

The scale dependence of permeability: Effects of pore-throat size distribution and pore connectivity

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Abstract

Scaling has been a long-standing challenge in subsurface hydrology, soil physics, and many other research disciplines. The effect of length scale (or sample dimension) has been known in the literature, and inconsistent results have been reported. For example, experimental measurements typically show that permeability k should increase with increasing scale. However, numerical simulations and some theoretical estimations appear to imply the opposite. In this study, we simulated permeability in twelve synthetic and four Fontainebleau pore networks with different pore-throat size distributions. For each pore network, simulations were carried out for ten pore coordination numbers $Z = 1.5, 1.65, 1.75, 2, 3, 3.25, 3.5, 4, 5,$ and 6 . We found a transition in the scale dependence of the permeability in the synthetic pore networks. More specifically, our results showed that the permeability increased with the scale for larger pore coordination numbers, while it decreased with the scale for smaller Z . In Fontainebleau pore networks, however, the trends were decreasing permeabilities regardless of the value of Z . We invoked concept of finite-size scaling analysis, a vigorous theoretical framework from physics, to address the effect of scale on the permeability. Although the plot of the permeability versus the network size for each pore network appeared scattered, the data collapsed together by applying finite-size scaling analysis. Our results demonstrated that finite-size scaling analysis is a promising approach to address the effect of scale on permeability.

The scale dependence of permeability: Effects of pore-throat size distribution and pore connectivity

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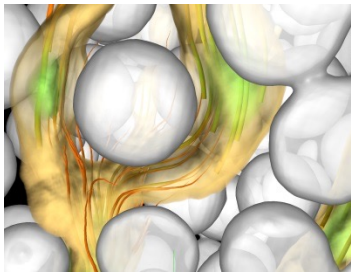
Department of Chemical Eng. & Materials Sci., University of Southern California

