

# Baroclinic control of Southern Ocean eddy upwelling near topography

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November 24, 2022

## Abstract

In the Southern Ocean, mesoscale eddies contribute to the upwelling of deep waters along sloping isopycnals, helping to close the upper branch of the meridional overturning circulation. Eddy energy is not uniformly distributed along the Antarctic Circumpolar Current (ACC). Instead, ‘hotspots’ of eddy energy that are associated with enhanced eddy-induced upwelling exist downstream of topographic features. This study shows that, in idealized eddy-resolved simulations, a topographic feature in the ACC path can enhance and localize eddy-induced upwelling. However, the upwelling systematically occurs in regions where eddies grow through baroclinic instability, rather than in regions where eddy energy is large. Across a range of parameters, along-stream eddy growth rate is a more reliable indicator of eddy upwelling than traditional parameterizations such as eddy kinetic energy, eddy potential energy or isopycnal slope. Ocean eddy parameterizations should consider metrics specific to the growth of baroclinic instability to accurately model eddy upwelling near topography.

# Baroclinic control of Southern Ocean eddy upwelling near topography

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## Key Points:

- Topography can support localised, enhanced cross-jet isopycnal transport driven by eddies.
- Transport occurs where baroclinic instability energizes eddies, not where eddy energy is high.
- In most cases, zonal growth of eddy energy is a more reliable indicator of cross-jet transport than metrics traditionally used.

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**Abstract**

In the Southern Ocean, mesoscale eddies contribute to the upwelling of deep waters along sloping isopycnals, helping to close the upper branch of the meridional overturning circulation. Eddy energy is not uniformly distributed along the Antarctic Circumpolar Current (ACC). Instead, ‘hotspots’ of eddy energy that are associated with enhanced eddy-induced upwelling exist downstream of topographic features. This study shows that, in idealized eddy-resolved simulations, a topographic feature in the ACC path can enhance and localize eddy-induced upwelling. However, the upwelling systematically occurs in regions where eddies grow through baroclinic instability, rather than in regions where eddy energy is large. Across a range of parameters, along-stream eddy growth rate is a more reliable indicator of eddy upwelling than traditional parameterizations such as eddy kinetic energy, eddy potential energy or isopycnal slope. Ocean eddy parameterizations should consider metrics specific to the growth of baroclinic instability to accurately model eddy upwelling near topography.

**Plain Language Summary**

The Southern Ocean plays an essential role in redistributing heat, salt and biogeochemical tracers of importance in the climate system. In particular, locations in which strong ocean currents interact with large topographic features are hotspots for eddy-driven upward transport, and are crucial pathways to bring deep, carbon- and nutrient-rich waters to the surface. The processes which set the location and magnitude of this eddy ‘upwelling’ remain challenging to understand. This study uses a series of high-resolution idealized simulations in which an ocean jet encounters a piece of topography to investigate what controls the eddy upwelling near topography. We find that the upwelling due to eddies occurs in regions where the eddies are growing through a mechanism called ‘baroclinic instability’, rather than in regions where eddies are highly energetic or energized by other mechanisms. Regions of growing eddy energy are a simple, first-order indicator of regions of eddy upwelling, but future parameterisations of transport should consider the mechanism of instability to be more accurate.

**1 Introduction**

The Southern Ocean is an essential component of the global overturning circulation, which redistributes heat, salt and biogeochemical tracers of importance in the cli-

47 mate system (J. Marshall & Speer, 2012). In particular, sloping density surfaces (isopy-  
48 cnyals) in the Southern Ocean provide an adiabatic route for deep waters to be upwelled  
49 to the surface. This along-isopycnal transport brings cold, carbon-rich waters to the sur-  
50 face (Le Quéré et al., 2007), imposing an important control on the Southern Ocean CO<sub>2</sub>  
51 sink and contributing to delayed warming of Southern Ocean waters (Armour et al., 2016).

52 Mesoscale eddies, which are particularly energetic in the Southern Ocean (Fu et  
53 al., 2010), play a dominant role in this along-isopycnal transport and therefore can have  
54 a critical influence on the associated mass, carbon and heat transports. Eddy activity  
55 in the Southern Ocean is not uniform in time or space. Zonal variations along the path  
56 of the Antarctic Circumpolar Current (ACC) are punctuated by regions of elevated eddy  
57 energy downstream of where the ACC interacts with major topographic features (Sokolov  
58 & Rintoul, 2009; Thompson, 2010; Frenger et al., 2015; Foppert et al., 2017), visible both  
59 at the surface (e.g. Fu et al., 2010) and at depth (e.g. Thompson & Naveira Garabato,  
60 2014). These hotspots of eddy energy are favourable to stronger cross-jet exchange (Thomp-  
61 son & Sallée, 2012; Dufour et al., 2015) and enhanced upwelling of deep and interme-  
62 diate waters (Viglione & Thompson, 2016; Tamsitt et al., 2017; Foppert et al., 2017).

63 Regions of elevated eddy energy are typically co-located with stationary meanders  
64 downstream of a topographic obstacle. The presence of these meanders, which are formed  
65 by arrested Rossby waves (Hughes & Ash, 2001), introduces non-zonal velocities, which  
66 lead to departures from the traditionally-assumed dynamical balances derived from a zonally-  
67 integrated view. The stationary meanders play an essential role in balancing zonal mo-  
68 mentum and provide a mechanism for rapid barotropic adjustment of the flow to changes  
69 in forcing (Thompson & Naveira Garabato, 2014). These meanders appear to dominate  
70 the meridional heat transport (Dufour et al., 2012), but such heat transport predomi-  
71 nantly occurs through transient eddies acting along the meander structure (Abernathey  
72 & Cessi, 2014). The essential role of transient eddies in this heat transport is visible when  
73 the transport is calculated in density-depth space (Zika et al., 2013) or following stream-  
74 lines (Abernathey & Cessi, 2014).

