Brewster's Incident Angle: Extension of Asymmetric Backward Peaking Radiation Pattern from a Relativistic Particle Accelerated by Lightning Leader Tip Electric Field

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November 22, 2022

Abstract

Previously the radiation patterns of combined parallel and perpendicular motions from the accelerated relativistic particle at low and high frequencies of the bremsstrahlung process with an external lightning electric field were explained. The primary outcome was that radiation patterns have four relative maxima with two forward peaking and two backward peaking lobes. The asymmetry of the radiation pattern, i.e., the different intensities of forwarding and backward peaking lobes, is caused by the Doppler effect. A novel outcome is that bremsstrahlung has an asymmetry of the four maxima around the velocity vector caused by the curvature of the particle's trajectory as it emits radiation. Previously stated bremsstrahlung asymmetry, R was an asymmetry in the radiation lobe pairs about particles velocity vector. Bremsstrahlung asymmetry used to occur at the same level in both forward radiation lobe pairs and backward radiation lobe pairs. However, in high-density mediums where the emitted wave can lag behind the speed of the particle, symmetry of the magnitude of bremsstrahlung asymmetry, R differs between forward peaking radiation lobe pairs relative to backward peaking radiation lobe pairs. This is another novel asymmetry and it causes bremsstrahlung asymmetry, R to be larger in the forward peaking compared to backward peaking radiation. The outcome is the shrink in radiation length that occurs in the backward peaking lobes. This extended work reports, changes in the radiation pattern as the emitted wave propagates through different mediums. Two novel formulas are derived from Snell's law for a particle entering the medium horizontally and from any other angle between \Pi/2 and -\Pi/2 radians. The novel outcome is the change in angle between forward peaking radiation lobe pair and backward peaking radiation lobe pair defined as bremsstrahlung angle, \theta_{brem}. When the bremsstrahlung particle crosses different mediums, change in angle between the forward and backward radiation lobe pairs, bremsstrahlung angle, \theta_{brem} breaks into its components as each lobe changes angle at different magnitudes from the particle's velocity vector. Therefore, bremsstrahlung angle, \theta_{brem} between forward-backward peaking lobes transforms into individual angles \Omega_{1}, \Omega_{2}, \Omega_{3}. \Omega_{4} all measuring from the particle's velocity vector.

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Key Points:

- Received final angle of maximum transmission for forward peaking radiation lobe after crossing different dielectric medium was found to be $\frac{1}{\sqrt{\left(\frac{\eta_2}{\eta_1}\right)^2+1}}$
- Received final angle of maximum transmission for backward peaking radiation lobe after crossing different dielectric medium was found to be $\frac{\eta_2^2}{\eta_1^2\sqrt{\left(\frac{\eta_2}{\eta_1}\right)^2+1}}$

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Abstract

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1 Introduction

Bremsstrahlung radiation patterns were predicted to be forward and backward peaking with associated novel bremsstrahlung asymmetry, R (Yücemöz & Füllekrug, 2021). In addition, the time evolution of dipole radiation pattern into forward-backward peaking was demonstrated. The reasoning for the existence of forward-backward peaking due to the collapse and separation of the lobes of the dipole radiation pattern was associated with the conservation of symmetry axes (Yücemöz & Füllekrug, 2021). Furthermore, the bremsstrahlung process of high-density mediums revealed a new outcome that symmetry of the bremsstrahlung asymmetry, R about the axis perpendicular to the direction of particle's motion, between the forward and backward peaking side is broken. Increasing refractive index causes bremsstrahlung asymmetry, R to exist at different ratios between forward and backward peaking radiation lobe pairs. Increasing refractive index increases the difference in bremsstrahlung asymmetry between the front radiation lobe pairs compared to backward radiation lobe pairs. Furthermore, increasing the refractive index shortens the radiation length in the backward peaking radiation side. This extended modelling predicts the changes in the emitted radiation pattern as the accelerating particle transits to different mediums. Novel equations 12 and 13 extended from Snell's law combining previous knowledge of the bremsstrahlung asymmetry parameter, R predicts how the symmetric bremsstrahlung angle, θ_{brem} between forward and backward peaking lobe pairs and how symmetric half bremsstrahlung angle, $\frac{\theta_{brem}}{2}$ from the particle's velocity vector within each lobe pair breaks. Novel symmetry break of bremsstrahlung angle, θ_{brem} into individual angles Ω_1, Ω_2, Ω_3, Ω_4, all describing the angle for each radiation lobe from particle's velocity vector. This novel symmetry break causes the radiation pattern to be a distorted combination of both either forward or backward peaking radiation pattern and the distorted dipole radiation pattern.

