The uncertainty of the calculative value of the volumetric flow rate in open channels

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Abstract

The well-known Manning formula is usually used for the calculation of the volumetric flow rate (discharge) in a river or open canal. The discharge depends on the geometry of the channel, i.e. the water area, the wetted perimeter and the slope, as well as on the roughness coefficients. All these quantities are determined with some uncertainty. If these uncertainties are taken into consideration in the process of discharge calculation, then, as has been demonstrated for a hypothetical river channel, the ratio of the uncertainty to the calculated value of the discharge will change from several dozen percent in case of small flows to about ten percent in case of big, flood flows

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Key Points

- The article presents a method of calculating the uncertainty of the flow rate in an open channel.
- The formulas necessary to calculate the uncertainty of the flow rate have been given.
- The uncertainty of flow rate decreases with its increase.

Abstract

The well-known Manning formula is usually used for the calculation of the volumetric flow rate (discharge) in a river or open canal. The discharge depends on the geometry of the channel, i.e. the water area, the wetted perimeter and the slope, as well as on the roughness coefficients. All these quantities are determined with some uncertainty. If these uncertainties are taken into consideration in the process of discharge calculation, then, as has been demonstrated for a hypothetical river channel, the ratio of the uncertainty to the calculated value of the discharge will change from several dozen percent in case of small flows to about ten percent in case of big, flood flows.

Index Terms and Keywords

Uncertainty assessment, Streamflow, River channels

1. Introduction

The issue of discharge uncertainty is increasingly broadly discussed in hydrology. The developed methods focus mainly on expressing the measurement uncertainty of the discharge value, e.g. Kyutae L., Hao-Che H., Muste M., Chun-Hung W. (2014). A review of the applied methods of uncertainty calculation was provided by Muste M., Lee K. and Krajewski J-L. (2012). Expressing discharge measurement uncertainty has also been included in the ISO 1088 (2007) standard. However, in engineering practice, for example during the designing of levees, an element which plays an important role are not only measurements, but also the calculation of the value of the discharge in a river or open canal. The issue of the uncertainty of the calculated discharge value has not been discussed in scientific and technical literature until now. The present article has been devoted to the methodology of calculating the calculative uncertainty of the discharge value.

1. Theoretical methods

    (a) The basic formulas in the theory of uncertainty

The notion of uncertainty is associated with a measured quantity, whereas the issue discussed in this article refers to a calculated quantity. Therefore, metrological notions referring to uncertainty require some clarification. The definitions presented below are based on the expressions included in GUM 1993:

- uncertainty – a parameter associated with the result of a calculation; it characterizes the dispersion of the calculated value;
- standard uncertainty – uncertainty expressed in the form of standard deviation;
- expanded uncertainty – the interval around the result of a calculation, which is expected to encompass a large fraction of the distribution of the calculated value;
- expansion coefficient – a numerical coefficient used as a multiplier of combined standard uncertainty in order to obtain expanded uncertainty;
- combined uncertainty – standard uncertainty of the calculated result, determined when that result is obtained from the values of a certain number of other quantities, equal to the square root of the sum of terms which are the variances or covariances of these other quantities with weights depending on how the measurement result varies together with changes in these quantities. Combined uncertainty is calculated in the following way: if quantity y depends on x_i quantities whose uncertainties are known and are $u(x_i)$, then the uncertainty $u(y)$ is calculated according to the formula:

$$u(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i}\right)^2 u\left(x_i\right)^2} \quad (1)$$

In the further part of the article, standard uncertainty is going to be, for short, referred to as uncertainty.

