

Characterizing Continental US Hurricane Risk: Which Intensity Metric is Best?

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Abstract

The damage potential of a hurricane is widely considered to depend more strongly on an integrated measure of the hurricane wind field, such as Integrated Kinetic Energy (IKE), than a point-based wind measure, such as maximum sustained wind speed (V_{\max}). Recent work has demonstrated that minimum sea level pressure (MSLP) is also an integrated measure of the wind field. This study investigates how well historical continental US hurricane damage is predicted by MSLP compared to both V_{\max} and IKE for continental United States hurricane landfalls for the period 1988–2020. We first show for the entire North Atlantic basin that MSLP is much better correlated with IKE ($r_{\text{rank}} = 0.50$) than V_{\max} ($r_{\text{rank}} = 0.26$). We then show that continental US hurricane normalized damage is better predicted by MSLP ($r_{\text{rank}} = 0.81$) than either V_{\max} ($r_{\text{rank}} = 0.65$) or IKE ($r_{\text{rank}} = 0.68$). For Georgia to Maine hurricane landfalls specifically, MSLP and IKE show similar levels of skill at predicting damage, whereas V_{\max} provides effectively no predictive power. Conclusions for IKE extend to power dissipation as well, as the two quantities are highly correlated because wind radii closely follow a Rankine vortex. The physical relationship of MSLP to IKE and power dissipation is discussed. In addition to better representing damage, MSLP is also much easier to measure via aircraft or surface observations than either V_{\max} or IKE, and it is already routinely estimated operationally. We conclude that MSLP is an ideal metric for characterizing hurricane damage risk.

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Key Points:

- Minimum sea level pressure better predicts continental US hurricane damage than maximum winds or integrated kinetic energy.
- Maximum winds have historically been a poor predictor of damage caused by hurricanes making landfall from Georgia to Maine.
- Minimum sea level pressure is intrinsically an integrated wind field metric and is easy to measure, ideal for categorizing hurricane risk.

Abstract

The damage potential of a hurricane is widely considered to depend more strongly on an integrated measure of the hurricane wind field, such as Integrated Kinetic Energy (IKE), than a point-based wind measure, such as maximum sustained wind speed (V_{\max}). Recent work has demonstrated that minimum sea level pressure (MSLP) is also an integrated measure of the wind field. This study investigates how well historical continental US hurricane damage is predicted by MSLP compared to both V_{\max} and IKE for continental United States hurricane landfalls for the period 1988–2020. We first show for the entire North Atlantic basin that MSLP is much better correlated with IKE ($r_{\text{rank}} = 0.50$) than V_{\max} ($r_{\text{rank}} = 0.26$). We then show that continental US hurricane normalized damage is better predicted by MSLP ($r_{\text{rank}} = 0.81$) than either V_{\max} ($r_{\text{rank}} = 0.65$) or IKE ($r_{\text{rank}} = 0.68$). For Georgia to Maine hurricane landfalls specifically, MSLP and IKE show similar levels of skill at predicting damage, whereas V_{\max} provides effectively no predictive power. Conclusions for IKE extend to power dissipation as well, as the two quantities are highly correlated because wind radii closely follow a Modified Rankine vortex. The physical relationship of MSLP to IKE and power dissipation is discussed. In addition to better representing damage, MSLP is also much easier to measure via aircraft or surface observations than either V_{\max} or IKE, and it is already routinely estimated operationally. We conclude that MSLP is an ideal metric for characterizing hurricane damage risk.

Plain Language Summary

For decades, maximum sustained winds have been used to categorize potential hurricane impacts. Recent work argues that an integrated hurricane wind field measure better represents risk. Here we use historical continental U.S. hurricane and economic damage data to show that minimum sea level pressure better correlates with damage than integrated kinetic energy, a measure of hurricane vortex size and strength, or maximum sustained wind. Maximum sustained wind has been a poor damage predictor for Georgia to Maine landfalling hurricanes. Since minimum central pressure is an integrated wind field measure that only requires storm center measurements, and is already routinely estimated, we propose that minimum sea level pressure replace maximum sustained wind as the primary hurricane categorization method.

1 Introduction

Hurricanes are one of the most damaging natural catastrophes, causing hundreds to thousands of fatalities and billions of US dollars (USD) in damage globally each year (Mendelsohn et al., 2012; Klotzbach et al., 2018; Grinsted et al., 2019). Damage from hurricanes has grown in recent years, with a primary driver being an increase in population and wealth along the coast. Given the large impacts that hurricanes cause, ideally their intensity should be categorized using metrics that best represent their potential impacts when communicating risk to the public.

For more than 40 years, North Atlantic (hereafter Atlantic) and eastern North Pacific hurricanes have been categorized using the Saffir–Simpson Hurricane Scale (Simpson, 1974), although the utility of this scale has been called into question during the past ~15 years. In 2010, the National Hurricane Center removed storm surge and minimum sea level pressure (MSLP) from the scale, resulting in the modified Saffir–Simpson Hurricane Wind Scale (SSHWS; Schott et al., 2012), which categorizes hurricanes purely based on maximum sustained wind (V_{\max}).

Powell and Reinhold (2007) advocated for an integrated kinetic energy (IKE) metric to categorize wind potential destruction from hurricanes. Many follow-up studies have also used IKE to categorize both individual hurricanes as well as entire hurricane seasons (e.g., Maclay et al., 2008; Misra et al., 2013; Kozar et al., 2014; Buchanan et al., 2018). Unlike V_{\max} , which simply represents a point-based estimate of the maximum sustained winds in a hurricane, IKE assesses the strength of the overall hurricane circulation. For a given V_{\max} , larger storms typically have increased storm surge (Irish et al., 2008) and larger wind and rainfall footprints (Lonfat et al., 2007).

Chavas et al. (2017) demonstrated that MSLP also intrinsically represents an integrated measure of the wind field that captures the combined effect of V_{\max} and storm size. Specifically, the relationship between the hurricane’s central pressure deficit (e.g., the difference in pressure between the center of the hurricane and the surrounding environment) and V_{\max} can be understood through gradient wind balance. The central pressure deficit increases predominantly with increasing V_{\max} (the canonical “wind–pressure relationship”; Knaff and Zehr (2007)) but also with increasing storm size as well as background rotation rate. Hence, MSLP ought to be more similar to an IKE-type metric than V_{\max} .

Klotzbach et al. (2020) showed that MSLP had a statistically significant improvement in correlation with normalized continental US (CONUS) landfalling hurricane damage (Weinkle et al., 2018) relative to V_{\max} from 1900–2018 as well as direct fatalities from 1988–2018. In addition to CONUS landfalling hurricane damage, they also found a stronger relationship between MSLP and a hurricane’s average 34-kt wind radii at landfall, providing additional verification of Chavas et al. (2017)’s study and further evidence that MSLP may be more similar to IKE than V_{\max} . To date, though, a full comparison of the utility of MSLP, V_{\max} , and IKE at predicting historical damage has yet to be undertaken.

The purpose of this manuscript is to examine how well MSLP predicts historical damage as compared to V_{\max} and IKE for CONUS landfalling hurricanes. We first compare the three metrics for all Atlantic hurricanes, then likely well-monitored hurricanes in the southwestern portion of the basin and then lastly for CONUS landfalling hurricanes. We then compare how well each quantity predicts historical damage both overall and for Texas to Florida vs. Georgia to

Maine events. We also discuss the physical relationship between V_{\max} , MSLP, IKE and power dissipation (PD; Bister & Emanuel, 1998; Emanuel, 1999).

2 Data and Methodology

The primary dataset for the analysis that follows is the Extended Best Track (Demuth et al., 2006) that consists of intensity, location and various wind radii measurements. The location and intensity information in the extended best track are the same as in HURDAT2 (Landsea & Franklin, 2013) - NOAA's official Atlantic hurricane database. The Extended Best Track also provides 34-kt, 50-kt, and 64-kt wind radii as well as the radius of maximum winds at 6-hourly temporal resolution since 1988. Wind radii from 1988–2003 in the Extended Best Track are from operational estimates, while the National Hurricane Center has best-tracked wind radii since 2004. Here we investigate the relationship between MSLP, V_{\max} and IKE in both the Extended Best Track for all Atlantic hurricanes, hurricanes in the southwest Atlantic that were likely well measured, as well as CONUS landfalling hurricanes specifically, from 1988–2020.

The southwest Atlantic hurricane dataset is classified using the following criteria from Chavas and Knaff (2022):

- 1) Take only hurricanes from 2004 onwards, as wind radii have been best tracked by the National Hurricane Center since that time
- 2) Select only hurricane positions where the center was located at or south of 30°N, to reduce any signal from extratropical transition
- 3) Take only hurricanes where the center was located at or west of 50°W, since these storms are more likely to have been observed by aircraft reconnaissance
- 4) Remove any hurricane locations whose distance to land is less than its mean R_{34kt} value, to reduce potential land interaction impacts on wind radii

Continental US landfalling hurricane MSLP and V_{\max} are taken from the Atlantic Oceanographic and Meteorological Laboratory website:

https://www.aoml.noaa.gov/hrd/hurdat/UShurrs_detailed.html that is based on HURDAT2. As was done in Klotzbach et al. (2020), we do count Sandy (2012) as a hurricane landfall, since it brought severe damage to the mid-Atlantic states and was a hurricane until just a few hours before landfall when it became extratropical.

