Frictional and Hydraulic Properties of Plate Interfaces Constrained by a Tidal Response Model Considering Dilatancy/Compaction

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Abstract

Tidal triggering of tectonic tremors has been observed at plate boundaries around the circum-Pacific region. It has been reported that the response of tremors to tidal stress during episodic tremor and slow slip (ETS) changes between the early and later stages of ETS. Several physical models have been constructed, with which observations for the tidal response during ETS have been partly reproduced. However, no model has been proposed that reproduces all the observations. In this study, a model adopted in previous studies is extended to include the effects of dilatancy/compaction that occur in the fault creep region. The analytical approximate solution derived in this study and numerical computational results reveal how the tidal response depends on the physical properties of the fault. Furthermore, the model reproduces all the above observations simultaneously for a specific range of fault parameters. Of particular importance is that the occurrence of dilatancy/compaction is essential to reproduce the tidal response at the early stage of the ETS. The value of the critical distance dc is constrained to be approximately 1~10 cm. This agrees with the values that have been widely used in seismic cycle numerical simulations rather than those obtained in laboratory experiments. The fluid pressure diffusivity is constrained to be at least 10^(-5) m^2/s or less, and the effective normal stress is constrained to $10^{(5-6)}$ Pa. In conclusion, this study shows that reproducing the tidal response of tectonic tremors during the ETS is useful for estimating fault physical properties, including hydraulic properties.

1	Frictional and Hydraulic Properties of Plate Interfaces Constrained by
2	a Tidal Response Model Considering Dilatancy/Compaction
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7	Key points:
8	• The spring-slider model with dilatancy/compaction reproduces the observed tidal
9	response of tremors during episodic tremor and slip (ETS).
10	• The critical slip distance, diffusivity and effective stress are constrained by a comparison
11	between the model and the observation.
12	• The pore fluid pressure change due to dilatancy/compaction is dominant at the early stage
13	of the ETS, while it is negligible at the later stage.
14	

15 Abstract

16 Tidal triggering of tectonic tremors has been observed at plate boundaries around the circum-Pacific region. It has 17 been reported that the response of tremors to tidal stress during episodic tremor and slow slip (ETS) changes 18 between the early and later stages of ETS. Several physical models have been constructed, with which observations 19 for the tidal response during ETS have been partly reproduced. However, no model has been proposed that 20 reproduces all the observations. In this study, a model adopted in previous studies is extended to include the effects 21 of dilatancy/compaction that occur in the fault creep region. The analytical approximate solution derived in this 22 study and numerical computational results reveal how the tidal response depends on the physical properties of the 23 fault. Furthermore, the model reproduces all the above observations simultaneously for a specific range of fault 24 parameters. Of particular importance is that the occurrence of dilatancy/compaction is essential to reproduce the 25 tidal response at the early stage of the ETS. The value of the critical distance d_c is constrained to be approximately 26 $1 \sim 10$ cm. This agrees with the values that have been widely used in seismic cycle numerical simulations rather than those obtained in laboratory experiments. The fluid pressure diffusivity is constrained to be at least 10^{-5} m²/s or 27 less, and the effective normal stress is constrained to 10^{5-6} Pa. In conclusion, this study shows that reproducing the 28 29 tidal response of tectonic tremors during the ETS is useful for estimating fault physical properties, including 30 hydraulic properties.

31 Plain Language Summary

32 Slow earthquakes, which are slower fault slips than ordinary earthquakes, have been observed at many plate 33 boundaries around the Pacific Rim. To understand how slow earthquakes occur, we need to know the exact physical 34 fault properties that cause slow earthquakes. Previous studies have reported that the rate of occurrence of tectonic 35 tremors, which are slow earthquakes, varies periodically in response to subsurface stress changes induced by tides. 36 However, the detailed mechanism of the periodic behavior is still unclear. In this paper, we develop a theoretical 37 model to explain this periodic behavior. A comparison between the observations in the Nankai Trough and Cascadia 38 with our model shows that the pore fluid pressure in the vicinity of the fault changes significantly when tremors 39 respond relatively weakly to tides. Furthermore, for the model to explain the observed tidal response of tremors, we 40 find that the scale of the surface roughness of the fault should be much larger than those obtained by laboratory 41 experiments and that the fault should have a low permeability.

42 **1 Introduction**

43 Recent geodetic and seismological observations have revealed that slow earthquakes occur in the transition zone, 44 which is located at the deeper extension of the locked megathrust zone in a subduction zone. Slow earthquakes have 45 various timescales, which are classified into low-frequency earthquakes (LFEs) with a major frequency of 2 - 8 Hz 46 (Obara, 2002), tectonic tremors, which are aggregations of LFEs (Shelly et al., 2007a), very low-frequency 47 earthquakes (VLFEs) with a major frequency of 20 - 200 Hz (Ito et al., 2007), and slow slip events (SSEs), which do not radiate seismic waves and continue to slip for more than a few days (Dragert et al., 2001; Hirose et al., 1999). 48 49 The focal mechanism of these slow earthquakes indicates that they accommodate shear slip on the plate interface 50 (e.g., Ide et al., 2007; Shelly et al., 2006). This focal mechanism coincides with that of ordinary earthquakes, which 51 are caused by fast slip. It is well known that the fast slip behavior of an ordinary earthquake reflects the physical 52 properties of the fault, which consist of friction, effective normal stress and dilatancy/compaction (e.g., Proctor et 53 al., 2020; Scholz, 2019; Segall and Rice, 1995). The coincidence of the focal mechanism with those of slow 54 earthquakes means that the slip behaviors of slow earthquakes should also reflect such fault physical properties. 55 Therefore, it is important to clarify the physical fault properties in the transition zone to reveal the mechanism of 56 slow earthquakes on various timescales. 57 In numerical simulation studies, several models have been proposed to reproduce slow earthquakes. These models 58 usually adopt the rate- and state-dependent friction law (RSF) (e.g., Dieterich, 1979; Marone, 1998) as the frictional 59 law on the plate interface. Examples of such models are those assuming near-neutral stability (e.g., Liu and Rice, 60 2005; Matsuzawa et al., 2010), dilatant strengthening of the shear zone (e.g., Liu, 2013; Segall et al., 2010), 61 transition from velocity weakening (VW) at a low slip rate to velocity strengthening (VS) at a high slip rate (e.g., Im 62 et al., 2020; Peng and Rubin, 2018; Shibazaki and Iio, 2003), spatial heterogeneity of frictional properties and effective normal stress (Luo and Ampuero, 2018), and sudden negative Coulomb stress change in the VS region due 63 to fault valve action (Perfettini and Ampuro, 2008). Comparisons between such models and observed slow slip 64 behaviors have allowed us to estimate the physical fault properties in the transition zone, which cannot be observed 65 66 directly (e.g., Beeler et al., 2018; Luo and Liu, 2019; Nakata et al., 2012; Shibazaki et al., 2012). 67 In this study, we focus on tectonic tremors because they occur more frequently than other slow earthquakes, and it is 68 easier to obtain more data to investigate the physical properties of faults. Tremors are classified into episodic 69 families that accompany a SSE and continuous families that consist of tremors that occur almost daily (Thomas et

70 al., 2018). The former is called episodic tremor and slip (ETS) (Obara et al., 2004; Rogers and Dragert, 2003). An 71 important observation is that, for an ETS, there is a correlation between the slip rate of the SSE and the tremor 72 occurrence rate (e.g., Bartlow et al., 2011; Hirose and Obara, 2010; Thomas et al., 2018; Villafuerte et al., 2017), even though the cumulative moment magnitude M_W of tremors is orders of magnitude smaller than that of SSEs 73 74 (Kao et al., 2010). This correlation has been modeled by assuming that a tremor source is driven to failure by the 75 stress loading due to aseismic slip that occurs in the region surrounding the tremor source (Shelly et al., 2007a). 76 Based on this model, Shelly et al. (2011) interpreted the delayed dynamic triggering of tremors as a result of 77 transient creep induced by the passage of seismic waves. Similarly, Tan and Marsan (2020) interpreted that the 78 spatial anisotropy of the SSE during an ETS causes anisotropy in the power law describing a spatial decay of 79 tremors. 80 Another important observational fact revealed by global observations of tremors is that tremors are sensitive to tidal stress (e.g., Chen et al., 2018; Hoston, 2015; Ide and Tanaka, 2014; Ide et al., 2015; Nakata, 2008; Rover et al., 81 82 2015; Rubinstein et al., 2008; Shelly et al., 2007b; Thomas et al., 2009, 2012; Van Der Elst et al., 2016; Yabe et al., 83 2015). 84 In general, stress changes on faults due to semidiurnal and diurnal tides are a few kPa or smaller. These stress 85 changes are much smaller than the stress drop of ordinary earthquakes. Observational studies have reported that, in some cases, a weak correlation can be seen between earthquakes and tidal stresses (e.g., Cochran et al., 2004; 86 Métivier et al., 2009; Tanaka 2010, 2012). The tidal response of earthquakes has also been studied by laboratory 87 88 experiments using a stress perturbation to rock or granular materials (e.g., Bartlow et al., 2012; Beeler and Lockner 2003; Chanard et al., 2019; Lockner and Beeler, 1999; Noël and Passelègue et al., 2019; Noël and Pimienta et al., 89 90 2019; Savage and Marone, 2007). For example, the results of Chanard et al. (2019) and Noël and Pimienta et al. 91 (2019) are consistent with the observation results of Tanaka (2010, 2012) regarding the tidal response that appears 92 before a large earthquake. Other studies have reported that no correlation is seen between earthquakes and tidal 93 stresses (e.g., Heaton 1982; Vidale et al., 1998; Wein and Shearer 2004). 94 The tidal response of tremors is clearer because the pore fluid pressure on the plate interfaces is much higher in the transition zone than in the seismogenic zone, and hence, the effective normal stress is extremely low (Audet et al., 95

96 2009; Shelly et al., 2006).