1.1 Aims & Objectives

This theoretical approach aims to extend the previous bremsstrahlung model to different mediums. Previously, the bremsstrahlung electron was propagating inside a vacuum. This report puts bremsstrahlung electron into crossing different mediums and mathematically models changes in the radiation pattern as a result of wave refraction by extending Snell's law for the waves that have larger wavelengths than the particles present in that specific medium. Therefore, mediums where wave refraction dominates wave scattering.

2 Brewster's Law for Forward-Backward Peaking Radiation Pattern

Considering forward peaking part of the overall radiation pattern. Radiation intensity, I of top lobe (I_T) can be related to the bottom lobe (I_B) with bremsstrahlung asymmetry,R mathematically by

$$I_T = \frac{I_B}{(1 - R)} \tag{1}$$

This information can be used to find the bremsstrahlung angle, θ_{Brem} between the two forward peaking radiation lobes that are bremsstrahlung asymmetric, with asymmetry value R.

By knowing the radiation length difference or in other terms, linear distance in the form of radiation intensity, Ξ $[Js^{-1}]$ between the maximum points of two radiation intensities, I in the forward peaking lobes, bremsstrahlung angle between the two forward peaking lobes can be written as,

$$cos(\theta_{Brem}) = \frac{I_T^2 + I_B^2 - \Xi^2}{2 I_T I_B} \tag{2}$$

The equation 1 and 2 can also be used for backward peaking radiation lobes.

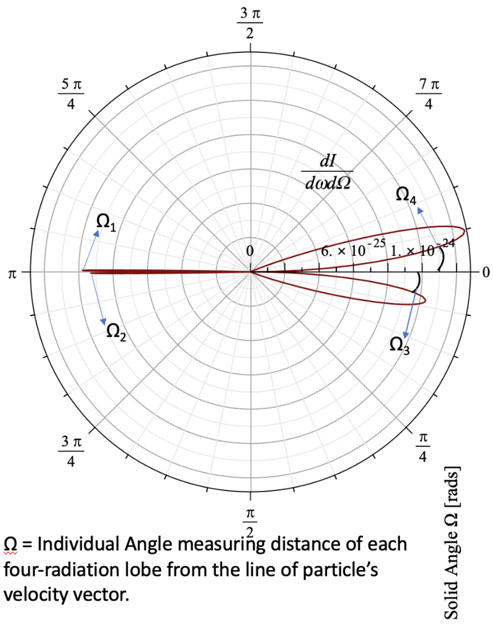

Figure 1. The Radiation patterns are emitted by the anti-clockwise rotating charged particle - bremsstrahlung process. High density (particle travelling inside the water) medium causes a novel asymmetry about a line perpendicular to the direction of motion of a particle. This novel asymmetry causes bremsstrahlung asymmetry, R to occur at different proportions in forward and backward peaking lobe pairs. Therefore, bremsstrahlung asymmetry, R is higher in forward peaking radiation and lower in backward peaking radiation. Moreover, a high-density medium also causes radiation length to shorten in backward peaking radiation. Plot is in Polar co-ordinates. Horizontal axis gives the radiation intensity per Solid angle, Ω, per emitted angular radiation frequency, ω. In addition, angle of the Polar plot is the Solid angle, Ω. The values used for plotting are: mean free time τ= 30 μs, number of charges z= 1, a=100 μm, b= 1 nm (a and b are related to mean free path), s_{ft}= 1, s_f= 1, $S_{SpecialR}$ = 1, velocity-time scaling factor s_{ftv}= 1×10^9 and velocity scaling factor s_{fv}= 8.19 $\times10^{-11}$. Finally, the bremsstrahlung asymmetry is R= 1/8. In addition, $\frac{1}{9} \leq R \leq \frac{1}{3}$., medium conductivity, σ = 0.005, relative permeability, μ_r = 0.99

3 Application of Snell's Law on Bremsstrahlung Asymmetric Radiation Passing Through a Medium with Positive Refractive Index