75 The strength of eddy-induced transport in the Southern Ocean is often assumed  
76 to scale linearly with eddy kinetic energy (EKE) along the lines of the classical mixing  
77 length hypothesis (Prandtl, 1925; Holloway, 1986). For example, studies investigating  
78 the response of Southern Ocean circulation to changes in forcing often examine the re-

79 sponse of EKE (e.g. Meredith & Hogg, 2006; Hogg et al., 2015; Patara et al., 2016), but  
80 few studies diagnose eddy-induced transport. Dufour et al. (2012) noted the increased  
81 southward transport due to transient and stationary eddies under increased wind forc-  
82 ing, but did not relate its response to that of EKE. However, there are no direct obser-  
83 vations or modelling studies which support a direct, local proportionality between EKE  
84 and eddy-induced upwelling. On the contrary, Tamsitt et al. (2017) reports enhanced  
85 upwelling upstream of EKE maxima, but does not provide a dynamical explanation for  
86 this spatial separation. Likewise, Foppert et al. (2017) noted an offset between eddy heat  
87 fluxes and EKE in the Drake Passage, and suggest that the sea surface height deviation  
88 (a proxy for eddy potential energy, EPE) is a better indicator of the divergent eddy heat  
89 flux and, by extension, eddy upwelling owing to a direct connection to baroclinic insta-  
90 bility (Watts et al., 2016). This offset is also found in the idealised simulations of Bischoff  
91 & Thompson (2014), which notes that EKE is not co-located with the steepest isopyc-  
92 nal slopes. An examination of how topography modulates eddy-induced upwelling and,  
93 further, an identification of the relationship between eddy energy and the mechanisms  
94 controlling upwelling location and magnitude are needed, in particular to inform our de-  
95 sign of eddy upwelling proxies.

96 This study focuses on how a single unstable jet in a 2-layer system supports intense,  
97 localised, isopycnal upwelling associated with transient eddies. This jet is an analogue  
98 for a single filament of the ACC; the simplicity of this system allows unambiguous def-  
99 inition of cross-jet volume transport to quantify eddy upwelling, revealing insights that  
100 are not possible in a more comprehensive model. In particular, the question of whether  
101 local eddy energy (EE), or one of its constituents (EKE or EPE), is a good indicator of  
102 local eddy-induced upwelling is examined. Lastly, we show that a simple parameterisa-  
103 tion of eddy-induced upwelling based on the zonal evolution of eddy energy provides a  
104 better representation of the zonal variability of upwelling around topography, compared  
105 with other proposed parameterisations based on EKE, EPE or time-mean isopycnal slope.

## 106 **2 Methods**

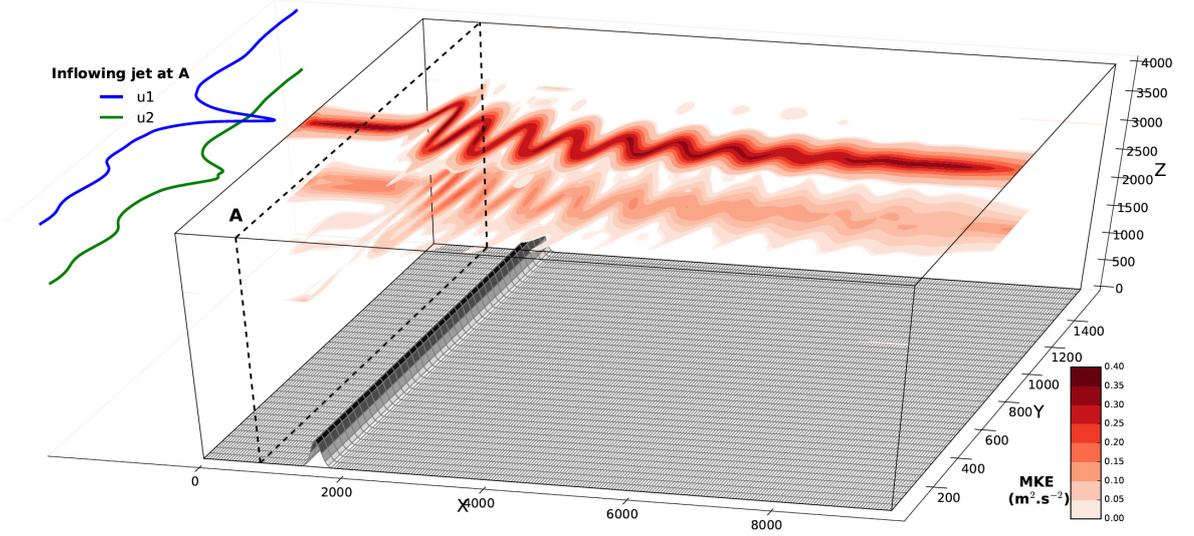
### 107 **2.1 Model configuration**

108 Our model simulations are designed to represent the interactions between a baro-  
109 clinic ocean jet and an isolated topographic feature, in a configuration relevant to the

110 Southern Ocean (see Fig. 1). The set-up used is identical to that of Barthel et al. (2017).  
 111 The model configuration is a channel on a  $\beta$ -plane, with dimensions of 9600 km  $\times$  1600  
 112 km and a horizontal resolution of 4 km. It consists of two isopycnal layers. We use MOM6  
 113 (Adcroft et al., 2019) to solve the hydrostatic thickness-weighted primitive equations un-  
 114 der the Boussinesq approximation. The background horizontal viscosity is parameterised  
 115 with a biharmonic horizontal viscosity of  $A_4 = 1.5 \times 10^9 \text{ m}^4 \text{ s}^{-1}$  to ensure numerical  
 116 stability, while bottom friction is modelled by a weak quadratic bottom drag (with  $C_{drag} =$   
 117  $5 \times 10^{-4}$ ). The dynamics in the interior of the channel are purely adiabatic.

118 The channel is forced to sustain an eastward-flowing jet (Fig. 1) by restoring the  
 119 stratification at the western boundary. The jet characteristics are representative of a typ-  
 120 ical frontal jet observed in the Southern Ocean, with a 50-to-150-km-wide jet core con-  
 121 taining peak velocities of 0.5-1  $\text{m s}^{-1}$  (in the upper 1000 m), while velocity below 1000  
 122 m is of order 0.1  $\text{m s}^{-1}$  (Waterman et al., 2013; Sheen et al., 2014). The eastern bound-  
 123 ary also features a ‘sponge’ region where isopycnal heights are restored to allow the flow  
 124 to readjust to the inflowing conditions. This boundary forcing of the flow provides a di-  
 125 rect control of the jet structure at the inflow, as well as prescribing the total zonal trans-  
 126 port. In this regard, this study differs from wind-driven channel studies which rely on  
 127 a wind-friction equilibration (e.g. Bischoff & Thompson, 2014; Chapman et al., 2015) and  
 128 can feature significantly different zonal transports depending on the presence of bottom  
 129 topography (see Abernathey & Cessi, 2014, their Fig. 8). Stratification is also restored  
 130 at the northern and southern boundaries, thus sustaining a large-scale meridional isopyc-  
 131 nal slope, with the upper layer shoaling southward. This combination of forcing allows  
 132 a non-zero residual overturning circulation to emerge in the domain, as it does in the South-  
 133 ern Ocean (e.g. G. J. Marshall, 2003; Lumpkin & Speer, 2007).

