

# Ambiguous stability of glaciers at bed peaks

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## Abstract

Increasing ice flux from glaciers retreating over deepening bed topography has been implicated in the recent acceleration of mass loss from the Greenland and Antarctic ice sheets. We show in observations that some glaciers have remained at peaks in bed topography without retreating despite enduring significant changes in climate. Observations also indicate that some glaciers which persist at bed peaks undergo sudden retreat years or decades after the onset of local ocean or atmospheric warming. Using model simulations, we show that glacier persistence may lead to two very different futures: one where glaciers persist at bed peaks indefinitely, and another where glaciers retreat from the bed peak suddenly without a concurrent climate forcing. However, it is difficult to distinguish which of these two futures will occur from current observations. We conclude that inferring glacier stability from observations of persistence obscures our true commitment to future sea-level rise under climate change.

# Ambiguous stability of glaciers at bed peaks

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## Key Points:

- Many glaciers persist at peaks in bed topography while enduring considerable changes in climate.
- Bed peaks are locations of enhanced stability, not vulnerability, contrary to some prior theories which neglect the effect of bed slope.
- Persistence of glaciers at bed peaks may give way to sudden retreat without a concurrent climate change.

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## Abstract

Increasing ice flux from glaciers retreating over deepening bed topography has been implicated in the recent acceleration of mass loss from the Greenland and Antarctic ice sheets. We show in observations that some glaciers have remained at peaks in bed topography without retreating despite enduring significant changes in climate. Observations also indicate that some glaciers which persist at bed peaks undergo sudden retreat years or decades after the onset of local ocean or atmospheric warming. Using model simulations, we show that glacier persistence may lead to two very different futures: one where glaciers persist at bed peaks indefinitely, and another where glaciers retreat from the bed peak suddenly without a concurrent climate forcing. However, it is difficult to distinguish which of these two futures will occur from current observations. We conclude that inferring glacier stability from observations of persistence obscures our true commitment to future sea-level rise under climate change.

## Plain Language Summary

Glaciers that flow into the ocean may retreat when the ocean or atmosphere warms. These glaciers often sit on top of solid ground that is bumpy with sharp peaks. Satellites and geological indicators on the seafloor have recorded the retreat of glaciers in the past, and show correlation between the location of bed peaks and positions where glacier retreats have paused. Computer simulations confirm that sharp bed peaks may pause the retreat of a glacier, giving the appearance of stability. However, such a pause in glacier retreat may only be temporary. Some glaciers will suddenly retreat from a bed peak without a recent change in climate. This behavior may lead to mistaken interpretations of a glacier’s future based on its current state or mistaken attributions of past glacier changes to changes in climate. Being able to predict whether a glacier will retreat or stay at these bed peaks is important for predicting future sea level rise from glacier change.

## 1 Introduction

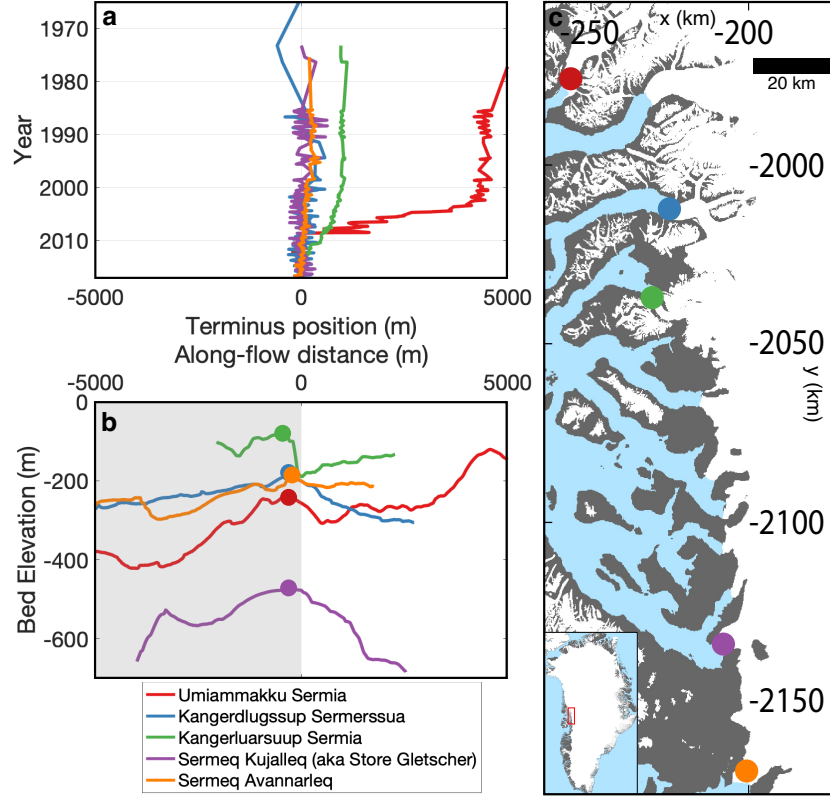
Mass loss from the Greenland and Antarctic ice sheets has accelerated in recent decades, driven by increasing surface melt and discharge of ice from glaciers (Shepherd et al., 2018; Mouginot et al., 2019). The increase in glacier discharge is driven, in part, by glacier retreat over deepening (retrograde) bed topography, which may initiate a positive feedback known as the “marine ice sheet instability” (Weertman, 1974). However,

the climate forcing needed to initiate this positive feedback depends on a range of other process including ice shelf buttressing and subglacial friction (Gudmundsson et al., 2012; Robel et al., 2016; Haseloff & Sergienko, 2018; Pegler, 2018; Sergienko & Wingham, 2019), which are not all represented accurately in theories of marine ice sheet stability and even in complex ice sheet models which are used to project future ice sheet changes. In particular, bed topography that fluctuates on length scales of tens to hundreds of kilometers leads to behaviors that are not accurately predicted using classical theories of marine ice sheet stability (Sergienko & Wingham, 2021).

As observations of subglacial bed topography and glacier retreat have improved, we are learning that bed topography is bumpy at a wide range of length scales (Jordan et al., 2017; Morlighem et al., 2017, 2020) and that many glaciers in Greenland and Antarctica have undergone large retreats (Tinto & Bell, 2011; Smith et al., 2017; Catania et al., 2018). Still, many glaciers have not retreated during the observational era, even while nearby glaciers have retreated in response to regional warming of the ocean and atmosphere. Notable examples includes Thwaites Glacier, West Antarctica, where geological evidence recorded the persistence of the grounding line at a bed peak for hundreds to thousands of years (Tinto & Bell, 2011), even amidst significant fluctuations in ocean temperatures (Hillenbrand et al., 2017). Nearby, observations show that Pine Island Glacier persisted at a bed peak until the 1970's, even though regional warming of the ocean began in the 1940's (Smith et al., 2017). As we will discuss further in section 2, large portions of the Greenland coast have also been subject to incursions of warm ocean water, though different glaciers have responded to these incursions in different ways (Catania et al., 2018), with the presence of sharp bed peaks being a key factor both in Greenland and in Antarctica.

