

# Bayesian calibration of a natural state geothermal reservoir model, Krafla, north Iceland

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## Abstract

The Krafla area in north Iceland hosts a high-temperature geothermal system within a volcanic caldera. Temperature measurements from boreholes drilled for power generation reveal enigmatic contrasts throughout the drilled area. While wells in the western part of the production field indicate a 0.5-1 km thick near-isothermal ( $\sim 210$  °C) liquid-dominated reservoir underlain by a deeper boiling reservoir, wells in the east indicate boiling conditions extending from the surface to the maximum depth of drilled wells ( $\sim 2$  km). Understanding these systematic temperature contrasts in terms of the subsurface permeability structure and overall dynamics of fluid flow has remained challenging. Here, we present a new numerical model of the natural, pre-exploitation state of the Krafla system, incorporating a new geologic/conceptual model and a version of TOUGH2 extending to supercritical conditions. The model shows how the characteristic temperature distribution results from structural partitioning of the system by a rift-parallel eruptive fissure and an aquitard at the transition between deeper basement intrusions and high-permeability extrusive volcanic rocks. As model calibration is performed using a Bayesian framework, the posterior results reveal significant uncertainty in the inferred permeability values for the different rock types, often exceeding two orders of magnitude. While the model shows how zones of single-phase vapor develop above the deep intrusive heat source, more data from deep wells is needed to better constrain the extent and temperature of the deep vapor zones. However, the model suggests the presence of a significant untapped resource at Krafla.

1            **Bayesian calibration of a natural state geothermal**  
2            **reservoir model, Krafla, north Iceland**

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8            **Key Points:**

- 9            • We perform MCMC calibration of a 3-D natural state reservoir model of the Krafla  
10            geothermal system
- 11            • Posterior results indicate that the uncertainty of inferred permeability values is  
12            underestimated using deterministic approaches
- 13            • One of the first reservoir models of an exploited geothermal system extending to  
14            the deep, supercritical roots near the heat source

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**1 Introduction**

Geothermal systems develop in response to elevated heat fluxes in areas of active magmatism and volcanism (Stimac et al., 2015; Jolie et al., 2021). Hydrothermal convection in permeable rocks results in cooling of subsurface intrusions and the development of boiling zones (Hayba & Ingebritsen, 1997; Scott et al., 2016). Natural state geothermal reservoir models describe the subsurface temperature/pressure distribution, the location and depth of boiling zones, and the rates of heat and mass transport prior to the onset of exploitation (M. O’Sullivan et al., 2001; M. O’Sullivan & O’Sullivan, 2016). The sustainable level of power generation that a geothermal system can support is intimately linked to the subsurface dynamics of fluid flow and heat transfer, which are elucidated by the natural state model (Gunnarsson et al., 2010; M. O’Sullivan & O’Sullivan, 2016). Even in geothermal systems such as Krafla that have been exploited for several decades, there is a paucity of data needed to constrain the system structure, and the uncertainty of natural state models is significant. As consideration of this uncertainty can help generate more realistic predictions of the impact of production on the system behavior, it is thus essential to ensuring that reservoir models effectively contribute to sustainable management of geothermal resources.

The foundation of the natural state model is the conceptual model describing the structure of a geothermal system based on an integration of available geologic, geophysical, hydrologic and geochemical data (Cumming, 2016a). The calibration of natural state geothermal reservoir models involves finding geologically-reasonable model parameters consistent with the conceptual model as well as field observations prior to exploitation, including measured temperatures/pressures, the locations of discharge zones, and rates of surface heat transport. Modern software tools such as Leapfrog Geothermal facilitate the development of a three-dimensional geologic model based on surface mapping and downhole data, and the design and export of numerical grids populated with rock types according to the geologic model (Newson et al., 2012; Milicich et al., 2018; Popineau et al., 2018). This ensures a close coupling between the conceptual/geologic model and the reservoir model.

One of the key parameters requiring calibration in natural state geothermal reservoir models is the anisotropic permeability of the different rock types and their spatial distribution. The other key parameter involved in natural state model calibration cons-

ists of the heat and mass sources applied at the base of the computational model, which control the strength and location of fluid upflow zones. The inherent uncertainty of these parameters makes calibration of natural state geothermal reservoir models challenging (M. O’Sullivan & O’Sullivan, 2016). While manual adjustment of input parameters remains the dominant approach to model calibration (M. O’Sullivan & O’Sullivan, 2016), a variety of inverse modeling tools are available to automate the calibration process, including iTOUGH2 (Finsterle, 2007) and PEST (Doherty, 2015). These methods use gradient-based methods to find the minimum of a sum-of-squares type objective function based on matching the measured and simulated temperatures. Uncertainty metrics for model parameters can be constructed from the derivatives of the local cost function (Aster et al., 2018). However, the objective function may have many local minima (Plasencia et al., 2014), and the global minimum may reside in a part of the parameter space that is inconsistent with field and/or laboratory measurements.

The Bayesian approach to inverse modeling naturally allows for specification of prior uncertainty of input parameters and quantification of uncertainty in the estimated parameters (Mosegaard & Sambridge, 2002; Tarantola, 2005; Kaipio & Somersalo, 2006; Gelman et al., 2013). In the Bayesian framework, the solution to the inverse problem is a posterior probability density over the model parameters, the statistics of which are usually obtained using Markov chain Monte Carlo (MCMC) sampling methods (Andrieu et al., 2003). One major drawback of the MCMC approach for inverse modeling of geothermal systems is the intensive computational cost required to repeatedly evaluate the forward model describing multi-phase fluid flow in a porous medium. This is particularly true for natural state models, which are run to long times (ca.  $10^6$  years) in order to achieve a steady-state configuration. Presently, MCMC sampling for geothermal reservoir model calibration is only feasible through the use of a coarsened, less computationally-expensive model for MCMC sampling instead of a finer, more computationally-expensive models (Cui et al., 2011; Cui, Fox, & O’Sullivan, 2019; Cui, Fox, Nicholls, & O’Sullivan, 2019; Maclaren et al., 2020). Different strategies have been developed to deal with the additional error introduced by using the coarsened model in place of the finer model. Cui et al. (2011) developed a sophisticated delayed acceptance algorithm that adaptively builds a stochastic model of this error. Maclaren et al. (2020) present an alternative approach that incorporates the posterior-informed approximation errors into a hierarchical framework. One advantage of the latter approach is its relative simplicity, as only a relatively small number of realizations of the fine model are required to enable sampling from the target posterior. Furthermore, the approach of Maclaren et al. (2020) is essentially independent of the specific MCMC sampling algorithm used.

In this study, we present a new natural state reservoir model of the Krafla geothermal system in northeast Iceland. Although the Krafla system has been under exploitation for more than 4 decades, the last published natural state reservoir model of the area dates back to the 1980s and was a 2-dimensional model (Bödvarsson et al., 1984). Since that time, the drilling of additional boreholes has provided significantly more geologic and hydrologic data, and computational abilities have increased tremendously. For example, recent enhancements to TOUGH2 extend the applicability of the numerical simulation software to the deep, supercritical ( $>375$  °C) roots (Magnusdóttir & Finsterle, 2015; J. O’Sullivan et al., 2015; M. O’Sullivan & O’Sullivan, 2016; J. O’Sullivan et al., 2020). Such conditions are present at depth in Krafla, as evidenced by the drilling of the Iceland Deep Drilling Project (IDDP) well IDDP-1, which encountered a shallow magmatic intrusion at  $\sim 2$  km depth and discharged a 440 °C single-phase vapor (Ármannsson et al., 2014). We adapt the Bayesian hierarchical approach of Maclaren et al. (2020) to perform the model calibration and quantify the uncertainty in the inferred permeability structure. This study highlights the potential for Bayesian approaches to enable better consideration of geologic uncertainty during natural state model calibration.

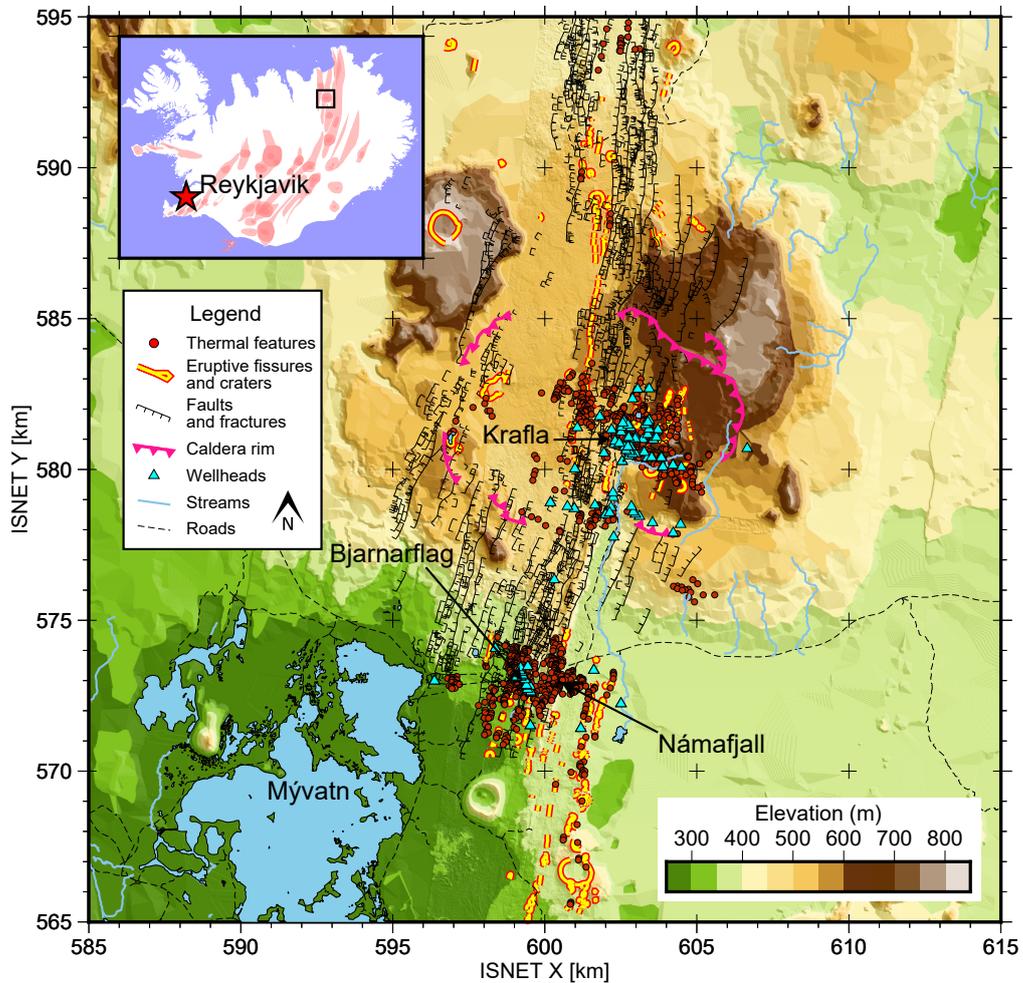
## 2 The Krafla Geothermal System

### 2.1 Geologic background

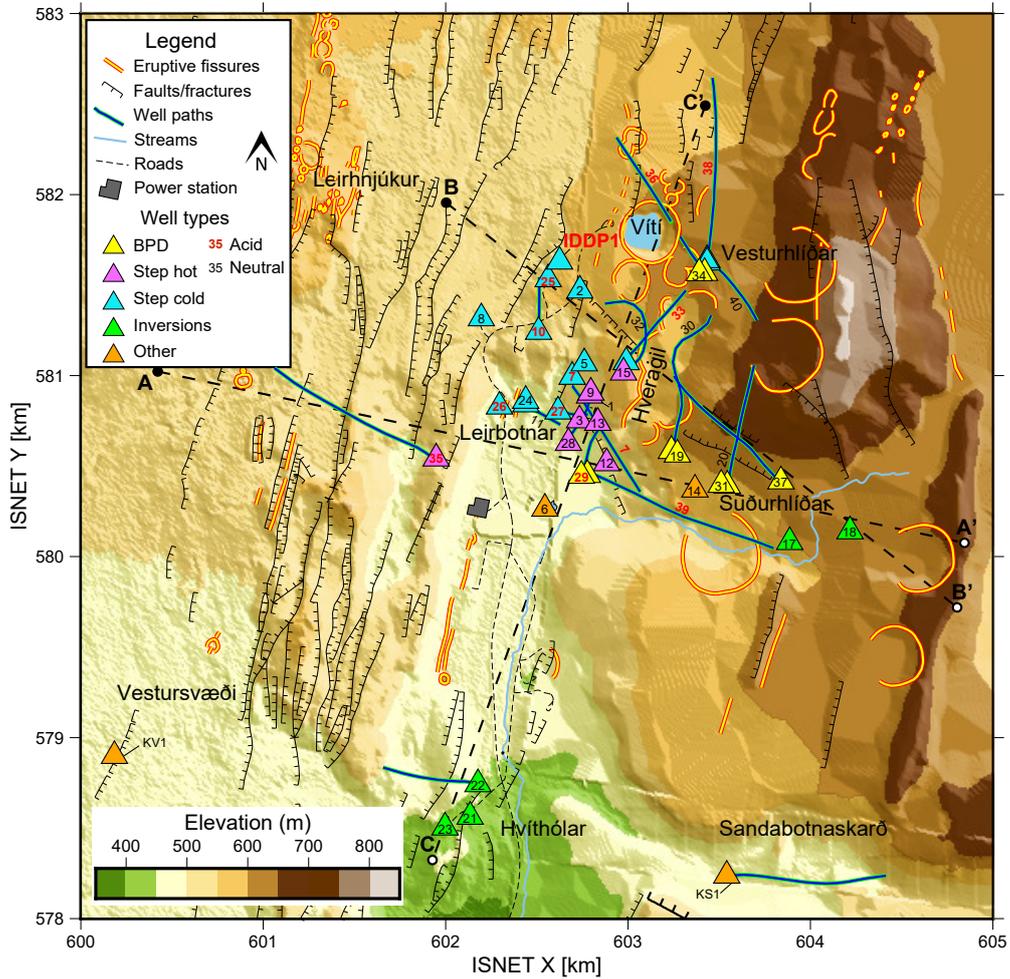
The Krafla area hosts a central volcano approximately 20 km in diameter sitting astride a 90 km long NNE-SSW trending fissure swarm (Sæmundsson, 1991; Hjartardóttir et al., 2012) (Figure 1). Krafla is one of five active volcanic centers arranged in en-echelon fashion within the northern volcanic zone (NVZ) in northeast Iceland along with Kverkfjöll, Askja, Fremrinámur and Theistareykir (P. Einarsson, 2008; Hjartardóttir et al., 2016). The central volcano has developed an 8-10 km diameter caldera structure that formed around 110 ky ago (Sæmundsson, 1991). Volcanism in the caldera dominantly occurs as basaltic fissure eruptions and dike injections, along with intermittent eruptions of more silicic magmas (Jónasson, 1994). About 20 eruptions have taken place within the Krafla caldera during the postglacial era (Sæmundsson, 1991), most recently in 1975-1984 C.E. (A. Björnsson et al., 1977; P. Einarsson, 1991). During this eruption sequence (often referred to as the ‘Krafla fires’), seismic studies delineated volumes between 3-7 km depth in the subsurface with S-wave shadows, which were interpreted to represent a network of basaltic sills and intrusions underlying the central part of the caldera (P. Einarsson, 1978). More recently, rhyolitic magma bodies were encountered around 2 km depth during the drilling of the wells KJ-39 and IDDP-1 (Mortensen et al., 2010; Elders et al., 2011). Petrologic data suggest a link between the IDDP-1 rhyolite and the rhyolite erupted in 1724 C.E. during the formation of Víti, a small (~300-m-diameter) maar located 0.5 km to northeast of IDDP-1, suggesting the subsurface rhyolite may extend throughout the center of the caldera (Rooyackers et al., 2021).

