Untangling the solar wind and magnetospheric drivers of the radiation belt electrons

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November 22, 2022

Abstract

Many solar wind parameters correlate with one another, which complicates the causal-effect studies of solar wind driving of the magnetosphere. Conditional mutual information (CMI) is used to untangle and isolate the effect of individual solar wind and magnetospheric drivers of the radiation belt electrons. The solar wind density (n_{sw}) negatively correlates with electron phase space density (PSD) (average energy ˜ 1.6 MeV) with time lag (t) = 15 hr. This effect of n_{sw} on PSD has been attributed to magnetopause shadowing losses, but when the effect of solar wind velocity (V_{sw}) is removed, t shifts to 7–11 hr, which is a more accurate time scale for this process. The peak correlation between V_{sw} and PSD shifts from t = 38 to 46 hr, when the effect of n_{sw} is removed. This suggests that the time scale for electron acceleration to 1–2 MeV is about 46 hr following V_{sw} enhancements. The effect of n_{sw} is significant only at L* = 4.5–6 (L* > 6 is highly variable) whereas the effect of V_{sw} is significant only at L* = 3.5–6.5. The peak response of PSD to V_{sw} is the shortest and most significant at L* = 4.5–5.5. As time progresses, the peak response broadens and shifts to higher t at higher and lower L*, consistent with local acceleration at L* = 4.5–5.5 followed by outward and inward diffusion. The outward radial diffusion time scale at L* = 5–6 is ˜40 hr per R_E.

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Abstract. Many solar wind parameters correlate with one another, which complicates the causal-effect studies of solar wind driving of the magnetosphere. Conditional mutual information (CMI) is used to untangle and isolate the effect of individual solar wind and magnetospheric drivers of the radiation belt electrons. The solar wind density (n_{sw}) negatively correlates with electron phase space density (PSD) (average energy ~ 1.6 MeV) with time lag (τ) = 15 hr. This effect of n_{sw} on PSD has been attributed to magnetopause shadowing losses, but when the effect of solar wind velocity (V_{sw}) is removed, τ shifts to 7–11 hr, which is a more accurate time scale for this process. The peak correlation between V_{sw} and PSD shifts from τ = 38 to 46 hr, when the effect of n_{sw} is removed. This suggests that the time scale for electron acceleration to 1–2 MeV is about 46 hr following V_{sw} enhancements. The effect of n_{sw} is significant only at L* = 4.5–6 (L* > 6 is highly variable) whereas the effect of V_{sw} is significant only at L* = 3.5–6.5. The peak response of PSD to V_{sw} is the shortest and most significant at L* = 4.5–5.5. As time progresses, the peak response broadens and shifts to higher τ at higher and lower L*, consistent with local acceleration at L* = 4.5–5.5 followed by outward and inward diffusion. The outward radial diffusion time scale at L* = 5–6 is ~40 hr per R_E.

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Plain Language Summary

Many solar wind parameters correlate with one another, which complicates the causal-effect studies of solar wind driving of the magnetosphere. We use conditional mutual information (CMI), which is part of information theory, to untangle and isolate the effect of individual solar wind and magnetospheric drivers of the radiation belt electrons. For example, the solar wind density negatively correlates with electron phase space density (PSD) (average energy ~ 1.6 MeV) with the response time lag of 15 hours. This has been attributed to the electron loss process called magnetopause shadowing. The time lag suggests the time scale for this process is 15 hours. However, when the effect of solar wind velocity is removed, the time lag is 7–11 hours, which is a more accurate time scale for this process. As another example, the time lag of the correlation between solar wind velocity and PSD shifts 38 to 46 hours, when the effect of solar wind density is removed. This suggests that the time scale for electron acceleration to 1–2 MeV is about 46 hours following the solar wind velocity enhancements. We also show that the effects of solar wind velocity and density have dependence on radial distance.

Keywords: radiation belt, relativistic electrons, solar wind drivers, nonlinear relationships, information theory, local acceleration, diffusion time scale, electron acceleration, magnetopause shadowing.

Index terms: 2774, 2784, 2720, 2730, 4499

Major science question:

New Science knowledge:

Broad Implications: Information theoretical tools can be useful to untangle and isolate individual solar wind and magnetospheric drivers of the radiation belt.

Key points: (1) The effect of n_{sw} on radiation belt electrons is significant only at L* = 4.5–6 and not significant at L* < 3. (2) The effect of V_{sw} on radiation belt electrons is significant at L* = 3.5–6.5 and not significant at L* < 3.5. (3) The radiation belt response time lag to V_{sw} suggests local acceleration at L* = 4.5–5.5 followed by outward and inward diffusion.

1. Introduction

The Earth's radiation belt is populated by electrons having energies of hundreds of keVs to >10 MeVs. These electrons are hazardous to satellites that encounter them in the inner-magnetosphere r ~2–8 R_E, including at the geosynchronous orbit (GEO), and at their foot points at low earth orbit (LEO) in the ionosphere, where 1 R_E = radius of the Earth = 6372 km. The MeV electrons can penetrate deep into spacecraft leading to spacecraft or instrument malfunctions while those with energies < 1 MeV can accumulate on the surface of the spacecraft bodies, leading to electrical discharges.

It has long been recognized that the variabilities of the radiation belt electrons, to a large extent, are driven by the solar wind (e.g., *Baker et al.*, 1990, 2018; 2019a; *Li et al.*, 2001; 2005; *Reeves*, 2007; *Ukhorskiy et al.*, 2004; *Turner and Li*, 2008, *Reeves et al.*, 2013; *Xiang et al.*, 2017; *Pinto et al.*, 2018; *Zhao et al.*, 2017, *Alves et al.*, 2017). However, many solar wind parameters positively and negatively correlate with one another, which can complicate the interpretation of the solar wind drivers of the radiation belt (e.g., *Hundhausen et al.*, 1970; *Wing et al.*, 2016; *Wing and Johnson*, 2019; *Borovsky*, 2012; 2016; 2018; 2020; *Maggiolo et al.*, 2017). For example, solar wind velocity (V_{sw}) positively correlates with radiation belt electron fluxes (J_e) (e.g., *Baker et al.*, 1990; *Reeves et al.*, 2011; *Balikhin et al.*, 2011; *Paulikas and Blake*, 1979; *Li et al.*, 2001; 2005; *Wing et al.*, 2016). Solar wind density (n_{sw}) negatively correlates with radiation belt J_e (e.g., Li et al., 2005; *Lyatsky and Kazanov*, 2008a; *Kellerman and Shprits*, 2012; *Rigler et al.*, 2007; *Balikhin et al.*, 2011; *Wing et al.*, 2016). However, V_{sw} negatively correlates with n_{sw} (e.g., *Wing et al.*, 2016; *Borovsky*, 2020). This raises the question that given the V_{sw}-n_{sw} negative correlation, if V_{sw} positively correlates with radiation belt electron J_e, then the negative correlation of n_{sw} with radiation belt electron J_e may simply be coincidental. Conversely, given the solar wind property, if n_{sw} negatively correlates with J_e, then the positive correlation of V_{sw} and radiation belt J_e may simply be coincidental. Of course, n_{sw} and V_{sw} may independently exert influence on the radiation belt electrons. In that case, how can one isolate the effect of an individual solar wind parameter?

A few studies attempted to separate the effects of n_{sw} from V_{sw} by using methods that bin the data into small intervals of V_{sw} and n_{sw} or explicitly select events when one parameter is nearly constant while the effect of the other parameter is investigated (e.g., *Lyatsky and Khazanov*, 2008a). This type of analysis has offered insights into the solar wind driving of the radiation belt J_e. However, holding one parameter nearly constant, either explicitly or through small binning, in order to investigate the second parameter does not completely eliminate the effect of the first parameter. For example, selecting events when V_{sw} is nearly constant to investigate the effect of n_{sw} does not completely eliminate the effect of V_{sw} because V_{sw} or its effect is not zero. Nearly constant but high V_{sw} can still affect the correlation of n_{sw} and radiation belt J_e. Moreover, it does not address the question of how much additional information n_{sw} provides to J_e, given V_{sw} and vice versa. Many studies have shown that other solar wind parameters and magnetospheric parameters can also contribute to J_e variations [e.g., *Balikhin et al*., 2011; *Rigler et al*., 2007; *Vassiliadis et al*., 2005; *Li et al*., 2005; *Onsager et al*., 2007; *Simms et al.*, 2014;], but presently, it is not entirely clear quantitatively given a main driver, e.g., V_{sw} (or n_{sw}), how much additional information these parameters provide to J_e.

The solar wind–radiation belt systems have been shown to be nonlinear [e.g., *Wing et al*., 2005; *Johnson and Wing*, 2005; *Reeves et al*., 2011; *Kellerman and Shprits*, 2012; *Wing et al*., 2016]. For nonlinear system, linear correlational analysis can be misleading [e.g., *Balikhin et al*., 2010; 2011].

Information theory has been shown to be quite useful for studies of the Earth's magnetosphere (*Balasis et al*., 2008; 2009; 2011; 2013; *Stumpo et al*., 2020; *Materassi et al*., 2011, *De Michelis et al*., 2011, 2017; *March et al*., 2005; *Johnson and Wing*, 2005; 2014, *Wing et al*., 2016; *Johnson et al*., 2018; *Wing and Johnson*, 2019; *Runge et al*., 2018; *Papadimitriou et al*., 2020; *Manshour et al*., 2021), Kronian magnetosphere (*Wing et al*., 2020), and the Sun (*Consolini et al*., 2009; *Wing et al*., 2018; *Snelling et al*., 2020). Information theory can help identify nonlinearities in the system and information transfer from one variable to another. Moreover, information theory can also help untangle the drivers that are positively or negatively correlated with one another (*Wing et al., 2016*; *Wing and Johnson*, 2019).

Wing et al. (2016) used information theoretic tools to study the solar wind driving of the radiation belt electrons. The study used the publicly available Los Alamos National Laboratory (LANL) satellite data, which provide geosynchronous electron flux measurements at daily resolution. When the study began in early 2015, the Radiation Belt Storm Probes (RBSP) or Van Allen Probes satellites had only been operational for a few years, and there was not enough data for a statistical study. Usage of the daily resolution of the radiation belt electron data prevented *Wing et al*. (2016) from resolving any electron response lag time to the solar wind drivers that is shorter than 24 hours. For example, the LANL MeV electron J_e negatively correlates with daily averaged n_{sw} with a lag time (τ) of 1 day. Zhao et al. (2017) correlated daily averaged radiation belt electron phase space density (PSD) with n_{sw} and also obtained τ = 1 day for MeV electrons (μ > 700 MeV G–1). However, if the effect of V_{sw} is properly removed, τ shifts to 0 day (*Wing et al*., 2016). In other words, the radiation belt electron response to n_{sw} is less than 24 hr. However, the study could not pinpoint exactly how much less than 24 hr with the daily resolution LANL data. Another limitation of *Wing et al*. (2016) study is that the LANL data only provide the electron observations at a fixed radial distance from the Earth, at the geosynchronous orbit.

Since *Wing et al.* (2016) study, RBSP has gathered seven years of radiation belt electron data (2013-2019) at high time resolution (< 1 min) from 2 < L* < 7. Hence, the time is ripe for a follow up study that uses the RBSP data. As in *Wing et al*. (2016), the present study uses information theory to determine the solar wind and magnetospheric drivers of the radiation belt electrons and the response time scales. In order to focus on the drivers of the nonadiabatic heating and acceleration, the present study examines the response of the radiation belt electron phase space density (PSD) to the drivers.

2. Data set

Van Allen Probe (or RBSP) mission, which was launched in 2012, had two identically instrumented spacecraft in near-equatorial orbit (about 10° inclination) with perigee at 600 km altitude and apogee at 5.8 R_E geocentric (*Mauk et al.*, 2013). The MAGnetic Electron Ion Spectrometer (MagEIS) and Relativistic Electron-Proton Telescope (REPT) instruments are part of the Energetic particle, Composition, and Thermal plasma Suite (ECT) instrument on board of RBSP (*Spence et al.,*, 2013). MagEIS measured the energy range of 30 keV to 4 MeV for electrons and 20 keV to 1 MeV for ions (*Blake et al*., 2013) while REPT measured electrons with energy range 1.5 to ≥ 10 MeV and protons with energy range 20 to 75 MeV (*Baker et al*., 2012).

The present study focuses only on the electron data. Radiation belt electron dynamics can often be described by their adiabatic invariants and PSD (μ, K, L*) where μ = the first adiabatic invariant related to the gyromotion perpendicular to the magnetic field line, K = the second adiabatic invariant related to the bounce motion along the field line, and L or L* = the third adiabatic invariant related to the curvature and gradient drift motion around the Earth (actually L* is inversely proportional to the third invariant Φ) (*Roederer*, 1970; *Schulz and Lanzerotti*, 1974;

Lejosne and Kollman, 2020).

Data from the RBSP-ECT Combined dataset (Boyd et al., 2021) are used to obtain electron PSD as a function of adiabatic invariants across the full MagEIS and REPT energy range. The PSD is calculated using the techniques outlined in (*Turner et al.*, 2014a; 2014b; *Boyd et al.*, 2014) at ~5 min time cadence. We select the electrons with μ = 725–875 MeV G^{-1} and K = 0.09–0.13 R_E $G^{-0.5}$. These electrons have an average energy of ~1.6 MeV, but they range from 480 keV to 4.8 MeV spanning over L* of 2.5 to 6.8.

The solar wind, AL, and SYM-H data 2013-2019 come from OMNI 1 min resolution data provided by NASA (http://omniweb.gsfc.nasa.gov/). Both the PSD and OMNI data 2013-2019 are averaged with 30 min sliding window.

We merge each OMNI solar wind parameter (V_{sw}, n_{sw} etc.) with the RBSP electron PSD. As described in Section 3, we perform time shifted correlation and conditional mutual information analysis to determine the radiation belt electron response lag time up to 120 hr. Depending on the solar wind parameter, typically the merged datasets have approximately 60,000 to 85,000 points.

3. Methodology

Mutual information and conditional mutual information are briefly described below, but they are also described in *Balasis et al*. (2013), and *Wing et al*. (2016, 2018).

Mutual information (MI) (*Tsonis*, 2001; *Li*, 1990; *Darbellay and Vajda*, 1999) between two variables, x and y, compares the uncertainty of measuring variables jointly with the uncertainty of measuring the two variables independently. The uncertainty is measured by Shannon entropy. In order to construct the entropies, it is necessary to obtain the probability distribution functions, which in this study are obtained from histograms of the data based on discretization of the variables (i.e. bins).