Here we demonstrate both observationally and using model simulations that the retreat of a marine-terminating glacier may pause at bed peaks for prolonged time periods even while the glacier continues to lose mass in response to a current or previous climate forcing. The persistence of glaciers at bed peaks ultimately leads to one of two very different futures: one in which the glacier continues to persist without losing mass (section 3), and another where retreat occurs suddenly without a concurrent change in climate and leads to a significant acceleration in mass loss. However, it is difficult to distinguish which of these two possible futures will occur from current observations of persistent glaciers (section 4). We also discuss how glacier stability is not necessarily im-





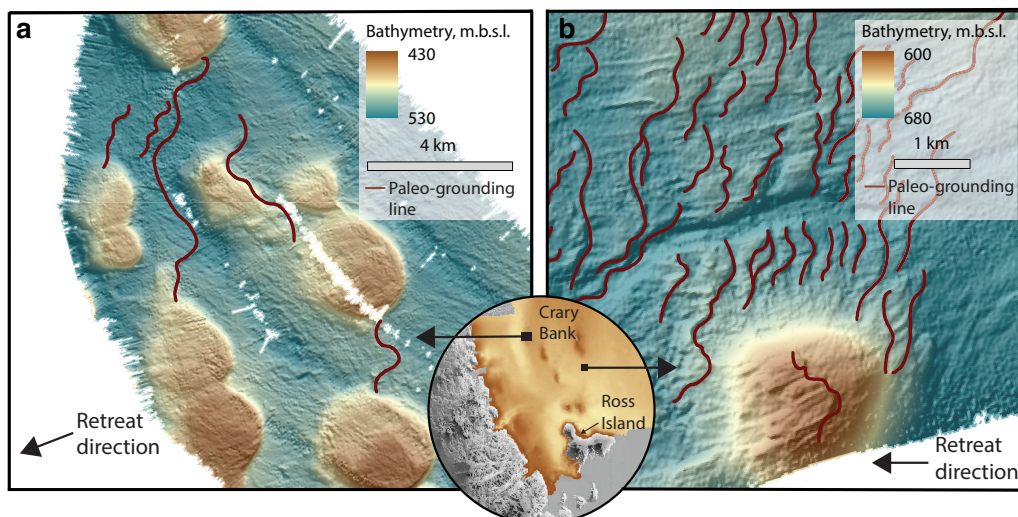
**Figure 1.** Observational evidence of terminus and grounding line persistence at bed peaks in Central West Greenland (CWG). (a) Terminus positions ( $x$ -axis) over time ( $y$ -axis) at five CWG glaciers derived from satellite-based sensors (Catania et al., 2018). (b) Along-flow bed topography at CWG glaciers in panel (a), with the  $x$ -axis is the along-flow distance relative to recent (2016) terminus position, with  $x = 0$  representing the present position of the glacier termini and gray shading indicating where there is currently grounded glacier ice ( $x < 0$ ). Nearest bed peaks upstream of the current terminus denoted by a filled circle in each case. For glaciers with strong cross-fjord variations in topography (Kangerluarsuup, Kujalleq), the minimum cross-fjord bed topography is used; for the others, mean cross-fjord topography is used. Bathymetry from BedMachine data compilation (panel b) (Morlighem et al., 2017). (c) Location of CWG glaciers in panels (a) and (b) on polar stereographic north projection (EPSG:3413).

plied by observations of glacier persistence (section 5). Ultimately, this ambiguous behavior of seemingly “stable” glaciers obscures the true commitment to future sea level rise under anthropogenic climate change.

## 2 Observations of glacier persistence

Central West Greenland provides a particularly well-observed laboratory for understanding glacier retreat over bumpy beds. As in most of Greenland, surface melting has been persistently intensifying since the 1970's (or potentially earlier; Trusel et al. (2018)). In the late 1990's an influx of warm water from the North Atlantic arrived in glacier fjords in this region (Holland et al., 2008). A compilation of terminus positions recorded by visible satellite imagery (Catania et al., 2018) show that many glaciers in Central West Greenland (CWG) retreated between the late 1990's and the early 2000's when ocean temperatures were warm. However, some glaciers in this region have not retreated during the observational record, despite experiencing the same influx of warm ocean water. Figure 1a shows observations of terminus positions at four such persistent glaciers, Kangerdlugssup Sermerssua (blue), Kangerluarsuup Sermia (green), Sermeq Kujalleq (purple, aka Store Gletscher), and Sermeq Avannarleq (orange). Figure 1b shows the along-flow bed topography at these same glaciers from the BedMachine v3 dataset (Morlighem et al., 2017). These glaciers have persisted less than one kilometer downstream of bed peaks, indicating the critical importance of peaks in bed topography in potentially delaying or preventing rapid glacier retreat. Glaciers in this region that did retreat following the ocean warming event in the late 1990's mostly retreated away from bed peaks (on which they had previously persisted) and all have since ceased retreat upon reaching a new bed peak. In Figure 1, we show one example of such a glacier, Umiammakku Sermia (red), which began rapidly retreating approximately 5 years after the arrival of warm waters in the region, before ceasing retreat at a bed peak around 2010.

Geological evidence from regions of past glacier retreat further demonstrates the importance of bed peaks in the response of glaciers to climate change. The bathymetry of the Ross Sea, Antarctica is composed of smooth, flat troughs separating large plateaus. Amid this smooth bathymetry, localized recessional moraines and grounding zone sediment wedges record locations where the deglacial retreat of glaciers in the Ross Sea embayment paused for prolonged time periods (Simkins et al., 2017; Greenwood et al., 2018). Figure 2 focuses on two particular locations in the Ross Sea where high-resolution multi-beam bathymetric observations show pervasive grounding zone wedges connecting, parallel to, and on top of seamounts on otherwise flat topography. Such evidence of grounding line persistence is not present in surrounding flat portions of the seafloor, indicating that these bed peaks (which are generated by non-glaciological processes) exert an