Introduction

Scaling: A long-standing issue in subsurface hydrology

Field

Pore



<http://subsurface.pnl.gov>

nanometers

Core



<http://physics.aps.org>

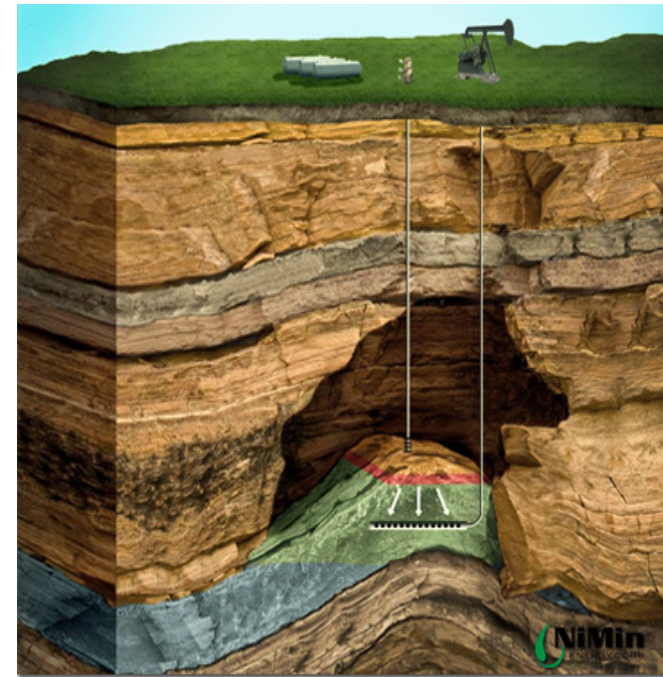
centimeters

Lysimeter



<http://www.gfz-potsdam.de>

meters



Oxygen, Carbon Dioxide, & Steam Combustion Zone Oil Water

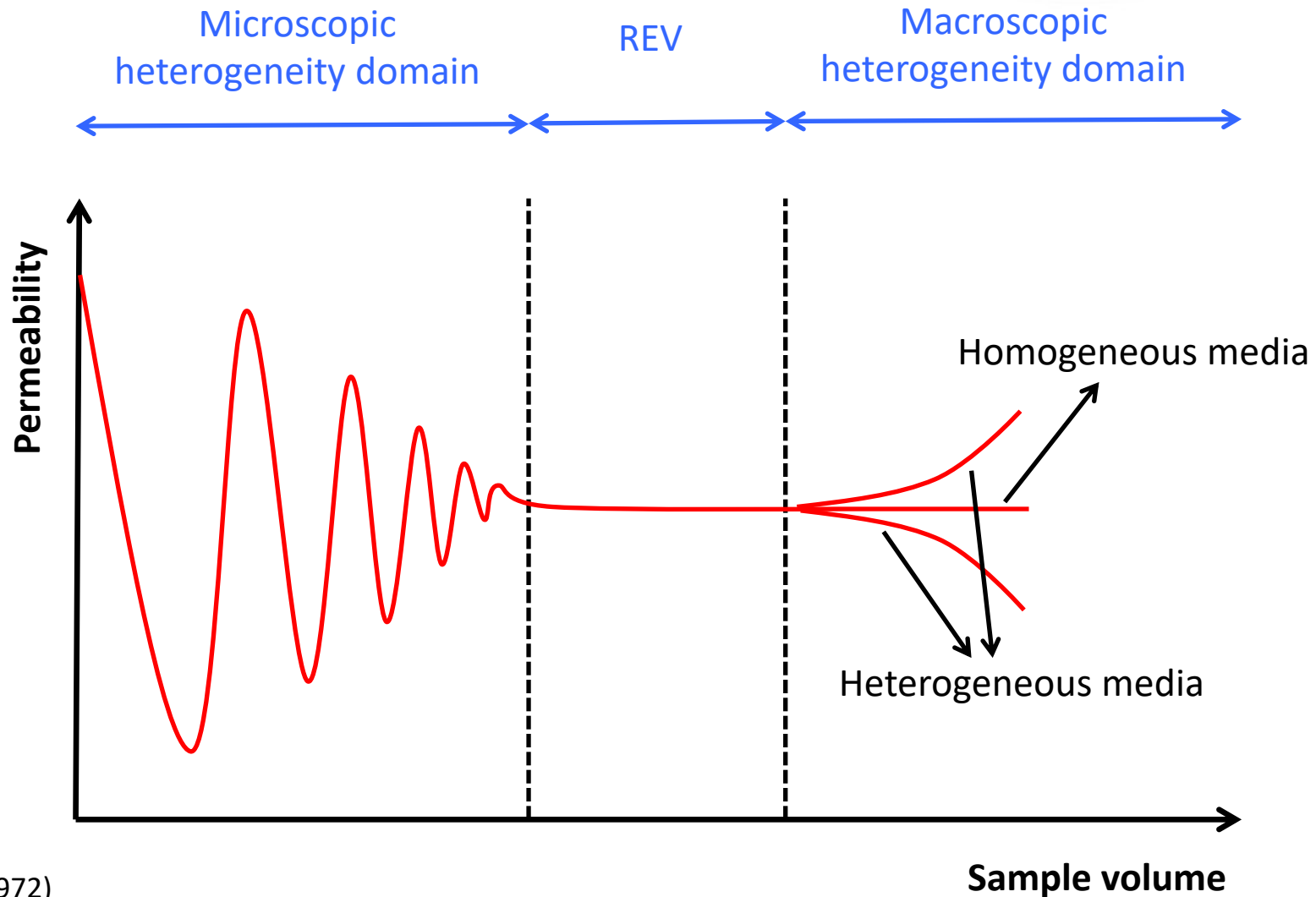
<http://www.siliconinvestor.com>

kilometers

↔
7 orders of magnitude

↔
5 orders of magnitude

Introduction



Bear (1972)

- (1) Investigating the effect of small-scale heterogeneity on scale-dependent permeability**
