

# Modeling Soil Organic Carbon Dynamics, Carbon Sequestration, and the Climate Benefit of Sequestration

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## Abstract

Mathematical models are essential for integrating different processes that control rates of soil C dynamics and for assessing C sequestration and related climate benefits. Many models have been proposed in the literature to predict C stocks and fluxes, with no overall consensus on the best model that can provide relevant insights at a large range of scales and for multiple questions. We reviewed general groups of models with their expected ranges of application. We also reviewed recent advances in using models of any level of detail to compute C sequestration, and the climate benefit of C sequestration. Using agricultural soils from Sweden and Hawai'i as examples, we show that new C inputs to the soil do not remain for long timescales, and only small proportions are stabilized. Although soils are a promising reservoir to store C and mediate emissions, long timescales are required to store amounts of C of relevance to mitigate climate change. The magnitude of climate benefit to mitigate warming through soil C sequestration is less than that of avoiding direct emissions, however, remains an important component of climate change mitigation and adaptation portfolios. Beyond the direct warming mitigation benefits, improved soil health through soil organic matter aggradation brings many co-benefits to the environment and local communities. Improved production practices and locally sourced food and energy feedstocks are associated directly with avoided emissions elsewhere in the food and energy system.

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# Introduction

Soil organic carbon is a complex mix of organic molecules with different biological, chemical and physical properties. This carbon (C) can be part of dead plant matter, live and dead microorganisms, or organic molecules dissolved in water or attached to the surface of minerals. Soil is an open system with respect to organic C, where it forms from the remains of plant parts or their root exudates, and it is lost as CO<sub>2</sub> from the respiratory activity of microorganisms, dissolved C in water leachates, or soil erosion [Swift et al., 1979, Sollins et al., 1996, von Lützow et al., 2006]. No single measurement or experiment can capture all this complexity in a comprehensive form that allows one to make predictions on the future dynamics of a particular soil. For a predictive understanding of soil C dynamics over time, the development of mathematical models is essential.

Mathematical models can integrate information that captures the heterogeneity of process rates, transformations of organic matter, and different input-output processes [Manzoni and Porporato, 2009, Sierra and Müller, 2015]. Models, however, are imperfect tools that sometimes are a crude approximation of reality and often make overly simplified assumptions in order to be useful. There has been a long debate in the soil science literature about the most appropriate models to represent C dynamics and their response to climate change and to different management practices. There are hundreds of soil C models proposed by many scientists, each developed for a specific purpose, to address a particular question, or to explain certain pattern in a dataset. There is no single model that can explain all observations, that can be used to explore every single question, or to analyze patterns at multiple scales.

We must accept and embrace the diversity of soil C models, including those that have been proposed already and those that will be proposed in the future. Each model has a particular advantage, either because it can predict well results from a certain experiment, address a scale or process of interest, or because it offers the best compromise between accuracy and parsimony.

In this chapter, instead of reviewing particular soil organic carbon (SOC) models, we review families of models with common mathematical properties. We will also review a set of metrics that can be applied to any model in order to obtain a holistic understanding of the main factors that influence C sequestration and the climate benefit of C sequestration. With a set of examples, we will show how different predictions by different models can be used to assess C sequestration and the timescale at which C remains in soils.

## Modeling soil organic carbon dynamics

### Landscape of soil carbon models

In a comprehensive review of soil carbon and nitrogen models, Manzoni and Porporato [2009] identified 250 mathematical models developed over a period of 80 years. The models differed in their level of detail and spanned over a wide range of temporal and spatial scales of application. Up to that time (year 2009), the number of models were growing at an exponential rate of 6% per year, and it is likely that the number of proposed models continues growing exponentially. Given this diversity of models, it is important to develop a conceptual map of the landscape of available models, so they can be studied systematically.

For the purpose of classifying models according to fundamental mathematical structures, we will consider here the variable  $x$  as the mass of SOC in a specific soil. We are interested in studying how this variable changes over time, which may depend on four fundamental variables that define model structure

$$\frac{\partial x}{\partial t} = f(\psi, z, x, t),$$

where

1.  $\psi$ : SOC heterogeneity with respect to process rates. This heterogeneity can be characterized with continuous functions, or with discrete compartments [Sierra et al., 2011]. The continuous function can represent complex shapes of the distribution of SOC along a continuum of process rates using only a few parameters [Carpenter, 1981, Ågren and Bosatta, 1996]. However, the solution of these systems is complex, and solutions are often derived for the average quality of SOC. In contrast, discrete pool models are very common, but require that total SOC is partitioned in discrete categories according to certain physical, chemical, or biological characteristic that do not necessarily represent the continuous distribution of rates. As the criteria to split SOC into discrete categories is somewhat

arbitrary, there has been a long debate on whether these criteria should be based on the chemistry of the substrates, the mineralogical composition of the soil matrix, or the microbial communities that consume the C.

2.  $z$ : Vertical dimension, which can be characterized with continuous functions, or with discrete layers. Continuous functions of the vertical dimension lead to systems of partial differential equations, and the equations of advection and diffusion are commonly used as templates for SOC vertical transfer. For modeling discrete layers, the system of equations can be simplified to first order differential equations [Koven et al., 2013, Metzler et al., 2020].
3.  $x$ : SOC mass. If the system of differential equations includes dependencies among the mass of multiple compartments, the system would be nonlinear. This is necessary for modeling the priming effect, or processes resembling Michaelis-Menten kinetics for substrate-enzyme interactions. In case the masses of multiple compartments do not interact, the system of differential equations would be linear.
4.  $t$ : Time dependencies. Environmental variables not explicitly modelled, and included as a driving factor, are usually modelled as time-dependent functions that modify process rates. Inputs of SOC to the soil can also be modelled as a time-dependent process, particularly if a data set of litter and root inputs is available. When time dependencies are explicitly included, the system of differential equations is classified as a non-autonomous dynamical system. If all coefficients are constant in the model and do not change over time, the system of differential equations is classified as an autonomous dynamical system. Additional details about the classification of models as autonomous and non-autonomous systems is provided below.