Suppose that two variables, x and y, are binned so that they take on discrete values, $\hat{x}$ and $\hat{y}$, where

$$x \in \{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n\} \equiv \aleph_1; \quad y \in \{\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_n\} \equiv \aleph_2 \qquad (1)$$

The variables may be thought of as letters in alphabets $\aleph_1$ and $\aleph_2$, which have n and m letters, respectively. The extracted data can be considered as sequences of letters. The entropy associated with each of the variables is defined as

$$H(x) = -\sum_{\aleph_1} p(\hat{x}) \log p(\hat{x}) ; \qquad H(y) = -\sum_{\aleph_2} p(\hat{y}) \log p(\hat{y}) \qquad (2)$$

where $p(\hat{x})$ is the probability of finding the word $\hat{x}$ in the set of x-data and $p(\hat{y})$ is the probability of finding word $\hat{y}$ in the set of y-data. To examine the relationship between the variables, we extract the word combinations $(\hat{x}, \hat{y})$ from the dataset. The joint entropy is defined by

$$H(x, y) = -\sum_{\aleph_1 \aleph_2} p(\hat{x}, \hat{y}) \log p(\hat{x}, \hat{y}) \qquad (3)$$

where $p(\hat{x}, \hat{y})$ is the probability of finding the word combination $(\hat{x}, \hat{y})$ in the set of (x, y) data.

The mutual information is then defined as

$$\mathrm{MI}(x, y) = \mathrm{H}(x) + \mathrm{H}(y) - \mathrm{H}(x, y) \qquad (4)$$

While MI is useful to identify nonlinear dependence between two variables, it is often useful to consider conditional dependency with respect to a conditioner variable z that takes on discrete values, $\hat{z} \in \{ z_1, z_2, \ldots z_n\} \equiv \aleph_3$. The conditional mutual information (*Wyner*, 1978)

$$\mathrm{CMI}(x, y \mid z) = \sum_{\aleph_1 \aleph_2 \aleph_3} p(\hat{x}, \hat{y}, \hat{z}) \log \frac{p(\hat{x}, \hat{y} \mid \hat{z})}{p(\hat{x} \mid \hat{z})\, p(\hat{y} \mid \hat{z})} = \mathrm{H}(x,z) + \mathrm{H}(y,z) - \mathrm{H}(x,y,z) - \mathrm{H}(z) \qquad (5)$$

determines the mutual information between x and y given that z is known. In the case where z is unrelated, $\mathrm{CMI}(x, y \mid z) = \mathrm{MI}(x,y)$, but in the case that x or y is known based on z, then $\mathrm{CMI}(x,y \mid z) = 0$. CMI therefore provides a way to determine how much additional information is known given another variable. CMI can be seen as a special case of the more general conditional redundancy that allows the variable z to be a vector (e.g., *Prichard and Theiler*, 1995; *Johnson and Wing*, 2014).

Herein, we use the short hand Pearson's linear correlation corr($x(t)$, $y(t + \tau)$) as corr($x \rightarrow y$). Likewise, CMI($x(t)$, $y(t + \tau)$| $z(t)$) is denoted as CMI($x \rightarrow y$| z). We define i_{tr} = information transfer = CMI($x \rightarrow y$| z) – mean noise, where noise = CMI(sur(x) $\rightarrow y$| z), sur(x) is the surrogate data of x and is obtained by randomly permuting the order of the time series of array x. Mean and σ of the noise are calculated from an ensemble of 100 values of CMI(sur(x) $\rightarrow y$| z). The mean noise and σ estimate are valuable diagnostics included on all of the CMI data presented here: any CMI outside the 3σ noise range are significant and CMI less than the 3σ from the noise is considered not significant. Furthermore, we define $i_{tr\_max} = i_{tr}$ at the peak τ and significance = i_{tr}/σ.

4. Applying information theory to radiation belt MeV electron data

4.1 A simple example of an application of conditional mutual information (CMI)

CMI can be quite useful to untangle the effects of multiple drivers of a system. Figure 1 presents a simple example that illustrates this point. Figure 1a plots corr($V_{sw} \rightarrow$ PSD). The figure shows that V_{sw} positively correlates with PSD and the correlation peaks at τ = 38 hr. The correlation is significant with n = 84,729 points and correlation coefficient (r) = 0.47 and $p < 0.01$. Previous studies have also found good correlations between V_{sw} and radiation belt electrons with ~2 days lag and the lag time has been attributed to the time scale to accelerate the electrons to 1–2 MeV due to local acceleration, radial transport, or some other acceleration mechanisms (e.g., *Baker et al*., 1990; *Shprits et al.*, 2008; 2009; *Reeves et al*., 2011; 2013; *Li et al*., 2005; *Ukhorskiy et al*., 2005; *Wing et al.*, 2016; *Summers et al*., 2007; *Thorne et al*., 2013; *Newell et al*., 2016;

Turner and Li, 2008; *Boyd et al.*, 2016; 2018). Figure 1b plots the corr($n_{sw} \rightarrow$ PSD), which shows that n_{sw} negatively correlates with PSD with a minimum at τ = 15 hr, r = –0.22, n = 84,729, p < 0.01. *Lyatsky and Khazanov* (2008a) also found the same negative correlation at τ = 15 hr. The negative correlation has been previously attributed to the magnetopause shadowing effect: an increase in n_{sw} would increase solar wind dynamic pressure (P_{dyn}), which would compress the magnetosphere leading to radiation belt electron losses (e.g., *Li et al.*, 2001; *Kellerman and Shprits*, 2012; *Turner et al.*, 2012; *Ukhorskiy et al.*, 2006). Figure 1c plots the corr($P_{dyn} \rightarrow$ PSD), which shows that the radiation belt electron response to P_{dyn}, which is ~ $n_{sw} V_{sw}^2$, has dual modes. At small τ, τ < ~20 hr, P_{dyn} negatively correlates with PSD, which is similar to the effect of n_{sw} and can be attributed to the magnetopause shadowing effect. However, at large τ, τ > 40 hr, P_{dyn} positively correlates with PSD, which is similar to the effect of V_{sw}. The correlations are significant at p < 0.01 and n = 82,652. Zhao et al. (2017) correlated P_{dyn} with PSD at daily time resolution and also found a dual response mode of the PSD to P_{dyn}. Figure 1d plots CMI($P_{dyn} \rightarrow$ PSD| n_{sw}), which shows the dependence of PSD on P_{dyn}, given n_{sw}. It shows that if we remove the effect of n_{sw}, the effect of P_{dyn} on PSD is similar to that of V_{sw} in Figure 1a, as expected. The CMI curve does not match exactly the correlation curve in Figure 1a because the CMI curve takes into account the nonlinearities in the data.

In Figure 1d, the green solid and dashed curves are mean noise and 3σ from the noise, respectively. The significance at peak τ = 51 hr is 203σ and hence it is significant.

4.2 Isolating the effects of the solar wind velocity from density and vice versa

Wing et al. (2016) isolated the effects of V_{sw} and n_{sw} on the radiation belt electron J_e using CMI. They found that CMI($V_{sw} \rightarrow J_e$| n_{sw}) peaks at τ = 2-3 days while CMI($n_{sw} \rightarrow J_e$| V_{sw}) peaks at $\tau = 0$ day. However, the lag times, τ, in *Wing et al*. (2016) are imprecise due to the usage of the daily resolution LANL electron data. Furthermore, LANL data are limited to GEO, but the outer belt is not accurately represented by data at GEO alone, especially for the heart of the outer belt between $4 \leq L \leq 5$ (e.g., *Baker et al*., 2019b). In the present study, we recompute the CMIs using 30 min resolution RBSP PSDs representative of ~1 MeV electrons throughout the entire outer belt and solar wind data. Furthermore, by using electron PSD for fixed values of the first and second adiabatic invariants in place of J_e as a function of energy, the data used here further deconvolute the energy and pitch angle dependencies of the underlying physical processes that drive radiation belt enhancements and losses.

Figures 2a and 2b replot corr($V_{sw} \rightarrow$ PSD) and corr($n_{sw} \rightarrow$ PSD), which are plotted in Figures 1a and 1b, respectively. However, V_{sw} negatively correlates with n_{sw} and corr($V_{sw} \rightarrow n_{sw}$) has a minimun at $\tau = 15$ hr ($r = -0.48$, $n = 105{,}459$, $p < 0.01$), as shown in Figure 2c. For completeness, Figure 2c also plots corr($n_{sw} \rightarrow V_{sw}$) (red curve), which has $r = 0.10$, $p < 0.01$. The figure shows that $|\text{corr}(V_{sw} \rightarrow n_{sw})| > |\text{corr}(n_{sw} \rightarrow V_{sw})|$. The negative correlation between V_{sw} and n_{sw} have been previously reported with similar τ (e.g., *Wing et al*., 2016; *Maggiolo et al*., 2017; *Borovsky*, 2020). Note that τ may vary from year to year (*Wing et al*., 2016), leading to an overall broadening of the peak when considering an ensemble of intervals across the solar cycle.

Given that V_{sw} negatively correlates with n_{sw}, if n_{sw} negatively correlates with PSD (Figure 2b), then the positive correlation between V_{sw} and PSD may be deemed just coincidental. To check on this possibility, we compute CMI($V_{sw} \rightarrow$ PSD| n_{sw}), which is plotted in Figure 2d. The figure shows that even after the effect of n_{sw} has been removed, there is still strong information transfer from Vsw to PSD, which peaks at $\tau = 46$ hr ($n = 78{,}811$, significance = 378σ). Apparently, removing the effect of n_{sw}, shifts the peak to the right. The lag time of $\tau = 46$ hr obtained from

CMI is considered a more accurate radiation belt electron response time to the V_{sw} (the time scale for electron acceleration to 1-2 MeV energy range) than the lag time of τ = 38 hr obtained from Pearson's correlation.

We can also check whether or not corr($n_{sw} \rightarrow$ PSD) is coincidental, given the V_{sw} negative correlation with n_{sw} and V_{sw} positive correlation with PSD. Figure 2e plots CMI($n_{sw} \rightarrow$ PSD| V_{sw}), which shows two peaks. The primary peak at τ = 7–11 hr (n = 78,811, significance = 52σ) can be compared to the minimum in corr($n_{sw} \rightarrow$ PSD) in Figure 2b. Apparently, removing the effect of V_{sw}, shifts the peak to the left. The lag time of τ = 7–11 hr is considered a more accurate radiation belt electron response time to the magnetopause shadowing effect than the τ = 15 hr obtained from the correlational analysis. Note that CMI only gives positive values and does not distinguish negative from positive correlations. As such, CMI is analogous to $|r|$.

Figure 2e shows that there is a secondary broad peak at τ = 80–120 hr (or even larger). Unlike the primary peak, which is a negative correlation, the secondary peak is a positive correlation with n_{sw}. In order to show this, we plot PSD(t + τ) vs. V_{sw}(t) vs. n_{sw}(t) for τ = 0, 5, 10, 40, 80, 100, and 120 hr in Figure 3 panels a to g, respectively. In all panels, it can be seen that at high V_{sw}, V_{sw} > ~500 km s^{-1}, V_{sw} positively correlates with the radiation belt electron PSD as previously reported (*Reeves et al.*, 2011; *Li et al.*, 2001; *Kellerman and Shprits*, 2012; *Wing et al.*, 2016). However, for V_{sw} < ~450 km s^{-1}, and small τ (τ = 0, 5, and 10 hr), n_{sw} negatively correlates with the radiation belt electron PSD, which is consistent with the first and primary peak of CMI(n_{sw} $\rightarrow$ PSD| V_{sw}) in Figure 2e, which can be attributed to the magnetopause shadowing effect. At τ = 40 hr, there is little or no correlation between n_{sw} and the PSD, which is consistent with the minimum of CMI($n_{sw} \rightarrow$ PSD| V_{sw}) in Figure 2e. However, at large τ, τ = 80, 100, 120 hr and V_{sw} < ~450 km s^{-1}, n_{sw} positively correlates with PSD, which is consistent with the secondary peak in

Figure 2e. This positive correlation between n_{sw} and electron PSD at large τ cannot be seen in the corr($n_{sw} \rightarrow$ PSD) in Figure 2b because the effect has been smeared or cancelled out by the effects for all V_{sw} where high electron PSD can correspond to high and low n_{sw} (Figure 3 panels e–g).

It is not clear what causes the positive linear and nonlinear correlation between n_{sw} and electron PSD at large τ. n_{sw} may be a proxy for another parameter. As an example, n_{sw} positively correlates with |IMF B| (peak τ = 6 hr, r ~ 0.40, n = 105,923, p < 0.01) as plotted in Figure 4a, which has implications for the radiation belt electrons. This correlation has been previously observed (*Borovsky*, 2020; *Maggiolo et al*., 2017). Larger n_{sw} would correspond to more negative IMF B_z when IMF B_z is southward (IMF B_z < 0) and more positive IMF B_z when IMF B_z is northward (IMF B_z > 0), as shown in Figures 4b and 4c, respectively. Figure 4b shows that n_{sw} negatively correlates with southward IMF (peak τ = 6 hr, r = 0.3, n = 54,265, p < 0.01) while Figure 4c shows that n_{sw} positively correlates with northward IMF (peak τ = 8 hr, r = 0.3, n = 51,388, p < 0.01). The n_{sw} negative correlation with IMF B_z < 0 would be relevant to the discussion of the effect n_{sw} on the radiation belt electrons because prolonged southward IMF can lead to subtorms (e.g., *Birn and Hones*, 1981; *Rostoker*, 1983; *McPherron et al*., 1986; *McPherron*, 1991; *Lui*, 1991; *Lyons*, 1995; *Lyons et al*., 1997; *Johnson and Wing*, 2014; *Birn et al*., 1997; 2006), which in turn can lead to injections of seed electrons, enhanced chorus wave activity, and the growth of the MeV radiation belt electrons (e.g., Miyoshi et al., 2013; Jaynes et al., 2015; see Section 5.3).

n_{sw} may also be correlated or be a proxy for other solar wind parameters. For example, *Kepko et al*. (2019) reported that a sharp density increase can be accompanied by quasiperiodic density fluctuations that can lead to the growth of the globally coherent magnetospheric ultralow frequency (ULF) waves, which can, in turn, accelerate radiation belt electrons (e.g., *Matthie and*

Mann, 2000; *O'Brien et al.*, 2003; *Ukhorskiy et al.*, 2005; *Hudson et al.*, 2008; *Li et al.*, 2009; *Mann et al.*, 2016; *Zhao et al.*, 2018; *Jaynes et al.*, 2018). Hence, the long range correlation of (n_{sw}, PSD) and (IMF $B_z > 0$, PSD) may be attributed to the density fluctuations and ULF waves. Whatever the mechanism is, our result shows that the time scale for such process to energize electrons to 1–2 MeV is slow, > 70 hr.

