Resolving the Force Term of the Electron Vlasov Equation with MMS

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Abstract

Widely employed to model collisionless plasma phenomena occurring naturally in Earth's magnetic environment, throughout the heliosphere, and in laboratory fusion devices, the Vlasov equation self-consistently describes the fundamental kinetic dynamics of plasma particles as they are accelerated through phase space via electric and magnetic forces. The Fast Plasma Investigation (FPI) onboard NASA's Magnetospheric Multiscale (MMS) four-spacecraft mission sufficiently resolves the seven spatial, temporal, and velocity-space dimensions of phase space needed to directly observe terms in the Vlasov equation, as recently demonstrated by *Shuster et al.* [2021] in the context of electron-scale current layers at the reconnecting magnetopause. These results motivate novel exploration of the types of distinct kinetic signatures in [?]f_e/[?]t, v[?][?]f_e, and (F/m_e)[?][?]$_\mathrm{v}f_e$ which are associated with the magnetic reconnection process, where F = -e(E + v×B) represents the Lorentz force on an electron, and f_e specifies the electron phase space density. We apply this approach to characterize the structure of the velocity-space gradient terms in the electron Vlasov equation measured by MMS. Discussion of the uncertainties which arise when computing the velocity-space gradients of the FPI phase space densities is presented, along with initial validation of the (F/m_e)[?][?]$_\mathrm{v}f_e$ measurements by comparison to the [?]f_e/[?]t and v[?][?]f_e terms. Successful measurement of the force term (F/m_e)[?][?]$_\mathrm{v}f_e$ in the Vlasov equation suggests a new technique for inferring spatial gradients from single spacecraft measurements which may be applied to improve the spatial resolution of the electron pressure divergence [?][?]P$_\mathrm{e}$ necessary to understand the microphysics of the electron diffusion region of magnetic reconnection. Reference: Shuster, J. R., et al. (2021), Structures in the terms of the Vlasov equation observed at Earth's magnetosphere, *Nature Physics*, doi:10.1038/s41567-021-01280-6.

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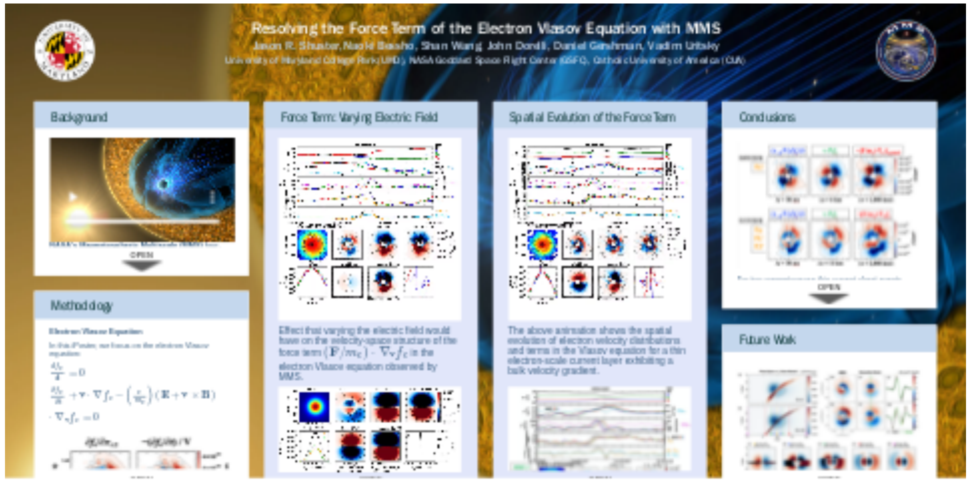

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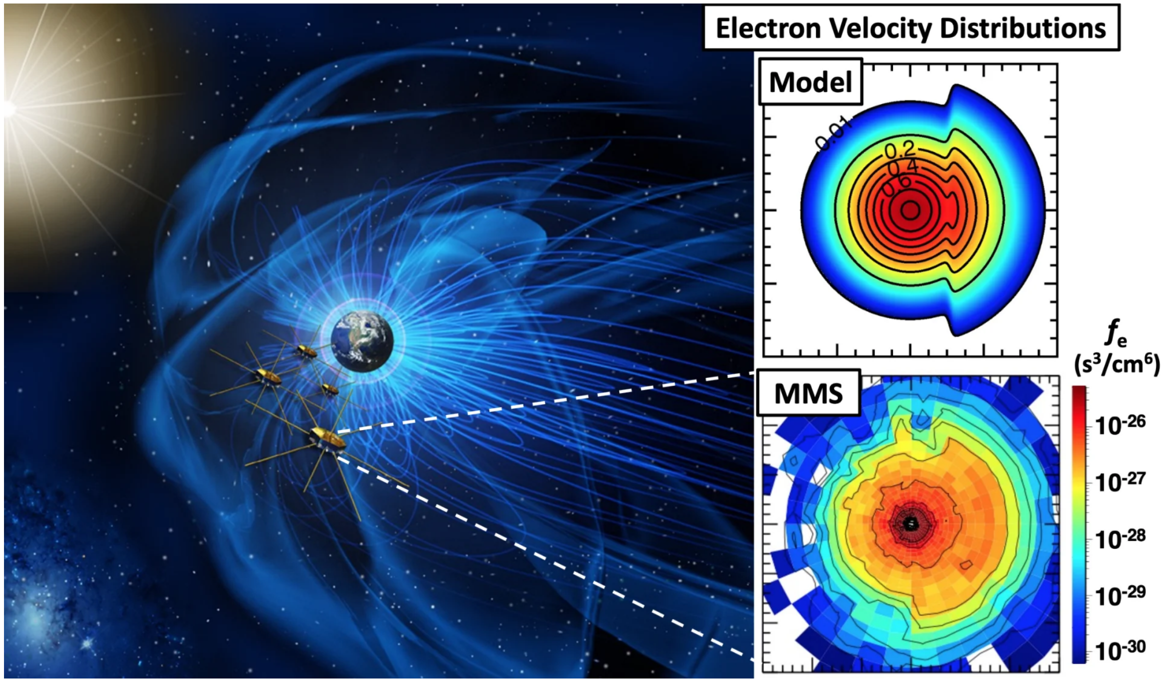
PRESENTED AT:

BACKGROUND

[VIDEO] https://res.cloudinary.com/amuze-interactive/video/upload/vc_auto/v1638818428/agu-fm2021/9B-F6-2A-9D-4D-04-23-0F-2D-C9-A6-C4-E0-99-DA-4A/Video/12901_TurbulentMagnetosheath_appletv_mp4d16.mp4

NASA's Magnetospheric Multiscale (MMS) four-spacecraft mission has recently detected very thin, electron spatial-scale current layers embedded within Earth's dayside magnetopause, the boundary between the turbulent magnetosheath (gold) and inner magnetosphere (blue) regions depicted in the above movie [*video credit*: Conceptual Animation: Exploring Turbulent Space Around Earth (https://svs.gsfc.nasa.gov/12901)].

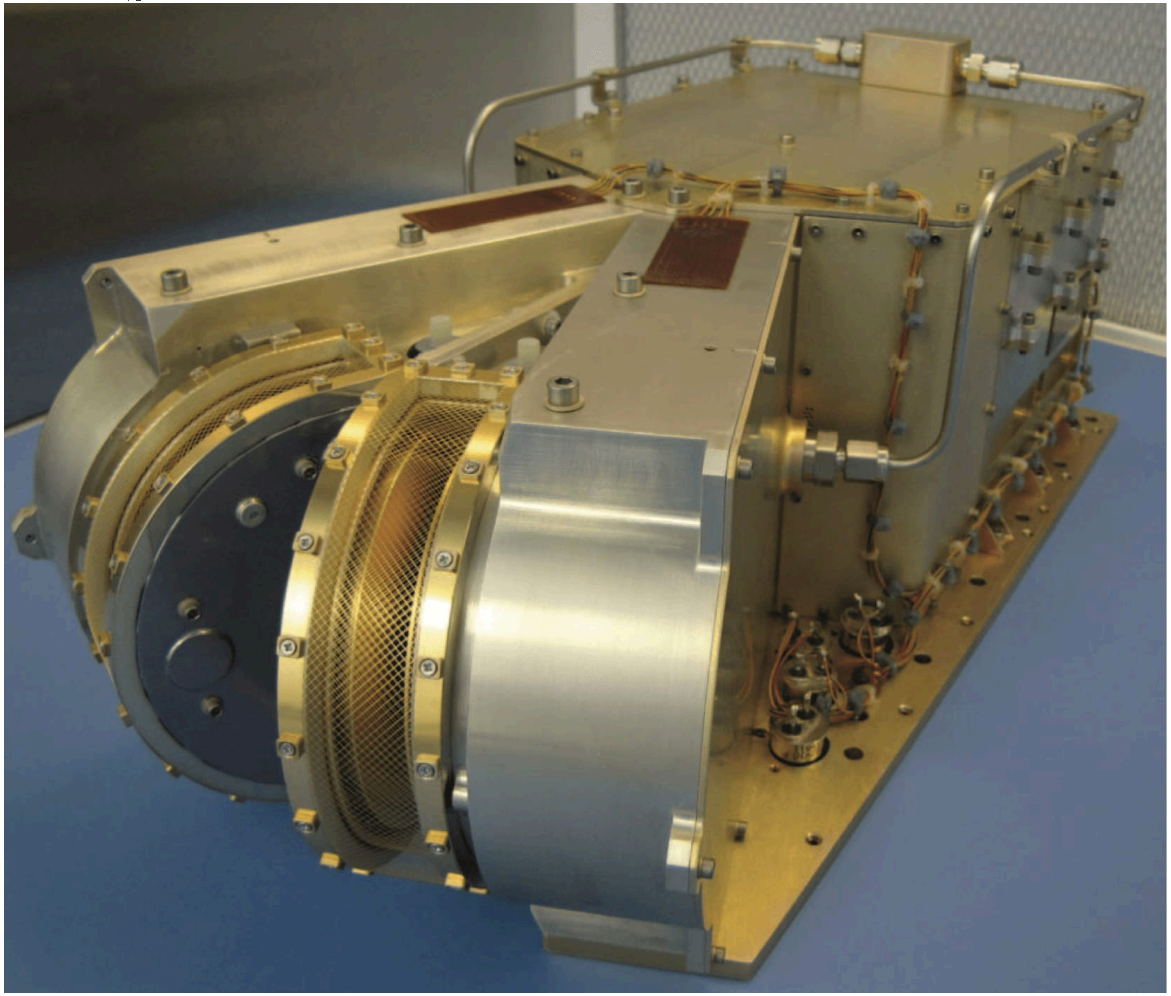


Within these thin, asymmetric electron-scale current layers, the electron velocity distribution sometimes exhibits a discernible crescent-shaped structure in velocity space (as illustrated above), especially when there is a strong electron temperature gradient across the layer [*Shuster et al.*, 2019 (https://doi.org/10.1029/2019GL083549)] [*Shuster et al.*, 2021 (https://doi.org/10.1063/5.0069559)].

Image credit: NASA artist's rendition of MMS (https://www.nasa.gov/content/goddard/mms-in-space-concept).

Fast Plasma Investigation (FPI)

With the unprecedented spatiotemporal resolution offered by the MMS mission's Fast Plasma Investigation (FPI) [*Pollock et al.*, 2016 (https://doi.org/10.1007/s11214-016-0245-4)], space scientists can now study the kinetic properties of ion and electron velocity distributions in remarkable detail. The 64 dual ion spectrometers (DIS) and dual electron spectrometers (DES) of the FPI instrument suite measure full 3D plasma distributions at time cadences of 150 ms for ions and 30 ms for electrons, respectively. These rapid measurement speeds, combined with the record-breaking close flying formation of the four MMS observatories, enables the direct measurement of terms in the Vlasov equation for the first time in the history of plasma physics research [*Shuster et al.*, 2021, Nature Physics (https://doi.org/10.1038/s41567-021-01280-6)].



One of the many Dual Electron Spectrometer (DES) units onboard the MMS spacecraft tetrahedron.

Image credit: [*Collinson et al.*, 2012 (https://doi.org/10.1063/1.3687021), Figure 1].