The presence or absence of any of these variables determines the particular structure of a model. For example, a model without vertical dimension  $f(\psi, x, t)$  describes the dynamics of SOC in different pools with possible nonlinear interactions, but without explicitly describing spatial patterns of C transfers along a soil profile. Although the structure of the equation above is useful to identify important components of models, it is too general to understand particular behaviors or mathematical properties of models. In the following section, we will describe a general equation for the classification of models into four groups with common mathematical properties, which in turn lead to different methods to study C sequestration in soils.

## Basic ecological principles to classify and analyze SOC models

Despite the large variety of SOC models proposed in the literature, there is a set of six ecological principles that can be used to mathematically generalize all models. These principles are: mass balance, heterogeneity in process rates, substrate dependence of decomposition, internal transfers and transformations of SOC, environmental variability effects, and substrate interactions [Sierra and Müller, 2015]. These principles lead to a general mathematical form expressed as a system of first order differential equations

$$\frac{d\mathbf{C}}{dt} = \mathbf{I}(t) + \mathbf{T}(\mathbf{C}, t) \cdot \mathbf{N}(\mathbf{C}, t) \cdot \mathbf{C}(t) \quad (1)$$

where  $\mathbf{C}$  is a vector of SOC contents in  $n$  discrete C pools;  $\mathbf{I}$  is a vector of C inputs to the soil;  $\mathbf{T}$  is a  $n \times n$  matrix of transfers and transformations of organic C among the  $n$  pools;  $\mathbf{N}$  is a  $n \times n$  diagonal matrix of process rates that characterize how fast C is processed in each of the  $n$  pools. If any of the terms in this equation is time dependent, the system is called non-autonomous. If any of the terms of this equation has an interaction among different C pools, the entire system is nonlinear. This distinction between linearity and autonomy is fundamental to classify and analyze models. Some diagnostic metrics can only be obtained for certain groups of models. For example, only autonomous models may have a steady-state, while non-autonomous models, by definition, have no steady-state solution because they have time dependencies that change C inputs and process rates all the time.

Almost all SOC models can be written in the form of equation (1), including models that represent vertical dynamics [Metzler et al., 2020]. Most models with partial differential equations that include vertical C transfers cannot be solved analytically, and instead are solved numerically in computers for discrete vertical intervals, then equation (1) can also be used to generalize any continuous model that has been discretized [Sierra et al., 2011]. This general mathematical form is also called a compartmental dynamical system, because due to mass constraints, the system of equations follows strict mathematical constraints [Anderson, 1983, Jaquez and Simon, 1993, Sierra et al., 2018a].

## Families of models

### Linear autonomous compartmental models

The simplest models are those that can be classified as linear autonomous models, i.e., those that have constant coefficients and no interactions among different types of SOC. For example, models with constant C inputs to the soil and constant decomposition rates that do not change over time can be classified into this category. These models are simple to analyze and can provide interesting insights, e.g., about the maximum C storage capacity of a soil or the average time it takes for C atoms to travel through the system. Models in this category converge to a steady-state solution, which can give an approximation of the maximum amount of C storage in a soil (assuming that C inputs are nearly constant and rates do not change significantly over time). Although these assumptions may not hold for very long time-scales, the steady-state C storage concept is useful to consider as a potential upper limit of maximum soil C stocks as determined by the set input and loss rates. Linear autonomous models may help land managers to estimate potential changes in the maximum C stocks that could be obtained if significant changes in C inputs or rates are changed.

### Nonlinear autonomous compartmental models

Models classified as nonlinear autonomous are useful to represent interactions among different forms of SOC. For example, it has been shown experimentally that the addition of fresh C promotes the decomposition of old C, a phenomenon called the priming effect [Kuzyakov, 2010]. This type of interaction can be modeled as an interaction among different forms of SOC that leads to nonlinear systems of differential equations [Wutzler and Reichstein, 2008, Blagodatsky et al., 2010]. This type of system may have one or multiple steady-states. However, in some cases steady-state solutions may violate the basic principles mentioned above and lead to unrealistic solutions, e.g., negative SOC stocks. In most cases, the interactions among different SOC pools are represented with multiplicative terms or with Michaelis-Menten functions that express the saturation effect imposed by one type of substrate (e.g., enzymes) that limits decomposition [Schimel and Weintraub, 2003, Allison et al., 2010, Wang et al., 2012, Wieder et al., 2014]. The predictions of these mathematical models often show oscillations among the different types of SOC [Wang et al., 2014]; e.g. fluctuations of enzymes and substrates resembling Lotka-Volterra cycles, mostly because the equations in these models are in essence the same equations used to model predator-prey interactions.

### Linear non-autonomous systems

Linear non-autonomous models usually take a linear autonomous model with constant coefficients as a template and add functions that change the values of these coefficients as other environmental variables change over time. These models are very common in many biogeochemical and land surface models. For example, it is common to use functions that depend on soil moisture and soil temperature to predict how changes in the environment change the values of decomposition rates over time [Sierra et al., 2015]. It is also common to use a dataset of past C inputs to the soil derived from crop yields or model-based estimates of net primary productivity as a main driver in linear non-autonomous models. It is always necessary to hold knowledge of the temporal dynamics of the system to be able to run these models. Because inputs and rates are always changing, these models do not converge to a steady-state solution, but rather show the trajectory of SOC stocks imposed by the time-dependent inputs and rates [Sierra et al., 2018a]. The linear non-autonomous system is the predominant paradigm in SOC models such as Century [Parton et al., 1987], RothC [Jenkinson and Rayner, 1977], MEMS [Robertson et al., 2019], Millenium [Abramoff et al., 2018], among many others. These models are very useful to predict how spatial patterns of SOC change over time, and to predict the effects of environmental change on SOC stocks. Most Earth system models include a soil C module built as a linear non-autonomous system.

