Earthquakes and Volcanic Eruptions Driven by Magma Solitons

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November 26, 2022

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Earthquakes and Volcanic Eruptions Driven by Magma Solitons

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Abstract

A magma soliton is derived to compute the timing of earthquakes on the Reykjanes ridge in Iceland and the occurrence and duration of the volcanic eruption in Geldingadalir by Fagradalsfjall in February 2021. The velocity of the magma soliton is computed using the earthquakes observed underwater on the Reykjanes ridge in November 2019 and the earthquakes that occurred by Fagradalsfjall in October 2020. This velocity also determines the shape, height and spatial extent, of the magma soliton. The width of the magma soliton is then derived using a Gaussian profile, which allows the volume of lava in the Geldingadalur-Fagradalsfjall eruption to be computed and compared to actual measurements. The timing of build-up of stress and subsequent earthquake clusters—which is caused by the magma soliton passing through the remaining volcanic systems in the Reykjanes peninsula—is then predicted. Reykjanes peninsula.

1 Introduction

Volcanic-tectonic earthquakes are caused by the vibrations generated by moving magma. As magma flows beneath the Earth's surface along mid-ocean ridges, it can flow into fissures in the volcanic rock and from one volcanic chamber into another. The seismic activity is caused by the liquid magma's widening the volcanic fissures and chambers, as well as the collapsing of surrounding rock when when an empty space is left to be filled. Such earthquakes occur frequently along volcanic systems, where the crust is already weakened and the mass of the volcanic material supplements the strain in the Earth's surface. They can cause ground deformation and damage man-made structures, but are usually much smaller than earthquakes generated by non-volcanic sources (McNutt & Roman, 2015).

Although occasional volcanic-tectonic earthquakes do not necessarily indicate an incoming eruption, a large cluster of them in rapid succession typically signifies that new magma is being introduced into the system, and therefore can signal impending volcanic activity. Powerful volcanic-tectonic earthquakes can weaken magma chambers and perturb gases inside such chambers, both of which increase the likelihood of volcanic eruptions. They can also produce new fissures extending to the surface and aid the magma's emergence from the volcanic system. As a result, scientists have long been observing volcanic-tectonic earthquakes to monitor volcanic activity (Eggert & Walter, 2009).

Researchers have also studied seismic mechanisms and tried to construct mathematical models to understand the basic mechanisms of earthquakes. In the 1960s, Burridge and Knopoff (1967) constructed a block-spring model that imitated the behavior of earthquake faults in order to study the role of friction in major earthquakes and their aftershocks. In this model, a block on a rough surface (representing one side of the fault) is attached by a spring to a loader plate (representing the other side of the fault). The friction term is a linear function of the loader plate's velocity and a viscous term that is proportional to the block's velocity.

Later empirical research by Marone (1998) suggested that the friction term should not be a single valued function of the block's velocity; and, indeed, Ruina (1983) instead proposed that the friction term should be modeled as rate-and-state friction. Conducting numerical simulations of the spring-block model with a single block and the full nonlinearity of the friction term, Erickson, Birnir, and Lavallée (2008) discovered that the system remains stationary for a set of parameter values. However, increasing the parameter ϵ leads to a Hopf bifurcation from the stationary state to a periodic orbit that imitates seismic mechanisms, where ϵ is the ratio between the stress dropped during an earthquake and the increase in stress that results from the change in fault velocity. Thus, once the former is sufficiently large relative to the latter, ϵ will be large enough to cause a sequence of chaotic earthquake motions.

To understand the multiple block cases, Erickson, Birnir, and Lavallée (2011) introduced equations for the discrete and continuous formulations of the BK spring-block model. In the former, they observed chaotic behavior for $N > 20$ and periodic behavior otherwise, where N is the number of blocks in the system, as long as $\epsilon = 0.5$. The requirement for ϵ is much smaller than what is needed for chaotic motion in the single block case, where $\epsilon = 11$, indicating that chaotic motion can emerge under a wide range of parameter values as long as the system size is sufficiently large. In the continuous model, a bifurcation from a stationary state to periodic behavior to chaotic behavior emerges as ϵ is gradually increased. The parameter value of $\epsilon = 0.4$ that is required for chaos is much smaller than what is required in the single-block case. Both the discrete and continuous formulations of the spring-block model demonstrate solutions where an initial Gaussian pulse splits into two traveling waves that propagate as solitons (Erickson et al., 2011).

However, limited research has been conducted to predict the occurrence of volcanic-tectonic earthquakes and subsequent timing and duration of volcanic eruptions. Thus, the research focus of this paper lies in the modeling of magma flow relative to volcanic systems; by predicting when large volumes of magma, or large liquid fractions, reach such volcanic systems, we can make conclusions about when volcanic-tectonic earthquakes will emerge, and therefore, when volcanic eruptions will occur. We focus our investigation on the Reykjanes Peninsula of Iceland, which lies along the mid-Atlantic ridge and is frequently the site of a significant number of volcanic-tectonic earthquakes and volcanic eruptions.