134 To explore topographic control of eddy-driven isopycnal upwelling, we compare flat-  
 135 bottom simulations with cases which include either a circular seamount, or a meridional  
 136 ridge, with a range of heights (0-500m). The range of topographic heights is small (com-  
 137 pared with the Southern Ocean) because the topography has a disproportionately large  
 138 effect in a two-layer system.



139 **Figure 1.** Model domain. A prescribed 2-layer jet flows eastward over topography, leading to  
 140 stationary meanders downstream of topography. The maximum inflow velocities at section A are  
 141  $0.7 \text{ m.s}^{-1}$  for the upper layer and  $0.3 \text{ m.s}^{-1}$  for the lower layer.

## 142 2.2 Quantifying meridional transport

143 We diagnose the eddy-driven upwelling by quantifying the southward volume trans-  
 144 port due to transient eddies in the upper, southward-shoaling isopycnal layer. Import-  
 145 tantly, we account for the presence of stationary meanders downstream of topography  
 146 by isolating the volume transport across the time-mean jet axis (hereafter the cross-jet  
 147 transport). As our simulations have only one southward-shoaling layer, we sidestep the  
 148 difficulties of defining a depth-dependent jet axis and focus on the transport by eddies  
 149 across the contour of maximum upper-layer time-mean velocity. The transport,  $T$ , per-  
 150 pendicular to the time-mean velocity field is written as

$$151 \quad T(x, y) = \overline{h_1 \mathbf{u}_1} \times \frac{\overline{\mathbf{u}'_1}}{|\overline{\mathbf{u}_1}|} = \overline{h'_1 \mathbf{u}'_1} \times \frac{\overline{\mathbf{u}_1}}{|\overline{\mathbf{u}_1}|}, \quad (1)$$

152 where  $h_1$  is the thickness and  $\mathbf{u}_1$  is the horizontal velocity in the upper layer. The over-  
 153 bar indicates the time-mean of a quantity, and the prime is the deviation from that mean  
 154 (i.e. the eddy component). By construction, only the eddy quantities contribute to the  
 155 net transport across the time-mean velocity field. The cross-jet transport,  $X_{jt}$ , is defined  
 156 on the jet axis:

$$157 \quad X_{jt} = T(x, y_m(x)), \quad (2)$$

158 where  $y_m$  is the value of  $y$  for which  $|\overline{\mathbf{u}}_1|$  is maximal. We count the transport as posi-  
 159 tive when it is southward, i.e. when it is associated with upwelling along the isopycnal  
 160 layer.

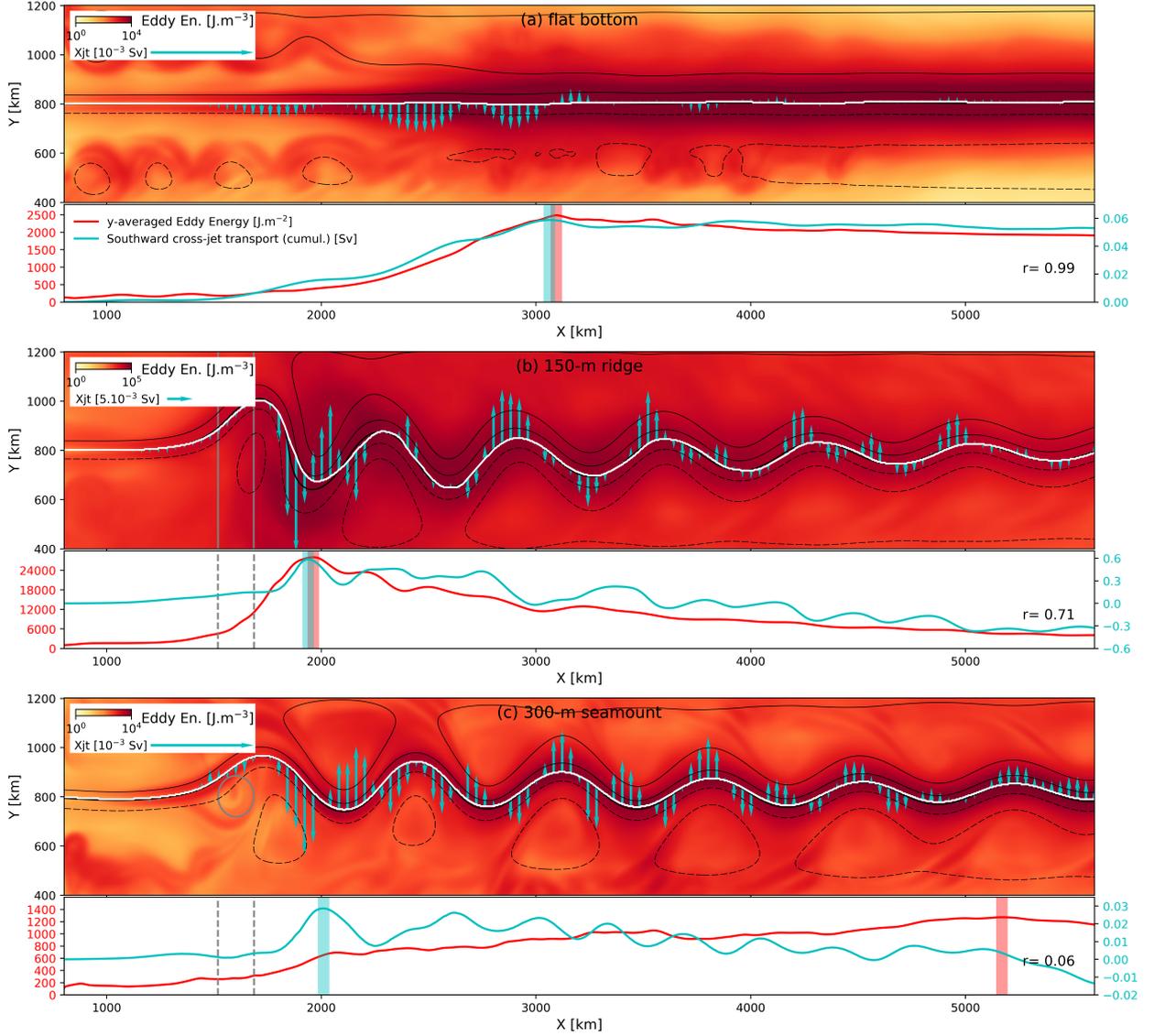
161 The advantage of the above definition is that it allows us to robustly compare the  
 162 net transport by eddies in the presence stationary meanders that form downstream of  
 163 topographic obstacles. As stationary meanders have significant time-mean meridional  
 164 velocities, they would have an alternating signal in southward and northward transport  
 165 across a fixed latitude line (see Hallberg & Gnanadesikan, 2001, for a discussion on trans-  
 166 port across streamlines versus fixed contours). Calculating the cross-jet transport at the  
 167 jet axis allows a more meaningful comparison of net cross-jet transport between cases  
 168 with and without jet meanders.