- (2) Providing a rigorous theoretical foundation for the interpretation of the scale dependence of permeability**
- (3) Investigating the increasing or decreasing trend of permeability with scale**

Generating pore networks

We used regular cubic lattices with coordination number $Z = 1.5, 1.65, 1.75, 2, 3, 3.25, 3.5, 4, 5,$ and 6 .

- **Networks 1.1 – 1.4**

Pore-throat radius: $0.1\text{-}10\ \mu\text{m}$

- **Networks 2.1 – 2.4**

Pore-throat radius: $1\text{-}50\ \mu\text{m}$

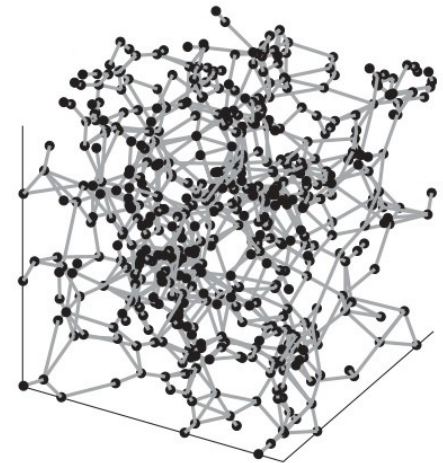
- **Networks 3.1 – 3.4**

Pore-throat radius: $10\text{-}75\ \mu\text{m}$

In networks 1-3, pore-throat length $l_t = 100\ \mu\text{m}$.