97	The tidal response of tremors can be characterized by a tidal sensitivity and a phase difference. The tidal sensitivity,
98	α , characterizes the magnitude of tidal modulation of the tremor rate (i.e., the number of observed tremor events per
99	unit time); the relationship between the tremor rate and the tidal Coulomb stress change is described by
100	$R = R_0 e^{\alpha \Delta S(t)},\tag{1}$
101	where <i>R</i> denotes the tremor rate, $\Delta S(t)$ is the tidal Coulomb stress, and R_0 is the reference tremor rate when
102	$\Delta S(t) = 0$. In equation (1), the order of the tidal sensitivity is 0.01~1 kPa ⁻¹ (e.g., Houston, 2015; Ide et al., 2015;
103	Royer et al., 2015; Thomas et al., 2012; Yabe et al., 2015). The phase difference, defined as δ_n represents the phase
104	shift between the tremor rate peak (i.e., the phase at which R is maximum) and the tidal stress peak (i.e., the phase at
105	which $\Delta S(t)$ is maximized). δ is positive when the tremor rate reaches its maximum before the tidal stress reaches
106	its maximus. For example, when the peak of R precedes the peak of $\Delta S(t)$ in the semidiurnal tide (approximately 12)
107	hour cycle) by 3 hours, $\delta \sim \pi/2$. Previous studies have reported that α and δ change at the early and later stages of
108	the ETS (Houston, 2015; Royer et al., 2015; Yabe et al., 2015). At the early stage of an ETS, $\alpha \leq 0.1 \text{ kPa}^{-1}$
109	(meaning that tidal modulation of the tremor rate is smaller) and $\delta \sim \pi/2$. At the later stage of the ETS, $\alpha \sim 0.7$ kPa ⁻¹
110	(meaning that the tidal modulation of the tremor rate is larger) and $\delta \sim 0$. In addition, the number of tremors
111	occurring at the later stage of ETS is approximately 1/10 or less than at the early stage of ETS (Houston, 2015;
112	Royer et al., 2015).
113	Constructing a model that reproduces such observed tidal responses of tremors is an effective method to infer the
114	physical properties of faults because tidal stress change, which serves as an "input" to a fault slip model to
115	reproduce the tidal response, is much easier to estimate. Previous studies have proposed several models to interpret
116	the observed tidal response of tremors (Ader et al., 2012; Beeler et al., 2013; Beeler et al., 2018; Hawthorne and
117	Rubin, 2013; Houston, 2015). These models are classified into deterministic models that adopt the physical model
118	proposed by Shelly et al. (2007a) and a stochastic model that adopts the Weibull distribution as the failure strength
119	of tremor sources. Furthermore, deterministic models are classified into two models: one considers the change in the
120	pore fluid pressure at the plate interface, and the other does not (Table 1).
121	Specifically, under the assumption that the tremor rate is proportional to fault creep velocity, Ader et al. (2012),
122	Beeler et al. (2013) and Hawthorne and Rubin (2013) investigated the tidally modulated tremor rate. Ader et al.

- 123 (2012) adopted the RSF for the VS to describe the tidal modulation of the fault creep velocity. They showed that the
- tidal sensitivity and the phase difference (the phase difference between the fault creep velocity peak and the tidal

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125 stress peak) depend on the tidal period, fault creep velocity and frictional properties. Based on this result, they 126 provided a framework to explain the tidal sensitivity and the phase difference in terms of tidal period, fault creep velocity and frictional properties. Beeler et al. (2013) compared dislocation creep, dislocation glide and the RSF for 127 128 the VS region to explain the exponential relationship of equation (1) and concluded that the exponential behavior 129 was derived from the RSF for the VS region. Based on a model that follows the RSF assuming a transition from the 130 VW at a lower slip velocity to the VS at a higher slip velocity, Hawthorne and Rubin (2013) showed that the tidal 131 modulation of the fault creep velocity increases as the fault creep average velocity decreases. Based on a 132 probabilistic model, Houston (2015) interpreted that the tidal response of tremors is different between the early stage 133 and later stages of ETS due to a gradual decrease in the fault strength for the tremor sources. Beeler et al. (2018) 134 reproduced the observed tidal response of tremors of the continuous families based on a model assuming the RSF of the VW and estimated fault physical properties of the transition zone, such as the fluid pressure diffusivity and 135 136 dilatancy coefficient.

137

138 Table 1. A summary of previ	ious models and our model
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Paper that proposed	Model type	Introduction of fluid	Reproduction of tidal	Reproduction of
the model		pressure change by	response of tremor at	tidal response of
		dilatancy/compaction	the early stage of ETS	tremor at the later
				stage of ETS
Ader et al. (2012)	Deterministic	No	Tidal period > 10^8 s	Yes
Beeler et al. (2013)	Deterministic	No	Cannot reproduce	Yes
			phase difference	
Hawthorne and	Deterministic	No	Cannot reproduce	Yes
Rubin (2013)			phase difference	
Houston (2015)	Probabilistic	—	Yes ^a	Yes ^a
Beeler et al. (2018)	Deterministic	Yes	b	b
This study	Deterministic	Yes	Yes	Yes

^a The model assumption may not be valid. ^b The model is not applicable to the tidal modulation of tremors during

140 ETS.

141

142	Most of the above models only partially explain the observations of the tidal response of tremors during ETS with
143	several advantages (Table 1). Ader et al. (2012) investigated the slip behavior of a one-degree-of-freedom spring-
144	slider system under a harmonic stress perturbation with different periods. Their model could reproduce the tidal
145	response of tremors at the later stage of the ETS when considering diurnal and semidiurnal tidal periods ($\sim 10^5$ s).
146	However, the model could not reproduce the phase difference of $\delta \sim \pi/2$ observed at the early stage of ETS unless
147	adopting much longer tidal periods (e.g., 18.6 years tide) (> 10^8 s). Beeler et al. (2013) and Hawthorne and Rubin
148	(2013) modeled the tidal modulation of fault creep velocity using a purely rate-dependent friction law and a
149	frictional law assuming a transition from the VW to VS region, respectively. Their results show that the fault creep
150	velocity increases as the tidal stress increases. This suggests that these models could explain the tidal response of
151	tremors at the later stage of the ETS but not the phase difference observed at the early stage of the ETS ($\delta \sim \pi/2$).
152	The model of Houston (2015) assumes that breakage of mineral precipitates due to slip accumulation during the ETS
153	might weaken the fault strength. This model may contradict the idea of dilatant strengthening, which assumes that
154	fault strength increases with increasing pore space due to breakage of precipitating minerals (Audet and Bürgmann,
155	2014). Beeler et al. (2018) modeled the correlation between LFE clusters and tidal stress for continuous families.
156	Their model focused only on the onset of the clusters. Therefore, their model cannot be applied to the tidal
157	modulation of tremors that lasts for 1~2 weeks during ETS. Therefore, the models proposed thus far cannot fully
158	explain the aspects of the observed tidal responses of tremors.
159	Here, we propose a new model in which dilatancy/compaction occurs in the VS region to explain the observed tidal
160	response of tremors during the ETS. We find that the pore fluid pressure changes due to dilatancy/compaction
161	caused by tidal stress change in the transition zone, where the effective normal stress is low, have a significant
162	influence on the sliding behavior in the VS zone. We present governing equations for this problem in section 2. In
163	the next section, we derive an approximate solution to quantitatively describe α and δ and clarify how the model
164	responds to tidal stress changes. We reveal the physical reason for the dependence of α and δ on the fault physical
165	properties. In section 4, we estimate fault physical properties based on a comparison between the observations and
166	our model results and discuss the validity of the estimated properties. In section 5, we summarize the results.

167 **2 Methods**

168 2.1 Modeling the rate of tremor occurrence

Similar to previous studies, we assume that tremors are generated by the rupture of small brittle patches on the fault plane due to the aseismic shear slip of a larger-scale surrounding fault (Ader et al., 2012; Beeler et al., 2013; Shelly et al., 2007a). This assumption means that the tremor source is very small and that the tremor rate, R, serves as a passive meter of the creep velocity of the surrounding fault, V:

173
$$\frac{V}{V_r} = \frac{R}{R_r},$$
 (2)

where R_r and V_r denote the tremor rate and the creep velocity at a reference state, respectively. Based on this assumption, we can regard a change in the tremor rate as a change in the creep velocity. Equation (2) has been adopted in previous studies that modeled the tidal response of tremors and LFEs (e.g., Ader et al., 2012; Beeler et al., 2013).

- 178 2.2 Governing equations
- 179 2.2.1 Rate- and state-dependent friction law

180 We model the above fault creep, assuming a one-degree-of-freedom spring-slider system and employ the RSF as a 181 friction law (Ader et al., 2012). According to the RSF, the friction coefficient μ can be written as:

182
$$\mu = \mu_0 + a\log\left(\frac{V}{V_0}\right) + b\log\left(\frac{V_0\theta}{d_c}\right),\tag{3}$$

183 where μ_0 denotes the friction coefficient at a reference slip velocity V_0 , V is the slip velocity, d_c is the critical slip 184 distance, θ is the state variable, which is often interpreted as the average contact time for an asperity, and a and b 185 are fault constitutive parameters (e.g., Scholz, 1998). To represent fault creep, the constitutive parameters must 186 satisfy a > b. This regime is called VS. In equation (3), the second term on the right-hand side (RHS) represents the 187 "direct effect", which is caused by a change in the slip velocity, and the third term on the RHS represents the "evolution effect", which is caused by the temporal change in the state variable. Fault slip behavior evolves to a new 188 steady state when a sudden slip velocity change occurs and the fault slips over a distance of d_c (Dieterich, 1979). 189 190 This process can be expressed in several ways. In this study, we adopt the slip law proposed by Ruina (1983):

191
$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{V\theta}{d_c}\log\left(\frac{V\theta}{d_c}\right). \tag{4}$$

192

2.2.2 Dilatancy/Compaction

193 Dilatancy/compaction is a mechanism that relates fault gouge deformation to the behavior of the pore fluid. A 194 shear zone exists at and near the plate interface where shear slip is localized and fault gouge is present (e.g., Rice, 195 2006). We assume that the porosity change in the shear zone is caused by dilatancy/compaction (e.g., Segall et al., 196 2010; Suzuki and Yamashita, 2009). The associated behavior of pore fluids can be modeled as in the following two 197 cases. The first is an undrained model, which assumes that the pore fluid pressure changes only within the shear 198 zone (Figure S1a in the supporting information), and the second is a drained model, which assumes a "homogeneous 199 diffusion" of the pore fluid into a region adjacent to the shear zone (Segall et al., 2010) (Figure S1b in the supporting 200 information). Since our model is a one-degree-of-freedom system, the pore fluid pressure is uniform in the direction 201 of the slip plane, and the pore fluid pressure diffuses in the direction perpendicular to the slip plane. The pore fluid 202 pressure in the shear zone is spatially uniform in both the drained and undrained models. Let a tidal period be T 203 (e.g., 12.4 hours) and the characteristic timescale at which the pore fluid pressure diffuses through the shear zone be t_w . The undrained model is valid when $T \ll t_w$ (Beeler et al., 2018), and the drained model is valid when $T \gg t_w$ 204 205 (Segall et al., 2010). As described later (section 4.2), the undrained model can explain the tidal response at the early 206 and later stages of the ETS, while the drained model cannot explain the tidal response at the early stage of the ETS. 207 We focused on the results for the undrained model in the main text, which can reproduce more observations than the 208 drained model. The derivation of the governing equations and results for the drained model are shown in the 209 supporting information (Texts S1-S4, Figures S3, S4 and S6).

