# Global System for Atmospheric Modeling: Model Description and Preliminary Results

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#### Abstract

The extension of a cloud-resolving model, the System for Atmospheric Modeling (SAM), to global domains is described. The resulting global model, gSAM, is formulated on a latitude-longitude grid. It uses an anelastic dynamical core with a single reference profile (as in SAM), but its governing equations differ somewhat from other anelastic models. For quasi-hydrostatic flows, they are isomorphic to the primitive equations in pressure coordinates, but with the globally uniform reference pressure playing the role of actual pressure. As a result, gSAM can exactly maintain steady zonally symmetric baroclinic flows that have been specified in pressure coordinates, produces accurate simulations when initialized or nudged with global reanalyses, and has a natural energy conservation equation, despite the drawbacks of using the anelastic system to model global scales. gSAM employs a novel treatment of topography using a type of immersed boundary method, the Quasi-Solid Body Method (QSBM), where the instantaneous flow velocity is forced to stagnate in grid cells inside prescribed terrain. The results of several standard tests designed to evaluate accuracy of global models with and without topography as well as results from real-Earth simulations are presented.

## Global System for Atmospheric Modeling: Model Description and Preliminary Results

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#### 1 Abstract

2 The extension of a cloud-resolving model, the System for Atmospheric Modeling (SAM), to global 3 domains is described. The resulting global model, gSAM, is formulated on a latitude-longitude grid. It uses an anelastic dynamical core with a single reference profile (as in SAM), but its governing equations 4 5 differ somewhat from other anelastic models. For quasi-hydrostatic flows, they are isomorphic to the 6 primitive equations in pressure coordinates, but with the globally uniform reference pressure playing the 7 role of actual pressure. As a result, gSAM can exactly maintain steady zonally symmetric baroclinic flows 8 that have been specified in pressure coordinates, produces accurate simulations when initialized or nudged 9 with global reanalyses, and has a natural energy conservation equation, despite the drawbacks of using 10 the anelastic system to model global scales. gSAM employs a novel treatment of topography using a type 11 of immersed boundary method, the Quasi-Solid Body Method (QSBM), where the instantaneous flow 12 velocity is forced to stagnate in grid cells inside prescribed terrain. The results of several standard tests 13 designed to evaluate accuracy of global models with and without topography as well as results from real-14 Earth simulations are presented.

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#### 16 Plain Language Summary

17 The System for Atmospheric Modeling (SAM), a model widely used to study evolution of clouds 18 and small-scale atmospheric motions using computational grids with horizontal spacings of several meters 19 to a few hundred kilometers, has been extended to simulations of the whole Earth with global grid spacings 20 of 1-5 km that realistically represent intense and localized rain and snow storms. The resulting global atmosphere model, gSAM, is unique in its use of a computationally efficient approximation to the exact 21 22 equations of air motion and its method of representing flow over mountains, which is well suited to steeply 23 sloped terrain. The paper documents gSAM's updated equations, representation of physical processes, 24 and computational methods. Despite its algorithmic simplicity, gSAM produces realistic global weather 25 forecasts and cloud distributions and agrees well with other more complicated global models on standard 26 benchmarking tests.

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- 32 Key Points
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The anelastic System for Atmospheric Modeling (SAM) is extended to a global latitude-longitude
 grid.

The model uses a novel treatment of topography forcing the flow to instantaneously stagnate in
 the cells inside prescribed terrain.

The model performs well in several standard tests that evaluate the accuracy of global models
 and in real-Earth simulations.

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#### 41 **1. Introduction**

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Global climate change has become one of the most pressing issues of our time and threatens the way of life and even lives themselves of billions of people. Global climate models (GCMs) have been at the forefront of the effort to predict the extent of the future climate change as documented in a steady stream of the IPCC reports (e.g., IPCC 2013; 2021). Although there has been a fair amount of progress on this long and arduous path to the truth, models with parameterized convection have struggled to represent the fundamental feedbacks of the climate system to anthropogenic forcing, particularly the feedbacks associated with clouds (e.g., Randall et al 2003; Stevens and Bony 2013).

50 One way to further advance our understanding of how the climate system responds to the external 51 influences is to continuously push against the limits of the currently available computer technology by 52 widening the range of resolved scales of atmospheric motion and improving the realism with which 53 physical processes are represented. Even though innovative models that test the limits of present-day 54 computational resources may not be practical for the long simulations required for climate prediction, they 55 may provide us with clues what is wrong with the current generation of the GCMs, and perhaps, even help 56 to improve them. One such innovative models that appeared impractical at first but eventually led to 57 staggering progress in the field of computer modeling of the Earth's general circulation is NICAM (Tomita 58 and Satoh 2004). By using a globe-spanning grid with a spacing of less than 5 kilometers, NICAM was 59 able to explicitly simulate deep convection, thereby becoming the first global storm-resolving model 60 (GSRM). NICAM was so far ahead of its time that several years passed before other models of similar 61 class started to appear. Today, more than a dozen global non-hydrostatic models with 2-5 km grid 62 resolution can run for at least several months at a time.

63 The development of one such GSRM, the global System for Atmospheric Modeling (further 64 gSAM) was motivated by near-global simulations with its predecessor, the original SAM (Khairoutdinov 65 and Randall 2003; further SAM), which is a conventional cloud-resolving model on a Cartesian grid. Those near-global simulations were used to better understand the physics of tropical deep convection and 66 67 its interactions with the equatorially trapped waves (Bretherton and Khairoutdinov 2015; Narenpitak et al 2020), the Madden-Julian Oscillation (Khairoutdinov and Emanuel 2018), and tropical cyclogenesis 68 69 (Narenpitak et al 2020) within the idealization of an aquaplanet. In those simulations, SAM was applied 70 to tropical convection over very large domains, which are comparably wide as the actual circumference 71 of the Earth. The domain was limited in the latitudinal direction by placing solid walls at about 45° latitude, as further extension using a Cartesian grid would distort the spherical geometry. The walls significantly 72 73 distort the nearby atmospheric structure and may affect the simulated Hadley circulation throughout the 74 domain.

75 Conceptually, it is straightforward to make SAM a truly global model by replacing its Cartesian 76 grid with a grid based on the spherical coordinates and including simple treatments of terrain and land 77 surface processes. A major technical obstacle was the solver for the global elliptic equation for pressure. 78 SAM uses a fast Fourier transform in both horizontal directions, which works only for Cartesian grids 79 with constant horizontal grid spacing. With colleagues at the Institute for Advanced Computational Science at Stony Brook University, an efficient hybrid FFT-Geometric Multigrid Solver (FFT-GMG), 80 81 based on Lu (2016), has been implemented in gSAM to address this issue. The terrain is treated using a 82 simple, novel immersed boundary method, unlike in other current GSRMs, which typically use terrain-83 following coordinates.

84 The first version of gSAM was completed just in time for DYAMOND (DYnamics of the 85 Atmospheric general circulation On Non-hydrostatic Domains; Stevens et al 2019), the first GSRM 86 intercomparison project. DYAMOND was based on a 40-day free run of each GSRM initialized from 87 reanalysis for 1 Aug. 2016 with specified sea-surface temperatures. The gSAM simulation was quite 88 competitive with other GSRMs (Satoh et al 2019; Stevens et al 2019), even though little time was available 89 for model refinement or debugging before the production run. The gSAM is unique among the 90 DYAMOND GSRMs in the formulation of its dynamical core and terrain, and in being developed 91 independent of a major modeling center. It is designed to be an easy-to-use extension of SAM, which 92 itself has a broad user community of cloud-resolving and large eddy simulation modelers.