- **Fontainebleau sandstone networks**

(Lindquist et al., 2000)

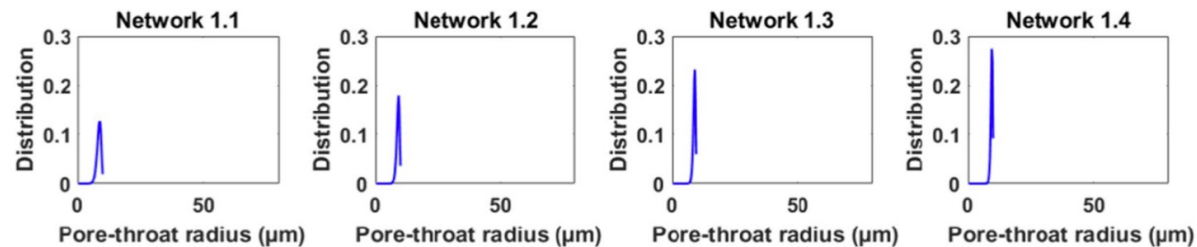


A 3D pore-network composed of pore throats and pore bodies (after Valvatne, 2004)

Pore-network simulations

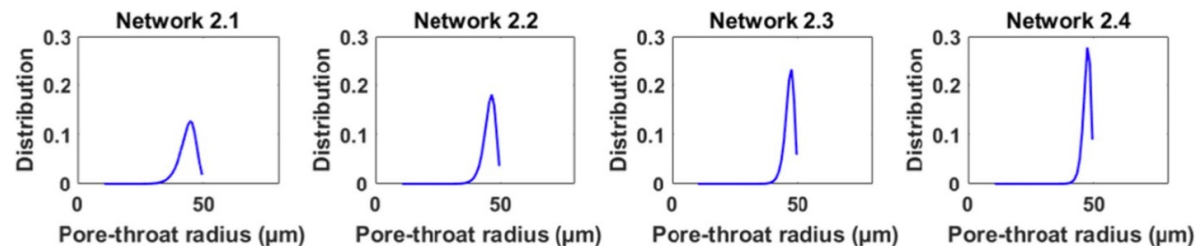
- Networks 1.1 – 1.4

Pore-throat radius: 0.1-10 μm