### 169 **3 Results**

#### 170 **3.1 Eddy-driven upwelling**

171 The mean flow state from three selected runs is presented in Fig. 2. In each case  
 172 the inflowing jet becomes unstable as it evolves eastward. The eddy energy (sum of EKE  
 173 and EPE; indicated by colours in the upper panel of each subplot) has a distinct spa-  
 174 tial pattern, growing with  $x$  as the flow evolves, with an along-stream maximum (high-  
 175 lighted by the red vertical bar in the lower panel). Eddy energy then remains constant  
 176 or decays with further distance downstream. The qualitative evolution of eddy energy  
 177 is similar in all three cases, although the zonal extent of the eddy energy growth region  
 178 and the magnitude of the eddy energy depends on the nature of the topography. Sim-  
 179 ilar results are obtained if we examine EKE and EPE individually (not shown).

188 The transient motions lead to an eddy-induced upwelling, quantified by the eddy-  
 189 induced thickness transport across the time-mean jet axis (cyan arrows in Fig. 2). This  
 190 transport also has zonal variations along the jet axis. In the case of the jet evolving over  
 191 a flat bottom (Fig. 2a), the transport is southward, and preferentially takes place in a  
 192 limited region ( $1500 \text{ km} < x < 3100 \text{ km}$ ). Further downstream, both southward and  
 193 northward flux can occur locally, but these fluxes contribute little to the net transport.  
 194 The bulk of the eddy-induced southward transport is localised in the region of eddy en-  
 195 ergy growth, with a 99% correlation between the zonal variations in the zonally integrated  
 196 cumulative transport and local eddy energy (Fig. 2a, lower subpanel).



180 **Figure 2.** Eddy energy and cross-jet transport for a) flat bottom, b) 150m-tall  
 181 tall seamount. In each subplot, the upper panel shows eddy energy (colours), the time-mean  
 182 upper layer streamfunction (black contours), the time-mean jet axis (white line) and southward  
 183 cross-jet volume transport (cyan arrows); the lower panel shows eddy energy integrated across  
 184 the jet (red line, with the maximum value indicated by the vertical red bar) and the zonally in-  
 185 tegrated cumulative southward cross-jet transport (cyan line, with the maximum value indicated  
 186 by the vertical cyan bar). Note that the 150 m ridge case (panel b) has significantly higher eddy  
 187 energy and cross-jet transport, and thus has a different colour scale and arrow length scale.

197 In the presence of topography (illustrated by the 150 m ridge case; Fig. 2b), localised  
 198 regions of enhanced eddy-induced cross-jet transport persist. The signature of the sta-  
 199 tionary meanders is visible in the eddy-induced cross-jet transport variability (manifested  
 200 as alternating regions of southward and northward transport), making it difficult to dis-  
 201 tinguish the net effect of eddies. It is therefore especially helpful in this case to consider  
 202 zonally integrated cumulative transport (cyan line, Fig. 2b, lower subpanel). This met-  
 203 ric shows that the region immediately downstream of the ridge ( $x = 1500 - 2000$  km)  
 204 contributes significantly to the net southward transport relative to the regions further  
 205 downstream. Around  $x \approx 2000$  km (highlighted by the cyan vertical bar), there is a  
 206 transition between a region of net southward transport ( $x < 2000$  km) and a region of  
 207 net northward transport ( $x > 2000$  km). In some cases, the cumulative transport at  
 208  $x = 6000$  km is northward, which may be due to the lack of disturbances to break down  
 209 the meanders downstream. The close relationship between zonal growth of eddy energy  
 210 and southward cross-jet transport, seen in the flat bottom case, also holds in the 150m  
 211 ridge case (71% correlation).

212 Most of the simulations with topography conducted in this study provided results  
 213 that are qualitatively similar to the 150m ridge case (not shown). However, the third case  
 214 presented in Fig. 2, that with a 300 m high seamount, is one of the exceptions. This case  
 215 is consistent with the results above in that it shows a qualitatively similar zonal evolu-  
 216 tion of eddy energy, and regions of preferential cross-jet transport immediately down-  
 217 stream of topography, but differs in the lack of correlation between along-stream eddy  
 218 growth and cumulative southward transport. The break-down in this relationship pro-  
 219 vides insights into the underlying dynamics at play, and is explored in more detail in the  
 220 next section.

221 In summary, two main points emerge from examination of the along-stream vari-  
 222 ations of eddy energy and transport in these idealised simulations. First, the presence  
 223 of topography leads to enhanced eddy-induced cross-jet transport localised immediately  
 224 downstream of the topographic obstacle, relative to the same jet evolving over a flat bot-  
 225 tom. The magnitude and location of the eddy-induced transport depend on the prop-  
 226 erties of the topography present. Second, eddy energy and eddy-induced cross-jet trans-  
 227 port ('eddy upwelling') have distinct zonal distributions. This transport tends to be lo-  
 228 calised in the region of along-stream eddy growth, but exceptions can occur where eddy-  
 229 induced transport occurs in a region of smaller zonal extent than eddy energy growth.

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### 3.2 Mechanism for topographic control

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We probe the dynamics underpinning the differing spatial distributions of eddy energy and eddy-induced transport by looking at the two instability mechanisms that energise the eddy field in the 300 m seamount case (Fig. 3b), and comparing them with the flat-bottom case (Fig. 3a). Following the thickness-weighted energetics approach used in Aiki & Richards (2008) and Barthel et al. (2017), we diagnose the eddy-mean flow energy conversions due to 1) the work of interfacial form stress,  $-\widehat{\mathbf{u}}_1 \cdot \overline{h'_1 \nabla \phi'_1}$  (where  $\phi_1$  is the Montgomery potential and ' denotes the anomaly from the  $\tau$  time mean), responsible for the generation of eddy energy in baroclinic instability (dark blue lines), and 2) the work of Reynolds stress associated with horizontal convergence of momentum in the upper layer, associated with barotropic instability (black lines)  $\rho_0 \widehat{\mathbf{u}}_1 \cdot \nabla \cdot \overline{(h_1 \mathbf{u}''_1 \otimes \mathbf{u}''_1)}$ , where  $\widehat{\mathbf{u}}_1$  and  $\mathbf{u}''_1$  are the thickness-weighted mean upper-layer velocity and the deviation from that mean, respectively. The outer product of two vectors is denoted by  $\otimes$ , and  $\rho_0$  is the reference density of the Boussinesq approximation (see Barthel et al., 2017, for the full derivation).