The geologic structure of the Krafla area has been intensively studied (Stefánsson, 1981; Sæmundsson, 1991; Ármannsson et al., 1987; Sæmundsson, 2008; Mortensen et al., 2009; Weisenberger et al., 2015). Above 0.4-1 km b.s.l., the subsurface lithology in the central part of the caldera mainly consists of extrusive igneous rocks, including sub- or intraglacially-erupted basaltic hyaloclastite (Jakobsson & Guðmundsson, 2008) and sub-aerially erupted basaltic lava flows. Doleritic and gabbroic intrusions dominate at greater depths, along with minor granophyre and other silicic intrusions (Mortensen et al., 2015). The depth to the intrusive basement varies from about 0.8-1.1 km in the central part of the caldera to about 1.5-1.6 km in the southern part (Weisenberger et al., 2015). The rocks at Krafla undergo alteration as a result of high-temperature fluid-rock interaction, with the alteration mineralogy following the typical depth- and temperature-dependent zonation characteristic of Icelandic geothermal systems (Kristmannsdóttir, 1979; Á. Sveinbjörnsdóttir, 1992). With increasing depth and temperature, these alteration zones are the smectite-zeolite zone, the mixed-layer clay zone, the chlorite-epidote zone, the epidote zone, and the epidote-actinolite zone. The intensity of alteration is very variable but often increases with depth (Mortensen et al., 2014). Alteration and compaction reduces the high primary porosity and permeability of the extrusive volcanic rocks (Weisenberger & Selbekk, 2009; Thien et al., 2015; Heap et al., 2020; Eggertsson, Lavallée, et al., 2020).

A total of 43 deep geothermal wells have been drilled in the central part of the Krafla caldera to provide steam to a geothermal power plant that currently generates 60 MWe (Figure 2). A smaller geothermal power plant currently generating 3 MWe is located in Bjarnarflag (Fig. 1), approximately 11 km to the south of Krafla. Bjarnarflag is located close to Námafjall, an area with abundant fumarole degassing (Oskarsson, 1984), and was the site of an explosive eruption of a basaltic tephra through a geothermal borehole during the Krafla fires (Larsen et al., 1979). While the Krafla and Bjarnarflag systems may be hydraulically connected via the high permeability NNW-SSE trending fissure swarm, a separate dyke complex is believed to be the heat source for Námafjall (Gylfadóttir, 2013; Drouin et al., 2017).



**Figure 1.** Topographic map of the Krafla area, showing the locations of drilled wells (blue triangles), thermal features (red circles), eruptive fissures (yellow/red lines), faults and fractures (hatched black lines), the caldera rim (pink toothed line), streams and water bodies (light blue), and roads (dashed black lines). The inset shows the location of the Krafla area in north-east Iceland and the locations of other volcanic areas and fissure swarms. Coordinates are shown in ISNET16 (same as Lambert 95).



**Figure 2.** Topographic map of the main production area in the center of the Krafla caldera. Locations of wellheads used to calibrate the numerical model in this study are shown as colored triangles. The wells are grouped according to the characteristic measured temperature profiles: 1) Yellow triangles indicate wells with temperatures following the boiling point with depth (BPD) to  $\geq 1.5\text{-}2$  km depth. 2) Purple triangles indicate wells that feature an isothermal zone with temperatures  $210\text{-}225$  °C (Step hot). 3) Light blue triangles indicate wells with an isothermal zone at temperatures  $\leq 210$  °C (Step cold). 4) Green triangles indicate wells that show temperature inversions, i.e. decreasing temperatures with increasing depth below a certain depth. 5) Orange triangles indicate wells with other characteristics. The traces of directional wells are shown as dark blue lines. The surface traces of the cross-sections shown in Figure 4 are shown with dashed black lines. The numbers indicate the well numbers, with red numbers corresponding to acid wells (see text).

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## 2.2 Conceptual model of the Krafla geothermal system

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Several conceptual models of the geothermal resource in the Krafla area have been published (Stefánsson, 1981; Ármannsson et al., 1989; Mortensen et al., 2015; Weisenberger et al., 2015; Árnason, 2020). The conceptual model below synthesizes insights from these previous studies and others. On the basis of differing production characteristics and temperature-depth relations, the drilled area at Krafla can be divided into five subfields: (1) Leirbotnar, located to the west of the Hveragil eruptive fissure, (2) Suðurhlíðar, located to the east of Hveragil, (3) Hvíthólar, located on the southern rim of the caldera, (4) Vestursvæði, located to the north of Suðurhlíðar and (5) Sandabotnaskarð, located to the south of Suðurhlíðar. The natural state temperatures from 40 deep geothermal wells grouped by subfield are shown in Figure 3.

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Downhole temperature measurements in Leirbotnar reveal two distinct reservoirs: an upper sub-boiling reservoir extending from -0.5 to -1 km a.s.l. with a remarkably constant temperature between 190-220 °C, and a deeper boiling reservoir that reaches temperatures up to 350 °C (Fig. 3a). In this study, we distinguish between wells in the Leirbotnar subfield with temperatures 210-225 °C (Step hot in Fig. 2) or  $\leq 210$  °C (Step cold) within the isothermal zone. Wells with a hotter isothermal zone tend to cluster around the southwest of the Hveragil eruptive fissure, while the wells with the cold isothermal zone are spread further to the northwest, suggesting a gradient with increased mixing of cooler recharging fluids to the northwest. The steep, near-linear temperature gradient at the base of the isothermal zone suggests the presence of a low permeability aquitard separating the upper liquid reservoir from the deeper boiling reservoir (Stefánsson, 1981; Bödvarsson et al., 1984). Comparison of the temperature measurements with the geologic data (Mortensen et al., 2015; Weisenberger et al., 2015) suggests that the transition between the upper sub-boiling reservoir and the deeper boiling reservoir coincides with a transition between shallower extrusive volcanics and deeper crystalline basement intrusions.

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In contrast to the characteristic step-increase seen in Leirbotnar, wells in the Suðurhlíðar subfield show downhole temperatures indicating boiling conditions from the surface to depths of at least -1.5 km a.s.l. (Fig. 3b). The lack of the step increase in temperature for the wells in Suðurhlíðar suggests that the inferred aquitard underlying Leirbotnar does not extend across the Hveragil eruptive fissure (Stefánsson, 1981; Bödvarsson et al., 1984), or that higher subsurface fluid pressures in areas beneath Krafla mountain prevents cold recharge from the north (Árnórsson, 1995). The Suðurhlíðar subfield is believed to be bounded to the east by a sub-vertical, rift-parallel fracture system (Stefánsson, 1981; Bödvarsson et al., 1984) and to the south by a WNW-ESE oriented transform structure, which has been inferred along the on the basis of structural, gravity and resistivity data (Árnason, 2020; Liotta et al., 2021). The intersection of such transform structures with the rift is believed to enhance permeability and localize fluid upflow at depth (Khodayar et al., 2018; Liotta et al., 2021). The Vesturhlíðar subfield is located to the north of Suðurhlíðar and to the east of the Víti maar, and shows wells that follow the BPD over the entire depth range with the characteristic “step” increase.

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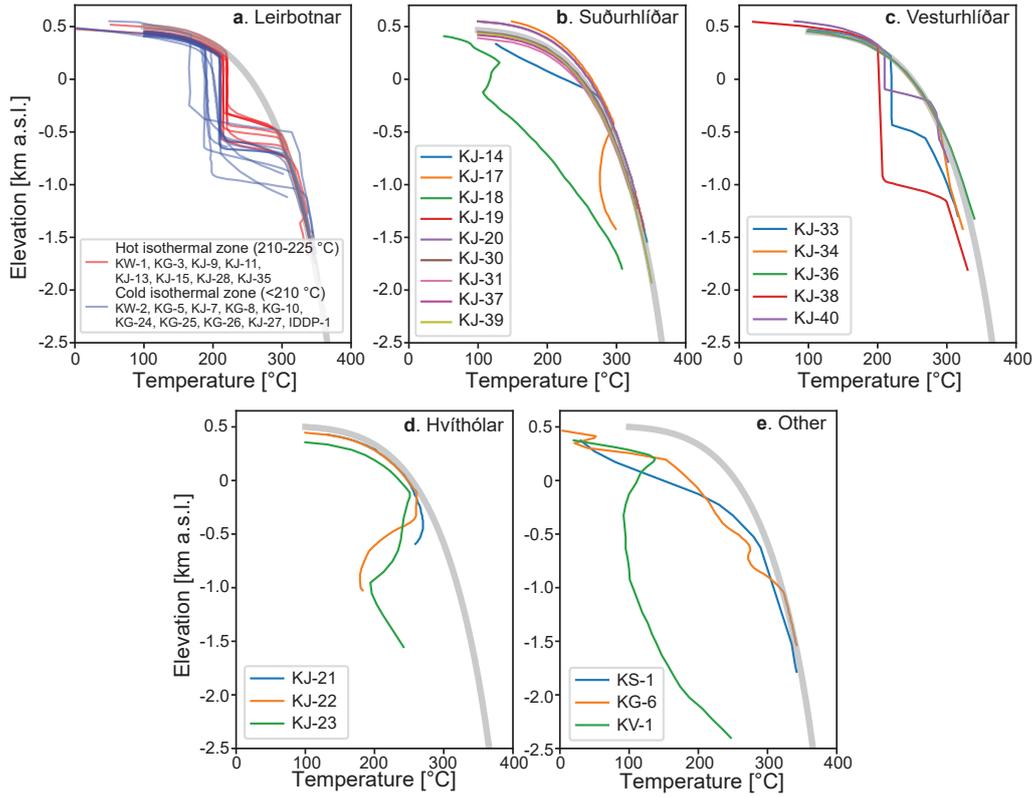
Several wells throughout Krafla have encountered high acidity fluids in the deeper boiling reservoir (K. Einarsson et al., 2010; Ármannsson et al., 2015) that are believed to originate from condensation of liquid from HCl-bearing vapor during ascent, depressurization, and mixing with cooler fluids (Heřmanská et al., 2019). The relatively wide geographic distribution of acid wells (labeled with red text in Figs. 2 and 4) suggests that HCl-bearing superheated/supercritical vapor is present at depths  $> 2$  km b.s.l. over a wide area in the center of the Krafla caldera (K. Einarsson et al., 2010; Ármannsson et al., 2015).

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Wells drilled in the Hvíthólar area show temperature inversions, with a boiling reservoir reaching 250-260 °C extending to approximately -0.5 km a.s.l. underlain by sharply decreasing temperatures to 170-190 °C. Such temperature inversions are often associa-



**Figure 3.** Natural state temperature-depth profiles for wells used to calibrate the numerical model. Wells are grouped by subfield and the overall style of temperature-depth profile: a) Leirbotnar, grouped on whether the temperature in the isothermal zone is 210-225 °C (Step hot in Fig. 3) or  $\leq 210$  °C (Step cold in Fig. 3), b) Suðurhlíðar c) Vesturhlíðar, d) Hvíthólar, and e) Other, including wells from Vestursvæði, Sandabotnaskarð, and KJ-06, the southernmost well in Leirbotnar. The boiling point with depth curve is shown in light gray, assuming a water table depth of 0.5 km a.s.l.

220 ted with lateral outflow of hot fluid below the clay cap, accompanied by recharge of cooler  
 221 waters at greater depths (Grant et al., 2011). However, Arnórsson (1995) suggested that  
 222 the temperature reversals represent ascending fluid along the rims of the caldera, and  
 223 Árnason (2020) argued that the temperature inversions represent the waning stages of  
 224 a geothermal system after the deep heat source has been extinguished by hydrothermal  
 225 convection. Well KS-1 (Fig. 3e) drilled in the Sandabotnaskarð area, east of Hvíthólar,  
 226 shows temperatures following the BPD between -0.5 and -1.5 km a.s.l. but lower tem-  
 227 peratures at shallower depths. These high temperatures suggest the potential that the  
 228 high-temperature system may extend to the south of Suðurhlíðar. In contrast, well KV-  
 229 1 in Vestursvæði, to the west of Hvíthólar, shows temperatures up to 150 °C above 0 km  
 230 a.s.l. and lower temperatures at greater depths, suggesting shallow outflow of hot fluid  
 231 and circulation of cooler surface-derived waters at depth.