The purpose of this paper is to formally document gSAM and to report the results of several standard tests of the global dry dynamical core as well as real Earth simulations. The paper is organized as follows. Section 2 describes the model equations, finite-difference approximation, physics packages, implementation of terrain, and numerical methods. Results of several tests of the dry dynamical core and real Earth simulations are presented in Section 3, followed by conclusions in Section 4.

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- 101 **2. Model equations and methods**
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- 103 2.1 Dynamical core
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In gSAM, the Cartesian model equations of SAM are reformulated for latitude-longitude coordinates on a sphere by redefining the horizontal metric coordinates x and y as  $x = r\lambda$  and  $y = r\varphi$ , where  $\lambda$  is longitude,  $\varphi$  latitude, and r the Earth's radius. The vertical coordinate is height z. The prognostic anelastic conservation equations for the zonal (u), meridional (v), and vertical (w) wind components, and for mass are:

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$$\frac{\partial u}{\partial t} + \frac{1}{\mu} \frac{\partial}{\partial x} uu + \frac{1}{\mu} \frac{\partial}{\partial y} \mu vu + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \bar{\rho} wu = -\frac{1}{\mu} \frac{\partial \Phi}{\partial x} + \left(f + \frac{u}{r} tan\phi\right) v - f^* w + F_u \tag{1}$$

$$\frac{\partial v}{\partial t} + \frac{1}{\mu} \frac{\partial}{\partial x} uv + \frac{1}{\mu} \frac{\partial}{\partial y} \mu vv + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \bar{\rho} wv = -\frac{\partial \Phi}{\partial y} - \left(f + \frac{u}{r} tan\phi\right) u + F_v \tag{2}$$

$$\frac{\partial w}{\partial t} + \frac{1}{\mu} \frac{\partial}{\partial x} uw + \frac{1}{\mu} \frac{\partial}{\partial y} \mu vw + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \bar{\rho} ww = -\frac{\partial \Phi}{\partial z} + B + f^* u + F_w$$
(3)

$$\frac{1}{\mu}\frac{\partial}{\partial x}u + \frac{1}{\mu}\frac{\partial}{\partial y}\mu\nu + \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}\bar{\rho}w = 0$$
(4)

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112 where  $\mu = \cos\varphi$ ,  $f = 2\Omega \sin\varphi$ ,  $f^* = 2\Omega \cos\varphi$ ,  $\Omega$  is the Earth's angular velocity of rotation,  $\bar{\rho}(z)$  is the 113 prescribed reference density profile, which depends on height only,  $\Phi = p'/\bar{\rho}$ , where p' is the 114 perturbation of pressure from the reference profile, buoyancy acceleration  $B = g\alpha'/\bar{\alpha}$ , where g is 115 gravitational acceleration and  $\alpha'$  is the perturbation of specific volume (inverse density) at the reference 116 pressure from its reference profile  $\bar{\alpha}(z)$ ,  $F_u$ ,  $F_v$ ,  $F_w$  are accelerations due the subgrid-scale (SGS) eddy 117 viscosity. The pressure-gradient terms in equations (1)-(3) generally follow the kinetic-energy-conserving 118 version of the anelastic approximation of Lipps and Hemler (1982). The prognostic equation for an 119 arbitrary advected scalar *s* is given by

$$\frac{\partial s}{\partial t} + \frac{1}{\mu} \frac{\partial}{\partial x} us + \frac{1}{\mu} \frac{\partial}{\partial y} \mu vs + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \bar{\rho} ws = S_s + F_s$$
(5)

121 where  $S_s$  denotes local sources and sinks, and  $F_s$  the divergence of SGS fluxes of s.

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#### 2.2 Buoyancy and conservation of energy

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For an anelastic model without moist processes, the thermodynamic equation is naturally 125 126 formulated as prognosis of the potential temperature  $\theta$ . However, for accurate simulation of moist and 127 radiative processes, the absolute temperature T must also be known (e. g. for calculating the saturation 128 vapor pressure of water). Over regional domains, the reference pressure profile is a good approximation 129 for the pressure field over the entire domain, so the absolute temperature can be estimated from potential 130 temperature using the reference pressure profile (Lipps and Hemler, 1992). However, over a global 131 domain, the local pressure can differ from the reference pressure at the same height by as much as 10%, 132 especially in the middle and upper troposphere. For a given potential temperature, this could change the 133 implied absolute temperature by as much as 10 K, which is far too inaccurate for calculating moist 134 thermodynamics.

Thus gSAM uses a different approach, following Pauluis (2008), who replaced conservation of 135 potential temperature by conservation of frozen moist static energy  $h_f = c_p T + gz + L_c q_v - L_f q_{ice}$  (the 136 sum of moist enthalpy and geopotential energy), where  $c_p$  is the specific heat of air,  $L_c$ ,  $L_s$  and  $L_f = L_s - L_s$ 137  $L_c$  are the latent heats of vaporization, sublimation, and freezing,  $q_v$  is water vapor mixing ratio, and  $q_{ice}$ 138 139 is the frozen water mixing ratio. This approximation can be consistently derived from the conservation of 140 moist entropy by replacing the total pressure with the reference pressure and justified by the scaling 141 assumptions used to derive the anelastic approximation. In this same spirit, where pressure is required in 142 a moist thermodynamic formula in addition to temperature, gSAM uses the reference pressure rather than 143 the full pressure. While pressure perturbations are neglected in moist thermodynamics here, their impact 144 was explored in an anelastic framework by Kurowski et al (2013).

145 In a height-coordinate model like gSAM, the conservation equation for frozen moist static energy 146 involves tendencies of T and q, natural choices for accurately formulating moist thermodynamics, rather 147 than  $\theta$ . Like SAM, gSAM is formulated in terms of a liquid/ice water static energy  $h_L = h_f - L_c(q_v + q_{liq} + q_{ice}) = c_p T + gz - L_c q_{liq} - L_s q_{ice}$ , where  $q_{liq}$  is the total liquid water mixing 149 ratio. Like frozen moist static energy,  $h_L$  is conserved in nearly hydrostatic air motions even with 150 reversible phase transitions between water vapor, liquid and ice, but it separates off the water contribution 151 to  $h_f$ , which is treated in a separate conservation equation. The prognostic equation for  $h_L$  is a special case 152 of equation (5) with the source term:

$$S_{h_L} = -\frac{1}{\bar{\rho}}\frac{\partial}{\partial z}(-L_c P_{liq} - L_s P_{ice}) - wB + c_p Q_{rad}$$
(6)

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Here  $P_{liq}$  and  $P_{ice}$  are total sedimentation fluxes for liquid and frozen water;  $Q_{rad}$  is the radiative heating rate. The buoyancy itself is expressed using absolute temperature as

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$$B = g\alpha'/\bar{\alpha} = g((T - \bar{T})/\bar{T} + 0.608q_v - q_{liq} - q_{ice})$$
(7)

157 An improvement of gSAM over SAM is the inclusion of the effect of the buoyancy flux wB in (6). As 158 emphasized by Pauluis (2008), it is important to include this term for conservation of total energy, as it 159 opposes the generation/sink of the vertical kinetic energy by the buoyancy. Note that use of temperature, 160 rather than potential temperature, in *B* differs from the Lipps-Hemler anelastic equations by neglecting 161 the contribution of pressure perturbations to potential temperature perturbations. It is consistent with 162 approximating the total pressure with the reference pressure in the dynamical equations, which requires 163 that buoyancy contributions due to relative horizontal Exner function perturbations  $(R/c_p)(p'/\bar{p})$  be much less than those due to relative horizontal temperature perturbations  $T'/\overline{T}$ . On a global scale, this 164 approximation holds well near the surface but less well in the upper atmosphere; nevertheless, we will see 165 166 that it does not substantially degrade the accuracy of gSAM in comparison to models that use fully 167 compressible formulations of the flow equations.