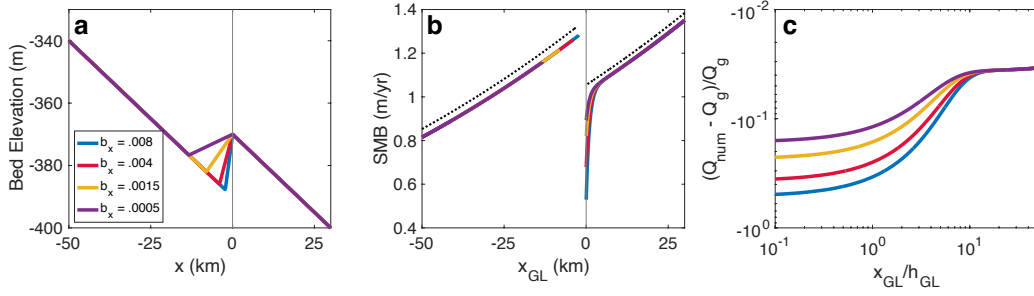


**Figure 2.** Bathymetry (in meters below sea level) of the south-western Ross Sea (inset bottom) with grounding zone wedges indicated by brown lines, which are generally perpendicular to the local retreat direction (indicated by black arrows). Multibeam echo sounding bathymetry collected on cruise NBP1502A (Simkins et al., 2017; Greenwood et al., 2018).

important control on glacier retreat. Other marine geophysical surveys of the seafloor in regions of past glacier retreat also reveal widespread geological evidence for prolonged periods of terminus persistence at bed peaks over a wide range of time periods and local conditions (Stoker et al., 2009; Greenwood et al., 2017; Todd & Shaw, 2012).

### 3 Enhanced glacier stability at bed peaks

To understand how bed peaks affect glacier stability, we consider first their effect on steady-state terminus positions, before considering their effect on transient evolution of the terminus position in the next section. To simulate a typical marine-terminating glacier near a bed peak, we use a one-dimensional flowline model of a marine-terminating glacier which solves the shallow stream/shelf approximation (SSA) and mass conservation equations to determine the evolution of ice thicknesses, velocities, and terminus position (as described in Schoof (2007a) and many other studies). The glacier is assumed to terminate where it goes afloat in seawater, and in the cases we consider in this study, ice flow is dominated by sliding over a moderately slippery bed (parameters listed in Table 1). We simulate the glacier velocity, thickness, and terminus position for prescribed surface mass balance (SMB, net annual snowfall and surface melt), which we assume to



**Figure 3.** Simulated stable grounding line positions in the vicinity of a bed peak. (a) Four idealized bed topographies with differing bed slope just upstream of bed peak. (b) Bifurcation diagrams showing steady-state grounding line positions over a range of surface mass balance and initial grounding line positions. The dotted line is the stable grounding line positions predicted from an approximation for ice flux based on neglecting the effect of local slope,  $Q_g$  (Schoof, 2007b). (c) The proportional difference between the ice flux at the terminus predicted from our numerical solution,  $Q_{num}$ , and the ice flux that would be predicted on neglect of the effect of local slope,  $Q_g$ , as a function of distance from the bed peak (normalized by grounding line ice thickness).

be uniform over the surface of the glacier. The position of the terminus is accurately modeled using a refined moving mesh. Glacier steady-states are determined by numerically solving for glacier states with rates of change that are zero to within machine precision (and with transient perturbations to glacier state to confirm stability). We have also replicated the substance of the steady-state and transient results described hereafter using very high-resolution simulations of the Elmer/Ice Full-Stokes numerical glacier model (see Figure S1), indicating that the SSA simplification does not affect the substance of the conclusions in this study.

The four idealized bed topographies we consider (Figure 3a) all have a single bed peak, but with different reverse bed slopes just upstream of the peak, and otherwise the same forward-sloping bed (i.e. shallowing towards the interior). Simulations show that over a wide range of SMB, glacier termini persist indefinitely (i.e. reside at a stable steady-state) near bed peaks (Figure 3b). We find that at the sharp bed peaks we consider in this study, which entail a rapid spatial transition from a forward-sloping bed to a sufficiently reverse sloped bed (i.e. over a horizontal length scale of several kilometers), lead to glacier stability over a wider range of SMB than what is predicted in prevailing the-

**Table 1.** Parameter values for steady-state and transient retreat simulations (unless otherwise specified in text)

Parameter	Description	Value
$A_g$	Nye-Glen Law coefficient ( $\text{Pa}^{-n} \cdot \text{s}^{-1}$ )	$4.2 \times 10^{-25}$
$b_x$	Prograde bed slope	$1 \times 10^{-3}$
$C$	Basal friction coefficient ( $\text{Pa} \cdot \text{m}^{-1/n} \cdot \text{s}^{1/n}$ )	$1 \times 10^6$
$g$	Acceleration due to gravity ( $\text{m} \cdot \text{s}^{-2}$ )	9.81
$m$	Weertman friction law exponent	1/3
$n$	Nye-Glen Law exponent	3
$\Delta t$	Time step (yr)	1
$\rho_i$	Ice density ( $\text{kg} \cdot \text{m}^{-3}$ )	917
$\rho_w$	Seawater density ( $\text{kg} \cdot \text{m}^{-3}$ )	1028

ories of terminus stability (dotted line in Figure 3b, reproduced from Schoof (2012)). The steeper the reverse sloped bed upstream of the bed peak, the wider the range of SMB over which the glacier will remain stable. For the steepest reverse slope (shown in blue), the glacier remains stable a short distance downstream of the topographic high for a significant range of SMB from from 0.5 to 1.0 m yr<sup>-1</sup>.

The cause of enhanced glacier stability at bed peaks can be explained by examining how ice flux out of the glacier changes as the glacier gets close to the bed peak. As the terminus approaches within  $\sim 10$  ice thicknesses of the bed peak, the ice flux (Figure 3c) decreases much more rapidly than is predicted under the assumption of negligible bed slope near the terminus. Indeed the magnitude of this reduction in ice flux near the bed peak (10-50% in these examples) is comparable to the effect of ice shelf buttressing on grounding line ice flux (Reese et al., 2018; Mitcham et al., 2021). The cause of this rapid decline in ice flux is a lowered driving stress on the ice flowing uphill to the bed peak, which lowers ice velocity just upstream of the bed peak and influences terminus ice flux through longitudinal viscous stresses. This reduced terminus ice flux is in balance with the total ice flux arriving at the terminus from upstream, maintaining a stable terminus position, even under dramatically lower SMB.