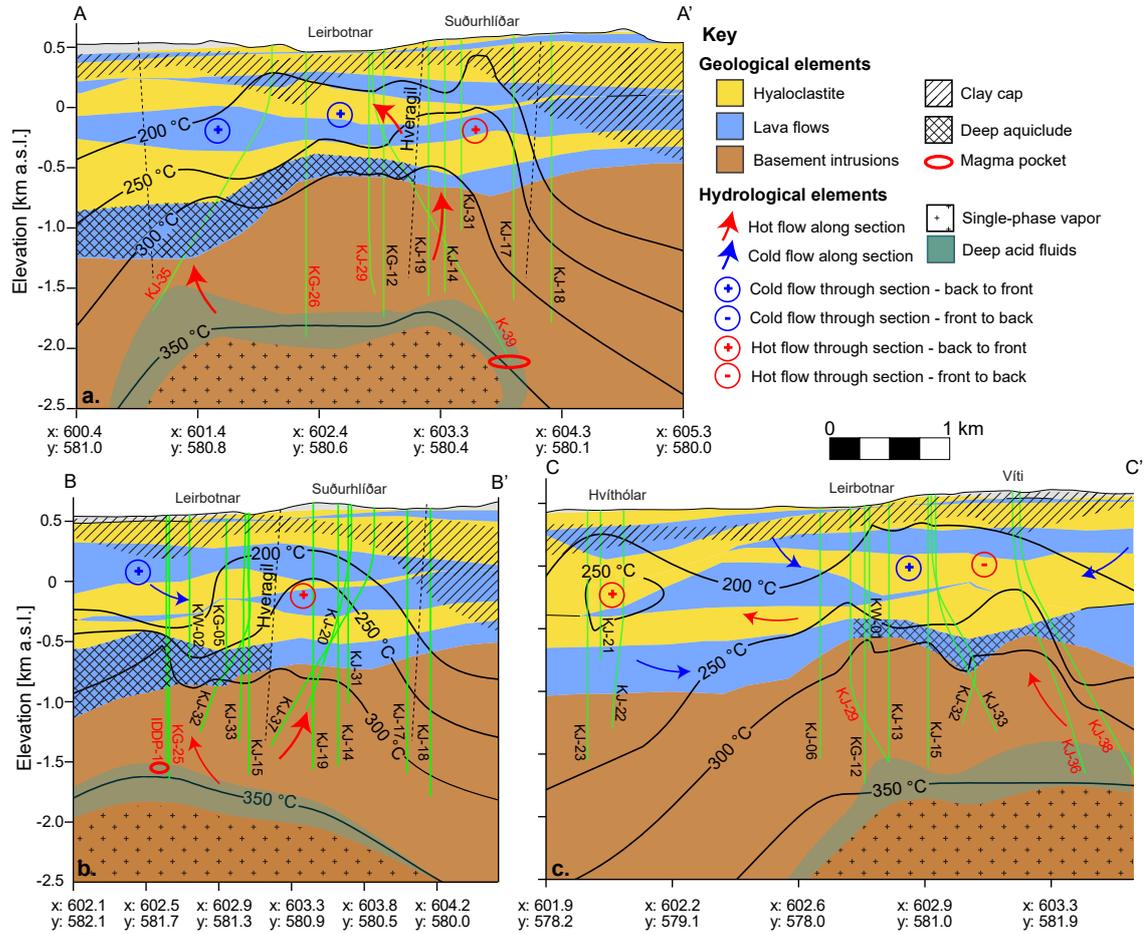
232 The conceptual model for the Krafla system is summarized in Figure 4. The mag-  
 233 matic heat source is a complex of basaltic sills and dykes located at  $\geq 2.5$ -3 km b.s.l., cor-  
 234 responding to the attenuating body identified during the volcano-tectonic episode between  
 235 1977-1984 (P. Einarsson, 1978). The rhyolitic magma encountered during drilling of IDDP-  
 236 1 in the northern part of Leirbotnar and KJ-39 along the southern rim of Suðurhlíðar  
 237 corresponds to local magma pockets with thickness  $\leq 0.2$  km, an interpretation consistent  
 238 with evidence from seismic (Kim et al., 2017, 2018; Schuler et al., 2016) and resistivity  
 239 (Lee et al., 2020) studies. At depths  $\geq 2$  km b.s.l., this fluid ascending above the heat  
 240 source is a single-phase vapor, which depressurizes, cools, and condenses to form an acidic  
 241 liquid condensate (Heřmanská et al., 2019), representing the deep reservoir fluids produced  
 242 by the 'acid' wells (labeled in red in Fig. 4). The main location of fluid upflow is along  
 243 and west of the Hveragil eruptive fissure, which features the high vertical and rift-parallel  
 244 permeability but relatively low cross-rift permeability. A low permeability aquitard situa-  
 245 ted at the transition between the underlying basement intrusions and shallower volcanics  
 246 that effectively separates the upper sub-boiling reservoir from the deeper boiling reser-  
 247 voir in Leirbotnar (Stefánsson, 1981; Bödvarsson et al., 1984). To the west of the Hveragil,  
 248 this aquitard is not present or not as tight, allowing the formation of a vertically-extensive  
 249 boiling zone in the Sudurhlíðar area. Variable mixing of fluid ascending along Hveragil  
 250 with cold water from the north (blue + sign in Fig. 4a) controls the spatial distribution  
 251 of temperature in the upper sub-boiling aquifer. We interpret the temperature inversions  
 252 measured in the Hvíthólar area as the result of southward outflow of hot water originating  
 253 from the main production field along the axis of the rift. The deuterium and oxygen-18  
 254 composition of the geothermal fluid at Krafla (A. E. Sveinbjörnsdóttir et al., 1986; Darling  
 255 & Ármannsson, 1989; Pope et al., 2015) indicate that the main recharge area for the geo-  
 256 thermal system comes from the highlands to the north of the main production field. However,  
 257 further deep recharge from the south and west, as suggested by the relatively cold tem-  
 258 peratures measured in Vestursvæði indicating cold cross-flow.

### 259 3 Methodology

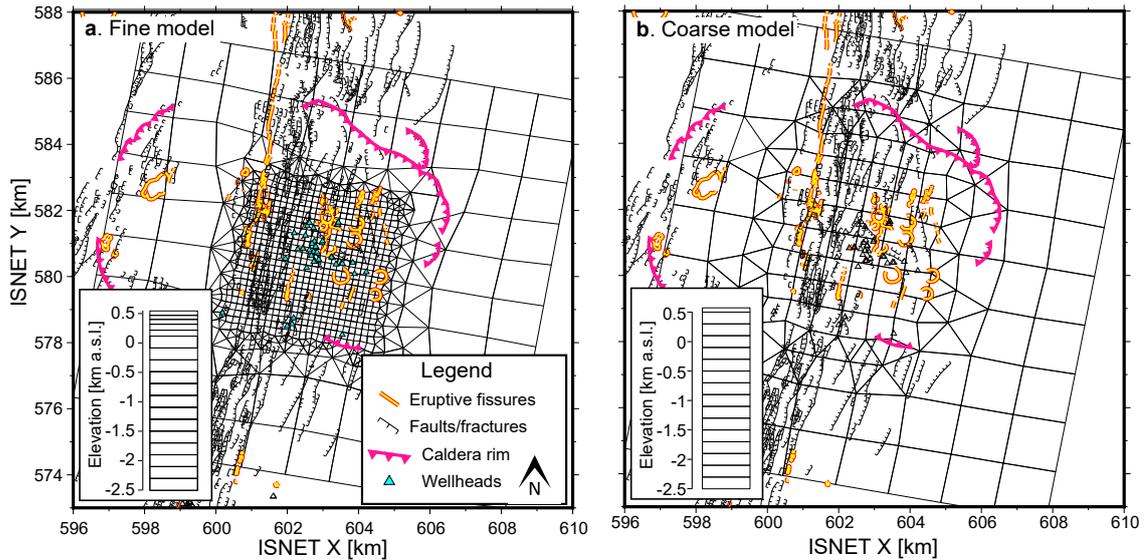
260 The model developed in this study incorporates several components:

- 261 1. Fine-scale and coarse-scale reservoir models based on the conceptual model of the  
 262 system (Figure 5)
- 263 2. A numerical method that accurately models flow of multi-phase, variably miscible  
 264 fluids, with a range of applicability extending to  $>375$  °C
- 265 3. A hierarchical Bayesian approach incorporating a posterior-informed approxima-  
 266 tion error model

267 Here, we briefly describe the numerical method and hierarchical Bayesian approach, which  
 268 have been published previously (Pruess et al., 2012; Maclaren et al., 2020), and focus on  
 269 description of the model setup.



**Figure 4.** Geologic-conceptual model of the Krafla system. The geologic and alteration model is based on Weisenberger et al. (2015). The surface traces of the cross sections are shown in Figure 2. **a.** An ESE-WNW cross-section across Leirbotnar and Suðurhlíðar. **b.** A NW-SW cross-section across the northern part of Leirbotnar and Suðurhlíðar. **c.** A N-S cross-section across the Víti caldera to Hvíthólar. The lithologic model describes a succession of hyaloclastite (yellow) and lava flows (blue) underlain by basement intrusions (brown). The cap rock (hatching) is based on the mapped distribution of smectite-zeolite and mixed-layer clay alteration facies. The deep aquitard (cross-hatching) is inferred to correlate with the basal lava flow west of Hveragili. Cross sections also show well traces (green lines), temperature isotherms derived from natural state temperature measurements (solid black lines), as well as select structural features (dashed black lines). Red/blue arrows indicate the schematic direction of flow of hot or cold water, respectively. Plus/minus sign enclosed in a circle indicates flow of hot or cold into or out of the section, respectively. Zones of single-phase vapor and deep acid fluids (see text) are shown with crosses and dark blue color, respectively.



**Figure 5.** Grid structure of the a) fine-scale and b) coarse-scale models used in this study.

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### 3.1 Model Structure

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Figure 5 shows the discretization and layer structure of the fine and coarse scale models developed in this study. The numerical grid extends  $\sim 14$  km in the N-S direction,  $\sim 12.5$  km in the E-W direction, and from the surface to  $-2.5$  km a.s.l. The grid is rotated  $11^\circ$  along the main axis of rifting. The coarse- and fine-scale models consist of 2551 and 17193 grid blocks in 17 and 22 layers, respectively, with increasing resolution towards the surface. The maximum horizontal resolution of the grid blocks in the fine- and coarse-scale model is  $0.15 \text{ km}^2$  and  $0.5 \text{ km}^2$ , respectively.

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The numerical grid was first designed in Leapfrog Geothermal <sup>®</sup>, refined using PyTOUGH (Croucher, 2011; J. O’Sullivan et al., 2015), and then imported back into Leapfrog Geothermal in order to populate rock types in the model based on the geologic model (Mortensen et al., 2015; Weisenberger et al., 2015; Scott et al., 2019). The numerical model considers 14 different rock types (Table 1). The extrusive volcanic rocks in the upper 1-1.5 km of the system consist of 4 lava flow units (LV001-LV004) and 3 hyaloclastite units (HY001-HY003). Two different basement intrusion rock types were defined for the geothermal system in the center of the caldera: BASE1, which comprises the majority of the basement, and BASE2, which was defined locally at the base of wells IDDP-1 and KJ-39 where magmatic intrusions were encountered. Two different fault rock types were defined: 1) TVFLT with relatively high cross-rift permeability, and low rift-parallel permeability, and 2) BARFT with relatively high rift-parallel permeability and low cross-rift permeability. A clay cap rock type (CLAYC) with relatively low permeability was defined and the geometry of the clay cap was constrained based on the alteration model and resistivity data. Two rock types were defined for the area outside of the main area of the geologic model: 1) OUTBS, defining the outer basement below 1 km b.s.l., and 2) OUTER, defining the upper volcanic sequence above 1 km b.s.l. As described in the conceptual model, the basal lava flow overlying the basement rocks (LV001) was modeled as an aquitard.

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Both the fine and coarse scale models were manually calibrated prior to MCMC sampling in order to achieve a relatively close fit between the model predictions and the measured natural state temperature data. This mainly involved manual adjustment of

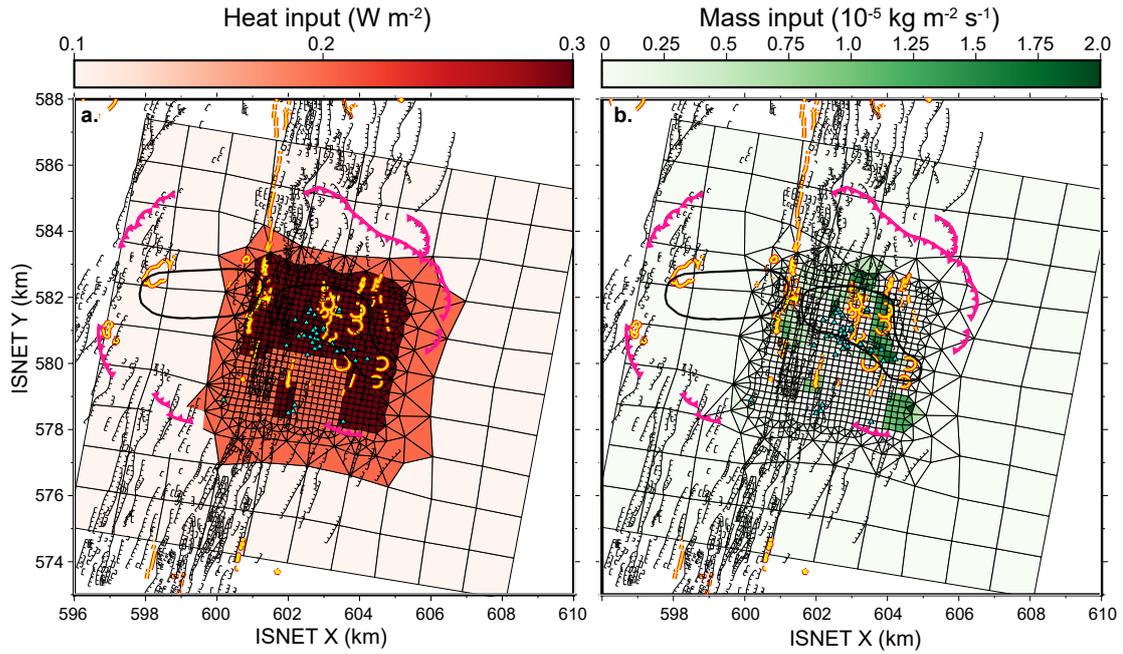
**Table 1.** Assumed density and porosity (fixed) and prior means for permeability for the 14 rock units included in the numerical model.