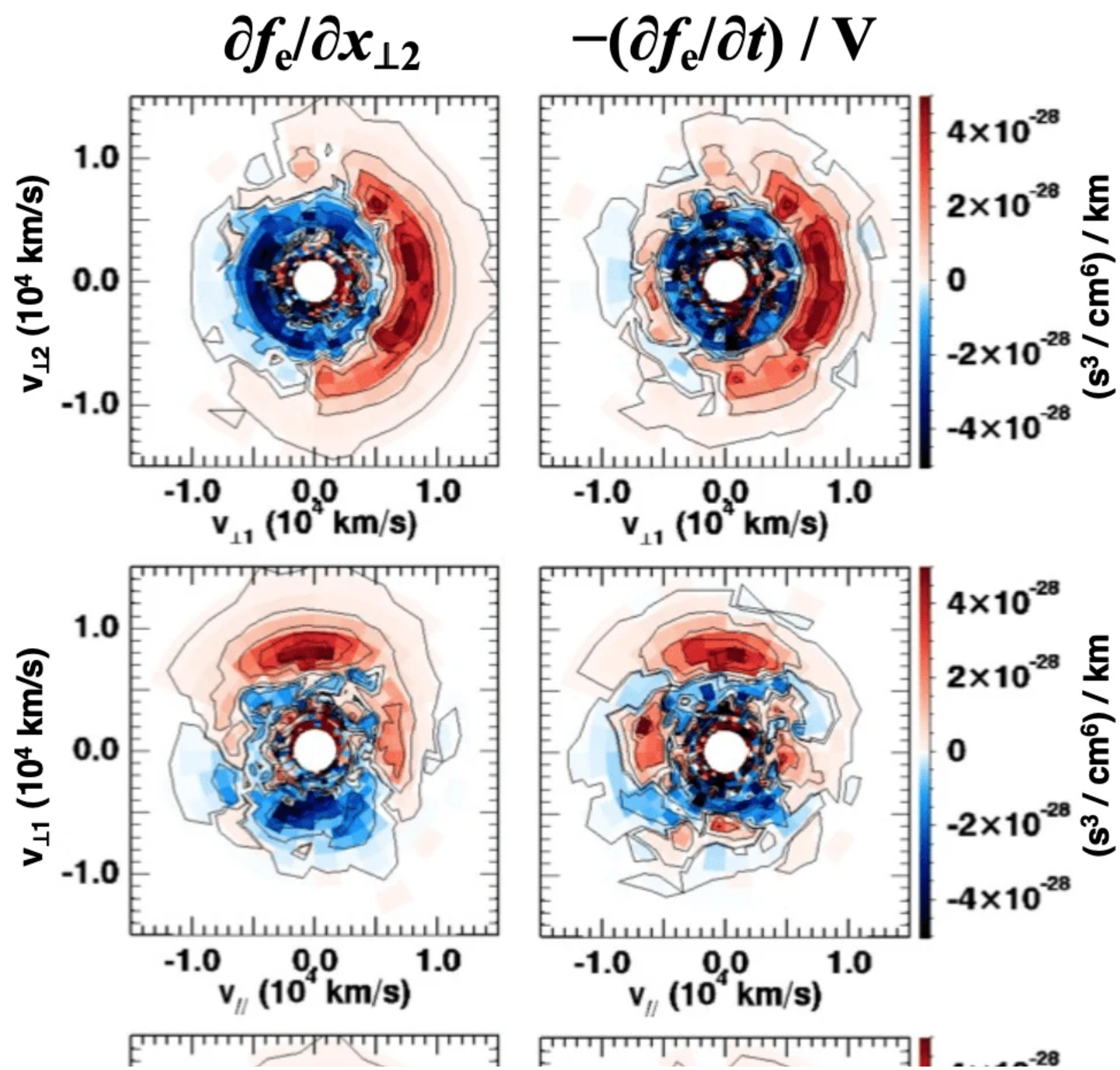
METHODOLOGY

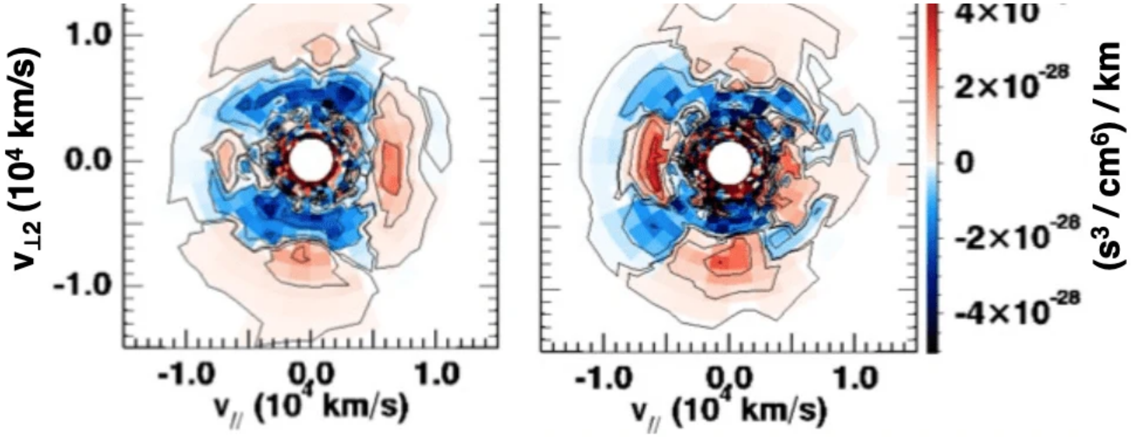
Electron Vlasov Equation

In this iPoster, we focus on the electron Vlasov equation:

$$\frac{df_e}{dt} = 0$$

$$\frac{\partial f_e}{\partial t} + \mathbf{v} \cdot \nabla f_e - \left(\frac{e}{m_e}\right)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_e = 0$$





For quasi-steady-state current layers moving quickly past the MMS spacecraft, the following approximation is valid:

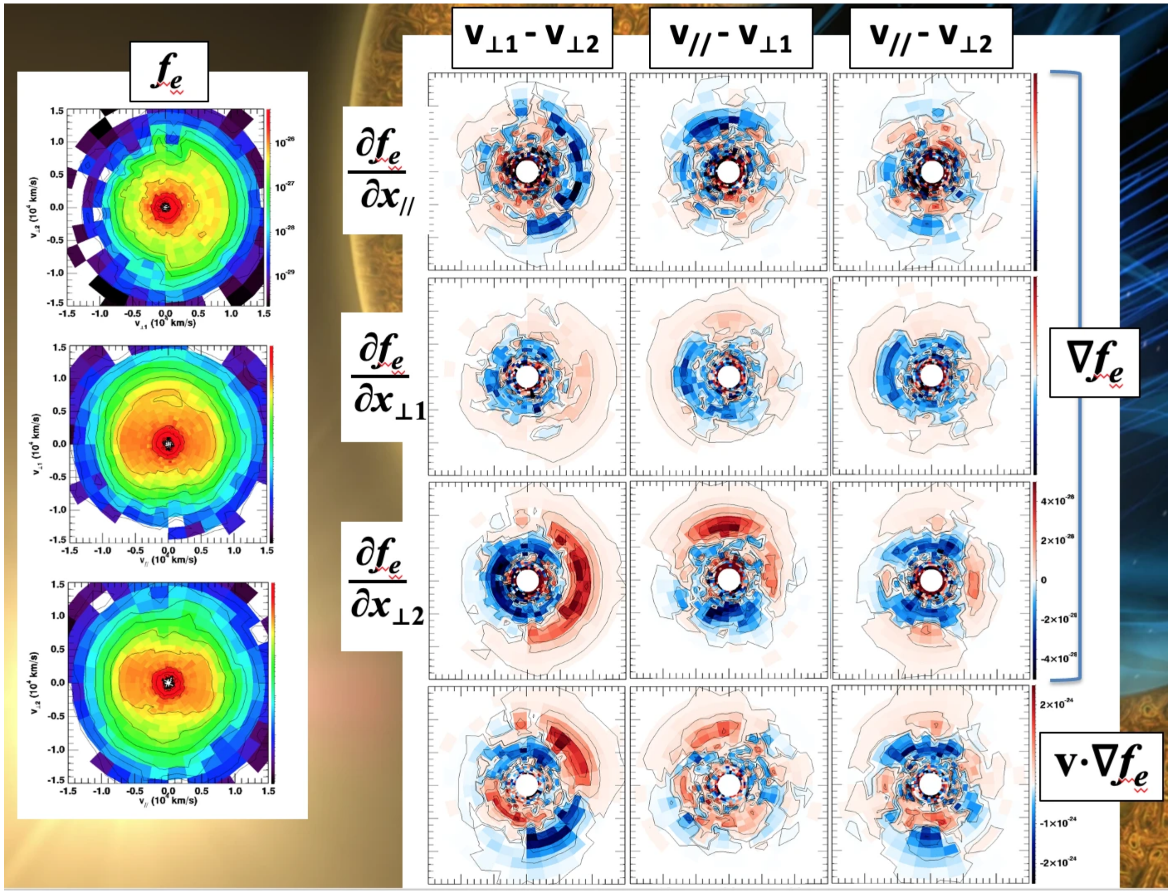
$$\frac{Df_e}{Dt} = \frac{\partial f_e}{\partial t} + \mathbf{V}_s \cdot \nabla f_e \approx 0$$