### Nonlinear non-autonomous systems

The most complex models have time-dependencies in inputs and process rates, and include nonlinear interactions among SOC pools. These nonlinear non-autonomous models sometimes lead to complex, chaotic behaviors and are difficult to generalize outside of the trajectories obtained from a particular simulation [Müller and Sierra, 2017]. There are few examples of these models to represent SOC and assess C sequestration. One interesting example of a nonlinear non-autonomous model is the Bacwave model [Zelenev et al., 2000], used to represent the dynamics of soil bacterial populations in response to the

time-dependent dynamics of the production of root exudates. The model reproduced well observations from a laboratory experiment, but to be able to run the model, a function that predicts exudate dynamics over time must be known. In general, it is challenging to use nonlinear non-autonomous models for large scale applications because considerable knowledge of the system and the expected behavior over time must be known a priori.

## Metrics to analyze models in the context of carbon sequestration for climate change mitigation

Despite the large variety of mathematical models used to represent the dynamics of SOC, it is possible to use metrics computed across diverse models and learn general system properties. In particular, we can use metrics to aggregate the combined effect of adding C to the soil with the amount of time this C will spend in the soil. The representation of models in the form of equation (1) is useful for this purpose because it allows us to use very general methods to compute aggregated metrics across all possible models. We will review two recently developed metrics [Sierra et al., 2021] that can be applied to any of the model categories mentioned above, these are Carbon Sequestration (CS) and the Climate Benefit of Sequestration (CBS).

### Carbon sequestration (CS)

Different perspectives on soil C sequestration exist. Carbon sequestration has been defined variously as the amount of C present in a soil at a given time, or the amount that enters the soil at a given time or during a particular period, or as a difference between the amount stored at one time versus the amount stored at a later time. Time is a particular aspect of utmost importance to consider when defining C sequestration. That is: Is new C entering the soil, or C already there, stored for a short period of time or at a longer timescale relevant for climate change mitigation? Researchers have struggle with this question before and pointed out that C sequestration in natural ecosystems is not permanent [Fearnside et al., 2000, Moura Costa and Wilson, 2000]. Therefore, a meaningful definition of C sequestration in the context of climate change mitigation must include a consideration of the time C is stored.

Here, we use a definition of C sequestration that integrates the amount of C inputs entering the soil at a given time and the amount of time that those C inputs are stored in the soil. We assume the soil to be an open system, where all C that enters eventually leaves in solid, gas, or liquid form. We base this definition on new C entering the soil, tracking the fate of those C inputs, and quantifying how long this C remains. Because decomposition is a continuous process that gradually degrades a substrate, we need to use a model that predicts the dynamics of these inputs and select a time horizon of interest. More precisely,

*Carbon sequestration is the storage of a certain amount of C input over a certain period of time within a system. It can be quantified as the area under the curve illustrating the fate of C input as it flows through an ecosystem over time.*

Graphically (Figure 1), we can see that an amount of C that enters at a time  $t_0$ , decreases in an almost exponential form until eventually most of it is decomposed. If we fix a time horizon  $T$ , we can compute the area under the curve of the remaining amount of inputs and obtain a metric that tells us how much C is retained in the soil over a fixed period of time. The approach is similar as in pharmacology, where the area under the curve of a drug in the blood system tells researchers about the total drug exposure across time. It is also the same approach used to study the absolute global warming potential of an emission of a greenhouse gas (GHG) to the atmosphere: the area under the curve of an amount of emissions tell us how much the GHG is exposed to the absorption of solar radiation [Rodhe, 1990]. We will see later that this approach is also useful to quantify the climate effect of sequestering C in soils.

CS is a metric that can be computed from any model and can be used to predict how much and for how long C will be stored in the soil. It can be computed for any model structure because its computational definition is independent of assumptions of linearity or the existence of a steady-state.

As an example, let's consider two different ecosystems, a perennial grass system growing on a Mollisol in Hawai'i, and an arable Cambisol soil in Sweden. A linear autonomous model with three C pools for the perennial grass cultivation in Hawai'i was developed by Crow et al. [2018]. Similarly, a linear autonomous model with two pools was developed for Swedish agricultural soils by Andren and Kätterer [1997] (the ICBM model). Both models can be written following the matrix form of equation (1), therefore we