The enhanced stability of simulated termini near bed peaks, as compared to prior theory, explains counterintuitive aspects of observations. There is a wide range of external forcing over which a terminus will persist at a bed peak, explaining why so many glacier termini are observed at bed peaks on bumpy bed topography. Indeed, repeating these steady-state simulations for corrugated bed topography (a regular series of peaks and troughs) indicates stable glacier terminus positions exist almost exclusively at bed peaks (Figure S2). The reduced glacier sensitivity to climatic changes at bed peaks also explains why many glaciers are observed to persist, seemingly on the precipice of instability, even while experiencing substantial fluctuations in local climate (Tinto & Bell, 2011; Hillenbrand et al., 2017; Catania et al., 2018). Such enhanced stability of glaciers at bed peaks is in contrast to the prevailing idea that glaciers at bed peaks are particularly “vulnerable” to fluctuations or trends in climate due to their proximity to reverse-sloping beds over which the marine ice sheet instability occurs (Gladstone et al., 2012; Ross et al., 2012; Morlighem et al., 2020).

The enhanced stability near points of destabilization and non-smooth changes in bed topography is also a hallmark of a “crossing-sliding bifurcation” (di Bernardo et al., 2008). The system behavior in the vicinity of such bifurcations is different from the canonical “saddle-node bifurcation”, which has previously been identified as the route through which grounding lines lose stability (Mulder et al., 2018; Pegler, 2018). In a saddle-node bifurcation, the loss of stability occurs smoothly and suddenly upon approach of the system state to the bifurcation point. In a crossing-sliding bifurcation, the loss of stability instead occurs due to an abrupt (i.e., non-smooth) change in the system properties. In the case of a sharp bed peak, smooth parameter variations causes the stable glacier state to reach the bifurcation point and then remains there, before eventually crossing the bed peak and initiating a large change in glacier state. This distinction in type of bifurcation can be important because it leads to larger “jumps” in the system state (i.e., ice volume loss) upon crossing the bifurcation. In other words, as the forcing gradually changes (i.e., SMB decreases), the onset of rapid ice loss is delayed, leading to a higher rate of ice loss if and when the terminus crosses the bed peak. In Figure 3b, this amounts to the difference between a 30 km jump in grounding line position for a relatively smooth bump (i.e. the dotted or purple lines), compared to a jump of 50-100 km for sharper peaks (yellow, red, blue lines). As we will show next, even when climate trends or fluctuations

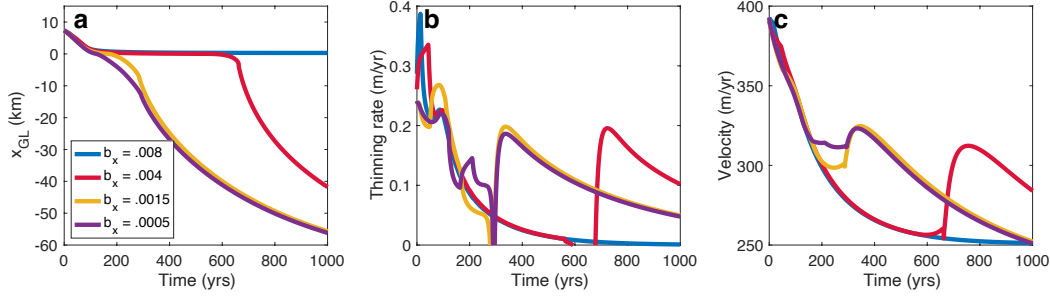
exceed the threshold for instability identified in Figure 3b, the onset of rapid glacier retreat may be substantially delayed.

#### 4 Distinguishing glacier stability from transient persistence

In transient simulations of terminus retreat over idealized bed peaks (the same as in the previous section), a glacier is initialized at a steady state with its terminus just downstream of a bed peak, and is then subjected to a 40% step reduction in SMB uniformly over the glacier catchment. Figure 4a shows that some of the simulated glaciers retreat up to, then transiently persist, just downstream of the bed peak for a period of time spanning decades to centuries (yellow and red lines), before eventually crossing the bed peak and rapidly retreating over the reverse sloping bed. There are also cases where there is merely a brief slowdown in the rate of retreat at the bed peak (purple line), and other cases where the persistence continues indefinitely (blue line). We define such an indefinitely persistent case as “stable” in the mathematical sense, where a system state persists forever with no change in forcing. Similar behaviors of transient and indefinite persistence of grounding lines at bed peaks also occur in equivalent full-Stokes simulations of grounding line retreat over bed peaks (Figure S3) and in SSA simulations of glacier retreat with different types of forcing and smoothed bed peaks (Figures S4-S6). These transient simulations show that even for SMB values that do not correspond to a stable glacier configuration (shown in Figure 3b), there may still be prolonged periods of transient terminus persistence. Longer periods of transient persistence lead to more rapid subsequent retreat, which continues even as the terminus encounters forward-sloping bed topography.

Glaciers that persist at bed peaks continue to lose mass through thinning upstream of the grounding line following changes in climate forcing, as seen in observations of recent thinning upstream of the terminus at persistent glaciers throughout Greenland (Kjeldsen et al., 2015; Mouginot et al., 2019; Shepherd et al., 2020; Felikson et al., 2017). At persistent glaciers in CWG, this thinning is mostly being driven by negative surface mass balance anomalies, which are largely offset by dynamic thickening bringing ice from upstream portions of glacier catchments (Felikson et al., 2017). Ultimately, this upstream-intensified thinning leads a decrease in ice surface slope and upstream slowing, which is captured in observations of persistent CWG glaciers (Joughin et al., 2010) and our simulations (Figure 4b-c). Though such thinning is less than that occurring at retreating





**Figure 4.** Simulated terminus retreat in the vicinity of a bed peak. (a) Evolution of a terminus from steady state, in response to an instantaneous 40% reduction in surface mass balance over the glacier catchment, for a variety of upstream bed slopes. Grounding line position ( $y$ -axis) is relative to bed peak location as in Figure 3a. (b) Thinning rate 50 km upstream of grounding line in transient simulations. (c) Ice velocity 50 km upstream of grounding line in transient simulations.

glaciers through dynamic thinning, it nonetheless shows that persistence of a glacier terminus is not necessarily indicative of a glacier in mass equilibrium.

We can compare our simulated glaciers which stabilize at bed peaks to those which merely pause at bed peaks to ascertain whether observations of persistent glaciers may provide evidence of their eventual fate. We find that, regardless of their eventual fate (remaining or retreating), the glaciers we simulate which persist at bed peaks have upstream thinning rates within millimeters/year of each other (Figure 4b and Figure S8), and ice velocities within meters/year of each other (Figure 4c and Figure S7). It would thus be exceedingly difficult to observationally distinguish glaciers that are merely paused from those that have stabilized indefinitely at bed peaks. Other studies have also found that, in realistic simulations of the future retreat of glaciers away from bed peaks, small uncertainties in the observed glacier state, bed topography, or the climate forcing produce large uncertainties in the timing of the onset of rapid glacier retreat which is then amplified by the divergence of retreat predictions due to marine ice sheet instability (Gladstone et al., 2012; Robel et al., 2019). Ultimately, the delicate balance between advection and thinning at persistent glaciers makes it exceedingly difficult to project retreat of glaciers over bumpy bed topography, and further emphasizes the need for more accurate observational constraints on glacier state and rate of change, bed topography, and local climate change.



