# Meteoroid Fragmentation in the Martian Atmosphere and the Formation of Crater Clusters

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## Abstract

The current rate of small impacts on Mars is informed by more than one thousand impact sites formed in the last twenty years, detected in images of the martian surface. More than half of these impacts produced a cluster of small craters formed by fragmentation of the meteoroid in the martian atmosphere. The spatial distributions, number and sizes of craters in these clusters provide valuable constraints on the properties of the impacting meteoroid population as well as the meteoroid fragmentation process. In this paper, we use a recently compiled database of crater cluster observations to calibrate a model of meteoroid fragmentation in Mars' atmosphere and constrain key model parameters, including the lift coefficient and fragment separation velocity, as well as meteoroid property distributions. The model distribution of dynamic meteoroid strength that produces the best match to observations has a minimum strength of 10–90 kPa, a maximum strength of 3–6 MPa and a median strength of 0.2–0.5 MPa. An important feature of the model is that individual fragmentation events are able to produce fragments with a wide range of dynamic strengths as much as ten times stronger or weaker than the parent fragment. The calibrated model suggests that the rate of small impacts on Mars is 1.5–4 times higher than recent observation-based estimates. It also shows how impactor properties relevant to seismic wave generation, such as the total impact momentum, can be inferred from cluster characteristics.

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## Key Points:

based estimates

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9	• Crater cluster observations used to calibrate model of meteoroid fragmentation	n
10	in Mars' atmosphere	
11	• Fragments exhibit a wide range of dynamic strengths both weaker and stronge	er
12	than parent	
13	• The rate of small impacts on Mars is 1.5–4 times higher than recent observation	on-

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#### 15 Abstract

The current rate of small impacts on Mars is informed by more than one thousand im-16 pact sites formed in the last twenty years, detected in images of the martian surface. More 17 than half of these impacts produced a cluster of small craters formed by fragmentation 18 of the meteoroid in the martian atmosphere. The spatial distributions, number and sizes 19 of craters in these clusters provide valuable constraints on the properties of the impact-20 ing meteoroid population as well as the meteoroid fragmentation process. In this paper, 21 we use a recently compiled database of crater cluster observations to calibrate a model 22 of meteoroid fragmentation in Mars' atmosphere and constrain key model parameters, 23 including the lift coefficient and fragment separation velocity, as well as meteoroid prop-24 erty distributions. The model distribution of dynamic meteoroid strength that produces 25 the best match to observations has a minimum strength of 10–90 kPa, a maximum strength 26 of 3–6 MPa and a median strength of 0.2–0.5 MPa. An important feature of the model 27 is that individual fragmentation events are able to produce fragments with a wide range 28 of dynamic strengths as much as ten times stronger or weaker than the parent fragment. 29 The calibrated model suggests that the rate of small impacts on Mars is 1.5–4 times higher 30 than recent observation-based estimates. It also shows how impactor properties relevant 31 to seismic wave generation, such as the total impact momentum, can be inferred from 32 cluster characteristics. 33

## <sup>34</sup> Plain Language Summary

Evidence of more than one thousand meteorite impacts on Mars has been detected 35 in spacecraft images of the surface of Mars taken over the last twenty years. In more than 36 half of these impacts, the meteoroid broke up under aerodynamic forces as it entered Mars' 37 thin atmosphere. The resulting fragments separated during flight to form a cluster of 38 craters on the ground rather than a single crater. Observations of these crater clusters, 39 including the number of individual craters, their sizes and their separations, provide clues 40 to the frequency and properties of meteoroids entering Mars' atmosphere, as well as the 41 nature of their break-up. In this paper, we use the crater cluster observations to test and 42 calibrate a numerical model of meteoroid fragmentation in Mars' atmosphere. The model 43 agrees well with observations and suggests that about 90–240 craters or clusters with a 44 diameter greater than 10 m are produced somewhere on Mars each year. Small mete-45 oroids entering Mars' atmosphere show a wide range of strengths and appear to be very 46 similar to those observed on Earth that form fireballs. Our calibrated model of mete-47 oroid entry will improve dating of surfaces on Mars and aid efforts to detect small im-48 pacts by the sound waves that they generate. 49

## 50 1 Introduction

Repeated imaging of the surface of Mars by orbiting spacecraft over the last two decades has revealed more than one thousand impact sites formed in this time period (Daubar et al., 2013, 2019, 2021). These observations provide important constraints on the current rate of small impacts on Mars, which are valuable for calibrating crater production models (Daubar et al., 2013; Williams et al., 2014), assessing the impact hazard to spacecraft on Mars, and determining the ratio of primary to secondary crater production rates (Hartmann et al., 2018).

Among the known recent impact sites, fewer than half are single craters; the majority are fields of craters, known as crater clusters (Daubar et al., 2013, 2019, 2021; Neidhart et al., 2021). The size and separation of individual craters within these clusters suggest that they are formed due to atmospheric break up of the primary meteoroid into a collection of fragments that separate and strike the ground almost simultaneously (Artemieva & Shuvalov, 2001; Popova et al., 2003, 2007). The diversity of crater clusters, in terms of the spatial distributions and size-frequency distributions of their craters, provide a unique opportunity to interrogate the processes of atmospheric entry and fragmentation, and
potentially constrain properties of the impactor population (Hartmann et al., 2018; Daubar
et al., 2019; Neidhart et al., 2021). Differences in small crater and crater cluster populations between surfaces of different ages may also help constrain historical variations
in atmospheric density (Popova et al., 2003).

A database of 634 recently formed crater clusters was mapped and characterized 70 in detail by Neidhart et al. (2021). This represents about 90% of the clusters in the up-71 dated catalog of new impact sites on Mars (Daubar et al., 2021). Here, we augment this 72 73 data set of clusters with a random sample of 90% (456) of new single craters from the same catalog. The resultant data set includes crater size and position information for 74 all individual craters larger than one meter in all detected clusters and single craters. 75 From the cluster data, a number of cluster properties were derived that characterize the 76 spatial distribution and size-frequency distribution of craters within the cluster (Daubar 77 et al., 2019; Neidhart et al., 2021). 78

On the basis that the present catalog of small impacts, while incomplete, is rep-79 resentative of recent crater and cluster production on Mars, here we use these quanti-80 tative cluster characteristics to calibrate a model of meteoroid fragmentation in Mars' 81 atmosphere (Artemieva & Shuvalov, 2001). This allows us to constrain key model pa-82 rameters and the distribution of meteoroid properties such as dynamic strength. The cal-83 ibrated model provides insight into the small impactor flux at Mars and how this com-84 pares to the observed flux at Earth (Bland & Artemieva, 2006). It also shows how im-85 pactor properties such as the total impact momentum can be inferred from cluster char-86 acteristics. 87

## <sup>88</sup> 2 Modelling meteoroid fragmentation in the martian atmosphere

Several semi-analytical atmospheric disruption models have been developed to sim-89 ulate the passage and fragmentation of meteoroids through a planetary atmosphere (e.g., 90 Passey & Melosh, 1980; Chyba et al., 1993; Hills & Goda, 1993; Artemieva & Shuvalov, 91 2016; Register et al., 2017). These can be broadly categorized based on their treatment 92 of fragmentation. Continuous fragmentation models treat the fragmented meteoroid as 93 a single, continuously deforming structure (e.g., Chyba et al., 1993; Hills & Goda, 1993). 94 They are well suited to describing airbursts where the meteoroid experiences catastrophic 95 break-up into many small fragments, leading to a dramatic increase in drag and depo-96 sition of energy in the atmosphere (McMullan & Collins, 2019). Discrete fragmentation 97 models, on the other hand, approximate fragmentation as a successive division of the me-98 teoroid into individual masses. Such models can track the resulting fragments until com-99 plete ablation or impact (e.g., Passey & Melosh, 1980; Artemieva & Shuvalov, 2001; Popova 100 et al., 2003; Bland & Artemieva, 2006). They are, therefore, ideally suited to modelling 101 the formation of crater clusters or strewn fields formed by near simultaneous impact of 102 a population of meteoroid fragments (Passey & Melosh, 1980; Popova et al., 2003; Bland 103 & Artemieva, 2006). A third, hybrid approach that combines elements of both models 104 has recently proved successful at replicating the energy deposition of a number of small 105 terrestrial bolides (Wheeler et al., 2017; Register et al., 2017; Wheeler et al., 2018). 106