168 The total energy is defined as  $E = K + gz + h_f$ , that is the sum of kinetic energy *K* and 169 thermodynamic energy. Similar to Pauluis (2008), it can be shown that the governing equations of gSAM 170 conserve total energy in the whole domain:

$$\frac{\partial}{\partial t} \iiint \bar{\rho} E \ \mu \ dx \ dy \ dz = SHF + LHF - L_f P_{sf} + R_{NET}$$
(8)

that is, globally-integrated total energy can only be changed by the total surface flux of frozen moist enthalpy  $SHF + LHF - (L_s - L_c)P_{sf}$  and net radiative flux convergence in the atmosphere  $R_{NET}$  (= 173  $R_{SFC} - R_{TOA}$ ), where *SHF*, *LHF*, and  $P_{sf}$  are the surface total sensible, latent and frozen precipitation 174 fluxes, respectively;  $R_{TOA}$  and  $R_{SFC}$  are the top-of-atmosphere and surface net radiative fluxes.

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#### 2.3 Consistency with primitive equations in pressure coordinates

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For quasi-hydrostatic flows (when horizontal scale of motions is much larger than the vertical), the system of anelastic equations (1)-(4) with thermodynamic equation (6) and buoyancy in form (7) is isomorphic to the system of primitive equations used by many GCMs. Assuming that the reference state is in hydrostatic balance, that is  $d\bar{p} = -g\bar{\rho}dz$ , defining pressure vertical velocity  $\omega = -g\bar{\rho}w$ , using the dry expression for buoyancy based on equation (7), neglecting terms with  $f^*$ , and neglecting the vertical acceleration, the dry system of equations (1)-(4) and (7) can be rewritten, using the horizontal Cartesian coordinates for clarity, as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial \bar{p}} = -\frac{\partial \Phi}{\partial x} + fv$$
(9)

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial \bar{p}} = -\frac{\partial \Phi}{\partial y} - fu$$
(10)

$$0 = -\frac{\partial(\Phi + gz)}{\partial\bar{p}} - \frac{RT}{\bar{p}}$$
(11)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial \bar{p}} = 0$$
(12)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \omega \left( \frac{\partial T}{\partial \bar{p}} + \frac{RT}{c_p \bar{p}} \right) = 0$$
(13)

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186 which is an exact form of the primitive equations (PE) in pressure coordinates based on the reference 187 pressure profile. Note that it is important for the derivation of the system (9)-(13) to have the buoyancy in 188 the form (7) and to include the buoyancy flux term as a source of  $h_L$  in equation (6).

The isomorphism of our anelastic system in the hydrostatic limit with the primitive equations can be an advantage both for dynamical reasoning and for initialization from or nudging to a global reanalysis. For instance, the equations of geostrophic balance are the same in both systems (although the interpretation of  $\phi$  is slightly different), and thermal wind balance can be expressed very similarly by replacing *p* by  $\bar{p}$  for gSAM, that is, for PE:

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$$f\frac{\partial u}{\partial p} = -\frac{\partial}{\partial p}\frac{\partial \Phi}{\partial y} = \frac{R}{p}\frac{\partial T}{\partial y},$$

195 while in gSAM:

$$f\frac{\partial u}{\partial \bar{p}} = -\frac{\partial}{\partial \bar{p}}\frac{\partial \Phi}{\partial y} = \frac{R}{\bar{p}}\frac{\partial T}{\partial y}$$

Based on this analogy, a good way to initialize gSAM from reanalysis fields is to interpolate the input data in pressure coordinates to the anelastic reference pressures  $\bar{p}(z_k)$  corresponding to the model height levels  $z_k$ . A suitable gSAM reference profile  $\bar{p}(z)$  is constructed by calculating the globally averaged geopotential height at the reanalysis pressure levels and interpolating this function to obtain the reference pressures at the known heights  $z_k$ . The reference density is computed from the hydrostatic balance of the reference pressure profile. This approach will preserve approximate dynamical balances (such as thermal wind balance) in the reanalysis, where they occur.

204 To illustrate the result of such a procedure, we initialized gSAM with the fields from the ERA5 205 reanalysis on pressure coordinates at 00Z on January 20, 2020, used for initialization of the most recent 206 DYAMOND-Winter project (https://www.esiwace.eu/services/dyamond-initiative/services-dyamond-207 winter). We ran gSAM on the same horizontal grid as the ERA5 for one hour to let the fields to adjust to the topography. Then we used gSAM's diagnosed  $\Phi = p'/\bar{\rho}$  field to obtain the pressure perturbation of 208 209 pressure p' which is added to the reference pressure profile to obtain the total pressure field. In Figure 1, 210 the resultant pressure at the surface of the topography is compared to the ERA5 surface pressure. gSAM 211 reproduces the surface pressure quite well, especially over the ocean. Some biases over land are mostly 212 explained by differences in the vertical model grid, topography dataset and interpolation techniques. The 213 difference map clearly shows the imprint of the "staircase" pattern of gSAM surface topography (to be 214 discussed further below) on its surface pressure.

- 215
- 216 *2.4 Physics*
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Many physics packages in gSAM are directly inherited from SAM. The formulation of the subgridscale (SGS) fluxes employs two approaches, the 1.5-order closure based on the prognostic SGS turbulent kinetic energy (TKE) and a simpler Smagorinsky-type closure, used in global simulations, which assumes perfect balance between shear/buoyancy production of TKE and its viscous dissipation. Several microphysics modules are available: the original SAM's single-moment microphysics, and three comprehensive bulk microphysics modules imported from the Weather Research and Forecasting model (WRF): the Morrison et al (2005), Thompson et al (2008), and P3 (Morrison and Milbrandt, 2015). There are two choices of radiation. The first is from the Community Atmosphere Model version 3 (CAM3;
Collins et al 2006), and the second is the Rapid Radiative Transfer Model (RRTM; Iacono et al 2008),
from the Community Earth System Model version 1 (Hurrell et al 2013). The former is faster than the
latter, but less accurate.

229 The land surface module is based on the Simplified Land Model (SLM), originally developed for 230 SAM by Lee and Khairoutdinov (2015). There are 16 land types as defined by the International 231 Geosphere-Biosphere Program (IGBP) classification system. Each type defined visible and near-infrared 232 albedos, leaf area index, characteristics of the root system, roughness length and displacement height for 233 surface flux computations, among others. The output of SLM includes land and ice surface sensible and 234 latent heat fluxes, surface albedo for visible and near-infrared radiation, surface skin temperature for 235 longwave radiation, and snow depth. The soil has 9 levels, with thicknesses gradually increasing from 1 236 cm at the surface to 1 m below. Soil hydraulic and heat conduction properties depend on the clay, sand, 237 and water contents and change sharply if a soil layer is frozen. In addition to soil, SLM computes heat 238 fluxes inside glaciers and sea ice, the depth of which is prescribed from observations. Also, there are 239 parameterizations of accumulation and melting of snow, accumulation of rainwater on tree leaves, surface 240 runoff and flooding. Planned future publications will provide more details on SLM in gSAM. Over the 241 ocean, surface flux and albedo parameterizations from CAM3 are used, as in SAM.

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#### 243 2.5 Numerics

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245 The finite-difference approximations for the equations in gSAM are formulated on Arakawa C-246 grid, like in SAM, that is, all scalars are defined at the grid-cell centers, while velocity components on 247 grid-cell's sides. Using coordinate transformations, gSAM's model equations can be written and 248 discretized in forms analogous to the Cartesian grid forms used by SAM. By simply setting  $\mu = 1$  and  $\tan \varphi = 0$  in all the equations, gSAM can seamlessly be switched from a global storm-resolving model to 249 250 a large-eddy simulation (LES) or a cloud-resolving model (CRM) and thereby include all the functionality 251 of original SAM. gSAM's latitude-longitude grid can also be used for large-area regional modeling where 252 a Cartesian grid would lead to a substantial loss in accuracy. For such regional modeling, gSAM uses 253 strong nudging to the prescribed fields at narrow zones along the domain boundaries.