4.3 The radial dependence of the radiation belt electrons on the solar wind density and velocity

The effects of the n_{sw} and V_{sw} on the radiation belt electrons have a dependence on the radial distance or L* (e.g., *Baker et al.*, 2019a; *Tang et al.*, 2017a; *Turner et al.*, 2019; *Zhao et al.*, 2017; *W. Li et al.*, 2014). In order to show this, PSD data are binned from L* = 3 to 6.5 into 7 bins with bin width = 0.5. The data coverage for the electrons with μ = 725–875 MeV G^{-1} and K = 0.09–0.13 R_E $G^{-0.5}$ is poor for L* < 3 and L* > 6.5. Figure 5 shows corr($n_{sw} \rightarrow$ PSD) as a function of L*. It shows that the correlation is near 0 at L* = 3–3.5, but slowly decreases with increasing L*. Except for L* = 3–3.5 (n = 8435, Figure 5a), the correlations are significant ($p < 0.01$) at the minimum τ (τ = 15, 16, 17, 17, 17, 13 hr), r = (–0.086, –0.17, –0.27, –0.31, –0.34, –0.50), (n = 8,302, 11,481, 17,7891, 27,060, 6,236, 528) for panels (b–g), respectively. The number of points in L* = 6–6.5 (Figure 5g) is the fewest, which results in a noisier correlation. However, as discussed in the Sections 1, 4.1, and 4.2, the correlation may be inaccurate because n_{sw} negatively correlates with V_{sw} (Figure 2c) and the system is nonlinear (Wing et al., 2016). Hence, we calculate CMI($n_{sw} \rightarrow$ PSD| V_{sw}) for the PSD data in the same bins. The results are plotted in Figure 6. The figure shows that the effect of n_{sw} on PSD is at the noise level at L* = 3–4.5, is significant at L* = 5–6, and back to the noise level at L* = 6–6.5. However, the result for L* = 6–6.5 is considered not reliable because of the small number of points (n = 528). The response lag times based on the first peak are τ = 9, 10, and 7 hr (n = 16,629, 25,238, and 5865; significance = 17, 42, and 5 σ) at L* = 4.5–5, 5–5.5, and 5.5–6, respectively. Notice that the response lag times are quite different in Figures 5 and 6.

For completeness, we examine the effect of P_{dyn} on the PSD. Figure 7 plots CMI($P_{dyn} \rightarrow$ PSD| V_{sw}) as a function of L* in the same format as Figure 6. It shows that the effect of P_{dyn} on PSD is similar to that of n_{sw}, as expected. The largest effect of P_{dyn} on electron PSD can be found at L* = 4.5–6. (significance = 32, 53, 17 σ for L* = 4.5–5, 5–5.5, and 5.5–6), respectively. The number of points in each bin in Figure 6 is the same as that in Figure 7. Taken together, Figures 6 and 7 suggest that the magnetopause shadowing is effective only at L* = 4.5–6. At L* = 6–6.5, there is a high variability of PSD and the bin has fewest number of points and hence the result is inconclusive.

Moreover, the significances are higher for the first peak in Figure 7 panels d–f than their counterparts for CMI($n_{sw} \rightarrow$ PSD| V_{sw}) in Figure 6 panels d–f, suggesting that the real causal parameter for the magnetopause shadowing is P_{dyn} rather than n_{sw}. However, the opposite is true for the secondary peak. This would suggest that the secondary peak may be more causally related to n_{sw} (or its proxy) than P_{dyn}.

We perform the same analysis with V_{sw} and electron PSD. Figure 8 shows corr($V_{sw} \rightarrow$ PSD) as a function of L* in the same format as Figure 5. The correlations are all significant at $p < 0.01$ at the maximum τ (τ = 40–120, 30–120, 38, 30, 37, 45, 30–90 hr), (r = 0.14, 0.27, 0.42. 0.57, 0.62, 0.64, 0.70) for L* = 3–3.5, 3.5–4, 4.–4.5, 4.5–5, 5–5.5, 5.5–6, and 6–6.5, respectively. Baker et al. (2019a) also found higher correlation with increasing L (r = 0.32, 0.51, and 0.61 for L = 3.5, 4.5, and 5.5, respectively). For comparison, Figure 9 plots CMI($V_{sw} \rightarrow$ PSD| n_{sw}) as a function of

L* in the same format as Figure 6. The number of points in each bin is the same as that in Figure 6. Figure 9 shows that the CMI is at the noise level at L* = 3–3.5 (panel a), unlike its counterpart in Figure 8a. At L* = 3.5–4.5, the CMI has a broad peak from $\tau \sim 50$–100 hr (or larger for the case L* = 3.5–4) and the peaks are significant (peak significance = 18 and 46 σ for L* = 3.5–4 and 4–4.5), respectively. At L* = 4.5–6, the CMI peaks are narrower (peak τ = 40, 46, 57 hr; significance = 236, 399, and 100 σ), respectively. Interestingly, at τ = 6–6.5, the peak broadens again but remains significant (peak significance = 19 σ). To help visualize the evolution of the CMI, red dashed vertical lines at τ = 40 hr is drawn in Figure 9.

The radiation belt electron response lag times as a function of L* is further examined in Figure 10. The figure shows the normalized i_{tr} for each L* bin (the blue curve subtracted by the solid green curve in Figure 9). The orange and yellow color correspond roughly to the top 20% of i_{tr} in each L* bin. At L* = 4.5–5.5, response lag time peaks around τ = 35–50 hours, which is consistent or close to the previously reported time scale of 2 days to accelerate electrons to 1-2 MeV (*Paulikas and Blake*, 1979; *Reeves et al*., 2011; *Li et al*., 2001; *Wing et al*., 2016). At higher L*, L* = 5.5–6, the peak broadens and shifts to larger τ, τ = 45–65 hr. At L* = 6–6.5, the peak is even broader at τ = 40–100 hr. At lower L*, L* = 4–4.5 and 3.5–4, one can also see successive broadening of the peak as the peak shifts to larger τ, τ = 35–55 hr and τ = 45–60 hr, respectively.

Our result is consistent with local acceleration where the peak of the electron acceleration region is located at L* = 4.5–5.5, from where electrons diffuse outward and inward (Section 5.3). This scenario would be consistent with the *O'Brien et al*. (2003) study that found the peak of the 1.5 MeV electron microburst precipitation is located at L = 4.5. It would also be consistent with the *Green and Kivelson* (2004) study that found evidence for local acceleration near L* = 5. More recently, in the RBSP era, Tang et al. (2017a) found that the peak flux of 1 MeV electrons is located mostly at L ~ 4–5 in 74 storm events and they also found evidence for local acceleration. *Boyd et al*. (2018) found the peak PSD is located mostly at L* = 4.5 – 5.5 in 80 storm events. They concluded that 70 out of 80 events show evidence of local acceleration based on the PSD vs. L* spectra. These results are consistent with our interpretation of Figures 9 and 10 that the L* band with the information peak at the smallest τ corresponds to the local acceleration region. The shifting of the peak to a larger τ at higher or lower L* suggests outward or inward diffusion, respectively.

However, there is also evidence that suggests localized acceleration in tandem with outward or inward diffusion originating from L* = 4.5–5.5 (e.g., Allison and Shprits, 2020). For example, at L* = 4–4.5, one can see that the i_{tr} starts increasing at τ = 25 hr, very much about the same time i_{tr} increases at L* = 4.5–5.5 hr, but the significance is lower (Figure 9). This may suggest that the whistler mode chorus waves are also present at L* = 4–4.5 and not all 1–2 MeV electrons are transported from L* = 4.5–5.5. The same dynamics can be seen at the outermost L*, L* = 6–6.5.

The radial diffusion time scale can be estimated from the peak τ at each L* band. In Figure 10, the peak τ increases from ~ 40 hr at L* = 4.5–5.5 to ~60 hr at L* = 5.5–6, suggesting outward diffusion time scale of 40 hr per R_E. The diffusion time scale of 40 hr (or about 2 days) per R_E can be compared with the theoretical estimate of 1–6 days that is attributed to ULF waves at L* = 6 (e.g., *Elkington et al*., 2003).

4.4 The dependence of the radiation belt electrons on the magnetospheric state

So far we have analyzed the dependence of the radiation belt electron PSD on the solar wind drivers, specifically V_{sw}, n_{sw}, and P_{dyn}. However, the radiation belt electrons depend not just on the external (solar wind) drivers but also the internal state of the magnetosphere (e.g., *Reeves et al*., 1998; *Baker et al.*, 2019a; *Lyatsky and Khazanov*, 2008b; *Borovsky and Denton*, 2014; *Tang et al*., 2017b; *Borovsky*, 2017; *Zhao et al*., 2017). In order to determine how the radiation belt electrons depend on the internal state of the magnetosphere, we examine the relationships of the electron PSD with AL and SYM-H indices. SYM-H index gives a measure of the strength of the ring current and storm (*Iyemori*, 1990) while AL gives a measure of the strength of the westward auroral electrojets and substorm (*Davis and Sugiura*, 1966). SYM-H is similar to Disturbance Storm Time (Dst) index (*Dessler and Parker*, 1959), except that SYM-H index is defined to have a one minute time resolution whereas Dst index has one hour resolution. Both SYM-H (proxy for storms) and AL (proxy for substorms) can be associated with plasma injections to the inner magnetosphere, which can enhance the whistler mode chorus waves and provide the seed population for the local acceleration (*Katus et al*., 2013; *Wing et al*., 2014).

Figure 11a plots corr(AL $\rightarrow$ PSD), which shows that AL negatively correlates with electron PSD with a minimum $\tau \sim 53$ hr (n = 70,125, r = –0.33, $p < 0.01$). Note that a smaller (more negative) AL corresponds to a more intense substorm and larger auroral electrojets. Figure 11b plots corr($V_{sw} \rightarrow$ PSD) (same as Figure 1a). Figure 11c plots corr($V_{sw} \rightarrow$ AL), which shows that the auroral electrojet response to V_{sw} is fairly quick $\tau = 0$ hr (< 30 min) (n = 82,995, r = –0.40, $p < 0.01$). Similar correlation was obtained by *Smirnov et al*. (2020). Given the positive correlation of V_{sw} and electron PSD, and the negative correlation of V_{sw} and AL, one may ask the question whether the negative correlation between AL and electron PSD may just simply be coincidental or whether AL can independently affect PSD. Figure 11d plots CMI(AL $\rightarrow$ PSD| V_{sw}), which shows that the radiation belt electrons still have strong dependence on AL even after the effect of V_{sw} has been removed. The CMI peaks at $\tau \sim$ 50–80 hr (significance = 58σ, n = 64,564), suggesting that perhaps the time scale to accelerate electrons to 1–2 MeV energy range from the time of substorm onset or substorm particle injection is about 50–80 hr. This time scale is an ensemble average for all L*. To help visualize the comparison of Figures 11a and 11d, a dashed vertical red line is drawn at τ = 53 hr (the minimum of corr(AL $\rightarrow$ PSD)). It shows that removing the effect of V_{sw}, shifts the peak to the right (to a larger τ) by a little bit (~10 hr).

Figure 12 displays the normalized i_{tr} as a function of L* and τ where i_{tr} = CMI(AL $\rightarrow$ PSD| V_{sw}) – mean noise in the same format as Figure 10. The i_{tr} is at the noise level at L* = 3–3.5 (n = 6409). At L* = 6–6.5, there is a high variability in PSD and the bin has the fewest number of points (n = 216) and hence the result is unclear and not shown. The region with the largest significance is L* = 4.5–5 and 5–5.5 with peak τ = 40–80 and 45–85 hr and peak significance = 33 and 38 σ (n = 13,825 and 20,527) respectively. The peak shifts to larger τ, τ = 75–100 hr, at L* = 5.5 –6 (n = 4686, peak significance = 18σ), suggesting outward diffusion from L* = 4.5–5.5. At L* = 4–4.5, the peak is broad, τ = 35–80 hr (n = 8802; peak significance = 19 σ). It is not clear what causes this broad peak. Because the peak starts at the same time or even earlier than at L*=4.5–5.5, it may suggest the whistler wave activity is present nearly simultaneously from L* = 4–5.5. However, the peak seems to decay slowly suggesting perhaps inward diffusion or longer lasting wave activity at L* = 4–4.5. At L* = 3.5–4, the peak is broad at τ = 60–110 hr, but the peak significance is relatively small (n = 6379, peak significance = 9 σ), which would be consistent with inward diffusion from higher L*.

Iles et al. (2006) examined a substorm event and found that peak PSD for electrons > 0.8 MeV is located at L* = 4.3–5.5, which is close to the L* band with the largest CMI significance, L* = 4.5–5.5. They also found evidence of local acceleration and radial diffusion.

We perform similar analysis with SYM-H. Radiation belt electrons have been found to show great variabilities during storms (e.g., *Reeves et al*., 2003; *Turner et al*., 2019; *Baker et al*., 2019a). Figure 13 is similar to Figure 11, except that it is for SYM-H instead of AL index. Figure 13a plots corr(SYM-H → PSD), which shows that SYM-H negatively correlates with electron PSD with a minimum at $\tau \sim 40$ hr (n = 91,589, r = –0.35, $p < 0.01$) while Figure 13b plots corr(V_{sw}→ PSD) (same as Figure 11b). V_{sw} negatively correlates with SYM-H and the correlation has a minimum at $\tau = 2$–4 hr (n = 91,589, r = –0.35, $p < 0.01$) (Figure 13c), suggesting that the ring current response to V_{sw} has a lag time of about 2–4 hr. Similar correlation was obtained in previous studies (e.g., *Maggiolo et al*., 2017). Figures 13 a–c pose the same dilemma as Figures 11 a–c do for AL. That is, given the positive correlation of V_{sw} and electron PSD and the negative correlation of V_{sw} and SYM-H, one may ask whether the negative correlation of SYM-H and PSD could just simply be coincidental or whether SYM-H provides additional information about PSD. Figure 13d plots CMI(SYM-H → PSD| V_{sw}), which shows that SYM-H indeed provides additional information to electron PSD even after the effect of V_{sw} has been removed. The CMI peaks at $\tau \sim$ 30–70 hr (n = 84,729, significance = 109 σ), suggesting that perhaps the time scale to accelerate electrons to 1–2 MeV energy range from the time of ring current enhancement is about 30–70 hr. However, the effect of SYM-H has a dependence on radial distance, as discussed next.

Figure 14 plots i_{tr} as a function of L* where i_{tr} = CMI(SYM-H → PSD| V_{sw}) – mean noise in the same format as Figure 12. The figure suggests a complicated relationship between SYM-H and radiation belt electrons. At L* = 6–6.5, i_{tr} is at the noise level, which can be attributed to high PSD variabilities as well as small number of points (n = 336). Studies found that storms can increase, decrease, or have little effect on radiation belt electron fluxes at geosynchronous orbit (e.g., *O'Brien et al*., 2001; *Reeves et al*., 2003). An increase in SYM-H (or ring current) would decrease magnetospheric |B| by greater proportion with increasing L* because magnetospheric |B|

decreases with r^{-3} (e.g., *Turner et al*., 2012). The effect of the reduction in |B| would be a decrease in PSD due to the outward transport because of the third adiabatic invariant. This Dst or SYM-H effect would counter the effect of the PSD enhancement due to the storm plasma injection and the ensuing local acceleration.