3.1 Two different Case Studies

3.1.1 Horizontal Incoming Particle

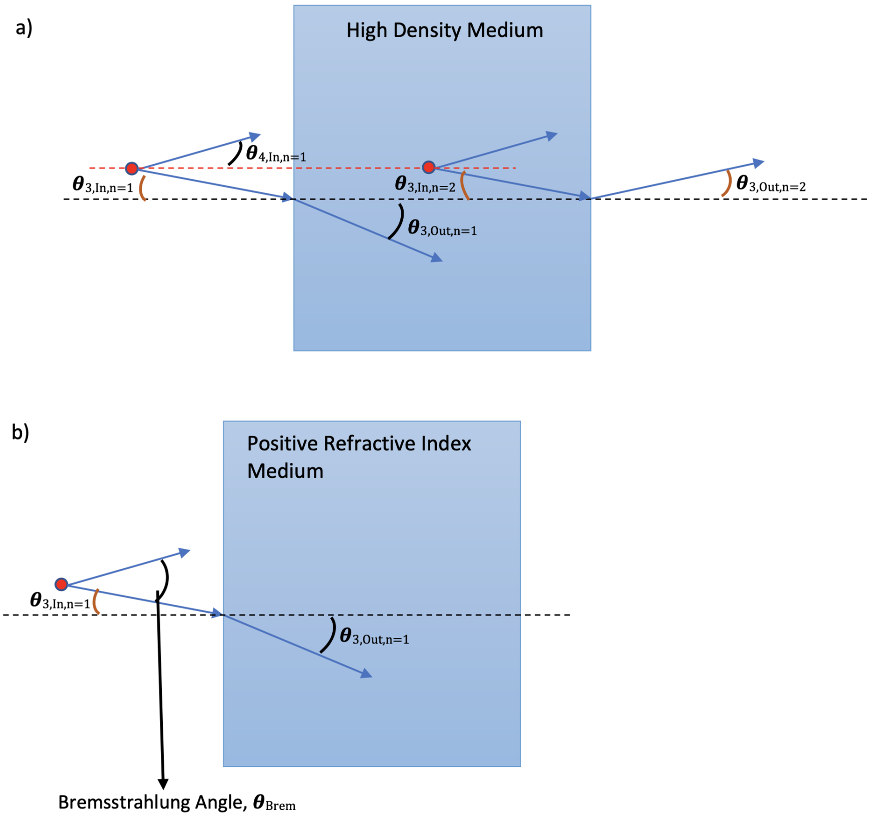

Figure 2. Case study A: the medium is high density such that the bremsstrahlung electron catches its own radiation and continues to emit radiation in the same medium. In this case, the bremsstrahlung electron could gain energy from its own radiation due to a curved trajectory that increases the likelihood of interaction with its own radiation. Case study B: the medium is not high density, the particle still lags its own emitted electromagnetic wave speed. In both cases bremsstrahlung angle, θ_{brem} between two forward peaking lobes increases. This is because of the bremsstrahlung asymmetry, R which changes the radiation intensity, I. Hence, causing a frequency difference between two forward peaking lobes which in turn causes the low-frequency wave to diffract more than the high-frequency lobe. More importantly, as the bremsstrahlung electron crosses more different mediums, the bremsstrahlung angle, θ_{brem} continues to grow such that eventually, the whole radiation pattern can transform back to a dipole radiation pattern. Extremely high frequencies such as X-rays diffract only by a tiny angle. However, forward peaking radiation patterns can start as early as in the MHz frequency range and peak in forward direction as the frequency increases.

When accelerated relativistic bremsstrahlung electron with bremsstrahlung asymmetric "R" forward-backward peaking radiation pattern enters into a medium with a refractive index of η_2 from a medium with refractive index η_1, the final direction of the emitted radiation in the final medium of η_2 can be predicted using Snell's law in combination with bremsstrahlung asymmetry, R.

$$sin(\Omega_{4,Out,n=1}) = \frac{sin(\Omega_{4,In,n=1})\eta_1}{\eta_2} \tag{3}$$

$$sin(\Omega_{3,Out,n=1}) = \frac{sin(\Omega_{3,In,n=1})\eta_1}{\eta_2} \tag{4}$$

Introducing the bremsstrahlung asymmetry, R parameter into equations 3 and 4.