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The 300 m seamount is a helpful case to disentangle the contributions of form stress and Reynolds stress because they have distinct zonal patterns (Fig. 3b). These patterns indicate that the eddy-induced transport is associated exclusively with baroclinic instability (i.e. positive conversion of energy into the eddy field via form stress). This relationship is consistent with our conceptual understanding that baroclinic instability contributes to flattening isopycnals, and with observations in Drake Passage that indicate the eddy heat flux is best aligned with the production of EPE (Watts et al., 2016; Foppert et al., 2017). These results further suggest that the zonally-averaged link between interfacial form stress and meridional thickness flux (e.g. Olbers et al., 2004) may apply at the local scale. Understanding that the mechanism for eddy-induced transport is baroclinic instability acting as a source of eddy energy is consistent with the alignment of the region of eddy upwelling with the region of along-stream eddy growth, rather than with regions of elevated eddy energy.

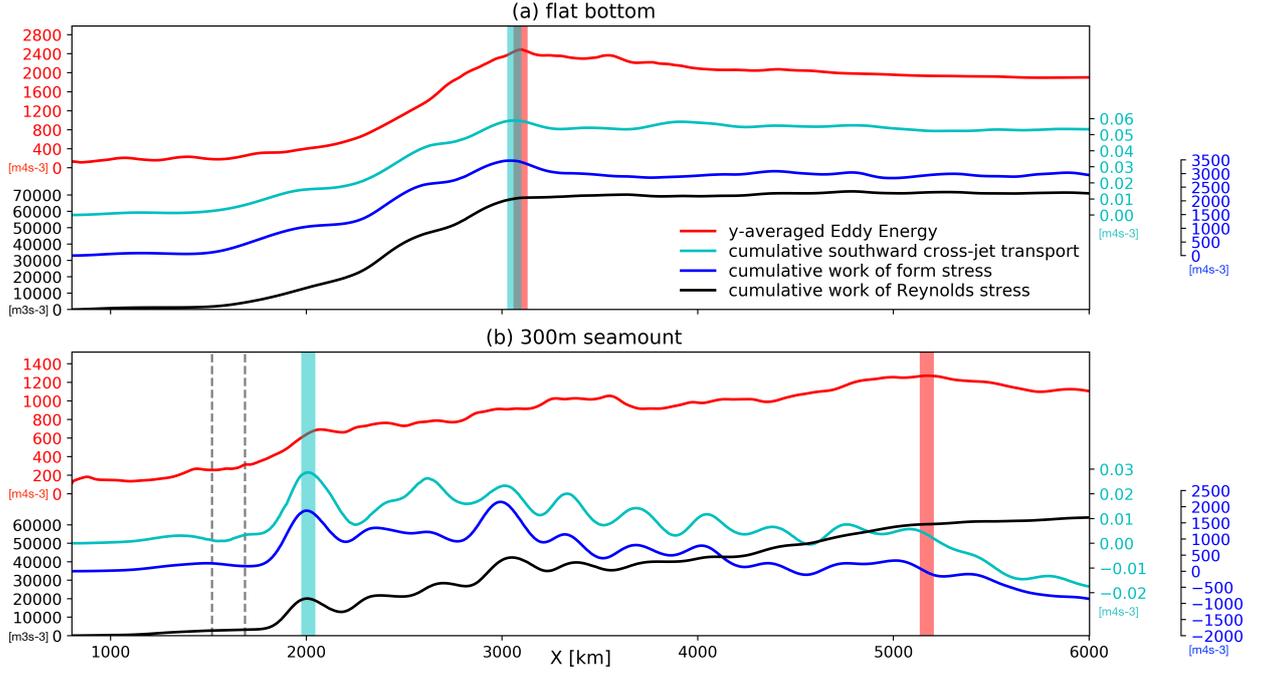
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The relationship between eddy upwelling and the action of eddy form stress in energising the eddy field is robust across all simulations, both with and without topography. In most cases, the region of southward eddy transport extends over the entire region of along-stream eddy growth because both energy conversion terms have the same



245 **Figure 3.** Zonal distributions of energy conversion terms when an unstable jet evolves over  
 246 (a) a flat bottom, (b) encounters a 300m-tall seamount. The total eddy energy averaged over the  
 247 channel (red), the along-jet cumulative eddy-driven southward transport across the time-mean  
 248 jet (‘eddy upwelling’) (light blue), the along-jet cumulative work of interfacial form stress due  
 249 to baroclinic instability (dark blue) and cumulative work by Reynolds stress due to barotropic  
 250 instability (black) are shown. The latter energy conversions terms are both calculated for the  
 251 upper layer, and are positive when energy is fluxed from the mean into the eddy field. The grey  
 252 dashed lines indicate the half-width of the topography. For each simulation, the location of maximum  
 253 total eddy energy is marked by the red shading (indicating the transition between regions  
 254 of along-stream eddy growth and decay), while the cyan shading marks a significant transition  
 255 between southward and northward cross-jet eddy transport.

273 zonal patterns, as illustrated by the flat bottom case (Fig. 3a). Nevertheless, it is im-  
 274 portant to keep in mind that baroclinic instability alone provides the dynamical mech-  
 275 anism to generate cross-jet transport and eddy upwelling. As such, it is possible that along-  
 276 stream eddy growth can occur in regions without net southward eddy transport (i.e. with-  
 277 out active baroclinic instability) when, for instance, horizontal shear instability is respon-  
 278 sible for eddy energy growth. This scenario is nicely illustrated by the 300 m seamount  
 279 case (Fig. 3b).

280 These examples speak to the method by which topography influences the eddy-induced  
 281 transport. We infer that the topographic obstacles affect the flow in such a way that ei-  
 282 ther baroclinic or barotropic instability, or both, are enhanced. In some, but not all, cases  
 283 there is a strong correspondence between these two different instability mechanisms. How-  
 284 ever, southward eddy-induced transport is only dependent on the action of baroclinic in-  
 285 stability, where isopycnal interfaces slump to release available potential energy into the  
 286 eddy field.