210 Mathematically, in the undrained model, the pore fluid pressure change in the shear zone can be described as

211
$$\frac{\mathrm{d}p}{\mathrm{d}t} = -M\frac{\mathrm{d}\phi}{\mathrm{d}t},\tag{5}$$

which is derived from the conservation of pore fluid mass (Segall et al., 1995), where dp/dt denotes a temporal change in the pore fluid pressure, *M* is the bulk modulus of the fluid and the pore space, and $d\phi/dt$ denotes a change in the porosity due to dilatancy/compaction. As described previously, for the friction coefficient (equation (3)), the porosity, which varies with dilatancy/compaction, also evolves from one steady state to another as the slip velocity changes. The evolution law for the porosity can be empirically described as

217
$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{\epsilon}{\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t},\tag{6}$$

using the state variable θ , where ϵ is a dilatancy coefficient (Segall and Rice, 1995). From equations (5) and (6), we obtain

220 $\frac{\mathrm{d}p}{\mathrm{d}t} = \epsilon M \frac{1}{\theta} \frac{\mathrm{d}\theta}{\mathrm{d}t} \,. \tag{7}$

221

222

2.2.3 A quasi-static equation of motion

The quasi-static equation of motion for the one-degree-of-freedom spring-slider model under tidal stress can bewritten as

225

 $\Delta \tau(t) + k\Delta u = \mu \sigma_{eff}(t), \tag{8}$

where $\Delta \tau(t)$ denotes the shear stress acting on the fault plane due to tides, $\sigma_{eff}(t)$ is the effective normal stress, Δu 226 227 is the relative displacement of the block to the spring pulling distance, k is the spring stiffness and μ is the friction 228 coefficient (Ader et al., 2012; Perfettini and Schmittbuhl, 2001). In our model, the effective normal stress is written as $\sigma_{eff}(t) = \sigma_{eff}^0 + \Delta\sigma(t) - \Delta p(t) - \Delta p'(t)$, where σ_{eff}^0 denotes a reference effective normal stress, $\Delta\sigma(t)$ is the 229 230 normal stress acting on the fault plane due to tides, $\Delta p(t)$ is the pore fluid pressure change due to 231 dilatancy/compaction in the shear zone, and $\Delta p'(t)$ is the pore fluid pressure change due to the tidal normal stress 232 change. When the tidal normal stress increases, $\Delta p'(t)$ also increases in proportion to the Skempton coefficient, B 233 (i.e., $\Delta p'(t) = B\Delta\sigma(t)$) (e.g., Beeler et al., 2018; Scholz et al., 2019). Using this relationship, the effective normal 234 stress can be rewritten as

$$\sigma_{eff}(t) = \sigma_{eff}^0 + (1 - B)\Delta\sigma(t) - \Delta p(t).$$
(9)

The observations show that there is almost no correlation between the tidal normal stress change and the tremor rate (Houston, 2015; Thomas et al., 2012), which indicates that the fault strength is almost unchanged due to the tidal normal stress change. This suggests that *B* is nearly equal to 1 (equation (9)). Therefore, we adopt B = 0.9 in our model. For simplicity, we assume that the tidal stresses $\Delta\sigma(t)$ and $\Delta\tau(t)$ have a common period with the same magnitude and phase (i.e., $\Delta\sigma(t) = \Delta\tau(t) = |\Delta\sigma(t)|e^{i\omega t}$).

241

235

242 2.3 Nondimensionalization of governing equations

Equations (3), (4), (7) and (8) constitute the governing equations for our model. For nondimensionalization of these equations, we selected a tidal period *T*, a reference effective normal stress σ_{eff}^0 , and a critical slip distance d_c as characteristic physical quantities (Table 2). Representing the dimensionless variables with a tilde, the result is written as:

247
$$\mu = \mu_0 + a \log\left(\frac{\tilde{V}}{\tilde{V}_0}\right) + b \log\left(\frac{\tilde{\theta}}{\tilde{\theta}_0}\right)$$

248
$$\Delta \tilde{\tau} + \tilde{K} \Delta \tilde{u} = \mu \tilde{\sigma}_{eff}$$

249
$$\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}\tilde{t}} = -\tilde{\theta}\tilde{V}\mathrm{log}\left(\tilde{\theta}\tilde{V}\right)$$

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}\tilde{t}} = \frac{U}{\tilde{\theta}}\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}\tilde{t}},\tag{10}$$

where $\theta_0 = d_c/V_0$ denotes the state variable at a reference slip velocity V_0 , $\tilde{K} = d_c k / \sigma_{eff}^0$ is the nondimensional 251 spring constant, and $U = M\epsilon/\sigma_{eff}^0$ is the dilatancy parameter. Substituting the last equation in equation (10) into the 252 nondimensionalized version of equation (9), we find that the larger U is, the more dominant the effect of $\Delta p(t)$ on 253 254 the effective normal stress is. In other words, the parameter U represents the relative importance of the 255 dilatancy/compaction to the effective normal stress change. Previous experiments and observations suggest that $\sigma_{eff}^0 \sim 10^{5 \sim 6}$ Pa (Nakata et al., 2008; Shelly et al., 2006; Yabe et al., 2015), $\epsilon \sim 10^{-4 \sim -5}$ (Samuelson et al., 2009), 256 and $M \sim 10^{10}$ Pa (Segall et al., 1995). This yields a possible range of U from 10^{0} to 10^{-2} . 257 258 The time evolution of each physical quantity is numerically calculated using the fourth-order Runge–Kutta method.

259

260	Table 2.	Parameters	of fault	physical	properties
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Parameter	Value
Reference velocity V_0	10 ⁻⁹ m/s
Spring pulling velocityV _{pl}	10 ⁻⁸ m/s
Reference frictional coefficient μ_0	0.7
Reference effective normal stress σ_{eff}^0	500 kPa

Skempton coefficient B	0.9
Spring stiffness k	10 ⁴ Pa/m
Magnitude of tidal shear stress $ \Delta \tau(t) $	1 kPa
Magnitude of tidal normal stress $ \Delta\sigma(t) $	1 kPa
Tidal period T	12.4 h
Frictional parameter a	0.003
Frictional parameter <i>b</i>	0.002
Dilatancy parameterU	10 ^{-2~0}

261

262 2.4 Definition of the tidal sensitivity (α) and the phase difference (δ)

In previous studies, α has been estimated using equation (1), and δ has been inferred using the phase difference between the tidal Coulomb stress peak and the tremor rate peak (Houston, 2015; Royer et al., 2015; Yabe et al., 2015); we define α and δ in the same way. In the following, we refer to these two parameters as the "tidal response".

To illustrate the definition of these two quantities and how to determine them, Figure 1 shows a result obtained by numerically solving the governing equations for the case of U = 0 and $d_c = 100 \,\mu\text{m}$. The solid yellow line in Figure 1a is the time evolution of $V(t)/V_{pl}$ during one tidal cycle. The solid yellow line in Figure 1b shows $V(t)/V_{pl}$ in Figure 1a against the tidal Coulomb stress change $\Delta S(t)$, and the solid green line shows the average of the upper and the lower values of $V(t)/V_{pl}$ at each $\Delta S(t)$ on the horizontal axis, where

272 $\Delta S(t) = \Delta \tau(t) - \mu_{pl}(1 - B)\Delta \sigma(t)$ (11)

273 (e.g., Beeler et al., 2018; Scholz et al., 2019), and μ_{pl} is the steady-state friction coefficient at velocity V_{pl} . The 274 reason why $\Delta S(t)$ is described by $\Delta \tau(t) - \mu_{pl}(1 - B)\Delta\sigma(t)$ instead of $\Delta \tau(t) - \mu_{pl}\Delta\sigma(t)$ is that, for a poroelastic 275 medium, the effective normal stress change due to tides is described by $(1 - B)\Delta\sigma(t)$ from equation (9). α is 276 obtained by fitting the following equation to the average of $V(t)/V_{pl}$ (solid green line in Figure 1b):

277
$$\frac{V(t)}{V_{pl}} = c e^{\alpha \Delta S(t)}.$$
 (12)

278 In the fitting, a constant c(< 1) is simultaneously determined. If c is not estimated, the following problem arises. 279 Since our model pulls the block through the spring with velocity V_{pl} , the block must slip with V_{pl} on a long-term average $(\int_0^T V(t) dt/T = V_{pl}$ with $T \gg 0$). Equation (12) shows $V(t) \propto e^{\Delta S(t)}$, indicating that the slip velocity 280 increases nonlinearly with $\Delta S(t)$. If $V(t) = V_{pl}$ at S(t) = 0 (equivalently c = 1), it is unphysical 281 because. $\int_0^T V(t) dt/T > V_{pl}$. Therefore, $V(t) < V_{pl}$ at S(t) = 0 (equivalently c < 1) is needed. The parameter c282 283 adjusts the average velocity but has nothing to do with the estimation of fault physical properties in the following. 284 Figure S2 shows the obtained value of c. The solid black line in Figure 1b shows the fitted result. δ is defined as the phase difference between the $\Delta S(t)$ peak and the $V(t)/V_{pl}$ peak (see δ of Figure 1a), where δ is positive when the 285 286 $V(t)/V_{pl}$ peak precedes the $\Delta S(t)$ peak (i.e., δ in Figure 1a is negative).



287

Figure 1. The numerical solution of equation (10) for U = 0 and $d_c = 100 \,\mu\text{m}$. (a) Determination of the phase 289 290 difference (δ). The horizontal axis denotes time normalized by the tidal cycle, and the value from 0 to 1 indicates 291 one tidal cycle. The vertical axis represents the slip velocity normalized by the reference velocity, $V(t)/V_{pl}$. The 292 solid yellow line shows $V(t)/V_{pl}$, and the dashed blue line shows the tidal Coulomb stress, $\Delta S(t) \times 10^3$, normalized by the reference effective normal stress, σ_{eff}^0 . The dashed black line represents the phase when $\Delta S(t)$ reaches the 293 294 maximum, and the solid black line represents the $V(t)/V_{pl}$ peak. The phase difference δ is defined so that it is 295 positive when the $V(t)/V_{pl}$ peak precedes the $\Delta S(t)$ peak. (b) Determination of the tidal sensitivity (α). The horizontal axis is $\Delta S'(t) = \Delta S(t) / \sigma_{eff}^0 \times 10^3$. The vertical axis represents the slip velocity normalized by the 296 297 reference velocity, $V(t)/V_{pl}$. The solid yellow line shows $V(t)/V_{pl}$ in (a). The solid green line shows the average of

the upper and lower velocities at each value of $\Delta S'(t)$. *c* and α in equation (12) are determined by a least squares method by fitting equation (12) against the green solid line. The solid black line shows the fitted result ($\alpha =$ 0.72, *c* = 0.89).

301

302	2.5 An	approximate	solution	for α	and δ
			001001011	101 00	

303 To clarify how the tidal responses depend on the fault physical properties, we analytically derived an approximate 304 solution for α and δ . The result is shown in section 3.1.