Normalized damage estimates, that is, the amount of damage a hurricane would likely cause if it were to make landfall today given inflation and changes in exposure, are taken from Weinkle et al. (2018) for hurricane landfalls from 1988–2017, while damage estimates for the ten CONUS landfalling hurricanes from 2018–2020 are taken from the National Hurricane Center best track reports on these storms (<https://www.nhc.noaa.gov/data/tcr/>). Normalized damage estimates from Weinkle et al. (2018) are provided in 2018 USD, while damage estimates from the hurricane landfalls of 2018–2020 are listed in USD of the year that they made landfall. Changes in inflation, population and exposure should be relatively minor factors from 2018–2020.

Multiple landfalls from the same hurricane are identified if there were two separate damage estimates recorded in the Weinkle et al. (2018) dataset. From 1988–2017, three hurricanes were recorded with two separate damage estimates: Andrew (1992), Erin (1995), and Georges (1998). The results would not change significantly if only one landfall per storm were considered. None of the ten CONUS landfalling hurricanes in 2018–2020 made multiple landfalls, defined in Klotzbach et al. (2018) and here to be two separate CONUS hurricane landfalls with at least 100 miles of open ocean between landfalls.

Integrated kinetic energy is defined as:

$$IKE = \int_0^{2\pi} \int_0^{r_0} \frac{1}{2} \rho h V^2 r dr d\theta \quad (1)$$

where r is radius, V is total wind speed, ρ is near-surface air density, and h is a fluid depth. The latter two may be assumed constant and so are not important for our analysis. We estimate IKE following the methodology of Misra et al. (2013), which sets $\rho=1 \text{ kg m}^{-3}$ and $h=1 \text{ m}$ and then uses the estimates of the radius of maximum wind (R_{max}) and the four quadrant estimates of the radius of 34-kt wind ($R_{34\text{kt}}$), radius of 50-kt wind ($R_{50\text{kt}}$) and radius of 64-kt wind ($R_{64\text{kt}}$) from the Extended Best Track. The method calculates the area within each quadrant between each pair of adjacent wind radii and uses a representative wind speed between the bounding wind speeds. IKE is then summed across all quadrant sub-regions. The algorithm is summarized in Table S1, which is identical to Table A1 of Misra et al. (2013), with one minor modification to clarify the criteria within the hurricane-force wind region (Misra personal communication. 2021–06–23). Approximately 1% of 6-hourly periods in the extended best track are excluded (all prior to 2003) either due to lack of radius of maximum winds or 34-kt wind radii which is necessary to calculate IKE.

Integrated kinetic energy at landfall was calculated as the IKE at the six-hourly period between 12–18 hours prior to landfall, since the wind radii necessary to calculate IKE are only given at six-hourly intervals recorded in the best track (e.g., 0, 6, 12, 18 UTC). Integrated kinetic energy at this time period had slightly higher correlations with V_{max} , MSLP and normalized damage than adjacent six-hour periods. As a hurricane gets closer to landfall, the outer circulation of the storm is already on land, likely causing deformation of the hurricane wind field. If different time periods were used to calculate landfalling IKE, the results would only change slightly.

We also compare results using IKE to those using power dissipation (PD; Bister & Emanuel, 1998). Power dissipation scales identically with IKE except with the wind speed cubed rather than squared, and is given by:

$$PD = \int_0^{2\pi} \int_0^{r_0} \rho C_d V^3 r dr d\theta \quad (2)$$

where ρ is near-surface air density and C_d is the surface drag coefficient, each of which may be taken as approximately constant and so are not important for our analysis. Here we set $\rho = 1 \text{ kg/m}^3$ and $C_d = 10^{-3}$. We calculate PD following the same methodology as IKE above, but cubing rather than squaring the wind speed.

Rank correlations (r_{rank}) are used as the predominant agreement metric between time series throughout the manuscript, in order to remove the influence of large outlying events (e.g., Katrina for normalized damage or Sandy for IKE). Higher ranks are defined to be higher V_{max} , lower MSLP (e.g., deeper storms), higher IKE and increased damage. We find that MSLP is a consistently better predictor of historical damage than both V_{max} and IKE, and we discuss the implications of this result given that MSLP is inherently an integrated measure of the wind field whose estimation is straightforward and already routinely measured.

Statistical significance is primarily calculated using bootstrap resampling methods and is reported at the 5% level (Efron, 1979; Hesterberg et al., 2003). Statistical significance of correlations are calculated by resampling with replacement 1000 times from the dataset being investigated. If fewer than 5% of the randomly resampled correlations are less than zero, the correlation is said to be significant. Statistical significance of correlation differences is calculated using the Fisher r to z transformation and accounting for the correlation between the two time series (Lee & Preacher, 2013).

3 Relationships between V_{max} , MSLP and IKE

3.1 Full Atlantic basin

We begin by investigating the relationship between MSLP, V_{max} , and IKE for all Atlantic hurricanes from 1988–2020 and find that IKE covaries strongly with MSLP but not V_{max} . Overall, for all Atlantic hurricanes, the correlation between MSLP and IKE is significantly stronger ($r_{rank} = 0.50$) than between V_{max} and IKE ($r_{rank} = 0.26$).

We visualize this closer relationship between MSLP and IKE for both Category 1–2 hurricanes and major (Category 3–5) hurricanes in Figure 1. Figure 1a displays a boxplot of IKE for the approximate quartiles of V_{max} for Atlantic hurricanes¹ classified as Category 1–2 based on MSLP, using the Klotzbach et al. (2020) definition (e.g., >960 hPa). There is no systematic variation in IKE across quartiles of V_{max} (Figure 1a), indicating that V_{max} provides little additional information about IKE beyond what is provided by MSLP.

In contrast, if we take Category 1–2 hurricanes by V_{max} (e.g., 64–95 kt) and plot quartiles of MSLP (Figure 1b), there is a pronounced trend towards larger IKE values at higher pressure intensity (i.e., lower MSLP). For example, mean IKE for the strongest quartile of MSLP (≤ 969 hPa) is three times larger than for the weakest quartile of MSLP (≥ 986 hPa).

Results are similar for major hurricanes defined by MSLP (≤ 960 hPa) and V_{max} (≥ 96 kt). V_{max} generally shows a weak relationship with IKE (Figure 1c), whereas lower MSLP generally is associated with larger values of IKE (Figure 1d).

¹Atlantic hurricanes are classified in 5 kt increments, which precludes a more precise stratification by quartiles. For example, 27% of all 6-hr periods for Category 1–2 hurricanes classified by MSLP are 65 kt, 18% are 70 kt, 17% are 75 kt, 11% are 80 kt, 8% are 85 kt, while hurricanes with $V_{max} \geq 90$ kt comprise the remaining 19% of the sample. The closest to a quartile breakdown that we can make is: 65 kt (27%), 70–75 kt (35%), 80–85 kt (19%) and ≥ 90 kt (19%).