The underlying hypothesis is that the rift between the tectonic plates that form the mid-Atlantic Ridge and pass along the Reykjanes peninsula on its way through Iceland forms a conduit for the flow of magma. Due to the geometry of the passage, its shape serves as a channel between two walls formed by the North-American and Eurasian plates. This is the earthquake zone, where frequent earthquakes make the rock porous and allow for the flow of magma. This channel is assumed to allow relatively unrestricted flow at some depth, while also being restricted by the surface crust—where the magma flow interacts with the structure of the crust—and particularly by earthquake faults and volcanic zones. Interestingly, the volcanic zones on the Reykjanes peninsula and those underwater to the southwest of the peninsula lie at a 45^o angle to the direction of the channel and the mid-Atlantic ridge (see Figures 1 and 8).

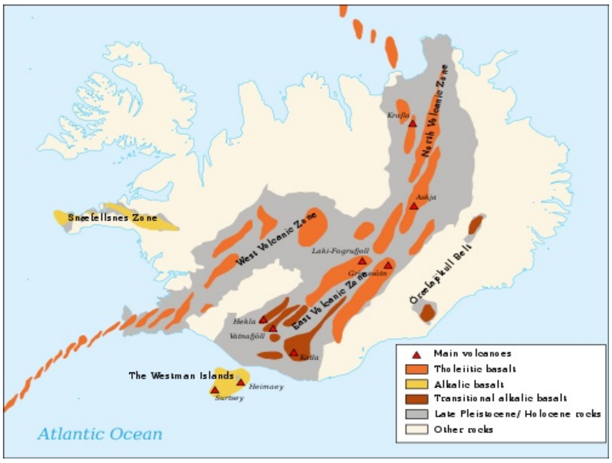


Figure 1: Iceland's volcanic zones (Mynott, 2013)

Many authors have studied the motion and phase transition of molten rock moving through the matrix of solid rock and the phase transitions of the rock (see McKenzie (1984); Scott and Stevenson (1986); Sleep (1975)). Stevenson and Scott proposed in 1986 an equation assuming the liquid and solid phases of the rock are fully incompressible and fully connected (Scott & Stevenson, 1986). They ignored phase transition and assumed that motion was only in one direction:

$$u_t = [u^n(u^{-m}u_{xx} - 1)]_x. \tag{1}$$

Here $u(x,t)$ is the mean volume fraction of the liquid phase. It is non-negative for all x and t. The exponents n and m denote the nonlinear dependence of permeability k and effective viscosity η on u: $k = k_0 u^n$ and $\eta = \eta_0 u^{-m}$, where k_0 and η_0 are constants. It was suggested in Scott and Stevenson (1986) that reasonable ranges for n and m are $2 \leq n \leq 5$ and $0 \leq m \leq 1$.

The equation (1) has solitary wave solutions (see Barcilon and Richter (1986); Scott and Stevenson (1986)) corresponding to a wave of an increased liquid volume fraction traveling through the porous media, which consists of the molted and solid phases of the rock. Our hypothesis is that earthquakes and eruptions along the Reykjanes ridge and the Reykjanes Atlantic rift zone are caused by such magma-solitary waves, or magmons. This is suggested by the geological record on Reykjanes ridge (see Figure 2).

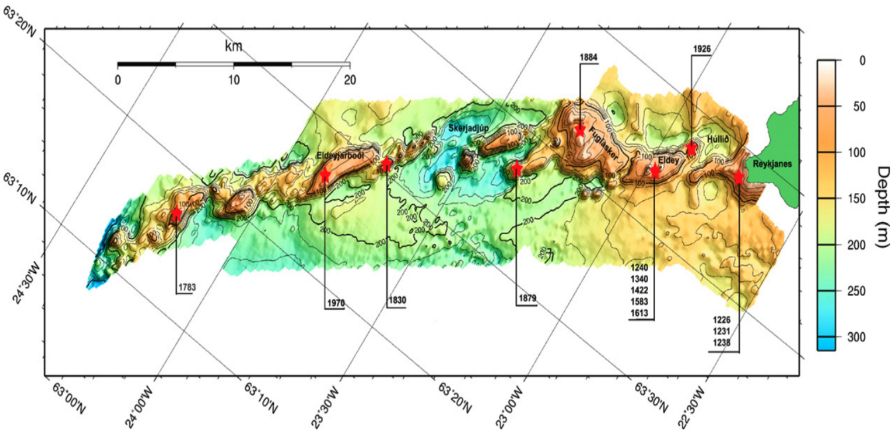


Figure 2: Volcanic eruptions (eldgos) on the Reykjanes Ridge (Hoskuldsson et al., 2007)

The driving force behind the movement of these solitons is simply the initial velocity with which the solitons are injected into the tectonic rift system. If there is a open passage forward, through rock shattered and made porous by earthquake activity, the magma solitons will continue to move forward. Their backward motion is hindered by the cooling magma, which seals the porosity of the passage through which the solitons travelled.