1. Uncertainty of discharge

In order to calculate the value of discharge, the empirical Manning formula is commonly used. It is a modification of the Chézy formula (Manning R.1895)

$$Q = \frac{1}{n} \bullet A \bullet R^{\frac{2}{3}} \bullet \sqrt{S} \qquad (2)$$

where:

Q – discharge

n – roughness coefficient

A – water area

R – hydraulic radius

P – wetted perimeter

S – slope

Taking into consideration that

$$R = \frac{A}{P} \qquad (3)$$

we receive:

$$Q = \frac{1}{n} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \bullet \sqrt{S} \qquad (4)$$

$$Q = \sum_{i=1}^{k} Q_i \qquad (5)$$

where k is the number of the channels of the watercourse.

Using the dependence (1), the uncertainty of the discharge $u(Q)$ will be expressed with the help of the following formula:

$$u(Q) = \sqrt{\left(\frac{\partial Q}{\partial n}\right)^2 \bullet u^2(n) + \left(\frac{\partial Q}{\partial A}\right)^2 \bullet u^2(A) + \left(\frac{\partial Q}{\partial P}\right)^2 \bullet u^2(P) + \left(\frac{\partial Q}{\partial S}\right)^2 \bullet u^2(S)} \qquad (6)$$

where

$u(Q)$ – uncertainty of the discharge

$u(n)$ – uncertainty of roughness

$u(A)$ – uncertainty of the water area

$u(P)$ – uncertainty of the wetted perimeter

$u(S)$ – uncertainty of the slope

By differentiating formula (2) we receive:

$$\frac{\partial Q}{\partial n} = \frac{\partial}{\partial n}\left(\frac{1}{n} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \bullet \sqrt{S}\right) = \frac{-1}{n^2} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \bullet \sqrt{S} = \frac{-1}{n^2} \bullet A \bullet R^{\frac{2}{3}} \bullet \sqrt{S}$$

$$\frac{\partial Q}{\partial A} = \frac{\partial}{\partial A}\left(\frac{1}{n} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \bullet \sqrt{S}\right) = \frac{5}{3} \bullet \frac{1}{n} \bullet \frac{A^{\frac{2}{3}}}{P^{\frac{2}{3}}}\sqrt{S} = \frac{5}{3} \bullet \frac{1}{n} \bullet R^{\frac{2}{3}} \bullet \sqrt{S}$$

$$\frac{\partial Q}{\partial P} = \frac{\partial}{\partial P}\left(\frac{1}{n} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \bullet \sqrt{S}\right) = -\frac{2}{3} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{5}{3}}} \bullet \frac{1}{n} \bullet \sqrt{S} = -\frac{2}{3n} \bullet R^{\frac{5}{3}} \bullet \sqrt{S}$$

$$\frac{\partial Q}{\partial S} = \frac{\partial}{\partial S}\left(\frac{1}{n} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \bullet \sqrt{S}\right) = \frac{1}{2} \bullet \frac{1}{n} \bullet \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \bullet S^{-\frac{1}{2}} = \frac{1}{2n} \bullet A \bullet R^{\frac{2}{3}} \bullet \frac{1}{\sqrt{S}}$$

After substituting the obtained results into formula (5) we receive:

$$u(Q) = \sqrt{\begin{array}{c}\left(\frac{-1}{n^2} \bullet A \bullet R^{\frac{2}{3}} \bullet \sqrt{S}\right)^2 \bullet u^2(n) + \left(\frac{5}{3} \bullet \frac{1}{n} \bullet R^{\frac{2}{3}} \bullet \sqrt{S}\right)^2 \bullet u^2(A) + \left(-\frac{2}{3n} \bullet R^{\frac{5}{3}} \bullet \sqrt{S}\right)^2 \bullet u^2(P) + \\ \left(\frac{1}{2n} \bullet A \bullet R^{\frac{2}{3}} \bullet \frac{1}{\sqrt{S}}\right)^2 \bullet u^2(S)\end{array}}$$