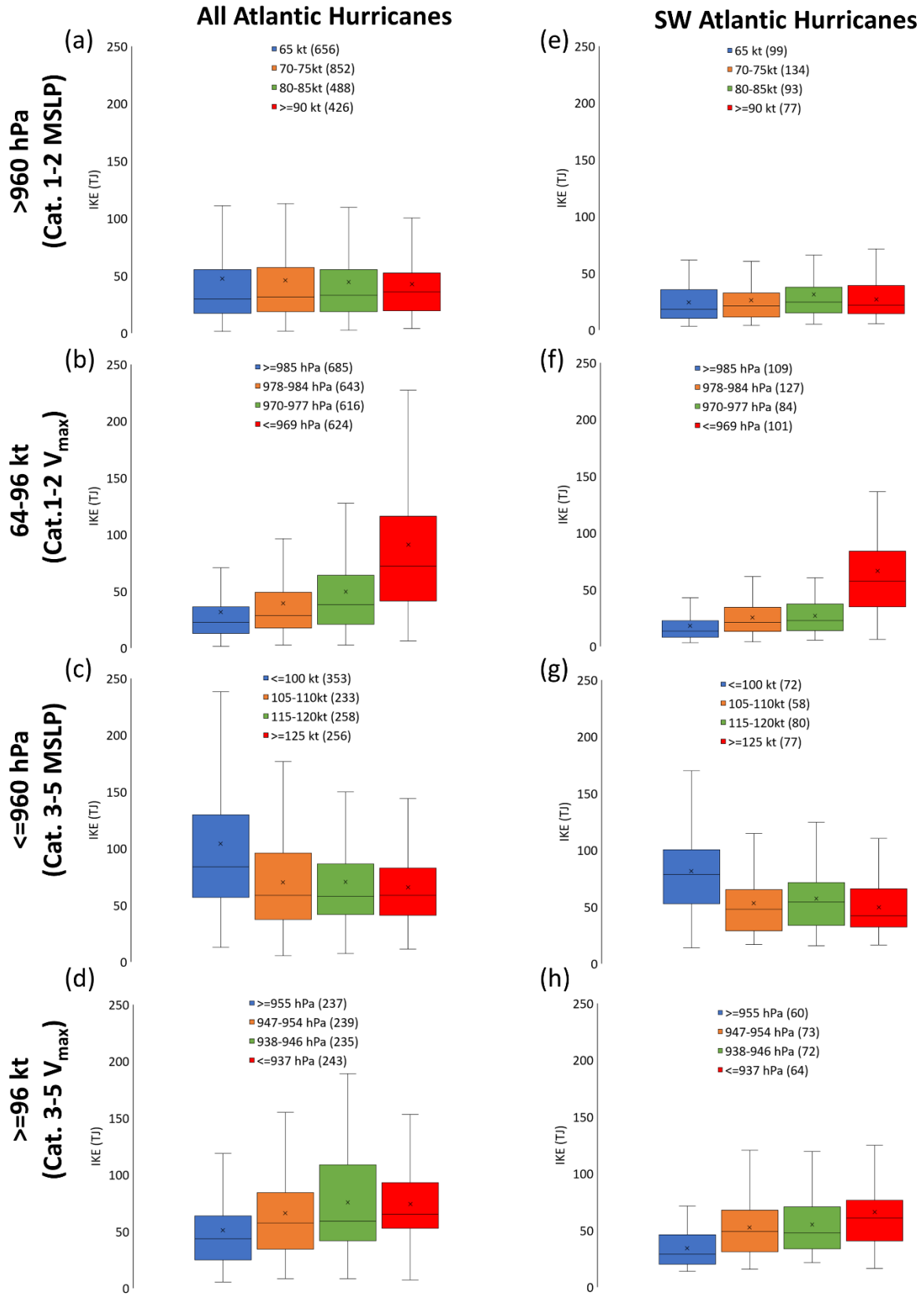


Figure 1. Quartile box plots showing relationships between MSLP, V_{\max} , and IKE for Atlantic hurricanes. (a) Box plot of IKE for approximate quartiles of V_{\max} for all Atlantic Category 1–2 hurricanes as classified by MSLP from 1988–2020. Numbers in parentheses represent the number of six-hourly hurricane observations in each quartile. (b) As in panel a but for Category 1–2 hurricanes classified by V_{\max} . (c) As in panel a but for all Atlantic major hurricanes classified by MSLP from 1988–2020. (d) As in panel a but for all Atlantic major hurricanes classified by V_{\max} from 1988–2020. (e–h) As in panels a–d but for southwest Atlantic hurricanes from 2004–2020. The middle line in all box plots represents the median value, while the ‘x’ in all box plots represents the mean value.

3.2 Southwest Atlantic hurricanes

Our results are similar when focusing on the subset of cases from the Extended Best Track dataset from the southwest Atlantic Ocean since 2004 that are expected to be well-sampled by aircraft (Figures 1e–h). The correlation between MSLP and IKE in the subset of the best sampled cases is stronger ($r_{\text{rank}} = 0.63$) than it was for the entire Atlantic basin over the longer record, and it remains significantly stronger than between V_{\max} and IKE ($r_{\text{rank}} = 0.45$). For Category 1–2 hurricanes, IKE again shows little systematic variation with V_{\max} (Figures 1e and 1g), while systematically increasing with decreasing MSLP (Figures 1f and 1h).

3.3 Continental United States landfalling hurricanes

We next show that these relationships extend specifically to CONUS landfalling hurricanes at landfall. Figures 2a–2c display scatterplots of the relationship between MSLP and IKE, V_{\max} and IKE, and V_{\max} and MSLP, respectively, for CONUS landfalling hurricanes. As was the case for basinwide hurricanes, there is a significantly stronger relationship between MSLP and IKE ($r_{\text{rank}} = 0.79$) than between V_{\max} and IKE ($r_{\text{rank}} = 0.40$) for CONUS landfalling hurricanes.

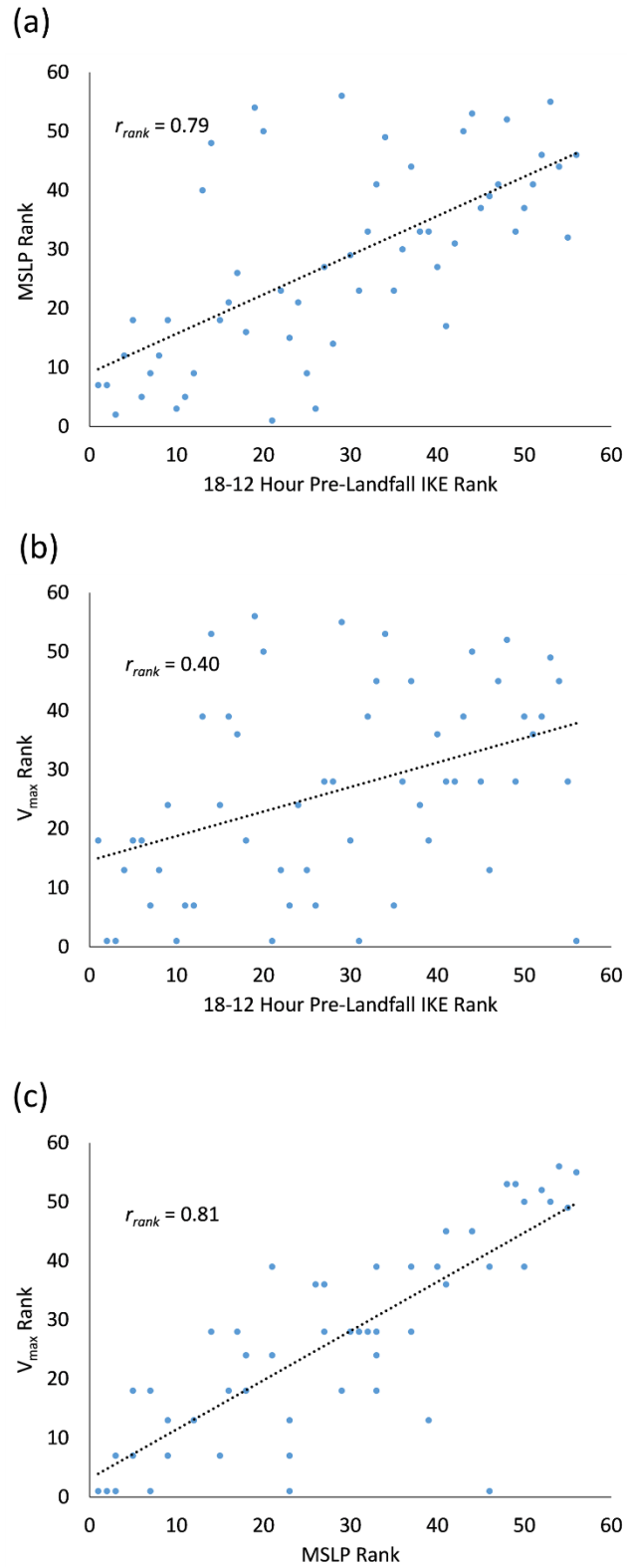


Figure 2. Relationship between MSLP, V_{max} and IKE for CONUS landfalling hurricanes from 1988–2020. (a) Rank scatterplot of MSLP and IKE for CONUS landfalling hurricanes. (b) As in panel a but for V_{max} and IKE. (c) As in panel a but for V_{max} and MSLP.

3.4 Texas to Florida vs. Georgia to Maine landfalling hurricanes

Results are similar when we decompose landfalls by region for Texas to Florida landfalls and Georgia to Maine landfalls. For Texas to Florida landfalls (Figures 3a–3c) the correlation between MSLP and IKE ($r_{\text{rank}} = 0.64$) is greater than the correlation between V_{max} and IKE ($r_{\text{rank}} = 0.49$). For Georgia to Maine landfalls (Figures 3d–3f) the correlation between MSLP and IKE ($r_{\text{rank}} = 0.76$) is again greater than the correlation between V_{max} and IKE ($r_{\text{rank}} = 0.38$), which is a starker contrast between MSLP and V_{max} than for Texas to Florida landfalls. While the relationship between V_{max} and MSLP is significant and strong for Texas to Florida landfalls ($r_{\text{rank}} = 0.92$), the correlation is weak and insignificant for Georgia to Maine landfalls ($r_{\text{rank}} = 0.28$). Hurricanes tend to grow in size as they move poleward (Knaff et al. 2014, Chavas et al. 2016, Klotzbach et al. 2020, Chavas and Knaff 2022), and have a larger radius of maximum wind as a result (Chavas and Knaff 2022), which increases variations in IKE that may be captured by MSLP but not V_{max} .