The advection scheme for all scalars is the fully 3-D monotonic and positive transport MPDATA (Smolarkiewicz 2006), as in SAM. Subgrid-scale fluxes and their divergence, as in equation (5), are

discretized using conventional centered finite differences. Also following SAM, the momentum equations 256 257 are integrated in time using the explicit third-order Adams-Bashforth (AB3) time scheme. One of the main 258 reasons for using the AB3 scheme is the use of the second order centered difference (SOC) scheme in flux 259 form for advection of momentum with the property of conservation not only of momentum but also of 260 kinetic energy, which also helps to control non-linear instability. The AB3 scheme allows a relatively high 261 maximum Courant number of 0.72 with SOC and is slightly dissipative, which helps to damp the two 262 computational modes inherent in AB3. A disadvantage of AB3 is its strict numerical instability threshold for diffusion  $\Delta t < 0.1 k/\Delta^2$ , where k is diffusion coefficient, and  $\Delta$  is grid spacing. This may cause a 263 264 serious problem when simulating deep convection and using relatively high vertical resolution in the 265 boundary layer, where relatively large SGS eddy-diffusivity coefficients, predicted by the SGS model, 266 could affect the model stability. AB3 also has also a rather restrictive stability criterion that in Newtonian 267 damping terms, the damping timescale must be longer than about two time-steps (Duran 1991). Therefore, 268 in gSAM, we introduced an additional implicit step for vertical diffusion and damping. The workflow 269 over each time step goes as follows. First, a provisional solution is found using the AB3 scheme as

270 
$$u^* = u^n + \Delta t^n (aA^{n-1} + bA^{n-1} + cA^{n-2})$$
(14)

where *A* denotes the advection, SGS diffusion (with the exception of the "fast" terms such as the vertical diffusion or strong-damping terms), Coriolis, but not the pressure-gradient terms; *a*, *b*, and *c* are the AB3 coefficients that depend on current  $\Delta t^n$  and past time steps  $\Delta t^{n-1}$  and  $\Delta t^{n-2}$  (Fiedler and Trapp, 1993). Next, a first correction is made to  $u^*$  to include vertical diffusion and damping, denoted here by *B*, using the implicit step:

$$u^{**} = u^* + \Delta t^n B(u^{**}) \tag{15}$$

Finally, a pressure-correction step is performed to enforce anelastic mass conservation (4):

278 
$$u^{n+1} = u^{**} - \Delta t^n (a \nabla \Phi^n + b \nabla \Phi^{n-1} + c \nabla \Phi^{n-2})$$
(16)

279 The unknown scaled perturbation pressure  $\Phi^n$  is diagnosed from the Poisson equation

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280 
$$\nabla(\rho\nabla\Phi^{n}) = \frac{1}{a\Delta t^{n}}\nabla\cdot\rho u^{**} - \frac{b}{a}\nabla(\rho\nabla\Phi^{n-1}) - \frac{c}{a}\nabla(\rho\nabla\Phi^{n-2})$$
(17)

The procedure above is identical to the time integration of the momentum equations employed by SAM except for the implicit step (15).

The Cartesian grid in SAM has constant horizontal grid spacings in x and y for which the elliptic equation (17) has coefficients that depend only on height; therefore, a bi-directional FFT in the horizontal directions is used to obtain a tridiagonal system of equations for the vertical structure of the horizontal Fourier coefficients. An inverse bi-directional FFT of their solutions yields  $\Phi^n$ . This direct method of solving (17) is non-iterative and can be efficiently parallelized across many computational cores.

288 For the latitude-longitude grid employed by gSAM, the coefficients of equation (17) also depend 289 on latitude, so the bi-directional FFT method can no longer be applied. However, in the usual case that 290 the horizontal grid spacing in the longitudinal direction is constant for each latitudinal circle, none of the coefficients depend on longitude. Then an FFT can still be applied in that direction, yielding a set of 291 292 independent two-dimensional (latitude-height) Helmholtz equations for the longitudinal Fourier 293 coefficients that can be solved in parallel. For gSAM, the height-latitude grid is highly anisotropic, with 294 thousands of grid cells in the latitudinal direction and only in the order of a hundred in the vertical. In 295 addition, the vertical grid is generally much finer, at least in the low troposphere, than the horizontal grid. 296 Attempts to use freely available generic multigrid solvers have failed because of very low convergence 297 rates for such anisotropic grids with Neuman boundary conditions which would make gSAM too 298 expensive to be practical. Therefore, a custom geometric multi-grid (GMG) solver for anisotropic 2-D 299 Helmholtz equations has been developed (Lu 2016). The resultant hybrid FFT-GMG solver is nearly as 300 efficient as the original bi-directional FFT solver when applied on a grid of similar size. When Cartesian 301 coordinates are used in gSAM, the conventional non-iterative bi-directional FFT solver is still used.

302 Because of the latitude-longitude grid, the equations have a singularity at the pole. In the early 303 development version of gSAM that participated in the DYAMOND project, this problem was 304 circumvented by placing a solid wall around each of the poles at about 89° latitude circle. In the current 305 version of gSAM, the grid goes all the way to the poles, so that the horizontal face of the cells in the last 306 row of the grid cells has a triangular shape because the poleward face of otherwise quadrilateral cells 307 degenerates into a point, so the corresponding fluxes, computed on C-grid, through that face are identically 308 zero. The other faces of the polar cells don't have the singularity problem. For other terms, such as 309 Coriolis, when the velocity at the pole is required, the nearest point with valid value of meridional velocity 310 is used instead. To minimize considerable reduction of the timestep because of potentially high CFL 311 numbers in the zonal direction right near the pole, the artificial damping of the horizontal velocity is 312 performed to prevent the CFL from exceeding the critical value. This, of course can distort the solution 313 right near the poles. This problem will be mitigated in the future by using local grid coarsening for the 314 advection in zonal direction (e.g., Asaithambi and Mahesh 2017).

One of the benefits of not using the FFT transform in the *y* direction on latitude-longitude grid is that the grid spacing in *y* can be variable. By varying grid spacing with latitude, the local (in meters, not in degrees) in *y* can be the same as resolution in *x*, effectively covering most of the Earth with a grid that is locally isotropic in the horizontal but has a somewhat broader grid spacing in the tropics than in the midlatitudes. In practice, such a grid is used only to about  $70^{\circ}$  latitude, and further refinements in *y* are avoided when approaching the poles, for stability reasons. Such an approach provides about twice as high resolution in midlatitudes than over the equator using the same number of latitudinal grid circles as would be used for a uniformly spaced grid.

- 323
- 324 2.6 Terrain
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A global model without representation of topography would be of little utility beyond idealized 326 327 aqua-planet studies. Note that SAM has no capability to represent topography. One of the most popular 328 methods for incorporating terrain in atmospheric models is to use a terrain following coordinate system. 329 However, this would mean abandoning both the bi-directional FFT solver for the Poisson equation (17) 330 for pressure in SAM and hybrid FFT-GMG solver in gSAM, replacing them with a fully 3-D multigrid 331 solver. Designing such a solver and making its computational and parallel performance as efficient as the 332 existing solvers in SAM and gSAM would be a very challenging task. Instead, a variant of a so-called 333 body force method has been implemented. In this approach, the velocity of the flow inside a solid obstacle 334 is forced to stagnate, similar to using a fictitious Newtonian damping (Chen and Leach, 2007; 335 Smolarkiewicz et al, 2007). However, we introduce a novel aspect to that approach. We cannot simply set 336 the velocity inside the topography to zero at the end of each time step, as this would violate mass 337 conservation (Eqn. 4). Instead, we damp the flow inside an obstacle using the implicit correction step (15), 338 used before applying the pressure gradient terms, which would look in the case of Newtonian damping 339 with time scale  $\tau$  as

340 
$$\boldsymbol{u}^{**} = \boldsymbol{u}^* - \boldsymbol{u}^{**} \frac{\Delta t^n}{\tau}$$
(18)

341 Step (18) is computationally stable for any value of  $\tau$ , and can be rewritten as

342 
$$u^{**} = u^* \frac{1}{1 + \frac{\Delta t^n}{\tau}}$$
 (19)

For instantaneous relaxation  $\tau = 0$ , the factor  $1/(1 + \Delta t^n/\tau)$  becomes also zero, so that the damping is equivalent to  $u^{**} = 0$  inside an obstacle and there is no need to specify a value for  $\tau$ . The modified  $u^{**}$  is used then in the right-hand-side of the Poisson equation (17), solution of which is used to compute the final velocity  $u^{n+1}$  for the current time step.