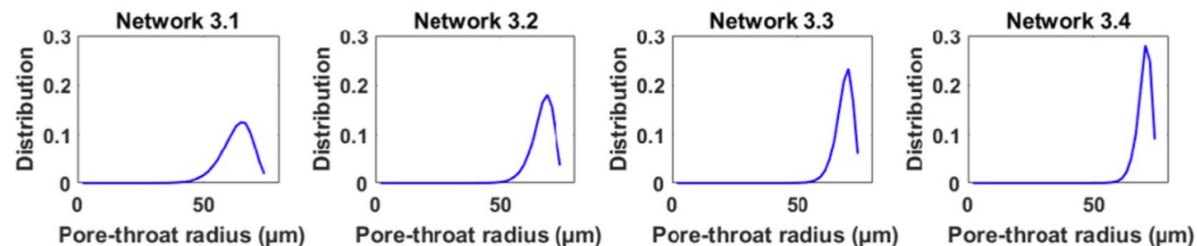
- Networks 2.1 – 2.4

Pore-throat radius: 1-50 μm

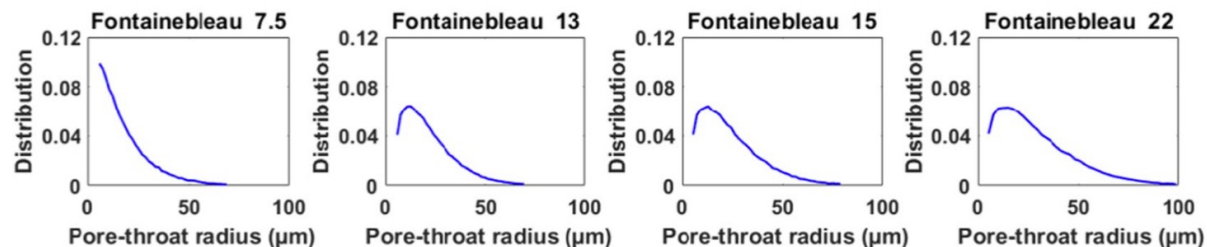


- Networks 3.1 – 3.4

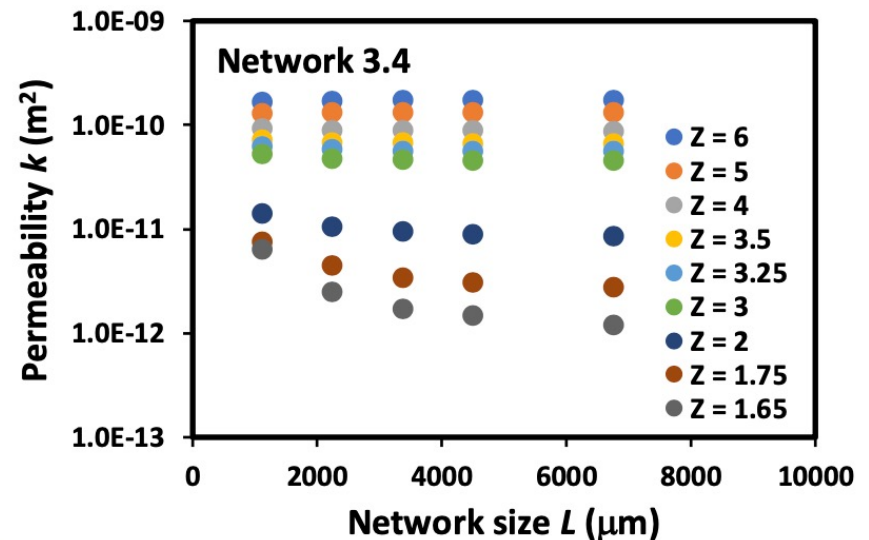
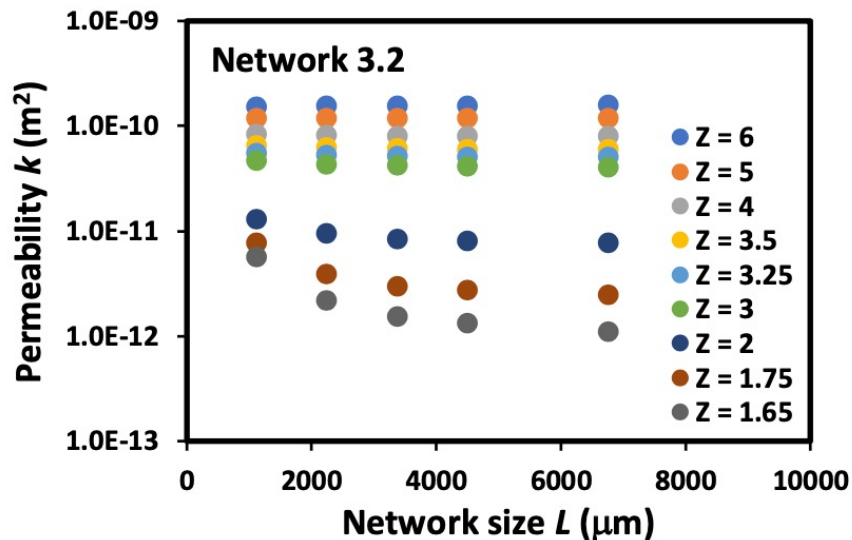
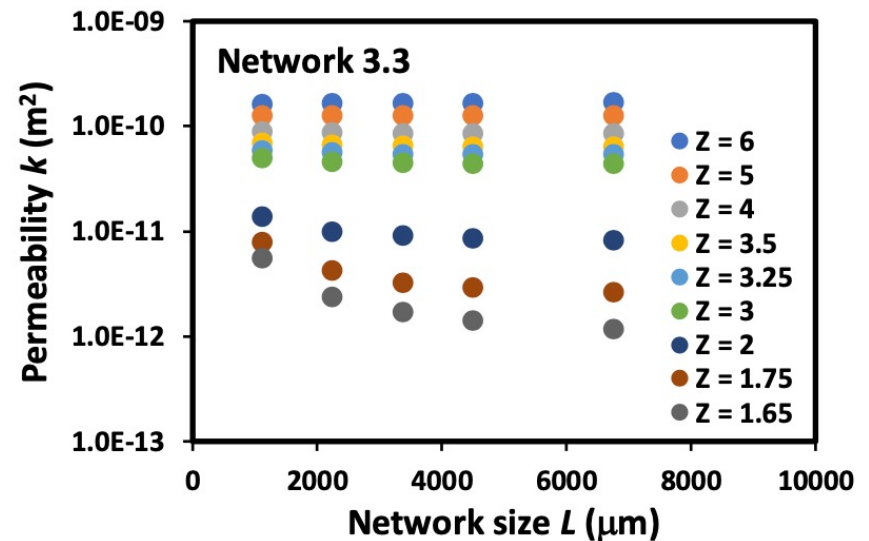
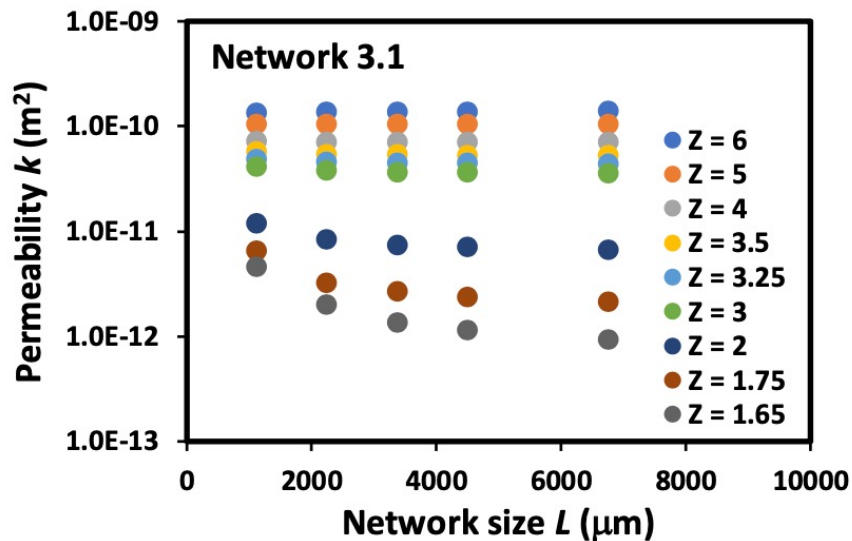
Pore-throat radius: 10-75 μm