Rock type	Density ( $\text{g cm}^{-3}$ )	Porosity (-)	Cross-rift permeability ( $k^x$ , $\text{m}^2$ )	Rift-parallel permeability ( $k^y$ , $\text{m}^2$ )	Vertical permeability ( $k^z$ , $\text{m}^2$ )
BASE1	2.75	0.05	7.5E-16	1.5E-15	1E-15
BASE2	2.75	0.03	7.5E-16	7.5E-16	5E-16
HY001	2.5	0.1	5E-14	1.5E-13	5E-14
HY002	2.5	0.15	5E-14	1.5E-13	1.5E-13
HY003	2.4	0.2	5E-14	1.5E-13	1E-13
LV001	2.6	0.05	1E-16	2.5E-16	5E-17
LV002	2.6	0.1	1E-13	2.25E-13	1.25E-13
LV003	2.6	0.1	1E-13	2E-13	1E-13
LV004	2.6	0.1	1E-13	2E-13	1.5E-13
OUTER	2.6	0.1	1E-15	1.5E-14	2E-15
OUTBS	2.75	0.05	5E-16	1E-15	7.5E-15
BARFT	2.6	0.1	2.5E-16	1E-15	1E-15
TVFLT	2.6	0.05	1.5E-15	7.5E-16	5E-15
CLAYC	2.6	0.2	2E-15	5E-15	5E-15

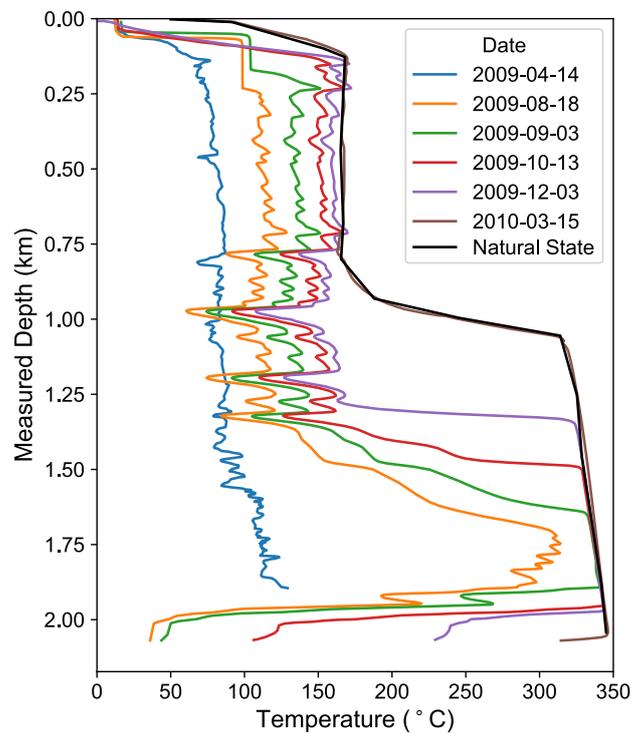
300 rock permeabilities and the distribution of heat and mass input at the base of the model,  
 301 along with minor modifications made to the spatial configuration of the rock types. The  
 302 anisotropic permeability of each rock unit along with other rock properties (density, porosity,  
 303 thermal conductivity) achieved during the initial calibration are shown in Table 1. These  
 304 values serve as the means of the prior probability distributions (assumed to be Gauss-  
 305 ian with a fixed standard deviation of 0.75 log units) specified for the MCMC sampling.

306 The distribution of heat and mass input at the base of the model is shown in Figure  
 307 6. Heat flow was adjusted from  $0.3 \text{ W m}^{-2}$  in the center of the caldera to  $0.1 \text{ W m}^{-3}$   
 308 on the perimeter of the model, in accordance with estimates of regional heat flow val-  
 309 ues in Icelandic volcanic systems (Flóvenz & Saemundsson, 1993). The specific enthalpy  
 310 of the input fluid is set to  $3.2 \text{ MJ kg}^{-1}$ , in accordance with measurements of the discharge  
 311 enthalpy of the IDDP-1 well (Ingason et al., 2014). Temperature and pressure were not  
 312 fixed at the base of the model. The total mass of high-enthalpy fluid added into the base  
 313 of the system is  $65.35 \text{ kg s}^{-1}$ .

314 The inferred natural state temperature profiles used to calibrate the simulations  
 315 were provided by Landsvirkjun in accordance with Iceland GeoSurvey (Mortensen et al.,  
 316 2009; Weisenberger et al., 2015). Figure 7 shows an example of an inferred natural state  
 317 temperature profile from measured downhole temperatures for well IDDP-1 (Mortensen  
 318 et al., 2015). With increasing time subsequent to drilling, the measured temperatures  
 319 more closely correspond to the inferred natural state temperature (black) as the well heats  
 320 up. After the IDDP-1 well began to discharge, fluid with a temperature of  $450 \text{ }^\circ\text{C}$  was  
 321 eventually produced at the surface (Elders et al., 2011). This is not recorded in the down-  
 322 hole temperature data, due to the likelihood that the permeable zone in the near vic-



**Figure 6.** (a) Heat and (b) mass input into the base of the fine-scale model. The black lines show the boundaries of the zone with S-wave shadows detected by P. Einarsson (1978).



**Figure 7.** Measured temperatures between 2009-2011 during heating up of well IDDP-1 and comparison with inferred natural state temperature (black line).

323 inity of the intrusion was not finished heating up before a restriction in the casing of the  
 324 well prevented downhole temperature logging (Axelsson et al., 2014).

### 325 3.2 Numerical Method

326 The simulations are carried out using AUTOUGH2 (Yeh et al., 2012), a version of  
 327 TOUGH2 (Pruess et al., 2012) developed at the University of Auckland. The EOS3sc  
 328 equation of state (EOS) (J. O’Sullivan et al., 2016, 2020), a modified version of the EOS1sc  
 329 EOS (Croucher & O’Sullivan, 2008; Magnúsdóttir & Finsterle, 2015), was used in order  
 330 to extend the applicability of the model to the supercritical roots of the system. Although  
 331 this EOS includes air and could in principle also model the vadose zone, the top of the  
 332 model is set to the depth of the water table.

333 TOUGH2 solves the governing equations of mass and energy balance using a finite  
 334 volume approach. For quantity  $\kappa$  (e.g. energy  $u$ , mass  $m$ ) within a finite volume  $V_i$ , bounded  
 335 by a surface  $\Omega_i$ , conservation is represented in integral form as

$$336 \frac{d}{dt} \int_{V_i} M_\kappa dV = - \int_{\Omega_i} \mathbf{F}_\kappa \cdot \mathbf{n} d\Omega + \int_{V_i} q_\kappa dV \quad (1)$$

337 where  $t$  is time,  $M_\kappa$  is mass or energy per unit volume ( $\text{kg m}^{-3}$  or  $\text{J m}^{-3}$ , respectively),  
 338  $F_\kappa$  is the flux of mass or energy,  $q_\kappa$  represents sink and source terms (e.g. deep inflows),  
 339 and  $\mathbf{n}$  denotes an outward-pointing unit normal vector to the surface  $\Omega_i$ . The amount  
 340 of mass ( $M_m$ ) and energy ( $M_u$ ) per unit control volume are given by

$$341 M_m = \Phi(\rho_l S_l + \rho_v S_v) \quad (2)$$

$$342 M_u = (1 - \Phi)\rho_r u_r T + \Phi(\rho_l u_r S_l + \rho_v u_v S_v) \quad (3)$$

344 where  $\Phi$  is porosity,  $S_\beta$  is the volumetric saturation of fluid phase  $\beta$  (liquid  $l$  or vapor  
 345  $v$ ),  $\rho_\beta$  and  $\rho_r$  the density of the fluid phase or rock (denoted by subscript  $r$ ),  $u_\beta$  fluid  
 346 phase internal energy,  $u_r$  rock specific heat, and  $T$  temperature. Fluid mass fluxes are  
 347 described by a two-phase version of Darcy’s law:

$$348 \mathbf{F}_\beta = -\mathbf{k} \frac{k_{r,\beta}}{\nu_\beta} (\nabla P - \rho_\beta \mathbf{g}) \quad (4)$$

349 where  $\mathbf{k}$  is the permeability tensor (assumed to be diagonal),  $\mathbf{g}$  the vector of gravitational  
 350 acceleration,  $P$  is pressure, and  $k_\beta$  and  $\nu_\beta$  are the relative permeability and kinematic  
 351 viscosity of fluid phase  $\beta$ . Energy flux includes the contribution of heat conduction and  
 352 advection of each fluid phase:

$$353 \mathbf{F}_u = -K \nabla T + \sum_{\beta} h_\beta \mathbf{F}_\beta \quad (5)$$

354 where  $K$  is the effective thermal conductivity of the fluid-rock medium and  $h_\beta$  is the specific  
 355 enthalpy of fluid phase  $\beta$ .

356 TOUGH2 implements the integral finite difference method (Narasimhan & Wit-  
 357 herspoon, 1976) for spatial discretization of the mass and energy conservation equati-  
 358 ons. A fully implicit scheme with adaptive time stepping is used for numerical integra-  
 359 tion in time, and upstream weighting of fluid properties is used for calculating flows between  
 360 adjacent blocks. TOUGH2 uses the Newton-Raphson method and a preconditioned conju-  
 361 gate gradient sparse matrix solver to solve the system of equations (Pruess, 1991). Newton-  
 362 Raphson iteration continues until the residual of the discretized version of the conserva-  
 363 tion equations are reduced to a small fraction of the mass accumulation terms, with the  
 364 convergence criterion set to  $1 \times 10^{-5}$ . If convergence is not achieved within 9 iterati-  
 365 ons, the timestep is reduced by a factor of 5 and solution re-attempted; if convergence  
 366 is achieved within 5 iterations, the next time-step is doubled. Once a steady-state natural

367 state configuration is reached, time-steps increase up to a large value and the simulation  
 368 runs to completion rapidly. We observed that convergence was not attained in some of  
 369 the simulations, potentially as a result of convective instabilities in the upper high per-  
 370 meability rocks or grid blocks that change between phase regions in response to very small  
 371 changes in pressure and temperature. Therefore, it was necessary to prematurely termi-  
 372 nate simulations that take longer than 5 minutes to converge and not use them for calculati-  
 373 on of the posterior statistics. However, this affected less than 5% of the simulations.

### 374 3.3 Bayesian Hierarchical Framework

375 According to the Bayesian framework, the solution to the inverse problem involving  
 376 identification of geologically reasonable model parameters  $\mathbf{k}$  consistent with measured  
 377 data  $\mathbf{y}_{\text{obs}}$  is a probability distribution calculated using Bayes' theorem, written schematically  
 378 as:

$$379 \quad p(\mathbf{k} \mid \mathbf{y}_{\text{obs}}) \propto p(\mathbf{y}_{\text{obs}} \mid \mathbf{k})p(\mathbf{k}) \quad (6)$$

380 Following standard terminology,  $p(\mathbf{k} \mid \mathbf{y}_{\text{obs}})$  refers to the posterior,  $p(\mathbf{y}_{\text{obs}} \mid \mathbf{k})$  the likeli-  
 381 hood and  $p(\mathbf{k})$  the prior. Bayes' theorem is written above as a proportionality relation-  
 382 ship, omitting a normalization factor that is not required by most sampling algorithms  
 383 (Gelman et al., 2013). For natural state geothermal reservoir models, the main adjust-  
 384 able model parameters include the spatial configuration of rock types, the anisotropic  
 385 permeability of each rock type, and the locations and magnitudes of deep hot inflows.  
 386 In this study, we limit the parameters of interest for uncertainty quantification to the  
 387 anisotropic permeability of each rock type; the model geologic structure (Figure 4) as  
 388 well as the locations and magnitudes of heat and mass input (Figure 6) are fixed. The  
 389 observed data consist of inferred natural state temperatures from 40 deep geothermal  
 390 wells (Figure 3), which were interpolated to layer centers in the coarse model, leading  
 391 to 261 total observation points.

392 The main challenge associated with the application of Bayes' theorem to geothermal  
 393 systems is the computational cost involved in the repeated evaluation of the forward model.  
 394 While a significant speed-up can be achieved by using a coarsened version of the forward  
 395 model in place of a finer, more accurate model, this introduces approximation errors that  
 396 can lead to incorrect estimation of model parameters and their associated uncertainties  
 397 (Kaipio & Somersalo, 2007). As described by Maclaren et al. (2020), the Bayesian hier-  
 398 archical framework allows the resulting model approximation error to enter into the calculati-  
 399 on of the likelihood as a probabilistic process error via decomposition of a full joint proba-  
 400 bility distribution over all quantities of interest into a measurement model, process model,  
 401 and parameter model:

$$402 \quad p(\mathbf{y}_{\text{obs}}, \mathbf{y}_{\text{process}}, \mathbf{k}) = p(\mathbf{y}_{\text{obs}} \mid \mathbf{y}_{\text{process}})p(\mathbf{y}_{\text{process}} \mid \mathbf{k})p(\mathbf{k}) \quad (7)$$

403 where the distribution parameters and distribution subscripts are suppressed for simplicity.  
 404 The latent process vector is assumed to be generated by the fine-scale model

$$405 \quad \mathbf{y}_{\text{process}} = f(\mathbf{k}) \quad (8)$$

406 The use of the coarsened model  $g(\mathbf{k})$  in place of a finer, more accurate model  $f(\mathbf{k})$  introduces  
 407 additional approximation errors, which are defined as

$$408 \quad \boldsymbol{\epsilon} = \mathbf{y}_{\text{process}} - g(\mathbf{k}) = f(\mathbf{k}) - g(\mathbf{k}) \quad (9)$$

409 Assuming additive error models (Maclaren et al., 2020), the measurement and process  
 410 model components correspond to a two-stage decomposition of the form

$$411 \quad \mathbf{y}_{\text{obs}} = \mathbf{y}_{\text{process}} + \mathbf{e} \quad (10)$$

$$412 \quad \mathbf{y}_{\text{process}} = g(\mathbf{k}) + \boldsymbol{\epsilon} \quad (11)$$

414 where  $\mathbf{e}$  is the measurement error (which may include correlations). Combining the ap-  
 415 proximation errors with the measurement errors to give the total error ( $\nu = \mathbf{e} + \epsilon$ ),  
 416 the measurement model can be written as:

$$417 \quad \mathbf{y}_{\text{obs}} = g(\mathbf{k}) + \epsilon + \mathbf{e} = g(\mathbf{k}) + \mathbf{v} \quad (12)$$

418 Obtaining the likelihood  $p(\mathbf{y}_{\text{process}} | \mathbf{k})$  using these functional relationships is done by  
 419 marginalizing over the total error, assuming that the measurement and approximation  
 420 errors are independent of the parameter vector (Maclaren et al., 2020), so that:

$$421 \quad p_{Y_0|K}(\mathbf{y}_{\text{obs}} | \mathbf{k}) = p_{\mathbf{v}}(\mathbf{y}_{\text{obs}} - g(\mathbf{k})) \quad (13)$$

422 Then, the posterior can be written as

$$423 \quad \begin{aligned} p_{K|Y_0}(\mathbf{k} | \mathbf{y}_{\text{obs}}) &\propto p_{Y_0|K}(\mathbf{y}_{\text{obs}} | \mathbf{k})p_K(\mathbf{k}) \\ &= p_{\mathbf{v}}(\mathbf{y}_{\text{obs}} - g(\mathbf{k}))p_K(\mathbf{k}) \end{aligned} \quad (14)$$

424 where the process error is absorbed into the likelihood.