The approach, which we called the Quasi-Solid Body Method (QSBM; Khairoutdinov et al 2022), has the virtue of simplicity, as it requires very little change to the model code (and no change to the structure of the Poisson solver), and can, in principle, be adopted by any anelastic model. Moreover, the method works not only for topography, but also buildings. For that case, the method has already been tested by comparing the simulated flow around a single rectangular building to the wind-tunnel observations with good results (Khairoutdinov et al, 2022).

353 There are drawbacks to the QSBM. First, the method does not guarantee that the velocity inside 354 the terrain remains completely stagnant, as some residual flow is unavoidable. However, the magnitude 355 of the residual velocities is found to be very small, typically one to two orders of magnitude smaller than 356 the velocity of the incoming flow, even in the case of a building. The simulation can further be improved 357 by reapplying the QSBM to the solution at the end of the time step, although each such iteration increases 358 the expense of the model because of the repeated use of the elliptic equation solver. Khairoutdinov et al 359 (2021) performed up to three such additional iterations and found that each iteration reduces the residual 360 velocities inside the building further by about a factor of two. For example, for an oncoming wind of 361 several meters per second, the maximum residual flow was only several centimeters per second. However, 362 it was also found that the additional iterations result in only minor simulation improvement compared to 363 the observations, and therefore, can generally be omitted in most cases.

Another drawback of the QSBM is that the topography can be represented only by whole grid cells, resulting in "staircase"-like mountain shapes and poor representation of gently sloped terrain. Therefore, the method works the best for buildings and very steep terrain. In the past, several methods have been developed to address such a problem, for example the "shaved cells" approach (Adcroft et al. 1997). However, such methods would result in an elliptic equation (17) having coefficients that are functions of longitude as well, which prevent the use of an FFT in zonal direction and would require a more complex 3-D multigrid solver.

371 "Staircase" terrain can generate short-wave (two-delta) noise around obstacles. In gSAM, a 372 numerically dissipative third-order upstream-biased scheme (TOB) is used to help to control the small-373 scale noise around topography. To minimize overdamping by the TOB scheme, the hybrid scheme is 374 employed to compute the momentum advection tendency a weighted superposition of the momentum 375 tendencies due to SOC and TOB schemes. The TOB scheme used together with the AB3 is somewhat less 376 stable than the SOC scheme, with a maximum stable Courant number of about 0.39 vs 0.72. However, the 377 stability threshold of the hybrid scheme changes almost linearly with the weighting coefficient  $\alpha$  between 378 the stability thresholds of the SOC and TOB schemes. Experimentally, using a 1-D advection test, the 379 following empirical relationship between maximum stable Courant number (CFL) and partitioning weight  $\alpha$  is found:  $CFL = 0.39 + 0.23\alpha + 0.09\alpha^2$ . For example, for  $\alpha = 0.7$ , that is 70% of momentum tendency 380 due to SOC and 30% due to TOB scheme, which is usually sufficient to control the short-wave noise, the 381 382 maximum stable Courant number is about 0.6, compared to 0.72 if only SOC is used. This reduces the 383 maximum time step required to maintain a stable solution by about 20%.

384

#### **385 3. Results**

386

### 387 3.1 Steady state test (gradient-wind balance)

388

389 To demonstrate that gSAM's anelastic approach indeed gives results equivalent to the primitive 390 equations in pressure coordinates in the hydrostatic limit, as discussed above in Section 2.4, the idealized 391 steady-state case developed by Jablonowski and Williamson (JW06; 2006) is used. The initial conditions 392 in JW06 describe a steady-state rotating atmosphere in the hydrostatic and gradient-wind balance as an 393 analytic solution to the primitive equations. The prescribed structure of the zonally symmetric temperature 394 distribution and corresponding zonal thermal wind are shown in Figs. 2*a*-*b* and closely resembles the real 395 atmosphere. A perfect model based on the primitive equations should maintain that initial gradient-wind 396 balance indefinitely without developing any instabilities or meridional circulation. For the test, we use a 397 relatively coarse 512x256 horizontal grid (approximately 0.7° uniform grid spacing) with 30 vertical levels 398 whose spacing gradually reduces from about 100 m near the surface to 5000 m near the domain top at 37 399 km. This is comparable to the GCM grids used in the JW06 study. The time step is 30 s. As suggested by 400 JW06, no explicit noise damping or SGS diffusion is applied. The non-dissipative SOC scheme is used 401 for advection of momentum. To minimize round-off errors, all computations are done in double precision. 402 Following the discussion in Section 2.3, the specified initial temperature and velocity fields are not 403 interpolated from pressure-sigma to height coordinates; instead, they are interpolated to the reference 404 pressures corresponding to the gSAM height levels, calculated following Sec. 2.3 with the global-mean 405 geopotential height as a function of pressure calculated by integrating the hydrostatic equation vertically 406 with the globally averaged temperature profile and surface pressure prescribed by JW06. This approach 407 enables gSAM to preserve the initial thermal wind balance of the unperturbed basic state specified in408 pressure coordinates.

409

410 Starting from the prescribed zonal velocity and temperature fields, the model is integrated for 30 411 days. The resultant zonal wind remains nearly steady, developing slight wind anomalies with the 412 amplitudes of less than about 0.1 m/s even in the region of jets in midlatitudes, as shown for day 30 in 413 Fig. 2c. Note that the reference pressure profile is used as the vertical coordinates on these plots. Even 414 smaller errors are found for the meridional wind (not shown). The slight imbalance can be explained by 415 the relatively coarse grid used in this test. As expected, the gSAM initialization procedure and numerics 416 almost precisely maintain the thermal wind balance of the prescribed temperature field and zonal wind, 417 even though they were designed for pressure/sigma-coordinate models. The corresponding geopotential height proxy  $\Phi/q + z$  computed by gSAM, shown in Fig. 3d, is nearly identical to the geopotential height 418 419 prescribed in JW06 (see their Fig. 1e).

420

#### 421 3.2 Baroclinic wave test

422

423 The zonally symmetric state in the previous test is baroclinically unstable, and baroclinic waves 424 can be triggered from a relatively small-amplitude initial zonal wind perturbation in mid-latitudes as 425 described in JW06. The model is run for 10 days with such an initial perturbation and the same grid as 426 described in the previous test. Snapshots of the surface pressure and of the temperature and vorticity at 427 850 hPa are shown in Fig. 3. As there is no analytical solution, we compared our results to numerous 428 results from other dynamical cores found in literature, including JW06. Overall, the magnitudes and the 429 shapes of the displayed fields compare quite well with other dynamical cores, with the notable exception 430 of the longitudes of the disturbances, which seem to be advanced forward by about 10° in gSAM. A similar 431 result was reported by Kurowski et al (2015), who directly compared results from compressible and 432 anelastic versions of the same model, although for a somewhat different moist baroclinic wave test. They 433 attributed that advance to the delayed onset of the rapid growth of the baroclinic wave in their anelastic 434 version of the model. In our case, the initial conditions may also play a role in the apparent phase bias of 435 the baroclinic wave, as the initial wind perturbation specified by JW06 does not satisfy the anelastic 436 continuity equation. As the result, gSAM immediately tries to make the flow non-divergent, reducing the

amplitude of the initial zonal wind perturbation from 1 m/s to 0.5 m/s, and also creating a rather strong
perturbation of the meridional wind, which is initially absent.