where Df_e/Dt is the convective derivative (*i.e.*, a time derivative measured in a frame moving at velocity $\mathbf{V}_s$ relative to the spacecraft frame), and where $\mathbf{V}_s$ is the structure velocity. Consequently, the time derivative term $\partial f_e/\partial t$ measured in the MMS frame is much smaller than the spatial gradient term $\mathbf{v} \cdot \nabla f_e$ (where here $\mathbf{v}$ is the velocity-space coordinate) throughout most of the electron's velocity space. This is because the speed of the structure moving past the spacecraft V_s (typically about 50 to 100 km/s) is significantly less than the electron thermal velocity v_{th} (typically on the order of 1,000 to 5,000 km/s) for a typical magnetosheath electron: $V_s << v_{th}$. Therefore, the following approximations can be made:

$$\frac{\partial f_e}{\partial t} \approx -\mathbf{V}_s \cdot \nabla f_e << \mathbf{v} \cdot \nabla f_e$$

where again the structure velocity $\mathbf{V}_s$ is not to be confused with the velocity-space coordinate $\mathbf{v}$.

Thus, when measuring terms in the MMS spacecraft frame, although the time derivative term is in general nonzero, it is negligible throughout most of the measured velocity space when compared to the spatial gradient term $\mathbf{v} \cdot \nabla f_e$ and force term $(\mathbf{F}/m_e) \cdot \nabla_{\mathbf{v}} f_e$. Structures in the spatial gradient term $\mathbf{v} \cdot \nabla f_e$ have been characterized as signatures of fundamental gradients in density (∇n_e), bulk velocity (∇U_e), and temperature (∇T_e) at the magnetopause [*Shuster et al.*, 2021 (https://doi.org/10.1038/s41567-021-01280-6)]. Here in this iPoster, we present direct measurements of the force term $(\mathbf{F}/m_e) \cdot \nabla_{\mathbf{v}} f_e$, which is expected to visually match and balance with the $\mathbf{v} \cdot \nabla f_e$ structures for a purely collisionless plasma that obeys the Vlasov equation.



The above figure shows each of the velocity-space slices of the full vector gradient term ∇f_e and $\mathbf{v} \cdot \nabla f_e$ distributions which are accessible to the FPI spectrometers.

FORCE TERM: VARYING ELECTRIC FIELD

[VIDEO] https://res.cloudinary.com/amuze-interactive/image/upload/f_auto,q_auto/v1638668991/agu-fm2021/9b-f6-2a-9d-4d-04-23-0f-2d-c9-a6-c4-e0-99-da-4a/image/mms_scane_l7y9ai.mp4

Effect that varying the electric field (from 0 to 20 mV/m) would have on the velocity-space structure of the force term $(\mathbf{F}/m_e) \cdot \nabla_{\mathbf{v}} f_e$ in the electron Vlasov equation observed by MMS.