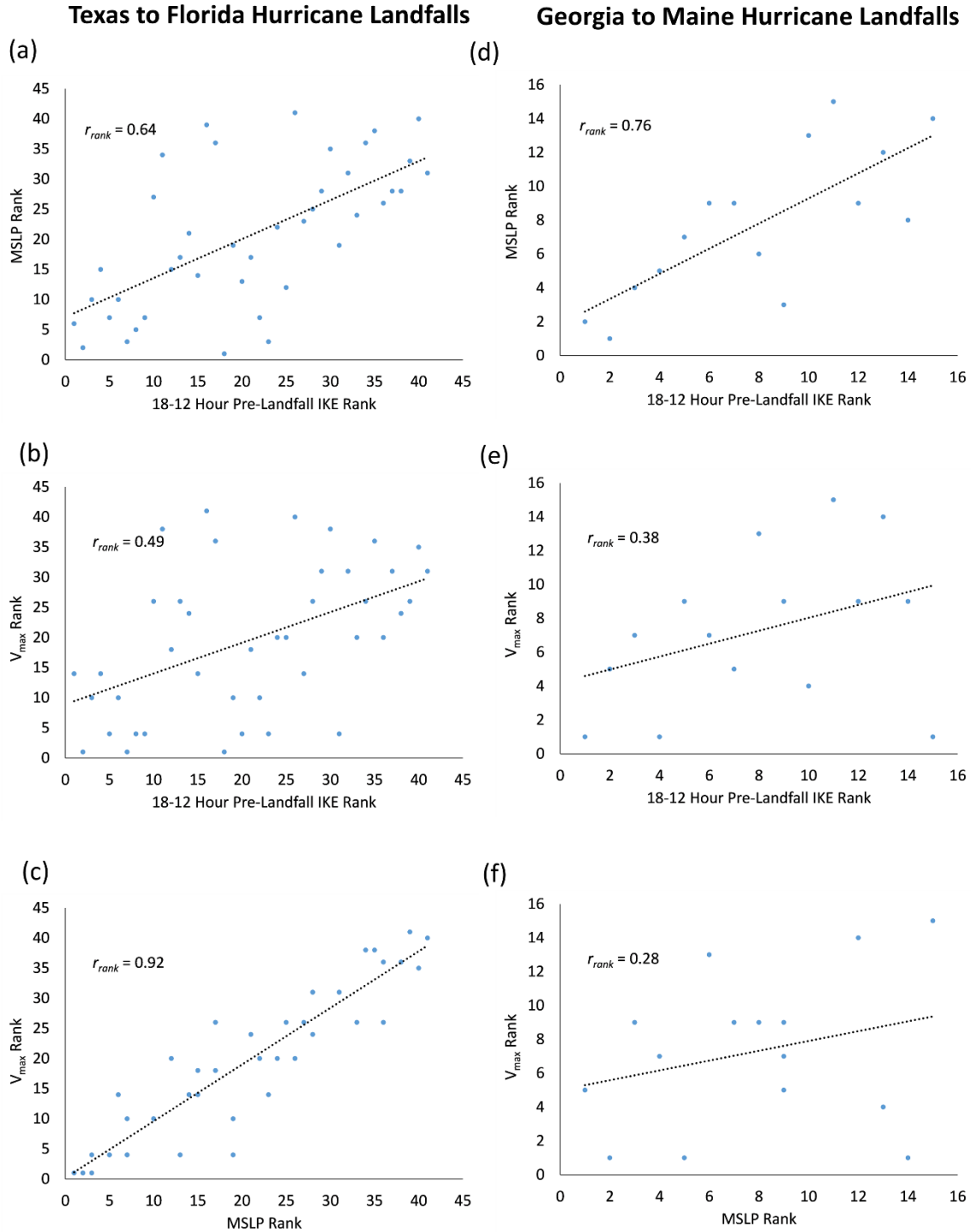
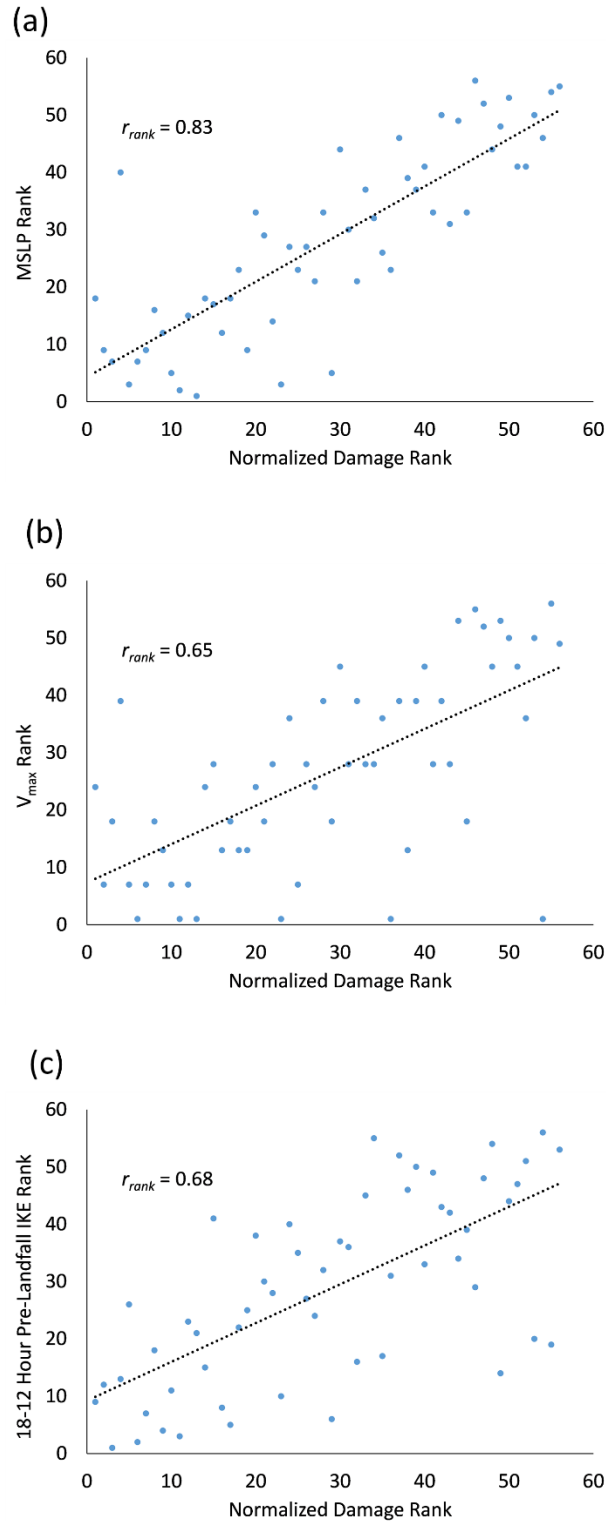


Figure 3. Relationship between MSLP, V_{\max} and IKE for Texas to Florida landfalling hurricanes (left column) and Georgia to Maine landfalling hurricanes (right column) from 1988–2020. (a) Rank scatterplot of MSLP and IKE for Texas to Florida landfalling hurricanes. (b) As in panel a but for V_{\max} and IKE. (c) As in panel a but for V_{\max} and MSLP. (d–f) As in a–c but for Georgia to Maine landfalling hurricanes.

4 Relationship between intensity metrics and normalized landfalling hurricane damage

4.1 Continental United States normalized landfalling hurricane damage

We now show that MSLP better predicts historical damage as compared to IKE or V_{\max} , beginning with the entire US coastline. Figures 4a–c display relationships between MSLP, V_{\max} and IKE with CONUS normalized damage, with higher ranks indicating stronger storms and increased damage. The correlation between MSLP and CONUS normalized damage ($r_{\text{rank}} = 0.83$; Figure 4a) is significantly stronger (as highlighted by the stronger slope of the best fit line) than the correlation between V_{\max} and CONUS normalized damage ($r_{\text{rank}} = 0.65$; Figure 4b). The MSLP-CONUS normalized damage correlation is also significantly stronger than the correlation between IKE and CONUS normalized damage ($r_{\text{rank}} = 0.68$; Figure 4c).



266

267 **Figure 4.** Relationship between intensity metrics and CONUS landfalling hurricane damage
 268 from 1988–2020. (a) Rank scatterplot of MSLP and damage from CONUS landfalling
 269 hurricanes. (b) As in panel a but for V_{max} and damage from CONUS landfalling hurricanes. (c)
 270 As in panel a but for IKE and damage from CONUS landfalling hurricanes.

4.2 Texas to Florida vs. Georgia to Maine normalized landfalling hurricane damage

Klotzbach et al. (2020) noted similar correlations for Texas to Florida hurricane landfalls between V_{\max} and normalized damage as between MSLP and normalized damage, while MSLP was a much more skillful predictor of damage than V_{\max} for Georgia to Maine hurricane landfalls. We now show that MSLP is also a better predictor for these two regions compared to both IKE and V_{\max} , particularly for Georgia to Maine.