425 Absorbing the process error into the likelihood in this manner requires the distribu-  
 426 tion of the total error,  $p_{\mathbf{v}}(\cdot)$ , to be known. In standard Bayesian approximation error ap-  
 427 proaches, the statistics of the model approximation errors are precomputed empirically  
 428 by drawing samples from the prior distribution. However, sampling from an insufficiently  
 429 informative prior and running the fine-scale geothermal reservoir model leads to a high  
 430 probability of practical issues such as model run failures, long model run times, and/or  
 431 extreme model inputs. Therefore, Maclaren et al. (2020) developed the posterior-informed  
 432 composite approximation error approach, where the statistics of the model approxima-  
 433 tion error are obtained via direct sampling from a *naïve posterior* computed by separ-  
 434 ate MCMC sampling using the coarse model without consideration of any approxima-  
 435 tion errors. Thus, the model approximation error is made using a posterior plug-in estima-  
 436 tion where the coarse model posterior is used to estimate the error distribution marg-  
 437 inalized over the parameter:

$$438 \quad p_{\epsilon}(\epsilon) \leftarrow \hat{p}_{\epsilon|Y_0}(\epsilon | \mathbf{y}_{\text{obs}}) = \int p_{\epsilon|K}(\epsilon | \mathbf{k})\hat{p}_{K|Y_0}(\mathbf{k} | \mathbf{y}_{\text{obs}})d\mathbf{k} \quad (15)$$

439 where the likelihood function in  $\hat{p}_{K|Y_0}$  is based on the coarse-scale model  $g(\mathbf{k})$  without  
 440 accounting for approximation errors. Once the error distribution has been estimated, the  
 441 composite model of the joint distribution along with the original prior is used:

$$442 \quad p_{\epsilon,K}(\epsilon, \mathbf{k}) \leftarrow p_{\epsilon}(\epsilon)p_K(\mathbf{k}) \quad (16)$$

443 Although the use of the naïve posterior for calculating of the model approximation error  
 444 may narrow the error distribution when compared to the distribution that results from  
 445 the prior, it has the benefit of providing more relevant estimates of the model error when  
 446 the posterior based on the coarse model is not too far from the true posterior, which is  
 447 the case for our models.

448 In this study, the statistics of the approximation error were calculated by drawing  
 449 an ensemble of  $q = 150$  samples from the naive posterior density  $\hat{\mathbf{p}}(\mathbf{k} | \mathbf{y}_{\text{obs}})$  with  $\mathbf{k}^{\ell}$   
 450 for  $\ell = 1, 2, \dots, q$ . Assuming the the approximation error is Gaussian, its distribution  
 451 can be calculated from the ensemble mean and covariance:

$$452 \quad \epsilon_* = \frac{1}{q} \sum_{\ell=1}^q \epsilon^{(\ell)}, \quad \Gamma_{\epsilon} = \frac{1}{q-1} \sum_{\ell=1}^q (\epsilon^{(\ell)} - \epsilon_*)(\epsilon^{(\ell)} - \epsilon_*)^T \quad (17)$$

453 As the total error  $\nu$  is the sum of both the noise and the approximation error, given the  
 454 normality assumption, the distribution of the total error is given by:

$$455 \quad \nu \sim \mathcal{N}(\nu_*, \Gamma_{\nu}) = \mathcal{N}(\mathbf{e}_* + \epsilon_*, \Gamma_e + \Gamma_{\epsilon}) \quad (18)$$

456 This enters into the likelihood as follows:

$$457 \quad p_{\nu}(\mathbf{y}_{\text{obs}} - g(\mathbf{k})) \propto \exp \left( -\frac{1}{2} \sum_{i=1}^N (g^N(\mathbf{k}) - \nu_*^N - \mathbf{y}_{\text{obs}}^N)^T \Gamma_{\nu}^{-1} (g^N(\mathbf{k}) - \nu_*^N - \mathbf{y}_{\text{obs}}^N) \right) \quad (19)$$

458 In our simulations, we observe that the covariance of the approximation errors differs greatly  
 459 for different measurements; namely, the approximation errors for temperature measurements  
 460 in the liquid isothermal zone show a high covariance, and the approximation errors for  
 461 temperature measurements in the boiling zones show low covariance. This influences the  
 462 relative weight of the residual of these measurements on the likelihood, increasing the  
 463 importance of the temperature measurements where a larger covariance is calculated by  
 464 the approximation error model. Therefore, to avoid this, we also perform MCMC sampling  
 465 using a likelihood function that only considers the means of the approximation errors  
 466 (i.e. offset or bias term) and a constant noise term:

$$467 \quad p_{\nu}(\mathbf{y}_{\text{obs}} - g(\mathbf{k})) \propto \exp \left( -\frac{1}{2} \sum_{i=1}^N \frac{[g^N(\mathbf{k}) - \nu_*^N - \mathbf{y}_{\text{obs}}^N]^2}{\sigma^2} \right) \quad (20)$$

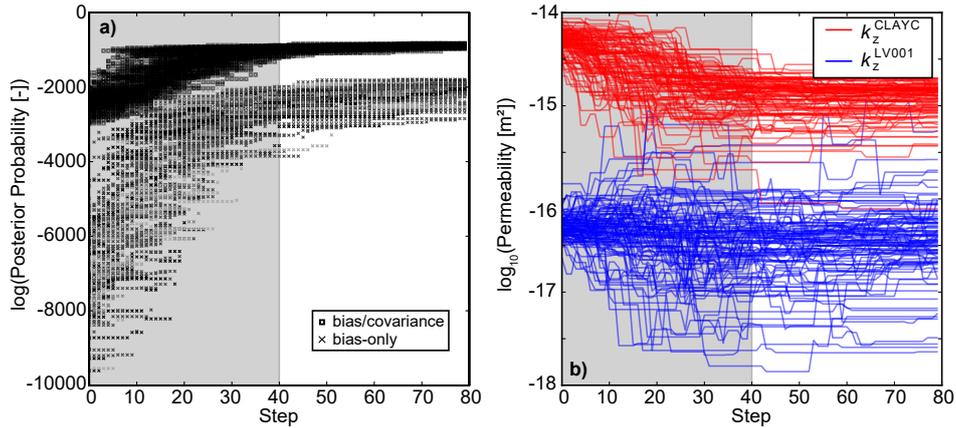
468 where the covariance of the temperature measurements is fixed (i.e.  $\sigma^2 = 10 \text{ }^{\circ}\text{C}$ ).

### 469 3.4 MCMC Sampling

470 Markov chain Monte Carlo (MCMC) sampling is carried out using the Python Pack-  
 471 age *emcee* (Foreman-Mackey et al., 2013), which implements an affine invariant ensemble  
 472 sampler (Goodman & Weare, 2010). The *emcee* package implements the Stretch Move  
 473 algorithm (Goodman & Weare, 2010), which is similar in principle to the standard Metropolis-  
 474 Hastings algorithm (Metropolis et al., 1953), with a proposal and an accept/reject step.  
 475 However, this method involves simultaneously evolving an ensemble of walkers, where  
 476 the proposal distribution for one walker is based on the current positions of the other  
 477 walkers in the complementary ensemble. Used in the context of our models, “position”  
 478 refers to a vector in the 42-dimensional parameter space (the  $x$ ,  $y$ , and  $z$  permeabilities  
 479 for the 14 different rock types). The property of affine invariance implies that the per-  
 480 formance of the algorithm is independent of the aspect ratio in highly anisotropic distri-  
 481 butions and is well-suited for skewed posterior distributions (Goodman & Weare, 2010).

482 The PyTOUGH library (Croucher, 2011; J. O’Sullivan et al., 2015) is used to in-  
 483 terface the *emcee* sampler with the AUTOUGH2 simulator. For determination of both  
 484 the ‘naive’ posterior and the discrepancy-informed posterior, 80,000 samples were compu-  
 485 ted (10 ensembles of 100 walkers taking 80 samples each). A total of 40,000 samples (4000  
 486 for each ensemble, 40 for each walker) were discarded as burn-in based on when the sam-  
 487 ples began to converge around a posterior certain probability (Fig 8a). The statistics of  
 488 the model error were computed by running 150 iterations of the fine and coarse model  
 489 using the statistics of the ‘naive’ posterior; this required approximately 15 days of compu-  
 490 ting time. All computations were carried out on a standard desktop computer with an  
 491 Intel Xeon E5-1620 3.50 GHz 8-core processor. Each simulation using the coarse model  
 492 took anywhere from 10 seconds to one hour to run; as noted above, in order to limit the  
 493 computing time, coarse models that took longer than 5 minutes to run (less than 5% of  
 494 all simulations) were terminated and not used for the calculation of posterior statistics.  
 495 Each ensemble of 100 walkers took approximately  $1.3 \times 10^6$  seconds (15 days) to run to  
 496 completion. Since each simulation using the fine model took up 30 minutes - 1 day to  
 497 run or more to run, MCMC sampling with an equivalently long chain and using the fine  
 498 model would take more than year. Thus, this represents a significant speed-up and em-  
 499 phasizes the fact that the MCMC computations are only feasible using the coarse model.

500 The evolution of the posterior probability of the computed models is shown in Fig  
 501 8a, with the position of each walker at each iteration shown as a marker. Each walker



**Figure 8.** Trace of the log posterior probability function and two parameters during MCMC sampling. The first forty steps were discarded as burn-in (grey shaded area). a) Trace of the posterior probability showing the positions of walkers during successive steps. Samples in MCMC chains in which the likelihood function considers the bias and covariance or the bias-only of the approximation error and shown as rectangles and crosses, respectively. b) Trace of the vertical permeability of LV001 (blue lines) and the vertical permeability of the clay cap (red lines).

502 was initialized at a point in the parameter space in the near vicinity of the prior means  
 503 for each parameter (Table 1). The example parameter trace for the vertical permeability  
 504 of LV001 (blue line in Fig 8b) show that the walkers tend to cluster around the prior mean,  
 505 which is the case for many of parameters. However, the trace of the vertical permeability  
 506 of the clay cap (red line in Fig. 8b) shows that the inversion process results in walkers  
 507 tending towards lower permeability values. The acceptance fraction varied between the  
 508 chains from  $\leq 0.05$  to 0.5, with an average of 0.26, close to the optimal range (Gelman  
 509 et al., 1996).

510 The correspondence between the prior and posterior means could be the result of  
 511 a highly informative prior model, but could also result from autocorrelation. Convergence  
 512 tests such as the autocorrelation time indicate that the chain is not long enough (Foreman-  
 513 Mackey et al., 2013), and an insufficient number of independent samples were obtained  
 514 to ensure representative sampling of the target density. However, due to practical lim-  
 515 itations imposed by the relatively long amount of time required to run the AUTOUGH2  
 516 forward models, which were run in serial and sequentially, it is not presently feasible to  
 517 run a chain long enough to ensure convergence to the target posterior. Despite this obvi-  
 518 ous shortcoming, we simply note that this approach allows for practical uncertainty quantifica-  
 519 tion of inferred rock permeabilities, assuming that the prior model is highly informative,  
 520 a necessary assumption when applying inverse modeling tools for natural state geothermal  
 521 reservoir model calibration (M. O’Sullivan & O’Sullivan, 2016).

## 522 4 Results

523 The Bayesian calibration scheme improves the fit between the model results and  
 524 the inferred natural state temperature profiles. Systematically different posterior results  
 525 are achieved if the MCMC sampling is carried out using the coarse model without any  
 526 approximation error model (the naïve posterior), or with an approximation error corrected  
 527 model. The approximation error model can account for the bias of approxima-  
 528 tion errors (the mean offset between the fine and coarse models) (Eqn. 20) or both the  
 529 bias and covariance of the approximation errors (Eqn. 19). Posterior predictive checks

530 compare the data generated using the fitted model with the observed data (Gelman &  
 531 Hill, 2006), show the ability of the stochastic model to match the inferred natural state  
 532 temperature distribution. Although MCMC sampling is performed using the coarse model,  
 533 all posterior predictive checks are computed using the fine model with parameter sets  
 534 from the respective posterior (naive, bias-only, or bias/covariance). The posterior proba-  
 535 bility distribution functions (pdfs) of the anisotropic permeability of the 14 different rock  
 536 types quantify the uncertainty underlying their estimation.

