Comment on “The global range of temperatures on convergent plate interfaces” by England and May

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Abstract

England and May (2021) present a model comparison for the forearc thermal structure in subduction zones that employs a finite element modeling technique and a new variation on approximate equations. We have some comments on various claims made in this published paper.

Comment on “The global range of temperatures on convergent plate interfaces” (England and May, 2021).

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England and May (2021) present a model comparison for the forearc thermal structure in subduction zones that employs a finite element modeling technique and a variation on approximate equations. These equations are based on analytical theory first introduced for this purpose for straight-dipping slabs by Molnar and England (1990). Their main conclusion is that one can introduce modifications to the original approximate equations for subduction zones with curvature. They state repeatedly that this conclusion is in contrast with a remark regarding the application of the approximate equations to subduction zones with significant variations in dip in the conclusion section of van Keken et al. (2019). It is important to note that this particular conclusion in van Keken et al. (2019) was based on an analysis of a specific transformation between time and depth as used in the literature (e.g., Tichelaar and Ruff, 1993) while employing the original Molnar and England (1990) equations. We showed that such a transformation is inconsistent with theory and leads to significant quantitative errors. Our finding is not actually disputed in England and May (2021). Rather, they show that newly-introduced changes to the approximate equations (compare, for example, equation (18) in England and May, 2021, with equation (A6) in England, 2018, or with equation (15) in Molnar and England, 1990) render them more suitable for subduction zones with curvature.

England and May (2021) suggest that the differences between the new equations and their finite element model predictions are relatively small (on the order of a few tens of degrees Celsius) for a few of the curved subduction geometries and parameter combinations considered. We are glad to see this analysis and to see that newly modified approximate equations appear to introduce added imprecision relative to the numerical results that is of modest magnitude. We have to assume here that the numerical method England and May (2021) employed provides sufficiently accurate solutions of the governing differential equations such that they can be reliably used to test the difference with the approximate equations. A full quantitative assessment of their numerical method is made impossible because any measures that normally would allow reproducing or testing results obtained with a new code are lacking. Such measures normally include resolution tests, reproductions of existing model solutions, comparisons against other numerical codes, or open source software; none of these are presented in England and May (2021). The claim that solving for a divergence free velocity field below the slab surface is novel and better than existing approaches (England and May, 2021) is factually incorrect as it has been commonly used before (see, among others, Wada and Wang, 2009, and Abers et al., 2020).

We find it unfortunate that England and May (2021) chose to ignore the very specific reason for our caution of the application of the Molnar and England (1990) approximate equations to models with variable dip. Our analysis is called out as incorrect without qualification or demonstration in multiple places in their manuscript. Our analysis remains correct as is quantitatively demonstrated in its application to the extension of the Molnar and England (1990) equations made by Tichelaar and Ruff (1993) (see Figure 6 in van Keken et al., 2019). We think it is an important point to make to the broader community who may be interested in using this particular form of the approximate equations in lieu of numerical methods. We very clearly state (our page 3269) that the approximate equations refer to those in Molnar and England (1990) and that our analysis is specific for the curved slab approximation used by Tichelaar and Ruff (1993) (our section 5.2). The repeated statements in England and May (2021) that suggest our analysis is 'in error' is misleading. One could only state this by taking out of context part of a specific sentence in the conclusions section in van Keken et al. (2019), by ignoring the actual tests in the paper, and by assuming that our acronym "AE" applies to any past, present, and future form of the approximate equations rather than to those in Molnar and England (1990). In 2019 we obviously could not analyze the new form of the approximate equations presented by England and May (2021).

The authors provide repeated editorial comments specifically in, but not limited to their section 2.4.1, that suggest we made several 'errors' and that we use 'erroneous expressions' in van Keken et al. (2019). This is also misleading. It misrepresents our work and, indeed, the work shown in Molnar and England (1990). These suggested 'errors' are in fact assumptions introduced in Molnar and England (1990) and merely reproduced by us for didactic purposes and further analysis. We have provided corrections to the approximate equations introduced in Molnar and England (1990) and have explicitly demonstrated that these make their application more accurate. The improvements made to the approximate equations in van Keken et al. (2019) and the results in Abers et al. (2020) are not affected by said 'errors'. This is, in fact, not disputed nor demonstrated by England and May (2021).