From equation 1, angle between two forward peaking lobes can be written as,

$$\Omega_{4,In,n=1} + \Omega_{3,In,n=1} = \theta_{Brem} = cos^{-1}\left[\frac{I_T^2 + I_B^2 - \Xi^2}{2I_T I_B}\right] \tag{5}$$

Hence,

$$sin(\Omega_{4,Out,n=1}) = \frac{sin\left(cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_T I_B}\right] - \Omega_{3,In,n=1}\right)\eta_1}{\eta_2} \tag{6}$$

$$sin(\Omega_{3,Out,n=1}) = \frac{sin\left(cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_T I_B}\right] - \Omega_{4,In,n=1}\right)\eta_1}{\eta_2} \tag{7}$$

As the emitted radiation could go through many different mediums until it is detected by the detectors, to find the final direction of the emitted radiation through all of its journey from source particle to receivers, equations 6 and 7 can be written in the form of series. For the detection after the "n^{th}" medium refractive index

$$sin(\Omega_{TopLobeakaNo.4,Out,nth}) = \sum_{n=1}^{\infty} \frac{sin\left(cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_T I_B}\right] - \Omega_{Top,In,1+n}\right)\eta_n}{\eta_{1+n}} \tag{8}$$

$$sin(\Omega_{BottomLobeakaNo.3,Out,nth}) = \sum_{n=1}^{\infty} \frac{sin\left(cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_T I_B}\right] - \Omega_{Bottom,In,n}\right)\eta_n}{\eta_{1+n}} \tag{9}$$

3.1.2 Incoming Particle at an Angle

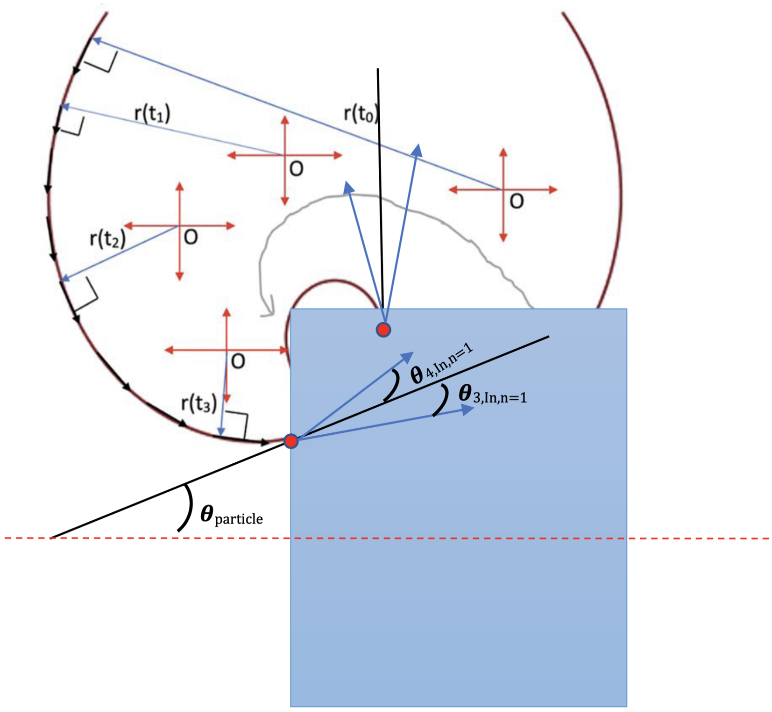

Figure 3. Accelerating bremsstrahlung electron following curved spiral trajectory transiting into a new medium coloured in blue. Orientation of the bremsstrahlung electron from the normal line (Horizontal line) of the new medium is described by the angle $\theta_{particle}$. When radiation is first emitted from the bremsstrahlung particle, each forward peaking lobe in the pair is symmetric about the particle's velocity vector, hence each forward lobe has an equal distance away from the particle's velocity vector. Hence, $\Omega_{3,In,n=1} = \frac{\theta_{Brem}}{2}$ and $\Omega_{4,In,n=1} = \frac{\theta_{Brem}}{2}$ from the particle's tangent velocity vector. Therefore, once the $\theta_{particle}$ is known, $\Omega_{3,In,n=1}$ and $\Omega_{4,In,n=1}$ are just $\mp\frac{\theta_{Brem}}{2}$ away from the $\theta_{particle}$. This symmetry of the angle of each lobe from the particle's velocity vector is broken once the radiation crosses from one medium into another. This is because of asymmetric refraction that occurs due to asymmetric radiation intensity caused by the bremsstrahlung asymmetry, R, and the curved trajectory.