#### 287 4 Implications for eddy parameterisations

288 Our results suggest that energy conversion terms are an unambiguous indicator of  
 289 eddy-induced cross-jet transport, however, we recognize that these are unlikely to be prac-  
 290 tical indicators of eddy upwelling in coarse resolution models. Our analysis also indicates  
 291 that the along-stream growth/decay of eddy energy may be a valuable predictor of eddy  
 292 upwelling in many cases, and hence may inspire new eddy parameterisations for coarsely-  
 293 resolved models. Thus, in this section, we assess whether a coarsely-resolved zonal pat-  
 294 tern of eddy energy may be used to estimate the cross-jet transport occurring in that  
 295 region. For that purpose, we compare transport estimates obtained from assuming trans-  
 296 port is proportional to the rate of along-stream eddy energy growth to those employing  
 297 other common parameterizations for eddy upwelling based on large-scale variables, such  
 298 as the mean isopycnal slope, EKE and EPE. Specifically we consider parameterizations  
 299 based on the following relationships:

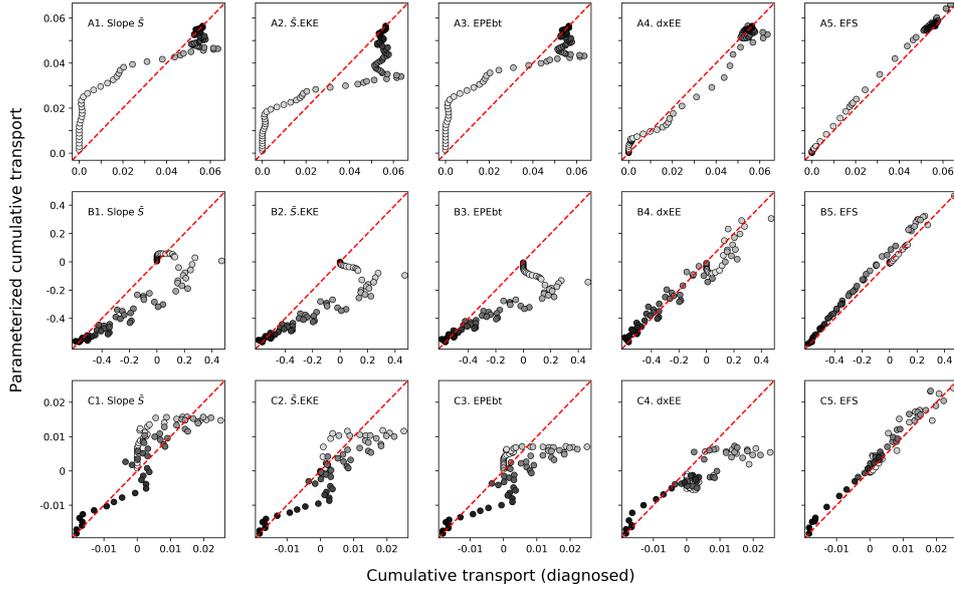
- 300 (a) Transport can be parameterised as a constant diffusivity applied to the time-  
 301 mean isopycnal slope ( $\bar{S}$ ):  $X_{jt}^{GM}(x) = \kappa\bar{S} + B$ , with a constant  $\kappa = A$ , in-  
 302 spired by Gent & McWilliams (1990);
- 303 (b) The diffusivity  $\kappa$  is proportional to EKE:  $X_{jt}^J(x) = A.EKE \cdot \bar{S} + B$ , inspired  
 304 by Jansen et al. (2015);
- 305 (c) Eddy transport is proportional to the barotropic EPE:  $X_{jt}^{EPE}(x) = A.EPEbt +$   
 306  $B$ , with  $EPEbt = \frac{\rho_0}{2}g\overline{\eta^2}$  ( $\rho_0$ : reference density;  $g$ : gravitational acceleration;  
 307  $\eta$ : sea surface height), inspired by Foppert et al. (2017);
- 308 (d) Eddy transport is proportional to the rate of along-stream eddy energy growth:  
 309  $X_{jt}^{dxE E}(x) = A.\frac{d}{dx}EE + B$  where EE denotes total eddy energy.

310 (e) Eddy transport is proportional to the local eddy form stress (EFS):  $X_{jt}^{EFS}(x) =$   
 311  $A.EFS + B$ , used as a reference in this exercise.

312 To compare the relative performance of these parameterisations in our model configu-  
 313 ration, the large-scale variable on which each parameterisation is based ( $\bar{S}$ , EKE, EPEbt,  
 314 EE or EFS) was smoothed and sub-sampled to a 80km horizontal resolution, roughly equiv-  
 315 alent to output from a 1° ocean model. In each case, a least-squares fit was performed  
 316 to determine the parameters  $A$  and  $B$  that minimise the total error in transport over the  
 317 domain. The parameterised transport is then compared to the modeled transport in four  
 318 different cases (Fig. 4). We place a caveat on the results that follow, which is that the  
 319 list of eddy parameterisations evaluated is by no means exhaustive. In addition, the com-  
 320 parison presented here allow the parameters to be fitted for each case, while a more sys-  
 321 tematic parameterisation may require the parameter values to work uniformly across cases.

322 Results from this exercise confirm the conclusions from the previous section. Lo-  
 323 cal values of energy reservoirs, such as EPE, are not a good indicator of cross-jet trans-  
 324 port (e.g. Fig. 4.A3 for the flat-bottom case). The zonal variations in EPE and trans-  
 325 port are so different that minimising the total error leads to applying a small southward  
 326 transport almost uniformly over the whole domain, leading to compensating over-estimated  
 327 transport upstream (light gray) and under-estimated transport further downstream (dark  
 328 gray). Similarly, the other relationships based on time-mean isopycnal slope and EKE  
 329 (Fig. 4A.1-2) fail to capture the zonal pattern of transport, with the best parameteri-  
 330 sation being an almost uniform transport of small magnitude.

331 In contrast, the zonal growth rate of total eddy energy,  $\frac{d}{dx}EE$ , is able to reproduce  
 332 the zonal variations in eddy transport, producing a parameterised transport which ad-  
 333 equately portrays regions of little to no transport, and regions of localised, enhanced trans-  
 334 port. Local eddy form stress is overall the best indicator for eddy transport, but is un-  
 335 likely to be readily available output from climate models or observations. In the absence  
 336 of eddy form stress, the zonal growth of eddy energy may be a valuable indicator of where  
 337 eddy-induced transport occurs, and outperforms commonly used parameterisations of  
 338 eddy upwelling, in most cases considered in this study (see Supporting Information). One  
 339 exception is the 300m seamount case (Fig. 4.C1-4) where the relationship between cross-  
 340 jet transport and the along-stream rate of change of total eddy energy breaks down (Fig.  
 341 3b) due to the influence of barotropic instability in generating eddy energy.