For Texas to Florida landfalls, the correlation between MSLP and normalized damage ($r_{\text{rank}} = 0.84$; Figure 5a) and V_{\max} and normalized damage ($r_{\text{rank}} = 0.81$; Figure 5b) are both strong and nearly equal. Meanwhile, the correlation between IKE and normalized damage is slightly weaker ($r_{\text{rank}} = 0.67$; Figure 5c). These results for the relationship between both V_{\max} and MSLP with normalized damage are similar to that of Klotzbach et al. (2020).

For Georgia to Maine landfalls, the correlation between MSLP and normalized damage is strong ($r_{\text{rank}} = 0.80$, Figure 5d). The correlation between IKE and normalized damage is a bit weaker ($r_{\text{rank}} = 0.67$, Figure 5f). Both of these correlations are similar to their Texas to Florida correlation values. However, the correlation between V_{\max} and normalized damage is extremely weak ($r_{\text{rank}} = 0.08$, Figure 5e) and is not significant. Hence, hurricane metrics that either explicitly (IKE) or implicitly (MSLP) have a size component are more skillful for hurricanes making landfall along the East Coast of the United States north of Florida. This result for damage aligns with the finding above that V_{\max} itself is poorly correlated with IKE for this landfall region ($r_{\text{rank}} = 0.38$, Figure 3e).

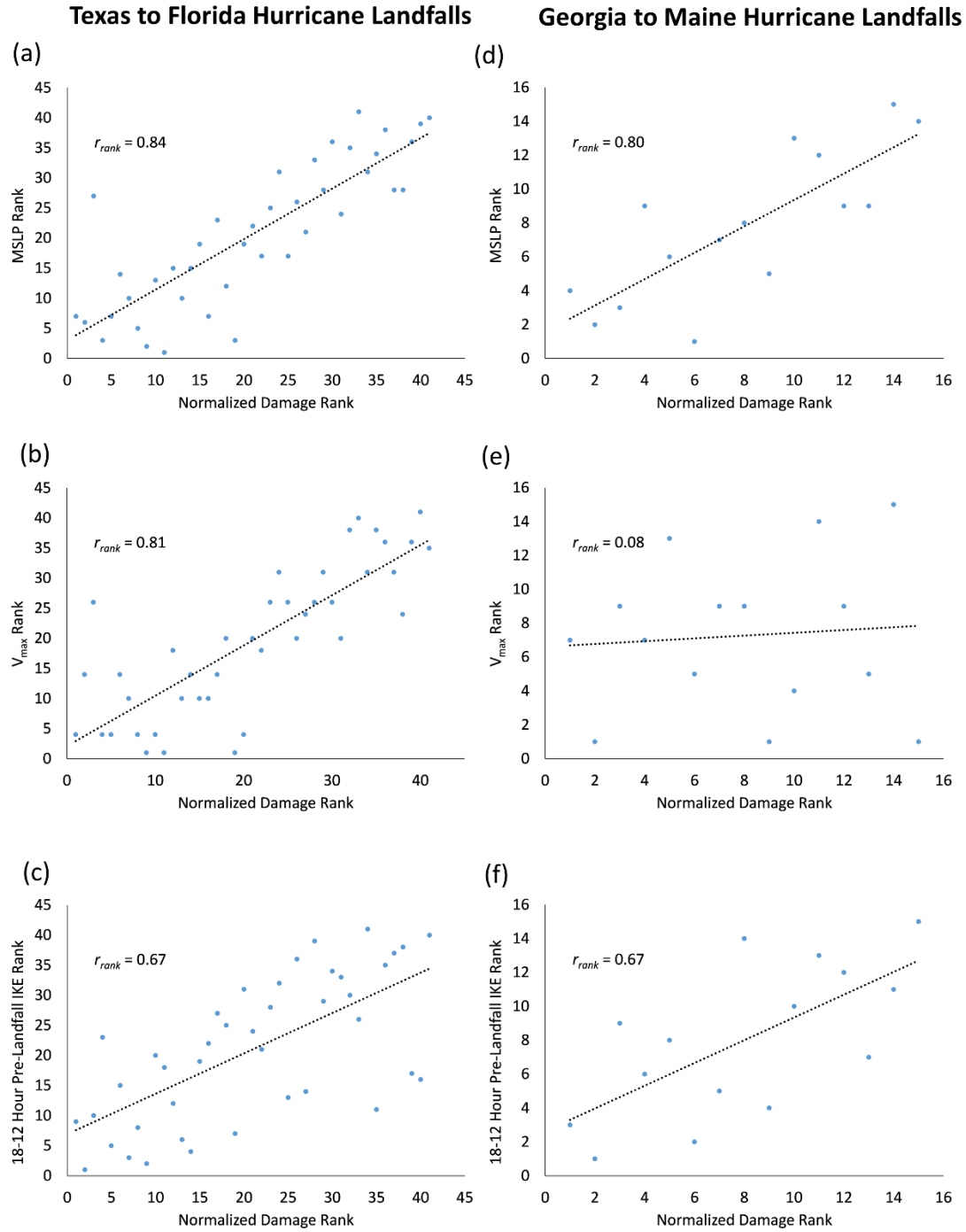


Figure 5. Relationship between MSLP, V_{max} and IKE with normalized damage for Texas to Florida landfalling hurricanes (left column) and Georgia to Maine landfalling hurricanes (right column) from 1988–2020. (a) Rank scatterplot of MSLP and normalized damage for Texas to Florida landfalling hurricanes. (b) As in panel a but for V_{max} and normalized damage for Texas to Florida landfalling hurricanes. (c) As in panel a but for IKE and normalized damage for Texas to Florida landfalling hurricanes. (d–f) As in a–c but for Georgia to Maine landfalling hurricanes.

A prime example of this is Sandy (2012), whose V_{\max} was barely at hurricane-equivalent intensity at landfall yet had a very low MSLP owing in part to its exceptionally large size (Halverson & Rabenhorst 2013; Chavas et al. 2018). We note that the correlation between V_{\max} and normalized damage for Georgia to Maine is considerably lower than what was found in Klotzbach et al. (2020) from 1900–2018 ($r_{\text{rank}} = 0.42$). The degradation in the correlation is due to a relatively smaller sample size of Georgia to Maine hurricane landfalls from 1988–2020 (e.g., 15 landfalls) that also includes Sandy. One outlier in a small sample can considerably impact a correlation value. If Sandy were excluded from the 1988–2020 analysis, the correlation between V_{\max} and normalized damage for Georgia to Maine would remain insignificant ($r_{\text{rank}} = 0.29$) but would be more in line with the correlation reported in Klotzbach et al. (2020).

4.3 Upper tercile of continental US landfalling hurricane damage

As an alternative way of demonstrating the value of MSLP as a damage predictor, we show that the historical damage caused by the strongest storms is systematically higher when storm strength is defined by MSLP. From 1988–2020, 18 hurricanes made landfall in the CONUS with a maximum intensity of 100 kt or greater - Category 3+ on the Saffir–Simpson Hurricane Wind Scale. Given that 56 CONUS landfalling hurricanes occurred from 1988–2020, this equates to the approximate upper tercile of landfalling hurricanes during the 33-year period. Figures 6a–c display the location of the 18 strongest landfalling hurricanes using MSLP (≤ 952 hPa), V_{\max} (≥ 100 kt) and IKE (≥ 71 TJ) criteria. While the spatial distribution of the upper tercile using MSLP (Figure 6a) and V_{\max} (Figure 6b) is similar, many more hurricanes from Georgia to Maine classify as upper tercile storms using IKE (Figures 6c). Using V_{\max} , two hurricanes from Georgia to Maine are in the upper tercile (Hugo (1989) and Fran (1996)). Using MSLP, in addition to Hugo and Fran, Sandy (2012) also is in the upper tercile. Using IKE, half of the 18 hurricanes in the upper tercile made landfall from Georgia to Maine. The larger number of high-IKE landfalls from Georgia to Maine is likely due to the growth in size of hurricanes as they move poleward and the relatively strong weighting of 34-kt and 50-kt wind radii in the IKE equation (discussed in more detail in the next section).

Finally, Figure 6d displays a box plot for normalized damage for the upper tercile of landfalling hurricanes with intensity defined using MSLP, IKE, or V_{\max} . The mean, median and high quantiles of normalized damage are all largest when using MSLP, second largest when using IKE, and smallest when using V_{\max} . This analysis again highlights the improved relationship using MSLP than either IKE or V_{\max} for representing the damage potential from hurricanes.