Slope of the line tangent to the particle's spiral trajectory given by

$$\frac{dy}{dx} = \frac{r(t)^{'}cos(t) + r(t)^{''}sin(t)}{-r(t)^{'}sin(t) + r(t)^{''}cos(t)} \tag{10}$$

Angle, $\theta_{particle}$ of an incident electron into a medium with respect to the horizontal line is,

$$\theta_{particle} = arctan\left(\frac{r(t)^{'}sin(t)}{r(t)^{'}cos(t)}\right) \tag{11}$$

$r(t)^{'}$ is particle's velocity vector and is given by, $\frac{dr}{dt} = \frac{b^R(\omega^{'})^R cos(\theta_{n,r(t)})^R c}{c^R\omega^{'}cos(\theta_{n,r(t)})}$

$\left(\frac{\tau^{2R}\left[t^{2R}\frac{2R}{t}+t^{2R}2R'ln(t)\right]-t^{2R}\left[\tau^{2R}\frac{2R}{\tau}+\tau^{2R}2R'ln(\tau)\right]}{\tau^{4R}}\right)-\frac{a\tau-at\tau'}{\tau^2}$ (Yücemöz & Füllekrug, 2021, Eq. 3)

The bremsstrahlung particle, hence the emitted radiation enters different mediums at different angles following a spiral trajectory, bremsstrahlung asymmetry, R and Doppler effect cause differences in frequency for each radiation lobe. Therefore, Each radiation lobe would have distinctive incoming $\Omega_{In,n=1}$ and exit $\Omega_{Out,n=1}$ angles. Incoming angles are given formulated in equations 12, 13, 14, and 15. For backward peaking waves, incoming angles to the different mediums are given in equations 14 and 15. In addition, the forward peaking radiation lobe with angle number 4 has the same entry angle as the backward peaking radiation lobe with angle number two. However, the radiation of angle number four and angle number two propagates in the opposite direction to each other. Similar is also true for the forward peaking radiation lobe with angle number three and backward peaking radiation angle number one. All these peaking radiation lobes with associated angle numbers are shown in Figure 4.

$$\Omega_{4,In,n=1} = \theta_{particle} + \frac{\theta_{Brem}}{2} = arctan\left(\frac{r(t)'\,sin(t)}{r(t)'\,cos(t)}\right) + \frac{cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_TI_B}\right]}{2} \tag{12}$$

$$\Omega_{3,In,n=1} = \theta_{particle} - \frac{\theta_{Brem}}{2} = arctan\left(\frac{r(t)'\,sin(t)}{r(t)'\,cos(t)}\right) - \frac{cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_TI_B}\right]}{2} \tag{13}$$

Bringing back the backward peaking radiation pattern,

$$\Omega_{2,In,n=1} = \Omega_{4,In,n=1} \tag{14}$$

and

$$\Omega_{1,In,n=1} = \Omega_{3,In,n=1} \tag{15}$$

At the point of the boundary of the medium transition region, the bremsstrahlung angle, θ_{brem} changes exactly the opposite but at the same rate and magnitude between pairs. Hence, if the bremsstrahlung angle, θ_{brem} increases between the forward peaking pairs it would decrease between backward pairs as the radiation crosses between the same mediums but in the opposite order (i.e one from Medium A to B other from medium B to A).

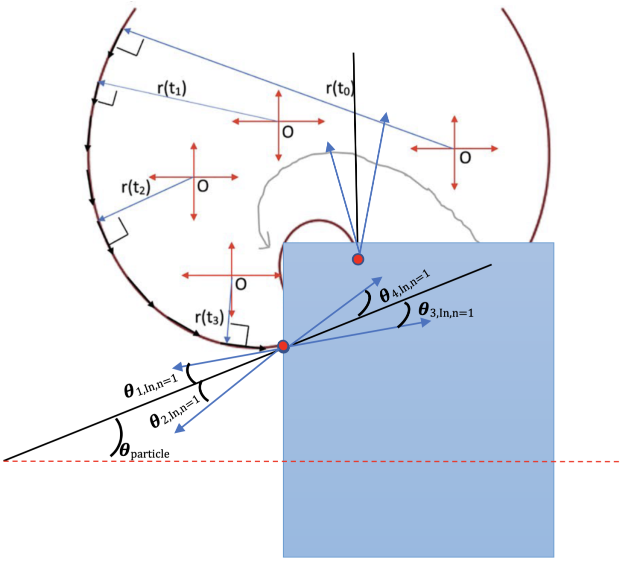

Figure 4. In addition to the Information presented in figure 3, This figure brings back the backward peaking radiation pattern into the equation. As can be seen, backward peaking radiation lobes propagate in the exact opposite directions compared to the forward peaking radiation lobes when radiation is first emitted from the particle. Backward peaking lobes have lower radiation intensity, hence, lower frequency and higher wavelengths due to the Doppler effect. This causes backward peaking radiation lobes to refract less than forward peaking radiation lobes. Which introduces another asymmetry about the line perpendicular to the particle's velocity vector and distortion in the overall radiation pattern.