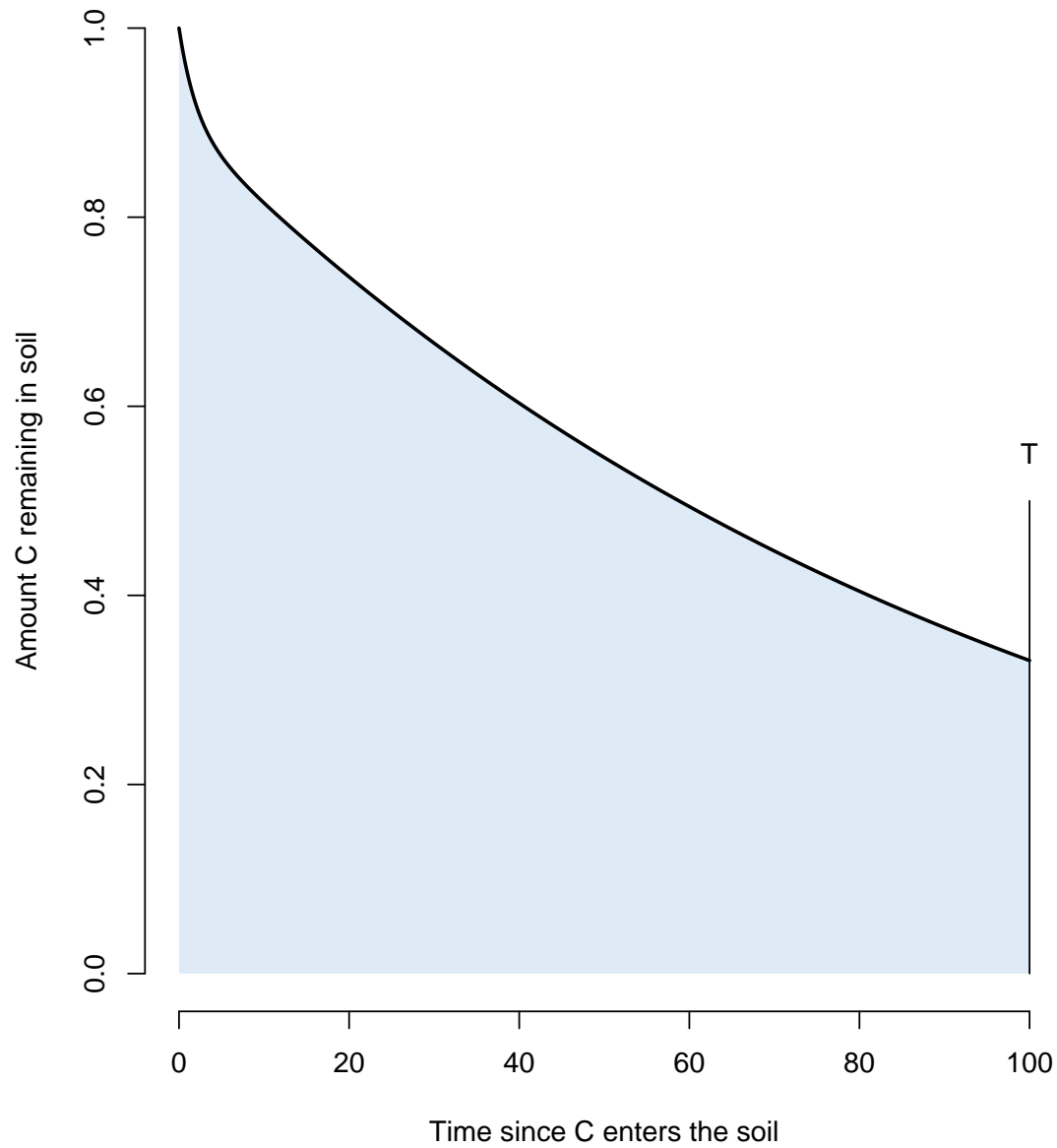


Figure 1: Carbon sequestration can be defined as the area under the curve of an amount of C inputs entering at  $t_0$  observed for a time horizon  $T$ . In this example, 1 unit of C enters at  $t_0 = 0$ , and the area under the curve (shaded blue area) of the remaining C is computed until  $T = 100$ .

can compute CS from both models. For simplicity, we will assume that rates and inputs are constant, consistent with the family of linear autonomous models.