305 **3 Result**

306 3.1 Derivation and verification of the approximate solution

When the magnitude of the tidal Coulomb stress change $|\Delta S(t)|$ is small enough $(|\Delta S(t)| \ll (a - b)\sigma_{eff}^0)$, we assume that the perturbation of each physical quantity caused by $|\Delta S(t)|e^{i\omega t}$ is proportional to $e^{i\omega t}$, where $\omega =$ $2\pi/T$ is the angular velocity of the tide (Segall, 2010; Ader et al., 2012). In other words, the physical quantities can be written as $V(t) = V_{pl} + \Delta V e^{i\omega t}$, $\theta(t) = \theta_{pl} + \Delta \theta e^{i\omega t}$ and $p(t) = p_0 + \Delta p e^{i\omega t}$, where θ_{pl} denotes the steadystate variable at $V = V_{pl}$, p_0 is the reference value of pore fluid pressure, and ΔV , $\Delta \theta$ and Δp are the magnitudes of the perturbation. Substituting these forms into equations (3), (4), (7), and (8), and after some algebra, the perturbation of the nondimensionalized slip velocity, $\Delta \tilde{V}$, can be written as

314
$$\frac{\Delta \tilde{V}}{\tilde{V}_{pl}} = \frac{2\pi i}{\tilde{K}\tilde{V}_{pl} + 2\pi iA} |\Delta \tilde{S}(t)|, \qquad (13)$$

315 where

316
$$A = a - \frac{1}{1 + i\frac{T_{\theta}}{T}} \left(b - \mu_{pl}U\right) \tag{14}$$

317 and

318
$$T_{\theta} = 2\pi \frac{d_c}{V_{pl}}.$$
 (15)

- Equation (15) represents a characteristic timescale on which the state variable evolves (Ader et al., 2012). From the relationship of $\Delta \tilde{V} e^{i\omega t} = \tilde{V}(t) - \tilde{V}_{pl}$, equations (13) and (14) can be rewritten as $\tilde{V}(t)/\tilde{V}_{pl} = 1 +$
- 321 $2\pi i \Delta \tilde{S}(t) / (\tilde{K}\tilde{V}_{pl} + 2\pi i A)$. We assume that this equation is the Taylor expansion of the RHS of

322
$$\frac{\tilde{V}(t)}{\tilde{V}_{pl}} = \exp\left(\frac{2\pi i}{\tilde{K}\tilde{V}_{pl} + 2\pi iA}\Delta\tilde{S}(t)\right)$$
(16)

to the first order. Then, comparing equation (16) with equation (1), we find that the tidal sensitivity (α) and the phase difference (δ) can be written as

325
$$\alpha = \operatorname{Re}\left(\frac{2\pi i}{(\widetilde{K}\widetilde{V}_{pl} + 2\pi iA)\sigma_{eff}^{0}}\right)$$
(17)

326
$$\delta = \arg\left(\frac{2\pi i}{\tilde{K}\tilde{V}_{pl} + 2\pi iA}\right)$$
(18)

For U = 0, where dilatancy/compaction is neglected, Ader et al. (2012) presented a linearized approximation

solution and a numerical solution. We confirmed that equations (13) and (18) are consistent with the

nondimensionalized version of equation (3) of Ader et al. (2012), who examined tidal responses for different values

of T. However, how the tidal response changes with different values of d_c was not studied in detail for the period of

~12 h, which is the dominant period of tides. Therefore, we examined how the tidal response changes with changes

in d_c or T_{θ} (equation (15)) for this period, since a comparison between our model and observations of the tidal

response enables us to infer d_c in the actual geophysical situation. Figures 2a and 2b show α and δ , respectively. In

these figures, the solid green line and green dots represent the numerical solution of equation (10) and the

approximate solution, respectively. The approximate solution and the numerical solution agree with each other

within 15% for most cases. When $T_{\theta}/T \sim 10^{-1}$, the approximate solution is less accurate for both α and δ . This

means that the accuracy of the approximate solution can deteriorate when the nonlinearity is stronger (i.e., α is

338 larger).

For $U \neq 0$, the approximate solution agrees with the numerical solution within 10% for all cases (the deviation between the approximate and numerical solutions is the maximum when $T_{\theta}/T \sim 140$ and U = 1. In this case,

- 341 $\delta \sim 1.134$ for the numerical solution and $\delta \sim 1.023$ for the approximate solution, resulting in a relative error of
- $\sim 9.7\%$). The good agreement is attributed to the fact that α is relatively small (at most $\sim 0.7 \text{ kPa}^{-1}$); thus, the
- nonlinearity is weaker. This indicates that the approximate solution is valid regardless of the value of T_{θ}/T when
- 344 $U \neq 0$.

345 3.2 Dependence of the tidal response on the fault physical properties

Based on the approximate solution and an analysis of the quasi-static equation of motion, we clarify how α and δ during an ETS depend on the fault physical properties. The specific range of physical properties that can explain the observations is discussed in section 4.

349

3.2.1 Factors governing the tidal response during the ETS

We assume that the VS region surrounding tremor patches slides at an average velocity of V_{pl} . In our

spring-slider model, this situation is represented by setting the velocity of the pulling spring to V_{pl} . We can

352 apply this model to fault creep during the ETS, which occurs over a shorter time span than secular plate subduction.

353 Geodetic observations show that the fault creep velocity during an ETS is $\sim 10^{-6 \sim -9}$ m/s (e.g., Meade and

Loveless, 2009; Schwartz and Rokosky, 2007). Therefore, we set V_{pl} as 10^{-8} m/s in the following numerical

simulation. In addition, the frictional parameters a and b are chosen so that a - b is small because it has been

suggested that a - b decreases in the transition zone (e.g., Liu, 2013; Matsuzawa et al., 2010). The other parameters

are similar to those used in previous studies (Ader et al., 2012; Hawthorne and Rubin, 2013). Table 2 shows the adopted parameters. For these parameters, we can confirm that $|\tilde{K}\tilde{V}_{pl}| \ll |2\pi i A|$. Then, the tidal response (equations

359 (17) and (18)) can be approximated as

$$360 \qquad \qquad \alpha \sim \operatorname{Re}\left\{\left(A\sigma_{eff}^{0}\right)^{-1}\right\}$$
(19)

361

$$\delta \sim \arg\{A^{-1}\}.$$
 (20)

Combining these equations with equation (14), we note that α and δ depend on T_{θ}/T and U. The former parameter T_{θ}/T prescribes the response of the state variable to the tide, which is uniquely determined once we determine V_{pl} , d_c and T. T_{θ}/T is constant throughout the tidal cycle. $T_{\theta}/T \ll 1$ means that the state variable is close to the steady state value (d_c/V) throughout a tidal cycle, and $T_{\theta}/T \gg |b - \mu_{pl}U|/a$ means that the state variable is almost constant throughout a tidal cycle (see section 3.2.3 for more details). In the following, we focus on these two parameters, T_{θ}/T and U, to discuss the tidal response.

368 3.2.2 A balance of the stress changes

From Figures 2a and 2b, we see a large difference between the cases for U = 0 and $U \neq 0$. The reason for this large difference can be understood by using the following equation, which is derived from the quasi-static equation of motion (equation (8)) (for the derivation, see Appendix A):

372
$$\Delta S(t) \sim \sigma_{eff}^0 \left(-\mu_{pl} U ln\left(\frac{\theta}{\theta_{pl}}\right) + a ln\left(\frac{V}{V_{pl}}\right) + b ln\left(\frac{\theta}{\theta_{pl}}\right) \right).$$
(21)

In equation (21), the left-hand side (LHS) and RHS correspond to the tidal Coulomb stress and frictional strength, respectively. The first, second, and third terms on the RHS represent the dilatancy/compaction effect, the direct effect, and the evolution effect, respectively. When U = 0, the first term vanishes, and the tidal response (equations (19) and (20)) obtained in this study is consistent with the result discussed in Chapter 4.1 of Hawthrome and Rubin (2013). Therefore, we analyze the tidal response for $U \neq 0$ below.

- 378 3.2.3 Analysis of the tidal response for $U \neq 0$
- Figures 2a and 2b show that the tidal response can be classified into three cases according to the value of T_{θ}/T
- because the value affects the degree to which the first term of $a\{1 (b \mu_{pl}U)/a(1 + iT_{\theta}/T)\}$ is dominant (see
- equation (14)). The condition for the first term on the RHS of equation (14) to be negligibly small is |b b| = 1
- 382 $\mu_{pl}U|/a|1 + iT_{\theta}/T| \ll 1$. Using the parameter set shown in Table 2, we obtain $|b \mu_{pl}U|/a \sim O(1)$, so $T_{\theta}/T \gg$
- $|b \mu_{pl}U|/a$ is required for the above inequality to hold. Conversely, the condition for the first term on the RHS of equation (14) becoming dominant is when the value of $a(1 + iT_{\theta}/T)$ becomes $\sim a$. In other words, $T_{\theta}/T \ll 1$. The
- other case is the intermediate region between these two limits.
- First, we consider the case where $T_{\theta}/T \gg |b \mu_{pl}U|/a$. We find that for larger values of T_{θ}/T , α and δ converge to the same values regardless of the value of $U(T_{\theta}/T \sim 10^4$ in Figures 2a and 2b). When $T_{\theta}/T \gg |b - \mu_{pl}U|/a$, the first term on the RHS of equation (14) can be ignored ($A \sim a$). Therefore, substituting $A \sim a$ into equations (19) and (20), α and δ become $a\sigma_{eff}^0$ and 0, respectively, regardless of the value of U. Because the state variable evolves more slowly than the tidal Coulomb stress change, the state variable is almost constant ($\theta \sim \theta_{pl}$) throughout a tidal cycle. Then, the dilatancy/compaction and the evolution effect term of equation (21) are almost zero. Therefore,
- equation (21) can be approximated as $\Delta S(t) \sim a\sigma_{eff}^0 \log(V/V_{pl})$ or $V \sim V_{pl} e^{\Delta S(t)/a\sigma_{eff}^0}$. This means that $\alpha = 1/a\sigma_{eff}^0$.

393 Moreover, the form of this equation indicates that the slip velocity peak agrees with the tidal Coulomb stress peak in 394 time, which means that $\delta = 0$.