The highest peak significance can be found at L* = 5 – 5.5 (peak τ = 20–55 hr, significance = 79σ, n = 27,060). The peak shifts to higher τ at higher L*, suggesting outward diffusion. At L* = 5.5–6, the peak can be found at τ = 60–75 hr (n = 6236, peak significance = 26 σ). At L* = 4–4.5 and 4.5–5, the i_{tr} peaks at τ = 30–60 and 30–75 hr (n = 11,495 and 17,924; peak significance = 38 and 47 σ, respectively). The large i_{tr} at L* = 4–5 would be roughly consistent with *Tang et al*. (2017a), which found that the peak flux of 1 MeV electron at L = 4–5 in 74 storm events and with *Green and Kivelson* (2004), which found local acceleration at L* = 5.

At L* = 3.5–4, the peak is very broad at τ = 20–120 hr or even higher (n = 8317, peak significance = 23 σ). It is not clear what causes such a broad peak. The response at τ < 30 hr suggests that the 1-2 MeV electrons at this time could not have originated from higher L* shell. Thus, it may suggest the presence of wave activity at this L* band. On the other hand, the response at τ > 80 hr, may suggest inward diffusion from higher L* shell or long-lasting wave activity. At L* = 3–3.5, the CMI peaks at τ = 110–120 hr or even higher (n = 8435, peak significance = 14). The analysis stops at τ = 120 hr. The large τ may suggest slow diffusion from outer L* shell. The slow diffusion at small L* would be consistent with the expected radial diffusion time scale that increases with decreasing |B| and L*, but this needs to be investigated further. It is worth noting that out of all the parameters that we have examined, only SYM-H can provide information about radiation belt electron PSD at L* = 3–3.5 albeit only a small amount. The CMI at L* = 3–3.5 and τ < 100 hr is low, which is consistent with *Turner et al*. (2019) study that found storms have little effect on 1–2 MeV electrons at L < 3.5 (see their Figure 2), but apparently at $\tau > 100$ hr, the storm effect is significant but only moderately.

The complicated response of the radiation belt to SYM-H or storm can be attributed to multiple factors. The general response of the storm plasma injection is PSD enhancement through local acceleration. However, this effect is countered by the Dst (SYM-H) effect due to the ring current enhancement, which would cause outward electron drift and a decrease PSD because of the the third adiabatic invariant. This Dst (SYM-H) effect is stronger with increasing L* because at |B| decreases with r^3. Moreover, different type storms can affect radiation belt electrons differently. For example, *Turner et al* (2019) reported that full coronal mass ejection (CME) storms cause MeV electron enhancements at L < ~5 while stream interaction region (SIR) storms cause enhancements at L > ~4.5. CME sheaths and CME ejecta can cause depletions throughout the outer radiation belt.

4.5 The rankings of solar wind and magnetospheric parameters by the information transfer to the radiation belt electrons

In the previous sections, we calculate the dependence of the PSD on V_{sw}, n_{sw}, P_{dyn}, AL, and SYM-H. V_{sw} transfers the most information to the PSD by significantly larger amount than any other solar wind variables. In this section, we calculate the CMI from other solar wind parameters to the PSD, given V_{sw}. Specifically, we calculate CMI($x \rightarrow$ PSD| V_{sw}) where $x =$ IMF |B|, $B_z < 0$, $B_z > 0$, B_y, Esw, and σ(IMF B).

Table 1 ranks these parameters based on the information transfer to the radiation belt electron PSD, given V_{sw} for L* = 3 – 6.5. The information transfer is calculated as i_{tr_max} = maximum of (CMI – mean noise). The information transfer from V_{sw} to the PSD is calculated from CMI($V_{sw} \rightarrow$ PSD| n_{sw}). It shows the dominance of V_{sw} in terms of information transfer to the PSD. SYM-H, which is ranked second, transfers only about a quarter as much information to the PSD. In Table 1, if the response lag time has a broad peak, τ is reported as having a range of values. Table 1 shows that the radiation belt electron response lag time to the solar wind and magnetospheric parameters fall into two categories. The electron response with a small τ ($\tau <$ 15 hr) is a decrease in PSD (electron loss) while the response at large τ ($\tau >$ 40 hr) is an enhancement in PSD.

Zhao et al. (2017) correlated PSD with solar wind (Vsw, nsw, Pdyn) and magnetospheric parameters (SYM-H, AL) and found that AL has the best correlation with PSD with t = 2-5 days for $\mu >$ 700 MeV G^{-1}. However, their study differs from the present study in two key aspects: (1) their study used daily resolution data; (2) more importantly, they did not remove the effect of Vsw from AL, SYM-H, and other parameters. The second point is particularly consequential because AL negatively correlates with V_{sw} (Figure 11c) and some of the good correlation between AL and PSD can be partially attributed to the good correlation between V_{sw} and PSD.

Many of the parameters, namely IMF |B|, IMF $B_z < 0$, IMF B_y, n_{sw}, and P_{dyn}, produce dual response modes in the radiation belt electrons. At small τ ($\tau <$ 15 hr), the response is a decrease in PSD or electron loss while at large τ ($\tau >$ 30 hr), the response is an enhancement in PSD. For these parameters, the ranking is based on the mode that has the higher i_{tr_max}. The response to IMF |B| has roughly the same i_{tr_max} at small and large τ, although Table 1 lists the response to the large τ. The response to IMF|B|, IMF $B_z < 0$, and IMF B_y at $\tau <$ 15 hr is electron loss and is mainly due to their correlations with n_{sw}. If the effect of n_{sw} is removed, this peak will diminish or disappear. An example for IMF $B_z > 0$ is illustrated in Figures 4e and 4f.

The ranking presented in Table 1 can be useful for modeling radiation belt electrons. The table may help modelers decide which parameters need to be considered as inputs to their models.

5. Discussion and conclusion

5.1 Untangling the solar wind and magnetospheric drivers

An important factor that is often ignored and underappreciated in many solar wind-magnetosphere interaction studies is that many solar wind parameters positively or negatively correlate with one another, which may introduce complications and ambiguities in the causal-effect interpretation of the data. In the present study, we use conditional mutual information, CMI, to untangle the effects of the solar wind and magnetospheric drivers of the radiation belt electrons PSD having μ = 725–875 MeV G^{-1} and K = 0.09–0.13 R_E $G^{-0.5}$ (average energy ~1.6 MeV).

The radiation belt electron response time lags to V_{sw}, n_{sw}, AL, and SYM-H obtained from correlational analysis differ from those obtained from CMIs that have removed the effect of the V_{sw} or n_{sw} as summarized in Table 2 (from Figures 2, 11, and 13). For the purpose of facilitating a more precise comparison, Table 2 lists only the peak τ even if the peak may be broad whereas Table 1 lists a range of τ, if the peak is broad. The response lag times obtained by CMIs are deemed more accurate because the effect of V_{sw} or n_{sw} has been removed. For example, CMI(n_{sw} $\rightarrow$ PSD| V_{sw}) peaks at t = 7–11 hr whereas corr(n_{sw} $\rightarrow$ PSD) has a minimum at τ = 15 hr (*Lyatsky and Khazanov*, 2008a). The smaller τ is deemed a more accurate time scale for magnetopause shadowing, which physically makes sense and and is consistent with observations (e.g., *Turner et al.*, 2014a; *Xiang et al.*, 2017; 2018; *Turner and Ukhorskiy*, 2020). The shift in the peak CMI depends on the conditional variable z in the CMI($x \rightarrow y| z$). If the (linear and nonlinear) correlation of z with y is smaller than that between x and y, then removing the effect of z would shift the peak to a larger value and vice versa.

The response of the radiation belt electrons to n_{sw} has dual mode. At small τ, n_{sw} negatively correlates with the electron PSD with a peak response time at τ = 7–11 hr, which can be attributed to the magnetopause shadowing effect. However, at large τ (τ > 80 hr), n_{sw} positively correlates with the electron PSD as shown in Figures 2e and 3. It is not clear what causes this positive correlation. n_{sw} may be a proxy for another solar wind parameter. n_{sw} negatively correlates with IMF $B_z < 0$, which may be the real driver. However, n_{sw} and IMF $B_z < 0$ may be both proxies for another parameter. For example, an increase in n_{sw} is sometimes accompanied by n_{sw} fluctuations, which can drive ULF waves in the magnetosphere and accelerate electrons (e.g., *Kepko and Viall*, 2019, *Ukhorskiy et al*., 2005). Interestingly, the secondary peak in CMI($n_{sw} \rightarrow$ PSD| V_{sw}) at L* = 4.5–5.5 (Figures 6d and 6e) is more prominent and significant than its counterpart in CMI($P_{dyn} \rightarrow$ PSD| V_{sw}) (Figures 7d and 7e), suggesting that the driver for the secondary peak may be more related to n_{sw} rather than P_{dyn}. Whichever parameter drives the electron acceleration, the result suggests a rather slow process for electron acceleration, τ > 80 hr. This will be investigated in our follow up study.

5.2 The radial dependences of the radiation belt electrons

The responses of radiation belt electrons to V_{sw}, n_{sw}, P_{dyn}, AL, and SYM-H have radial dependence. The data coverage for the electrons with μ = 725–875 MeV G^{-1} and K = 0.09–0.13 R_E $G^{-0.5}$ is poor for L* < 3 and L* > 6.5. Hence, the present study does not consider these L* ranges. The effect of n_{sw} and P_{dyn} on the radiation belt electron PSD appear to be significant only at L* = 4.5–6 and insignificant at L* = 3–4.5. This suggests that the magnetopause shadowing is effective mostly at L* = 4.5–6. At L* = 6–6.5, there is a high variability in PSD and the bin has the fewest number of points (n = 336) and hence the result is deemed unreliable. In contrast, the effect of V_{sw} on the electron PSD appears to be significant at a larger range of L*, L* = 3.5 to 6.5.

An increase in n_{sw} or P_{dyn} compresses the magnetosphere leading to the electron loss at high L*, e.g., L* > 7. However, ULF waves generated throughout the magnetosphere due to the compression would redistribute the loss to lower L*. Our result shows that the electron loss can be seen at L* = 4.5–6, consistent with understanding from observations and simulations (*Turner et al*. 2012, 2014a; *Xiang et al*. 2017; 2018; *Turner and Ukhorskiy*, 2020). At higher L*, the noise in the CMI($n_{sw} \rightarrow$ PSD| V_{sw}) is higher, which can be attributed to higher variability of the PSD. For example, the PSD initially decreases due to the magnetopause compression and then increase because of the outward diffusion (*Turner et al*., 2012; *Shprits et al*., 2006). Our result differs from Zhao et al. (2017) that found that P_{dyn} negatively correlates with PSD only at a small range of L* band near 6 (see their Figure 4c).

The radiation belt electrons also have strong dependences on the internal state of the magnetosphere. In the present study, this is explored and exemplified with AL and SYM-H, which can be used as proxies for magnetospheric state. However, the dependences on AL and SYM-H vary with radial distance or L*. The dependence of the radiation belt electrons on AL is significant at L* = 4–6 while the dependence on SYM-H is significant at L* = 3–6. The response of the radiation belt electrons to AL and SYM-H peak at τ = 40–80 and τ = 20–60 hr, respectively. These lag times are averaged for all L*. AL and SYM-H are proxies for substorms and storms, respectively. Substorm and storm injections can increase the temperature anisotropy in the inner magnetosphere, which are important for local acceleration mechanism discussed in Section 5.3. However, it is not clear why there is a large difference between the response lag times to AL and that to SYM-H. This difference in response lag times are also seen in the corr(AL $\rightarrow$ PSD) and corr(SYM-H $\rightarrow$ PSD) in Zhao et al. (2017), but their peak τ are smaller, which can be attributed, at least partly, to their usage of daily resolution data and their correlations did not remove the effect of V_{sw}.

5.3 Implications to electron acceleration mechanism and transport

One of the fundamental questions in radiation belt physics is how the electrons are accelerated to relativistic energies (> 1 MeV). There have been many proposed mechanisms, but most tend to fall into two categories: (1) local acceleration and (2) radial transport (see review in Friedel et al., 2002).

In the local acceleration mechanism, substorms or storms transport low energy electrons (a few to tens of keVs) from the plasma sheet into the inner magnetosphere, which are often referred to as the source population (e.g,, *Baker et al*., 1996; *Tang et al.*, 2017a; *Boyd et al*., 2016). The temperature anisotropy in the source population leads to the growth of the VLF whistler mode chorus waves (e.g., *Meredith et al*., 2001; *W. Li et al*., 2009). Substorms and storms also transport high energy electrons (a few tens to hundreds keVs) electrons from the plasma sheet into the inner magnetosphere, which are commonly referred to as seed population. Then, the chorus waves interact with the seed electrons and energize them to relativistic energies (e.g., *Summers et al*., 1998; 2002; *Horne et al*., 2005; *Omura et al*., 2007; Thorne, 2010; Reeves et al., 2013; *W. Li et al.*, 2014).

In the radial transport acceleration mechanism, electrons at larger L* get accelerated as they move inward to the inner magnetosphere through interactions with ULF waves (e.g., *Baker et al*., 1998; *Li and Temerin*, 2001; *Li et al*., 2005; *Ukhorskiy et al*., 2005; *Mathie and Mann*, 2000; *Elkington et al*., 1999; *Kepko and Viall*, 2019). These ULF waves can be associated with high V_{sw} and Kelvin-Helmholtz Instability (KHI) or n_{sw} or P_{dyn} fluctuations (e.g., *Johnson et al*., 2014;

Engebretson et al., 1998; *Vennerstrøm*, 1999; *Claudepierre et al.*, 2010; *Takahashi and Ukhorskiy*, 2007; *Liu et al.*, 2010).

The result of CMI($V_{sw} \rightarrow$ PSD| n_{sw}) as a function of L* (Figures 9 and 10) can be interpreted in terms of local acceleration mechanism. Figure 10 suggests that the radiation belt electrons at L* = 4.5–5.5 have the shortest response lag time with peak τ = 35–50 hr and the highest significance. The response lag time is larger and broader at higher L*, τ = 45–65 hr (L* = 5.5–6), τ = 40–100 hr (L* = 6–6.5), and at lower L*, τ = 35–55 hr (L* = 4–4.5) and τ = 45–60 hr (L* = 3.5–4). This would suggest that local acceleration peaks at L* = 4.5–5.5 and then subsequent outward and inward diffusion spreads enhancements from this L* band. Previous studies also found evidence for local acceleration at this L* band by examining the MeV PSD or electron fluxes as a function of radial distance (*Green and Kivelson*, 2004; *Boyd et al.*, 2018; *Tang et al.*, 2017a) or microburst MeV electron precipitation (*O'Brien et al.*, 2003). At L* = 4–4.5, there is evidence of weak local acceleration. Its i_{tr} starts increasing at about the same time as that for L* = 4.5–5.5, but its significance is lower than that for L* = 4.5–5.5.

Our finding differs from previous studies that attributed the good correlation between Vsw and PSD to ULF waves and radial transport (e.g., Rostoker et al., 1998; Elkington et al., 1999; Mathie and Mann, 2000; Zhao et al., 2017).