## 5 Discussion and Conclusions

We have shown that glaciers observed at bed peaks have two possible futures: they may remain at the bed peak indefinitely (i.e., stabilize) or initiate retreat, potentially long after the onset of climate change. Glaciers persisting at bed peaks may continue to lose mass in response to a previous or sustained climate change, though there will be an increasing “disequilibrium” between this mass loss and the total committed glacier mass loss implied by contemporaneous climate forcing (Christian et al., 2018). If the terminus does eventually cross the bed peak, terminus retreat and total glacier mass loss accelerates rapidly, relaxing the glacier disequilibrium between instantaneous and total committed mass loss. Eventually, the total sea level contribution from non-persistent and transiently persistent glaciers may be similar, though the timing and rate of peak mass loss may be very different (e.g., Figure 4).

In attempting to infer the future behavior of glaciers persisting at bed peaks, observations can be deceptive. We have shown that ice flux and thickness may change considerably with relatively little change in the terminus position. Thus, interpreting observations of terminus change requires accurate measurements of bed topography and the critical context of changes in other aspects of glacier state (particularly interior thickness and velocity) to assess whether the glacier is in balance. Additionally, the slow response time scale of glaciers, particularly those that have encountered bed peaks, indicates that the utility of “stability” as a tool for categorizing observed glacier changes is limited without the critical context of multi-centennial (or millennial) glacier changes, and the climate forcing over that time period. The scope of these challenges and potential impacts indicate that we should direct a similar degree of attention and resources to closely observing and carefully simulating persistent glaciers as we do to rapidly changing glaciers, as it is possible and perhaps likely that they will eventually contribute just as much to future sea level rise.

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# Supporting Information for "Ambiguous stability of glaciers at bed peaks"