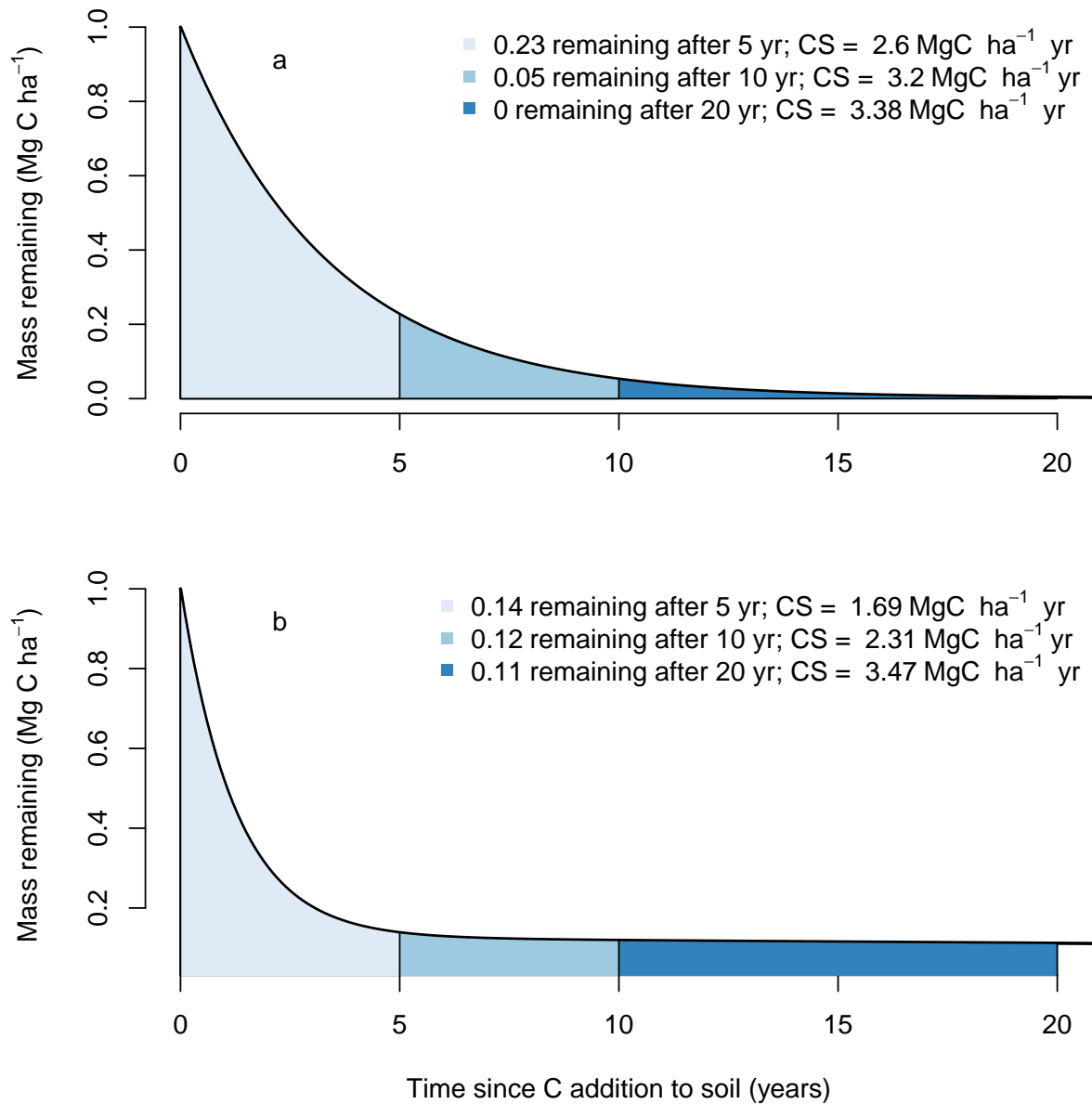


Figure 2: Mass remaining over time, and CS defined as the area under the curve for different time horizons, for two different soils: (a) a Mollisol under perennial cultivation in Hawai'i; (b) an arable Cambisol in Sweden. On a 5-year time horizon, more C remains from one Mg of C inputs in Hawai'i than in Sweden, therefore CS is higher for the tropical Mollisol. However, at a 20-year time horizon the amount of remaining C is higher in the Swedish Cambisol.

If we compare the fate of the same amount of inputs entering these soils (1 Mg C ha<sup>-1</sup>), we can observe the remaining C over time (Fig. 2). The comparison between the amount of C remaining and the area under this curve, which we define as carbon sequestration CS, depends strongly on the chosen time horizon for the comparison. We see that the 1 Mg C ha<sup>-1</sup> entering the Swedish soil decays fast at first, but then the remaining C persists for a longer time in comparison to the Hawaiian Mollisol. On a 5-yr time horizon, more C remains in the Hawaiian Mollisol (higher C sequestration) than on the Swedish Cambisol. However, on a 20-yr time horizon, C sequestration is slightly larger in the Swedish soil. Therefore, according to this conceptual framework, the timescale in which soil C persistence is evaluated plays a large role in the assessment of CS in soils.

In addition to the timescale of persistence, it is also important to consider differences in the amount



of inputs entering the soil. If we now consider the differences in plant productivity between the two sites, i.e. the amount of C inputs between the Hawaiian and the Swedish soil, we can see that the more productive Hawaiian soil with  $3.5 \text{ Mg C ha}^{-1} \text{ yr}^{-1}$  of inputs has a larger area under the curve for all time horizons in comparison to the less productive Swedish cropland with only  $2.0 \text{ Mg C ha}^{-1} \text{ yr}^{-1}$  of inputs (Fig. 3). The value of CS for the Hawaiian Mollisol is  $11.88 \text{ Mg C ha}^{-1} \text{ yr}$  on a 20-yr time horizon, which is more C remaining (higher CS) than in the Swedish Cambisol on the same time horizon ( $6.93 \text{ Mg C ha}^{-1} \text{ yr}$ ). Therefore, both amount of inputs and the timescale at which soil C persistence is evaluated play a large role in quantifying soil CS in this conceptual framework.