2 The Model

Extensive research has been performed to study and model the movement of molten rock. In particular, Takahashi and Satsuma (1988) propose the magma equation, choosing $n = 3$ and $m = 0$ in Equation (1):

$$u_t = [u^3(u_{xx} - 1)]_x \tag{2}$$

where x and t represent horizontal space coordinate and time, respectively, and $u(x,t)$ denotes the fraction of the magma that is molten rock (i.e. liquid). As outlined in the original paper, the variable transformations

$$\xi = \int^x u^{-1} dx \tag{3}$$

$$\tau = t \tag{4}$$

reduce equation (2) to

$$u_\tau = u_{\xi\xi\xi} - 6uu_\xi + 3\alpha u_\xi \tag{5}$$

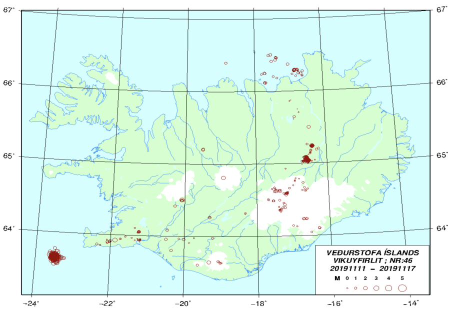
where a is a constant (for further details, see Appendix A). Equation (5) is the well-understood Korteweg-de Vries equation, which indicates that the solution of (2) can be given in the form of a soliton (or solitary wave) with equation

$$u(x,t) = -\frac{c}{2}\,\mathrm{sech}^2\left(\frac{\sqrt{c}}{2}(x - ct - a)\right) \tag{6}$$

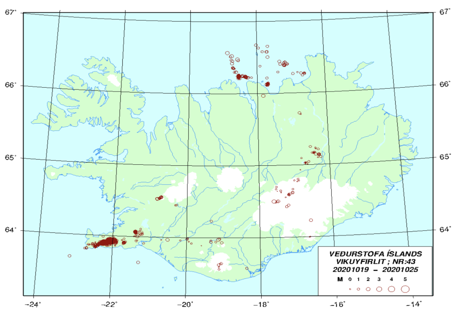
where sech is the hyperbolic secant, c is the velocity of the wave, and a is an arbitrary constant. 2D and 3D plots of a general soliton are shown below. The liquid volume fraction is the negative $v = -u$ of u.

2.1 Soliton Equation

We can use equation (6) as the basis for our modeling of magma flow. Because the entire equation of a soliton can be obtained through its velocity, we can derive the magma soliton by calculating its speed. To do this, we analyze seismic activity along the Reykjanes Peninsula observed by the Iceland Meteorological Office and identified two large clusters of volcanic-tectonic earthquakes, both along the Reykjanes Ridge. The first occurred in mid-November of 2019 and was located roughly 45 km southwest of the Reykjanes Peninsula; the second was in late October of 2020 and centered in the Fagradalsfjall volcanic system.



(a) Earthquake cluster on the week of November 11, 2019



(b) Earthquake cluster on the week of October 19, 2020

Figure 3: Earthquake clusters along the Reykjanes Peninsula (Icelandic Meteorological Office, 2021)

Since volcanic-tectonic earthquakes emerge as a result of moving magma beneath the volcanic systems, we interpret such clusters as indications that the front of the traveling magma soliton reached the volcanic fault and fissure system where the earthquakes were observed. As a result, we use the locations and dates of these two earthquake clusters to calculate the velocity of the traveling magma soliton.

Because the mid-Atlantic ridge—which the magma moves along—connects these two clusters almost perfectly, the distance traveled by the soliton was calculated to be 65.3 kilometers, using the coordinates of the two clusters. The difference between the two dates is 349 days. Therefore, the soliton travels at a rate of approximately 0.1872 km/day, producing the formula for the magma soliton as

$$v(x,t) = 0.0936\,\mathrm{sech}^2(0.216(x - 0.1872t)), \tag{7}$$

where x is the displacement in the direction of its propagation (along the ridge) in kilometers and t is time measured in days.

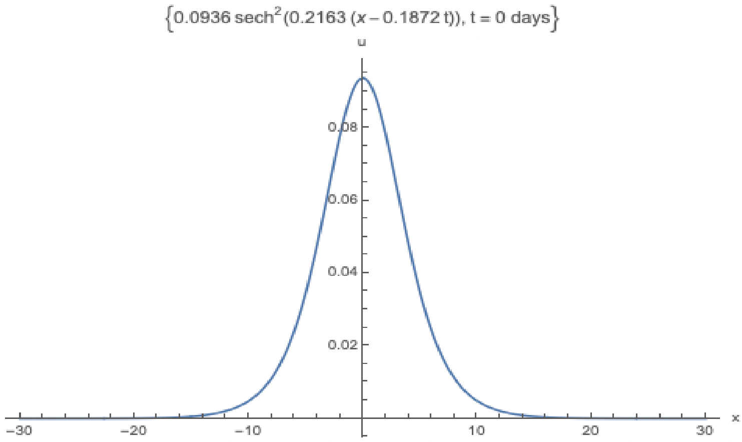
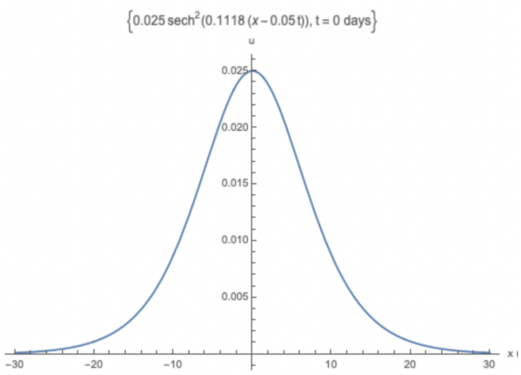