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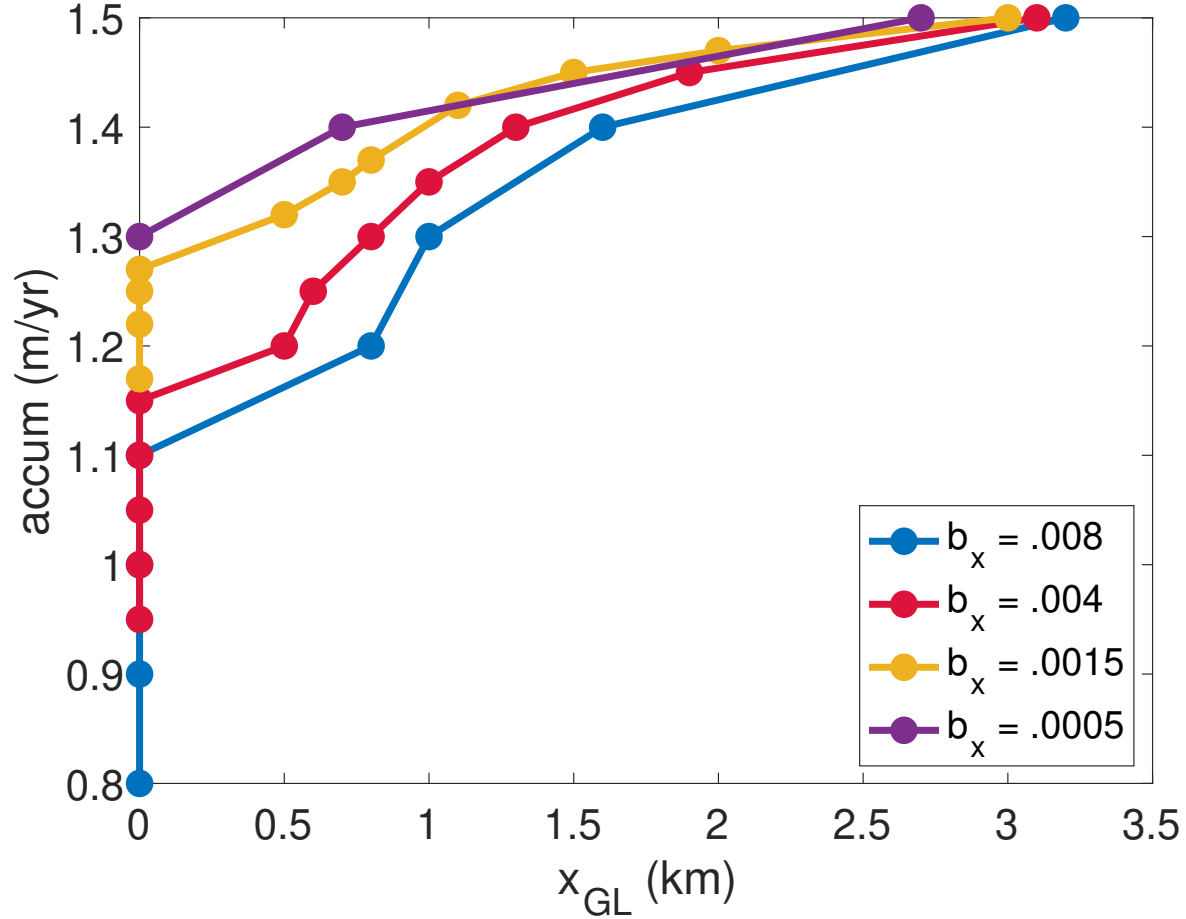
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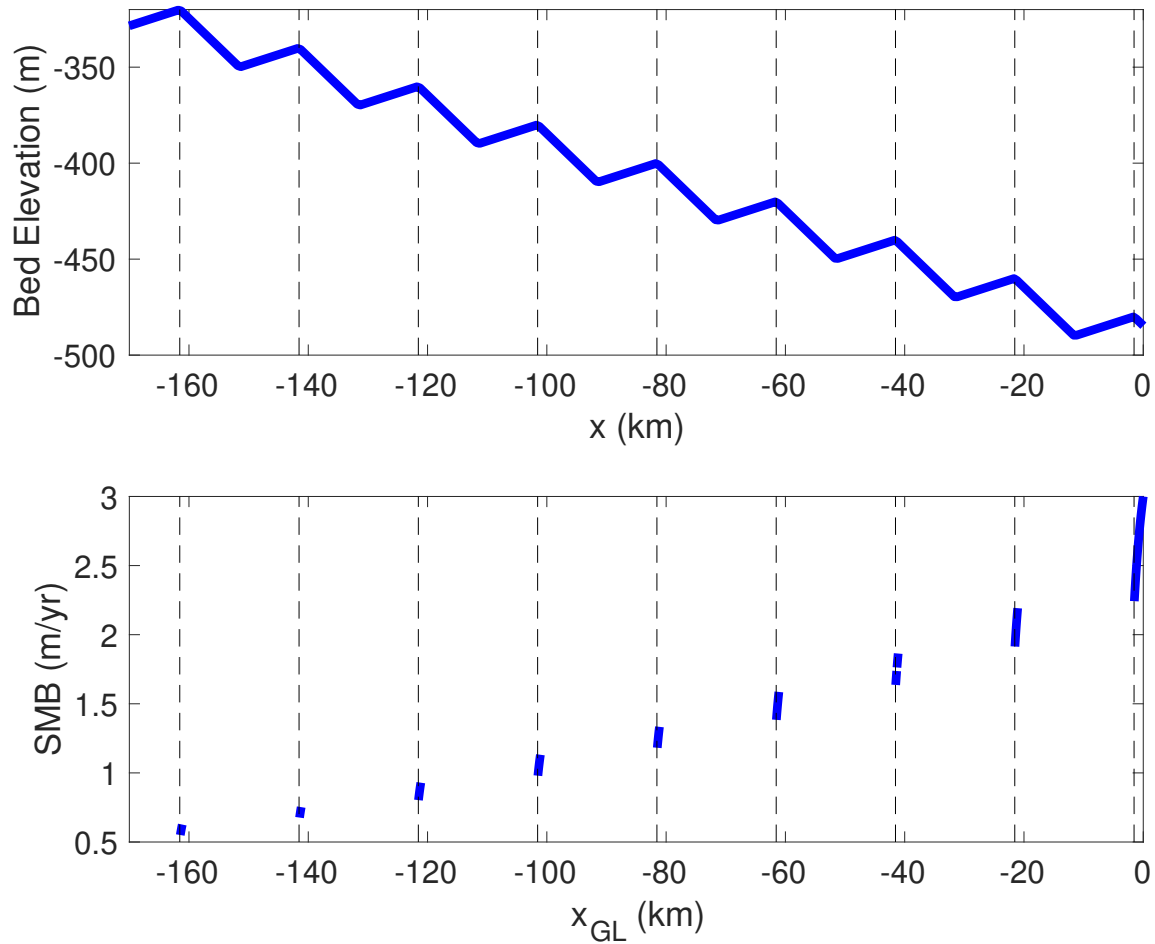
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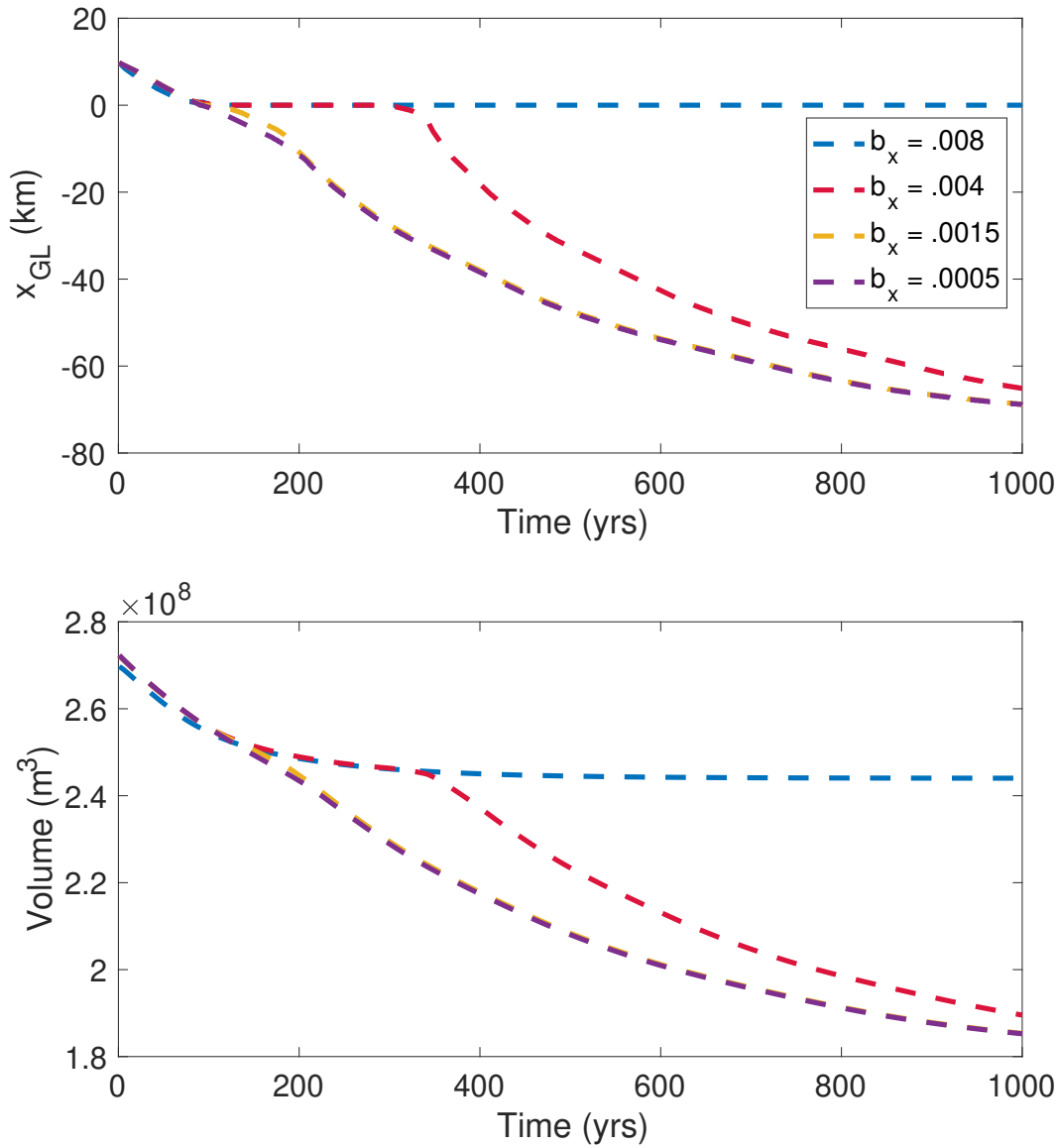


**Figure S1.** Stable grounding line positions in Elmer/Ice Full Stokes simulations (along a flowline) of grounding line retreat over sharp bed peaks located at  $x = 0$  on the x-axis. Bed topographies are identical to those use in simulations plotted in Figure 3. Horizontal resolution is 100 meters throughout domain. Simulations are variants on the the Elmer/Ice MISMIP benchmark simulations.