Here we apply a version of the discrete fragmentation model often referred to as 107 the Separate Fragments Model (SFM) of atmospheric break-up (Passey & Melosh, 1980; 108 Artemieva & Shuvalov, 2001; Bland & Artemieva, 2006) to the formation of small crater 109 clusters on Mars. This model was previously applied to the formation of crater strewn 110 fields on Earth by the break-up of strong iron meteorites (Passey & Melosh, 1980; Artemieva 111 & Shuvalov, 2001; Bland & Artemieva, 2006) and used to show that small crater clus-112 ters on Mars are consistent with the break-up of much weaker stony meteoroids in Mars' 113 more tenuous atmosphere (Artemieva & Shuvalov, 2001; Popova et al., 2003, 2007). A 114 similar model was also used to simulate the production of small craters on Mars using 115

the observed flux of terrestrial fireballs (Williams et al., 2014). The availability of new
observational data on the frequency and characteristics of small clusters and single craters
provides the opportunity for a rigorous calibration of these models from which new insight into the rate and nature of small impacts on Mars can be derived.

## 120 2.1 Flight Integration

<sup>121</sup> The basis for the SFM is the coupled set of ordinary differential equations of stan-<sup>122</sup> dard meteor physics (e.g., Baldwin & Sheaffer, 1971; Passey & Melosh, 1980), which de-<sup>123</sup> scribe the temporal evolution of the meteoroid speed v, mass m, trajectory angle to the <sup>124</sup> horizontal  $\theta$  and position in space x, y, z:

$$\frac{dv}{dt} = -\frac{C_D \rho_a v^2 \pi r^2}{2m} + g \sin(\theta), \qquad (1)$$

$$\frac{dm}{dt} = -\frac{\sigma \rho_a v^3 \pi r^2}{2}, \qquad (2)$$

$$\frac{d\theta}{dt} = \frac{g\cos(\theta)}{v} - \frac{C_L \rho_a \pi r^2 v}{2m} - \frac{v\cos(\theta)}{R_p + z}, \qquad (3)$$

$$\frac{dz}{dt} = -v\sin(\theta), \qquad (4)$$

$$\frac{dx}{dt} = v\cos(\theta)\cos(\phi)\frac{R_p}{R_p+z},$$
(5)

$$\frac{dy}{dt} = v\cos(\theta)\sin(\phi)\frac{R_p}{R_p+z},$$
(6)

$$\frac{d\phi}{dt} = 0. \tag{7}$$

In these equations,  $C_D$  and  $C_L$  are dimensionless coefficients of drag and lift, respectively,  $\sigma$  is an ablation parameter, r is the meteoroid radius (assumed to have a circular crosssection orthogonal to the trajectory),  $\rho_a$  is the air density,  $R_p$  is the planetary radius, and g is the gravitational acceleration, which is a function of altitude z. The introduction of the azimuth  $\phi$  and two coordinates for downrange distance x and cross-range distance y allows the calculation of impact locations on a two-dimensional planetary surface.

These equations are integrated numerically from a specified initial state with re-132 spect to time for the initial meteoroid and any fragment subsequently produced until the 133 meteoroid or fragment (a) strikes the ground; (b) ablates to a size sufficiently small to 134 be neglected; or (c) deflects off the atmosphere back into space. The form of the equa-135 tions adopted here account for the effects of drag, lift, ablation, planetary curvature and 136 the decrease of gravity with altitude. An atmospheric density-altitude table extracted 137 from the Mars Climate Database<sup>1</sup> (Forget et al., 1999; Millour et al., 2018) at a point 138 on the equator was used to provide a reference atmospheric density profile. The database 139 provides meteorological fields, including density, compiled from General Circulation Mod-140 els of the martian atmosphere, calibrated with available observational data. The specific 141 table used is included in the supporting information. Altitude is defined relative to the 142 MOLA zero altitude. As the density profile is approximately exponential and the den-143 sity table is relatively coarse, intermediate density points were calculated using expo-144 nential interpolation. 145

#### <sup>146</sup> 2.2 Fragmentation and separation

<sup>147</sup> During integration of the meteor equations each meteoroid or fragment is assigned <sup>148</sup> a dynamic strength Y. A widely used fragmentation criteria is then applied that the frag-

<sup>&</sup>lt;sup>1</sup> http://www-mars.lmd.jussieu.fr

ment breaks when the ram pressure (the product of the local air density and the fragment velocity squared) exerted on the meteoroid exceeds the dynamic strength Y:

$$\rho v^2 > Y. \tag{8}$$

In the version of the Separate Fragments Model used here, the parent meteoroid is always broken into two child fragments with a mass ratio chosen at random between specified limits. The two fragments are each assigned a strength, based partly on mass, and a relative velocity that acts to separate the two fragments in opposing directions, perpendicular to the direction of flight of the parent and at a random azimuth  $\phi$  to the parent trajectory. The magnitude of the lateral separation velocity  $v_S$  is defined by (Passey & Melosh, 1980):

$$v_S = v \sqrt{C_S \frac{3}{2} \frac{r_1}{r_2} \frac{\rho_a}{\rho_m}} \tag{9}$$

where v is the along trajectory velocity,  $C_S$  is a dimensionless separation velocity coefficient,  $\rho_a/\rho_m$  is the ratio of the atmospheric to fragment density and  $r_1/r_2$  is the ratio of the larger to the smaller fragment radii (where spherical geometry is assumed). The lateral velocity imparted to each fragment is determined so as to satisfy two constraints: linear momentum conservation and a net separation velocity of  $v_S$ .

The masses of the two child fragments are assigned randomly at each fragmentation event as a fraction of the parent fragment mass  $m_p$ . The first fragment is assigned a mass equal to:

$$m_1 = m_p (1 - x) f_m, \ x \sim \mathcal{U}[0, f_r],$$
(10)

where  $f_m$  represents the nominal mass fraction of the first fragment and  $f_r$  is a randomization factor (where  $f_r = 0$  implies  $m_1/m_p$  is always  $f_m$  and  $f_r = 0.9$  implies that the first fragment can take a mass fraction between  $0.1f_m$  and  $f_m$ ). The second fragment is assigned the remaining mass  $m_2 = m_p - m_1$ .

The strength of each child fragment is determined based on two principles from pre-170 vious implementations of the SFM (Artemieva & Shuvalov, 2001). First, we assume that 171 in general the child fragment acquires an increased strength relative to the parent frag-172 ment owing to the removal of larger structural weaknesses and faults within the parent 173 fragment. A second important principle is that the actual strength of the fragment is 174 allowed to vary about this nominal value so that child fragments can, occasionally, be 175 weaker than their parents. The strength  $Y_f$  of the fragment is therefore prescribed ac-176 cording to: 177

$$Y_f = Y_p \left(\frac{m_p}{m_f}\right)^{\alpha} 10^x, \ x \sim \mathcal{N}(0, \delta).$$
(11)

The first term on the right-hand side defines the nominal strength of the fragment based on Weibull statistics (Weibull, 1951), where  $m_p$  and  $m_f$  is the mass of the parent and child fragment, respectively,  $Y_p$  is the strength of the parent and  $\alpha$  is the strength scaling exponent (Popova et al., 2011). A value of  $\alpha$  close to zero implies little increase in strength as mass decreases. The second term on the right-hand side is a random strength scaling factor drawn from a log-normal distribution; i.e., where the exponent x is drawn from a normal distribution with a mean of zero a standard deviation of  $\delta$ .