By substituting equations 12, 13, 14 and 15 into equations 3 and 4, final exit angle can now be written for the forward peaking as follows

$$sin(\Omega_{4,Out,n=1}) = \frac{sin\left(arctan\left(\frac{r(t)'sin(t)}{r(t)'cos(t)}\right) + \frac{cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_TI_B}\right]}{2}\right)\eta_1}{\eta_2} \tag{16}$$

$$sin(\Omega_{3,Out,n=1}) = \frac{sin\left(arctan\left(\frac{r(t)'sin(t)}{r(t)'cos(t)}\right) - \frac{cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_TI_B}\right]}{2}\right)\eta_1}{\eta_2} \tag{17}$$

and exit angle for the backward peaking radiation lobes as follows

$$sin(\Omega_{2,Out,n=1}) = \frac{sin\left(arctan\left(\frac{r(t)'sin(t)}{r(t)'cos(t)}\right) + \frac{cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_TI_B}\right]}{2}\right)\eta_2}{\eta_1} \tag{18}$$

$$sin(\Omega_{1,Out,n=1}) = \frac{sin\left(arctan\left(\frac{r(t)'sin(t)}{r(t)'cos(t)}\right) - \frac{cos^{-1}\left[\frac{I_T^2+I_B^2-\Xi^2}{2I_TI_B}\right]}{2}\right)\eta_2}{\eta_1} \tag{19}$$

The Refractive index for the refraction of backward peaking radiation is inverted as the radiation propagates in opposite direction and out of every new medium the bremsstrahlung electron gets into. $n = 1$ means bremsstrahlung electron only crossed into one different medium. $n = nth$ meaning particle crossed nth different mediums.

To predict radiation refraction right from the particle, we only need equations 12 and 13 with only one medium property. However, to track radiation pattern crossing from different mediums over time since its first emission from the particle at medium $n = 1$, the equation needs to be used with multiple $n = nth$ mediums.

Moreover, for wave tracking from medium $n = 1$, until the $n = nth$ medium, an important step is to calculate the wave refraction pattern at medium $n = 1$ considering all asymmetries, Doppler effect, the Bremsstrahlung asymmetry in the travelling wave pattern. Once this information is known, this would be the starting wave incident direction for the second medium. After the calculations at the $n = 1$ medium with equations 16, 17, 18, and 19, until the $n = nth$ medium only Snell's law needs to be used which is given in equations 3 and 4 for the forward peaking radiation pattern. This is also true for refracted backward peaking radiation patterns where the only difference in the equation is the inverted refractive index as the radiation propagates in the opposite direction with respect to the forward peaking lobes.

4 Expected Radiation Patterns as a result of Wave Medium Crossing

When bremsstrahlung electron emits radiation patterns while transiting from one medium to another medium, the radiation pattern of the forward and backward side of the particle refracts in exactly the opposite directions. Therefore, if bremsstrahlung angle, θ_{brem} between forward peaking radiation lobes increases where bremsstrahlung angle, θ_{brem} separates into its components Ω_3, Ω_4 and $\Omega_3 \neq \Omega_4$. In addition, $\Omega_4 > \Omega_3$ as radiation lobe that is described by the angle Ω_4 in bremsstrahlung asymmetric, R hence it has a lower frequency, therefore higher wavelength that makes it refract more compared to its other forward peaking pair described by the angle Ω_3. The bremsstrahlung angle, θ_{brem} between backward peaking lobes should decrease as opposed to the increase in bremsstrahlung angle, θ_{brem} between forward peaking radiation lobes. This is because backward peaking radiation lobes are propagating in the opposite direction outside of the medium whereas, forward peaking lobes propagate into the new medium.

Overall, as sketched in figure 5, when forward peaking lobes tend towards forming a distorted dipole pattern because of $\Omega_3 \neq \Omega_4$. Backward peaking lobes tends to form distorted further backward peaking as a result of the decrease in bremsstrahlung angle, θ_{brem} where components that make bremsstrahlung angle, θ_{brem} are again not equal to each other. Hence, $\Omega_1 \neq \Omega_2$. Similarly, $\Omega_1 > \Omega_2$ because it has a larger wavelength and hence refracts at a larger angle.