395 Next, we consider the case where $T_{\theta}/T \ll 1$. In this case, α depends on U and takes a small value when U is large $(T_{\theta}/T \sim 10^{-2} \text{ in Figure 2a})$. However, δ converges to zero regardless of the value of $U(T_{\theta}/T \sim 10^{-2} \text{ in Figure 2b})$. 396 When $T_{\theta}/T \ll 1$, we obtain $\alpha = 1/(a - b + \mu_0 U)\sigma_{eff}^0$ and $\delta = 0$ from equations (14), (19) and (20). Because the 397 398 state variable evolves more rapidly than the tidal Coulomb stress change, the state variable is close to the steady 399 state value $(\theta \sim d_c/V)$ throughout a tidal cycle. This is derived by considering $d\theta/dt \sim 0$ in equation (4). Then, 400 equation (21) can be approximated as $\Delta S(t) \sim (a - b + \mu_{pl} U) \sigma_{eff}^0 \log(V/V_{pl})$. As before, the form of this equation 401 explains the above values of α and δ . Moreover, it is clear from the form of α that it decreases as U increases. 402 Finally, we consider the case of the intermediate region between the above two limit cases $(T_{\theta}/T \ll 1 \text{ and } T_{\theta}/T \gg$ $|b - \mu_{pl}U|/a$). Figure 2a shows that α varies smoothly and connects the limit values for $T_{\theta}/T \gg |b - \mu_{pl}U|/a$ and 403 $T_{\theta}/T \ll 1 \ (T_{\theta}/T \sim 10^{0})^{-1}$ in Figure 2a). Figure 2b shows that the maximum value of δ approaches $\pi/2$ as U 404 increases $(T_{\theta}/T \sim 10^{0})$ in Figure 2b). To clarify why this occurs, we compared the time variation of the tidal 405 406 Coulomb stress term ($\Delta S(t)$ of equation (21)), the dilatancy/compaction effect term and the evolution effect term in 407 equation (21). Figure 3 shows these three terms for U = 1 and U = 0.01. For U = 1, the amplitude of the dotted 408 blue line representing the tidal Coulomb stress and the amplitude of the solid black line representing the 409 dilatancy/compaction effect are almost the same, and there is a slight phase difference between them. Representing 410 this phase difference as $\beta \ll \pi$, we see from the balance between the solid black line and the dot blue line in Figure 3 that $-\mu_{pl}U\sigma_{eff}^0\log(\theta/\theta_{pl})\sim|\Delta S(t)|e^{i\omega(t-\beta)}$. The dashed black line representing the evolution effect is negligibly 411 small $(b\sigma_{eff}^0 \log(\theta/\theta_{pl}) \sim 0)$. Substituting these into equation (21), and after some algebra (Appendix B), we find 412 413 that

414
$$\log\left(\frac{V}{V_{pl}}\right) \propto \operatorname{Re}\left\{e^{i\omega\left(t+\frac{\pi}{2}\right)}\right\}.$$
 (22)

This formula indicates that the slip velocity peak agrees with the tidal Coulomb stress rate peak $(T_{\theta}/T \sim 10^1 \text{ in the})$ black line of Figure 2a). For U = 0.01, the phase difference δ is small $(T_{\theta}/T \sim 10^0 \text{ in the blue line of Figure 2a})$. This difference can be explained by considering the balance in equation (21). The amplitude of the solid yellow line representing the dilatancy/compaction effect in equation (21) is smaller than the amplitude of the dotted blue line representing the tidal Coulomb stress. Furthermore, the dashed yellow line representing the evolution effect 420 decreases when the dilatancy/compaction effect term (solid yellow line) is larger and vice versa. Therefore, the

- 421 amplitude of the sum of these two effects becomes even smaller than the amplitude of $\Delta S(t)$. For the stress balance
- 422 of equation (21) to be satisfied, the direct effect term (second term on the RHS) should balance the difference
- 423 between $\Delta S(t)$ and the sum of the above two effects. This means that for a smaller U, a larger direct effect is
- 424 needed. The dominance of the direct effect term indicates that δ is small, as we have seen for the case of $T_{\theta}/T \gg$
- 425 $|b \mu_{pl}U|/a$, which explains why δ is closer to zero for U = 0.01 than for U = 1, as shown by Figure 2 (Figure S5)
- 426 shows the time variation of the tidal Coulomb stress term ($\Delta S(t)$ of equation (21)), the dilatancy/compaction effect
- 427 term and the evolution effect term in equation at U = 0.1).
- 428 Figure S6 schematically illustrates how the pore fluid (dilatancy/compaction effect), tidal stresses and
- fault creep velocity generally evolve during one tidal oscillation. Because the case for U = 1 can
- 430 reproduce the observations at both stages of the ETS, we show only the case for U = 1.



431

Figure 2. (a) The numerical solution of α (dots) and the approximation solution (equation (19)) (solid line). From equation (19), we can derive $\alpha = 1/(a - b + \mu_0 U)\sigma_{eff}^0$ when $T_{\theta}/T \ll 1$ and $\alpha = 1/a\sigma_{eff}^0$ when $T_{\theta}/T \gg$ $|b - \mu_{pl}U|/a$. Specific values for U = 0.01 are displayed in the figure at $T_{\theta}/T \gg |b - \mu_{pl}U|/a$ and at $T_{\theta}/T \ll 1$. (b) The numerical solution of δ (dots) and the approximation solution (equation (20)) (solid line). The differences in

color represent the differences in the dilatancy parameter U.

437

436



438

Figure 3. Time evolution of the tidal Coulomb stress (tCs) (blue dotted line), the dilatancy/compaction effect (D/C) (solid lines), and the evolution effect (Evo) (dashed lines) in equation (21). The horizontal axis denotes time normalized by the tidal cycle, and the values from 0 to 1 indicate one tidal cycle. The vertical axis denotes the tidal Coulomb stress/frictional strength normalized by σ_{eff}^0 . The numerical solutions for $T_{\theta}/T = 14$ and U = 1 are shown in black, and those for $T_{\theta}/T = 1.4$ and U = 0.01 are shown in yellow.

444

445 **4 Discussion**

446

4.1 Application of the model to the observed tidal response during the ETS

447 As mentioned in the introduction, most of the previous models are unable to account for the phase difference of

448 $\delta \sim \pi/2$, which is observed at the early stage of ETS. As shown below, our model reproduces the tidal response

during the ETS, including the phase difference, for a specific range of fault physical properties constrained by

- 450 experiments, geological studies, and numerical modeling.
- 451 The observed tidal responses typically show $\alpha \leq 0.1 \text{ kPa}^{-1}$ and $\delta \sim \pi/2$ at the early stage of the ETS and

452 $\alpha \sim 0.7 \text{ kPa}^{-1}$ and $\delta \sim 0$ at the later stage of the ETS. The slip velocity of the fault, which rapidly increases at the

- 453 onset of the ETS, decreases to below steady-state subduction velocity with the progress of the ETS. In our model,
- ETS is represented by setting V_{pl} higher than the steady-state subduction velocity (Table 2). Considering that

455 $V_{pl} \sim 10^{-8 \sim -6}$ m/s, at the moment, we assume $V_{pl} \sim 10^{-6}$ m/s at the early stage of the ETS and $V_{pl} \sim 10^{-8}$ m/s at the 456 later stage.

457

4.2.1 The ranges of *U* reproduce the observation

We see from Figure 2a and 2b that the model reproduces the observed tidal response at the early stage of ETS ($\alpha \leq 0.1, \ \delta \sim \pi/2$) when $T_{\theta}/T \sim 10$ and $U \sim 1$. This case corresponds to the last of the three categories of T_{θ}/T presented in section 3.2.3. We have seen that the first term on the RHS of equation (21) (the dilatancy/compaction effect term), which has a phase delay with respect to the tidal Coulomb stress change, dominates in the frictional strength change, and δ becomes $\pi/2$. The dominance of the dilatancy/compaction effect term reduces the direct effect term, which results in a smaller variation in the slip velocity ($\alpha \leq 0.1$). For $V_{pl} = 10^{-6}$ m/s, we obtain $d_c = 10^{-1}$ m from the condition of $T_{\theta}/T \sim 10$ (equation (15)).

Up to this point, we have used the undrained model. The drained model assumes that the pore fluid pressure 465 466 diffuses outside the shear zone, as shown in equation (S1). This implies that the shear zone has a high permeability. From text S1 and S2 in the supporting information, we derived the dilatancy parameter (E_p) in the drained model 467 and estimated the valid range of the parameter. Within the range, the drained model can reproduce α for both the 468 initial and later stages by assuming $d_c = 10^{-2}$ m (the solid black line in Figure S3a in the supporting information). 469 470 However, the drained model cannot reproduce $\delta \sim \pi/2$ at the early stage (Figure S3b in the supporting information). 471 In addition, we confirmed that even when the dilatancy parameter is an order of magnitude larger than the 472 reasonable value, we cannot reproduce $\delta \sim \pi/2$ at the early stage (Figure S7 in the supporting information). The 473 result that only the undrained model can reproduce both α and δ suggests the low permeability of the shear zone. 474 This indicates the possibility that our model can constrain the frictional parameters and the dilatancy coefficient as 475 well as a hydraulic property of the fault through a comparison with observations of tidal response. Here, we return to the application of the undrained model. Focusing on the case of $U \sim 1$, which explains the early 476 stage, we see that the model can reproduce the observed tidal response at the later stage of the ETS ($\alpha \sim 0.7 \text{ kPa}^{-1}$ 477

478 and $\delta \sim 0$) when $T_{\theta}/T \gtrsim 10^3$. This case corresponds to $T_{\theta}/T \gg |b - \mu_{pl}U|/a$ described in the three categories in

479 section 3.2.3. As noted above, the phase advance disappears ($\delta \sim 0$) as the direct effect term (second term) on the

480 RHS of equation (21) becomes dominant in the frictional strength change, and α asymptotically reaches a value that

481 is independent of $U(\alpha \sim 1/a\sigma_{eff}^0)$. For $V_{pl} = 10^{-8}$ m/s, the condition of $T_{\theta}/T \gtrsim 10^3$ indicates that $d_c \gtrsim 10^{-1}$ m.

On the other hand, Figure 2a shows that the model for U = 0.01 and U = 0.1 can explain the tidal response at the 482 later stage of ETS when $d_c \gtrsim 10^{-3}$ m and $d_c \gtrsim 10^{-2}$ m, respectively. This means that if we apply the model only 483 to the tidal response at the later stage of ETS, d_c might be underestimated. 484

485 The above comparison between the model and observations shows that the dilatancy/compaction effect is dominant at the early stage of the ETS, while the dilatancy/compaction effect is negligible at the later stage of the ETS. Figure

487 4 schematically illustrates the physical process suggested by our model. First, we see the early stage of ETS (Figure

488 4a). A higher tide level increases the ocean load and reduces $\Delta S(t)$. At low tide ($\delta \sim 0$), $\Delta S(t)$ takes its maximum.

489 However, the effect generated by the low tide is almost canceled out by the significant increase in the normal stress

490 due to the dilatancy/compaction effect. At the mean tide $(\delta \sim \pi/2)$, $\Delta S(t)$ is zero. The dilatancy/compaction effect is

491 reduced but still able to decrease the normal stress. Consequently, the slip velocity or the tremor rate reaches the

492 maximum. Next, we see the later stage of ETS (Figure 4b). Since the dilatancy/compaction effect is always

493 negligible at this stage, the slip velocity is maximized when $\Delta S(t)$ becomes the largest (low tide).

494 Incidentally, we can reproduce the observed tidal response (α, δ) as well as the observation that the number of

495 tremors decreases by one or two orders of magnitude at the later stage of ETS compared to that at the early stage.