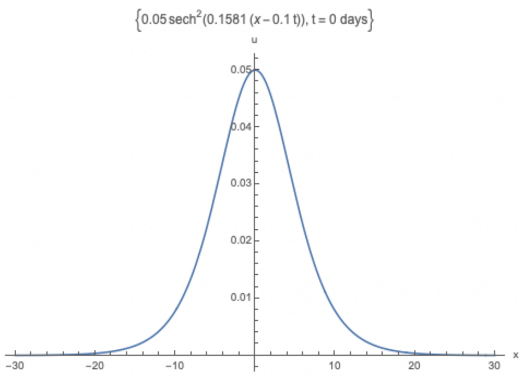
Figure 4: A 2D Graph of the Soliton at $t = 0$

Figure (4) reveals this solitary wave reveals that it is very long and flat, with a maximum amplitude of about 0.1 but with a width of about 36 kilometers. This indicates that the magma spans a large distance along the ridge.

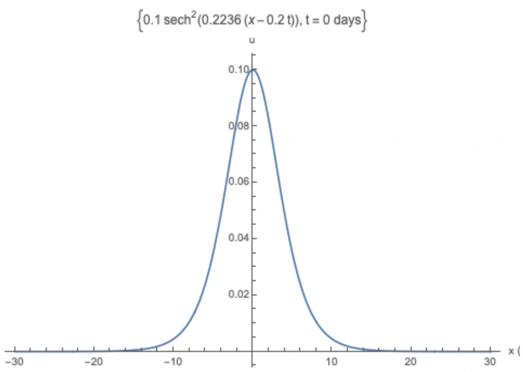
The amplitude of the soliton scales with its velocity (c):



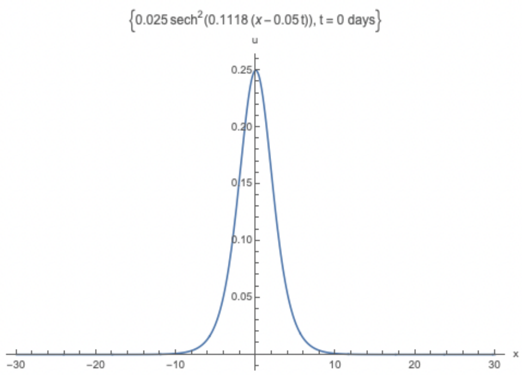
(a) $c = 0.05$



(b) $c = 0.1$



(c) $c = 0.2$



(d) $c = 0.5$

Figure 5: Solitons with Different Velocities

As the velocity increases, the soliton becomes thinner and taller, indicating that it spans a shorter distance along the ridge and that a greater proportion of the magma is molten and hotter than the surrounding rock.

2.2 Gaussian Profile in the Direction Perpendicular to Propagation

However, we do observe that the primary earthquake activity over time is concentrated tightly along the Atlantic-ridge and does not deviate significantly from the ridge line. Therefore, we can conclude that the magma does not demonstrate a similar behavior in the y-direction, the direction perpendicular to the magma soliton propagation along the ridge, as in the x-direction. In fact, the earthquakes farthest from the ridge are typically located only located 1 to 5 kilometers away from the ridge. Because we are modeling the movement of the molten lava, which lies in the center of the ridge, we will use the lower bound of this range and assume that the width of the traveling magma wave is approximately 1 kilometer in the y-direction, much less than its length in the x-direction.

As a result, we propose a Gaussian profile for the magma in the y-direction, which will generate a model with s smaller cross-section in the direction perpendicular to the ridge. The general Gaussian distribution function is

$$p(y) = \frac{e^{-\frac{1}{2}(\frac{y-\mu}{\sigma})^2}}{\sigma\sqrt{2\pi}} \tag{8}$$

This give the two-dimensional model for the magma soliton

$$v(x, y, t) = \frac{c}{2}\,\mathrm{sech}^2[\frac{\sqrt{c}}{2}(x - ct - a)]\frac{e^{-\frac{1}{2}(\frac{y-\mu}{\sigma})^2}}{\sigma\sqrt{2\pi}}, \tag{9}$$

depending on two parameters, velocity c and half-width σ, in addition to the translation parameter a and the centering parameter μ.