The radial diffusion time scale can be estimated by considering the peak τ of each L* bin in Figure 10. It is found that the outward diffusion from L* = 5 to 6 is about 2 days per.

The result of CMI(AL $\rightarrow$ PSD| V_{sw}) can also be interpreted as consistent with local acceleration at L* = 4–5.5 and inward and outward diffusion to lower and higher L*, respectively. This may not be too surprising because the link between V_{sw} and PSD involves substorm injections.

However, the radiation belt electron response lag time appears more complicated for

CMI(SYM-H → PSD| V_{sw}) (Figure 14). The peak τ is most significant and smallest at L* = 5–5.5 suggesting local acceleration peaks at this L* band. There is evidence for inward and outward diffusion from this L* band. However, there is also evidence for local acceleration at smaller L*. The complication may stem from the competing processes that would increase and decrease PSD and electron fluxes. A decrease of SYM-H would indicate increase in the ring current and the intensity of storms. The general response to storm plasma injections would be an increase in whistler mode chorus waves and electron acceleration, leading to an increase in PSD. However, storm would increase the ring current, which would reduce |B|. This would cause outward diffusion and reduction of PSD as the electrons would attempt to conserve the third adiabatic invariant (*Turner et al.*, 2012). This Dst or SYM-H effect would be stronger with increasing radial distance. This could be a contributing factor in the high noise and variability seen in L* = 6-6.5. Studies have shown that the radiation belt electron response at the outermost L* band can sometimes be enhancement, depletion, or no change (*O'Brien et al*., 2001; *Reeves et al*., 2003). These competing processes may contribute to this variability in the radiation belt response. Also, different types of storms would affect different L* differently (*Turner et al*., 2019).

5.4 Ranking of the solar wind and magnetospheric drivers

We rank the solar wind and magnetospheric parameters based on the information transfer to the radiation belt electron PSD. This ranking can be useful for modelers who would like to develop models that input solar wind and magnetospheric parameters and predict radiation belt electrons having energies 1–2 MeV. This is shown in Table 1. The table shows that V_{sw} transfers the most information to the radiation belt electrons and hence should be considered an important, if not the most important, input parameter to radiation belt models. However, SYM-H can also be an important input parameter for models for two reasons: (1) SYM-H transfers the second most information to the radiation belt electrons; and (2) Out of a long list of parameters (V_{sw}, n_{sw}, P_{dyn}, AL, SYM-H), only SYM-H has information the radiation belt electrons at L* = 3–3.5 albeit only a small amount of information. SYM-H can play a crucial role for models that predict the radiation belt electrons at L* = 3–3.5.

Acknowledgments. The solar wind, SYM-H, and AL dataset were obtained from NASA OMNIweb https://omniweb.gsfc.nasa.gov/. All the derived data products in this paper are available upon request by email (simon.wing@jhuapl.edu). All RBSP-ECT data are publicly available at the website http://www.rbsp-ect.lanl.gov. Simon Wing acknowledges support of NASA Van Allen Probe Contract NNN16AA09T and NASA Grants NNX16AQ87G, 80NSSC20K0704, 80NSSC19K0843, 80NSSC19K0822, 80NSSC20K0188, 80NSSC20K1279, and 80NSSC20K1271. Drew Turner is thankful for funding from NASA grants 80NSSC19K0280 and 80NSSC18K1377. This work has benefitted from discussions within the International Space Science Institute (ISSI) Team # 455 "Complex Systems Perspectives Pertaining to the Research of the Near-Earth Electromagnetic Environment."

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rank	solar wind and magnetospheric parameters	i_{tr_max}	peak τ (hour)
1	V_{sw}	0.12	46
2	SYM-H	0.030	20–60
3	AL	0.020	50–80
4	P_{dyn}[a]	0.018	7–11
5	IMF |B|[a]	0.018	50–110
6	IMF $B_z < 0$[a]	0.017	50–110
7	n_{sw}[a]	0.016	7–11
8	IMF B_y[a]	0.012	0–16
9	Esw	0.012	40–90
10	IMF $B_z > 0$	0.011	0–16
11	σ(IMF B)	0.0083	0–10

Table 1. Ranking of the the solar wind and magnetospheric parameters based on information transfer to radiation belt electron PSD. Parameters 2–11 are calculated from CMI($x \rightarrow$ PSD| V_{sw}) where x = IMF |B|, $B_z < 0$, $B_z > 0$, B_y, Esw, and σ(IMF B). Parameter 1 from CMI($V_{sw} \rightarrow$ PSD| n_{sw}). i_{tr_max} = peak CMI – mean noise where noise is calculated for surrogate data (see Section 4.1).

[a] the response has dual mode: at small τ (τ < 15 hr) the response is electron loss and at large τ (τ > 40 hr) the response is electron enhancement. The ranking is based on the larger i_{tr_max} of the two responses (see text for explanation).

Parameters	Correlation	Peak τ (hr)	Conditional Mutual Information (CMI)	Peak τ (hr)
V_{sw}	**corr($V_{sw} \rightarrow$ PSD)**	**38**	**CMI($V_{sw} \rightarrow$ PSD| n_{sw})**	**46**
n_{sw}	**corr($n_{sw} \rightarrow$ PSD)**	**15**	**CMI($n_{sw} \rightarrow$ PSD| V_{sw})**	**7**
AL	**corr(AL $\rightarrow$ PSD)**	**53**	**CMI(AL $\rightarrow$ PSD| V_{sw})**	**76**
SYM-H	**corr(SYM-H $\rightarrow$ PSD)**	**40**	**CMI(SYM-H $\rightarrow$ PSD| V_{sw})**	**55**

Table 2. Highlighting the differences between correlation and CMI. τ = is the radiation belt response lag time.

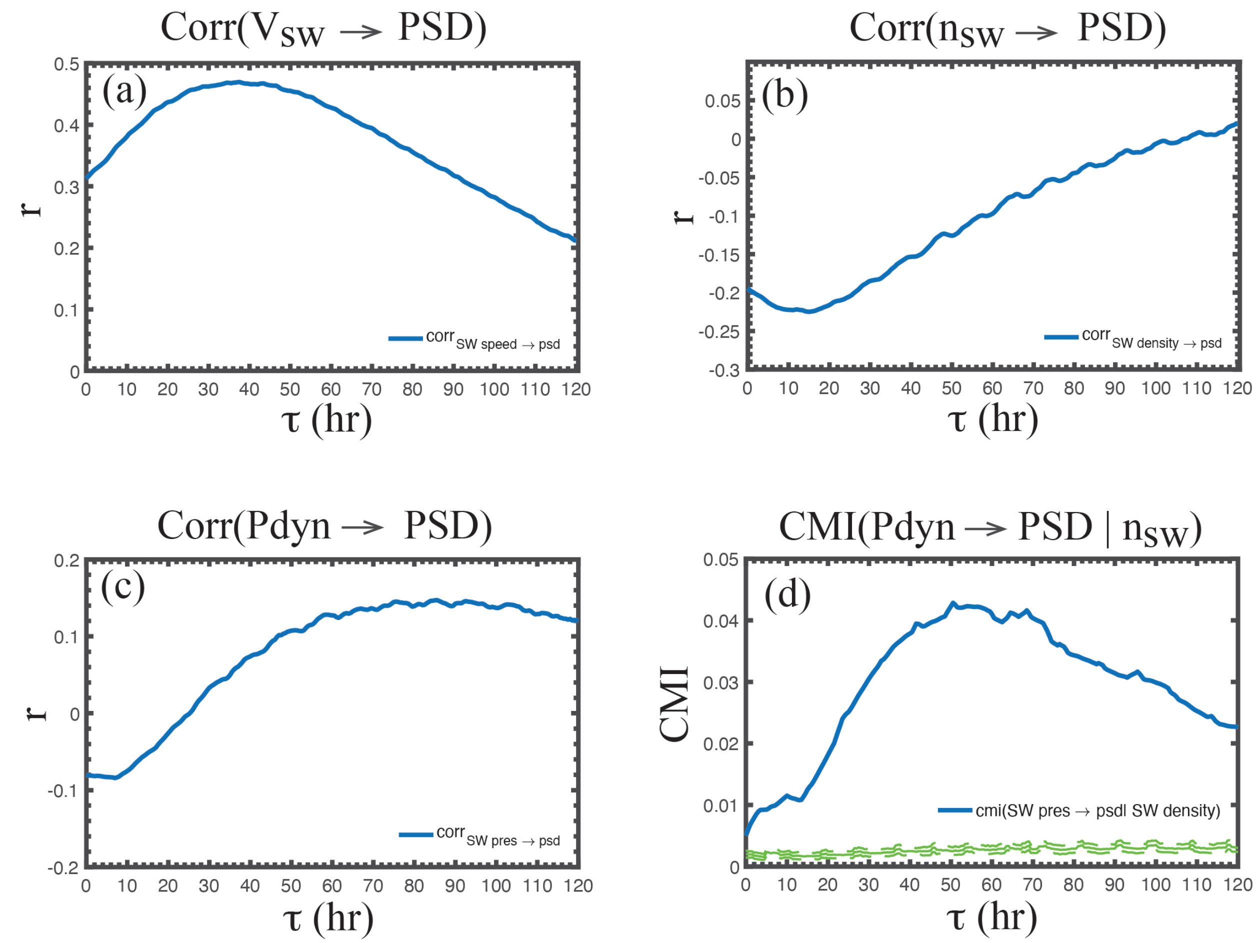

Figure 1. (a) V_{sw} positively correlates with radiation belt electron PSD. (b) n_{sw} negatively correlates with PSD. (c) The PSD response to solar wind dynamic pressure (P_{dyn}) has two modes: at small τ, P_{dyn} negatively correlates with PSD, similar to n_{sw} while at large τ, P_{dyn} positively correlates with PSD, similar to V_{sw}. (d) CMI($P_{dyn} \rightarrow$ PSD| V_{sw}) is plotted as the blue curve. It shows that removing the effect of V_{sw}, the information transfer from P_{dyn} to PSD is similar to V_{sw} correlation with PSD, as expected. The mean noise and 3σ from the noise are plotted as solid and dashed green curves, respectively. The peak of the blue curve is 203σ above the mean noise and hence significant.

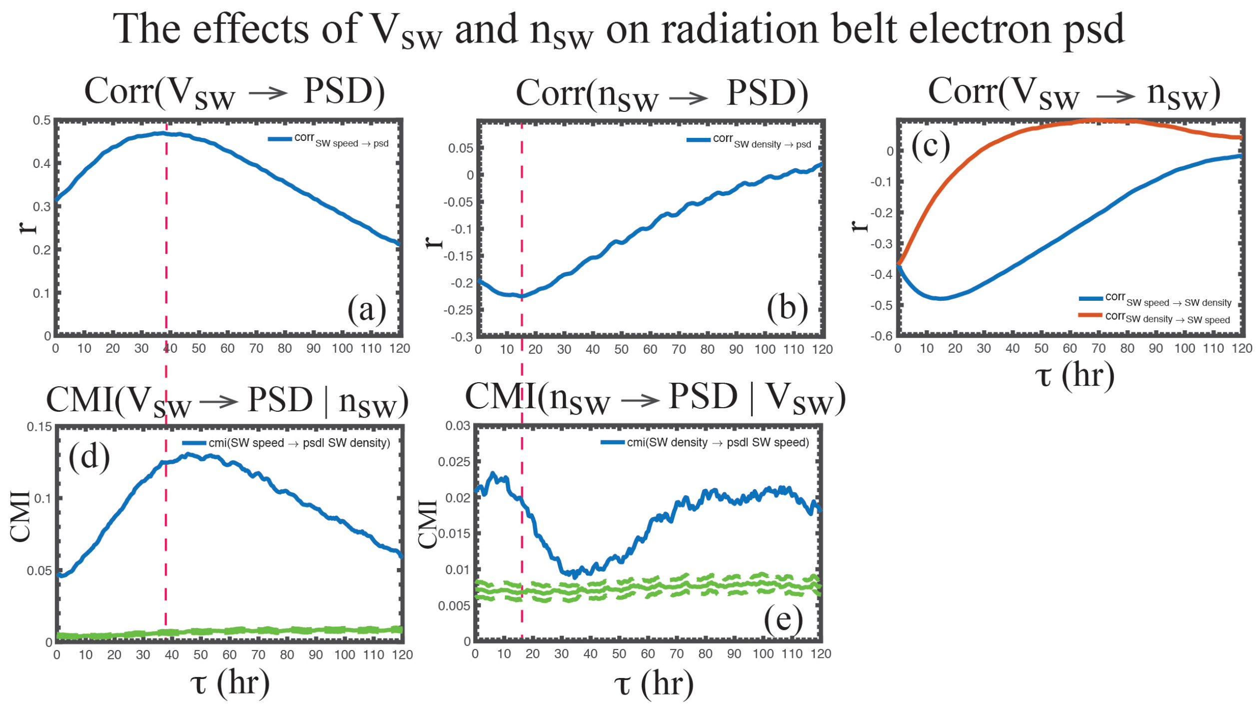

Figure 2. (a) V_{sw} positively correlates with PSD (same as Figure 1a). (b) n_{sw} negatively corelates with PSD (same as Figure 1b). (c) corr($V_{sw} \rightarrow n_{sw}$) = blue curve and corr($n_{sw} \rightarrow V_{sw}$) = red curve. The negative correlation between V_{sw} and n_{sw} raises the question that (a) or (b) may be coincidental. (d) CMI($V_{sw} \rightarrow$ PSD| n_{sw}) shows that (1) there is still information transfer from V_{sw} to PSD even after the effect of n_{sw} is removed and (2) removing the effect of n_{sw} shifts the peak of corr($V_{sw} \rightarrow$ PSD) to the right. (e) CMI($n_{sw} \rightarrow$ PSD| V_{sw}) shows that (1) there is still information transfer from n_{sw} to PSD even after the effect of V_{sw} is removed and (2) removing the effect of V_{sw} shifts the peak of corr($n_{sw} \rightarrow$ PSD) to the left. The red dashed vertical lines help visualize the shifts of the peaks in the correlations. CMI($n_{sw} \rightarrow$ PSD| V_{sw}) has a secondary peak at τ = 80–120 hr. The mean noise and 3σ from the noise are plotted as solid and dashed green curves, respectively.