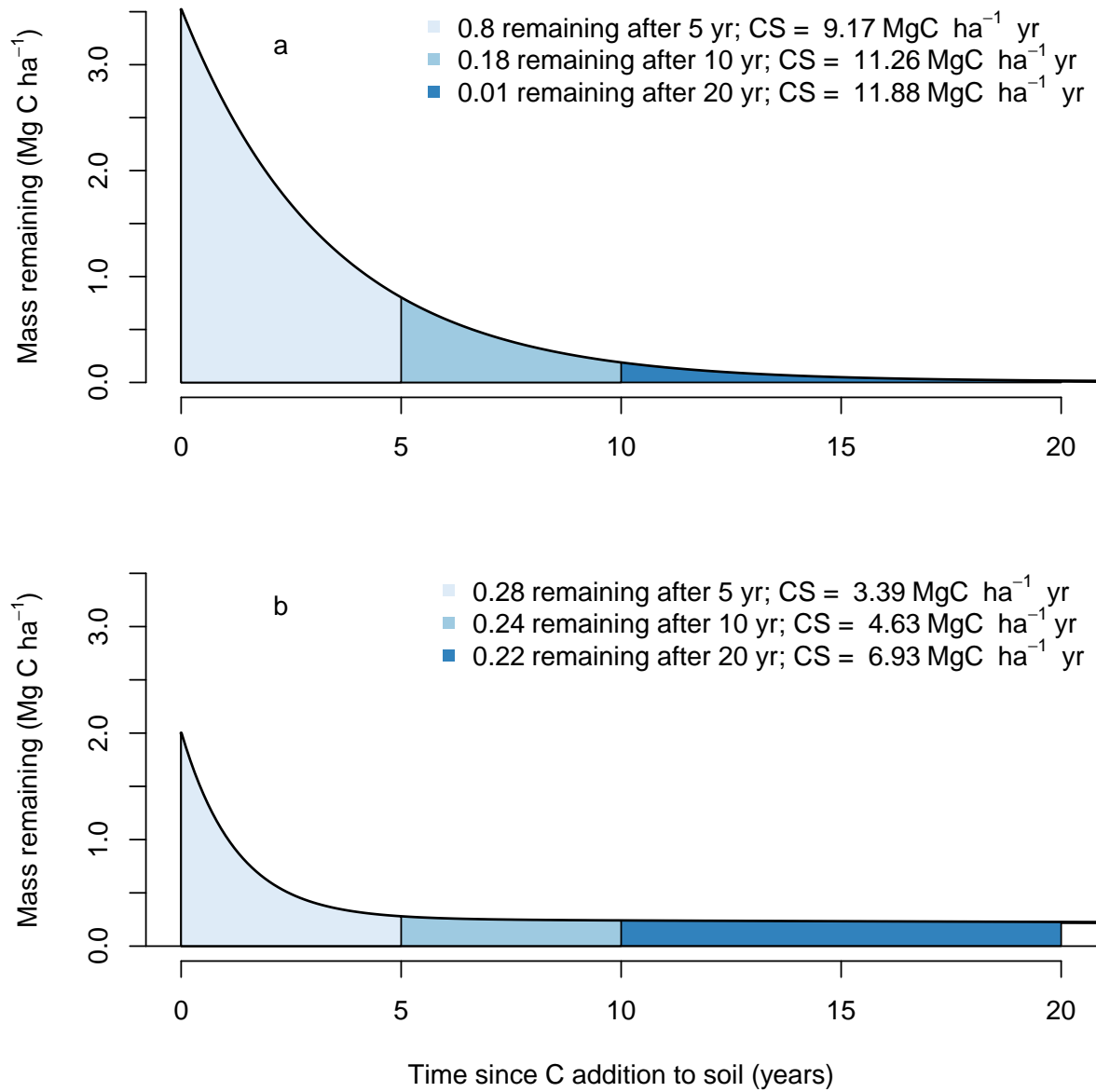


Figure 3: Mass remaining over time, and CS defined as the area under the curve for different time horizons, for two different soils with different productivity and annual C inputs: (a) a Mollisol under perennial cultivation in Hawai'i with inputs of  $3.5 \text{ Mg C ha}^{-1} \text{ yr}^{-1}$ ; (b) an arable Cambisol in Sweden with inputs of  $2.0 \text{ Mg C ha}^{-1} \text{ yr}^{-1}$ .

This simple example shows that we can assess C sequestration from the perspective of new C inputs entering the soils, taking into account how long it will stay stored. Because the fate of the inputs can be obtained from different models that may include different mechanisms leading to different model structures, the CS metric helps to make comparisons independent of the type of model being used. The amount of C inputs can change over time, and it is possible that one is interested in assessing the fate of C entering the soil during a particular time window. In this case, it is a simple matter of summing the CS values obtained

for discrete time units of say one year each. The areas under the curves of discrete cohorts of inputs can be summed to obtain an overall CS value that integrates multiple cohorts. A detailed description of the procedure including all required mathematical formulas can be found in Sierra et al. [2021].

## Climate benefit of sequestration (CBS)

Carbon sequestration tells us about the amount of C stored over a period of time, but it does not inform us about the greenhouse effect this C avoids while it is stored in soil. To quantify this, we use a different metric, which we call the Climate Benefit of Sequestration (CBS). We define it as

*The Climate Benefit of Sequestration is the radiative forcing (greenhouse) effect avoided by an amount of C inputs to the soil stored over a specific time horizon.*

CBS is reported in units of Watts per square meter year ( $\text{W m}^{-2} \text{ yr}$ ). It is very similar to the Absolute Global Warming Potential (AGWP) of  $\text{CO}_2$ , but instead of considering a C emission as an input to the atmosphere with a positive sign, it considers sequestration into a soil as a negative emission. Because most C that enters the soil returns to the atmosphere as heterotrophic respiration, CBS accounts for the temporary effect of storing a unit of C that enters at a particular time and returns to the atmosphere over a time horizon.

The concept of CBS can be used to compare different soils or different forms of management in terms of the avoided warming they can provide as an ecosystem service. For its computation, it is necessary to have a model that predicts the fate of the inputs of C to the soil and the timing of C return to the atmosphere from respiration. Practically any SOC model can provide these predictions, and it is possible to compare predictions of CBS from different models.

In addition to a SOC model, it is also necessary to use a so-called Impulse Response Function (IRF) of  $\text{CO}_2$  in the atmosphere. This is a function that predicts how the atmosphere reacts to additions or removals of  $\text{CO}_2$ . The most common IRF used today was developed by Joos et al. [2013], and we use this function for our analysis here.

A computation of CBS for the Swedish and the Hawaiian soils shows that the climate benefit of sequestration can be very different depending on the time horizon used for comparing these soils. For time horizons below 40 years, more warming is avoided by the Mollisol soil in Hawai'i than by the Cambisol in Sweden. This is because the more productive site in Hawai'i draws down more C from the atmosphere than the less productive Swedish soil. However, decomposition is also faster in the Hawaiian soil and C returns back to the atmosphere faster than the Swedish soil. Therefore, for time horizons longer than 40 years, CBS is larger for the Swedish Cambisol where a small proportion of the original inputs remains stored for a longer time (Fig. 4).

This example shows the importance of considering two key factors related to C sequestration in soils and its climate benefit: the amount of inputs added to the soil, and the speed of decomposition, which determines the time C persists. Although much research has been done in quantifying inputs and rates of decay of SOC, much less attention has been paid to the time C persists in soils.

From these examples it is also possible to infer that management of soils for C sequestration purposes can be focused in balancing the amount of inputs and the time they will remain. This book contains many examples and methods to increase inputs or decrease decomposition of SOC. The conceptual framework provided by the concepts of CS and CBS, together with simulation models, can give important insights on how to best achieve climate goals through C management in soils.