496 This is because in equation (2), the tremor rate is proportional to the fault creep velocity, meaning that the tremor

rate at the later stage of the ETS ($V_{pl} \sim 10^{-8} \text{ m/s}$) is two orders of magnitude less than the tremor rate at the early 497

stage of the ETS ($V_{nl} \sim 10^{-6} \text{ m/s}$). 498

499

486



Figure 4. A schematic illustration of the relationship between the fault creep velocity and tide level. For simplicity, only the normal stress change is represented. (a) Early stage of ETS. The sum of the normal stress due to the dilatancy/compaction effect (black arrows) and the tidal normal stress (white arrows) becomes the largest in the sense of enhancing fault slip at $\delta \sim \pi/2$. (b) Later stage of ETS. The dilatancy/compaction effect is negligible, and the fault creep velocity reaches its maximum at $\delta \sim 0$.

500

507 4.2.2 The ranges of d_c reproduce the observation

508 In section 4.2.1, we showed that the observation can be reproduced when $d_c \sim 10^{-1}$ m, assuming $V_{pl} \sim 10^{-8 \sim -6}$ m/s.

- 509 On the other hand, Figure 5 (a) and (b) show that the observation cannot be reproduced when d_c is other than
- 510 ~10⁻¹ m. For example, when $d_c = 10^{-3}$, $\alpha \sim 0$ and $\delta \sim 0$ at the early stage of ETS ($V_{pl} \sim 10^{-6}$ m/s) and $\alpha \sim 0$
- 511 and $\delta \sim \pi/2$ at the later stage of ETS ($V_{pl} \sim 10^{-8}$ m/s).
- 512 The above range of $d_c \sim 10^{-1}$ m, which explains the observation, was determined from the conditions that
- 513 $T_{\theta}/T \sim 10$ at the early stage of ETS and $T_{\theta}/T \gg |b \mu_{pl}U|/a$ (~100) at the later stage of ETS. In principle,
- these two conditions can be met for any V_{pl} if d_c is appropriately chosen, considering the form of equation (15). In
- 515 practice, however, d_c can be constrained based on V_{pl} estimated from observations and numerical simulations.
- Based on observations and simulations, the slip velocity of the SSE is 10^{-9-6} m/s (e.g., Goswami and Barbot 2018; Schwartz and Rokosky, 2007; Segall et al. 2010). The slip velocity of the fault, which rapidly increases at the onset of the ETS, decreases to the inter-ETS period velocity with the progress of the ETS. Based on the above conditions, we can reproduce the observation not only for $V_{pl} \sim 10^{-8-6}$ m/s but also for $V_{pl} \sim 10^{-9-7}$ m/s by employing $d_c = 10^{-2}$ (Figure S8). When d_c is other than 10^{-2} m, the observation cannot be reproduced.
- 521 Therefore, our model and observations are consistent for critical slip distances of 0.1-0.01 m in the transition







Figure 5. (a) The approximation solution of α (equation (19)) when $V_{pl} = 10^{-6}$ m/s (black) and 10⁻⁸ m/s (yellow). The horizontal axis denotes the critical slip distance d_c . The blue dots show that α for $d_c = 0.13$ m increases from ≤ 0.1 to ~0.7 as V_{pl} decreases from 10⁻⁶ m/s to 10⁻⁸ m/s. (b) The

same as in (a) but for the approximation solution of δ (equation (20)). The blue dots show that δ decreases from $-\pi/2$ to -0 as V_{pl} decreases from 10^{-6} m/s to 10^{-8} m/s.

529

530 4.3 The constrained physical fault properties

- 531 For our model to simultaneously reproduce the observed tidal responses at the early and later stages of the ETS, the
- following four conditions must be satisfied: $U(=M\epsilon/\sigma_{eff}^0) \sim 1$, $d_c \sim 10^{-1}$ m, the occurrence of

dilatancy/compaction in the fault creep region (i.e., a > b) and low permeability within the shear zone (undrained

- model). Below, we discuss the validity of these conditions.
- 535 4.3.1 The dilatancy parameter U

Samuelson et al. (2009) obtained a dilatancy coefficient, and Segall et al. (1995) obtained bulk moduli of the fluid and pore space. These results yield $\epsilon \sim 10^{-4 \sim -5}$ and $M \sim 10^{10}$ Pa (equation (7)). We assume that these experimentally obtained values are of the same magnitude in the transition zone. Substituting these values into $U(=\epsilon M/\sigma_{eff}^0) = 1$, which reproduces the observed tidal response, we obtain $\sigma_{eff}^0 = \epsilon M U \sim 10^{5 \sim 6}$ Pa, which supports a near-lithostatic pore fluid pressure (e.g., Audet et al., 2009; Nakata, 2008; Shelly et al., 2006; Yabe et al., 2015).

541 4.3.2 The critical slip distance d_c

The results of friction experiments on rocks and gouges show $d_c \sim 10^{-4 \sim -6}$ m (e.g., Marone, 1998). Our results 542 $(d_c \sim 10^{-1})^{2}$ m) are 2~5 orders of magnitude larger. The much larger critical slip distance can be explained by 543 544 considering the differences in roughness between laboratory surfaces and natural faults (Scholz et al., 1988) and the 545 differences in the thickness of the shear zone between experimental and natural faults (Marone and Kilgore, 1993). 546 Numerical models assuming the RSF also adopt a critical slip distance larger than that in the experimental results. 547 For example, Nakata et al. (2012) successfully modeled the SSE and aftershocks after the ~M7 earthquake in Hyuga-nada, Japan, with $d_c = 10^{-1 \sim 0}$ m. Maury et al. (2014) calculated a time evolution of shear stress for the SSE 548 549 in Mexico and estimated that the critical slip distance that can quantitatively reproduce the observed results is 5×10^{-2} m. Kawamura et al. (2018) applied a 1-D multidegree of freedom spring-slider model with $d_c = 10^{-2}$ m 550

551	to reproduce various types of fault slip, such as fast slip, source nucleation, aftershock, and SSE. Our analysis of the
552	tidal response during ETS also supports d_c with the order of $10^{-1 \sim -2}$ m.

553

4.3.3 The occurrence of dilatancy/compaction in the fault creep region

Numerical models that have been proposed thus far generally require the presence of a VW region (a - b < 0) to

reproduce SSE (e.g., Liu and Rice, 2005; Segall et al., 2010). Some models have proposed a mechanism by which

556 SSE occurs in the VS regime, such as the generation of a negative Coulomb stress change due to fault valve action

(Perfettini and Ampuro 2008) and the transition of the RSF from the VW at low speeds to the VS at high speeds

(e.g., Im et al., 2020; Peng and Rubin, 2018; Shibazaki and Iio. 2003). Our model employs the framework of the VS

and expresses the velocity of the slow slip by V_{pl} phenomenologically.

560 The above two models assuming the VS (e.g., Im et al., 2020; Peng and Rubin, 2018; Perfettini and Ampuro, 2008;

561 Shibazaki and Iio 2003) do not consider the time variation of pore fluid pressure. On the other hand, Beeler et al.

562 (2018) developed a model that considers the time variation of pore fluid pressure in the VW region. However, it

563 cannot be applicable to the tidal modulation of the tremor rate during ETS. Our results show that when we assume

the framework of the VS, the observed tidal response at the early stage of ETS cannot be reproduced unless

- 565 dilatancy/compaction occurs.
- 566

4.3.4 The fluid pressure diffusivity derived from the undrained condition

For the undrained model, $T \ll t_w$ must be satisfied (section 2.2.2). Using this condition, we can quantitatively constrain the fluid pressure diffusivity as follows. We assume that the thickness of the shear zone is w and the fluid pressure diffusivity in the shear zone is c_{hyd}^* . Then, a dimensional analysis shows that $w \sim \sqrt{t_w c_{hyd}^*}$, where t_w denotes the characteristic timescale on which the pore fluid pressure diffuses through the shear zone. Therefore, the condition of $T \ll t_w$ can be rewritten as $T \ll w^2/c_{hyd}^*$.

We estimate *w* in the transition zone in the following manner since it cannot be observed directly. A drilling investigation and structural analyses of drill cores on the Nojima Fault revealed that *w* in the seismogenic zone is $\sim 10^{1/2}$ m (Lin and Nishikawa, 2019). It is generally expected that *w* in the VS region is larger than in the VW region (e.g., Chen and Rampel, 2015). Therefore, we assume $w \sim 10^{0 \sim 1/2}$ m in the VS region. Then, the above undrained condition yields $c_{hyd}^* \ll 2 * 10^{-5 \sim -4}$ m²/s. This value of c_{hyd}^* is consistent with Branut (2021), who

- 577 reported that the observed rupture propagation of an SSE could be reproduced by a crack propagation model at
- 578 $w \sim 6 \ cm$ and $c_{hyd}^* \sim 10^{-6} \ m^2/s$. Previous studies have shown that the c_{hyd}^* of the seismogenic zone is

 $\sim 10^{-8-3} \text{ m}^2/\text{s}$ (Yamashita and Tsutsumi, 2018). Our results suggest that the shear zone in the transition zone is

580 probably as impermeable as that in the seismogenic zone.

581 4.4 Other effects than dilatancy/compaction

We have seen that the dilatancy/compaction effect is important to explain the phase difference ($\delta \sim \pi/2$) in the tidal response. In this section, we examine whether other effects could explain $\delta \sim \pi/2$. The following two possibilities are considered.

585 In the first case, a change in the state variable is introduced due to the normal stress acting on the fault plane 586 (Linker and Dieterich, 1992). In this case, the time variation of the state variable can be written as follows:

587
$$\frac{d\theta}{dt} = -\frac{V\theta}{d_c} \log\left(\frac{V\theta}{d_c}\right) - \frac{\gamma}{b}\frac{\dot{\sigma}}{\sigma}\theta,$$
 (23)

where γ is a constitutive parameter representing a normal stress dependence. In general, $\gamma \sim O(0.1)$. Therefore, we adopt $\gamma = 0.2$ and solve the governing equations of our model replacing the evolution law (equation (4)) with equation (23). The results indicate that the difference caused by considering the effect of normal stress on the state variable is less than 1%. Therefore, the influence of the Linker-Dieterich effect is small and does not provide a reason for the large phase difference.

In the second case, tidal Coulomb stress can directly destroy the tremor source instead of aseismic slip on the surrounding fault. This effect is ignored in our model. If this is the case, the tremor rate is proportional to the tidal Coulomb stressing rate (i.e., $\delta \sim \pi/2$) (Beeler et al., 2013; Lockner and Beeler, 1999). This direct effect of the tidal Coulomb stress should become clearer when the aseismic slip on the surrounding fault is smaller, i.e., at the later stage of the ETS (Royer et al., 2015). However, the observed result shows $\delta \sim 0$ at the later stage, indicating that the direct effect is smaller.

- None of the above effects can explain the phase difference of $\delta \sim \pi/2$, and thus, the pore fluid pressure change due
- to dilatancy/compaction is more likely to cause the large phase difference at the early stage of the ETS.