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- 440 3.3 Held-Suarez test
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442 Held and Suarez (1994; further HS94) developed a standard test of how well dry dynamical cores 443 simulate statistical properties of mid-latitude baroclinic eddies. The test prescribes highly idealized 444 diabatic heating and cooling as well as the boundary layer damping. The model then runs for very long 445 time to obtain the time-averaged statistics. As there is no analytical solution or observations, the model 446 results are typically compared to the results of other dynamical cores. We run gSAM for 1200 days using 447 a 288x144x30 grid (uniform 1.25° spacing in latitude and longitude) with the domain top at 37 km and a 448 20 s time step. The Smagorinsky closure is used for the SGS fluxes, with SOC differences of momentum 449 fluxes. Above 22 km, a sponge layer is used to control vertically propagating gravity waves. The initial 450 state is motionless isothermal atmosphere with the temperature of 300K.

451 After allowing 200 days for model spin-up, statistics are calculated over the last 1000 days and 452 shown in Fig. 4. Overall, the results agree well, both qualitatively and quantitatively, with the climatology 453 in HS94 as well as other models. For example, Figure 4 compares the zonal wind, meridional eddy-flux 454 of zonal momentum and meridional eddy flux of temperature to the results obtained by CAM with a 455 spectral Eulerian dynamical core (CAM-EUL) running with T85 truncation and 30 levels, comparable in 456 resolution to the grid used by gSAM in this test. The plots for CAM were obtained from its official website 457 https://www.cesm.ucar.edu/models/simpler-models/held-suarez.html. One can see that, in general, gSAM 458 and CAM-EUL agree quite well in the position, structure and strength of the westerlies in midlatitudes 459 and of the easterlies in the tropics and polar regions, as well the meridional eddy fluxes of momentum and 460 heat. The zonally averaged temperature also looks quite similar to HS94 results. A kinetic energy spectrum for midlatitudes shows the canonical  $k^{-3}$  power law for the large-scale eddies transitions to a  $k^{-5/3}$  power 461 462 law for relatively short scales, which is consistent with the observed spectra in the real atmosphere in mid-463 latitudes (e. g. Nastrom and Gage, 1985).

- 464
- 465 3.4 3-D Rossby-Haurwitz wave
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467 This test is an extension of a standard 2-D test for the shallow water models discussed by 468 Williamson et al (1992). We use the formulation of the 3-D test as described by Jablonowski et al (2008). 469 Despite being formally a 3-D test, the wave itself is essentially a 2-D barotropic wave, propagating 470 westward with a constant speed and preserving its shape. Therefore, it is a very useful test to check the 471 implementation of the Coriolis and metric terms and the ability of the model to maintain such a wave for 472 long time. The domain has 576x288x30 grid points (0.625° horizontal grid spacing and 30 levels), and 473 time step is set to 30 s. The number of wavelengths around the equator is set to 4, which corresponds to a 474 propagation speed of about -15.2° per day, or 24 days for the full revolution around the globe, which is 475 the model's integration time in this case. Figure 4 shows zonal and meridional wind components as well 476 as the surface pressure at the end of the run. The propagation speed computed from the Hovmöller diagram 477 (not shown) turned out to be very close to this theoretical value. The shape and magnitude of the simulated 478 fields in Fig 4 are barely different from the initial fields after 24 days of integration.

479

#### 480 3.5 Non-hydrostatic mountain waves

481

482 The tests presented thus far have dealt with quasi-hydrostatic large-scale flows. The present test 483 examines the non-hydrostatic response of gSAM to flow over the terrain profile proposed in Schär et al 484 (2002; further S02), which prescribes a series of bell-shaped mountains with the tops inside a cosine-like 485 envelope. Two cases are considered, a 2-D mountain range case following S02, who also provided an 486 analytical solution, and a 3-D circular mountain following the case 2.1 from the DCMIP 2012 (Ullrich et 487 al 2012), for which linear analytical solution has also been developed (Klemp et al 2015; further K15). 488 For this test we use gSAM in Cartesian coordinates as our goal here is primarily to test the implementation 489 of the terrain. The atmosphere is isothermal at 300 K, with uniform wind of 20 m/s. The domain has 490 512x512x80 grid points with 333.3 m grid spacing to make the domain similar in size to the domain in 491 S02, with the top at 37 km. The vertical grid spacing gradually increases with height, from 40 m near the 492 surface, to 500 m at 3600 m, staying constant up to 19 km, and then again monotonically increasing to 493 1400 m at 37 km. The maximum height of the terrain is 250 m with 5000 m mountain-range half-width 494 and 4000 m mountain wavelength, which represents the distance between individual mountain peaks. This 495 vertical grid is the same one used by gSAM in DYAMOND (Stevens et al, 2019) and was not specially 496 refined for this specific test. As the result, the terrain is represented rather coarsely in the vertical by only 6 grid levels below the highest mountain peak. To avoid reflection of gravity waves from the top of the 497

498 domain, a sponge layer is placed above 20 km. Vertical cross-sections of vertical velocity after two hours 499 of simulation are shown in Figs. 6a and 6b for the 2-D and 3-D cases, respectively. Both simulations agree 500 quite well with the analytical solutions Figs. 6c and 6d, which are adopted from K15. There is little 501 evidence of the effect of terrain "staircasing" on the simulation, mostly because of the use of hybrid SOC-TOB momentum advection in the vicinity of the mountain with  $\alpha$ =0.7 (see sec. 2.6 for details). There are 502 503 some additional perturbations compared to the linear analytical solution, probably because of nonlinear 504 effects. Horizontal cross-sections of vertical velocity for 3-D simulation at 8 and 16 km are shown in Figs. 505 7a and 7b. They also compare quite well to the analytical solutions Figs. 7c and 7d, also adopted from 506 K15.

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#### 508 3.6 Mountain induced Rossby wave train

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510 This is another test of the terrain implementation in gSAM, this time for hydrostatic flow over the 511 globe. The case set-up follows the specifications in the DCMIP 2008 report (Jablonowski et al, 2008). 512 The base state is an isothermal atmosphere in zonally symmetric solid-body rotation, which is constant 513 with height, with maximum wind at the equator of 20 m/s. This base state is perturbed by an idealized 514 bell-shaped 2000-m high mountain, with half-width of 1500 km, centered at 90°E and 30°N. The grid 515 configuration is similar to the Rossby-Haurwitz wave case. To minimize reflection of gravity waves from 516 the model top, a sponge layer in the upper third of the domain is employed. The solution for the zonal and 517 meridional wind components at 700 mb levels at days 5, 15 and 25 are shown in Fig. 8. There is no 518 analytical solution for this case, but the results compare well with those from other models shown in 519 DCMIP report by Jablonowski (2008; Fig. 20) and also by Ullrich and Jablonowski (2012). As noted by 520 the latter, the models in this test usually begin to diverge after 15 simulated days.

521

#### 522 3.7 DYAMOND-Winter real Earth simulation

523

524 So far, we presented the results of several idealized tests, just a few of many that have been 525 developed over the years. Such idealized tests have been routinely used by the weather and climate 526 modelling communities to identify the problems with models' formulation, their major weaknesses, and 527 for finding "bugs" in their code that would be difficult to find otherwise in realistic simulations, as many 528 of the problems and bugs could be simply hidden in the general clatter and noise of such simulations.