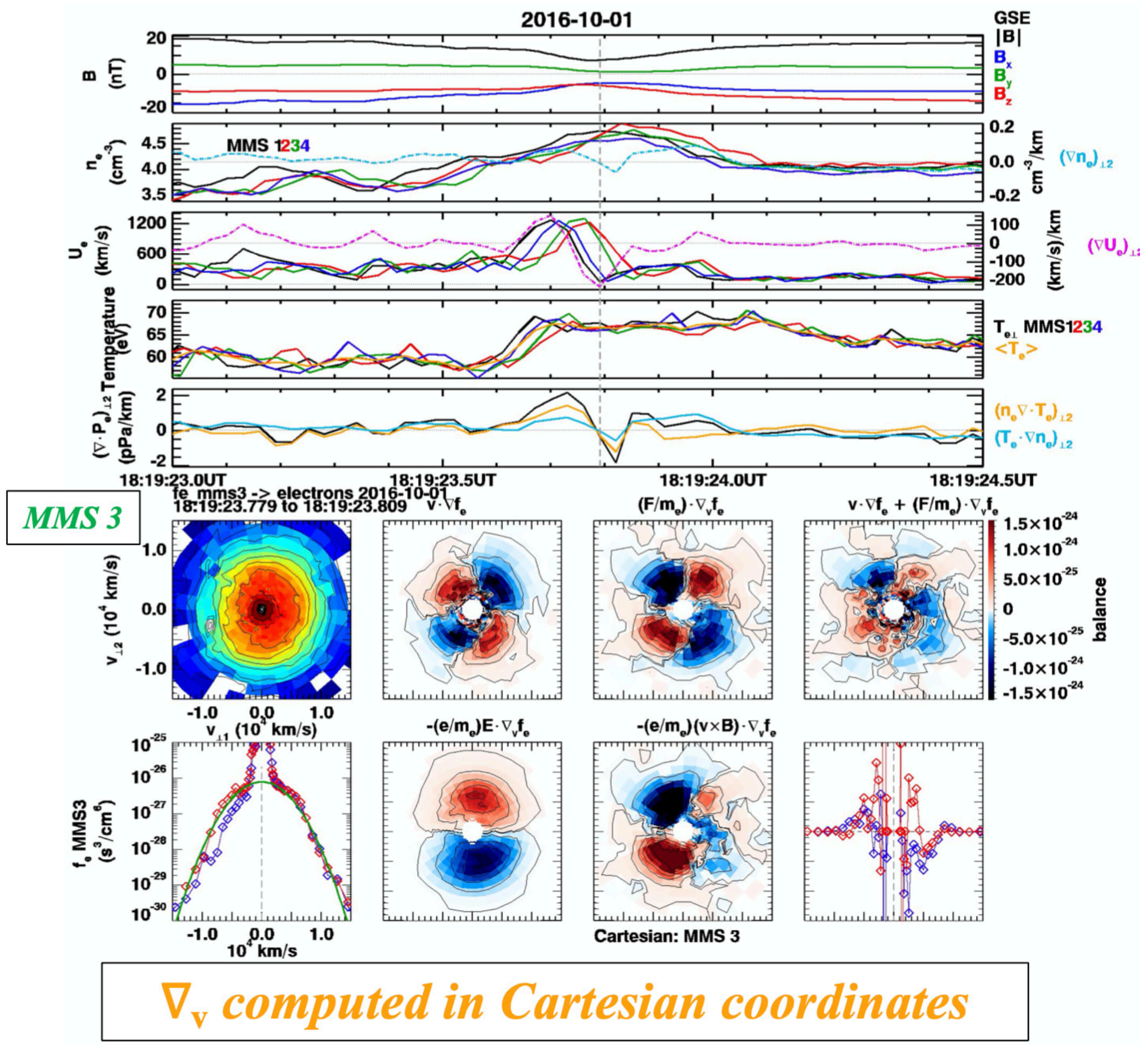
[VIDEO] https://res.cloudinary.com/amuze-interactive/image/upload/f_auto,q_auto/v1638668955/agu-fm2021/9b-f6-2a-9d-4d-04-23-0f-2d-c9-a6-c4-e0-99-da-4a/image/maxwellian_scane_w86tqh.mp4