- Fontainebleau sandstone networks (Lindquist et al., 2000)



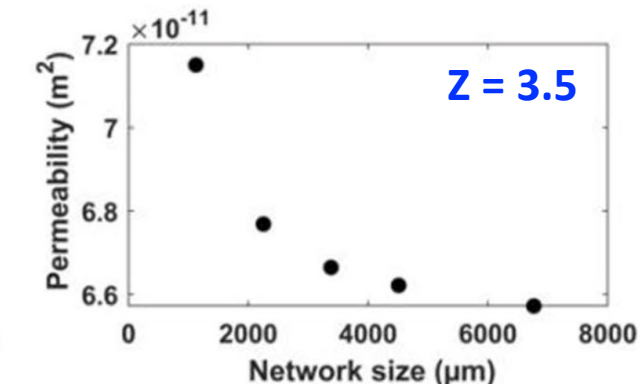
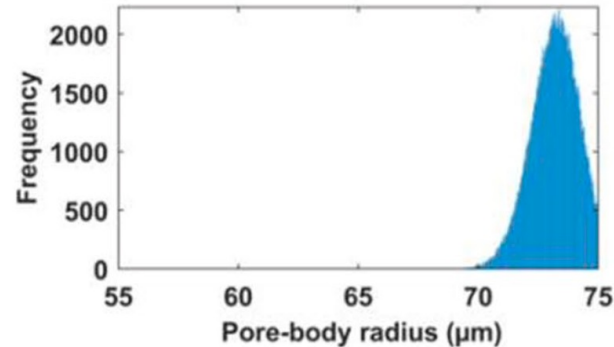
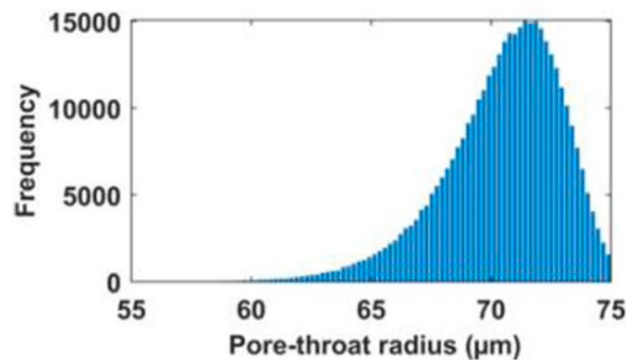
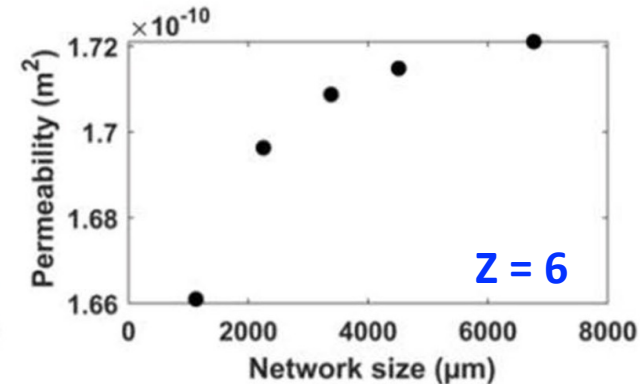
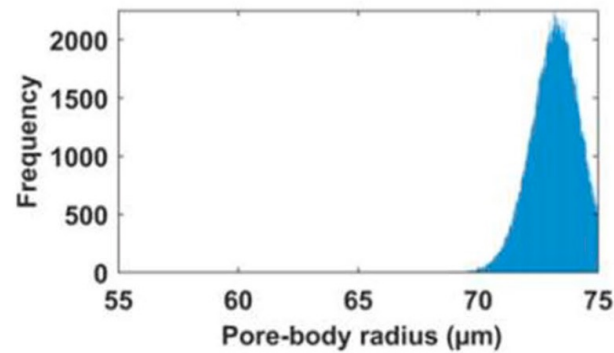
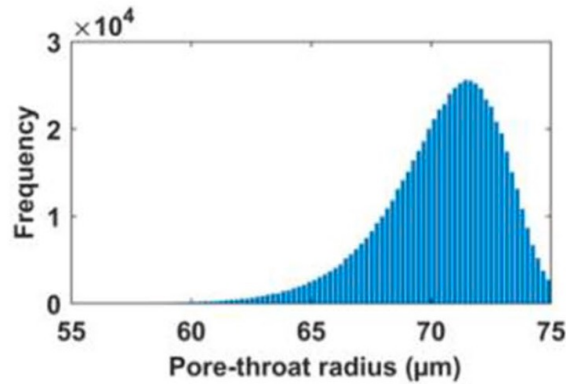
Results: Pore-network simulations