$$u(Q) = \sqrt{\frac{A^2 \bullet R^{\frac{4}{3}} \bullet S \bullet u^2(n)}{n^4} + \frac{25 \bullet R^{\frac{4}{3}} \bullet S \bullet u^2(A)}{9n^2} + \frac{4 \bullet R^{\frac{10}{3}} \bullet S \bullet u^2(P)}{9n^2} + \frac{A^2 \bullet R^{\frac{4}{3}} \bullet u^2(S)}{4n^2 \bullet S}}$$

$$u(Q) = \frac{R^{\frac{2}{3}}}{6n^2 \bullet \sqrt{S}}\sqrt{9A^2\left(4S^2 \bullet u^2(n) + n^2 \bullet u^2(S)\right) + 4n^2 \bullet S^2\left(25 \bullet u^2(A) + 4R^2 u^2(P)\right)} \quad (7)$$

If the channel is a main channel with two floodplains, then the Q uncertainty will be expressed by the following formula:

$$u(Q) = \sqrt{\sum_{i=1}^{k} \left(u(Q_i)\right)^2} \quad (8)$$

1. Uncertainty of roughness

The roughness coefficients have been collected and published by Ven Te Chow (1959) who provided mean, maximum and minimum values for appropriate types of watercourses and canals. Due to the fact that the statistical distributions of the values of these roughness coefficients are not known, it should be assumed that they are subject to uniform distribution, limited by the minimum and maximum value, i.e. distribution with probability density

$$f(x) = \frac{1}{n_{\max} - n_{\min}}.$$

The variance of which is (James F., 2006):

$$\sigma^2 = \frac{(n_{\max} - n_{\min})^2}{12}$$

Standard uncertainty $u(n)$ is equal to the standard deviation and is equal:

$$u(n) = \sqrt{3}\frac{n_{\max} - n_{\min}}{6} \qquad (9)$$

1. Uncertainty of distance

For engineering purposes, the distance l is calculated on the basis of the difference between the x (horizontal) and y (vertical) coordinates. If a segment is vertical, then:

$$l_x = |x_2 - x_1|$$

$$l_y = |y_2 - y_1|$$

Taking into consideration formula (1), the uncertainties of distance $u(l_x)$ and $u(l_y)$ will be expressed by the following formulas:

$$u\left(l_x\right) = \sqrt{\left(\frac{\partial l_x}{\partial x_1}\right)^2 u^2\left(x_1\right) + \left(\frac{\partial l_x}{\partial x_2}\right)^2 u^2\left(x_2\right)}$$

$$u\left(l_y\right) = \sqrt{\left(\frac{\partial l_y}{\partial y_1}\right)^2 u^2\left(y_1\right) + \left(\frac{\partial l_y}{\partial y_2}\right)^2 u^2\left(y_2\right)} \qquad (10)$$

We will also assume that the uncertainties of determining all horizontal coordinates x and vertical coordinates y are equal to one another and are equal $u(x)$, i.e. that:

$$u(x) = u\left(x_1\right) = u\left(x_2\right) = u(y_1) = u(y_2)$$

Then the formulas (10) determining the uncertainty of the length of a horizontal segment and a vertical segment take the following form:

$$u\left(l_x\right) = u\left(l_y\right) = \sqrt{u^2(x) + u^2(x)} = \sqrt{2} \bullet u(x) \qquad (11)$$

If a segment is oblique, then its length l_o is calculated using the Pythagorean theorem:

$$l = \sqrt{{l_x}^2 + {l_y}^2}$$

Taking into consideration formulas (1) and (11), the uncertainty of length of the oblique segment $u(l_o)$:

$$u\left(l_o\right)=\sqrt{\left(\frac{\partial\sqrt{{l_x}^2+{l_y}^2}}{\partial l_x}\right)^2 u^2\left(l_x\right)+\left(\frac{\partial\sqrt{{l_x}^2+{l_y}^2}}{\partial l_y}\right)^2 u^2\left(l_y\right)}$$