However, gSAM is not a new model per se: it is an extension of the well-established SAM to a global 529 530 domain. Aside from the topography implementation, the rest of the physics in gSAM is basically identical 531 to SAM's. Therefore, in addition to the tests of the dry dynamical core, cloud-resolving simulations of the 532 real Earth have been also performed and evaluated. Because of the considerable computational cost of 533 such realistic simulations, relatively few of them have been performed to date. The very first such 534 simulation was for the aforementioned DYAMOND project (Stevens et al, 2019), for which the results of 535 gSAM have been quite encouraging. The model has also performed short hindcast simulations using 536 different microphysical schemes during the SOCRATES (Southern Ocean Clouds, Radiation, Aerosol 537 Transport Experimental Study) field experiment, as reported by Atlas et al (2021) whose analysis focused 538 on a comparison of mixed-phase clouds over the Southern Ocean simulated by different microphysical 539 parameterizations implemented in SAM. Recently, gSAM has also participated, among several other 540 GSRMs, in the second phase of the DYAMOND project. Below, we present just a few highlights from 541 that simulation, reserving more substantial discussion and analysis for future publications.

The DYAMOND-Winter simulation covers a 40-day period from January 20<sup>th</sup> to March 1<sup>st</sup>, 2020. Initialized from ERA5 reanalysis at 00Z on January 20<sup>th</sup>, gSAM ran for 40 days using prescribed sea surface temperatures and sea ice cover. The CAM3 radiation and standard SAM's single-moment microphysics are used. The grid has 9216 x 4608 grid cells in the horizontal and 74 levels in the vertical. The grid spacing is locally isotropic in the horizontal (same in both meridional and zonal directions) until about 70°N/S with grid spacing of 4.25 km over the equator, gradually decreasing to 2-3 km in midlatitudes, then coarsening towards the poles to maintain numerical stability. The time step is 10 s.

Figure 9 compares the 5-day average (January 21<sup>st</sup> to January 25<sup>th</sup>) of precipitation, surface latent 549 550 heat flux (LHF), and top-of-atmosphere (TOA) outgoing longwave (OLR) and absorbed solar (ASR) 551 radiative fluxes to ERA5 reanalysis. These five days are used to make a quantitative comparison to 552 reanalysis before the model divergence from the observed state due to natural variability. The first day 553 has been neglected to allow for model spin-up. Overall, the geographic pattern of the mean precipitation 554 is reproduced quite well, and gSAM's global mean precipitation of 3.0 mm/day is similar to the 2.9 555 mm/day five-day average in ERA5. The most notable bias of gSAM is the relatively light precipitation in 556 the South Pacific Convergence Zone (SPCZ) in tropics. It is encouraging to see orographically enhanced 557 precipitation zones over the southern tip of South America and the Pacific coast of Colombia, as well as 558 over Norway, the Pacific Northwest, and South Africa, implying that gSAM's representation of the 559 topography works reasonably well. The geographic distribution of the surface LHF also compares quite

well to ERA5 estimates, with a global mean value of  $83.9 \text{ W/m}^2$  in gSAM and  $80.0 \text{ W/m}^2$  in ERA5. The performance of gSAM's land model is validated by good agreement with ERA5 for LHF over land, especially over the heavily vegetated areas of tropics and subtropics, where these fluxes vary strongly due to the diurnal cycle and also depend on partitioning of total flux between the evapotranspiration and sensible heat fluxes.

565 The simulated pattern of OLR compares well to ERA5, showing large areas filled with cold cloud 566 tops over tropical land masses, the SPCZ, and major storm-tracks in midlatitudes. In cloud-free areas over 567 subtropical oceans, the OLR tends to be overestimated, but is underestimated over convectively active 568 regions over land in tropics. The overall pattern of the ASR is simulated by gSAM reasonably well, but 569 there are many regional biases, most prominently over the Southern Ocean. Atlas et al (2021) showed that 570 the low bias in ASR over the Southern Ocean seen in Fig. 9d can be removed by using a microphysics 571 scheme in gSAM that more efficiently glaciates supercooled shallow cumuli. Global mean OLR is higher 572 in gSAM than in ERA5, 245.4 W/m<sup>2</sup> vs 240.2 W/m<sup>2</sup>, while at the same time, global mean ASR is considerably underestimated in gSAM relative to ERA5, 241.6 W/m<sup>2</sup> vs 248.8 W/m<sup>2</sup>. Therefore, the net 573 574 incoming radiative flux at TOA (ASR-OLR) is -3.8 W/m2 in gSAM vs 8.6 W/m<sup>2</sup> in ERA5, a difference of about 12 W/m<sup>2</sup>. While radiative fluxes from ERA5 should not be considered "ground truth", this large 575 discrepancy between gSAM and ERA5 warrants further investigation, including its sensitivity to the 576 577 choice of radiation and microphysics packages in gSAM.

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#### 579 *3.8 Hindcast of Hurricane Irma*

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581 Although weather forecasting was never a motivation for the extension of SAM to a global domain, 582 the ability of gSAM to forecast a tropical cyclone has been tested by performing a hindcast of Hurricane 583 Irma, one of the most powerful Atlantic hurricanes of the 2017 season. Irma reached Category 5, 584 devastated several Caribbean islands, struck Cuba and the Florida Keys, and eventually made landfall in 585 Florida. Global numerical weather prediction models had difficulty forecasting its path. For example, Fig. 586 10a shows a so-called "spaghetti" plot with predicted tracks of Irma as of September 5th, 2017 based on 587 forecasts from several NWP models, approximately 5-6 days before landfall in Florida. One can see a 588 wide spread of the trajectories with no clear indication on which coast of Florida (east or west) Irma would 589 make landfall. After initialization from ERA5 reanalysis at 00Z on September 5, gSAM was run for 7 590 days. After initialization, the sea-surface temperature was not updated since a real forecast model would

591 not have such information. An ensemble of 10 simulations was made with coarse, 17-km grid spacing 592 using a global 2304x1152 grid, along with one high-resolution deterministic run using 4.25-km spacing 593 using a 9216 x 4608 grid. All runs used 74 vertical levels and a domain top at 37 km, and no cumulus 594 parameterization was employed. The ensemble was used to determine the most probable path of Irma, 595 while the deterministic run was used to make the intensity forecast. All runs were performed on the 596 Chevenne supercomputer at NCAR using 1152 cores for the ensemble runs and 4608 cores for the 597 deterministic run. Each ensemble member ran about 50 times faster than wall-clock time, while the 598 deterministic simulation ran 6 times faster than wall-clock time. Note that gSAM could run on twice as 599 many processors as this, which would further increase the throughput by about 80%. For a 7-day forecast, 600 each of the ensemble runs (which could be done concurrently) would then take less than 2 wall-clock 601 hours, and the deterministic forecast would take about 16 hours.

602 Figure 10b compares the predicted tracks of Irma from all runs with the estimated best track from 603 observations. The ensemble generates a wide spread of tracks toward the end of the runs, with the 604 ensemble mean following the actual path of Irma quite well and correctly predicting that Irma would make 605 landfall on the east coast of Florida. Fig 10c shows one "lucky" ensemble member that predicted the actual 606 path of Irma almost perfectly. Figs 10e and 10f show the minimum surface pressure and maximum 607 sustained wind near the surface as well as best-track data. Because of ERA5's rather coarse resolution, 608 the initial depression associated with Irma was relatively weak, so it took several days for the model to 609 properly spin up the hurricane. As expected, 17-km resolution is not sufficient for predicting the minimum 610 pressure and intensity of the hurricane but can be useful for predicting its path. The 4-km deterministic 611 forecast predicts the intensity much better despite not being the best in predicting the hurricane's track. 612 Overall, the results produced by gSAM in this hurricane case are encouragingly realistic given the 613 approximations inherent in gSAM's dynamical equations. This gives a more favorable view of gSAM's 614 ability to simulate tropical cyclones than the DYAMOND analysis of Judt et al. (2021) and shows that 615 gSAM is suitable for both realistic and idealized global simulations requiring storm-resolving grid 616 resolution.