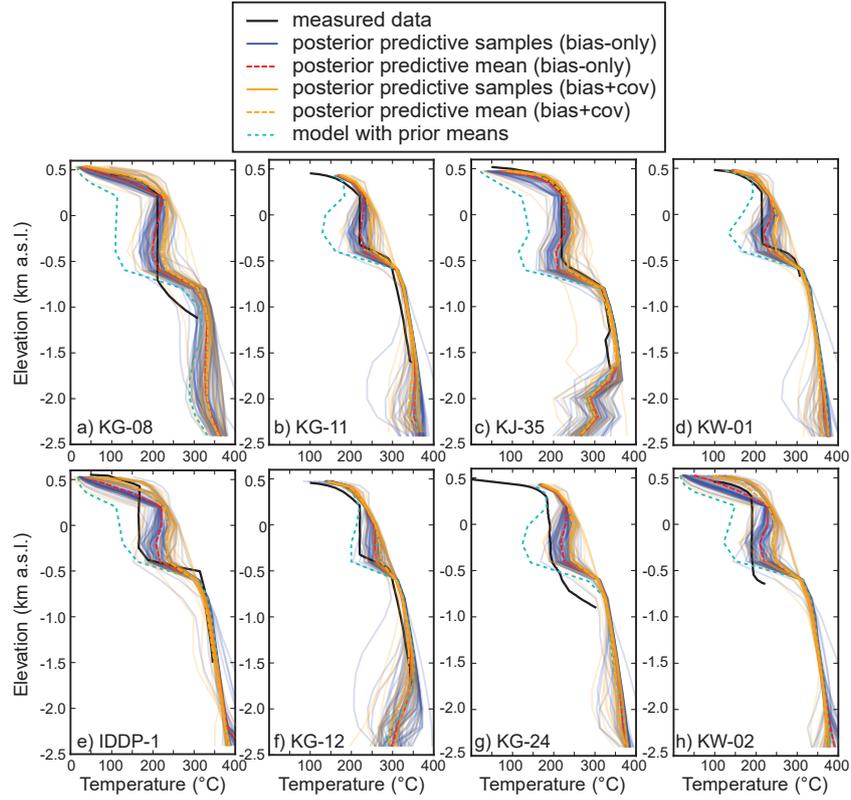
#### 537 4.1 Posterior predictive checks

538 The modeled natural state temperature distribution generally corresponds well to  
 539 the inferred natural state temperature field. Figure 9 compares model predictions with  
 540 measured data (posterior predictive checks) for 8 wells located in the Leirbotnar sub-  
 541 field. Compared to the fine model assuming prior means for all parameters (dashed blue  
 542 lines in Figs. 9-11), posterior results achieve a closer fit to the inferred natural state temp-  
 543 erature data (black lines). These temperature profiles reproduce the characteristic temperature-  
 544 depth relationship in Leirbotnar, with the upper isothermal zone at temperatures  $\sim 180$ -  
 545  $220$  °C underlain by a deeper boiling reservoir.

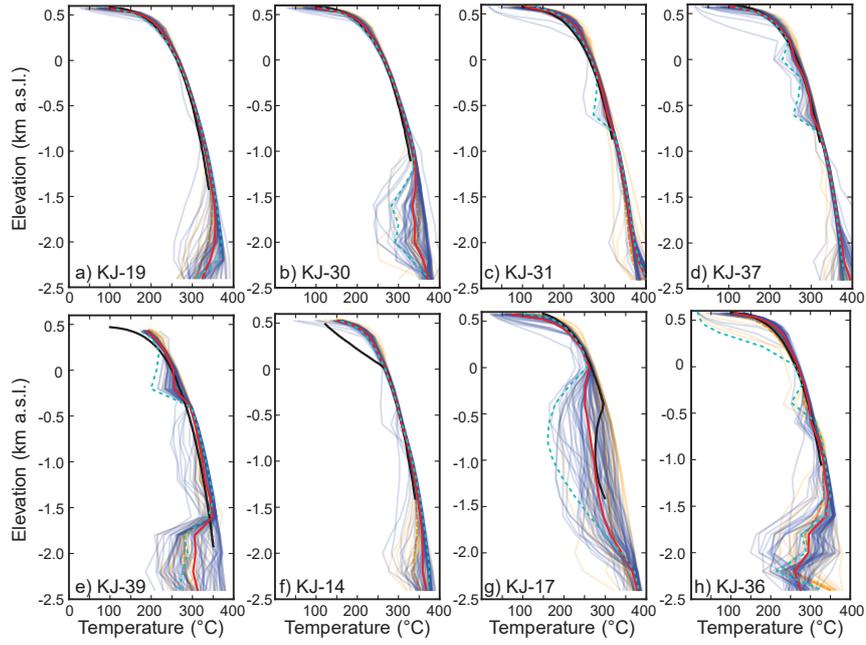
546 Several of the wells (Figure 9a-d) show a good correspondence between the mean  
 547 temperature of the isothermal zone computed by the set of posterior predictive samples  
 548 and the inferred natural state temperatures. However, the models predict 20-40 °C higher  
 549 temperatures in the isothermal zone than measured for several wells. Only considering  
 550 the bias in the approximation error model (orange lines in Fig. 9-11) results in a closer  
 551 approach to the inferred temperature of the isothermal zone than if both the bias/covariance  
 552 of the approximation errors are considered (blue lines in Fig. 9-11). Posterior samples  
 553 from the latter tend to systematically show temperatures 20-40 °C higher than inferred.  
 554 The model clearly reproduces the transition between the upper near-isothermal liquid  
 555 aquifer and deeper boiling aquifer, but the model results for wells KG-08, KG-24 and KW-  
 556 02 show this transition occurring at shallower depths than measured. The uncertainty  
 557 in posterior temperature is larger in the isothermal zone and at depths  $\leq -2$  km a.s.l. than  
 558 in the near surface ( $\geq 0$  km a.s.l.) or between 1-2 km b.s.l. This is at least partially a  
 559 result of the fact that temperatures in boiling zones following the boiling point with depth,  
 560 and are dependent mainly on the hydrostatic pressure.

561 Figure 10 shows posterior predictive checks for selected wells located in the Suður-  
 562 hlíðar subfield, The model clearly reproduces the vertically-extensive boiling zone between  
 563 the surface and -2 km a.s.l. in Suðurhlíðar. Due to the thermodynamic constraint imposed  
 564 by the co-dependence of temperature and pressure at boiling conditions, the uncertainty  
 565 in in temperatures in model predictions in boiling zones is low. The greatest uncertainty  
 566 in model predictions is at depths  $\leq -1.5$  km a.s.l., below the maximum depth of the drilled  
 567 wells, where the posterior predictive samples shows temperatures ranging between 200-  
 568  $400 \geq$  °C (e.g. Fig. 10e). Temperatures in excess of the critical temperature of pure water  
 569 ( $\geq 374$  °C) are predicted below -2 km a.s.l. in several wells (KJ-31, KJ-37). However, in  
 570 other wells (KJ-19, KJ-39, KJ-36) the model suggests the potential for significant deep  
 571 temperature inversions below -1.5 km a.s.l. A temperature inversion below -0.5 km a.s.l.  
 572 measured in well KJ-17 is generally reproduced by the model (Fig. 10g), but there is significant  
 573 uncertainty in the depth and magnitude of the temperature reversal. The model does  
 574 not reproduce the lower temperatures closer to the surface in well KJ-14 (Fig 10f).

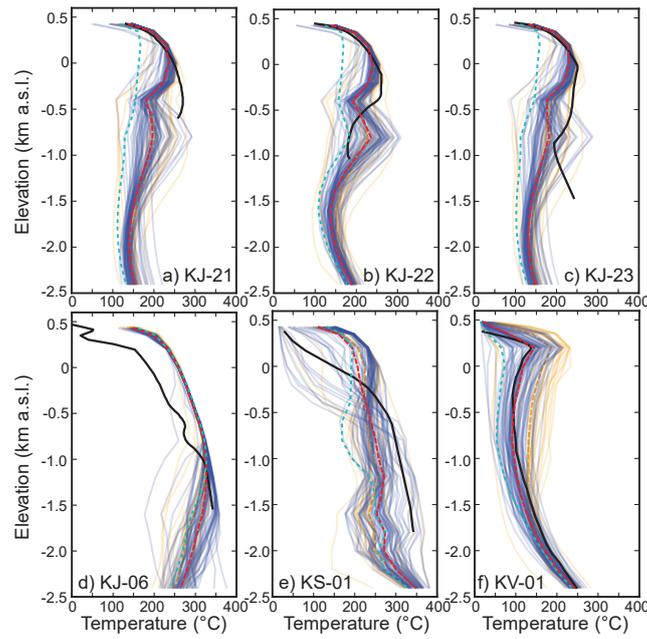
575 Figure 11 shows posterior predictive checks for 3 wells located in the Hvíthólar sub-  
 576 field as well some other wells on the margin or away from the main production field. The  
 577 model clearly reproduces the temperature inversion between -0.5 km a.s.l. as well as the  
 578 slight increase in temperature beneath -1 km a.s.l. in wells KJ-22 and KJ-23, although  
 579 the model tends to suggest a more rapid decrease in temperature with increasing depth  
 580 below 0 km a.s.l. There are certain wells that the model does poorly at reproducing mea-



**Figure 9.** Posterior predictive checks for Leirbotnar wells a) KG-08, b) KG-11, c) KJ-35, d) KW-01, e) IDDP-1, f) KG-12, g) KG-24, h) KW-02. Posterior predictive samples generated using the bias-only correction in the approximation error model are shown in blue (with the mean of the subset of posterior samples is shown with a red dashed line), and posterior predictive samples generated with an approximation error model considering both the bias and covariance are shown in orange, (with the mean as an orange dashed line). The inferred natural state temperatures are shown in black. Temperatures calculated assuming prior means for all parameters are shown with the dashed blue line.



**Figure 10.** Posterior predictive checks for Suðurhlíðar wells a) KJ-19, b) KJ-30, c) KJ-30, d) KJ-37, e) KJ-39, f) KJ-14, g) KJ-17, and h) KJ-36 in Vesturhlíðar. Symbology same as in Fig. 9.



**Figure 11.** Posterior predictive checks for Hvíthólar wells a) KJ-21, b) KJ-22, c) KJ-23, as well as d) KJ-06, e) KS-01, f) KV-01. Symbology same as in Fig. 9.

581       sured temperatures. For example, the model fails to reproduce the strong temperature  
 582       decrease above -1 km a.s.l. in KJ-06, located at the southern margin of Leirbotnar (Fig.  
 583       11d). Similarly, the model does not reproduce the near linear temperature gradient in  
 584       the upper 1 km in well KS-01 (Fig. 11e). The model closely matches the measured tem-

585 peratures in well KV-01 (Fig. 11f), which is located at the periphery of the producti-  
 586 on area and shows low temperatures  $\leq 200$  °C to 2 km depth, suggests that the model  
 587 is broadly capturing the temperature distribution of the groundwater system outside of  
 588 the main production field.

## 589 4.2 Posterior Parameter Distributions

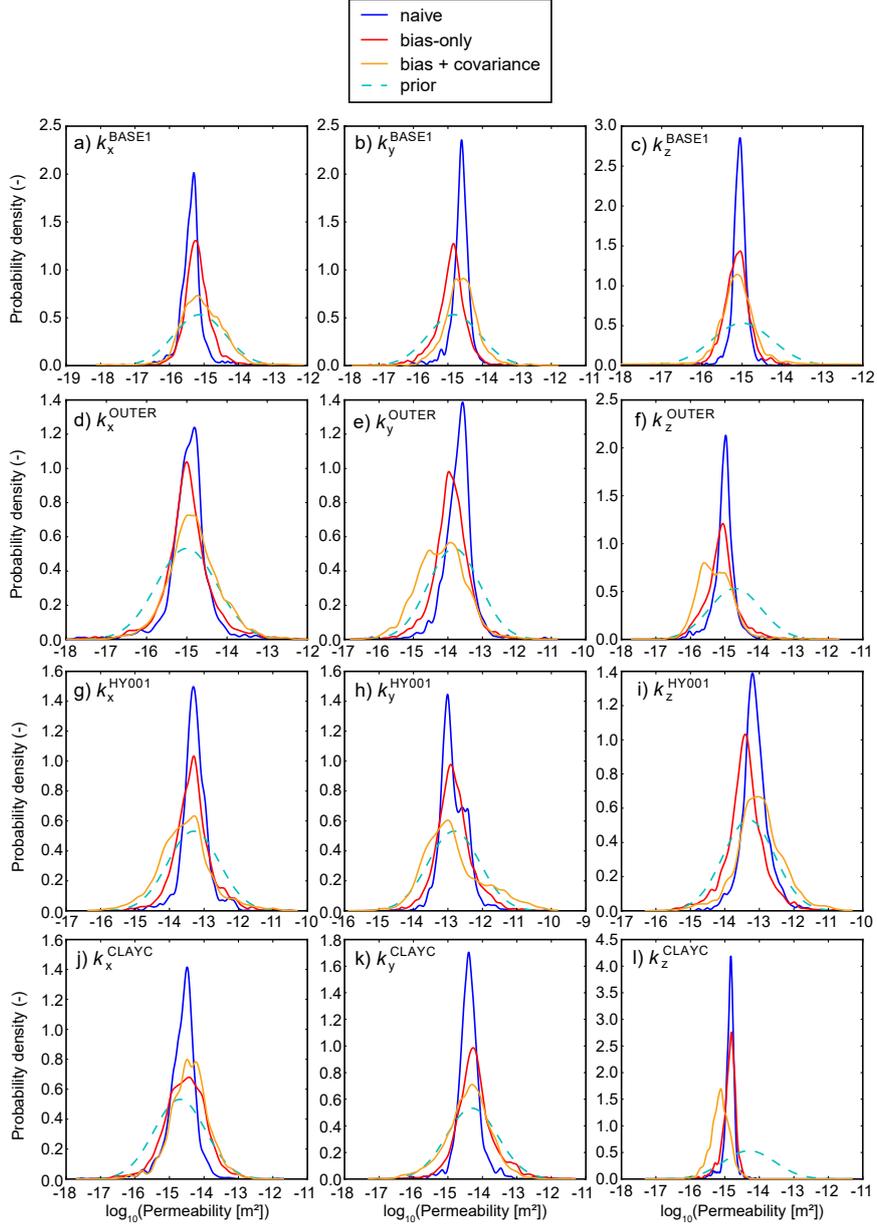
590 The uncertain parameters in this model consist of anisotropic permeabilities for 14  
 591 different rock types. Figure 12 shows the marginal posterior densities for 4 different rock  
 592 types (BASE1, OUTER, HY001, CLAYC) for each direction (x = cross-rift, y = rift-parallel,  
 593 z = vertical). This figure compares posterior pdfs corresponding to the naive (blue line)  
 594 and the discrepancy-informed models, either considering both the bias and covariance  
 595 terms in the approximation error (orange) or the bias-only (red lines).