A consequence of this random strength scaling factor, which turns out to be im-185 portant for reproducing the characteristics of crater clusters, is that the child fragment 186 can become either much stronger than its parent, potentially precluding any further frag-187 mentation, or weaker than the parent, implying that the child fragment immediately un-188 dergoes at least one further fragmentation. This latter scenario effectively allows for more 189 catastrophic fragmentation events, where the parent breaks into more than two fragments. 190 This feature of the model, therefore, may be replicated by a model that splits the par-191 ent into a random number of fragments rather than always a pair. The resulting frag-192 ments are traced until either their mass is too small to produce a crater or an impact 193 occurs. 194

#### <sup>195</sup> **2.3** Crater formation and cluster characterization

Once fragments of the meteoroid strike the ground, we use a well-established scaling relationship to determine the crater diameters D (Holsapple, 1993):

$$D = 2f_{\rm rim} K_r V^{1/3}, \qquad (12)$$

$$\frac{\rho_t V}{m} = K_1 \left[ \left( \frac{g_0 r}{v_z^2} \right) \left( \frac{\rho_t}{\rho_m} \right)^{\frac{6\nu - 2 - \mu}{3\mu}} + K_2 \left( \frac{Y}{\rho_t v_z^2} \left( \frac{\rho_t}{\rho_m} \right)^{\frac{6\nu - 2}{3\mu}} \right)^{\frac{2 + \mu}{2}} \right]^{-\frac{3\mu}{2 + \mu}}.$$

This relationship allows for a large range of potential surface properties and impact scenarios. V is the crater volume below the preimpact surface,  $g_0$  is the gravitational acceleration at the impact location,  $r, v_z, m$  and  $\rho_m$  are the radius, vertical impact velocity component, mass and density of the impacting fragment, respectively. The factor  $f_{\rm rim} =$ 1.3, converts the crater diameter at pre-impact surface level to the value at the rim level to be consistent with observational measurements (Holsapple, 1993).

Based on the sizes of craters formed by mass-balance devices released during the descent of Perseverance (Fernando et al., 2021), we modeled the martian surface with properties appropriate for a weak granular soil: density  $\rho_t = 1500 \text{ kgm}^{-3}$ , cohesive strength Y=50 kPa, and with scaling coefficients  $K_1 = 0.133$ ,  $K_2 = 1$ ,  $K_r = 1.25$ ,  $\mu = 0.41$ ,  $\nu = 0.4$  (Holsapple, 1993).

Due to the current resolution of satellite imagery, craters with a diameter smaller than 1 m cannot be resolved, therefore no craters below this limit are included in the observational data. To take this into account in the model applied here, small fragments that will not produce a crater larger than 1 m after accounting for overlap are removed from the model and not included in the results.

In cases where craters formed by two or more fragments overlap, they are combined into a single crater with a volume equal to the sum of the two crater volumes (justified by the approximately linear scaling between impactor mass and crater mass, Eq. 12). Based on the proximity of craters in observed clusters (Daubar et al., 2019), the overlap criterion used here is that the separation distance of crater centers,

$$d_{\rm sep} < 0.75 \max(R_1, R_2) + 0.25(R_1 + R_2), \tag{13}$$

where  $R_1$  and  $R_2$  are the rim radii of the two craters. This criterion is a weighted combination of two thresholds:  $d_{sep} < \max(R_1, R_2)$  implies that two craters are only merged if one of their centers lies within the other crater;  $d_{sep} < (R_1 + R_2)$  implies that two craters are merged when they touch.

For each cluster, a number of characteristics were calculated based on the sizes and spatial distribution of the craters (Neidhart et al., 2021). These characteristics include the number of craters  $N_c$  and the effective diameter,

$$D_{\text{eff}} = \sqrt[3]{\sum_{i} D_i^3}.$$
 (14)

Effective diameter is proportional to the cube root of the total volume of all craters in the cluster and, hence, is a measure of the equivalent crater size if fragmentation did not occur (Malin et al., 2006; Daubar et al., 2013).

To quantify the spatial distribution of a cluster, we define the dispersion of the cluster as the median separation of all crater pairs  $d_{med}$  (Neidhart et al., 2021) and the aspect ratio of the cluster as the eccentricity e of the ellipse of minimum area that encompasses the location of the majority of craters in the cluster (Daubar et al., 2019). Finally, to characterize the size-frequency distribution of craters within the cluster, we calculate several metrics: the maximum, median and minimum crater diameter  $(D_{\text{max}}, D_{\text{med}}, D_{\text{min}})$ ; the number and fraction of craters in the cluster with diameter larger than half the maximum diameter  $(N(>D_{\text{max}}/2), f_L = N(>D_{\text{max}}/2)/N_c)$ ; and the exponent of the powerlaw that best fits the cumulative size-frequency distribution of craters in the cluster (i.e.,  $\gamma$ , in  $N(>D) \propto D^{-\gamma}$ ). This exponent  $\gamma$  was determined by performing a linear regression between log N(>D) and log D for clusters with more than five craters.

Figure 1 shows two examples of observed crater clusters on Mars, together with a digitized representation of the same clusters based on mapping of Daubar et al. (2019). It also displays examples of clusters produced by the SFM used in this work that are comparable in terms of all characteristics, including the number of craters  $N_c$ , effective diameter  $D_{\text{eff}}$ , dispersion  $d_{\text{med}}$  and aspect ratio e.



Figure 1. a) HiRISE image of a crater cluster on Mars (ESP\_037706\_1765; NASA/JPL/University of Arizona) with number of craters  $N_c = 9$ , effective diameter  $D_{\rm eff}=11.4$  m, aspect ratio e = 0.3, dispersion  $d_{\rm med}=27.2$  m. b) Digitized version of the same crater cluster where each circle represents a single crater to scale. c) An example of a comparable model crater cluster produced by the SFM with  $N_c = 9$ ,  $D_{\rm eff} = 10.1$  m, e = 0.34,  $d_{\rm med}=33.7$  m. d) HiRISE image of a crater cluster on Mars (ESP\_038458\_2030; NASA/JPL/University of Arizona) with  $N_c = 133$ ,  $D_{\rm eff}=16.1$  m, e=0.77,  $d_{\rm med}=55.8$  m. e) Digitized version of the same crater cluster where each circle represents a single crater to scale. f) An example of a comparable model crater cluster produced by the SFM with  $N_c = 120$ ,  $D_{\rm eff} = 15.7$  m, e = 0.71,  $d_{\rm med} = 65$  m.

## 245 2.4 Monte Carlo Modeling

While the ability of the SFM to reproduce individual clusters is reassuring, a rigorous test of model performance requires that that model can replicate the statistical distribution of observed crater clusters as well as the proportion of events that result in formation of a cluster rather than a single crater.

To produce a set of synthetic crater clusters for comparison with those observed 250 on the surface of Mars, a number of Monte Carlo simulations were carried out. While 251 some observed cluster characteristics, such as effective diameter and number of craters, 252 show a dependency on surface elevation (Neidhart et al., 2021), the ratio of cluster-forming 253 to single-crater impacts is independent of surface elevation (Daubar et al., 2021) and about 254 50% of impact sites have surface elevations within 2 km of the MOLA 0 km reference. 255 For this study, therefore, all simulations used the same atmospheric density profile, with 256 a surface elevation of 0 km and began with the meteoroid at an altitude of 100 km. Fu-257 ture work will investigate the sensitivity of cluster characteristics to variation in ground 258 elevation. We also adopted the same drag coefficient  $C_D = 1$  and nominal fragment mass 259 fraction  $f_m = 0.5$ , throughout. 260

All other initial properties of the meteoroid were selected at random or according 261 to parameter probability distributions for Mars. The probability distribution for pre-entry 262 velocity was derived from Le Feuvre and Wieczorek (2011) and we used the canonical 263 probability distribution for impact angle  $P(>\theta) = \sin^2 \theta$  (Shoemaker, 1961). Mete-264 oroid mass was drawn from a Pareto distribution based on observations of terrestrial fire-265 balls for meteoroids > 3 kg (Halliday et al., 1996; Bland & Artemieva, 2006); P(>m) =266  $(m/m_{\rm min})^{-0.926}$ , where  $m_{\rm min}$  is the minimum mass meteoroid mass in the simulation. 267 For most simulations presented here a minimum mass of 15 kg was used as a compro-268 mise between computational expedience and generating a statistically complete synthetic 269 data set for craters or clusters with an effective diameter greater than 10 m. 270