For this model, y denotes displacement in the y-direction perpendicular to the Mid-Atlantic Ridge (in kilometers). Setting $\mu = 0$ (the magmon is centered along the ridge) and $\sigma = 0.5$ (the deviation in either direction of the ridge is approximately 0.5 km), we obtain the Gaussian profile of the magma to be

$$p(y) = \frac{e^{-\frac{y^2}{0.5}}}{0.5\sqrt{2\pi}} \tag{10}$$

We can now write the model for the liquid volume fraction forming a magma soliton as

$$v(x, y, t) = 0.0936\,\mathrm{sech}^2[0.216(x - 0.1872t)]\frac{e^{-\frac{y^2}{0.5}}}{0.5\sqrt{2\pi}}. \tag{11}$$

3 Application of the Model

With equation (11), we can generate predictions about future seismic and volcanic activity. Even though the model was derived from only two clusters of earthquakes, it can be applied to model eruptions along the entire ridge, even ones far away from the these two clusters, as seismic activity in one part of the ridge could be connected with past or future activity in another location on the ridge. Indeed, extensive research and observation have confirmed that earthquakes along the entire mid-Atlantic ridge are connected (Haroarson, 2015; Scordilis, C.B., & G.F., 2016).

3.1 Volume Calculations

As the magma soliton traveled across the Fagradalsfjall system, the Fagradalsfjall volcano experienced a powerful eruption that is ongoing at the time of writing. However, as the soliton traveled across the remaining volcanic systems, subsequent eruptions were not observed. This is most likely because there was an insufficient amount of magma remaining in the chambers after the Fagradalsfjall eruption; we can study this analytically with our model.

Data collected from University of Iceland Institute of Earth Sciences (2021) shows that 151 million cubic meters of magma have been released in the eruption, as of September 17, 2021. More magma has currently not been released.

To compute the total volume of magma in the soliton, we compute the area beneath the soliton and multiply it by the area beneath the Gaussian profile:

$$\begin{aligned} \text{Volume} &= 2 * \int_0^\infty 0.0936\,\text{sech}^2(0.216x)dx * \int_0^\infty \frac{e^{-\frac{x^2}{0.5}}}{0.5\sqrt{2\pi}}dx \\ &= 0.4327\text{km}^3 \\ &= 4.327 * 10^8\text{m}^3 \\ &= 432.7\,\text{million m}^3 \end{aligned} \tag{12}$$

Therefore, we estimate that the volume released in the Fagradalsfjall eruption is approximately one-third of the total magma in the soliton. We should note that, as discussed in section 3.2, molten lava in the soliton may have caused surrounding rock to melt in the eruption; therefore, some of the magma recorded in the eruption may be in addition to the original soliton. In fact, it is probably more appropriate to think of the soliton as a propagation of (liquid) phase rather than magma, although both can occur.

Still, the calculation offers qualitative insight into the volume of magma released in the eruption relative to the total magma in the chamber; and, because the volume released is a very significant portion of the total volume, it makes sense that no subsequent volcanic eruptions have been observed beyond Fagradalsfjall: there is simply an insufficient amount of magma remaining for the earthquakes to trigger eruptions.

It also makes sense to conclude that the eruption should end no later than October 2021, which is when half of the volume of magma carried by the soliton would have been emitted. Given that the soliton built significant pressure in Brennisteinsfjöll further east along its path and may have caused earthquakes in Hengill—the easternmost volcanic zone on the Reykjanes peninsula—on August 20th (Icelandic Meteorological Office, 2021), it seems likely that the soliton escaped from the Geldingadalur-Fagradalsfjall volcano with half of its magma volume intact. This means that roughly 200 million cubic meters can be emitted in the eruption; indeed, 151 million cubic meters of lava have already been emitted by September 17th 2021 (University of Iceland Institute of Earth Sciences, 2021).

It is unlikely that the soliton actually consists of magma flowing along the mid-Atlantic Ridge in the brittle part of the lithosphere made porous by earthquake activity, as such flow of magma would have been detected by satellite and GPS measurements. It is more likely that it is a phase transition from a mostly solid phase to a mostly liquid phase caused by a hot thermal pulse traveling as the soliton. The computation of the liquid magma created by this hot pulse is that same as above, so we can use the analogy of the magma soliton. This is what the thermal pulse creates in each location; in a volcanic zone, this liquid soliton can melt the lid of porous rock that normally prevents a volcanic eruption from taking place. These soliton-thermal pules are likely created by thermal pulses flowing in the opposite direction in the asthenosphere Jones et al. (2014), which is discussed in more detail in discussion section below.

3.2 Predicting Future Earthquakes

There are three more volcanic systems in the path of the soliton along the mid-Atlantic ridge beyond Fagradalsfjall: Krýsuvík, Brennisteinsfjöll, and Hengill. When the front of the traveling magma soliton enters these volcanic systems, we would predict that the pressure will build and volcanic-tectonic earthquakes can be observed.