596 Compared to the prior uncertainty (dashed blue lines), which extends over four or-  
 597 ders of magnitude, Figure 12 indicates that the MCMC process reduces uncertainty in  
 598 the inferred permeability values. The naïve and discrepancy-informed models reach fair-  
 599 ly consistent results, with a Gaussian-looking pdf often centered on or near the prior means.  
 600 Generally, many of the parameters show substantial uncertainty, with posterior densities  
 601 extending over 1-2 orders of magnitude or more. The uncertainty in parameter values  
 602 is higher in the discrepancy-informed model compared to the naïve model, particular-  
 603 ly if both the bias and covariance of the approximation errors are considered. In the latter  
 604 case, the boundaries of the prior seem to exercise some control on the limits of the poster-  
 605 ior uncertainty (e.g. Figure 12e,f).

606 Although there is a clear correspondence between the mode of the posterior and  
 607 the prior means, the high posterior uncertainty of many of the parameters suggests lim-  
 608 ited identifiability. However, certain parameters show narrower posterior distributions cen-  
 609 tered at an offset from the prior mean, This includes the particularly the vertical per-  
 610 meability of the clay cap (Fig. 12l). Thus, this appears to be one of the most strongly  
 611 identifiable parameters, with a maximum *a posteriori* value narrowly focused around  $\sim 10^{-15}$   
 612  $\text{m}^2$ .

613 The posterior pdfs indicate significant permeability anisotropy, particularly for the  
 614 upper extrusive rock types (hyaloclastites and lava flows) and the fault rock types. Figure  
 615 13a-b compares the posterior densities (computed using the approximation error model  
 616 considering the bias-only correction) in different orientations for the permeability of hyalo-  
 617 clastites (3 units), lava flows (4 stratigraphic units), Generally, with the exception of LV001  
 618 (which is modeled as an aquiclude), the predicted permeability of the hyaloclastites and  
 619 lava flows is very high ( $10^{-14}$  -  $10^{-12}$   $\text{m}^2$ ). For both the hyaloclastites and lava flows (Fig.  
 620 13a,b), the rift-parallel permeability (red lines) seems to be higher ( $10^{-13}$   $\text{m}^2$  than the  
 621 cross-rift (grey lines) or vertical (blue lines) permeability. The predicted permeability  
 622 of the lowermost lava flow LV001 is distinctly lower ( $10^{-17}$  -  $10^{-15}$   $\text{m}^2$ ).

623 Figure 13c shows that the fault rock types (barrier faults, shown by the solid lines,  
 624 and transform fault, shown by the dashed lines) show distinct anisotropic permeability  
 625 contrasts consistent with their strikes. The barrier faults (solid lines), which strike SW-  
 626 NE parallel to the rift, show intermediate rift-parallel (x) and vertical permeability (z)  
 627 around  $10^{-15}$   $\text{m}^2$ , but lower cross-rift permeability ( $\sim 10^{-16}$   $\text{m}^2$ ). The NNW-SSE-striking  
 628 transverse faults (dashed lines) oriented roughly perpendicular to the axis of rifting, show  
 629 intermediate rift-parallel permeability but higher cross-rift and particularly vertical per-  
 630 meability ( $10^{-15}$   $\text{m}^2$ ). These results indicate that these the faults are generally more per-  
 631 meable along the axis of their strike, retain moderate vertical permeability, and are less  
 632 permeable across strike.

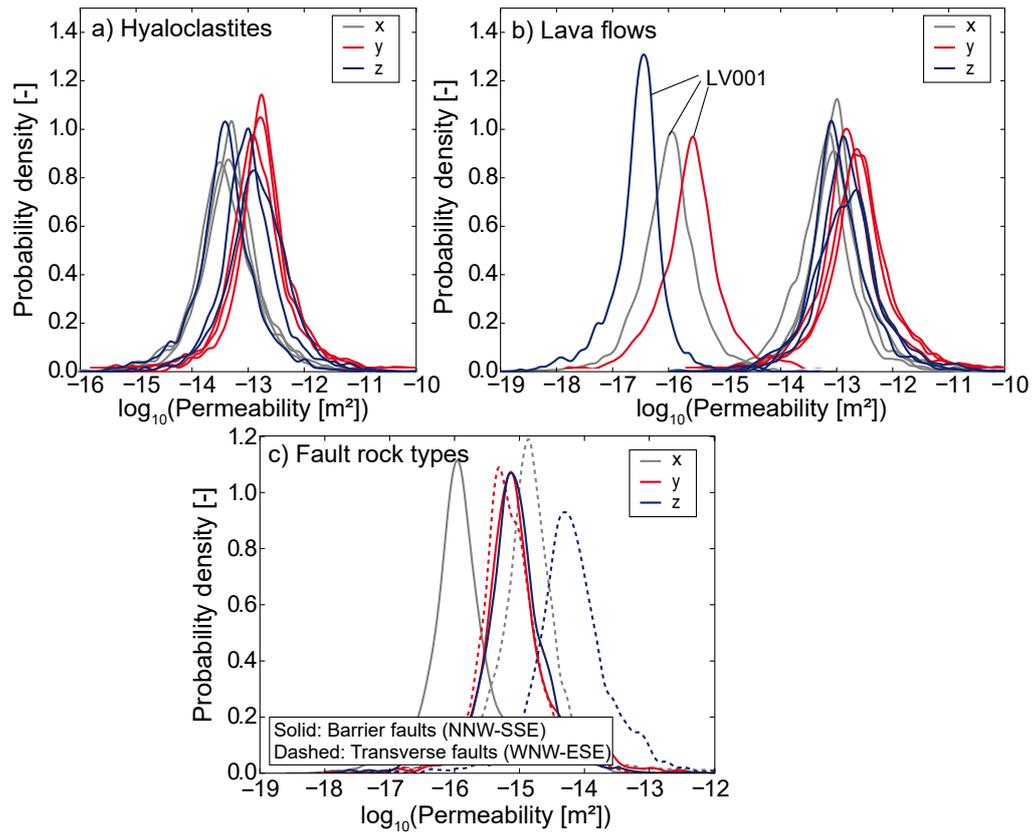


**Figure 12.** Marginal posteriors for permeability ( $x =$  cross-rift,  $y =$  rift-parallel, and  $z =$  vertical) for four rock types, a-c) BASE1, d-f) OUTER, g-i) HY001, j-l) CLAYC. The blue lines shows the naïve posterior, calculated by performing MCMC on the coarse model without any approximation error. The dashed light blue lines shows the prior. The solid red lines shows the posterior calculated by the discrepancy-informed model with bias-only correction to the likelihood function. The orange line shows the posterior calculated by the discrepancy-informed model considering both the bias and covariance in the model approximation error.

633 **5 Discussion**

634 **5.1 Calibration of natural state reservoir models**

635 We have presented an approach to inverse modeling of geothermal reservoirs that  
 636 incorporates the additional error resulting from performing MCMC sampling with a “coarse”,



**Figure 13.** Marginal posterior densities for permeability in different directions ( $x$  = cross-rift, shown with grey lines,  $y$  = rift-parallel, shown with red lines, and  $z$  = vertical, shown with dark blue lines) for a) Hyaloclastites (HY001, HY002, HY003) b) Lava flows (LV001, LV002, LV003, LV004) c) Fault rock types (barrier faults, BARFT, shown as solid lines, and transform faults, TVFLT, shown as dashed lines).

637 relatively inexpensive model instead of a finer, computationally expensive model into a  
 638 hierarchical Bayesian framework. Compared to other approaches that construct approx-  
 639 imate posterior distributions using reduced-order models in the context of natural state  
 640 geothermal reservoir models, such as delayed acceptance schemes that calculate the model  
 641 reduction error dynamically (Cui et al., 2011; Cui, Fox, & O’Sullivan, 2019; Cui, Fox,  
 642 Nicholls, & O’Sullivan, 2019), the approach of Maclaren et al. (2020) is relatively easy  
 643 to implement, requiring a relatively small number of fine model simulations to construct  
 644 the posterior-informed model approximation error. Despite a huge reduction in the di-  
 645 mensionality of the coarse model compared to the fine model (from 17193 to 2551 grid  
 646 blocks), the naïve posterior is strongly informative (Fig. 12), suggesting that the physics  
 647 of the fine model are adequately captured in the coarse model and that the approxima-  
 648 tion error calculated using the naïve posterior yields relevant error estimates.

649 Despite the computational intensity involved with running hundreds of thousands  
 650 of AUTOUGH2 models, the MCMC approach can be used to achieve a reasonably good  
 651 fit to temperature data derived from 40 deep geothermal wells. Such a fit has also been  
 652 obtained for large-scale reservoir models using manual calibration (Ratouis et al., 2016)  
 653 or optimization based schemes such as iTOUGH2 and PEST (Mannington et al., 2004;  
 654 Gunnarsson et al., 2010; Prasetyo et al., 2016). However, it is not uncommon that optim-  
 655 um inferred parameter values differ greatly from initial “best-guess” estimates, or are restricted  
 656 by reasonable bounds imposed on the inversion process (Reid & Wellmann, 2012; Kondo  
 657 et al., 2017). While iTOUGH can be useful for showing the most uncertain parameters  
 658 (Moon et al., 2014), when an optimization function can have several local minima, as  
 659 is likely for geothermal data, the algorithms often only converge to the minima closest  
 660 to the initial starting guess (Plasencia et al., 2014).

661 The data (downhole temperature profiles) and parameter (posterior parameter distri-  
 662 butions) space show considerable differences arising if the likelihood function is induced  
 663 by both the bias and covariance of the model approximation errors or the bias-only. This  
 664 results from the much higher covariance of certain model approximation errors for certain  
 665 measurements, particularly for measurement points in the isothermal zone, effectively  
 666 resulting in a lower weight for these measurements in the likelihood function. In contrast,  
 667 the covariance of model approximation errors in the boiling zones is very low due to the  
 668 boiling point with depth constraint. Therefore, we suggest the use of a bias-only correcti-  
 669 on can be justified in such cases where the variability of coarse and fine model results  
 670 is much greater in selected measurement points.

671 This study shows that a Bayesian framework to calibrate natural state geothermal  
 672 reservoir models allows quantification of the uncertainty of the inferred permeability struc-  
 673 ture while retaining strong prior beliefs about the geologic/hydrologic structure of the  
 674 system. Although the anisotropic permeability of each rock type are treated as stochastic  
 675 parameters, the arrangement of rock types and the applied mass and heat fluxes at the  
 676 base of the model are fixed. Other studies have also treated the input mass as an uncertain  
 677 variable during calibration and even treated each block as a separate rock type (Cui et  
 678 al., 2011). However, in our case this would result in a very large dimensional parameter  
 679 space that would limit the feasibility of using the affine invariant MCMC sampling scheme  
 680 (Foreman-Mackey et al., 2013). Rather, our approach ensures a close link between the  
 681 conceptual model and the reservoir model, in effect allowing the reservoir model to serve  
 682 as a test of the conceptual model. The relatively good match between measured data  
 683 and simulation results may suggest that the conceptual model is highly informative. More-  
 684 over, the general correspondence between the prior and posterior means for permeability  
 685 suggests that the *a priori* permeability ranges for the different rock types are realistic.

686 Inferred permeabilities in the range of  $10^{-16}$ - $10^{-12}$  m<sup>2</sup> are consistent with prev-  
 687 ious field measurements in geothermal systems (G. Björnsson & Bödvarsson, 1990; Ingebrit-  
 688 sen & Manning, 2010). Although permeabilities measured in intact core samples under  
 689 confining pressures are lower than inferred in this study (Eggertsson, Lavallée, et al., 2020;

690 Eggertsson, Kendrick, et al., 2020; Weaver et al., 2020), this discrepancy can generally  
691 be explained as permeability being fracture-controlled (Lamur et al., 2017). For a geo-  
692 thermal field such as Krafla, such fine-scale permeability heterogeneities will not be cap-  
693 tured in models that assume permeability is fixed for each rock type or within grid blocks  
694 of the resolution used in this study. Previous modeling studies of Icelandic geothermal  
695 systems have inferred similarly high permeability values for extrusive volcanic rocks (hyalo-  
696 clastites and lava flows). For example, using drawdown (pressure) and tracer data, Aradóttir  
697 et al. (2012) calculated lateral and vertical permeabilities of  $3 \times 10^{-13}$  and  $1.7 \times 10^{-12}$  m<sup>2</sup>  
698 for the basaltic lava flows hosting the Carbfix injection site at the Hellisheidi geothermal  
699 field. These values lie at the high end of the inferred permeability values in this study  
700 (Fig. 13a,b).

701 The inferred permeability values of the different rock types reflect the conceptual  
702 model. If a different conceptual model was assumed, with a significantly different geologic  
703 structure and distribution of heat and mass at the bottom boundary, different calibra-  
704 ted rock permeabilities would result. For example, tests were carried out to investiga-  
705 te results if the prior permeability of the aquitard (LV001) was set to be similar the other  
706 hyaloclastites and lava flows. Given this scenario, the model resulted in much higher natural  
707 state temperatures in the isothermal zone than measured, unless the prior permeability  
708 of the extrusive volcanic rocks was unrealistically high ( $\geq 10^{-11}$  m<sup>2</sup>). Such high rock per-  
709 meabilities are not justified by the data cited above. Given the inherent uncertainty of  
710 the conceptual model and data, a range of different conceptual models should be developed  
711 and tested using numerical models (Cumming, 2016b). Bayesian approaches are well-  
712 suited to incorporate future conceptual model reassessments and additional constraints  
713 as new geologic and thermohydrodynamic data becomes available.