By calculating the derivative and taking into consideration (11) we receive:

$$u\left(l_o\right)=\sqrt{\left(\frac{2l_x}{2\sqrt{{l_x}^2+{l_y}^2}}\right)^2 u^2\left(l_x\right)+\left(\frac{2l_y}{2\sqrt{{l_x}^2+{l_y}^2}}\right)^2 u^2\left(l_y\right)}=\sqrt{\frac{{l_x}^2+{l_y}^2}{{l_x}^2+{l_y}^2}}\bullet\sqrt{2}\bullet u(x)=\sqrt{2}\bullet u(x) \tag{12}$$

1. Uncertainty of the water area

In engineering practice, the water area in a river is approximated using a rectangle, triangle or trapezium. The particular areas and uncertainties calculated with the help of formula (3) are expressed by the formulas:

- the area of the rectangle A_r is:

$$A_r=a\bullet h$$

therefore, the uncertainty of the area of the rectangle $u(A_r)$ is provided by the formula:

$$u\left(A_r\right)=\sqrt{\left(\frac{\partial a\bullet h}{\partial a}\right)^2\bullet u^2(a)+\left(\frac{\partial a\bullet h}{\partial h}\right)^2\bullet u^2(h)}=\sqrt{h^2\bullet u^2(a)+a^2\bullet u^2(h)}$$

Due to the fact that the base a is a horizontal segment and the altitude h is a vertical segment, after taking into consideration formula (11) we receive:

$$u\left(A_r\right)=\sqrt{\left(\sqrt{2}\bullet h\bullet u(x)\right)^2+\left(\sqrt{2}\bullet a\bullet u(x)\right)^2}=\sqrt{2}\bullet u(x)\sqrt{a^2+h^2} \tag{13}$$

- the area of the trapezium A_{tz} is:

$$A_{\mathrm{tz}}=\frac{a+b}{2}\bullet h$$

The uncertainty of the area of the trapezium $u(A_{tz})$ is equal:

$$\mathrm{u}\left(A_{\mathrm{tz}}\right)=\sqrt{\left(\frac{\partial \frac{1}{2}(a+b)h}{\partial a}\right)^2 \bullet u^2(a)+\left(\frac{\partial \frac{1}{2}(a+b)h}{\partial b}\right)^2 \bullet u^2(b)+\left(\frac{\partial \frac{1}{2}(a+b)h}{\partial h}\right)^2 \bullet u^2(h)}=\sqrt{\left(\frac{h}{2}\right)^2 \bullet u^2(a}$$

$$u\left(A_{\mathrm{tz}}\right)=\sqrt{\frac{h^2}{4} \bullet 2 \bullet u^2(x)+\ \frac{h^2}{4} \bullet 2 \bullet u^2(x)+\frac{(a+b)^2}{4} \bullet 2 \bullet u^2(x)}=\sqrt{h^2+\frac{(a+b)^2}{2}} \bullet u(x) \quad (15)$$

- the area of the triangle A_{tg} is:

$$A_{\mathrm{tg}}=\frac{a}{2} \bullet h$$

Therefore, the uncertainty of the area of the triangle $u(A_{tg})$ is calculated using the formula:

$$u\left(A_{\mathrm{tg}}\right)=\sqrt{\left(\frac{\partial \frac{1}{2} a \bullet h}{\partial a}\right)^2 \bullet u^2(a)+\left(\frac{\partial \frac{1}{2} a \bullet h}{\partial h}\right)^2 \bullet u^2(h)}=\sqrt{\left(\frac{a}{2}\right)^2 u^2(a)+\left(\frac{h}{2}\right)^2 u^2(h)} \quad (16)$$

we take into consideration formula (11):

$$u\left(A_{\mathrm{tg}}\right)=\sqrt{\frac{h^2}{4} \bullet 2 u^2(x)+\frac{a^2}{4} \bullet 2 u^{2}(x)}=\sqrt{\frac{h^2+a^2}{2}} \bullet u(x) \quad (17)$$