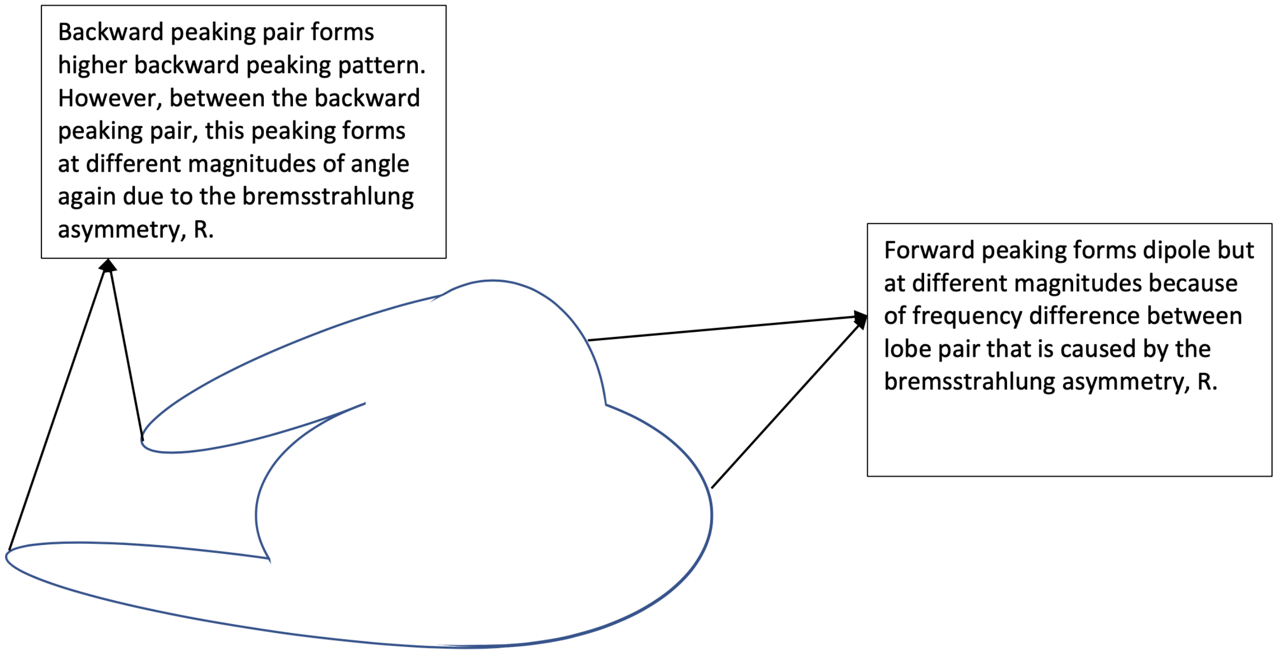

Figure 5. Sketch to show the expected overall refracted radiation pattern of the bremsstrahlung electron after transiting through one different medium. This sketch presents new generic information about the refracted radiation pattern that is expected to apply at all times for different mediums. This is not the wave track but rather important initial information required for the wave tracking. As can be seen, it is expected that forward and backward peaking radiation lobes should behave in opposite directions. So, when one leads toward dipole radiation other pair should lead more towards the direction of the particle's velocity vector (peaking). In this sketch, forward peaking tends towards a distorted dipole radiation pattern and backward lobes tend towards more peaking in the direction of the particle's velocity vector. Opposite tendencies are the result of pair lobes moving in the opposite direction where one enters into the new medium while the other lobe pair exits the new medium. Moreover, the distorted dipole tendency and backward distorted peaking tendencies are result novel symmetry break of bremsstrahlung angle, θ_{brem} into individual different angles Ω_1, Ω_2, Ω_3, Ω_4 where, $\Omega_3 + \Omega_4 = \theta_{brem-BetweenForwardPair} \neq \Omega_1 + \Omega_2 = \theta_{brem-BetweenBackwardPair}$. The cause of this new novel symmetry break of bremsstrahlung angle, θ_{brem} is the bremsstrahlung asymmetry parameter, R, curved trajectory, and the Doppler effect.

5 Transmission Angle for Incident Brewster's Angle

This section assumes emitted waves entering different medium at special incident Brewster's angle. For unpolarised wave, at this angle some of the radiation will refract (transmitted) and some part of the radiation will reflect. This incident angle can only be true for one of the forward peaking lobes and one of the backward peaking lobes as both forward peaking pair and backward peaking pair have angular difference in between them.