To select meteoroid density, we used a uniform probability distribution between 271 lower and upper bounds of 1400 and 4000 kg m<sup>-3</sup>, respectively. We adopted this sim-272 ple approach because the real distribution of meteoroid bulk densities is not well known. 273 The density distribution of common stony meteorites is well established (Britt & Con-274 solmagno, 2003), and while bulk meteoroid densities will be lower because of macrop-275 orosity, how the density of the surviving fragment maps to pre-entry bulk meteoroid den-276 sity is not well constrained, particularly for underrepresented meteorite types. Although 277 they may represent a substantial proportion of all meteoroids (Halliday et al., 1996; Ce-278 plecha et al., 1998), we do not consider meteoroids with density less than 1400 kg m<sup>-3</sup>, 279 including cometary meteoroids, because in our size range of interest these objects rarely 280 survive passage through the atmosphere to form craters on the ground larger than 1 m 281 (Williams et al., 2014). We also ignore the small population (< 5%) of iron meteoroids 282 as these are strong enough that their fate is either to ablate entirely or impact the ground 283 as a single fragment and form a single crater (Popova et al., 2003). Neglecting these iron 284 meteorite scenarios will therefore only overestimate the rate of cluster formation by a 285 small factor. 286

The probability distribution of the dynamic strength of meteoroids appropriate for our fragmentation criterion (Eq. 8) is also not well known. As this parameter is of critical importance for the frequency of cluster formation and the characteristics of clusters, we treat this distribution as one of the principal unknowns in our analysis. For simplicity, we adopt a log-uniform probability distribution, defined by a median dynamic strength  $Y_{\text{med}}$  and a width (in  $\log_{10} Y$ -space) of w. In other words, the log-uniform distribution has a minimum dynamic strength of  $Y_{\text{min}} = Y_{\text{med}} 10^{-w}$  and a maximum dynamic strength  $Y_{\text{max}} = Y_{\text{med}} 10^w$ .

The ablation parameter  $\sigma$  is another meteoroid property that is not well constrained. 295 Here we select this parameter from a uniform probability distribution between the lower 296 and upper bounds for stony meteorites  $(1 \times 10^{-8} - 4.2 \times 10^{-8} \text{ s}^2 \text{ m}^{-2})$ , respectively; Ce-297 plecha et al., 1998) and neglect any correlation with density, size or velocity (Baldwin 298 & Sheaffer, 1971; Brykina & Bragin, 2020). Model parameters that we vary to determine 299 the most appropriate for replicating Mars crater clusters are the strength-mass scaling 300 exponent  $\alpha$  and randomization factor  $\delta$  (Eq. 11), the lift coefficient  $C_L$  (Eq. 3), the frag-301 ment mass fraction randomization factor  $f_r$  (Eq. 10) and the fragment separation co-302

## efficient $C_S$ (Eq. 9). The ranges of these parameters explored in our simulations are shown in Table 1.

Parameter	Baseline Value	Variation Range
Initial density, $\rho_m$	-	$1400{-}4000 \text{ kg m}^{-3}$
Ablation parameter, $\sigma$	-	$1-4.2 \times 10^{-8} \text{ s}^2 \text{ m}^{-2}$
Median initial strength, $Y$	330  kPa	100–1000 kPa
Width of strength distribution, $w$	1.25	0.5 - 1.5
Lift coef., $C_L$	0.02	0.005 – 0.05
Separation velocity coef., $C_S$	0.6 - 2.2	0.1 – 2.2
Strength-mass scaling exponent, $\alpha$	0.25	0.01 – 0.8
Strength-mass scaling variance, $\delta$	0.5	0 - 1
Fragment split coef., $f_r$	0.9	0 - 0.99

 Table 1. Best-fit values and variation range for model parameters

To determine the optimum meteoroid and model parameters or parameter distri-305 butions that produce a synthetic set of crater clusters (and single craters) most consis-306 tent with observations (Neidhart et al., 2021; Daubar et al., 2021), we used a three-phase 307 approach. In a first exploratory phase, we performed a number of experiments with dif-308 ferent parameter/parameter distribution combinations to determine good parameter com-309 binations for more detailed study. In this phase, each Monte Carlo simulation was run 310 until the synthetic crater population included 200 singular craters or crater clusters with 311 an effective diameter greater than 10 m. This target was adopted as it is the approxi-312 mate number of such craters in the observational data set and is less sensitive to biases 313 in the observational data (Daubar et al., 2021; Neidhart et al., 2021). 314

Agreement between the observed distribution of single craters and crater clusters and the simulation results was assessed based on both a quantitative comparison of the frequency distributions of individual cluster characteristics (e.g.,  $D_{\text{eff}}$ ,  $N_c$ ) and a qualitative comparison of two-dimensional frequency distributions in various characteristicspaces (e.g.,  $D_{\text{eff}}$ - $N_c$ ).

A quantitative comparison of the model frequency distribution of a given cluster 320 characteristic with the observed distribution was made by binning the distribution and 321 calculating the chi-square statistic of the binned distributions, which quantifies the mis-322 match between the two binned distributions. For example, for the number of craters char-323 acteristic,  $N_c$ , we first bin the observed (normalised) frequency distribution of this char-324 acteristic to define  $F_i^o(N_c)$  using 10-15 bins, depending on the characteristic. We then 325 bin the model frequency distribution using the same bins to define  $F_i^m(N_c)$ . The chi-326 square statistic is then computed for this characteristic as: 327

$$\chi^{2}(N_{c}) = \sum_{i}^{n} \frac{\left[F_{i}^{m}(N_{c}) - F_{i}^{o}(N_{c})\right]^{2}}{F_{i}^{o}(N_{c})} \,. \tag{15}$$

A more qualitative comparison between the observed distribution of cluster characteristics (Neidhart et al., 2021) and those produced by the model was made by visual inspection of kernel-density plots of the distributions in various characteristic-spaces (e.g.,  $D_{\text{eff}}$ - $N_c$ , e- $d_{\text{med}}$ ). This provided confirmation that important correlations between cluster characteristics present in the observational data are replicated by the model.

As certain crater cluster characteristics require more than five craters for mean-333 ingful determination, we restricted our comparison to the 84% of observed clusters with 334  $N_c > 5$ . We also omit all clusters and craters with  $D_{\rm eff} < 5.65$  m (4 $\sqrt{2}$  m) from our 335 comparison to compromise between statistical power and likely completeness of the ob-336 servational dataset. An additional important statistic to determine model performance 337 was the ratio of the total number of crater clusters to singular craters (C-S ratio) with 338 effective diameters > 10 m produced by the model. The observed C-S ratio is 1.68 for 339 all clusters with  $D_{\text{eff}} > 10 \text{ m}$ , 1.46 for clusters with  $N_c > 5$  and  $D_{\text{eff}} > 10 \text{ m}$  and 1.39 340 for the entire catalog (Daubar et al., 2021). 341

In the second phase of our approach, simulations using the most promising param-342 eter combinations were performed with five times more samples to refine the best-fit model 343 parameters and quantify the sensitivity of model performance to the number of samples. 344 Model performance was measured in the same quantitative and qualitative way. The best-345 fit Monte Carlo model parameters from this phase are given in Table 1. In the final phase, 346 several suites of simulations were performed whilst systematically varying key model pa-347 rameters using orthogonal sampling. The purpose of this final phase was to confirm that 348 the most-promising model was a local optimum and to quantify the sensitivity of the model 349 performance to variation of key meteoroid properties and model parameters. 350

For brevity and clarity of presentation, here we describe the results of the latter two of these phases: we first present the results of our best-fit model; we then consider the sensitivity of model performance to the variation of influential individual model parameters.