342 **Figure 4.** Parameterised cumulative southward transport across the jet axis plotted against  
 343 the resolved cumulative transport, at each zonal gridpoint along the time-mean jet axis, for A.  
 344 flat bottom, B. 150m ridge, and C. 300m seamount. Data points are colored from light to dark  
 345 as we move downstream. In each case, the parameterisation is the best linear fit (by least-square  
 346 method) that minimises the total error between local values of transport and local 1) time-mean  
 347 cross-jet isopycnal slope  $\bar{S}$ , 2) EKE times  $\bar{S}$ , 3) barotropic EPE ( $EPEbt$ ), 4) zonal growth of  
 348 total eddy energy (dxEE), and 5) eddy form stress (EFS), where each variable was smoothed and  
 349 sub-sampled to a 80km resolution.

## 350 5 Discussion

351 This study highlights that eddy-driven cross-jet transport within a shoaling isopyc-  
 352 nal occurs in regions of eddy energy growth through baroclinic instability. The pres-  
 353 ence of topography leads to enhanced eddy upwelling in the region immediately down-  
 354 stream of the obstacle (especially in the first meander) because it modifies the growth  
 355 of baroclinic instability. The idealised set-up allows exact calculations of quantities not  
 356 usually diagnosed in global climate models, and the simulations performed in this study  
 357 provide a plausible mechanism explaining the location of the upwelling pathways from  
 358 Tamsitt et al. (2017) which occurs in regions upstream of EKE maxima, and further the  
 359 offset between the divergent eddy heat flux and EKE discussed by Foppert et al. (2017).  
 360 Simple parameterisations based on mean isopycnal slope, EKE and EPE fail to repro-

361 duce this strong, localised, eddy transport near topography. In most cases, the along-  
362 stream growth of eddy energy is a good indicator for southward transport, with the ex-  
363 ception of cases where barotropic instability and baroclinic instability have distinct growth  
364 regions (e.g. a steep isolated seamount; Fig. 4C4).

365 The benefit of an idealized set-up is that it allows exact calculations of both the  
366 eddy form stress and the Reynolds stress, and we can thereby attribute dynamical rel-  
367 evance between the two without ambiguity (noting that these quantities are not usually  
368 diagnosed from global climate models or observations). However, the simplified verti-  
369 cal structure in this two-layer system leads to an exaggerated impact of topography, as  
370 small values of topography are more dynamically relevant to the ACC. Idealised simu-  
371 lations may also over-stimulate barotropic instability near topography (e.g., Barthel et  
372 al., 2017; Youngs et al., 2017). Despite this caveat, we argue that the two-layer set up  
373 provides useful dynamical insight, given that evidence of mixed barotropic-baroclinic in-  
374 stability is also observed in the Drake Passage (Foppert, 2019) and may be important  
375 for the momentum balance in the ACC (Constantinou & Hogg, 2019).

376 In this study, we focused only on eddy-induced isopycnal thickness fluxes and showed  
377 that eddy-driven upwelling does not occur in regions of high eddy energy, but rather in  
378 regions of along-stream eddy energy growth by baroclinic instability. However, the pres-  
379 ence of high eddy energy, and potentially high EKE in particular, may contribute to en-  
380 hancing other types of transport, such as the upwelling of tracers through increased isopy-  
381 cnal stirring (Abernathey & Ferreira, 2015; Dufour et al., 2015). In addition, the net merid-  
382 ional transport in the Southern Ocean is forced by a combination of factors, including  
383 wind stress, surface buoyancy fluxes and diabatic processes in the surface mixed layer;  
384 these factors are dominant where layers outcrop at the surface and emphasise the role  
385 of the vertical structure of eddy processes in the ACC that are omitted from this study.  
386 Results from the adiabatic simulations considered in this study best inform on interior  
387 upwelling processes, away from frictional boundaries such as the surface and bottom Ek-  
388 man layers, and away from locations where diabatic mixing dominates (e.g. close to rough  
389 topography).

390 Keeping in mind the above caveats, the detailed dynamical analysis of these ide-  
391 alised simulations provides an important insight: assuming that high values of EKE and/or  
392 EPE indicate regions of strong eddy-driven transport is a misconception. In the South-

ern Ocean there is increasing evidence of mixed instability near topography, where both barotropic and baroclinic instability mechanisms contribute to the dynamics (Youngs et al., 2017; Barthel et al., 2017; Foppert, 2019). The distinct role of each instability mechanism, and their interaction, need to be considered when developing eddy transport parameterisations that will respond physically to changes in ocean dynamics.

## 6 Open Research

The simulation data and scripts used in the study are freely available on the Zenodo repository at DOI: 10.5281/zenodo.2542957.

## Acknowledgments

AB was partially supported by the RGCM program of the US Department of Energy. SW acknowledges support from the Australian Research Council (DE120102927) and the National Science and Engineering Research Council of Canada (22R23085). SRK was supported by a UNSW Silverstar research grant. This research was undertaken on the NCI National Facility in Canberra, Australia, supported by the Australian Commonwealth Government.

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# Supporting Information for “Baroclinic control of Southern Ocean eddy upwelling near topography”

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## Contents of this file

This supporting information provides details of the energy equations in a two-layer isopycnal framework.

## Energy budget in a two-layer isopycnal framework

### 1. Definitions and decompositions

In this study, we use a thickness-weighted energy framework similar to that used by Barthel et al. (2017). The two-layer system has four main energy reservoirs, defined as follows.  $APE_{bt}$  is the available potential energy due to the free surface elevation  $\eta_0$  (or ‘barotropic’ potential energy)

$$APE_{bt} = \frac{\rho_0}{2} g \eta_0^2, \quad (1)$$

$APE_{bc}$  is the available potential energy due to the motions of the interface separating the upper and lower layers  $\eta_1$  (or ‘baroclinic’ potential energy)

$$APE_{bc} = \frac{\rho_0}{2} g' \eta_1^2, \quad (2)$$

and  $KE_i$  is the kinetic energy in each layer ( $i = 1, 2$ ),

$$KE_i = \frac{\rho_0}{2} h_i |\mathbf{u}_i|^2, \quad (\text{for } i = 1, 2). \quad (3)$$

Here,  $\rho_0$  is the reference density of the Boussinesq approximation,  $g$  is the acceleration due to gravity,  $g' = \frac{g\Delta\rho}{\rho_0}$  is the reduced gravity of the interface between the two layers,  $h_i$  is the  $i$ -th layer thickness, and  $\mathbf{u}_i = [u_i, v_i]$  is the horizontal velocity in layer  $i$ .