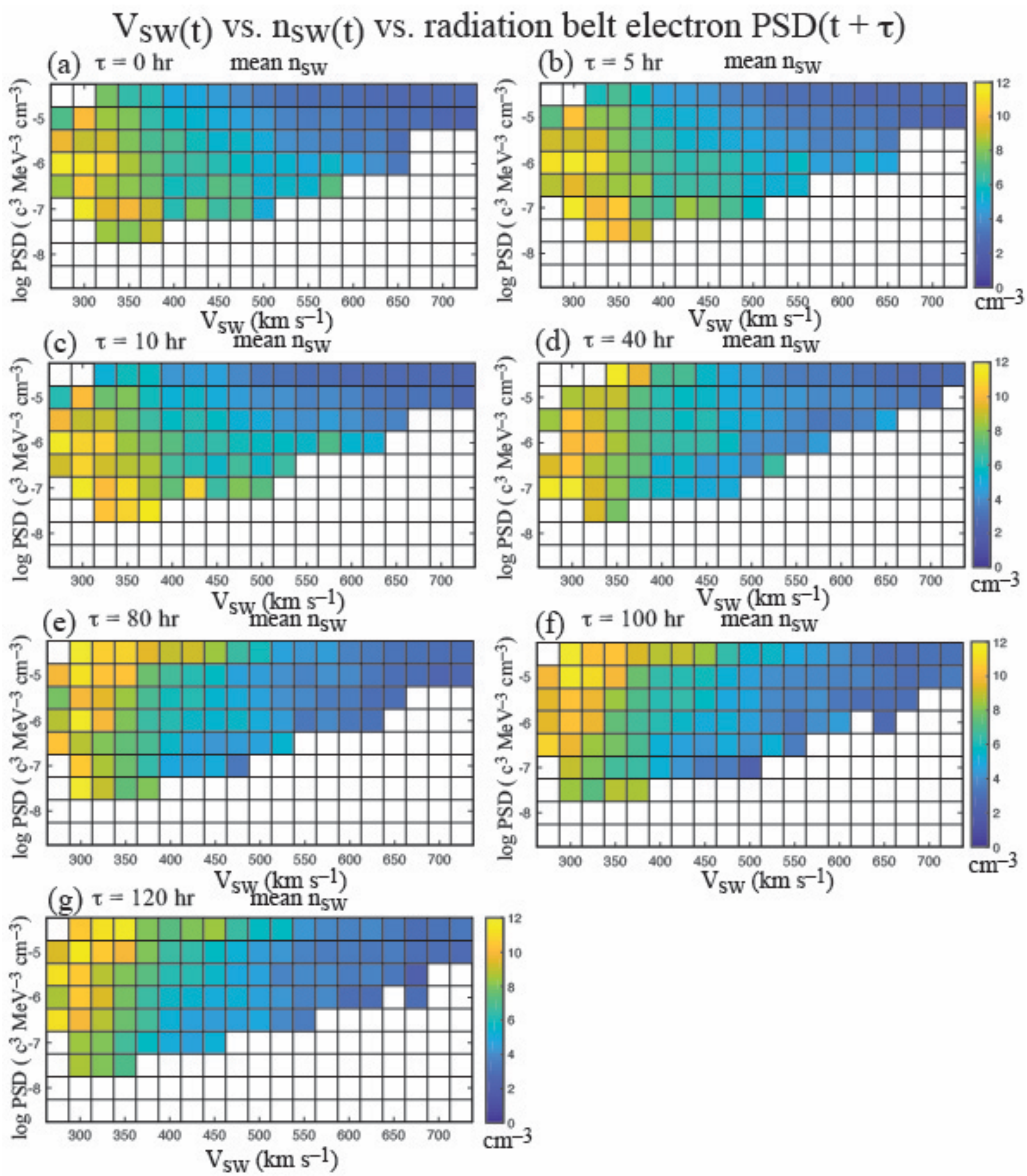

Figure 3. (a–g) log PSD(t + τ) vs. V_{sw}(t) vs. n_{sw}(t) for $\tau = 0$, 5, 10, 40, 80, 100, and 120 hr, respectively. The color is n_{sw}. Large V_{sw} ($V_{sw} > 450$ km s^{-1}) corresponds to high PSD. For small V_{sw} ($V_{sw} < 450$ km s^{-1}), at small τ (τ = 0, 5, 10 hr), n_{sw} negatively correlates with PSD, but at large τ (τ = 80, 100, 120 hr), n_{sw} positively correlates with PSD. At τ = 40, the correlation is weak. Figure 3 is consistent with CMI($n_{sw} \rightarrow$ PSD| V_{sw}) plotted in Figure 2e.

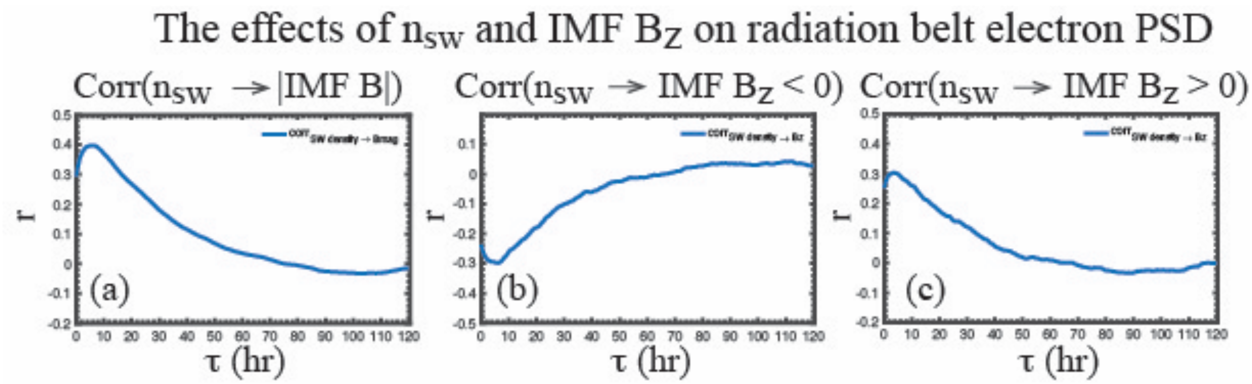

Figure 4. (a) n_{sw} positively correlates with |IMF B|. (b) n_{sw} negatively correlates with IMF $B_z < 0$. (c) n_{sw} positively correlates with IMF $B_z > 0$.

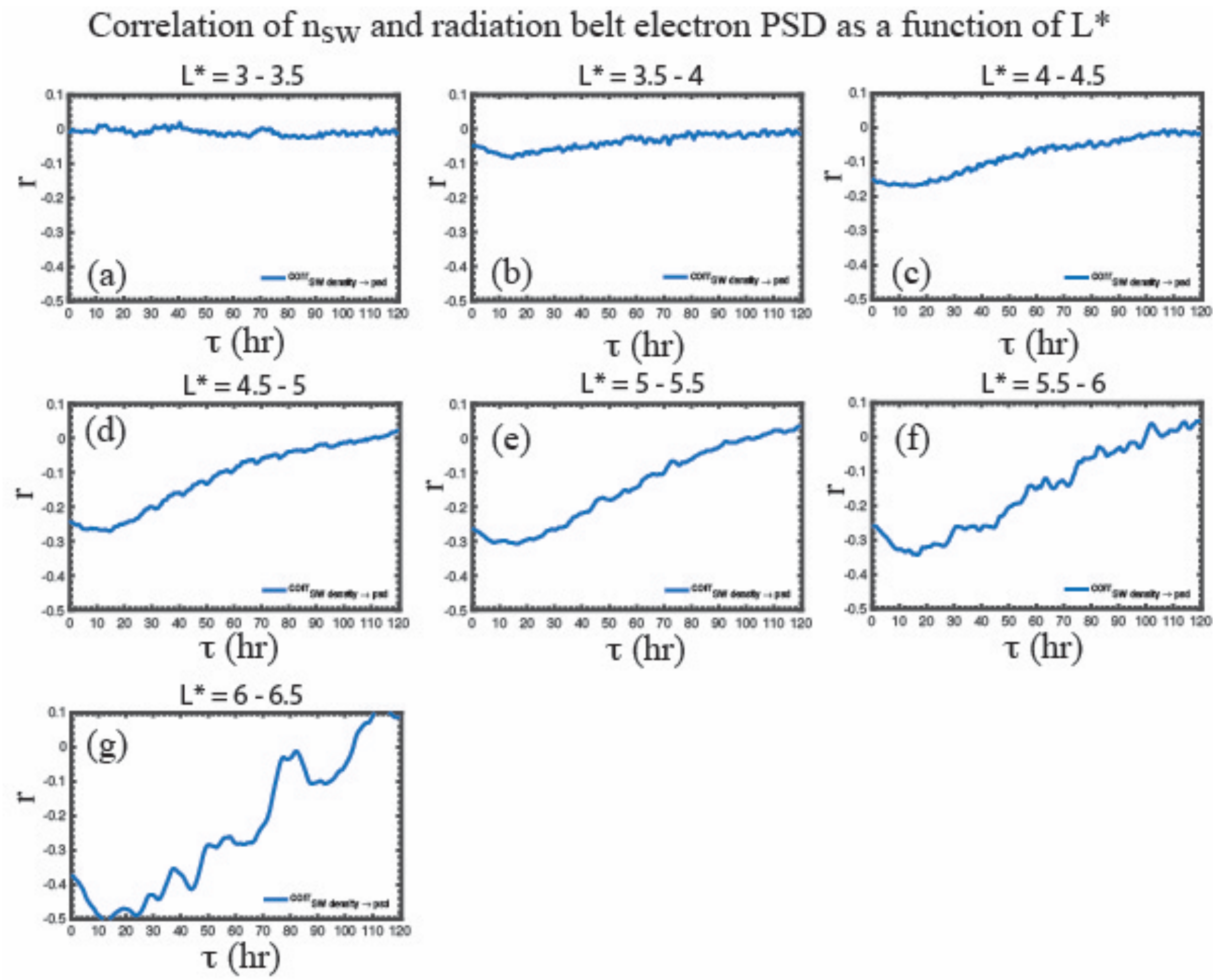

Figure 5. corr(n_{sw} → PSD) from L* = 3 to 6.5 in seven bins each with width = 0.5. (a) The correlation is insignificant at L* = 3–3.5, but slowly increases with increasing L*. (b–g) The correlations are significant ($p < 0.01$) at the minimum τ (τ = 15, 16, 17, 17, 17, 13 hr), r = (–0.086, –0.17, –0.27, –0.31, –0.34, –0.50), and n = (8,302, 11,481, 17,7891, 27,060, 6,236, 528), respectively.

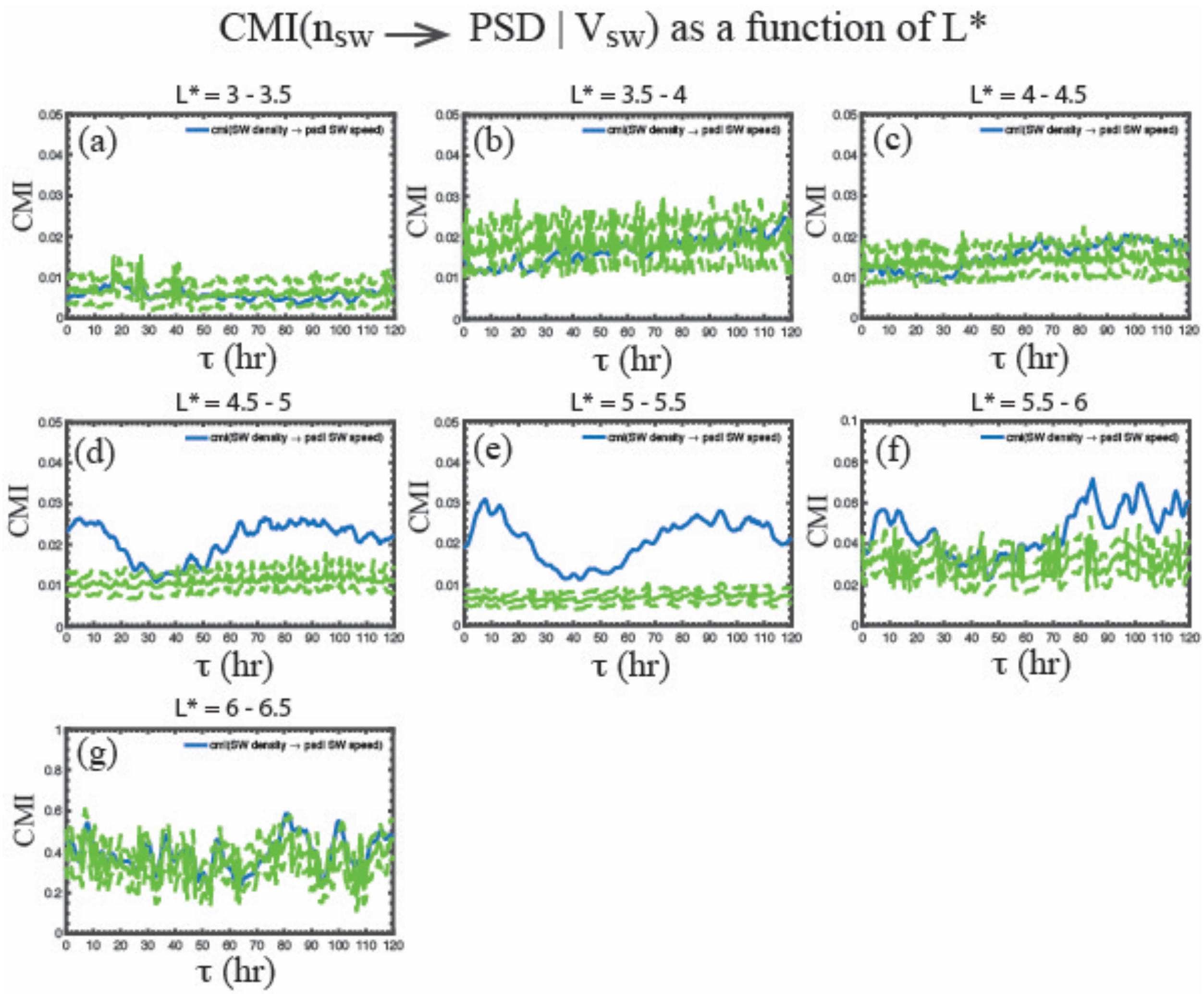

Figure 6. CMI($n_{sw} \rightarrow$ PSD| V_{sw}) from L* = 3 to 6.5 in seven bins each having width = 0.5. The mean noise and 3σ from the noise are plotted as solid and dashed green curves, respectively. (a) The CMI is at the noise level at L* = 3–3.5, which is consistent with the correlation in Figure 5a. (b–c) CMI is at the noise level at L* = 3.5–4.5, unlike the correlation in Figures 5a and 5b. (d–e) The peak CMI is significant at L* = 4.5–5.5 and (f) barely significant at L* = 5.5–6. (g) The CMI is at the noise level at L* = 6–6.5 where there is a large variability in PSD at this outermost L* layer.