- an area composed of two figures

In the case when the water area A is composed of two figures, A_1 and A_2, the uncertainty of the area A is calculated using the formula:

$$u(A)=\ \sqrt{u^2\left(A_1^2\right)+u^2\left(A_2^2\right)}$$

For example, if area A is composed of a rectangle and a trapezium, then:

$$u(A)=\ \sqrt{u^2(A_r)+u^2(A_{\mathrm{tz}})} \quad (18)$$

1. Uncertainty of the wetted perimeter

The wetted perimeter P is a broken line composed of j segments. This includes k vertical segments, m horizontal segments and n oblique segments. The length of the broken line l_{bl} is calculated as the sum of the lengths of the particular segments. Let us mark the length of the i$^{\mathrm{th}}$ vertical segment as $l_{y(i)}$, the length of the i$^{\text{th}}$ horizontal segment as $l_{x(i)}$, and the length of the i$^{\text{th}}$ oblique segment as $l_{o(i)}$. Then the length of the wetted perimeter may be written down as:

$$P = \sum_{i=1}^{k} l_{y(i)} + \sum_{i=1}^{m} l_{x(i)} + \sum_{i=1}^{n} l_{o(i)} \quad (19)$$

If any of the types of segments is not present, then the appropriate element of the sum disappears. In accordance with the dependence (1), the uncertainty of the length of the wetted perimeter $u(P)$ will be expressed using the formula:

$$u(P) = \sqrt{\sum_{i=1}^{k} \left(\frac{\partial}{\partial l_{y(i)}} l_{y(i)} \right)^2 u^2\left(l_y\right) + \left(\sum_{i=1}^{m} \frac{\partial}{\partial l_{x(i)}} l_{x(i)} \right)^2 u^2\left(l_y\right) + \left(\sum_{i=1}^{n} \frac{\partial}{\partial l_{o(i)}} l_{o(i)} \right)^2} = \sqrt{k \bullet u^2\left(l_y\right) + m \bullet u^2 (l}$$

We take into consideration formulas (11) and (12) and we receive:

$$u(P) = \sqrt{k \bullet 2u^2(x)^2 + m \bullet 2u^2(x) + n \bullet 2u^2(x)} = \sqrt{2} \bullet u(x)\sqrt{k+m+n}$$

$$u(P) = \sqrt{2} \bullet j \bullet u(x) \quad (20)$$

1. Uncertainty of the slope

Hydraulic gradient S (slope) is the ratio of the water table slope y to the distance L on which this slope occurred

$$S = \frac{y}{L} \quad (21)$$

The uncertainty of the hydraulic gradient (slope) $u(S)$ is calculated using formula (1):

$$u(S) = \sqrt{\left(\frac{\partial}{\partial L}\frac{y}{L}\right)^2 u^2(L) + \left(\frac{\partial}{y}\frac{y}{L}\right)^2 u^2(y)} = \sqrt{\left(\frac{1}{L}\right)^2 u^2(y) + \left(\frac{-y}{L^2}\right)^2 u^2(L)}$$

Due to the fact that we assumed that all the distances are measured with the same uncertainty $u(x)$, therefore, taking into consideration the dependence (11) we receive:

$$u(S) = \frac{1}{L} \bullet u(x)\sqrt{1 + \left(\frac{-y}{L}\right)^2} = \frac{1}{L}\sqrt{1+S^2} \bullet u(x) \quad (22)$$

1. Material and methods

   (a) Assumptions for the calculations

Let us assume that we are designing the channel of a lowland river (fig. 1) together with levees. The dimensions of the designed canal are:

- the width at the bottom: 5.0 m
- the slope of the river banks: 1:2
- the depth of the river channel: 1.0 m
- the width of the floodplain: 10.0 m
- the height of the levee: 2.0 m
- the slope of the levee: 1:3

uncertainty of vertical and horizontal coordinates $u(x) = 0.01$ m

1. Calculations

Exemplary calculations will be carried out for the designed river channel presented in fig. 1 for the depth H of from 0 to 2.00 m with an increment of 0.1 m. The calculations were carried out using a spreadsheet. An example of the calculations for the depth $H = 1.5$ m has been included in Appendix A.

