa s $< \eta < 10^8$ Pa s show strong 265 deviations from expected behaviour for some melt compositions (Figure 7). This is not 266 surprising given the fact that the ANN is not trained in this η range due to the exper-267 imental gap discussed in the Introduction. 268

5.1 Creating a Synthetic Model

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²⁷⁰ Mimicking the fitting of experimental data, we create a set of synthetic data from ²⁷¹ the ANN in the H η and L η ranges and fit the physically motivated MYEGA equation ²⁷² (Mauro et al., 2009)

$$\log \eta = A + (12 - A)\frac{T_{\rm g}}{T} \exp\left[\left(\frac{m}{12 - A} - 1\right)\left(\frac{T_{\rm g}}{T} - 1\right)\right] , \qquad (6)$$

to them. The viscosity at infinite T, $A := \log \eta_{\infty}$, melt fragility m and glass transition temperature $T_{\rm g}$ (for $\log \eta = 12$) are fitting parameters. We fix A = -2.9, following our previous work and discussion (Langhammer et al., 2021), creating a two-parameter model equation for a specific composition that can be used to calculate η over a wide T range.



Figure 8. Distribution of viscosities in the training/validation database in the $H\eta$ and $L\eta$ regions, binned with $\log \eta = 0.5$ intervals. Data sets in both regions are split into compositions with $SiO_2 < 60 \text{ wt\%}$ (blue) and $SiO_2 \ge 60 \text{ wt\%}$ (red). Purple indicates the overlap of distributions. The vertical dashed lines indicate the $\log \eta$ ranges over which we create synthetic data, i.e., [9.5, 11.5] at $H\eta$ for all compositions, and at $L\eta$ [0, 2] or [2, 4.5] for melts containing $SiO_2 < 60 \text{ wt\%}$ and $SiO_2 \ge 60 \text{ wt\%}$, respectively.

Determining the $H\eta$ and $L\eta$ intervals for which synthetic data are created is an im-279 portant step in the process, as this choice strongly influences the model parametrisation. 280 We examine the distributions of $\log \eta$ for the whole database (Figure 8) to choose ap-281 propriate intervals for the H η and L η ranges. In the H η region, the data density is the 282 highest at $\log \eta$ between 10 and 11, and we create synthetic data in 1-log around the max-283 imum, i.e., in the interval [9.5, 11.5] with a step size of $\log \eta = 0.5$. Data coverage in 284 the $L\eta$ region varies with SiO₂ content, and we split the data sets at 60 wt% SiO₂. With 285 broad maxima for $\log \eta$ between 0.5 and 1.5 (SiO₂ ≤ 60 wt%) and $\log \eta$ between 2.5 286 and 3.5 (SiO₂ > 60 wt%), we create synthetic data in the log η intervals [0, 2] and [2, 4.5], 287

respectively, again in steps of $\log \eta = 0.5$. Such a split is not necessary in the H η range due to very similar distribution of data (Figure 8).

Technically, we use the ANN in conjunction with the bisection method to calcu-290 late T for the previously discussed $\log \eta$ values to a precision of 10^{-5} . The $\log \eta - T$ de-291 pendence is fit using the MYEGA model (equation 6 with A = -2.9). A web applica-292 tion which follows this scheme and calculates the MYEGA parameters and a viscosity 293 value for a desired T can be found at https://share.streamlit.io/domlang/visc_calc/ 294 main/final_script.py. It is important to mention, that if the composition which is en-295 tered into the app only reports $\text{FeO}_{\text{total}}$ one must split it according to $\text{FeO} = \text{FeO}_{\text{total}}/2$ 296 and $Fe_2O_3 = FeO_{total}\frac{1.11}{2}$. On the other hand, if the compositions reports the total iron 297 as Fe_2O_{3total} it must be split according to $FeO = \frac{Fe_2O_{3total}}{(2\cdot1.11)}$ and $Fe_2O_3 = Fe_2O_{3total}/2$. 298 Fitting parameters m and $T_{\rm g}$ can be used for further calculations. An Excel table to cal-299 culate η values from the fit parameters m and T_g for various temperatures is supplied 300 in the Supporting Information. 301



Figure 9. Comparison of fits to synthetic data to measurements for the compositions also used in Figure 7 (training/validation sets in panel a, test sets in panel b). Crosses are the synthetic data and lines are MYEGA fits (Mauro et al., 2009) to them. Circles are the respective measurements. Data references can be found in Tables 1 and 2.