#### 355 **3 Results**

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#### 3.1 Best-fit model

The best-fit Monte Carlo simulation produced in this work used the best-fit param-357 eters listed in Table 1. The simulation produced a synthetic data set that includes 1000 358 craters/clusters with  $D_{\rm eff} > 10$  m. The incremental frequency distributions of (a) di-359 ameter of singular craters; (b) effective diameter of crater clusters (with  $N_c > 1$ ); (c) 360 number of craters in a cluster (with  $D_{\rm eff} > 5.65$ ) in this synthetic data set are compared 361 with observations (Neidhart et al., 2021) in Fig. 2 after scaling to the same population 362 size (200 craters/clusters with  $D_{\rm eff} > 10$ ). The relative frequency of singular craters 363 and clusters is very consistent with observations above 10 m in (effective) diameter, which 364 is the threshold chosen as the most robust measure of the ratio of clusters to singular 365 craters. The best-fit model produces a cluster-singular crater ratio of 1.62 for  $D_{\rm eff}$  > 366 10 m and 1.35 for those clusters with  $N_c > 5$  (c.f. observed values of 1.68 and 1.46, re-367 spectively). 368

Below an effective diameter of 5.65 m  $(4\sqrt{2} \text{ m})$  there is a clear drop-off in the observed number of craters and clusters, which is attributed to observational detection limitations (Daubar et al., 2021). The drop-off in simulated crater/cluster numbers at approximately the same diameter is a consequence of the minimum meteoroid mass (15 kg) used in the models and not the effects of atmospheric filtering, which become most important at < 1 m diameter (Williams et al., 2014).

Apart from a small excess of clusters with large numbers of craters, the simulation 375 agrees well with the frequency distribution of  $N_c$  for  $N_c > 5$ . For  $N_c < 5$  (and  $N_c =$ 376 2 specifically) the simulation predicts substantially more clusters than observed. This 377 is likely a consequence of the frequency with which the SFM splits the meteoroid into 378 only two fragments, with no further fragmentation events. For these reasons, in the sub-379 sequent analysis we quantitatively compare the synthetic and observed distributions only 380 for craters/clusters with an effective diameter larger than 5.65 m (with  $D_{\rm eff} > 5.65$ ) 381 and for clusters with more than five craters (with  $N_c > 5$ ). 382



Figure 2. Incremental frequency distributions of (a) diameter of singular craters; (b) effective diameter of crater clusters (with  $N_c > 1$ ); (c) number of craters in a cluster (with  $D_{\text{eff}} > 5.65$ ) predicted by the best-fit model compared with observations. Error bars for the models and observations are  $\sqrt{N}$ . The dotted lines indicate the threshold effective diameter (a), (b) and number of craters in the cluster (c) above which the models and observations are quantitatively compared.

The synthetic dataset produced by the best-fit Monte Carlo simulation has the lowest aggregated chi-square statistic for all cluster characteristics in our investigation. The best-fit model cumulative frequency distributions for eight cluster characteristics show an excellent agreement with the observed normalized frequency distributions (Fig. 3). For almost all characteristics, small discrepancies are explainable by statistical variations. The thin blue curves in Fig. 3 show 40 model frequency distributions obtained by subsampling from the full synthetic data-set to a sample size equivalent to the observational data-set.

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The model tends to produce clusters with a slightly larger number of craters than observed (Fig. 2c and Fig. 3b) and with slightly fewer large craters than observed (Fig. 3f). The latter discrepancy may suggest that a proportion of the meteoroid population produce stronger large fragments than tend to be produced by the SFM model used here.



Figure 3. Normalised cumulative frequency distributions of various crater cluster characteristics for observed crater clusters on Mars (grey) and simulated crater clusters (blue). The CFDs are for all clusters with  $N_c > 5$  and  $D_{\text{eff}} > 5.65$  m. Dark blue curve represents the CFDs of all the clusters in the full synthetic data set, which comprises 1000 clusters/crater with  $D_{\text{eff}} > 10$  m; light blue curves represent CFDs of 40 sub-samples of the full data set with approximately the same number of  $D_{\text{eff}} > 10$  m craters/clusters as observed (200).

The former discrepancy, on the other hand, is likely to be at least in part because of uncertainty in crater scaling combined with the inability to resolve all the smallest craters in an observed cluster image (Daubar et al., 2019; Neidhart et al., 2021). A factor of two uncertainty in the cohesion of the martian surface translates to a  $\approx 13\%$  uncertainty in crater diameter (Eq. 12). Increasing the minimum crater size threshold in the model by only 10-20% (equivalent to one pixel on a HiRISE image) is sufficient to bring the model into agreement with the observations.

As an additional test of the consistency between the simulation and observations, 402 we compared two-dimensional Kernel Density plots of the cluster characteristic distri-403 butions in various spaces (Figs. 4 and 5). While this comparison does not provide a quan-404 titative metric of mis-fit, it visually confirms that correlations between observed char-405 acteristics are replicated by the Monte Carlo simulation and that the model does not pro-406 duce clusters that are inconsistent with observations. In particular, the simulation repli-407 cates very well the variation in dispersion, number of craters, aspect ratio and large crater 408 fraction among clusters with an effective diameter of 5–10 m, and how this variation changes 409 as effective diameter increases. Also well replicated is the positive correlation between 410 effective diameter and number of craters and the negative correlation between the large 411 crater fraction  $f_L$  and effective diameter (Fig. 4). 412

Similarly, the variation in dispersion, aspect ratio and the exponent of the cumulative size-frequency distribution of craters in the cluster ( $\gamma$ ) with increasing number of craters and the correlations between these characteristics are all very consistent between the simulated and observed clusters (Fig. 5). One interesting trend that is present in the synthetic clusters, but not obviously apparent in the observational data is a negative correlation between dispersion and aspect ratio. In the synthetic data, clusters that are more circular tend to be slightly less dispersed than those clusters that are more elongated.



Figure 4. Kernel density comparison of various observed and simulated crater cluster characteristic distributions as a function of effective diameter  $D_{\text{eff}}$ .



Figure 5. Kernel density comparison of various observed and simulated crater cluster characteristic distributions as a function of number of craters in the cluster  $N_c$  or aspect ratio.

The number of samples required for a robust assessment of the model was determined by aggregating the chi-square statistic for the eight cluster characteristics shown in Fig. 3 and analyzing the change in this quantity, as well as the C-S ratio, with increasing sample size in the Monte Carlo simulation (Fig. 6). For each sample size, multiple samples were drawn from the complete synthetic data-set to quantify the variance in each

statistic, shown as error bars in Fig. 6. A synthetic sample size of 200 events with  $D_{\text{eff}} >$ 425 10 m, which is approximately equivalent to the observed sample size, is insufficient to 426 give a robust comparison between the model and observations; however, a sample size 427 of 1000 appears to be a good compromise between robustness and computational expe-428 dience. We note that as this sample size is greater than that of the observational data-429 set, the 200-sample variance of the C-S ratio and chi-square statistic are the most rel-430 evant measures of uncertainty between the simulation and observations for assessing model 431 sensitivity. 432



Figure 6. Convergence of the aggregate chi-square statistic for all cluster characteristics (a) and cluster-singular ratio  $(N_c > 5)$  (b) with increasing sample size. The number of samples on the x-axis is the number of simulated impact events that produce a single crater or cluster with  $D_{\text{eff}} > 10$  m (the equivalent sample size of observed clusters is approximately 200). The total number of impacts simulated is much larger.

#### 433 434

## 3.2 The influence of meteoroid properties and model parameters on cluster characteristics

Using the best-fit Monte Carlo simulation as a baseline, we then explored the sensitivity of model performance to various influential meteoroid and SFM parameters using orthogonal sampling (i.e., running additional Monte Carlo simulations, varying one parameter while setting all other parameters as the baseline).

The C-S ratio is most sensitive to the initial dynamic strength of the meteoroid, which is governed in the SFM by the median strength  $Y_{med}$  and the strength range parameter w (Fig. 7). Sensitivity analysis suggests that the optimum median strength is between 200 and 500 kPa and the width (in log-space) of the log-uniform distribution is between 0.75 and 1.5. This range of optimum meteoroid strength distribution parameters imply that in all scenarios consistent with observations the upper bound of the initial meteoroid strength distribution is 3–6 MPa, while the lower bound of the strength distribution ranges from 11–90 kPa.