To separate the mean and eddy terms, we define the traditional Reynolds decomposition for most variables in our model. For example, the layer thickness becomes:

$$h_i \equiv \overline{h_i} + h'_i, \quad (4)$$

where the overbar and prime symbols denote a three-year time mean and the associated deviation respectively. Following the methodology used in Aiki & Richards (2008), the velocity variable is decomposed into a thickness-weighted mean (TWM) velocity  $\widehat{\mathbf{u}}$  and deviation from the TWM mean  $\mathbf{u}''$ ,

$$\mathbf{u}_i \equiv \widehat{\mathbf{u}}_i + \mathbf{u}''_i, \quad \text{with } \widehat{\mathbf{u}}_i \equiv \frac{\overline{h_i \mathbf{u}_i}}{h_i}. \quad (5)$$

In a thickness-weighted framework, each energy reservoir can be decomposed into contributions from the mean and eddy, as proposed by ?,

$$\overline{APE}_{bt} = \frac{\rho_0}{2} \overline{g\eta_0^2} = \underbrace{\frac{\rho_0}{2} \overline{g\bar{\eta}_0^2}}_{MPE_{bt}} + \underbrace{\frac{\rho_0}{2} \overline{g\eta_0'^2}}_{EPE_{bt}}, \quad (6)$$

$$\overline{APE}_{bc} = \frac{\rho_0}{2} \overline{g'(\eta_1)^2} = \underbrace{\frac{\rho_0}{2} \overline{g'(\bar{\eta}_1)^2}}_{MPE_{bc}} + \underbrace{\frac{\rho_0}{2} \overline{g'\eta_1'^2}}_{EPE_{bc}}, \quad (7)$$

$$\overline{KE}_i = \frac{\rho_0}{2} \overline{h_i |\mathbf{u}_i|^2} = \underbrace{\frac{\rho_0}{2} \overline{h_i |\hat{\mathbf{u}}_i|^2}}_{MKE_i} + \underbrace{\frac{\rho_0}{2} \overline{h_i |\mathbf{u}_i''|^2}}_{EKE_i}, \quad (\text{for } i = 1, 2). \quad (8)$$

Note that the kinetic energy is decomposed using the TWM decomposition of velocity. Note also that this eddy-mean decomposition is based on the separation between stationary (i.e. mean) and transient (i.e. eddy) features. Thus, the contribution of stationary meanders, or stationary eddies, are included in the contribution of the time-mean flow.

## 2. Time evolution of the mean and eddy energy

The equations governing the two-layer system can be derived from the incompressible hydrostatic equations of motion in isopycnal coordinates (see Barthel et al. (2017) for the full derivation). In particular, the time-mean energy reservoirs are governed by:

$$\partial_t MPE_{bt} = (\bar{h}_1 \hat{\mathbf{u}}_1 + \bar{h}_2 \hat{\mathbf{u}}_2) \cdot \nabla \bar{\phi}_1 - \nabla \cdot (\bar{\phi}_1 (\bar{h}_1 \hat{\mathbf{u}}_1 + \bar{h}_2 \hat{\mathbf{u}}_2)), \quad (9)$$

$$\partial_t MPE_{bc} = \bar{h}_2 \hat{\mathbf{u}}_2 \cdot \nabla (\bar{\phi}_2 - \bar{\phi}_1) - \nabla \cdot ((\bar{\phi}_2 - \bar{\phi}_1) \bar{h}_2 \hat{\mathbf{u}}_2), \quad (10)$$

$$\begin{aligned} \partial_t MKE_i &= -\nabla \cdot (\hat{\mathbf{u}}_i MKE_i) - \bar{h}_i \hat{\mathbf{u}}_i \cdot \nabla \bar{\phi}_i - \hat{\mathbf{u}}_i \cdot \bar{h}_i \nabla \bar{\phi}_i' \\ &\quad - \rho_0 (\hat{\mathbf{u}}_i \cdot \nabla) \cdot (\overline{h_i \mathbf{u}_i'' \mathbf{u}_i''}) + \rho_0 \overline{h_i \mathbf{F}_{\tau i}} \cdot \hat{\mathbf{u}}_i, \quad (\text{for } i = 1, 2). \end{aligned} \quad (11)$$

The equation governing the mean component of the layer MP flux divergence is

$$\nabla \cdot (\bar{\phi}_i \bar{h}_i \hat{\mathbf{u}}_i) = -\bar{\phi}_i \partial_t \bar{h}_i + \bar{h}_i \hat{\mathbf{u}}_i \cdot \nabla \bar{\phi}_i, \quad (\text{for } i = 1, 2). \quad (12)$$

Likewise, the eddy energy reservoirs are governed by the following equations:

$$\partial_t EPE_{bt} = \overline{\phi'_1 \partial_t h'_2} + \overline{\phi'_1 \partial_t h'_1} \quad (13)$$

$$\partial_t EPE_{bc} = \overline{\phi'_2 \partial_t h'_2} - \overline{\phi'_1 \partial_t h'_2} \quad (14)$$

$$\begin{aligned} \partial_t EKE_i = & -\nabla \cdot (\widehat{\mathbf{u}}_i EKE_i) - \nabla \cdot (\overline{\mathbf{u}''_i EKE_i}) - \overline{\mathbf{u}''_i \cdot h_i \nabla \phi'_i} \\ & + \rho_0 \widehat{\mathbf{u}}_i \cdot \nabla \cdot (\overline{h_i \mathbf{u}''_i \otimes \mathbf{u}''_i}) + \rho_0 \overline{h_i \mathbf{F}_{\tau i} \cdot \mathbf{u}''_i}, \quad (\text{for } i = 1, 2), \end{aligned} \quad (15)$$

where  $\otimes$  denotes the outer product of two vectors. The associated eddy MP flux divergence equation is:

$$\nabla \cdot (\overline{\phi'_i h'_i \widehat{\mathbf{u}}_i} + \overline{\phi'_i h'_i \mathbf{u}''_i}) = -\overline{\phi'_i \partial_t h'_i} + \widehat{\mathbf{u}}_i \cdot \overline{h'_i \nabla \phi'_i} + \overline{h_i \mathbf{u}''_i \nabla \phi'_i} \quad (16)$$

These equations include advective terms, expressed as flux divergences, and local conversion terms. Only two terms locally convert energy between the time-mean and the eddy components of the system in layer  $i$ :

1. the work of interfacial form stress,  $-\widehat{\mathbf{u}}_i \cdot \overline{h'_i \nabla \phi'_i}$ , responsible for the generation of eddy energy in baroclinic instability, and
2. the work of Reynolds stress due to the horizontal convergence of momentum,  $\rho_0 \widehat{\mathbf{u}}_i \cdot \nabla \cdot \overline{(h_i \mathbf{u}''_i \otimes \mathbf{u}''_i)}$ , which is associated with barotropic instability.

These terms can be bi-directional but are here defined as positive when converting energy from the mean to the eddy field.

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