302 5.2 Testing Synthetic Models

A comparison between the MYEGA fit to ANN viscosities (synthetic model) and direct experimental measurements for the compositions in the training/validation set (Figure 9a) already used in Figure 7 and discussed in Section 4 shows that the experimental measurements agree well with the ANN predictions where they overlap, both in the ³⁰⁷ L η and H η ranges. The synthetic fit describes the experimental T dependence of η mea-³⁰⁸ surements reasonably well, with the exception of Mercato 1600 (Giordano et al., 2009) ³⁰⁹ that shows a more Arrhenian behavior in experiments. This discrepancy stems from the ³¹⁰ fact that, with an SiO₂ content of 58.84 wt%, the synthetic model is based on ANN vis-³¹¹ cosities in the log η interval [0, 2], while experiments cover log η in a range 2–4.6. How-³¹² ever, the direct ANN predictions reproduce measured η for Mercato 1600 quite well (Fig-³¹³ ure 7b).

A similar behaviour can be seen for samples from the test set in Figure 9b. The 314 315 viscosity for three of the four compositions is described well, but values for the Dacite by Hofmeister et al. (2016) are predicted lower than the measurements by 0.9-0.7 log-316 units (decreasing with T) in the H η range, while η of two Fe-free synthetic dacites in the 317 training/validation set (Hellwig, 2006; Whittington, Hellwig, et al., 2009) is reproduced 318 well. Evaluating the same compositions with the global model by Giordano et al. (2008) 319 reveals that it has a similar problem predicting η for the Dacite by Hofmeister et al. (2016) 320 although to a slightly smaller extent (0.7-0.5 log-units). As our model reproduces the 321 experimentally measured η for the Dac-and by Hofmeister et al. (2016) with a similar 322 iron content well, the reason for this specific discrepancy remains unclear. 323

The RMSE value for η in the training/validation set is 0.20 (Figure 10), slightly worse than those from the ANN directly (0.09 and 0.12 for training and validation, respectively). For the test set the values are 0.45 for the ANN predictions and 0.52 for the synthetic model, respectively. In comparison to RMSE values of 1.36 and 1.18 for the entire database by the global models of Hui and Zhang (2007) and Giordano et al. (2008), respectively, the synthetic model provides quite accurate results, and has a physical basis with the MYEGA equation.

The parameters for the MYEGA model (equation 6) do not differ significantly for 331 $T_{\rm g}$ from the individual isochemical fits (Figures 11 and S2 in the Supporting Informa-332 tion) and the direct ANN inversion. For a reasonable comparison, only samples which 333 include measurements in the H η and L η range from the training/validation set are in-334 cluded in Figures 11 and S2 in the Supporting Information. RMSE values of 18.3 K and 335 7.16 K for the direct ANN inversion and the synthetic model, respectively, reflect the nar-336 row distribution in Figures 11 and S3 in the Supporting Information, and an average dif-337 ference of $\delta T_{\rm g} = -0.5$ K between the direct MYEGA fit and the synthetic model (Fig-338 ure S2 in the Supporting Information) show that the synthetic model approach used here 339 does not introduce any bias. 340

Values of melt fragility m derived from synthetic data tend to be slightly overestimated (Figure S3 in the Supporting Information), with the average deviation from direct MYEGA fits of $\delta m = 1.04$. A quasi-linear trend of increasing m with structural parameter SM discussed in Langhammer et al. (2021) is reproduced well.



Figure 10. Comparison of viscosity values predicted by the synthetic models to measurements of the training/validation (top) and test (bottom) sets. The solid line shows the one to one correspondence, with the dashed lines ± 1 log-unit deviations from identity. The respective RMSE values are 0.2 (top) and 0.52 (bottom).



Figure 11. Comparison of calculated $T_{\rm g}$ values for compositions from the training/validation database. $T_{\rm g}$ on the x-axis are from isochemical MYEGA fits of the experimental data, on the y-axis $T_{\rm g}$ calculated from the ANN directly (squares), from the synthetic data approach (crosses) and from the model of Giordano et al. (2008) (GRD, diamonds) are shown. Only samples with measurements in both the L η and H η ranges in Table 1 are used. RMSE values are 18.3 K for the ANN, 7.16 K for the fit to synthetic data, and 57.53 K for the GRD model.