The discharge in the river channel as well as in the floodplain was calculated according to formula (2), and their uncertainties were calculated on the basis of formula (7). The total discharge, which is the sum of the flows in the channel and in the floodplains, was calculated in accordance with formula (5), and its uncertainty – in accordance with formula (8).

1. Results

   As a result of the performed calculations, the following was obtained:

- the rating curve $Q(H)$ as a function of filling the river channel H together with the confidence interval (fig. 2). The values of the calculations have been presented in tab. 1a graph of the percentage dependence of the $u(Q)/Q$ ratio (fig. 3)
- the value of the percentage change of the discharge uncertainty depending on the value of the percentage change of the uncertainty of roughness coefficients and of the location uncertainty (fig. 4)

Fig. 4. The dependence of the influence of the percentage change of the uncertainty of roughness coefficients of the channel, of the floodplain, and of the uncertainty of the geometric dimensions of the river channel water area on the uncertainty of the discharge

1. Discussion

From the rating curve presented in fig. 2 we can see that together with the increase of the discharge, there is an increase of the interval of its expanded uncertainty. At the same time, relative uncertainty calculated as the ratio of the uncertainty of discharge to the discharge $u(Q)/Q$ expressed as a percentage, decreases together with the increase of the discharge and it asymptotically heads to the value of 7.1% (fig. 3).

As has been demonstrated in theoretical calculations, the uncertainty of the calculative value of the discharge in an open canal is impacted by two factors. The first factor is the determination of the dimensions of the channel water area. Due to the fact that the channel dimensions, adopted for the calculations of the discharge, are dimensions that are designed, this means that the uncertainty of the transverse diameter of the channel should be identical with the assumed accuracy of creation of the channel of the canal or river. In exemplary calculations it was taken into consideration that the uncertainty of distance is 1 cm. In practice, such accuracy of channel creation is impossible to achieve. However, as has been presented in fig. 4, this accuracy has the smallest influence out of the remaining factors. Nevertheless, if we even assume $u(x) = 5$ cm, then the total discharge, for the depth of 2 m $Q(H{=}2\text{m})$ will only increase by 5%. The second factor is the selection of the roughness coefficients of both, the channel as well as of the floodplain. As has been demonstrated in fig. 4, even a 10% change of the uncertainty of the roughness coefficient causes an about 5% change of the uncertainty of the total discharge.

It is obvious that the obtained value depends on the adopted geometry of the channel and on the flow conditions, i.e. the roughness coefficients and the hydraulic gradient (slope). The way these factors influence the $u(Q)/Q$ ratio requires further research.

The presented line of reasoning does not take into consideration the uncertainty with which human chooses the values of roughness coefficients. And the uncertainty of the roughness coefficients is the main factor influencing the value of discharge uncertainty. Fig. 4 presents the influence of the percentage change of the uncertainty of roughness coefficients of the channel and of the floodplain and of the uncertainty of the geometrical dimensions of the river water area on the value of the discharge uncertainty. One can notice that the percentage change of the roughness coefficients has got an about 3.5 times bigger influence than the percentage change of the accuracy of the measurement of the channel geometry.

1. Data availability

All data is contained in the text of the article and may be used by other authors.

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Appendix A - exemplary calculations for depth *H*= 1.50 m.

The calculations were carried out for a river, the water area of which has been demonstrated in fig. no. 1. Due to the fact that floodplains are congruent, the discharge and its uncertainty were calculated for a single floodplain. The discharge in the whole river is the sum of the flows in its particular parts. For the purpose of the transparency of the calculations, units were taken into consideration only in case of the final result.