<sup>447</sup> Optimization of the model also allows for the calibration of SFM parameters that <sup>448</sup> are unknown or not well constrained. The fit to cluster aspect ratio e is sensitive to the <sup>449</sup> lift coefficient  $C_L$  (Fig. 8a, b). A lift coefficient that is too low leads to clusters that are <sup>450</sup> too circular on average, while a lift coefficient that is too high leads to too many highly <sup>451</sup> elongated clusters compared with observations. To minimize the chi-square statistic for <sup>452</sup> cluster aspect ratio requires a lift coefficient of 0.015–0.02.



Figure 7. Cluster-singular ratio ( $N_c > 5$ ) as a function of (a) the median meteoroid strength  $Y_{\text{med}}$  (for a fixed width of the meteoroid strength distribution w); and (b) width of the meteoroid strength distribution w (for fixed median meteoroid strength  $Y_{\text{med}}$ ). Error bars represent the standard deviation in C-S ratio from 40 200-sample subsets of the best-fit model (Fig. 6).

The fit to dispersion  $(d_{\text{med}}, \text{ the median separation between all crater pairs in a clus$  $ter) is sensitive to the fragment separation coefficient <math>C_S$  (Fig. 8c, d). A  $C_S$  coefficient that is too low results in crater clusters that are not as dispersed in space as observed and vice-versa. To minimize the chi-square statistic for dispersion appears to require a  $C_S > 1$ , although a range of values give equally good performance.

The exponent of the cumulative size-frequency distribution of craters in the clus-458 ter ( $\gamma$ ) is influenced most sensitively by the strength-mass scaling parameter  $\delta$ , which 459 dictates the variation in child fragment strength relative to its parent (Fig. 8e, f). In par-460 ticular, the model performs very poorly for  $\delta = 0$ , which implies that the child frag-461 ment strength is always stronger than the parent by a factor related to the child/parent 462 mass ratio. In this case, the clusters tend to have crater size-frequency distributions that 463 are too steep on average, with a median  $\gamma \approx 3$ . If  $\delta$  is too large, on the other hand, the 464 clusters tend to have crater size-frequency distributions that are too shallow. To min-465 imize the chi-square statistic for  $\gamma$  we found  $\delta \approx 0.5$ , although a range of similar val-466 ues give good performance. This value of  $\delta$  implies that the random variation in child 467 fragment strength should be such that there is a 95% chance the fragment strength is 468 within a factor of 10 times lower than or higher than the nominal (mass-dependent) value. 469

The aspect ratio of the best-fit ellipse that encompasses most craters in the clus-470 ter has been proposed as a proxy for impact angle (Daubar et al., 2019). A simple ge-471 ometric model that projects a cloud of meteoroid fragments that is axially symmetric 472 about the main trajectory onto the surface to produce the cluster predicts a cluster as-473 pect ratio  $e = \sin \theta$ . However, while our model results confirm that aspect ratio cor-474 relates with impact angle, particularly for clusters with many craters, impacts at a given 475 angle can produce clusters with a wide range of aspect ratios (Fig. 9). Our results sug-476 gests that the simple geometric projection model is not a good predictor of impact an-477 gle, but does provide a reasonable lower bound for the range of possible impact angles 478 that could have formed the cluster. In other words, the observed cluster aspect ratio e479 can be used to preclude an impact angle less than  $\sin^{-1} e$ . 480



Figure 8. Chi-square statistic (left) and cumulative frequency distributions (right) for three crater cluster characteristics as a function of the most influential fragmentation model parameter. Aspect ratio as a function of the lift coefficient  $C_L$  (a), (b); dispersion as a function of fragmentation separation coefficient (c), (d); and the exponent of the cumulative size-frequency distribution of craters in the cluster ( $\gamma$ ) as a function of the strength-mass scaling parameter  $\delta$  (e), (f). Error bars in (a), (c), (e) represent the standard deviation in chi-square statistic for the given cluster characteristic from 40 200-sample subsets of the best-fit model (Fig. 6). Cumulative frequency distributions in (b), (d), (f) are for all clusters with  $N_c > 5$  and  $D_{\text{eff}} > 5.65$  m.

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## 3.3 Meteoroid properties as a function of outcome

The calibrated and optimized SFM provides insight into the probability distribution of initial meteoroid properties in the 15–10,000 kg mass range as a function of out-



Figure 9. Aspect ratio e of the best-fit ellipse that encompasses most craters in the cluster as a function of initial meteoroid entry impact angle to the horizontal  $\theta$ . Results shown are for the best-fit model described in the text and colored by  $\log_{10} N_c$ . The dotted line  $e = \sin \theta$  is a simple geometric model described in the text.

come (Fig. 10). In the synthetic data-set produced by the best-fit model, crater cluster
is the dominant outcome for meteoroid strengths less than 650 kPa, consistent with previous estimates (Williams et al., 2014), while single crater formation dominates for stronger
meteoroids. However, it is evident that some weak meteoroids can still produce a single crater and meteoroids as strong as 3 MPa will occasionally fragment to produce a
cluster.

Airbursts, which represent about 14% of the impacts simulated, are slightly more 490 likely among low-strength meteoroids. Among the other meteoroid properties, favorable 491 parameters for airburst are high velocity, high ablation parameter, low mass and shal-492 low angle impacts. Favorable scenarios for crater clusters are steeper angle impacts, with 493 low ablation parameter and moderate-to-low entry velocity, while formation of a single 494 crater is favored by impacts at intermediate angles and low entry speeds. In the mete-495 oroid mass range considered in our model the relative proportion of clusters craters and 496 airbursts is relatively independent of mass; however, extending the model to lower masses 497 would result in a transition to a predominance of single craters and then a predominance 498 of airbursts (Popova et al., 2003; Williams et al., 2014). 499



**Figure 10.** Probability distributions of meteoroid properties by outcome. Histograms of the relative frequency with which individual meteoroid properties result in a single crater, a crater cluster (meteoroid fragments in the atmosphere to form multiple craters) and an airburst (meteoroid is entirely ablated or forms no crater larger than 1-m diameter on the ground).

## 500 4 Discussion

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## 4.1 Constraints on meteoroid properties and SFM parameters

The dynamic meteoroid strength distribution used in our work that produces a syn-502 thetic cluster population that best matches the observed population is a log-uniform dis-503 tribution with a minimum strength of 10-90 kPa, a maximum strength of 3-6 MPa and 504 a median strength of 0.2-0.5 MPa. This strength range is very consistent with the in-505 ferred dynamic strengths of stony meteoroids observed to disrupt in Earth's atmosphere 506 (e.g., Ceplecha et al., 1998; Popova et al., 2011; Wheeler et al., 2018; Borovička et al., 507 2019, 2020). It is possible that other strength distributions may produce a cluster pop-508 ulation that is a similar or even better fit to observations, but they would have to have 509 the same (approximate) minimum, median and maximum values. 510

Investigating alternative strength distributions would be a fruitful avenue for fur-511 ther work. In particular, evidence from terrestrial meteoroid entry observations suggests 512 that the strength distribution and fragmentation behaviour of carbonaceous chondrites 513 and ordinary chondrites may differ (Borovička et al., 2020). Carbonaceous chondrite me-514 teoroids tend to exhibit a low initial dynamic strength of  $\sim 50$  kPa and then undergo 515 a relatively continuous sequence of fragmentation events until deceleration is complete 516 (Borovička et al., 2019). Ordinary chondrites, on the other hand, often exhibit a two-517 stage fragmentation process, which may suggest that these meteoroids possess a bi-modal 518 internal strength (Borovička et al., 2020). While most such meteoroids experience an ini-519 tial break-up at a dynamic strength of  $\sim 100$  kPa, for a substantial proportion of events 520 there is then a hiatus in fragmentation until the ram pressure exceeds  $\sim 1$  MPa. This 521 has been interpreted as evidence that many ordinary chondrite meteoroids possess two 522 strengths: a very low bulk strength that may be related to weak cementation of fragments 523 and a stronger internal strength that is required to break the (partially fragmented) frag-524 ments themselves (Borovička et al., 2020). 525