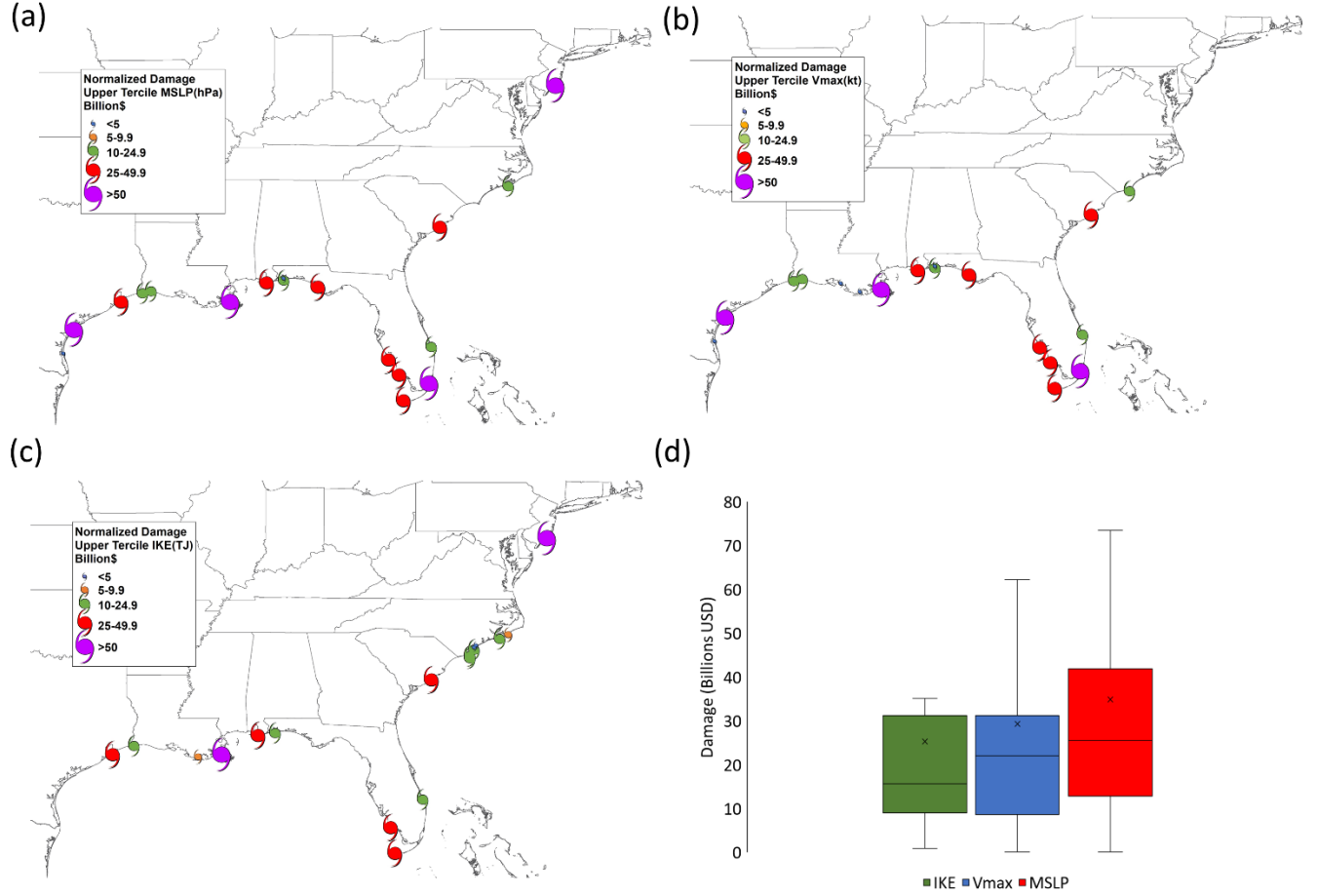


Figure 6. Location and relationship between the upper tercile of hurricane intensity categorized by MSLP, V_{\max} , and IKE and normalized damage. (a) Location of upper tercile CONUS landfalling hurricanes from 1988–2020 based on MSLP with the size of the hurricane symbol proportional to the normalized damage. (b) As in panel a but for the upper tercile based on V_{\max} . (c) As in panel a but for the upper tercile based on IKE. (d) Box plot showing the distribution of normalized damage for the upper tercile of CONUS landfalling hurricanes classified by IKE, V_{\max} and MSLP.

5 Physical discussion

As shown above, an integral measure of the storm wind field (MSLP or IKE) is preferable to a point estimate of the maximum wind speed for predicting potential damage, with MSLP performing best. Here we show how MSLP represents a radial integral of the wind field, and how that integral weights wind speeds at different radii differently from IKE.

For an axisymmetric field, a radially-integrated quantity, X , may be written as:

$$X = \int_0^{r_0} x \, dr \quad (3)$$

where x is the integrand and r_0 is some larger radius (e.g., R_{34kt}). For ease of interpretation we may neglect multiplicative factors in each equation that may be taken as approximately constant, as we use these quantities purely as statistical predictors of damage. Thus, absolute magnitudes do not matter.

MSLP represents a reduction in pressure at the storm center relative to the ambient environmental pressure P_{env} at the outer edge of the storm. This pressure difference is commonly referred to as the central pressure deficit:

$$dP = P_{env} - MSLP \quad (4)$$

and is related to the wind field via gradient wind balance (Knaff and Zehr 2007, Chavas et al 2017). Hence, for dP , the integrand is given by:

$$x_{dP} \sim \frac{V^2}{r} + fV \quad (5)$$

where we drop the density factor (ρ). For IKE, from Eq. (1) the integrand is given by:

$$x_{IKE} \sim rV^2 \quad (6)$$

where r arises from the polar integral, and we drop the factor $\frac{1}{2}\rho h$.

To show how each quantity weights wind speeds at different radii, each integrand x may be normalized by its maximum value, and the result analyzed as a function of radius normalized by the radius of maximum wind. An example calculation is shown in Figure 7 for a characteristic hurricane wind profile defined by the model of Chavas et al. (2015). This model has been shown to capture the observed structure of the complete hurricane wind field as well as the basic structural relationships between R_{max} , R_{34kt} , and V_{max} in the historical record (Chavas and Knaff 2022). For this example, the model is defined using parameter values taken as the median values of southwest Atlantic hurricanes: $R_{max} = 28$ km, $V_{max} = 90$ kt, and latitude at $23.7^\circ N$. The central pressure deficit is weighted towards the strongest wind speeds in the inner core ($r < 2R_{max}$), and its maximum weighting is at R_{max} itself. Integrated kinetic energy has a similar qualitative structure but more strongly weights weaker wind speeds at larger radii towards R_{34kt} , with its maximum value at about $1.7R_{max}$. This difference arises because V^2 is weighted inversely by radius in the centrifugal term $\frac{V^2}{r}$ in x_{dP} , and so x_{dP} decreases rapidly beyond R_{max} , whereas V^2 is weighted proportionally to radius in x_{IKE} .