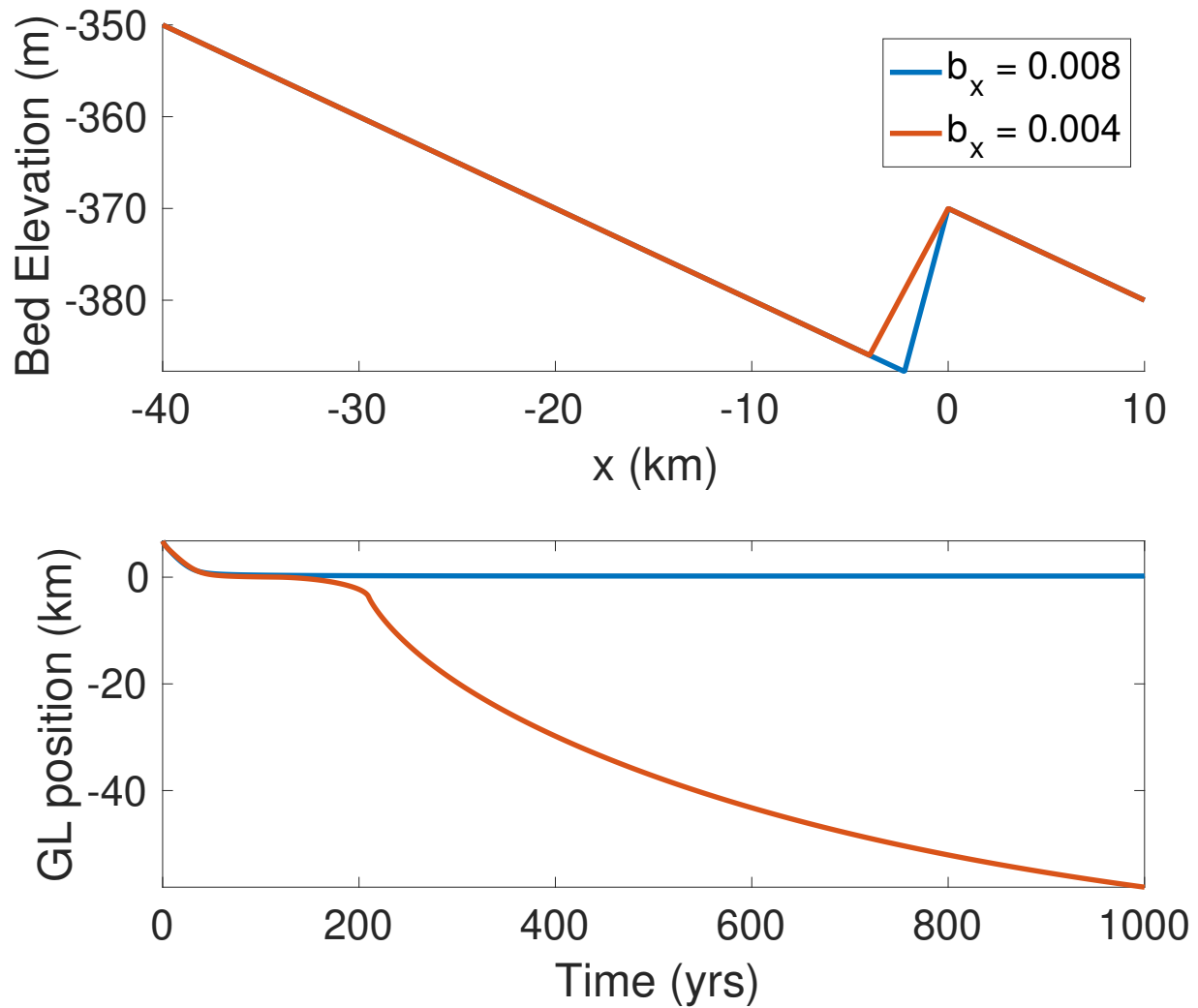


**Figure S2.** Steady-state grounding line positions over a range of surface mass balance (panel b) on a corrugated bed with many bed peaks (panel a). Bed peaks are indicated with black dashed lines.