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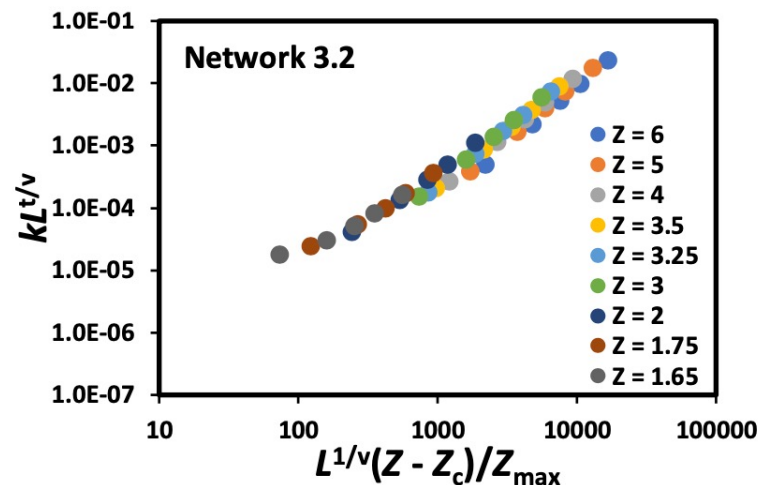
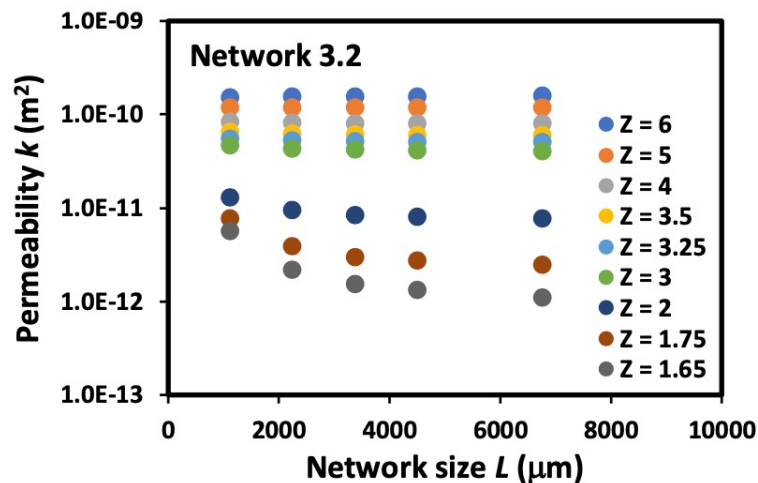
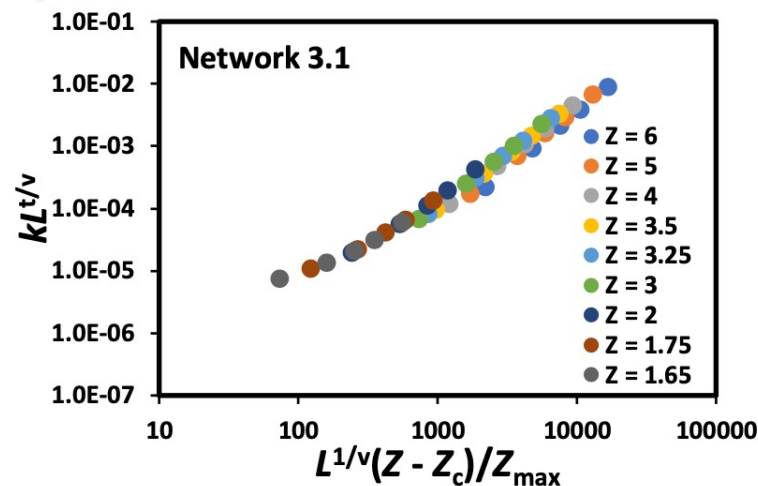
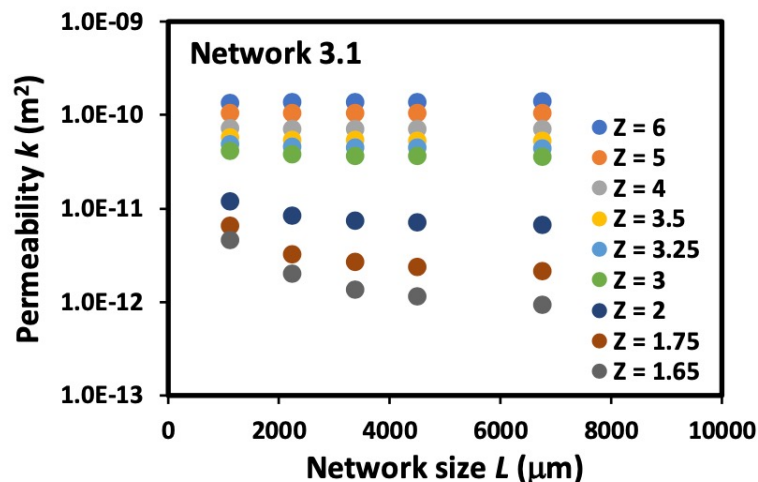
For the synthetic pore networks, we find a transition in the scale dependence of the permeability, with our results indicating that the permeability increases with the scale for larger pore coordination numbers, whereas the opposite is true for smaller Z .