4.5 Application to the tidal response of continuous families

By setting the value of V_{pl} to a steady-state plate convergence velocity (e.g., 10^{-9} m/s), we can examine the range 602 603 of d_c and U in which our model reproduces the tidal response of continuous families. The observations show that the tidal response of continuous families is $\delta \sim 0$ (Ide and Tanaka, 2014; Thomas et al., 2012) and $\alpha \sim 1.5$ kPa⁻¹ 604 605 (Thomas et al., 2012), for example. We examine whether these observations can be reproduced with parameters that reproduce the tidal response of episodic families ($d_c \sim 10^{-1}$ m, $U \sim 1$) (section 4.2). In the case of $d_c = 10^{-1}$ m, 606 $V_{pl} = 10^{-9}$ m/s and $T_{\theta}/T \sim 10^4$, we obtain $\alpha \sim a \sigma_{eff}^0$ (= 0.67 kPa⁻¹) and $\delta \sim 0$ (section 3.2.2). Therefore, by slightly 607 reducing the value of σ_{eff}^0 , the tidal responses of continuous families and episodic families can be reproduced with 608 609 similar values of the fault physical properties.

610 4.6 Limitations of our model

611 Our model, in which fault creep is represented with a one-degree-of-freedom spring block of the VS regime (a - a)

b > 0), necessarily fails to include the occurrence of slow slip accompanying a rapid release of the accumulated

613 stress. For this reason, we expressed the occurrence of slow slip by the difference in V_{pl} and assumed that the tremor

614 rate during ETS is proportional to the fault creep velocity (equation (2)). Our model accounted for the tidal response

of tremors during ETS, but this does not hold if the frictional law requires a - b < 0 during SSE. In such a case, our

model needs to be extended to include unstable regions (a - b < 0) by increasing the degrees of freedom.

Models that assume a - b < 0, which have been proposed thus far, include models with complex fault geometries

618 (Romanet et al., 2018), 3-D elastic media (Matsuzawa et al., 2010), heterogeneous fault physical properties (Luo and

Ampuero, 2018), and nonuniform permeability in space and time (Bizzarri, 2012; Cappa, 2011; Dunham and Rice,

620 2008). They account for more complex effects that are not considered in our model. However, the tidal response of

621 these models has not yet been investigated.

622 Because our model adopts a one-degree-of-freedom (one-DOF) spring-slider system, it cannot simulate the

- 623 spatiotemporal variation in stress during an ETS. Such spatiotemporal changes in stress have been modeled using a
- 624 two-dimensional system (e.g., Hawthorne and Rubin, 2013), which can reproduce observations such as a spatial
- 625 propagation of ETS and temporal changes in the slip velocity during ETS. Hawthorne and Rubin (2013) examined
- the tidal response of ETS based on such a 2-D model.

However, Hawthorne and Rubin (2013) reported that the tidal response during ETS obtained by a 2-D simulation qualitatively agrees with the tidal response of the one-DOF ramp block slider model. Their model does not include the effect of dilatancy/compaction. To confirm whether the one-DOF and 2-D simulation results are in agreement for a model including the dilatancy/compaction effect, we need to extend our model to a 2-D system. One approach to do so would be to incorporate the dilatancy/compaction effect considered in our model into the model of Hawthorne and Rubin (2013).

633 **5 Conclusions**

634 Tremors in the transition zone are sensitive to tidal stress. In this study, we propose a physical model to explain the 635 tidal response of tremors observed during the ETS. Following previous studies (Ader et al., 2012; Beeler et al., 2013; Shelly et al., 2007a), we assumed that tremors are generated by the rupture of a small brittle patch on the fault 636 637 plane due to the aseismic shear slip of a larger-scale surrounding fault. As in Ader et al. (2012), we adopted a one-638 degree-of-freedom spring-slider that follows the RSF for the VS and set up the governing equations to describe the 639 slip behavior of the block, considering a pore fluid pressure change in the shear zone (section 2). We considered 640 drained (high-permeability) and undrained (low-permeability) models and presented results are mainly of the 641 undrained model, which could reproduce more observations than the drained model. The inclusion of pore pressure 642 changes due to dilatancy/compaction in the VS regime is in remarkable contrast to previous theoretical models 643 describing tidal modulation. 644 In our model, the tidal response is expressed with the tidal sensitivity (α), which represents the amplitude of the

tidal modulation of fault creep velocity, and the phase difference (δ) of the fault creep velocity peak relative to the 645 646 tidal Coulomb stress peak. We analytically derived an approximate solution to reveal how the tidal response depends 647 on the fault physical properties in section 3. We note that the slip behavior is primarily controlled by the characteristic timescale $T_{\theta} \left(= 2\pi d_c / V_{pl}\right)$ at which the state variable evolves, where d_c is the critical slip distance 648 and V_{pl} is the background fault creep. We found that the behavior of α and δ can be classified into three cases 649 according to the magnitude of T_{θ}/T $(T_{\theta}/T \gg |b - \mu_{pl}U|/a, T_{\theta}/T \sim 1 \sim |b - \mu_{pl}U|/a, T_{\theta}/T \ll 1)$, where T is the 650 651 tidal cycle (~12 hours), a and b are frictional constitutive parameters, μ_{pl} is the frictional coefficient and U is the 652 dilatancy parameter. This classification reflects the degree to which the dilatancy/compaction effect is dominant in

the frictional strength change. We showed that the smaller T_{θ}/T is, the more dominant the dilatancy/compaction effect is in the friction strength change.

655 We applied the model to ETS, assuming that V_{pl} changes between the early and later stages of the ETS. The model 656 successfully reproduced the tidal response observed at both stages of the ETS. Adopting this undrained model, we constrained the effective normal stress to be $10^{5\sim 6}$ Pa, the critical slip distance to be $10^{-1\sim -2}$ m, and the fluid 657 pressure diffusivity to be 10^{-5} m²/s or less. Of particular importance is the use of the phase difference in the 658 659 estimation of the fault properties. Without considering the dilatancy/compaction effect, the phase difference at the early stage cannot be reproduced. Moreover, using the tidal response data obtained during only the early stage or the 660 661 later stage produces different estimates of the fault properties. The range of the fault properties obtained in our study are in the ranges inferred by independent studies. Our model supports a critical slip distance of $\sim 10^{-1}$ m, which 662 663 has been used in numerical simulations of earthquake cycles. This study shows that the physical modeling of the 664 tidal response of tremors during the ETS is an effective method to retrieve the fault properties in the transition zone, 665 including hydraulic properties.

666

667 Appendices

Appendix A: Derivation of equation (21)

669 Substituting equation (3) into equation (8) and transforming the result, we obtain

670
$$k\Delta u + \Delta \tau = \left\{ \mu_0 + a\log\left(\frac{V_{pl}}{V_0}\right) + b\log\left(\frac{\theta_{pl}}{\theta_0}\right) + a\log\left(\frac{V}{V_{pl}}\right) + b\log\left(\frac{\theta}{\theta_{pl}}\right) \right\} \sigma_{eff}$$

671
$$= \left\{ \mu_{pl} + a \log\left(\frac{V}{V_{pl}}\right) + b \log\left(\frac{\theta}{\theta_{pl}}\right) \right\} \sigma_{eff}.$$
(A)

672 We represent the relative displacement of the block at the steady state without the tide as Δu_{no} . Then, $k\Delta u_{no} =$

673 $\mu_{pl}\sigma_{eff}^0$ holds, where the RHS is obtained by setting $\Delta\sigma(t) = 0$ and $\Delta p(t) = 0$ in equation (9). We can confirm that

- 674 $k\Delta u \sim \mu_{pl} \sigma_{eff}^0$ as follows. For the parameter set in Table 2, $k\Delta \dot{u} \sim O(kV_{pl})$ is three orders of magnitude smaller than
- 675 $\Delta \dot{\tau} \sim O(2\pi |\Delta \tau|/T)$. This means that $k\Delta \dot{u}$ on the LHS of the time derivative of equation (A) is negligibly small,
- 676 suggesting that $\Delta u \sim \Delta u_{no}$. Replacing $k\Delta u$ with $\mu_{pl}\sigma_{eff}^0$ on the LHS and using equations (9) and (11), equation (A)
- 677 can be rewritten as

678
$$\Delta S(t) = -\mu_{pl} \Delta p(t) + a\sigma_{eff} \log\left(\frac{v}{v_{pl}}\right) + b\sigma_{eff} \log\left(\frac{\theta}{\theta_{pl}}\right). \tag{B}$$

679 In equation (B), the LHS corresponds to the tidal Coulomb stress and the RHS corresponds to the frictional strength.

680 Furthermore, equation (B) can be written as

681
$$\Delta S(t) \sim -\mu_{pl} U \sigma_{eff}^0 \log\left(\frac{\theta}{\theta_{pl}}\right) + a \sigma_{eff}^0 \log\left(\frac{V}{V_{pl}}\right) + b \sigma_{eff}^0 \log\left(\frac{\theta}{\theta_{pl}}\right) \tag{C}$$

by using equation (7), where $\Delta p = 0$ is taken at $\theta = \theta_{pl}$, and it is assumed that the changes in the effective normal

stress in the second and third terms on the RHS of equation (C) are sufficiently small compared to σ_{eff}^0 .

Appendix B: Derivation of equation (22)

Substituting $-\mu_{pl}U\sigma_{eff}^0\log(\theta/\theta_{pl})\sim\Delta Se^{i\omega(t-\beta)}$ and $b\sigma_{eff}^0\log(\theta/\theta_{pl})\sim0$ into equation (21), as described in section 3.2.3, we obtain

687
$$\log\left(\frac{V}{V_{pl}}\right) \sim |\Delta S(t)| \operatorname{Re}\left(e^{i\omega t} - e^{i\omega(t-\beta)}\right). \tag{D}$$

688 When $\theta_1 = \omega t - \beta/2$, $\theta_2 = \beta/2$, we can write $Re(e^{i\omega t} - e^{i\omega(t-\beta)}) = \cos(\theta_1 + \theta_2) - \cos(\theta_1 - \theta_2) = \cos(\theta_1 - \theta_2)$

689 $\sin(\theta_1)\sin(\theta_2)$. Using $\sin(\theta_1) = \cos(\pi/2 + \theta_1)$, we obtain $\sin(\theta_1)\sin(\theta_2) = \cos(\omega t + (\pi - \beta)/2)\sin(\beta/2)$.

690 That is, $\log(V/V_{pl}) \sim |\Delta S(t)| \sin(\beta/2) \cos(\omega t + (\pi - \beta)/2)$. Furthermore, since $\beta \ll \pi$, equation (D) can be

691 rewritten as

692
$$\log\left(\frac{V}{V_{pl}}\right) \sim |\Delta S(t)| \sin\left(\frac{\beta}{2}\right) \operatorname{Re}\left\{e^{i\left(\omega t + \frac{\pi}{2}\right)}\right\}.$$
(E)

693 **Open Research**

694 The source code is available from zenodo (10.5281/zenodo.6403829).

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Supporting Information for

Frictional and Hydraulic Properties of Plate Interfaces Constrained by

a Tidal Response Model Considering Dilatancy/Compaction

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Introduction

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TextS3 presents a numerical method for solving the governing equation of the drained model. TextS4 presents the approximate solution for the tidal response of the drained model.