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#### 622 **4. Conclusions**

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This paper describes the extension of the cloud-resolving and large eddy simulation model, SAM, to a global domain. The resulted global storm resolving model, gSAM, is formulated on a latitudelongitude grid but can seamlessly switch back to Cartesian coordinates. Hence, it can still be used as a conventional large-eddy simulation or cloud-resolving model over limited domains. The latitudelongitude grid allows gSAM to run both globally and over very large regional domains for which a Cartesian grid is not sufficiently accurate because of the Earth's curvature.

630 gSAM is unique among GSRMs in using an anelastic dynamical core with a single reference profile, like past versions of SAM. However, the anelastic equations used in gSAM, which follow Pauluis 631 632 (2008), differ from most other anelastic models in ways that are helpful for global modeling. Buoyancy 633 in gSAM is based on the absolute temperature, not the potential temperature, and the prognostic 634 thermodynamic equation is based on a moist form of the static energy, rather than potential temperature; 635 these modifications are convenient for microphysical parameterization. The thermodynamic equation also 636 includes the vertical flux of buoyancy (Pauluis, 2008). The resulting system is shown to conserve the 637 global integral of the sum of the kinetic, thermodynamic, and potential energies. For quasi-hydrostatic 638 flows, the anelastic system in gSAM is isomorphic to the primitive equations in pressure coordinates, but 639 with the globally-uniform reference pressure (which is a known function of height) in gSAM playing the 640 role of actual pressure. Therefore, to preserve geophysically important relationships such as geostrophic 641 and hydrostatic balance, initialization of gSAM by external data should be done by mapping horizontal 642 velocity, temperature and other data in pressure coordinates to the reference pressures of each grid level, 643 instead of using a direct column-by-column interpolation of the data to height coordinates. If this is done 644 in a standard gradient-wind balance test, gSAM can exactly maintain steady zonally symmetric baroclinic 645 flows that have been specified in pressure coordinates. This isomorphism helps rationalize how gSAM 646 can simulate realistic global temperature and wind distributions even though assumptions underlying the 647 anelastic approximation, such as the smallness of horizontal pressure perturbations, become shaky on 648 global scales.

649 Several tests of gSAM's accuracy in simulating global-scale flows without topography were 650 presented. Results of a standard baroclinic-wave test compare quite well to other models' results. In a 651 Held-Suarez test, the mean general circulation agrees well with results from the Community Atmosphere 652 Model with Eulerian dynamical core at similar horizontal grid resolution. In a barotropic equatorial Rossby-Haurwitz wave test, gSAM accurately maintains the expected shape and magnitude of the wave
very well and propagates the wave in the westward direction at a speed very close to the theoretically
derived value.

The treatment of topography in gSAM is novel among global atmospheric models, which generally 656 657 use terrain-following coordinates. gSAM uses altitude as the vertical coordinate. This considerably 658 simplifies the discretized fluid transport equations, but it means that some grid points lie within regions 659 of elevated terrain. The surface boundary condition is enforced using a type of immersed boundary method, the Quasi-Solid Body Method (QSBM), in which the flow velocity in grid cells inside prescribed 660 661 terrain is stagnated using "instantaneous" relaxation to zero. The QSBM method works well, as 662 demonstrated by two tests: linear non-hydrostatic flow over an idealized mountain range and a global 663 simulation hydrostatic Rossby waves induced by a large circular mountain. Starting with a no-topography 664 model version, implementing the QSBM requires minimal code changes and no algorithmic change to the 665 sophisticated multigrid anelastic Poisson solver used in gSAM.

666 Several results were presented from a real-Earth simulation performed for the DYAMOND-Winter 667 project. We compared several important fields with ERA-5 reanalysis, such as precipitation, surface latent 668 heat flux and top-of-atmosphere OLR and solar radiation, averaged over January 21-25, 2020. Despite 669 several notable biases, the gSAM results track the reanalysis well over this relatively short period.

We also presented results of a hindcast of Hurricane Irma in 2017. We used a 17-km grid to perform a 10-member ensemble runs and high-resolution 4-km for a single deterministic run initialized from the ERA5 data five days before Irma's landfall in Florida. The ensemble predicts the hurricane track quite well but considerably underpredicts the hurricane intensity just before landfall. In contrast, the deterministic run predicts an intensity similar to that observed, both in minimum surface pressure and maximum surface wind.

Our future research with gSAM will be conducted on multiple fronts. We will continue working on improving the accuracy of simulations around the poles and on achieving optimal computational performance on evolving high-performance computing architectures. We plan to run gSAM in weathernudged mode, forced by reanalysis data, with different microphysical packages, to improve representation of clouds and radiation and better understand how different cloud microphysics parameterizations affect the results. We will evaluate the simulation of the diurnal cycle over land, especially of precipitation, and work on further improvement of the Simplified Land Model and especially its treatment of snow cover.

- We will further evaluate the suitability of gSAM (including regional versions that can be used with finer grids) for forecasting extreme weather events in present and future climates.

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Figure 1. Comparison of surface pressure computed by gSAM (top) and ERA5 reanalysis (middle),
along with their difference (bottom) after one-hour simulation starting at 00Z on January 2020.



**Figure 2.** Initial a) temperature, b) zonal wind, c) pressure, d) geopotential height (zero level is shown by thicker isoline), and deviation from initial e) zonal and f) meridional wind after 30 days of SAM integration in the steady state test.





**Figure 3.** (top) Surface pressure, (middle) temperature at 850 hPa, and (bottom) relative vorticity at 850 hPa at (left) day 9 and (right) day 10 of the evolution of baroclinically unstable wave.



**Figure 4.** 1000-day mean climatology from the Held-Suarez test for various fields for gSAM (top panels) and CAM-EUL (three left bottom panels). The right-bottom panel shows the midlatitude zonal-wind variance spectrum for gSAM. The panels for CAM-EUL have been adopted from the CESM website https://www.cesm.ucar.edu/models/simpler-models/heldsuarez.html .





**Figure 6.** Vertical cross section of vertical velocity over a) 2-D and b) 3-D Schär-type mountain ranges. The plots c) and d) for corresponding analytical linear solutions are adopted from Klemp et al (2015).



linear solutions are adopted from Klemp et al (2015).

Figure 7. Horizontal cross section of vertical velocity over 3-D Schär-type mountain

ranges at c) 8 km and d) 16 km height. The plots c) and d) for corresponding analytical



**Figure 8.** The snapshots of wind components at 700 mb at (top) day 5, (middle) day 15, and (bottom) day 25 of the Rossby wave-train induced by a circular mountain. The center of the 2-km heigh and 1500-km wide bell-shaped mountain is at 90°E and 30°N.



**Figure 9.** The 5-day (00Z Jan 21 – 00Z Jan 26, 2020) average precipitation rate, latent heat flux, TOA outgoing longwave and absorbed solar radiative fluxes for gSAM (left panels) and ERA4 (right panels).



**Figure 10.** (a) "Spaghetti" forecast from NWP models as of September 5; (b) tracks predicted by gSAM: 10-member ensemble (red) and ensemble mean (green), deterministic (blue), and best-track (black); (c) track of one of the "lucky" ensemble members; (d) simulated satellite view of Irma using predicted fields from gSAM; e) minimum surface pressure and (f) maximum surface wind for ensemble members (green) and deterministic run (blue). Best-track data are denoted by black dots.