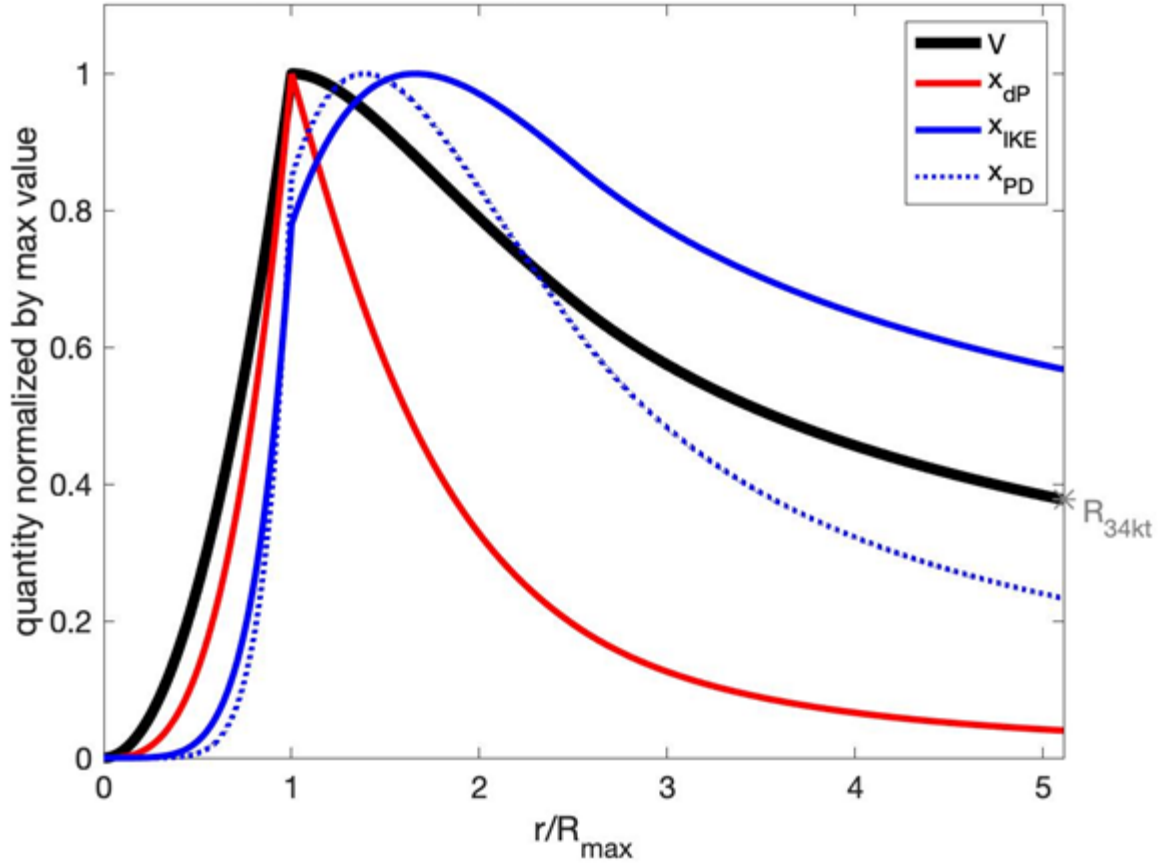


Figure 7. Radial structure of the pressure deficit (dP; red), integrated kinetic energy (IKE; blue), and power dissipation (PD; cyan) calculated from an example tropical cyclone wind profile (V ; black). Each quantity is normalized by its maximum value, and radius is normalized by the radius of maximum wind, R_{\max} . The wind profile is defined using the physical model of Chavas et al. (2015) taking as input the median values of southwest Atlantic hurricanes: $V_{\max} = 90$ kt, $R_{\max} = 28$ km, and latitude at 23.7°N . A simple quadratic profile is used in the eye for $r < R_{\max}$. R_{34kt} is marked with a gray star. Each colored curve represents the integrand whose radial integral scales with the given quantity. This quantity is normalized by its maximum value to allow for direct comparison across dP, IKE, and PD (see text for details).

A viable alternative integral quantity to IKE is PD. Power dissipation scales identically with IKE except with the wind speed cubed rather than squared. While IKE (units of Joules) is much more widely used, PD (units of Watts) has physical appeal for damage potential because it represents the rate of transfer of kinetic energy from the near-surface air into the surface due to surface friction. For PD, from Eq. (2) the integrand is given by:

$$x_{PD} \sim rV^3 \quad (7)$$

where r again arises from the polar integral, and we drop the factor ρC_d . Power dissipation yields a weighting of the radial structure that lies in between dP and IKE (Figure 7). This behavior arises because V^3 more strongly weights higher wind speeds than V^2 .

However, our results are nearly identical when applying our methodology for PD rather than IKE. Despite their different weighting structures, variations in PD and IKE correlate very strongly with one another ($r_{\text{rank}} = 0.99$; Figure S1). The close relationship between IKE and PD arises because the inner wind field is well-approximated by a Modified Rankine vortex (Rappin et al. 2013), given by $\tilde{V} = \tilde{r}^\alpha$, where $\tilde{r} = r/R_{\text{max}}$ and $\tilde{V} = V/V_{\text{max}}$. The statistics of the Extended Best Track wind radii data maps closely onto a Rankine vortex with an exponent $\alpha = -0.55$ (Figure 8a). For this wind profile solution, the ratio of PD to IKE between R_{max} and $R_{34\text{kt}}$ can be derived analytically, and may be written as:

$$\frac{\text{PD}}{\text{IKE}} \sim \left(\frac{2\alpha+2}{3\alpha+2} \right) V_{\text{max}} \left(\frac{\tilde{V}_{34\text{kt}}^{3+\frac{2}{\alpha}} - 1}{\tilde{V}_{34\text{kt}}^{2+\frac{2}{\alpha}} - 1} \right) \quad (8)$$

where $\tilde{V}_{34\text{kt}} = V_{34\text{kt}}/V_{\text{max}}$, $V_{34\text{kt}}$ is simply the gale force wind speed, and we have neglected the constants in each quantity as described above. This solution neglects winds within the eye ($r < R_{\text{max}}$). Eq. 8 shows that, for fixed values of α and wind speed of the bounding radius ($V_{34\text{kt}}$), the ratio of PD to IKE depends only on V_{max} ; it does not depend on R_{max} . Moreover, the dependence on V_{max} is weak (Figure 8b), following a scaling of approximately $V_{\text{max}}^{0.35}$. As a result, IKE and PD scale very closely together and are nearly equivalent as predictors for historical damage. A more detailed analysis of the relationship between IKE and PD in observed storms may be an interesting avenue for future research.

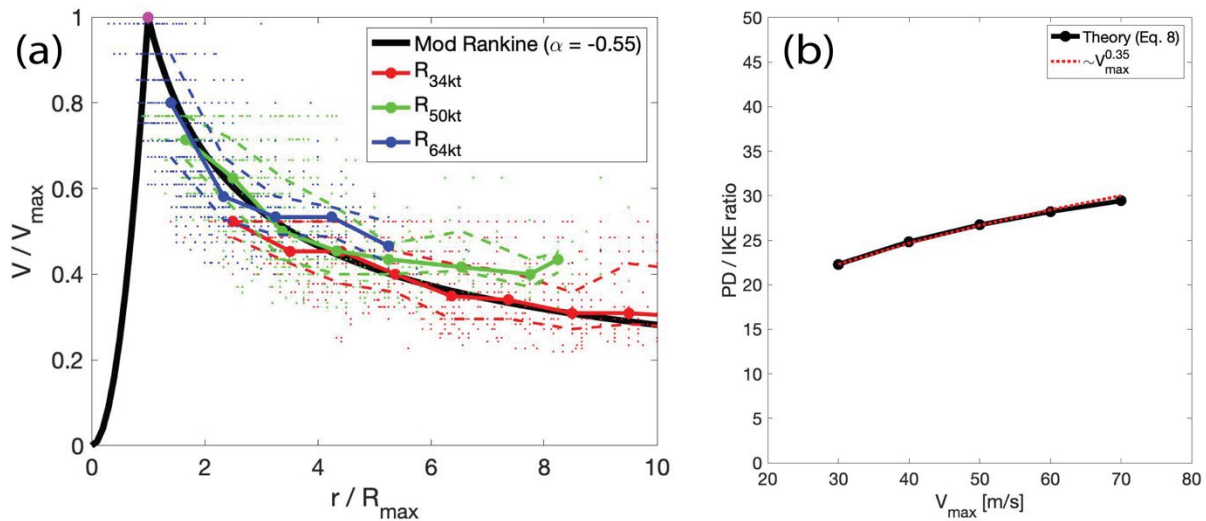


Figure 8. (a) Statistics of Extended Best Track wind radii ($R_{34\text{kt}}$ in blue, $R_{50\text{kt}}$ in green, $R_{64\text{kt}}$ in red) plotted with radius normalized by the radius of maximum wind speed and wind speed normalized by maximum wind speed, for the 2004-2020 southwest Atlantic subset. Median (solid) and interquartile range (dashed; 25th-75th percentile) values of $(r/R_{\text{max}}, V/V_{\text{max}})$ calculated within unit bins of r/R_{max} (i.e. 1-2, 2-3, etc.); values plotted in bins with at least 10 datapoints. Modified Rankine profile shown (black) with $\alpha = -0.55$. (b) Ratio of PD to IKE for the Modified Rankine solution between R_{max} and $R_{34\text{kt}}$ (Eq. 8) as a function of V_{max} (black), with approximate scaling (red) for comparison.

Note that technically the weighted-average wind speeds (Table S1) should be recalculated for V^3 , but doing so using a piecewise-linear model of the wind field has a negligible change to this outcome (not shown).

Ultimately there is likely no single “correct” weighting of the radial structure when relating the wind field to damage potential, as storm hazards (wind, surge, and rainfall) each depend on different aspects of the wind field in addition to an array of other environmental factors that can vary from storm to storm. Indeed, our results indicate that IKE and PD are equally useful as predictors of damage potential despite their different weighting structures. We find that MSLP is slightly more useful as a damage predictor, suggesting that its weighting structure may be better suited for representing damage potential or other direct/indirect societal disruptions. Explanations for why that might be are highly complex, though, and hence we leave this topic for future work.

6 Summary and conclusions

Here we have investigated the relationship between IKE, V_{\max} and MSLP for both Atlantic basin hurricanes and for CONUS landfalling hurricanes, specifically from 1988–2020. We find that IKE has a stronger relationship with MSLP than with V_{\max} , both for basinwide hurricanes and CONUS landfalling hurricanes. This finding is likely due to the robust relationship between storm size and central pressure deficit, as the central pressure is itself an integrated measure of the wind field. When focusing specifically on well-measured southwest Atlantic hurricanes and using rank correlations, V_{\max} explains ~25% of the variance in IKE, while MSLP explains ~40% of the variance in IKE.