In addition, CBS can be compared with atmospheric metrics that quantify the warming effect of an emission of a GHG ( $\text{CH}_4$ ,  $\text{N}_2\text{O}$ ,  $\text{O}_3$ , etc), namely the Absolute Global Warming Potential (AGWP), which is also expressed in units of  $\text{W m}^{-2} \text{ yr}$ . AGWP and CBS (with negative sign) can be added to obtain the net climate effect of simultaneous emissions and sequestration in a particular system, which can be useful to assess overall climate effects of multiple forms of ecosystem and soil management. In general, the AGWP of  $\text{CO}_2$  is much larger for 1 unit of C emissions than for 1 unit of sequestration for any timescale because fossil fuel emissions stay for much longer in the atmosphere-ocean-biosphere system [Sierra et al., 2021]. The emission of 1 Mg C to the atmosphere leads to a potential warming of  $3.4 \times 10^{-10} \text{ W m}^{-2} \text{ yr}$  on a 100-year time horizon [Joos et al., 2013], one order of magnitude higher than the potential avoided warming in any of the two soils from our example.

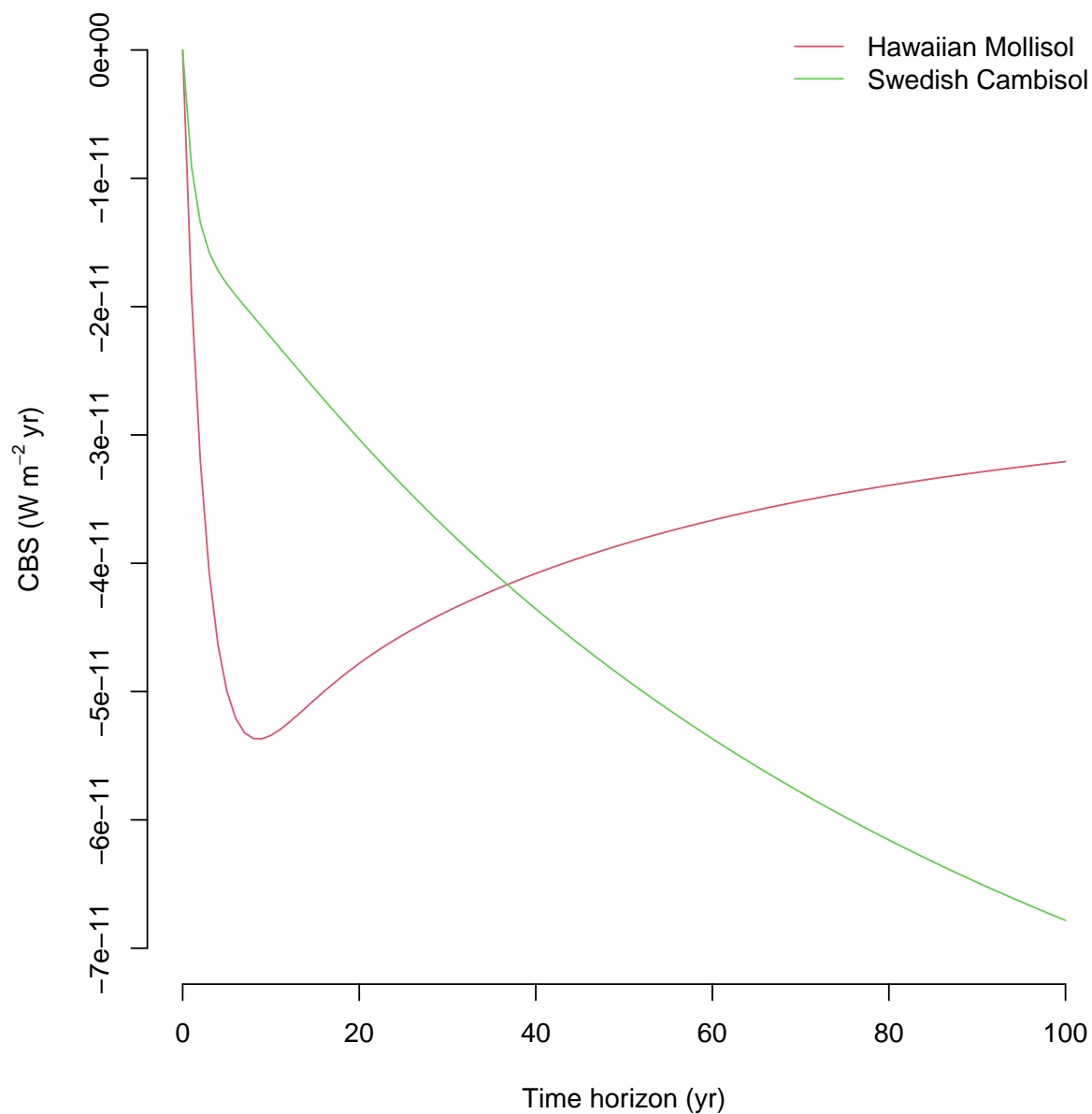


Figure 4: Climate benefit of sequestration (CBS) for a Mollisol in Hawai'i with  $3.5 \text{ Mg C ha}^{-1} \text{ yr}^{-1}$  of inputs versus a Cambisol in Sweden with inputs of  $2.0 \text{ Mg C ha}^{-1} \text{ yr}^{-1}$ . The negative CBS values indicate the warming avoided by storing C in these soils for different time horizons. Because inputs are larger in the Mollisol, more warming is avoided for time horizons below 40 yrs, but because a small proportion of the original input stays in the Cambisol for a long time, CBS is larger in the Cambisol for time horizons longer than 40 yrs.