Figure S1 shows a schematic of the undrained and drained models.

Figure S2 shows the numerical solution of c in equation (12) for the undrained model.

Figure S3 shows the numerical and approximate solutions for the tidal responses α and δ of the drained model.

Figure S4 shows the numerical solution of c in equation (12) for the drained model.

Figure S5 shows the time variation of the tidal Coulomb stress term ($\Delta S(t)$ of equation (21)), the dilatancy/compaction effect term and the evolution effect term in equation (21).

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Figure S8 shows the dependence of d_c on the tidal sensitivity and phase difference when $V_{pl} = 10^{-7}$ and 10^{-9} m/s.

Text S1. Derivation of the governing equations for the drained model

We explain a drained model in which pore fluids flow out of the shear zone (Figure S1b). The difference between the undrained and drained models is the presence of fluid flow. The governing equations of the drained model are the same as those in the undrained model (equations (3), (4) and (8)), except for the governing equation for pore fluids.

Following the work of Segall et al. (2010), we assume homogeneous diffusion (HD), which holds under the condition that $T \gg t_w$. In the HD case, the effect of the finite shear zone thickness can be neglected, so the width of the shear zone can be formally defined as $w \to 0$ (Segall et al., 2010). The direction of fluid flow (Figure S1b) is parallel to the z-axis, and the shear zone lies on z = 0. c_{hyd} denotes the fluid pressure diffusivity at $z \neq 0$. Then, the governing equation for pore fluids can be written as (Segall et al., 2010)

$$\frac{\partial p}{\partial t} = c_{hyd} \frac{\partial^2 p}{\partial z^2} \left(\frac{\partial p}{\partial z} \Big|_{z=0} = \frac{Mw\dot{\phi}}{2c_{hyd}} \right).$$
(S1)

Text S2. Nondimensionalization of the governing equations of the drained model

The governing equations for the drained model are the upper three in equation (10) and the nondimensionalized equation (S1). When we adopt $\sqrt{c_{hyd}T}$ as the representative length in the z-axis direction, the nondimensionalized equation (S1) can be written as

$$\frac{\partial \tilde{p}}{\partial \tilde{t}} = \frac{\partial^2 \tilde{p}}{\partial \tilde{z}^2} \left(\frac{\partial \tilde{p}}{\partial \tilde{z}} \Big|_{\tilde{z}=0} = -E_p \frac{1}{\tilde{\theta}} \frac{d\bar{\theta}}{d\tilde{t}} \right), \tag{S2}$$

where $E_p = M\epsilon/2\sigma_{eff}^0 \sqrt{w^2/Tc_{hyd}} = U/2 \sqrt{w^2/Tc_{hyd}}$. E_p represents the relative importance of the dilatancy/compaction effect to the effective normal stress change in the drained model.

Previous experiments and observations suggest that $c_{hyd} \sim 10^{-1 \sim 3} \text{ m}^2/\text{s}$ (Yamashita and Tsutsumi, 2018) and $U \sim 10^{0 \sim -2}$ (Section 2.2.3). Using $10^{0 \sim 1/2}$ m as the value of w (Section 4.3.4), we obtain a possible range of E_p as 10^{-2} to 10^{-4} .

Text S3. A numerical method for solving the governing equation of the drained model

The upper three equations in equation (10) are calculated numerically using the third-order Adams-Bashforth method as in the undrained case. Equation (S2) is calculated numerically using the method presented in Appendix B of Segall et al. (2010). In the following, we discuss the latter method. Near the shear zone, the discretization needs to be sufficiently fine to capture a steep gradient of the pore fluid pressure. On the other hand, for a region far from the shear zone, the discretization does not need to be fine because the pore fluid pressure gradient is small. Thus, we use the following coordinate transformation between z and r (Segall et al., 2010):

$$z(r) = -c + e^{r}$$
 or, equivalently, $r(z) = \ln(c + z)$

We solve equation (S2) numerically in a new coordinate system using the Crank-Nicolson method. Specifically, we solve

$$\left\{ 1 + \frac{\gamma}{2} e^{-r_k} \left(e^{-(r_k - \delta)} + e^{-(r_k + \delta)} \right) \right\} p_k^{i+1}$$

= $p_k^i + \frac{\gamma}{2} e^{-r_k} e^{-(r_k - \delta)} \left(p_{k-1}^i + p_{k-1}^{i+1} \right) - \frac{\gamma}{2} e^{-r_k} p_k^i \left(e^{-(r_k - \delta)} + e^{-(r_k + \delta)} \right),$ (S3)

where $\gamma = \Delta t / \Delta r^2$, $\delta = \Delta r / 2$, and p_i^k is the value of the pore fluid pressure of the *k*-th grid in the new coordinate system at time step *i*. Δt and Δr represent increments in time and space, respectively. In this study, the number of grids is 35, and the starting position of the grid is $r(0) = \ln(0)$, $\Delta r = 0.3$, and $c = 10^{-4}$. Therefore, the grid farthest from the shear zone in the numerical calculation using $c_{hyd} \sim 10^{-1}$ m²/s, $T \sim 12.4$ h is approximately z = 170 m, while the grid farthest from the shear zone in the numerical calculation using $c_{hyd} \sim 10^{-3}$ m²/s, $T \sim 12.4$ h is approximately z = 17 m.

Text S4. The approximate solution for the tidal response of the drained model

The approximate solution for the drained model is derived in the same manner as in Section 3.1. The pore fluid pressure change due to fluid flow is represented as $p(z,t) = p_0 + \Delta p(z)e^{i\omega t}$, where p(0,t) corresponds to the pore fluid pressure in the shear zone. The equations for the drained model corresponding to equations (13) and (14) for the undrained model are

$$\frac{\Delta \tilde{V}}{\tilde{V}_{pl}} = \frac{2\pi i}{\tilde{K}\tilde{V}_{pl} + 2\pi iC} |\Delta \tilde{S}(t)| \tag{S4}$$

and

$$C = a - \frac{1}{1 + i\frac{T_{\theta}}{T}} (b - \mu_{pl}E_p\sqrt{2\pi i}), \text{ respectively.}$$
(S5)

The equations of the drained model corresponding to equations (17) and (18) for the undrained model are

$$\alpha = Re\left(\frac{2\pi i}{(\tilde{K}\tilde{V}_{pl} + 2\pi iC)\sigma_{eff}^0}\right)$$
(S6)

and

$$\delta = \arg\left(\frac{2\pi i}{\tilde{K}\tilde{V}_{pl} + 2\pi iC}\right), \text{ respectively.}$$
(S7)

Furthermore, by applying the argument from which equations (19) and (20) were derived for the drained model, the approximate solutions of α and δ can be expressed as

$$\alpha \sim Re\left\{ \left(C\sigma_{eff}^{0} \right)^{-1} \right\}$$
(S8)

and

$$\delta \sim arg\{\mathcal{C}^{-1}\},\tag{S9}$$

respectively.



Figure S1. A schematic of the undrained (a) and drained (b) models. The difference between the two models is whether fluid flows outside the shear zone.



Figure S2. The numerical solution of c (dots). The differences in color represent differences in the dilatancy parameter U.



Figure S3. (a) The numerical solution of α (dots) and the approximation solution (i.e., equation (S8)) (solid line). (b) The numerical solution of δ (dots) and the approximation solution (i.e., equation (S9)) (solid line). The differences in color represent differences in the dilatancy parameter E_p for the drained model.



Figure S4. The numerical solution of c (dots). The differences in color represent differences in the dilatancy parameter E_p for the drained model.



Figure S5. The time evolution of the dilatancy/compaction (D/C) term and the evolution (Evo) term for U = 1 and U = 0.1 (the variation from the time average over one tidal cycle is shown). The horizontal axis denotes the time normalized by the tidal period, and the values from 0 to 1 indicate one tidal cycle. The vertical axis denotes the tidal Coulomb stress (tCs) × 10³, normalized by the frictional strength σ_{eff}^0 . The numerical solutions for $T_{\theta}/T = 14$ and U = 1 are shown in black, and those for $T_{\theta}/T = 4.2$ and U = 0.1 are shown in yellow. The amplitudes of the D/C term and tCs term are almost the same for U = 0.1 as well as U = 1. This means that the argument in Lines 407-414 in the body text can be applied to U = 0.1 as well as U = 1. Furthermore, the D/C term for U = 0.1 has a phase shift to the right of that for U = 0.1. Thus, $\beta(\ll \pi)$ at U = 0.1 is larger than $\beta(\ll \pi)$ at $U = 1 (\log(V/V_{pl}) \sim |\Delta S(t)| \sin(\beta/2) \cos(\omega t + (\pi - \beta)/2)$. See Appendix B). Since the tidal Coulomb stress peak corresponds to $\omega = \pi/2$, the larger β is, the smaller δ is. In other words, δ is smaller for U = 0.1 than for U = 1.



Figure S6. A schematic illustration of the relationship between the fault creep velocity and tide level when U = 1. For simplicity, only the normal stress change (white arrows) is represented. (a) Tidal modulation of fault creep when $T_{\theta}/T \ll 1$. Since the dilatancy/compaction effect (black arrows) decreases the amplitude of $\Delta S(t)$, the fault creep velocity reaches its maximum at $\delta \sim 0$. (b) Tidal modulation of fault creep when $T_{\theta}/T \ll 1$ and $T_{\theta}/T \gg |b - \mu_{pl}U|/a$. The sum of the normal stress due to the dilatancy/compaction effect and the tidal normal stress becomes the largest in the sense of enhancing fault slip at $\delta \sim \pi/2$. (c) Tidal modulation of fault creep when $T_{\theta}/T \gg |b - \mu_{pl}U|/a$. The dilatancy/compaction effect is negligible, and the fault creep velocity reaches its maximum at $\delta \sim 0$.



Figure S7. The dependence of T_{θ}/T on the phase difference at $E_p = 0.1$ The numerical solution of δ (dots) and the approximation solution (i.e., equation (S9)) (solid line). We see that the maximum phase difference is $\delta \sim \pi/6$ at $T_{\theta}/T \sim 10$, which cannot explain the observed phase difference of $\pi/2$.



Figure S8. (a) The approximation solution of α (equation (19)) when $V_{pl} = 10^{-7}$ m/s (black) and 10^{-9} m/s (yellow). The horizontal axis denotes the critical slip distance d_c . The blue dots show that α for $d_c = 0.13$ m increases from ≤ 0.1 to ~ 0.7 as V_{pl} decreases from 10^{-7} m/s to 10^{-9} m/s. (b) The same as in (a) but for the approximation solution of δ (equation (20)). The blue dots show that δ decreases from $\sim \pi/2$ to ~ 0 as V_{pl} decreases from 10^{-7} m/s to 10^{-9} m/s.