One of the 'errors' that is listed in section 2.4.1 relates to a value of a constant named b that arises from the underlying theory relevant to this problem (Carslaw & Jaeger, 1959). Full derivation and details are described in Molnar and England (1990), England (2018), and van Keken et al. (2019). This parameter b is around one and differs for the various mechanisms that heat the slab surface (such as basal heating or shear heating). It has long been common in the literature (e.g., Molnar and England, 1990; England, 2018) to assume that b is a single constant for all heating modes, with a value that is approximately average between that for basal heating and depth-dependent shear heating. England (2018), for example, assumes a constant $b=1.12$ for models that use a combination of basal, radiogenic, and depth-dependent shear heating. For several comparisons with the numerical models in Molnar and England (1990) b was chosen by those authors as $b=1$. We showed in van Keken et al. (2019): that it is better to treat the heating modes separately; that one should assign different b values for each heating mode; and that radiogenic heating in the crust cannot be accurately described by any choice of *b* in the approximate equations. We are glad to see that England and May (2021) have adopted our approach since they now use different values for *b* for the shear heating and basal heating modes and ignore radiogenic heating altogether. As we showed in van Keken et al. (2019), this significantly improves the match between the thermal predictions from the approximate equations and the finite element models. We do not consider that assuming *b* is constant or the exact choice of its value necessarily as 'errors' but rather as simplified assumptions that, while rendering the algebraic expressions more compact, also make the approximate equations unnecessarily imprecise. England and May (2021) are correct in pointing out that we reproduced results from Molnar and England (1990) with *b=1*, but that is, of course, because Molnar and England (1990) chose *b=1* for many of their models (see their various comments on the *b* value and the captions of their Figures 9 and 10). If this is indeed an 'error' then the approach in Molnar and England (1990) clearly suffers from the same problem. Choosing a constant *b=1.12* as in England (2018) does not make this less 'erroneous' as this particular value is not appropriate for any heating mode assumed in that paper (see analysis in van Keken et al., 2019).

A second 'error' that we are claimed to have made by England and May (2021) is also just a straightforward reproduction of an assumption in Molnar and England (1990). In van Keken et al. (2019) we posited a caveat regarding the application of the approximate equations in Molnar and England (1990) for settings with low thermal parameter and quantified the depth limit to which the approximate equations could be used (where the thermal parameter is the product of lithospheric age, convergence velocity, and sine of the dip of subduction). At the time we had not fully appreciated that in a later publication (Molnar and England, 1995) a solution to this issue was provided. Or, to use the terminology of England and May (2021), that Molnar and England (1995) had recognized and corrected an 'error' they had made in their 1990 paper. Our analysis of this issue remains correct as far as the Molnar and England (1990) equations are concerned.

Curiously, England (2018) ignores the useful modification made in Molnar and England (1995) (see equations (4) and (8) in England, 2018). Our analysis therefore applies to England (2018) equally well and one should approach with significant caution the model predictions made in England (2018) for subduction zones with low thermal parameter. England and May (2021) should have realized and clarified in their paper that the particular 'errors' we are claimed to have made originated in Molnar and England (1990), that they were repeated in England (2018), and that they were not introduced, as is suggested, in van Keken et al. (2019).

Acknowledgments

We are grateful to Profs. England and May for spotting a typographical error in equation (7) of Abers et al. (2020). This concerns a negative sign which should read positive. This was a typographical error in the text only and does not affect any of the work presented as can be easily verified. We have published an erratum in *Geosphere* to clarify this (Abers et al., 2021).

References

Abers, G. A., van Keken, P. E., & Wilson, C. R. (2020). Deep decoupling in subduction zones: Observations and temperature limits. *Geosphere, 16*, 1408–1424. https://doi.org/10.1130/GES02278.1

Abers, G. A., van Keken, P. E., & Wilson, C. R. (2021). Erratum: Deep decoupling in subduction zones: Observations and temperature limits. *Geosphere*, published August 30. https://doi.org/10.1130/GES02278e.1

Carslaw, H. S., & Jaeger, J. C. (1959). *Conduction of heat in solids* (2nd Ed., 510 pp.). Oxford, UK: Oxford University Press.

England, P. (2018). On shear stresses, temperatures, and the maximum magnitude of earthquakes at convergent plate boundaries. *Journal of Geophysical Research: Solid Earth, 123*, 7165–7202. https://doi.org/10.1029/2018JB015907

England, P., and May, D. (2021). The global range of temperatures on convergence plate interfaces. *Geochemistry, Geophysics, Geosystems, 22*, e2021GC009849. https://doi.org/10.1029/2021GC009849

Molnar, P., & England, P. (1990). Temperatures, heat flux, and frictional stresses near major thrust faults. *Journal of Geophysical Research: Solid Earth, 95*, 4833–4856. https://doi.org/10.1002/2017JB014445

Molnar, P., & England, P. (1995). Temperatures in zones of steady-state underthrusting of young oceanic lithosphere. Earth and Planetary Science Letters, 131, 57–70. https://doi.org/10.1016/0012-821X(94)00253-U

Tichelaar, B. W., & Ruff, L. J. (1993). Depth of seismic coupling along subduction zones. *Journal of Geophysical Research: Solid Earth, 98*, 2017–2037. https://doi.org/10.1029/92JB02045

van Keken, P. E., Wada, I., Sime, N., & Abers, G. A. (2019). Thermal structure of the forearc in subduction zones: A comparison of methodologies, *Geochemistry, Geophysics, Geosystems, 20*, 3268–3288. https://doi.org/10.1029/2019GC008334

Wada, I., & Wang, K. (2009). Common depth of slab-mantle decoupling: Reconciling diversity and uniformity of subduction zones. *Geochemistry, Geophysics, Geosystems, 10,* Q10009. https://doi.org/10.1029/2009GC002570