Minimum sea level pressure is a better predictor of CONUS landfalling hurricane damage than IKE and especially V_{\max} . While all three metrics show strong skillful correlations for hurricanes making landfall from Texas to Florida, the correlation between V_{\max} and landfalling hurricane damage is small and insignificant for hurricanes making landfall from Georgia to Maine. The degradation in the relationship between V_{\max} and normalized damage for hurricanes making landfall along the East Coast of the United States north of Florida is likely due to the growth in size of hurricanes as they move poleward. Hence, our analysis indicates that the use of MSLP to categorize hurricane strength would have especially high value for potential landfalls along the East Coast. Very similar results are obtained when using PD as an integrated wind field quantity as opposed to IKE because the wind profile is well-approximated by a Modified Rankine profile, for which the two quantities scale closely with each other.

Importantly, an additional benefit of using MSLP to categorize hurricanes is that it is already routinely measured operationally. Furthermore, it is much simpler to estimate than either the full hurricane wind field or even V_{\max} given its relatively noisy nature. In essence, MSLP is a storm-integrated quantity that can be measured directly (in principle) at a single point at the center of the storm. In contrast, IKE requires estimating the wind field over a large range of radii along multiple azimuths. Since MSLP is found to be the best predictor of historical hurricane damage and is relatively easy to measure, we conclude that MSLP is an ideal metric for categorizing damage potential for hurricanes. Based on these findings, we advocate for efforts to improve

forecasts and interpretation of MSLP as an intensity metric when communicating tropical cyclone societal risk to the general public.

Acknowledgments

P. Klotzbach would like to acknowledge a grant from the G. Unger Vetlesen Foundation. D. Chavas acknowledges NSF Grants 1826161 and 1945113. M. Bell was supported by Office of Naval Research Grant N000142012069. C. Schreck was supported by NOAA through the Cooperative Institute for Satellite Earth System Studies under Cooperative Agreement NA19NES4320002. E. Gibney's research at the National Hurricane Center is supported by NOAA's Science Collaboration Program and administered by UCAR's Cooperative Programs for the Advancement of Earth System Science (CPAESS) under award NA21OAR4310383.

Data Availability Statement

All data used in this study are publicly available at the following locations:

Extended Best Track:

https://rammb2.cira.colostate.edu/research/tropical-cyclones/tc_extended_best_track_dataset/

Continental US Hurricane Landfalls:

https://www.aoml.noaa.gov/hrd/hurdat/UShurrs_detailed.html

Normalized Continental US Hurricane Damage (1988–2017):

https://static-content.springer.com/esm/art%3A10.1038%2Fs41893-018-0165-2/MediaObjects/41893_2018_165_MOESM2_ESM.xlsx

Normalized Continental US Hurricane Damage (2018–2020):

<https://www.nhc.noaa.gov/data/tcr/>

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Characterizing Continental US Hurricane Risk: Which Intensity Metric is Best?

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Contents of this file

Text S1

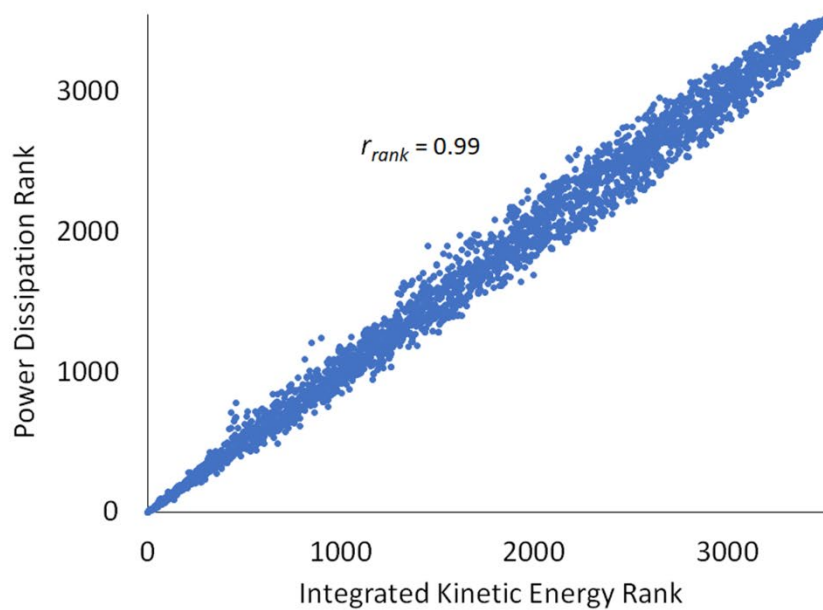
Figure S1

Table S1

Introduction

This supporting information includes brief additional text, one additional figure and one additional table. Text S1 discusses the physical reasoning behind the high correlation between IKE and PD. Figure S1 displays the relationship between IKE and PD from 1988–2020 for all Atlantic hurricanes and for southwest Atlantic hurricanes. Table S1 highlights the methodology used to calculate IKE in each quadrant.

(a)



(b)

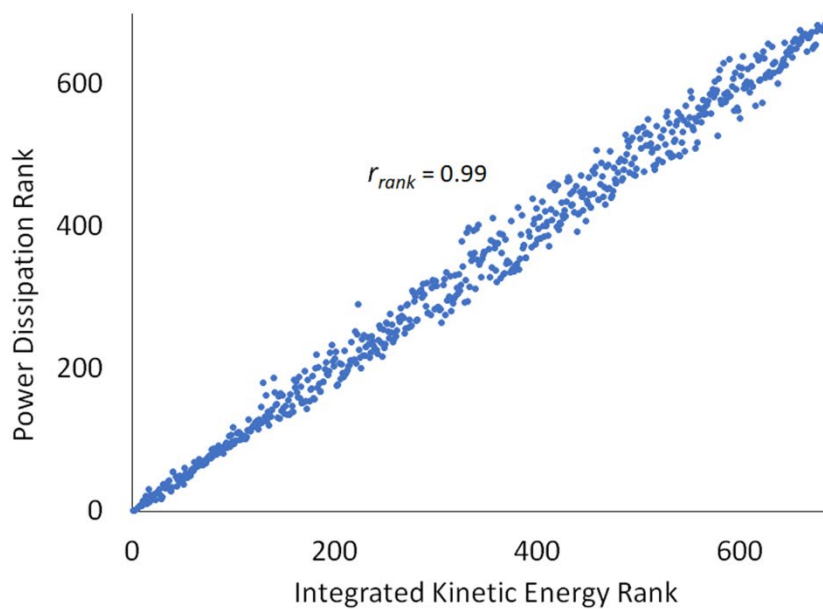


Figure S1. Relationship between PD and IKE. (a) Scatterplot of PD rank and IKE rank for all Atlantic hurricanes from 1988–2020. (b) As in panel a but for southwest Atlantic hurricanes from 2004–2020.

Table S1. Methodology for computing IKE in each quadrant at each 6-hourly timestep for each hurricane in the Extended Best Track database. The table format is identical to Table A1 from Misra et al. (2013). One adjustment is made for IKE_H to use three simple criteria; the criteria referencing “max quadrant” in their original methodology were unclear and unnecessary, and as a result the fourth criterion is no longer needed as it is subsumed in the previous three. All wind speeds are listed in units of ms^{-1} .

Quadrant IKE contribution	Criteria	Mean wind (ms^{-1})	Area (m^2)
IKE_{18-26}	$R_{26} > 0$	20	$0.25\pi(R_{18}^2 - R_{26}^2)$
	No R_{26} , $V_{max} > 26$, $R_{max} < R_{18}$	20	$0.25\pi(R_{18}^2 - (0.75R_{max})^2)$
	No R_{26} , $V_{max} < 26$, $R_{max} < R_{18}$	$0.25V_{max} + 0.75(18)$	$0.25\pi(R_{18}^2 - (0.75R_{max})^2)$
	No R_{26} , $R_{max} = R_{18}$	18	$0.25\pi(R_{18}^2 - (0.5R_{18})^2)$
IKE_{26-33}	$R_{33} > 0$	27.75	$0.25\pi(R_{26}^2 - R_{33}^2)$
	No R_{33} , $V_{max} > 33$, $R_{max} < R_{26}$	27.75	$0.25\pi(R_{26}^2 - (0.75R_{max})^2)$
	No R_{33} , $V_{max} < 33$, $R_{max} < R_{26}$	$0.25V_{max} + 0.75(26)$	$0.25\pi(R_{26}^2 - (0.75R_{max})^2)$
	No R_{33} , $R_{max} \geq R_{26}$	26	$0.25\pi(R_{26}^2 - (0.5R_{33})^2)$
IKE_H	$R_{max} < R_{33}$	$0.25V_{max} + 0.75(33)$	$0.25\pi(R_{33}^2 - (0.75R_{max})^2)$
	$R_{max} = R_{33}$	$0.25V_{max} + 0.75(33)$	$0.25\pi(R_{33}^2 - (0.75R_{33})^2)$
	$R_{max} > R_{33}$	$0.1V_{max} + 0.9(33)$	$0.25\pi(R_{33}^2 - (0.75R_{33})^2)$