It is possible that a bi-modal meteoroid strength distribution may improve the agree-526 ment between model results and observations. Fewer meteoroids with an intermediate 527 strength may reduce the number of meteoroids that fragment only once to produce two 528 craters, which is a clear deficiency of the model (Fig. 2). Similarly, this may also reduce 529 the number of clusters with a large number of craters without at the same time reduc-530 ing the proportion of meteoroids that produce clusters compared with singular craters, 531 producing a better match to the observations. Modifying the SFM to account for a two-532 phase fragmentation process may also improve the model in regards to the number of 533 large craters within the cluster. This could be achieved, for example, by representing the 534 meteoroid with two structural components (e.g., Wheeler et al., 2018; Borovička et al., 535 2020)—a weaker component that represents the fragile bonding of the bulk of the me-536 teoroid and a stronger component that represents large fragments within the meteoroid 537 that contain internal fractures but are more-or-less intact. Such a meteoroid would un-538 dergo one phase of fragmentation at high altitude to form a well-separated cluster, and 539 a second fragmentation phase much closer to the ground that would tend to produce larger 540 craters. 541

Apart from the dynamic strength distribution of impacting meteoroids, the suc-542 cess of the model employed here in replicating cluster formation on Mars provides in-543 sight into several important aspects of the model and its parameters. Given the simplic-544 ity of the discrete fragmentation model with pair-wise separation that we employ, the 545 excellent agreement with observations suggests that more elaborate models may not be 546 necessary to explain atmospheric break-up of stony meteorites on Mars. On the other 547 hand, our analysis shows that a critical feature of the model is that the dynamic strength 548 of child fragments must be highly variable and not always greater than the strength of 549 the parent. This seems to be more important than any inverse correlation between frag-550 ment size and strength. In other words, the first fragmentation event must produce a 551

variety of scenarios that include (a) one or both child fragments being weaker than the
original fragment, resulting in immediate further break-up; and (b) one or both child fragments being substantially stronger than the original fragment, and hence not undergoing any subsequent fragmentation. The importance of this aspect of the model suggests
that alternative fragmentation models that account for heterogeneous internal strength
(e.g., Wheeler et al., 2018) may also be very successful at replicating crater clusters on
Mars.

The dimensionless lift coefficient  $(C_L, \text{Eq. 3})$  that relates the change of trajectory 559 angle during flight to the differential force of the air flowing around the meteoroid is a 560 poorly constrained model parameter that is often either neglected (Register et al., 2017; 561 Wheeler et al., 2017, 2018) or assumed to be a very small value (e.g.,  $C_L < 10^{-3}$ ; Passey 562 & Melosh, 1980). While this parameter has very little influence on most cluster char-563 acteristics, a good match to the observed distribution of cluster aspect ratios requires 564 a lift coefficient  $\sim 10^{-2}$ . This new constraint on the lift coefficient may be useful for im-565 proving predictions of meteorite strewn fields that are critical for successful recovery ef-566 forts on Earth following large fireballs. 567

The optimal fragment separation coefficient ( $C_S \sim 1-2$ , Eq. 9), which relates the 568 separation speed of fragments to the along-trajectory speed at break-up, is substantially 569 larger than the commonly used estimate of  $C_S \approx 0.13$  based on theoretical consider-570 ation of bow-shock interaction as well as shock physics modeling of interaction between 571 two identical fragments and atmosphere (Artemieva & Shuvalov, 2001). It is closer to, 572 but larger still than the estimate of  $C_S \approx 0.67$  obtained from shock physics modeling 573 of atmospheric interaction of a fragmented chondritic asteroid comprising 13–27 iden-574 tical fragments (Artemieva & Shuvalov, 2001). It is also at the upper bound of previ-575 ous empirical estimates based on the separation of craters in terrestrial strewn fields and 576 martian clusters (Passey & Melosh, 1980; Popova et al., 2007). 577

On the other hand, the implied separation speeds of our successful model are broadly consistent with observations of terrestrial bolides (Popova et al., 2007). For a typical breakup altitude on Mars of 10 km (where air density is approximately equivalent to 40 km altitude on Earth), the separation speeds of equal sized fragments implied by our bestfit model are in the range 0.2–0.3% of the pre-entry velocity. This is comparable with the median separation velocity inferred from detailed observations of the Morávka meteorite fall, where break-up occurred at an altitude of 30–45 km altitude, although maximum relative separation speeds of up to 1.5% were observed (Boroviĉka & Kalenda, 2003).

Taken together, the implied high separation speed may indicate that a supplementary separation mechanism, such as explosive release of volatiles or volumetric evaporation (Nemtchinov et al., 1999; Popova et al., 2007), acts to push the fragments apart more quickly than bow-shock interaction alone.

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## 4.2 Implications for the rate of small impacts on Mars

A subset of 44 recent impact sites imaged before and after impact by Mars Recon-591 naissance Orbiter's Context Camera have been used to derive an estimate of the current 592 rate of small impacts on Mars that accounts for the area and time period over which de-593 tection was possible (Daubar et al., 2013). A later update using 110 impacts and an updated area-time factor resulted in a nearly identical rate (Daubar et al., 2014). The es-595 timated cumulative production rate of craters with an effective diameter larger than 10 m 596  $(N_{>10})$  is  $4.06 \times 10^{-7}$  craters per km<sup>2</sup> per year, which is a factor of three to five lower than 597 the crater production models of (Hartmann, 2005) and (Neukum et al., 2001), respec-598 tively, based on the formation rate of much larger lunar craters. 599

The impactor mass-frequency distribution employed in the Monte Carlo model used here is based on fireball observations on Earth (Halliday et al., 1996; Bland & Artemieva,

2006). Comparison between the implied cratering rate of the best-fit model and the in-602 ferred cratering rate from observations therefore represents an independent test of the 603 small impact rate on Mars (Williams et al., 2014). The best-fit model can be used to es-604 timate the small impact cratering rate on Mars if two important parameters are known: 605 (a) the Earth/Mars impact flux ratio  $R_{ME}$ ; and (b) the proportion of impactors that 606 are of asteroidal origin  $f_A$  (i.e., are not cometary). As discussed above, cometary me-607 teoroids are not considered in the Monte Carlo model because they very rarely form craters 608 or clusters in our model. This implies that the fraction of cometary impacts at Mars is 609 not possible to constrain with observations of this sort. 610

A predicted production rate of craters or clusters on Mars with an effective diam-611 eter greater than 10 m,  $N_{>10}$ , is obtained from the cratering rate from the best-fit model 612 times the product  $f_A R_{ME}$  (Fig. 11b). To match the cratering rate inferred from obser-613 vations of  $N_{>10} = 4.06 \times 10^{-7}$  (Daubar et al., 2014) requires  $f_A R_{ME} \approx 0.5$ , shown by 614 the thick line in Fig. 11b. When scaled by this factor, a comparison between the pre-615 dicted cumulative size-frequency distribution of craters/clusters and the observed cumu-616 lative size-frequency distribution of craters/clusters in the recent catalog (Daubar et al., 617 2021) shows good agreement for craters and clusters larger than 8-m effective diameter 618 (Fig. 11). 619

The slope of the cumulative size-frequency distribution CSFD of the observed craters 620 and clusters is  $2.39\pm0.07$  ( $2.38\pm0.05$  for clusters alone) for effective diameters > 8 m. 621 The equivalent CSFD slope for the model is  $2.52\pm0.04$  ( $2.41\pm0.06$ ) in reasonable agree-622 ment with observations, particularly with respect to clusters. An equivalent Monte Carlo 623 simulation with no atmosphere shows the important effect of atmospheric filtering on 624 the CSFD slope (dashed line, Fig. 11). The CSFD slope of the synthetic crater popu-625 lation with no atmosphere is  $2.94 \pm 0.07$ , which is more than 15-20% steeper than the 626 CSFDs of the best-fit Monte Carlo simulation and observed craters. The effects of at-627 mospheric entry reduce the cumulative number of craters with effective diameter greater 628 than 10 m by more than a factor of two relative to the airless case. 629

Estimates of the Mars/moon impact flux ratio based on dynamical models range from 2.04 to 3.20 (Ivanov, 2001; Le Feuvre & Wieczorek, 2011; Marchi, 2021). Adopting an Earth/Moon impact flux ratio of 1.58 (Le Feuvre & Wieczorek, 2011) implies a Mars/Earth flux ratio in the range 1.3–2 (blue region; Fig. 11b).