345 6 Conclusion

In this work we have trained an artificial neural network (ANN) on a database con-346 taining 3194 temperature-dependent viscosity (η) measurements for volcanic melts span-347 ning a large chemical domain, including extraterrestrial model systems. The neural net-348 work takes melt composition, H_2O content, temperature T, the chemical parameters SM, 349 reflecting melt polymerisation, and the alkaline ratio $K_2O/(Na_2O + K_2O)$ as input to 350 predict η . We show that the trained neural network describes the data in the database 351 very well, significantly better than commonly used global models (Hui & Zhang, 2007; 352 Giordano et al., 2008). In this context it is worth emphasising that – contrary to such 353 models – the ANN relies on data only, and makes no assumption on the functional de-354 pendence of viscosity. As such, interactions between different compositional components 355 are taken into account implicitly. This suggests that despite the relatively scarce viscos-356 ity data for volcanic melts the success of ANN previously demonstrated for technical glasses 357 (Cassar, 2021; Tandia et al., 2019) can be transferred to volcanology. To facilitate an easy 358 use of our trained ANN, we make an online viscosity calculator available at https:// 359 share.streamlit.io/domlang/visc_calc/main/final_script.py. 360

The lack of training data in an η range $10^5 - 10^8$ Pa s for T characteristic for volcanic eruptions results in unphysical behaviour of η . We therefore combine the reliable ANN predictions of η in the ranges where training data is available at high and low viscosity with a fit using the MYEGA equation to achieve a physically sound interpolation for geologically relevant conditions. The relevant parameters can be calculated using the application mentioned in the previous paragraph. An Excel file to calculate viscosites using these parameters and the MYEGA equation (Mauro et al., 2009) is supplied in the
 Supporting Information.

Creating more accurate and versatile neural networks for melt viscosity, and other 369 properties in general, is only limited by the quality and quantity of data. The time-consuming 370 task of guessing a model equation and assuming critical parameters for melt character-371 istics is partly eliminated, while physical behaviour of the melt is implicitly included in 372 the modelling process. With more data and further analysis, these implicit physical re-373 lations may reveal themselves and lead to a better understanding on composition-structure-374 375 property relations. Our results hopefully encourage other researchers to further explore machine learning algorithms in the context of natural silicate melts and volcanology. Fi-376 nally, our model supports studies for numerical modelling of eruption scenarios and thus 377 forecasting volcanic eruptions on a probabilistic basis. 378

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- and trained model can be found at https://github.com/DomLang/Visc_Calc. An Ex-
- cel file to calculate further viscosities from the MYEGA equation is included in the Sup porting Information.

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Supporting Information for "Modelling Viscosity of Volcanic Melts with Artificial Neural Networks"

D. Langhammer¹, D. Di Genova¹, G. Steinle-Neumann¹

 $^1\mathrm{Bayerisches}$ Geoinstitut, Universität Bayreuth, 95440 Bayreuth, Germany

Contents of this file

- 1. Figures S1 to S3
- 2. Table S3

Additional Supporting Information (Files uploaded separately)

1. Captions for Datasets S1 (ds01) and S2 (ds02)

Introduction The Excel files contain information on the compositions used for training, validation and testing of the artificial neural networks. Figure S1 depicts the performance of literature models (Giordano et al., 2008; Hui & Zhang, 2007) on the training/validation dataset we compiled (Tables 1 and S1), Figures S2 and S3 show the behaviour of fit parameters $T_{\rm g}$ and m derived from our synthetic data approach compared to a direct fit. Table S3 contains the scaling parameters used in the input of the artificial neural network (Secton 3.1).

Data Set S1. The excel file ds01 includes two sheets. The sheet "wt%" contains the compositions of data used for training and validation in wt% of oxides and the base composition name from the respective reference. The sheet "mole frac input" contains the relevant input compositions in mole fractions (excluding Cr_2O_3 as it is not used to train the ANN) and the input parameters SM and $K_2O/(Na_2O + K_2O)$.