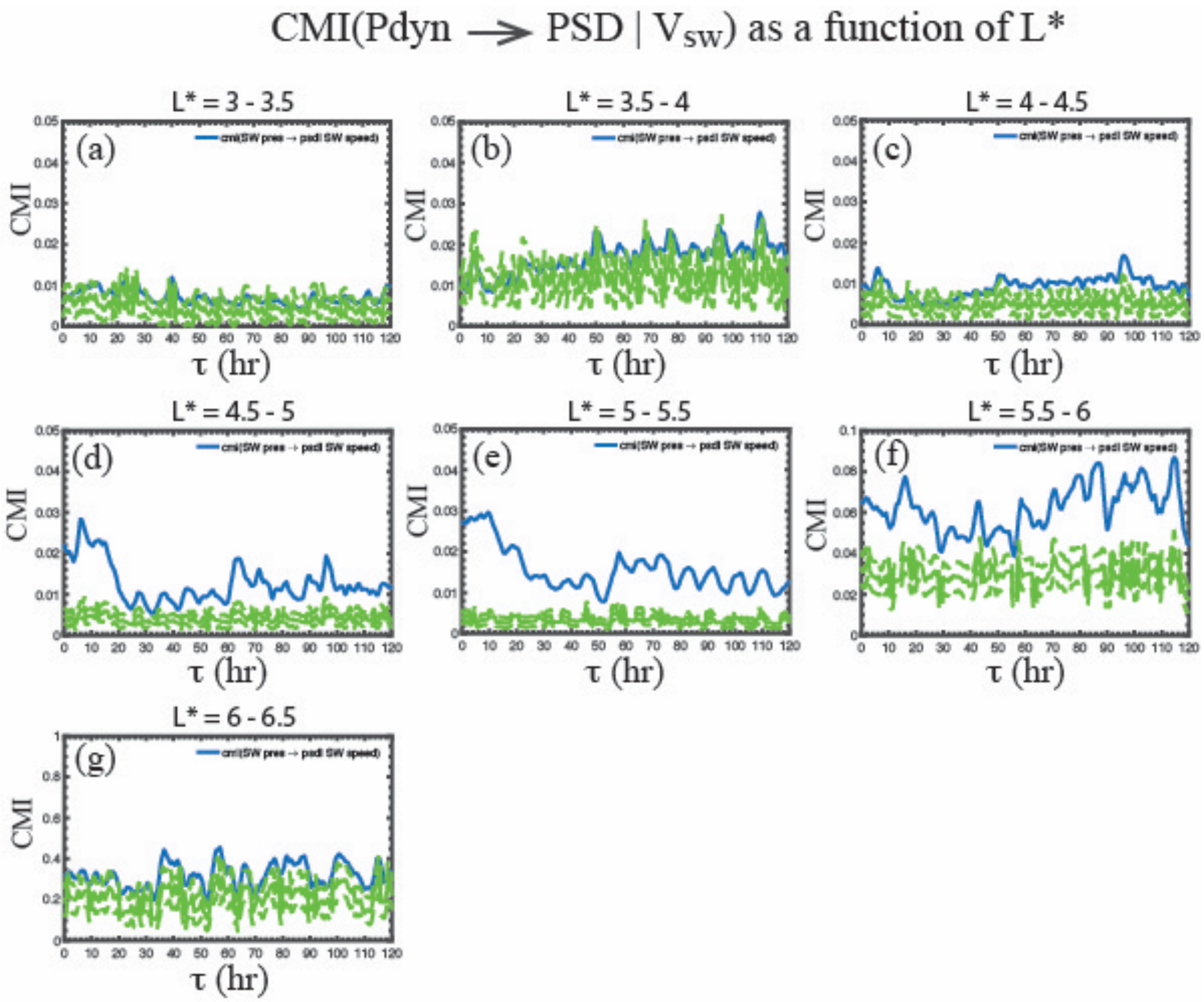

Figure 7. CMI($P_{dyn} \rightarrow$ PSD| V_{sw}) from L* = 3 to 6.5 in seven bins in the same format as in Figure 6. The mean noise and 3σ from the noise are plotted as solid and dashed green curves, respectively. Similar to their counterparts in Figure 6 panels d–f, the primary peak CMIs are significant only at L* = 4.5–5.5 (d–e) and barely significant at L* = 5.5–6. The significances at L* = 4.5–6 are higher than their counterparts in Figures 6 panels d–f for CMI($n_{sw} \rightarrow$ PSD| V_{sw}), suggesting that P_{dyn} is the real causal variable rather than n_{sw}. The opposite is true for the secondary peak, suggesting the causal variable is related more to n_{sw} rather than P_{dyn}.

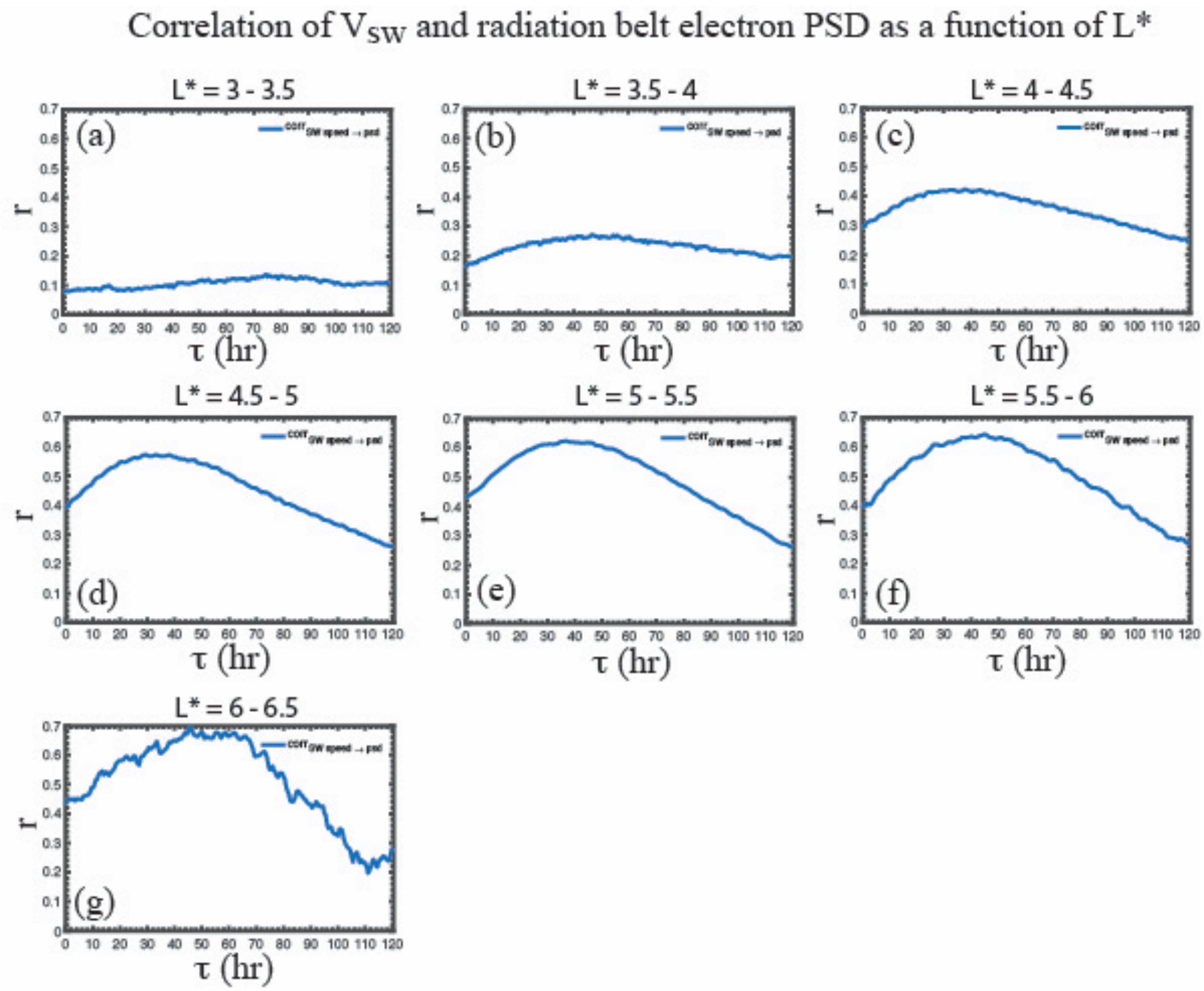

Figure 8. corr(V_{sw} → PSD) from L* = 3 to 6.5 in seven bins in the same format as Figrue 5. (a–g) The correlations are all significant at $p < 0.01$ at the maximum τ (τ = 40–120, 30–120, 38, 30, 37, 45, 30–90 hr), (r = 0.14, 0.27, 0.42. 0.57, 0.62, 0.64, 0.70) for L* = 3–3.5, 3.5–4, 4.–4.5, 4.5–5, 5–5.5, 5.5–6, and 6–6.5, respectively.

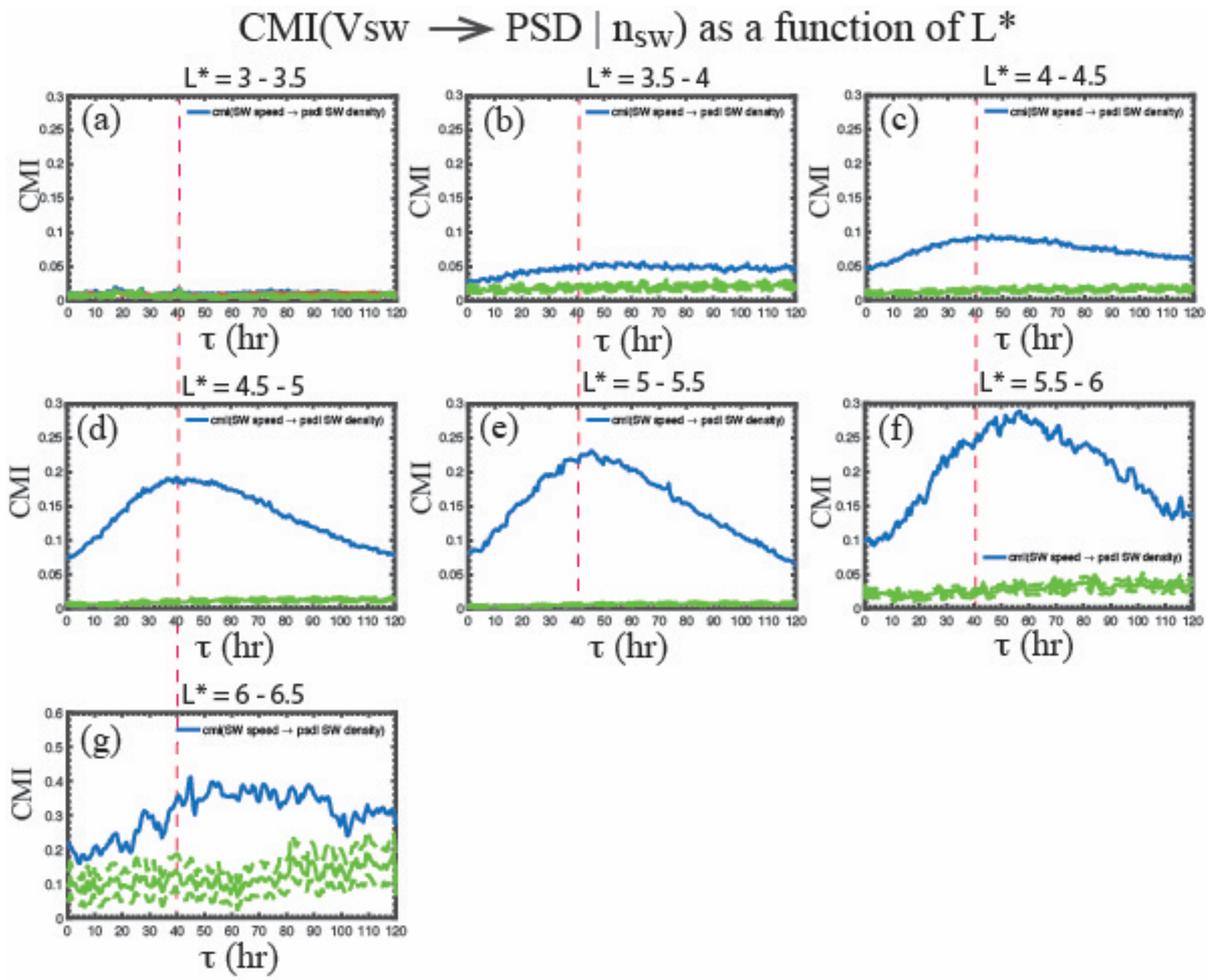

Figure 9. CMI($V_{sw} \rightarrow$ PSD| n_{sw}) from L* = 3 to 6.5 in seven bins in the same format as Figure 6. The mean noise and 3σ from the noise are plotted as solid and dashed green curves, respectively. (a) The CMI is at the noise level at L* = 3–3.5, unlike the correlation in Figure 8a. (b–c) At L* = 3.5–4.5, the CMI has broad peaks from τ ~ 50 to 100 hr (or larger in the case L* = 3.5 – 4) and the peaks are significant. (d–f) At L* = 4.5–6, the CMI peaks are narrower (peak τ = 40, 46, 57 hr, respectively). (g) At τ = 6–6.5, the peak broadens again but remains significant.

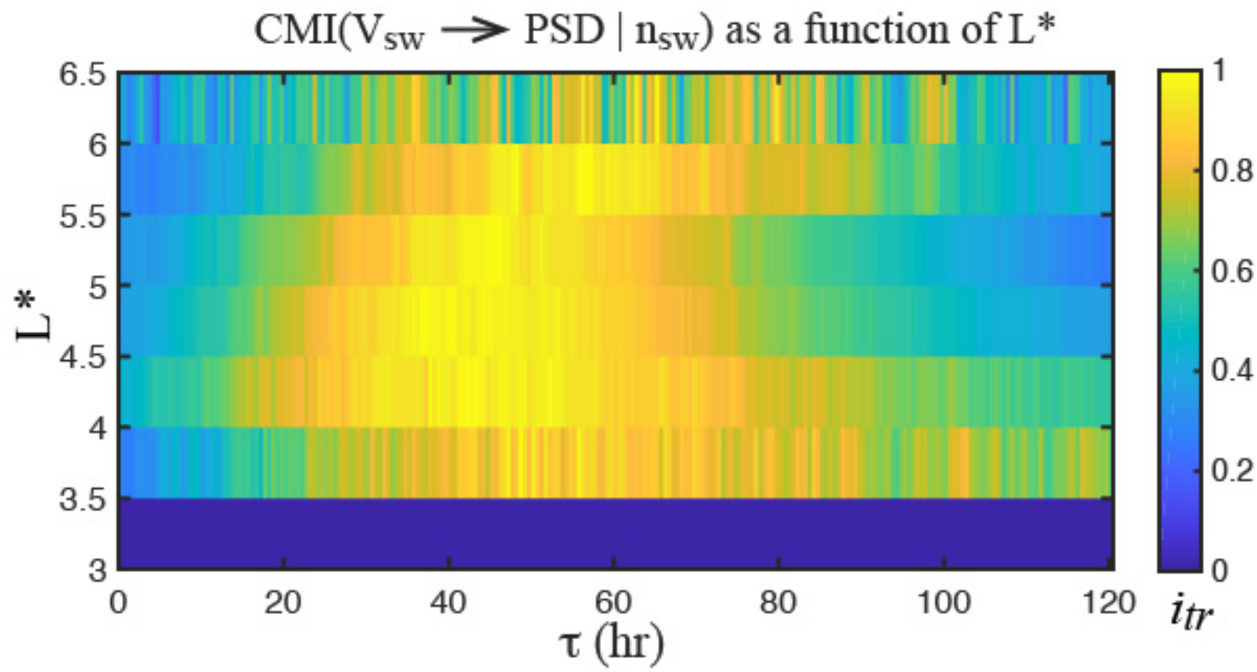

Figure 10. The normalized i_{tr} as a function of L* and τ where i_{tr} = the amount of information transferred = CMI(V_{sw} → PSD| n_{sw}) – mean noise (the blue curve subtracted by the solid green curve in Figure 9). The orange and yellow correspond roughly to the top 20% of i_{tr} in each L* bin. The smallest peak τ can be found at L* = 4.5 –5.5 where τ = 35–50 hr. The peak τ broadens and shifts to larger τ at higher L* (t = 45–65 and 40–100 hr for L* = 5.5–6 and 6–6.5, respectively) and lower L* (τ = 35–55 and 45–60 hr for L* = 4–4.5 and 3.5–4, respectively). The broadening and shifting of the peak to higher τ may suggest outward and inward diffusion from L* = 4.5–5.5. At L* = 4–4.5, peak τ starts about the same time as that at L* = 4.5–5.5, which is suggestive of local acceleration at this L* band as well.