Terrestrial fireball observations suggest that the proportion of the impactor flux 634 in the 1-10 kg mass range that is of asteroidal origin may be in the range 65-80% (Halliday 635 et al., 1996; Ceplecha et al., 1998). Assuming that the composition of the impactor flux 636 is the same at Mars and Earth, the combined constraints on  $R_{ME}$  and  $f_A$  suggest a Mars 637 small crater production rate that is 1.5 to 4 times higher than the observation-based es-638 timate of Daubar et al. (2014), which could reflect the number of current impacts miss-639 ing from the observational data set. This range encompasses the production rate pro-640 posed by Williams et al. (2014), based on a similar modeling approach, and the crater 641 production model of (Hartmann, 2005). 642

<sup>643</sup> On the other hand, to match the cratering rate inferred from observations of  $N_{>10} =$ <sup>644</sup> 4.06×10<sup>-7</sup> (Daubar et al., 2014) requires either a Mars-Earth impact flux ratio less than <sup>645</sup> one or a much higher proportion of cometary (low density) impactors at Mars than the <sup>646</sup> proportion inferred at Earth.

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## 4.3 Implications for the seismic detection of crater clusters on Mars

A well calibrated model of crater cluster formation allows us to empirically relate impactor properties of interest to cluster properties. In future, this may be useful for interpreting newly formed impacts on Mars and their seismic detectability. For example, as the effective diameter of the crater correlates very well with the vertical component



Figure 11. The estimated rate of small impacts on Mars. (a) A comparison of the simulated cumulative size-frequency distribution of small impact craters and crater clusters on Mars per square km per year with an estimate based on observed craters and clusters (Daubar et al., 2014) (D2014). Also shown is the corresponding model outcome for no atmosphere. (b) The predicted cumulative number of craters or clusters with effective diameter larger than 10 m as a function of the fraction of the impact flux that is asteroidal and the Mars/Earth impact flux ratio. The symbols in (a) show the D2014 estimated annual cumulative size-frequency distribution of small craters. The thick curves in (a) show the CFD of craters and clusters from Neidhart et al. (2021) and Daubar et al. (2021) scaled to match the D2014 cumulative cratering rate for craters larger than 10 m. The thin curves in (a) show the CFD of synthetic craters and clusters scaled to match the observed CFD for craters/clusters with an effective diameter larger than 10-m, which requires a  $f_A$ - $R_{ME}$  combination denoted by the thick line in (b). In (b) the blue shaded region denotes the range of proposed Mars/Earth impact flux ratios; the green region indicates an approximate estimate of the fraction of impact flux that is asteroidal, based on fireball observations.

of the total momentum of all fragments impacting the ground (Fig. 12a), it serves as an 652 excellent predictor of the total seismic moment of the impact, regardless of whether it 653 forms a cluster or a single crater (Schmerr et al., 2019; Wójcicka et al., 2020). Our re-654 sults also provide insight into the median time interval between fragments striking the 655 ground (Fig. 12b), which is important for the frequency content of impact generated seis-656 mic signals (Schmerr et al., 2019). For the size range of impacts in our model, which are 657 those most relevant for possible seismic detection by NASA's InSight lander (Daubar et 658 al., 2018), the time interval between fragment impacts are typically less than one sec-659 ond, although for some highly dispersed clusters the time interval can be as long as 10 660 seconds. 661

It is also evident from analysis of our model results that a cluster and a single crater of the same effective diameter can differ dramatically in terms of their initial meteoroid properties (Fig. 13) because fragmented meteoroids undergo substantially greater deceleration and ablation in the atmosphere. While the fragments that produce the largest craters within a cluster undergo the same degree of deceleration as meteoroids that form single craters (Fig. 13a), the more abundant smaller fragments are decelerated much more dramatically (Fig. 13b). Consequently, the total meteoroid mass that strikes the ground



Figure 12. The total vertical impactor momentum for single craters and crater clusters (a) and the median time separation between fragment impacts (b) as a function of effective crater diameter for the distribution of impact scenarios from the best-fit simulation.

to form a crater cluster of a given effective diameter is typically a much smaller proportion of the initial meteoroid mass than is the case for the equivalent size single crater (Fig. 13c). This also implies that newly formed crater clusters represent excellent targets for meteorite detection and recovery on Mars.

## **5 Conclusions**

The Separate Fragments Model of meteoroid fragmentation during atmospheric en-674 try involves the sequential, pair-wise division of the meteoroid into numerous fragments 675 and their subsequent separation, deceleration and ablation before striking the ground. 676 In this paper, we have shown that this model is able to reproduce the observed popu-677 lation of crater clusters on Mars in terms of all their observed characteristics. An im-678 portant feature of the successful fragmentation model is that each fragmentation event 679 is able to produce child fragments with highly variable dynamic strength that can be both 680 much stronger and much weaker than the strength of the parent fragment. 681

Our work assumed a log-uniform distribution of meteoroid strengths between an upper and lower bound. To best reproduce the observed distribution of crater clusters, including the ratio of clusters to single craters, required a meteoroid strength distribution with a minimum strength of 10–90 kPa, a maximum strength of 3–6 MPa and a median strength of 0.2–0.5 MPa. This range is very consistent with the inferred dynamic strengths of terrestrial fireball meteoroids.

Optimization of the model also places new constraints on the separation speed of 688 fragments after break-up and the lift coefficient that relates the change of trajectory an-689 gle during flight to the differential force of the air flowing around the meteoroid fragments. 690 To match the observed distribution of cluster aspect ratios requires a lift coefficient  $\sim$ 691  $10^{-2}$ , which is several times higher than previous estimates. Similarly, to match the ob-692 served median separation distance between crater pairs (dispersion) requires fragment 693 separation speeds of 0.2-0.3% of the pre-entry meteoroid velocity. Such speeds are at the 694 upper end of previous model estimates but are comparable with the median separation 695 velocity inferred from detailed observations of terrestrial meteorite falls. 696



Figure 13. A comparison of deceleration and ablation between cluster and single-crater impacts. The ratio of the maximum fragment velocity to the pre-entry velocity (a); the ratio of the median fragment velocity to the pre-entry velocity (b); and the ratio of the final mass to the pre-entry mass (c) for both cluster-forming and crater-forming impacts as a function of effective diameter.

The calibrated model suggests that the rate of small impacts on Mars is 1.5–4 times 697 higher that current observation-based estimates (Daubar et al., 2013, 2014). Lower small 698 crater production rates require a significantly lower Mars/Earth flux ratio than current 699 estimates based on dynamical models (Ivanov, 2001; Le Feuvre & Wieczorek, 2011) or 700 an unexpectedly high (>50%) proportion of low-density, cometary meteoroids among the 701 Mars impactor flux. It therefore seems likely that a substantial proportion of meter-to-702 decameter craters forming on Mars are not being detected with current orbital imaging 703 techniques. 704

The success of the Separate Fragments model in reproducing the diversity of observed clusters suggests that future efforts to infer impactor properties from observed cluster characteristics by inversion may be fruitful. The formation of crater clusters is favored by low-to-moderate impact speeds, low meteoroid strengths and relatively steep

- <sup>709</sup> trajectories to the surface. Newly formed crater clusters represent excellent targets for
- <sup>710</sup> meteorite detection and recovery on Mars.

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- <sup>714</sup> cloud. The Monte Carlo code, post-processing code and synthetic data are all available
- at https://github.com/gsc10/Mars-clusters (DOIs for these repos will be minted prior
- to publication). The observational data is available as a supplement to (Neidhart et al.,
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