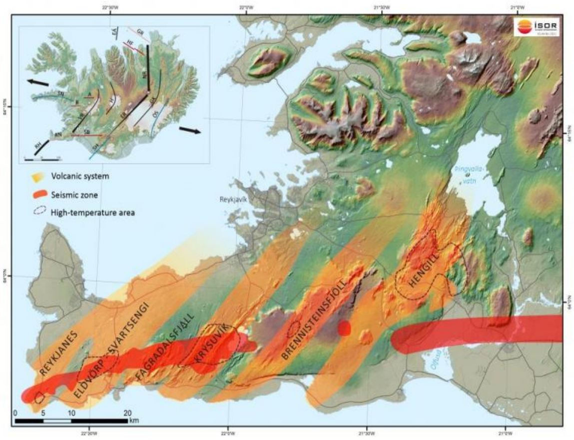


Figure 6: Volcanic Systems on the Reykjanes Peninsula

Using our solitary wave equation, we can predict when the soliton will reach other volcanic systems across Iceland by dividing the distance between each system by the soliton's velocity.

However, when the soliton reaches a volcanic system and builds up pressure, it will also release a large amount of its magma, as observed with the Fagradalsfjall eruptions. Based on the calculations above, assuming that the soliton loses slightly more than one third of its magma in the eruption in Fagradalsfjall, we propose the following equation for the liquid volume fraction of the magma soliton between Fagradalsfjall and Hengill:

$$v(x,y,t) = 0.0514\,\mathrm{sech}^2[0.1815(x-0.1317t)]\frac{e^{-\frac{y^2}{0.39}}}{0.39\sqrt{2\pi}}, \tag{13}$$

This soliton magmon has lost more than one third of its volume and has the velocity 0.1317 km/day, and a narrower Gaussian profile in the y-direction with variance 0.39. Its equation is derived from Equation (6), which indicates that the equation of a soliton is entirely dependent on its velocity, and Equation (9), which connects the soliton equation and the Gaussian profile.

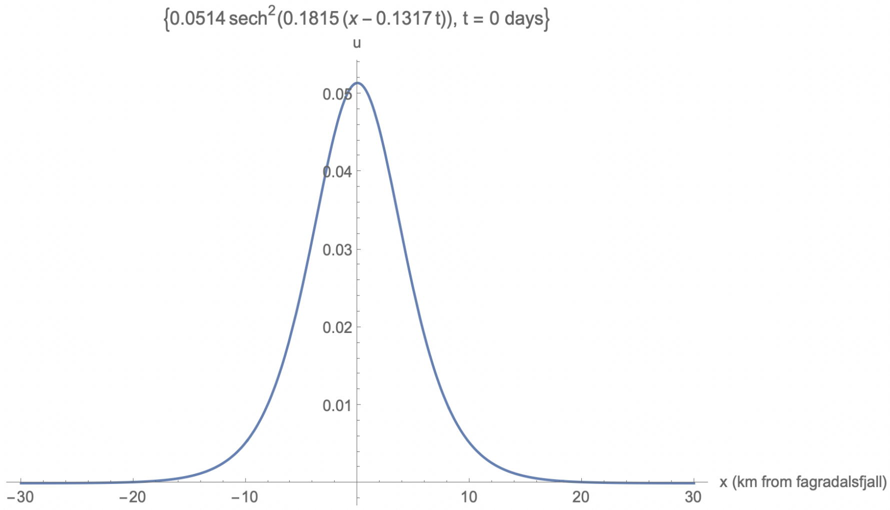


Figure 7: Soliton with new velocity

Its volume is:

$$
\begin{aligned}
\text{Volume} &= 2 * \int_0^{\infty} 0.0514\,\text{sech}^2(0.1815x)dx * \int_0^{\infty} \frac{e^{-\frac{x^2}{0.39}}}{0.39\sqrt{2\pi}}dx \\
&= 0.2831\text{km}^3 \\
&= 2.831 * 10^8\text{m}^3 \\
&= 283.1 \text{ million m}^3
\end{aligned} \tag{14}
$$

This is the volume of the magma that the thermal pulse escaping the eruption in Fagradalsfjall can melt.

Using the velocity from equation (13), we obtain:

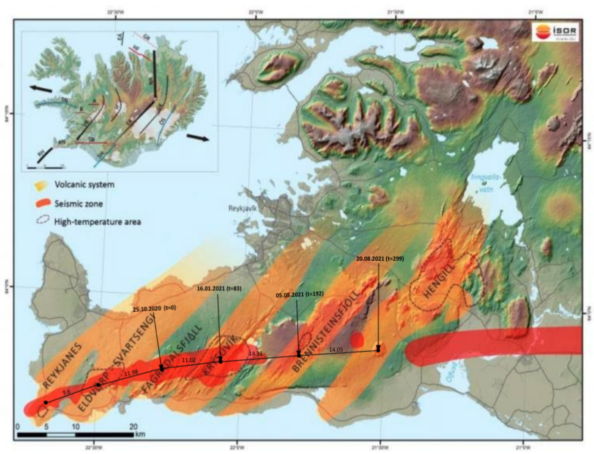
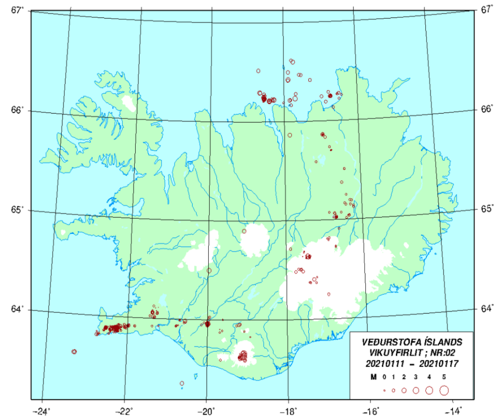
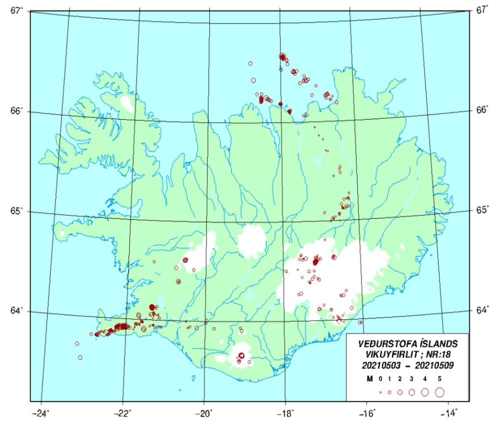