Network 3.4



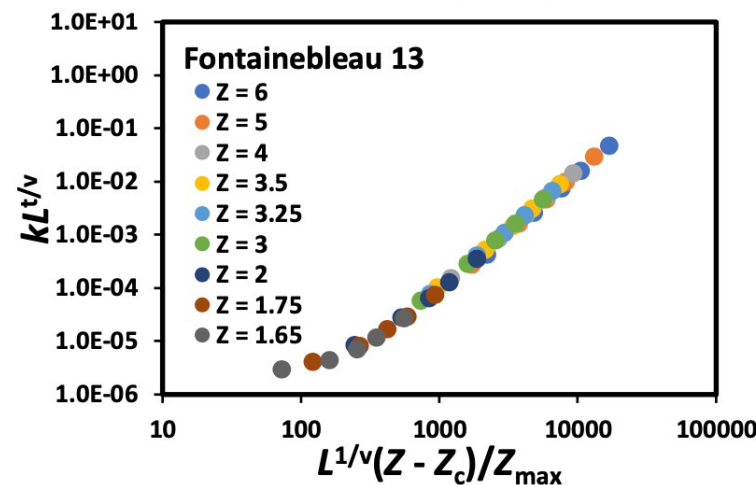
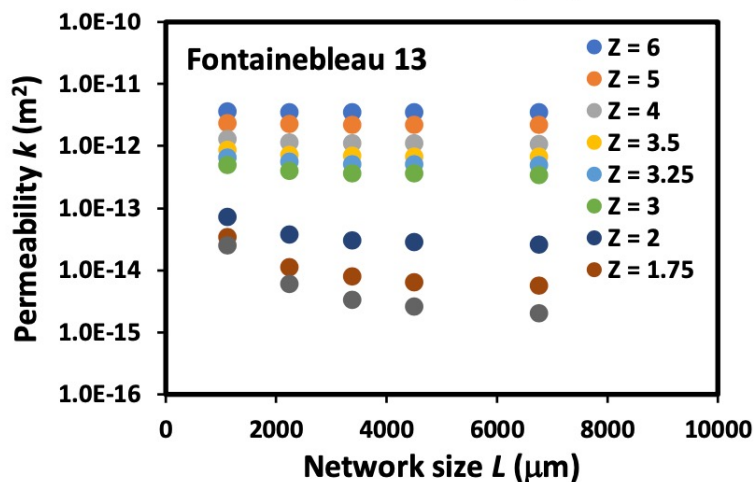
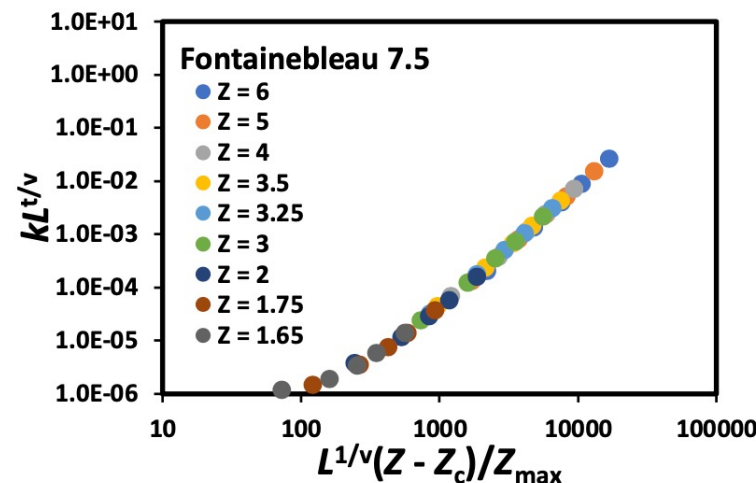
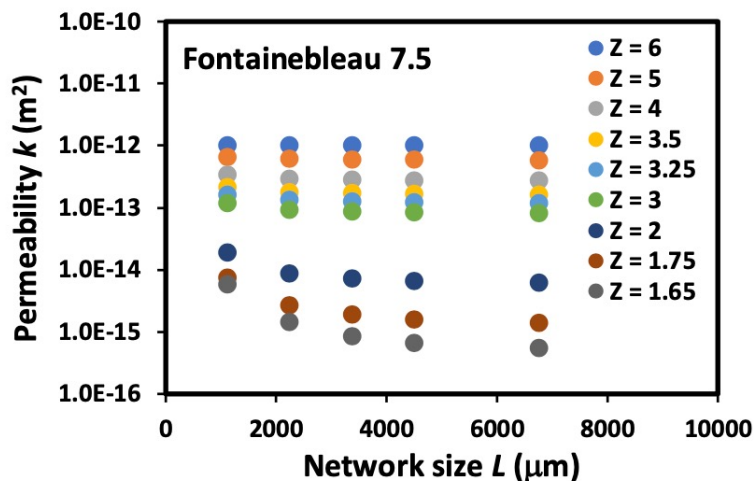
Results: Finite-size scaling analysis

$$k = L^{-\frac{t}{\nu}} f \left[L^{\frac{1}{\nu}} (p - p_c) \right] \quad \longrightarrow \quad k = L^{-\frac{t}{\nu}} f \left[L^{\frac{1}{\nu}} (Z - Z_c) / Z_{max} \right]$$



Results: Finite-size scaling analysis

$$k = L^{-\frac{t}{\nu}} f \left[L^{\frac{1}{\nu}} (Z - Z_c) / Z_{max} \right]$$



- **We found a transition in the scale dependence of the permeability.**
- **Depending on pore-throat radius distribution broadness, permeability may increase or decrease with scale.**
- **The modified finite-size scaling analysis results in perfect collapse in data indicating a quasi-universal trend.**



Questions?