Assuming one of the forward peaking lobe with Ω_4 is incident at Brewster's angle. Hence, equation 16 can be rewritten as:

$$sin(\Omega_{4,Out,n=1}) = \frac{sin\left(tan^{-1}(\frac{\eta_2}{\eta_1})\right)\eta_1}{\eta_2} = \frac{\eta_2}{\eta_1\sqrt{\left(\frac{\eta_2}{\eta_1}\right)^2+1}}\frac{\eta_1}{\eta_2} = \frac{1}{\sqrt{\left(\frac{\eta_2}{\eta_1}\right)^2+1}} \tag{20}$$

Hence for the equivalent backward peaking lobe, $\Omega_{2,In,n=1}$ with Brewster's exit angle, the output angle, $\Omega_{2,In,n=1}$ is,

$$sin(\Omega_{2,Out,n=1}) = \frac{\eta_2}{\eta_1\sqrt{\left(\frac{\eta_2}{\eta_1}\right)^2+1}}\frac{\eta_2}{\eta_1} = \frac{\eta_2^2}{\eta_1^2\sqrt{\left(\frac{\eta_2}{\eta_1}\right)^2+1}} \tag{21}$$

At this incident angle, hence the final transmission angles for the forward and backward peaking radiation lobes transmission should be at maximum. Difference in final transmission angles of forward and backward peaking radiation lobes is due to their opposite direction of propagation. When forward peaking radiation lobes enter a new medium, backward peaking radiation lobes exit a new medium.

Equations 20 and 21 can mean more than just the ratio of refractive indexes of two different mediums. It means a lot in terms of the bremsstrahlung process and bremsstrahlung asymmetry, R. As a result of the bremsstrahlung process and asymmetry, no two forward-backward peaking radiation can enter and exit the new medium at the same time. This allows us to write refractive indices as the instantaneous wave speed of each peaking radiation lobe at the boundary as $\eta_1 = cv_T^{-1}$ and $\eta_2 = cv_B^{-1}$ for anti-clockwise rotating particle. cv_T^{-1} is the velocity of the top radiation lobe.

This approach allows to bring bremsstrahlung asymmetry, R parameter into the transmission equations 20 and 21. This in turn allows to predict how bremsstrahlung asymmetry, R as a result of curved trajectory can affect the overall wave pattern as the wave transmits through different mediums.

Assuming same, $v_T = \frac{v_B}{(1-R)}$ relationship between radiation intensities of top and bottom forward peaking lobes apply in terms of wave velocities of top and bottom lobes. Equation 20 can be re-written as:

$$sin(\Omega_{4,Out,n=1}) = \frac{1}{\sqrt{\left(\frac{\eta_2}{\eta_1}\right)^2+1}} = \frac{1}{\sqrt{\left(\frac{v_T}{v_B}\right)^2+1}} = \frac{1}{\sqrt{\left(\frac{1}{(1-R)}\right)^2+1}} \tag{22}$$

and for the the output angle, $\Omega_{2,Out,n=1}$

$$sin(\Omega_{2,Out,n=1}) = \frac{1}{(1-R)^2\sqrt{\left(\frac{1}{(1-R)}\right)^2 + 1}} \quad (23)$$

6 Summary

When the bremsstrahlung particle was in a vacuum, peaking lobes were all equally away from the particle's velocity vector by half of the bremsstrahlung angle, $\frac{\theta_{brem}}{2}$. When bremsstrahlung particle transits between multiple different mediums, peaking lobes are all at different angles away from the particle's velocity vector where each is defined as, Ω_1, Ω_2, Ω_3. Ω_4 from the particle's velocity vector. This is a novel bremsstrahlung angle, θ_{brem} symmetry break. They are defined in equations 12 and 13 that apply to all cases.

Most importantly, the whole radiation emission process should be divided into frames where each frame would represent the emission of radiation for each tangential point of the particle's trajectory. Therefore, even though the emitted radiation would be distorted and would have different angles from the particle's velocity vector, all are described by the exit angles given in equations 16, 17, 18, and 19. Because of the frame perspective, the whole process restarts again with new input angles given in equations rather than using the output angles of the distorted radiation pattern from the previous frame as the new input angles for the next frame.

Finally, from the example in this report, forward peaking tends toward a distorted dipole radiation pattern and backward lobes tend more peaking in the direction of the particle's velocity vector. The opposite tendencies should always be the case, however, it is medium and particle trajectory that defines whether either forward or backward peaking lobes would tend towards refracted dipole radiation pattern or peak more in the direction of the particle's velocity vector.

Acknowledgments

EPSRC and MetOffice sponsor my PhD project under contract numbers EG-EE1239 and EG-EE1077. I would like to thank a lot to my family for their support and good wishes. The Maple worksheets used to simulate the particle trajectory, external lightning leader tip electric field, particle velocity, bremsstrahlung radiation pattern in high density medium and the radiation patterns are openly available from the University of Bath Research Data Archive.

References

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