Figure 8: Path of the Soliton Along the Mid-Atlantic Ridge

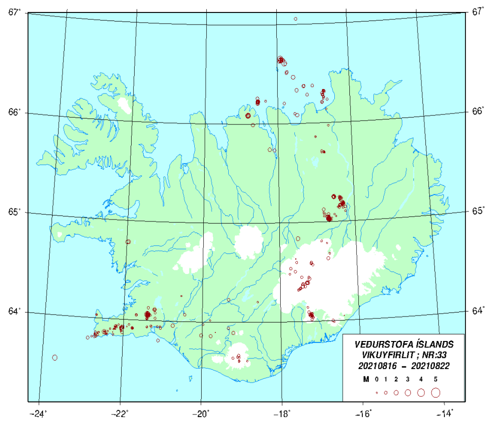
The dates listed in the figure are when the front of the soliton reaches the volcanic system. Because they are before the time of writing, we can analyze seismic activity across the Reykjanes Peninsula to verify or refute the predictions.



(a) 16.01.2021



(b) 05.05.2021



(c) 20.08.2021

Figure 9: Earthquake Clusters (Icelandic Meteorological Office, 2021)

The observed earthquake activity affirms our model's predictions in Krýsuvík, Brennisteinsfjöll, and Hengill. In figure 9a, we can see that on the week of January 16, 2021, earthquakes are beginning to occur beyond Fagradalsfjall in the Krýsuvík system. Earthquakes are also still occurring in Fagradalsfjall; this aligns with our model's predictions, as the soliton has a width of nearly 40 kilometers in the direction of propagation, and therefore will continue to cause earthquakes long after the front of the soliton crosses the system.

Figure 9b shows seismic activity is also being observed in Brennisteinsfjöll on the week of May 5, 2021, as the model predicts. Relative to the observations in Fagradalsfjall or Krýsuvík, the seismic activity is smaller; this observation, though, does not refute the model's validity. Because there are many underground channels connecting Brennisteinsfjöll to the other volcanic systems, not of all which flow directly through Brennisteinsfjöll, a large portion of the magma in the soliton could have taken another path and therefore could not have led to earthquakes or volcanic eruptions (Saemundsson, Sigurgeirsson, & Frioleifsson, 2020).

Similarly, Figure 9b shows large amounts of seismic activity in Hengill on the week of August 20, 2021, as our model predicts. These earthquakes were reasonably large for the Hengill, with some reaching magnitudes of almost 3.0, reaffirming that all the heat in the thermal pulse has arrived at the Hengill system at the forecasted time.

3.3 Predicting When Earthquakes Will End

Because the soliton spans almost 40 kilometers in the direction of propagation and takes a long time to empty a sufficient amount of its magma, earthquakes will continue to occur in the volcanic systems for long periods of time after the soliton first arrives. We can use the model to predict when such earthquakes will end by observing when the tail of the soliton fully crosses over each system.

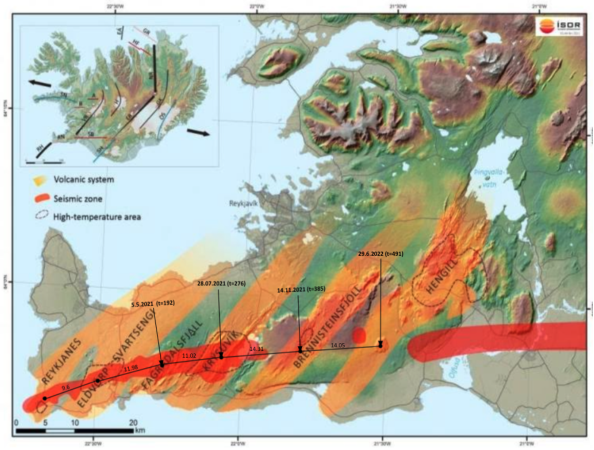


Figure 10: Path of the Tail of the Soliton

However, we must also consider that the molten lava in the soliton is extremely hot and, as it travels within volcanic chambers, can heat up surrounding solid rock into semi-liquid forms. Under sustained pressure and heat, this rock can fill the void left by the moving soliton and lead to earthquakes even after the soliton has left the volcanic chamber. Therefore, seismic activity may linger for months even after the soliton has exited the volcanic system, as the stress in the rock continues to relax. Indeed a cluster of earthquakes occurred in the Hengill volcanic zone on August 20th (Icelandic Meteorological Office, 2021) as mentioned above. This may have been caused by the arrival of the thermal soliton, but the earthquake activity is expected to last until February 2022, when the thermal soliton departs from the Hengill volcanic zone.