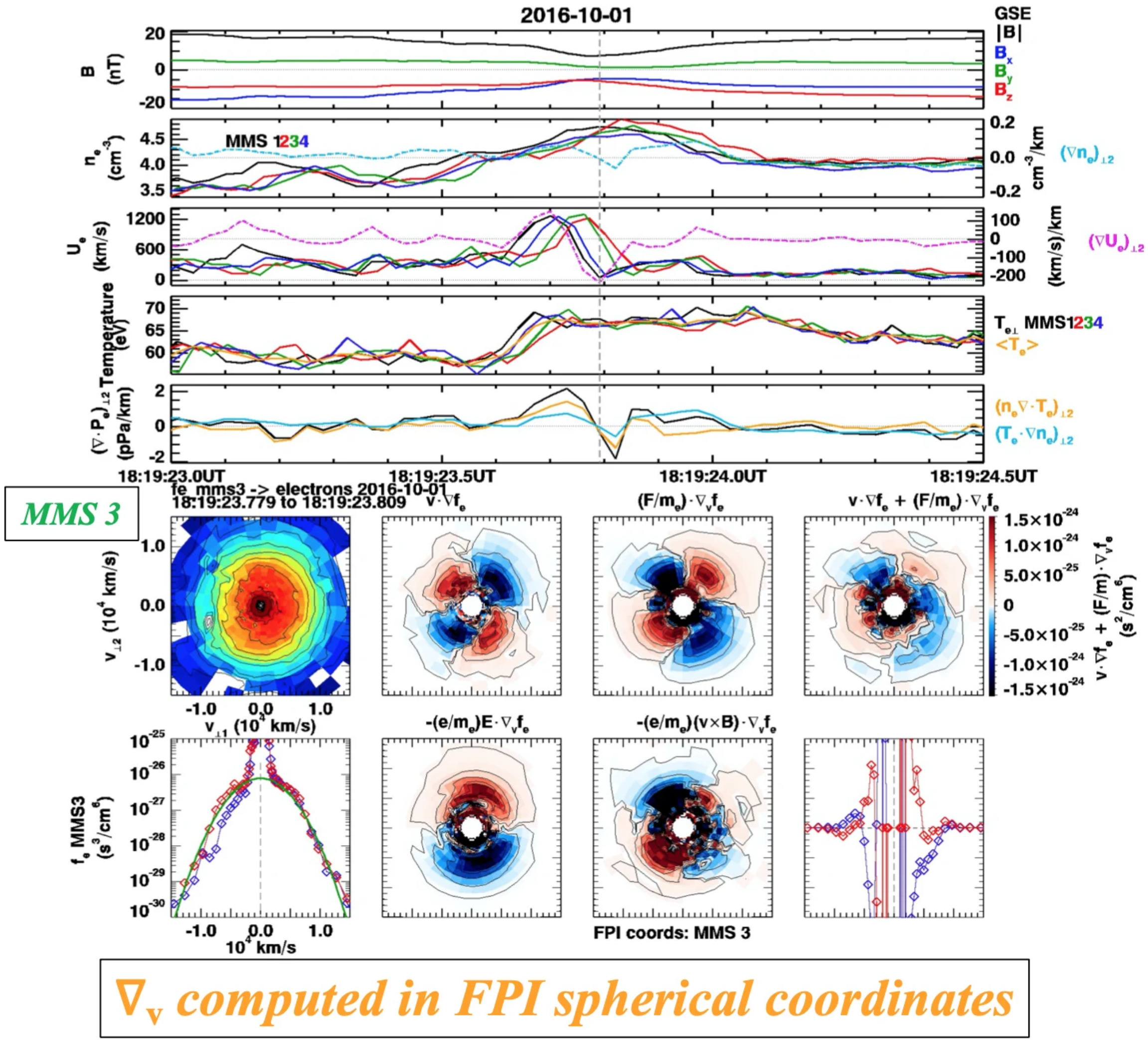
Analogous animation to the above MMS data where the electric field varies, here highlighting an intuition based on a distribution comprised of two Maxwellian populations of differing bulk velocity and temperature (with the electric field ranging from 0 to 16 mV/m).

SPATIAL EVOLUTION OF THE FORCE TERM

[VIDEO] https://res.cloudinary.com/amuze-interactive/image/upload/f_auto,q_auto/v1638669048/agu-fm2021/9b-f6-2a-9d-4d-04-23-0f-2d-c9-a6-c4-e0-99-da-4a/image/mms_gradu_scan_time_rfs09m.mp4

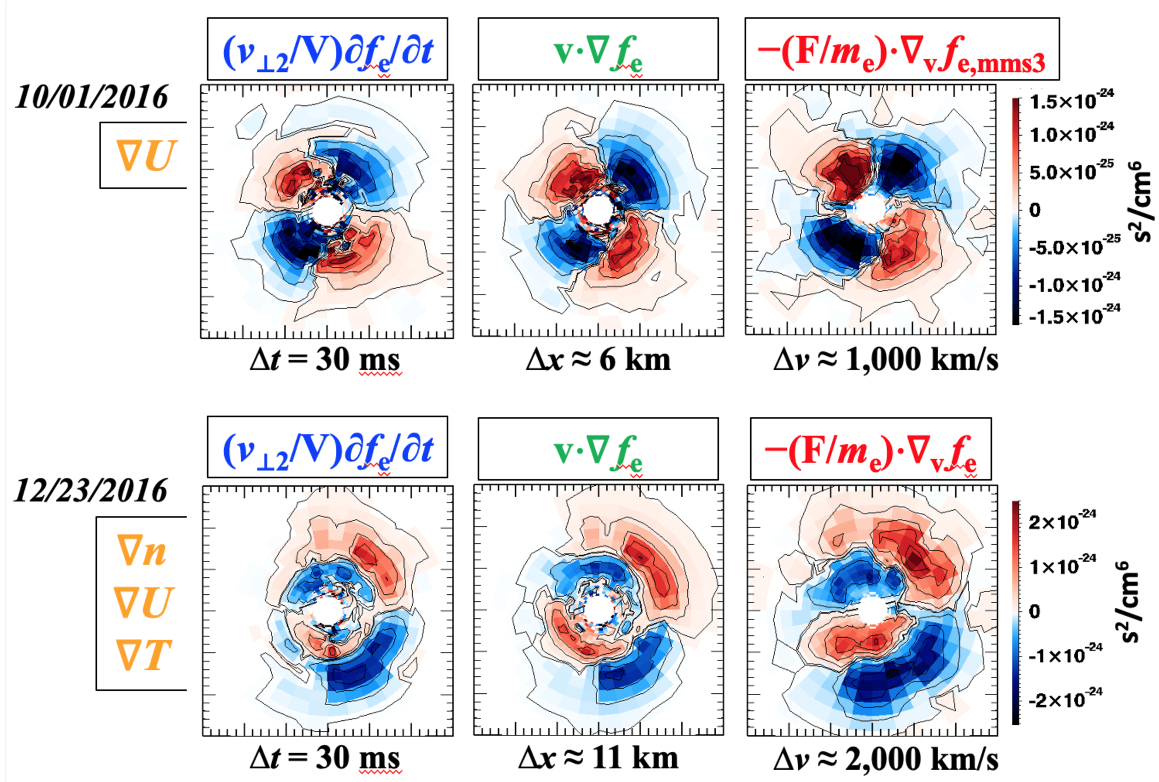
The above animation shows the spatial evolution of electron velocity distributions and terms in the Vlasov equation for a thin electron-scale current layer exhibiting a bulk velocity gradient.