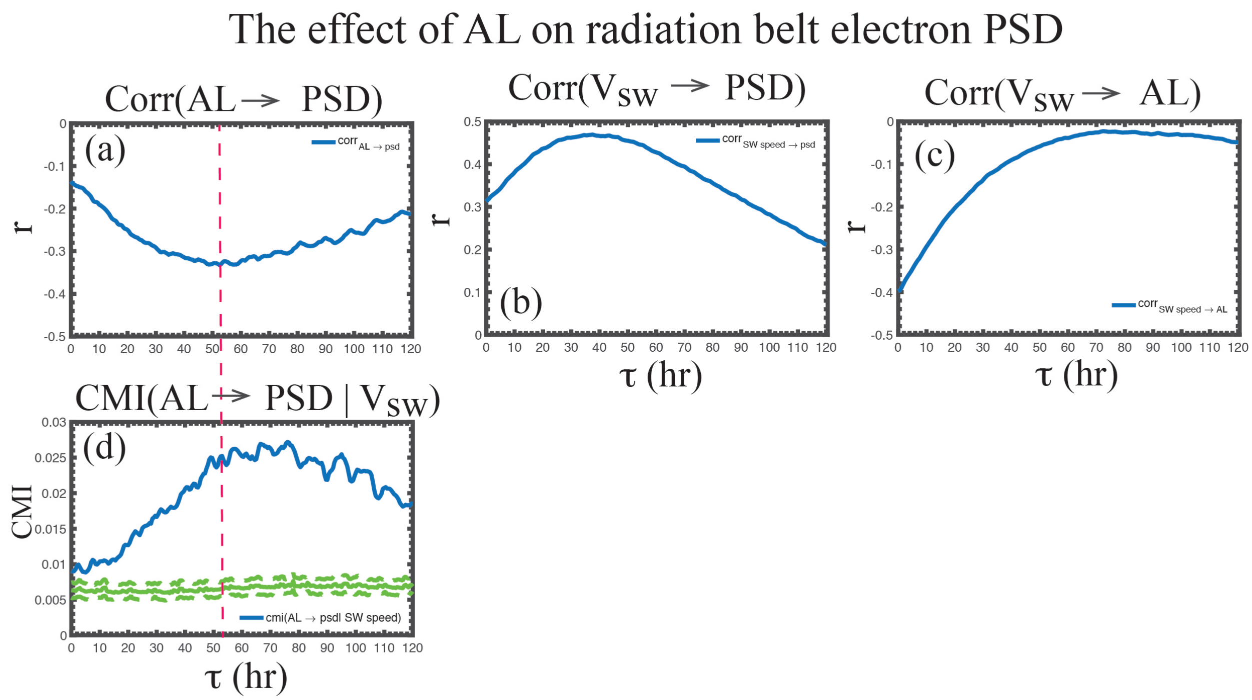

Figure 11. (a) AL negatively correlates with radiation belt electron PSD. (b) V_{sw} positively correlates with PSD (same as Figure 1a). (c) V_{sw} negatively corelates with AL. Given (b) and (c), the correlation in (a) may just be coincidental. (d) CMI(AL → PSD| V_{sw}) shows that even after the effect of V_{sw} has been removed, AL still has an effect on PSD. The mean noise and 3σ from the noise are plotted as solid and dashed green curves, respectively.

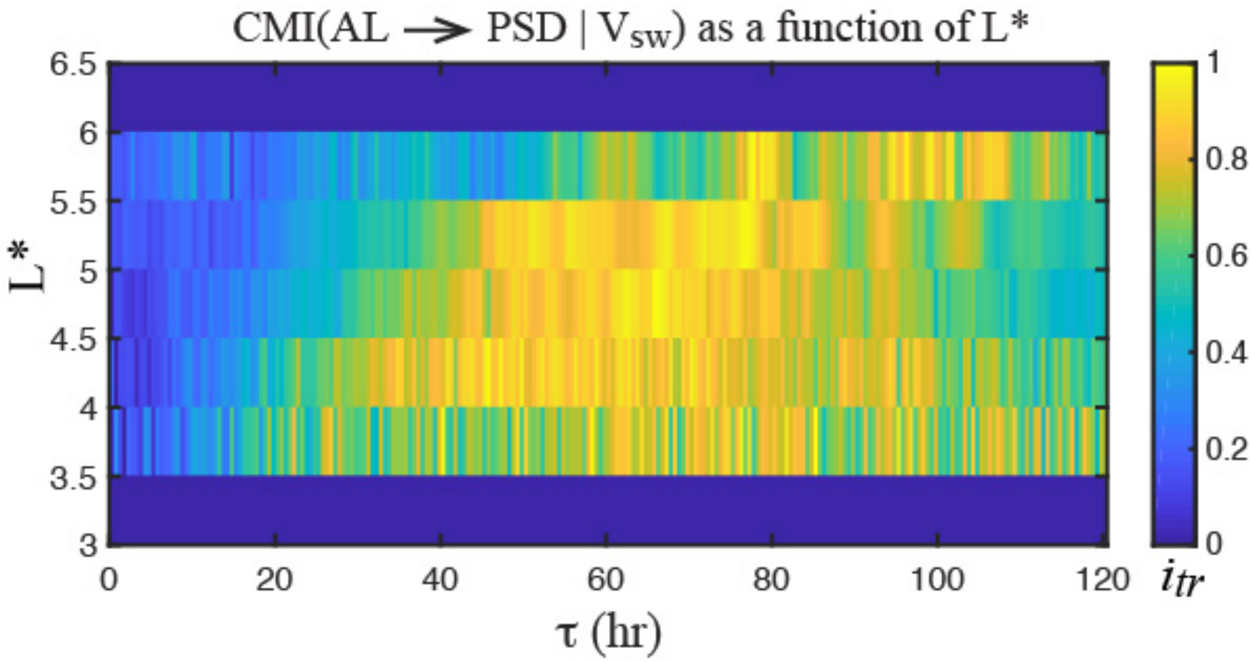

Figure 12. The normalized i_{tr} as a function of L* and τ where i_{tr} = the amount of information transferred = CMI(AL → PSD| V_{sw}) – mean noise in the same format as Figure 10. The orange and yellow correspond roughly to the top 20% of i_{tr} in each L* bin. The i_{tr} is at the noise level at L* = 3–4 and 6–6.5. The region with the largest significance is L* = 4.5–5 and 5 –5.5 with τ = 40–80 and 45–85 hr, respectively. The peak shifts to higher τ, τ = 75–100 hr, at L* = 5.5–6, suggesting outward diffusion from L* = 4.5–5.5. At L* = 4–4.5, the peak is the broad at τ = 35–80 hr. Because the peak τ starts about the same time as that at L* = 4.5–5, it may suggest local acceleration at this L*. The slow decay of τ suggests inward diffusion or longer lasting wave activity.

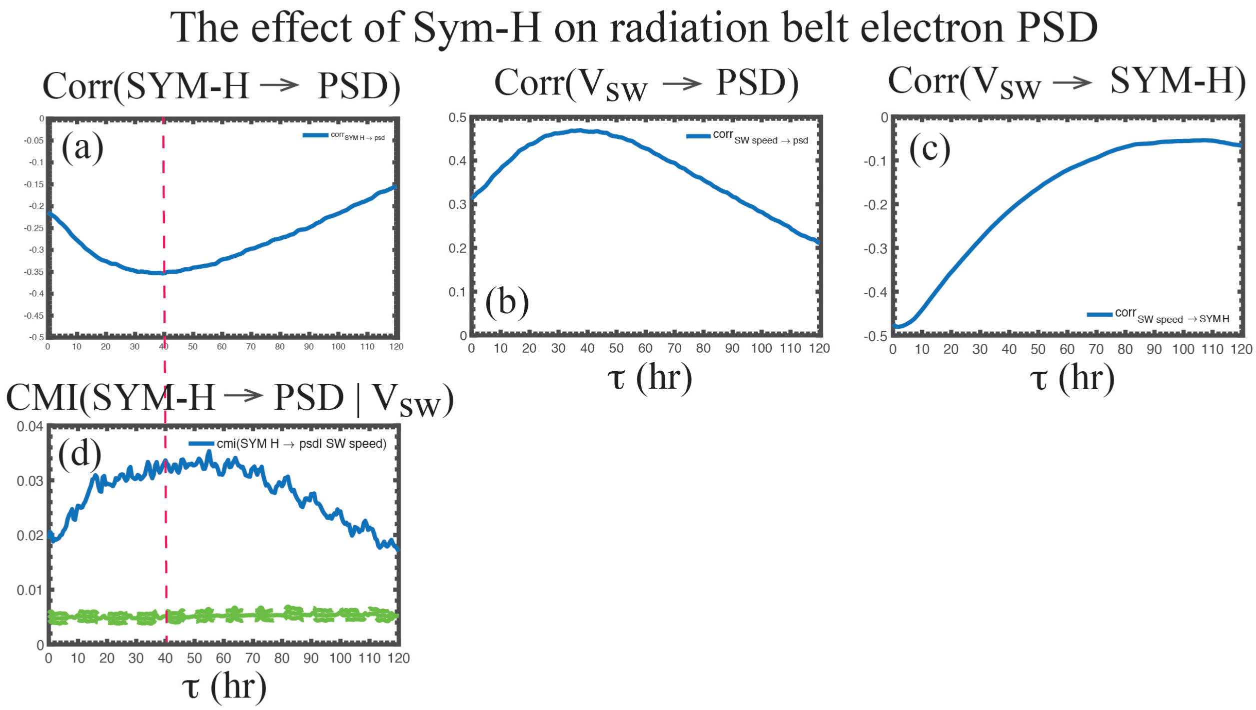

Figure 13. (a) SYM-H negatively correlates with radiation belt electron PSD. (b) V_{sw} positively correlates with PSD (same as Figure 1a). (c) V_{sw} negatively corelates with SYM-H. Given (b) and (c), the correlation in (a) may just be coincidental. (d) CMI(SYM-H → PSD| V_{sw}) shows that even after the effect of V_{sw} has been removed, SYM-H still has an effect on PSD. The mean noise and 3σ from the noise are plotted as solid and dashed green curves, respectively.

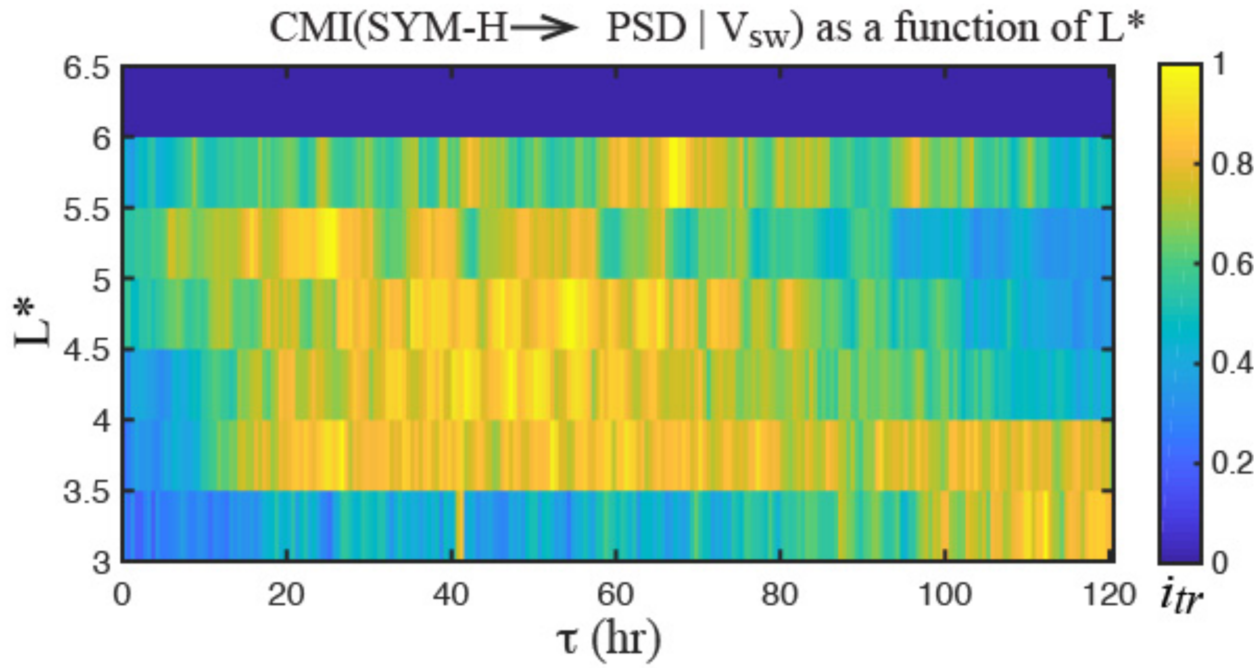

Figure 14. The normalized i_{tr} as a function of L* and τ where i_{tr} = the amount of information transferred = CMI(SYM-H → PSD| V_{sw}) – mean noise in the same format as Figure 10. The orange and yellow correspond roughly to the top 20% of i_{tr} in each L* bin. The i_{tr} is at the noise level at L* = 6–6.5 where there is a large variability in the radiation belt electron PSD. The highest peak significance can be found at L* = 5–5.5 with τ = 20–55 hr. The peak broadens and shifts to higher τ, τ = 60–75 hr at L* = 5.5–6, suggesting outward diffusion from L* = 5–5.5. At L* = 4–4.5 and 4.5–5, the i_{tr} peaks at τ = 30–60 and 30–75 hr. At L* = 3.5–4, the peak is very broad at τ = 20–120 hr, which may suggest local acceleration and inward diffusion from higher L*. At L* = 3–3.5, the i_{tr} peaks at τ = 110–120 hr or may be even higher, which may suggest slow diffusion from higher L*.