## Discussion

Models of SOC dynamics are important tools to quantify C sequestration and related climate benefits. There is a large variety of models that can be used for this purpose, and a systematic classification and analysis of these models helps to better integrate different models within one single conceptual framework. Mathematically, we can classify models according to the presence of nonlinearities (linear or nonlinear models) and time dependencies (autonomous and non-autonomous models). These classes help to define the type of analysis methods and metrics that can be used for each model. Two metrics to analyze models in the context of soil C sequestration are the concepts of CS and CBS we presented here. These two metrics take into consideration the amount of C entering the soil and for how long this C remains stored.

Decisions such as the location of a site, options for land use, or aspects of management can affect both the amount of C inputs and the time that C may be expected to remain in the system [Sierra et al., 2021]. The amount of inputs are profoundly affected by land use and management changes such as deforestation, afforestation, conversion of managed pasture to conservation lands, removal of crop residues, among others [Lal, 2016, Chabbi et al., 2017, Minasny et al., 2017]. The amount of time C inputs remain in the system is affected by a management choice that would affect SOC persistence; e.g., due to site selection based on climate or environmental factors [Sierra et al., 2018b, Shi et al., 2020], and/or soil mineralogy such as poorly or non-crystalline minerals [Torn et al., 1997], or selection of particular soil amendments such as biochar [Guo and Chen, 2014]. Likewise, management decisions that consider both the organic C inputs and the persistence of those inputs can maximize the climate change mitigation potential through sequestration as a result of those choices.

Examples presented here were specifically for soil C sequestration, but the CS and CBS concepts may also be applied to any other system that can remove C from the atmosphere and store it for a particular amount of time. Examples include single trees or forest stands, aquatic ecosystems, or engineered C reservoirs (carbon capture and storage systems). The concepts can also be applied to “cradle to grave” life cycle analyses that allow for systems-level understanding of the consequences of any form of C management. A system is defined here in a very general form (equation 1) as a set of compartments or reservoirs that exchange mass among each other and interact with the environment. The definition of this system can explicitly consider the way human interventions affect its functioning by modifying inputs, transfer among components, and process rates. Soil is a classic example of a social-ecological system [Kibblewhite, 2018], where humans interact directly with environmental factors to influence the outcome, including soil C management [Amin et al., 2020]. The climate benefit of soil C sequestration may only be one component of a system defined by the boundaries of a site, jurisdiction, or nation.

In system-level assessment, often there is a need to directly compare the benefit of sequestration to that of direct emissions avoidance elsewhere [Verkerk et al., 2020]. For example, in the Hawaiian islands undisturbed native forests with high productivity in volcanic ash derived soils (that have very high capacity to stabilize soil C on poorly and non-crystalline mineral surfaces) are likely to have high CS [Giardina et al., 2014]. In this case, the avoided warming quantified by CBS may provide the same or greater value than the warming predicted by AGWP from direct emissions avoidance, and argue for prioritizing the protection of these forests. Afforestation of rangelands is often assumed to promote climate change mitigation, however in some systems soil C loss during land preparation and planting is greater than the tree biomass produced during the same time frame [Crow et al., 2016]. Furthermore, the time C inputs persist in the system as a whole is dependent on the fate of the final wood product, e.g., if the forest is managed for pulp/paper or pellets/biofuel (short timeframe), construction or furniture (medium timeframe), or conservation (long timeframe) [Sahoo et al., 2019, Schulze et al., 2020].

Finally, consider a food system, which is the social-ecological system encompassing all the drivers, activities and resources that go into producing, distributing and consuming food and includes waste stream and disposal [Miles et al., 2017]. A third of anthropogenic greenhouse gas emissions originate from food systems globally [Crippa et al., 2021]. Increasingly, incentives are offered to support producers to implement practices promoting soil health, for example, that also improve multiple co-benefits including water quality, human health, and climate change [Lehmann et al., 2020]. Many practices that promote soil health will also build soil organic matter and C, but not predictably and unlikely in a form that would provide meaningful CS that persists over time [e.g., Gattinger et al., 2012, Leifeld et al., 2013]. However, the CBS computation allows a more holistic quantification of improved sustainability of the food system that includes CS and avoided emissions associated with local production of food rather than importing. This is particularly important in island systems such as Hawai’i and other areas where regional accounting

places system boundaries at geopolitical borders [Miles et al., 2017].

## Summary

Mathematical models are essential for integrating different processes that control rates of soil C dynamics and for assessing C sequestration and related climate benefits. Many models have been proposed in the literature to predict C stocks and fluxes, with no overall consensus on the best model that can provide relevant insights at a large range of scales and for multiple questions. We reviewed general groups of models with their expected ranges of application. We also reviewed recent advances in using models of any level of detail to compute C sequestration, and the climate benefit of C sequestration. Using agricultural soils from Sweden and Hawai'i as examples, we show that new C inputs to the soil do not remain for long timescales, and only small proportions are stabilized. Although soils are a promising reservoir to store C and mediate emissions, long timescales are required to store amounts of C of relevance to mitigate climate change. The magnitude of climate benefit to mitigate warming through soil C sequestration is less than that of avoiding direct emissions, however, remains an important component of climate change mitigation and adaptation portfolios. Beyond the direct warming mitigation benefits, improved soil health through soil organic matter aggradation brings many co-benefits to the environment and local communities. Improved production practices and locally sourced food and energy feedstocks are associated directly with avoided emissions elsewhere in the food and energy system.

## Further reading

- A comprehensive review of soil C models can be found in Manzoni and Porporato [2009]. A conceptual classification of soil C models based on ecological principles can be found in Sierra and Müller [2015], with a further classification scheme in Sierra et al. [2018a].
- The concepts of CS and CBS, including all required formulas for their computations for different type of models, can be found in Sierra et al. [2021]. Background information on the concept of AGWP can be found in Rodhe [1990] and Joos et al. [2013].

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