The two figures above show qualitative consistency between two different methods for computing the velocity-space gradients needed for evaluating the force term $(\mathbf{F}/m_e) \cdot \nabla_{\mathbf{v}} f_e$: (1) interpolating to a Cartesian grid, and (2) performing the gradient in spherical energy-space coordinates native to FPI.

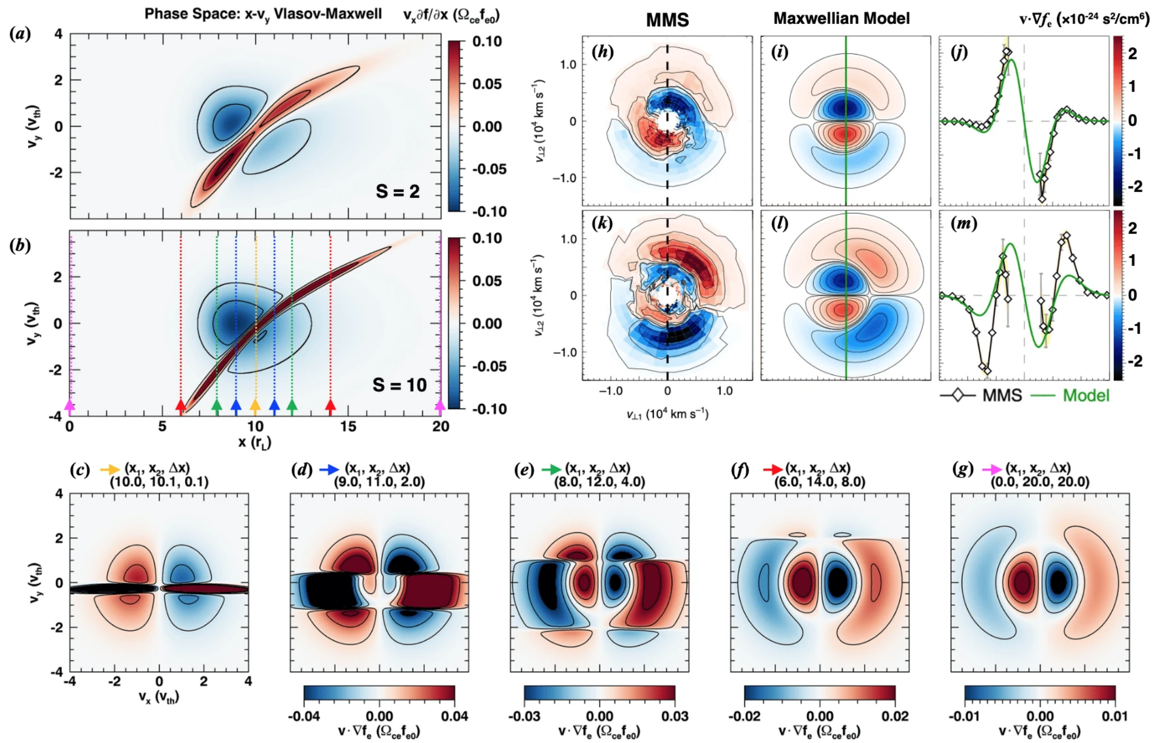
CONCLUSIONS



For two magnetopause thin current sheet events exhibiting density, bulk velocity, and temperature gradients, we find (1) qualitative agreement between the $\mathbf{v}\cdot\nabla f_e$ and $(\mathbf{F}/m_e)\cdot\nabla_{\mathbf{v}} f_e$ terms throughout velocity space, and that (2) the steady-state approximation for the time derivative term is valid, indicating $\partial f_e/\partial t$ is negligible for most of velocity-space when compared to the spatial gradient and velocity space gradient terms in the electron Vlasov equation.

These results imply that successful measurement of the force term from a *single spacecraft* can be used to infer localized spatial gradients and the full $\mathbf{v}\cdot\nabla f_e$ term throughout velocity-space.

FUTURE WORK



The above figure shows an example from the Vlasov-Maxwell temperature gradient model for how the velocity-space structure of the spatial gradient term $\mathbf{v} \cdot \nabla f_e$ measured from virtual spacecraft probes would vary with increasing interspacecraft separations in panels (c)-(g).

Combining the exact Vlasov-Maxwell solutions like the one shown above for an electron-scale temperature gradient, one can conduct direct comparisons between the MMS measurements and model predictions for the spatial gradient and force terms from the Vlasov equation. Such comparisons are useful validation techniques and effective for quantifying the accuracy of the spatial gradient computation for various current sheet thicknesses and spacecraft separations.

DISCLOSURES

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Reference:

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