4 Discussion and Concluding Remarks

In this paper, we apply the magma equations proposed by Takahashi and Satsuma (1988) to the Reykjanes peninsula in Iceland. We use clusters of earthquake activity to calculate the velocity of the thermal flow and produce a magma soliton that models the movement of a thermal pulse along the Reykjanes ridge. We also generate a Gaussian profile of the soliton in the direction perpendicular to its propagation along the ridge. This soliton-like thermal pulse is able to melt porous rock into a soliton-like liquid magmon, instead of a magmon moving along the plate boundaries. However, we can think about the melted magma as a soliton moving in the direction of the earthquake zone on the plate boundaries and the computations are the same for the magmon and the thermal pulse.

With this model, we predict when ongoing earthquakes in the Reykjanes peninsula will end. We also compute the total volume in the magma soliton and compare it to the amount of magma that has been released in the ongoing Fagradalsfjall eruption. We conclude that subsequent eruptions have not occurred as the soliton reaches the remaining volcanic systems because a significant proportion (at least one third) of the magma that the soliton is able to melt has been released already. Thus there is not enough heat remaining in the propagating thermal pulse to cause another eruption. In continuation, it would be interesting to link the mathematical models for earthquakes Erickson et al. (2008, 2011) and the model for magma in eruptions presented in the paper. This remains for future work.

The thermal-soliton model presents a new paradigm in volcanology where a thermal pulse moves a significant distance horizontally towards an eruption. It is unlikely to apply in the center of Iceland where volcanoes are connected to their magma chambers and the magma moves vertically. However, it may apply on the Reykjanes ridge and on Reykjanes and also on the continuation of the Atlantic ridge by Grímsey north of Iceland. It also presents an intriguing connection between earthquakes on volcanic zones and subsequent eruptions on different volcanic zones. This could help with long-time warnings systems for possible volcanic eruptions.

Much work has been done, see Jones et al. (2014), on termal plumes propagating in the asthenosphere and causing V-shaped ridges in the North-Atlantic. There plumes propagate horizontally from the Icelandic hotspot in the opposite direction to the thermal pulses discussed in this paper. Intriguingly, the velocity of the plume head, upward from 87 km/year Jones et al. (2014), is not very different from the velocity of the soliton thermal pulse that we have discussed in this paper. Its velocity is 68.3 km/year. This suggest that the thermal plumes in asthenosphere may be causing our thermal solitons in the plate-boundary region, made brittle by earthquakes, of the lithosphere. However, the volume flux of the heads is much greater 49 km^3/year, see Jones et al. (2014), than the amount of magma, 0.433 km^3/year, melted by the soliton thermal pulse. Thus the mechanism of generation of the soliton thermal pulses and the possible connection to the propagation of plumes remains to be explored.

Appendices

A Transformation of Magma Equation into KdV Equation

The transformations (3) and (4) produce the following relations:

$$\partial_x = \frac{\partial_\xi}{u} \tag{A.1}$$

$$\partial_t = (\frac{-u_{\xi\xi}}{u} + 3u - 3\alpha)\partial_\xi + \partial_\tau \tag{A.2}$$

where α is a constant. Applying (A.1) and (A.2) to (2) yields

$$\begin{aligned}
&(\frac{-u_{\xi\xi}}{u} + 3u - 3\alpha)\partial_\xi u + \partial_\tau u = [u^3(u_{xx} - 1)]_x, \\
&\frac{-u_{\xi\xi}u_\xi}{u} + 3uu_\xi - 3\alpha u_\xi + u_\tau = \frac{1}{u}[uu_{\xi\xi} - u_\xi^2 - u^3]_\xi, \\
&\frac{-u_{\xi\xi}u_\xi}{u} + 3uu_\xi - 3\alpha u_\xi + u_\tau = u_{\xi\xi\xi} - \frac{u_{\xi\xi}u_\xi}{u} - 3uu_\xi,
\end{aligned} \tag{A.3}$$

or

$$u_\tau = u_{\xi\xi\xi} - 6uu_\xi + 3\alpha u_\xi.$$

Acknowledgments

We want to thank Ólafur Ingólfsson, Ólafur Gudmundsson and Bryndís Brandsdóttir for useful comments. We also thank Vedurstofa Íslands (The Icelandic Meterological Office) for the permission to use Figures 3 and 9, Jardfrædistofnun of the University of Iceland for permission to use the information from University of Iceland Institute of Earth Sciences (2021) and Ármann Höskuldsson for permission to use Figure 2.

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