714 This study shows several practical limitations to using MCMC sampling for inverse  
715 modeling of the natural state of geothermal reservoirs. Running a single ensemble of 100  
716 MCMC walkers for 80 steps required computational time in excess of two weeks. In addi-  
717 tion, computation of the approximation error model and the posterior predictive checks  
718 required significant computational expense, as a single fine model iteration could take  
719 up to a day or more to compute. More troublingly, the integrated autocorrelation time  
720 suggests that the chain is not long enough and adequate coverage of the target poster-  
721 ior density is not ensured. While running 1000+ iterations in each chain wasn't possi-  
722 ble in the present study due to time constraints, ongoing efforts to parallelize TOUGH2  
723 may enable this in the future (J. O'Sullivan et al., 2019; Croucher et al., 2020). Another  
724 possible indication of inadequate uncertainty is the observation that limits of the prior  
725 pdfs for permeability coincide with the limits of the posterior uncertainty (e.g. Fig. 12g-  
726 i). However, widening the prior further would have the disadvantage that the chain would  
727 take a significantly longer time to run, without necessarily providing more realistic estima-  
728 tes. Thus, the prior means and bounds must be carefully selected based on considerati-  
729 on of realistic permeability ranges observed in similar types of geothermal systems. Al-  
730 though MCMC methods may be prone to underestimating uncertainty due to the effects  
731 of autocorrelation and an insufficiently broad prior, the value of the calibrated permea-  
732 bilities and uncertainty metrics ultimately hinges on the conceptual model and the assumpti-  
733 on of an informative prior model. This will inevitably be the case for natural state geo-  
734 thermal reservoir models, which are poorly suited to “black box” inverse modeling ap-  
735 proaches. In addition, the reliability of the inferred permeability structure depends greatly  
736 on the quality of data (both geologic and downhole measurements) used to calibrate the  
737 numerical model. Given the limited temperature data at depth  $\geq 2$  km, the uncertainty  
738 of the model predictions in the “deep roots” of the system is much greater than at shallower  
739 depths, as is reflected by the posterior predictive checks (Figs. 9-11).

## 740 5.2 The natural state of the Krafla geothermal reservoir

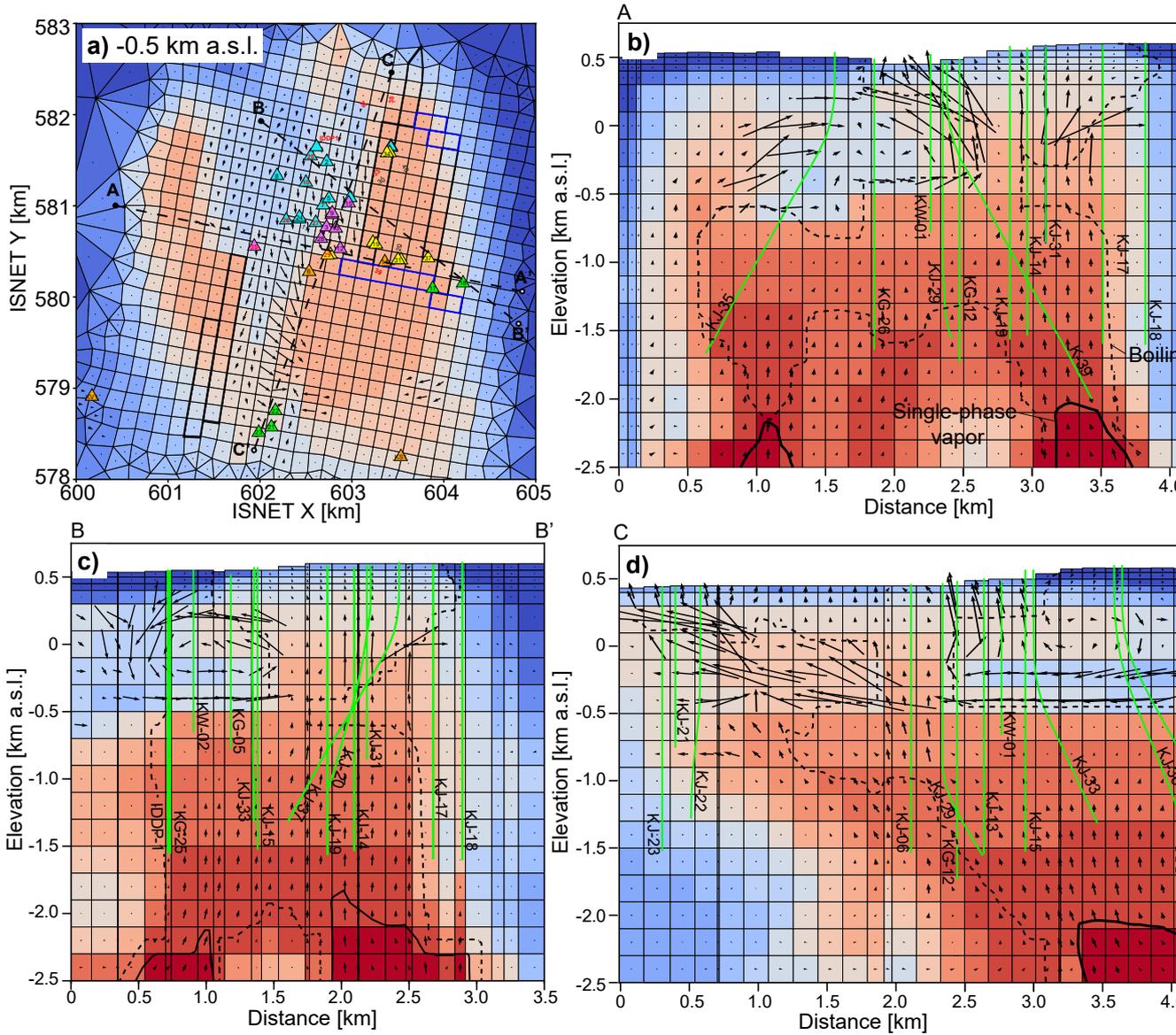
741 The foundation of the natural state geothermal reservoir model presented in this  
 742 study is the conceptual model described in Section 2.2. According to our conceptual model,  
 743 the distinctive spatial variability of temperature in Krafla is the combined result of heterogeneous  
 744 upflow of high-enthalpy fluid from the deep heat source, structural partitioning of the  
 745 reservoir by the Hveragil eruptive fissure, and the presence of a low permeability aqui-  
 746 tard isolating the shallow boiling zone in Leirbotnar from the deeper boiling zone. Superimposed  
 747 on these factors is the effect of topography-driven fluid flow associated with the Mt. Krafla,  
 748 which shields the Suðurhlíðar system from intensive cold recharge leads to the relatively  
 749 low temperatures in Leirbotnar.

750 Model results derived from the *maximum a posteriori* (MAP) estimate are presented  
 751 in Figure 14 with the same cross-sections used to develop the conceptual model. Schematic  
 752 flow arrows in Figure 14a indicate intensive southward-directed recharge into Leirbot-  
 753 nar at the depth of the isothermal zone (-0.5 km a.s.l.). Similarly intensive lateral rechar-  
 754 ge is lacking in the Suðurhlíðar subfield, likely due to the topographic high associated  
 755 with Mt. Krafla, which leads to higher hydrostatic pressures in this area. Increasing tem-  
 756 peratures within the isothermal zone to the SE of Leirbotnar result from increased mix-  
 757 ing of ascending fluid within the Hveragil fault to the SW. Thus, the model predicts that  
 758 topographic gradient from the NE to the SW drives fluid flow to the SW, resulting in  
 759 cooling of the Leirbotnar isothermal zone as well as the shallow outflow in Hvíthólar.

760 As a result of intensive cold water recharge from the north in the Leirbotnar area,  
 761 boiling is confined to depths  $\leq -0.5$  km. However, boiling extends to the surface in the  
 762 Suðurhlíðar area ( Fig. 14b). The distribution of boiling zones is supported by the temp-  
 763 erature measurements as well as production data. Wells that produce exclusively from  
 764 the upper reservoir in Leirbotnar have enthalpies  $0.8-1 \text{ MJ kg}^{-1}$ , indicative of a single-  
 765 phase liquid reservoir. In contrast, wells that produce from the deeper reservoir in Leirbot-  
 766 nar or in Suðurhlíðar develop much higher discharge enthalpies (up to  $\sim 2.7 \text{ MJ kg}^{-1}$ ,  
 767 corresponding to the specific enthalpy of saturated vapor), indicative of a boiling reser-  
 768 voir (Guðmundsson & Arnórsson, 2002).

769 Zones of single-phase vapor develop beneath Suðurhlíðar, Vestursvaedi, and the north-  
 770 ern part of Leirbotnar  $\leq -2$  km a.s.l. (Fig. 14b,c,d) Generally, the models predict single-  
 771 phase vapor zones at greater depths and lower temperatures than experienced during the  
 772 drilling of the IDDP-1 well, which discharged a ca.  $440 \text{ }^\circ\text{C}$  vapor with an enthalpy of  $3.2$   
 773  $\text{MJ kg}^{-1}$  (Axelsson et al., 2014). However, as the role of the heat source is represented  
 774 using boundary conditions, the structure of the vapor zones is controlled by the input  
 775 of high-enthalpy fluid at the bottom boundary (Fig. 6b). Reproducing the rapid temp-  
 776 erature in the proximity of shallow intrusions like those encountered during the drilling  
 777 of IDDP-1 and KJ-39 requires explicit representation of fluid flow around intrusions (Scott  
 778 et al., 2015). Despite this limitation, the models show how zones of supercritical fluid  
 779 form at depth in the Krafla system and undergo ascent and decompression to form over-  
 780 lying boiling zones. The flow arrows at the base of the model in Fig. 14 indicate that  
 781 circulating liquid may undergo boiling prior to supercritical fluid formation, in agreement  
 782 with geochemical studies indicating that supercritical fluids can form from isobaric heating  
 783 of liquid around intrusions (Heřmanská et al., 2019).

784 The distinctive character of the temperature field in the various Krafla subfields  
 785 can be clearly linked to permeability structure. The isothermal zone temperatures in Leirbot-  
 786 nar require very high rock permeability ( $\geq 10^{-13} \text{ m}^2$ ), which allow for the intensive vertical  
 787 and horizontal convection that effectively smear out vertical temperature gradients. In  
 788 addition, the low permeability aquitard lying on top of the basement intrusions reduces  
 789 the input of high-enthalpy fluid from the deeper boiling reservoir to the shallow isothermal  
 790 zone. Previous studies that have suggested that vertically-extensive boiling zones extend-  
 791 ing to  $\geq 1$  km depth, such as are found in Suðurhlíðar, require intermediate rock permea-



**Figure 14.** Model results from *max a posteriori* estimate. a) Depth slice at -0.5 km asl. Grid blocks modeled as fault rock types are outlined, with barrier faults outlined in black and transverse faults in blue. Well-head locations shown with triangles with same symbology as in Figure 3. b-d) Vertical cross-sections through numerical model, with traces of cross-sections shown in a). Cross-sections correspond to same cross-sections shown in Figure 4. Boiling zones are highlighted in dashed lines, zones of single-phase vapor with solid black lines. Well traces shown in green.

bility ( $\sim 10^{-15} \text{ m}^2$ ) (Hayba & Ingebritsen, 1997; Scott et al., 2016). However, this study shows that such temperature contrasts can also result from spatial variations in the strength of the deep upflow, which is stronger beneath Suðurhlíðar due to higher deep mass input (Fig. 6b), as well as the lack of the low permeability aquitard in this area.

The numerical models suggest that a substantial untapped resource may be present in the Krafla area. For example, the models predict the shallow boiling zone in Suðurhlíðar extends further to the south, towards Sandabotnaskarð. Such an inference is also supported by resistivity measurements, which reveal that the areal extent of the shallow low-resistivity cap rock exceeds the present area exploited for power production, extending along the eastern rim of the caldera and to the south of Leirhnjúkur, located to the northwest of the main production area at Krafla (Árnason & Magnússon, 2001). Leirhnjúkur was a major locus of extrusive magmatism and magmatic gas discharge during the volcano-rifting event between 1975-1984 (Ármannsson et al., 1989). However, as there are no deep wells drilled into these area, it remains uncertain whether and to what extent the high-temperature system extends to this area.

## 6 Conclusions

This study has presented a new natural state model of the Krafla geothermal system. The Bayesian approach to model calibration quantifies the significant uncertainty in the model parameters. Even in an extensively drilled geothermal field such as Krafla, for which temperatures from many deep wells are available, the uncertainty in the inferred anisotropic permeability values of the different rock types is considerable, often exceeding two orders of magnitude. Moreover, fundamental restrictions on grid resolution and computational expense presently limit the ability for models to resolve small-scale permeability variations. Therefore, when harnessing modern machine learning techniques for the calibration of natural state geothermal reservoir models, a close link between the resource conceptual model and the reservoir model is essential to ensure adequate representation of the large-scale structure of the system.

The presented numerical model is one of the first to extend to the deep, supercritical roots of a system under production. However, as the role of the heat source is expressed in terms of bottom boundary conditions, fluid flow processes around the intrusive heat sources are not explicitly modeled. Moreover, geothermal systems such as Krafla undergo processes such as repeated intrusion, fracturing, and eruption over time scales of  $10^2$ - $10^3$  years (Thordarson & Larsen, 2007; Sparks & Cashman, 2017), and the heat of subsurface intrusions can be exhausted by hydrothermal fluid circulation on time scales of  $10^3$ - $10^4$  years (Scott et al., 2016), shorter than the time needed in order to reach a steady-state in the transient models (ca.  $10^6$  years). Other models of cooling intrusions have shown that transient effects play a key role in governing the thermo-hydraulic structure (Scott, 2020).

In addition to better consideration of transient effects, other possibilities for future improvements could include making every block a different rock type (Bjarkason et al., 2019) and treating input mass/enthalpy as uncertain parameters (Cui et al., 2011). However, as the number of uncertain parameters increases, the connection between the reservoir model and the conceptual model becomes more tenuous, and in geothermal fields that are well-constrained by geologic and well data, incorporating such constraints in the reservoir model is valuable. In addition, the model would benefit from more data, particularly below -2 km a.s.l. Such data could become available as a result of the ongoing Krafla Magma Tested project, which aims drill several additional deep wells into the magma body in Krafla to better constrain the interface between the magmatic heat source and overlying hydrothermal system (Eichelberger et al., 2018). While the production history of this system was not used to calibrate the numerical model, this natural state model could serve a basis for the calibration of such a model. In addition, it would help evaluat-

ing whether the exploitation of the system has led to changes in the temperature distribution of the system (Guðmundsson & Arnórsson, 2002). Generally, the development of natural state model should be seen as a dynamic, iterative process, that improves as additional data becomes available and the conceptual understanding of the system deepens.

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