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Data Set S2. The excel file ds02 includes two sheets. The sheet "wt%" contains the compositions of data used as test set in wt% of oxides and the base composition name from the respective reference. The sheet "mole frac input" contains the relevant input compositions in mole fractions (excluding Cr_2O_3 as it is not used to train the ANN) and the input parameters SM and $K_2O/(Na_2O + K_2O)$.

Giordano, D., Russell, J. K., & Dingwell, D. B. (2008). Viscosity of magmatic liquids: a model. Earth Planet. Sci. Lett., 271, 123–134. doi: /10.1016/j.epsl.2008.03.038

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Figure S1. Comparison of calculated and measured viscosities for the models by Giordano et al. (2008) (top, GRD08) and Hui and Zhang (2007) (HZ07, bottom), when applied to the training and validation set (Table 1). The solid black line indicates the identity while the dashed lines give a deviation of ± 1 log units.



Figure S2. Comparison of $T_{\rm g}$ derived from direct fits to data and predictions using the synthetic data approach. Only data that include measurements in the L η and H η regime from the training/validation set are used. The top panel (a) shows $T_{\rm g}$ values from direct MYEGA fits (blue) and the synthetic model (red) plotted against SM. The bottom panel (b) depicts the difference $\delta T_{\rm g} = T_{\rm g,MYEGA} - T_{\rm g,synth}$.



Figure S3. Comparison of m derived from direct fits to data and predictions using the synthetic data approach. Only data that include measurements in the L η and H η regime from the training/validation set are used. The top panel (a) shows m values from direct MYEGA fits (blue) and the synthetic model (red) plotted against SM. The bottom panel (b) depicts the difference $\delta m = m_{\text{MYEGA}} - m_{\text{synth}}$.

Table S3. Parameters μ and σ^2 for scaling of input data used for training/validation of the ANN.

 $\frac{\text{Parameters}}{\mu} \left[\begin{matrix} T & \text{SiO}_2 & \text{TiO}_2 & \text{Al}_2\text{O}_3 & \text{FeO} & \text{MnO} & \text{MgO} & \text{CaO} & \text{Na}_2\text{O} & \text{K}_2\text{O} & \text{P}_2\text{O}_5 & \text{Cr}_2\text{O}_3 & \text{Fe}_2\text{O}_3 & \text{H}_2\text{O} & \text{SM} & \text{K/(Na+K)} \\ \hline \sigma^2 & 0.031535 & 0.013667 & 7.55\text{E-05} & 0.001869 & 0.000335 & 7.07\text{E-07} & 0.00481 & 0.003721 & 0.001487 & 0.00103 & 1.63\text{E-06} & 9.68\text{E-08} & 0.00148 & 0.00094 & 0.01038 & 0.00745 \\ \hline \sigma^2 & 0.031535 & 0.013667 & 7.55\text{E-05} & 0.001869 & 0.000335 & 7.07\text{E-07} & 0.004981 & 0.003721 & 0.001487 & 0.0103 & 1.63\text{E-06} & 9.68\text{E-08} & 0.00148 & 0.00994 & 0.01038 & 0.047454 \\ \hline \sigma^2 & 0.031535 & 0.013667 & 0.05867 & 0.001689 & 0.00035 & 7.07\text{E-07} & 0.004981 & 0.003721 & 0.001487 & 0.0103 & 1.63\text{E-06} & 9.68\text{E-08} & 0.00148 & 0.00994 & 0.01038 & 0.047454 \\ \hline \sigma^2 & 0.031535 & 0.013667 & 0.05867 & 0.001689 & 0.000355 & 7.07\text{E-07} & 0.004981 & 0.003721 & 0.001487 & 0.01033 & 1.63\text{E-06} & 9.68\text{E-08} & 0.00148 & 0.00994 & 0.010378 & 0.047454 \\ \hline \sigma^2 & 0.031535 & 0.013667 & 0.05867 & 0.001689 & 0.000355 & 7.07\text{E-07} & 0.004981 & 0.003721 & 0.001487 & 0.01033 & 1.63\text{E-06} & 9.68\text{E-08} & 0.00148 & 0.00994 & 0.010378 & 0.047454 \\ \hline \sigma^2 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 \\ \hline \sigma^2 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 \\ \hline \sigma^2 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 \\ \hline \sigma^2 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 \\ \hline \sigma^2 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 & 0.00148 \\ \hline \sigma^2 & 0.00148$