1. Distance

The component distances, both, the vertical one and the horizontal one, are calculated as a difference of coordinates. Their uncertainties $u(l_x)$ and $u(l_y)$ are expressed by formula (11)

$$u\left(l_x\right) = u\left(l_y\right) \ = \sqrt{2} \bullet 0.01\ m = 1.14 \bullet 10^{-2} \text{ m.}$$

1. The hydraulic gradient (slope) S and its uncertainty $u(S)$, formulas (21) and (22)

$$S = \frac{0.5\ m}{250\ m} = 0.002$$

$$u(S) = \frac{1}{250}\sqrt{1 + 0.002^2} \bullet 0.01 = 4 \bullet 10^{-5}$$

1. The river channel

- Roughness coefficient

  The width of the river channel at the top is 9 m = (5 m + 2 m · 2 m). It is a value which is lower than 100 ft (32.8 m), so (Chow, 1959) the value of the roughness coefficients is:

$$n_{\text{minimum}} = 0.025;\ n_{\text{normal}} = 0.030;\ n_{\text{maximum}} = 0.033$$

  therefore

$$n = 0.03\ m^{-\frac{1}{3}} \bullet s$$

  and uncertainty $u(n)$ calculated in accordance with formula (9) is:

$$u(n) = \sqrt{3}\frac{0.033 - 0.025}{6} \quad = 2.31 \bullet 10^{-3}\ {}^{m^{-\frac{1}{3}} \bullet s}$$

- The water area of the main channel

  The water area of the main channel $\mathrm{A_{mc}}$ is the sum of the areas:

- of the rectangle A_r of the following dimensions: $a_r = 9.0$ m and $h_r = 0.5$ m
- of the trapezium A_{tz} with the following bases: $a_{tz} = 5.0$ m and $b_{tz} = 9.0$ m, and the altitude $h_{tz} = 1.0$ m

  The A_{mc} area is:

$$A_{\text{mc}} = A_r + A_{\text{tz}} = 9.0 \bullet 0.5 + \frac{1}{2}(5.0 + 9.0\ ) * 0.5 = 9.0\ m^2$$

  The uncertainty of the area of the rectangle $u(A_r)$, formula (13) is:

$$u\left(A_r\right) = \sqrt{2} \bullet 10^{-2} \bullet \sqrt{9^2 + 0.5^2} = 0.127\ m$$

  The uncertainty of the area of the trapezium $u(A_{tz})$, formula (14) is:

$$\mathrm{u}\left(A_{\text{tz}}\right) = \sqrt{1^2 + \frac{(5+9)^2}{2}} \bullet 0.01\ m = 0.099\ m^2$$

  Using formula (18) let us calculate the uncertainty of the A_{mc} area.

$$u\left(A_{\text{mc}}\right) = \sqrt{0.127^2\ + 0.099^2} = 0.161\ m$$

- Wetted perimeter

In the discussed case, the wetted perimeter is the sum of three segments. Two of them are oblique segments and one is horizontal. Therefore, in accordance with formulas (19) and (20), the length and the uncertainty of the length of the wetted perimeter is:

$$P = 5 \ + \ 2 \bullet \sqrt{1^2 + 2^2} = 9.47 \ m$$

$$u(P) = \sqrt{2} \ \bullet 3 \bullet 0.01 = 4.24 \bullet 10^{-2} m$$

- Volumetric flow rate (discharge)

  The value and the uncertainty of the discharge is calculated based on formulas (2) and (7). It is:

$$R = \frac{9.0}{9.47} = 0.95 \ m; \qquad Q = \frac{1}{0.030} \bullet 9.0 \bullet 0.95^{\frac{2}{3}} \bullet \sqrt{0.002} = 12.97 \ \frac{m^3}{s}$$

$$u(Q) = \frac{0.95^{\frac{2}{3}}}{6 \bullet 0.03^2 \bullet \sqrt{0.002}} \sqrt{\begin{matrix} 9 \bullet 9^2 \bullet \left(4 \bullet 0.002^2 \bullet 0.0023^2 + 0.03^2 \bullet 0.00004^2\right) + \\ 4 \bullet 0.03^2 \bullet 0.002^2 \bullet \left(25 \bullet 0.161^2 + 4 \bullet 0.95^2 \bullet 0.0424^2\right) \end{matrix}} = 1.08 \ \frac{m^3}{s}$$

1. Floodplain

- The roughness coefficient for the floodplain covered with high grass is:

$$n_{\text{minimum}} = 0.03; \ n_{\text{normal}} = 0.035; \ n_{\text{maximum}} = 0.05$$

  therefore

$$n = 0.035 \ m^{-\frac{1}{3}} \bullet s$$

  and uncertainty $u(n)$ calculated in accordance with formula (9) is:

$$u(n) = \sqrt{3} \frac{0.05 - 0.03}{6} \quad = 5.78 \bullet 10^{-3} \ m^{-\frac{1}{3}} \bullet s$$

- The water area of the floodplain

  The water area of the floodplain A_{fp} is a trapezium of the following dimensions:

$$a = 10 \ m; \quad b = 10 \ m + 0.5 \ m \ \bullet 3 = 11.5 \ m; \quad h = 0.5 \ m$$

  The uncertainty of the trapezium area is calculated in accordance with formula (15), therefore:

$$A = \frac{10 + 11.5}{2} \bullet 0.5 = 5.375\ m^2$$

$$u(A) = \sqrt{0.5^2 + \frac{(10 + 11.5)^2}{2}} \bullet 0.01 = \ 0.152\ m^2$$

- Wetted perimeter

  The wetted perimeter is a broken line composed of two segments. In accordance with formulas (19) and (20):

$$P = 10.0 + \ \sqrt{0.5^2 + (3 \bullet 0.5)^2} = 11.58\ m$$

$$u(P) = \sqrt{2}\ \bullet 0.01 = 1.41 \bullet 10^{-2} m$$

- The discharge in the floodplain was calculated in accordance with formula (2), and its uncertainty – according to formula (7).

$$R\ = \frac{5.375}{11.58} = 0.464\ m; \qquad Q = \frac{1}{0.035} \bullet 5.375 \bullet 0.464^{\frac{2}{3}} \bullet \sqrt{0.002} = 4.12\ \frac{m^3}{s}$$

$$u(Q) = \frac{0.464^{\frac{2}{3}}}{6 \bullet 0.035^2 \bullet \sqrt{0.002}} \sqrt{\begin{array}{l} 9 \bullet 5.375^2 \bullet \left(4 \bullet 0.002^2 \bullet 0.00578^2 + 0.035^2 \bullet 0.00004^2\right) + \\ 4 \bullet 0.035^2 \bullet 0.002^2 \bullet \left(25 \bullet 0.152^2 + 4 \bullet 0.464^2 \bullet 0.0141^2\right) \end{array}} = 0.71 \frac{m^3}{s}$$

1. Total volumetric flow rate (discharge)

The total discharge and its uncertainty are calculated based on formulas (5) and (8).

$$Q = 12.97 + 2\ \bullet 4.12 = 21.21\ \frac{m^3}{s}$$

$$u(Q) = \sqrt{1.08\ ^2 + 2 \bullet 0.71^2} = 1.47 \frac{m^3}{s}$$

If we assume that the expansion coefficient $k = 2$, then expanded uncertainty is an interval [18.27; 24.15]. If additionally we assume that uncertainty u(Q) is subject to Gauss distribution, then the probability P that the total calculative discharge fits within the indicated interval is 0.98.

Appendix B - list of formulas

The basic value is the uncertainty of coordinates $u(x)$. We assume that the uncertainties of vertical and horizontal coordinates are equal. The formulas necessary for the calculation of the uncertainty of the volumetric flow rate (discharge) have been listed in tab. 2