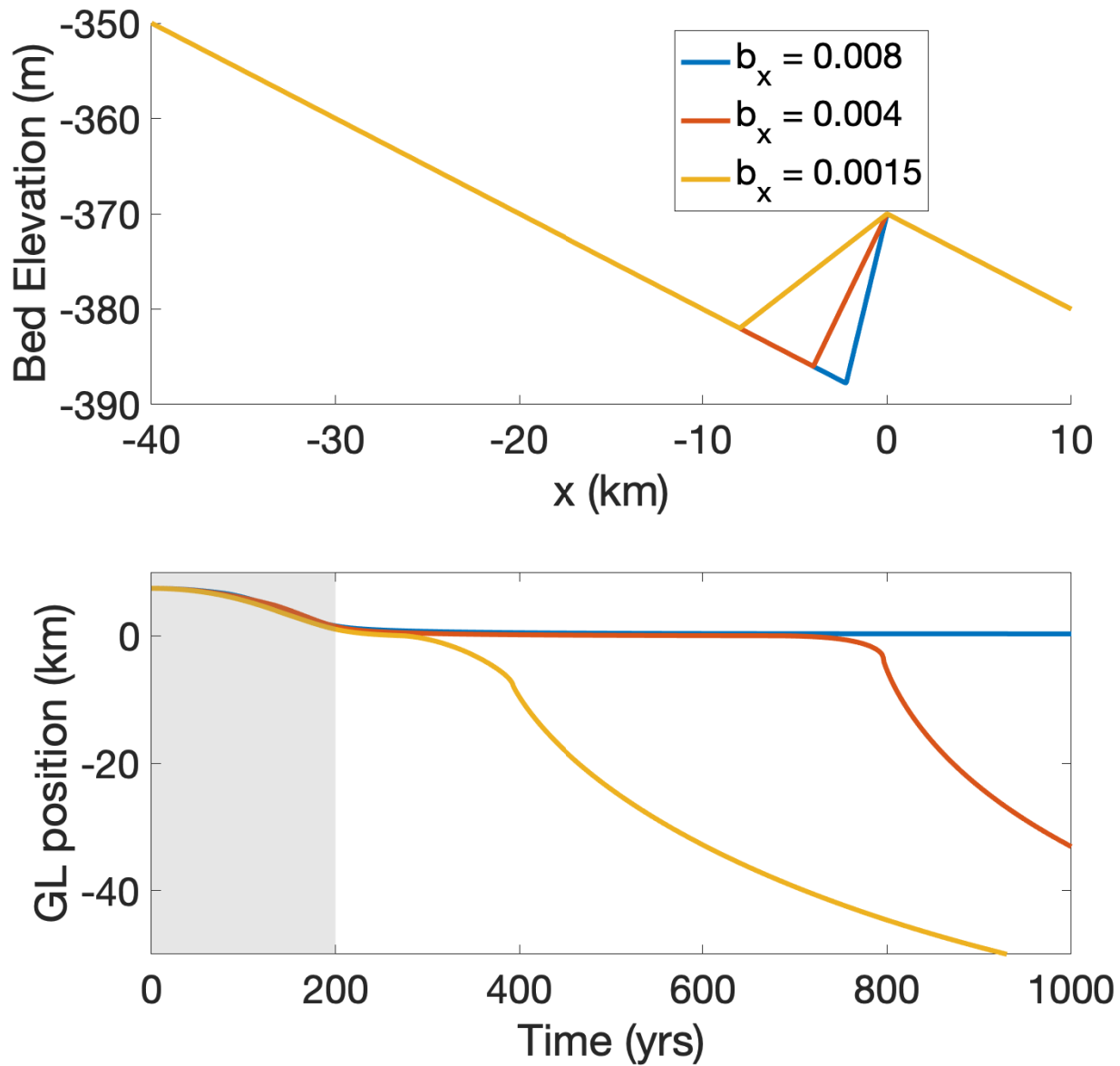




**Figure S3.** Transient grounding line (top panel) and glacier volume (bottom panel) evolution in Elmer/Ice Full Stokes simulations of retreat over sharp bed peaks (simulated along a flowline). Bed topographies are identical to those used in simulations plotted in Figure 4. Due to small quantitative differences in steady-state grounding line positions in full Stokes simulations, initial SMB is set to 1.5 m/yr (compared to 1.1 m/yr in SSA simulations) and then reduced by 40% at beginning of simulation, as in the SSA simulations). Horizontal resolution is 100 meters throughout domain.

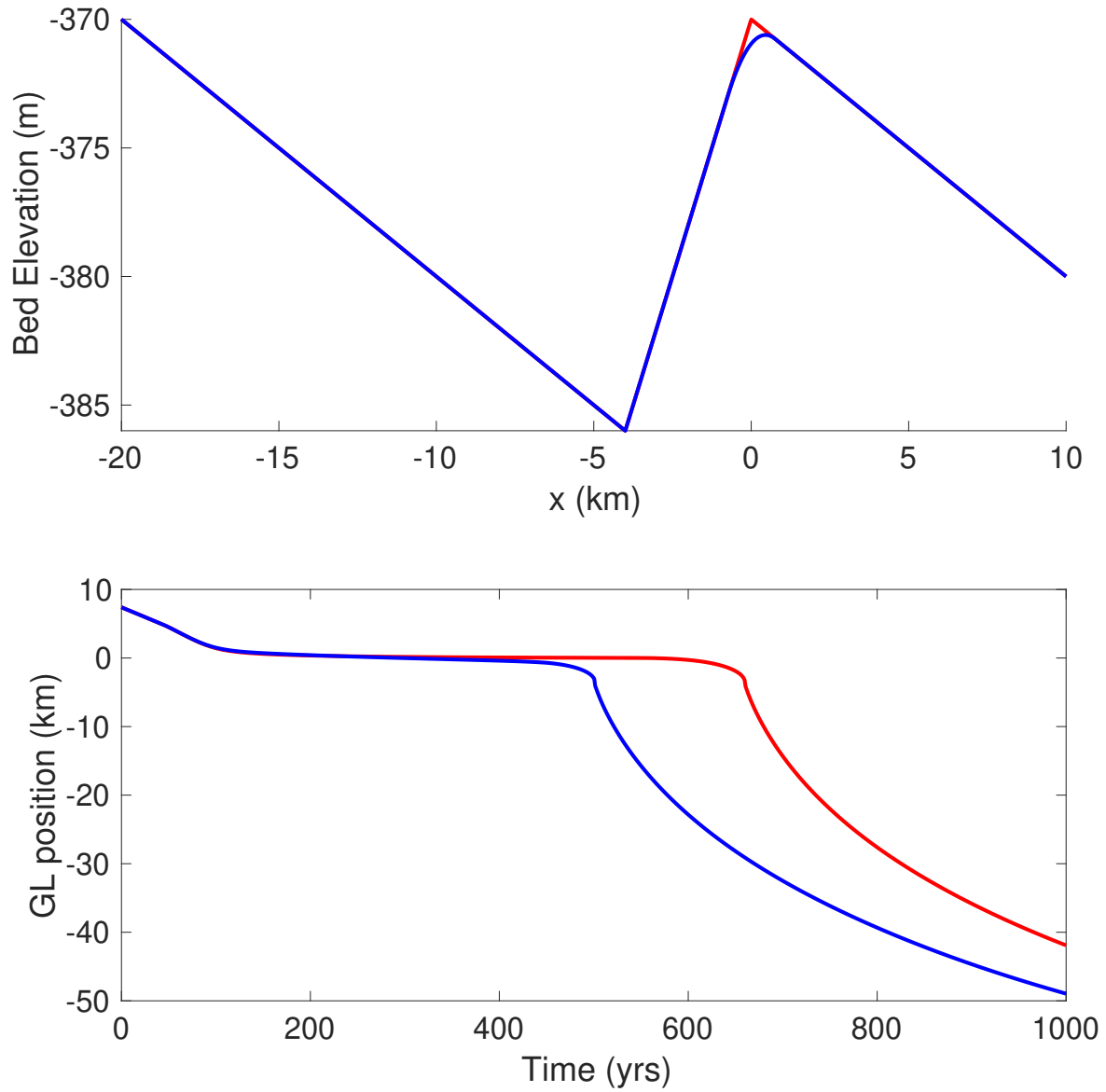


**Figure S4.** Comparison between simulated transient persistence and retreat over bed peaks with different upstream bed slope under changing ocean forcing. Simulations are the transient response to a step change from zero ocean melting to 50 m/yr basal melt rate at the terminus. Submarine ocean melt is imposed as a basal melt rate at the node corresponding to the grounding line. The idealized bed topographies here correspond to the two steepest bed peaks plotted in Figures 3 and 4.

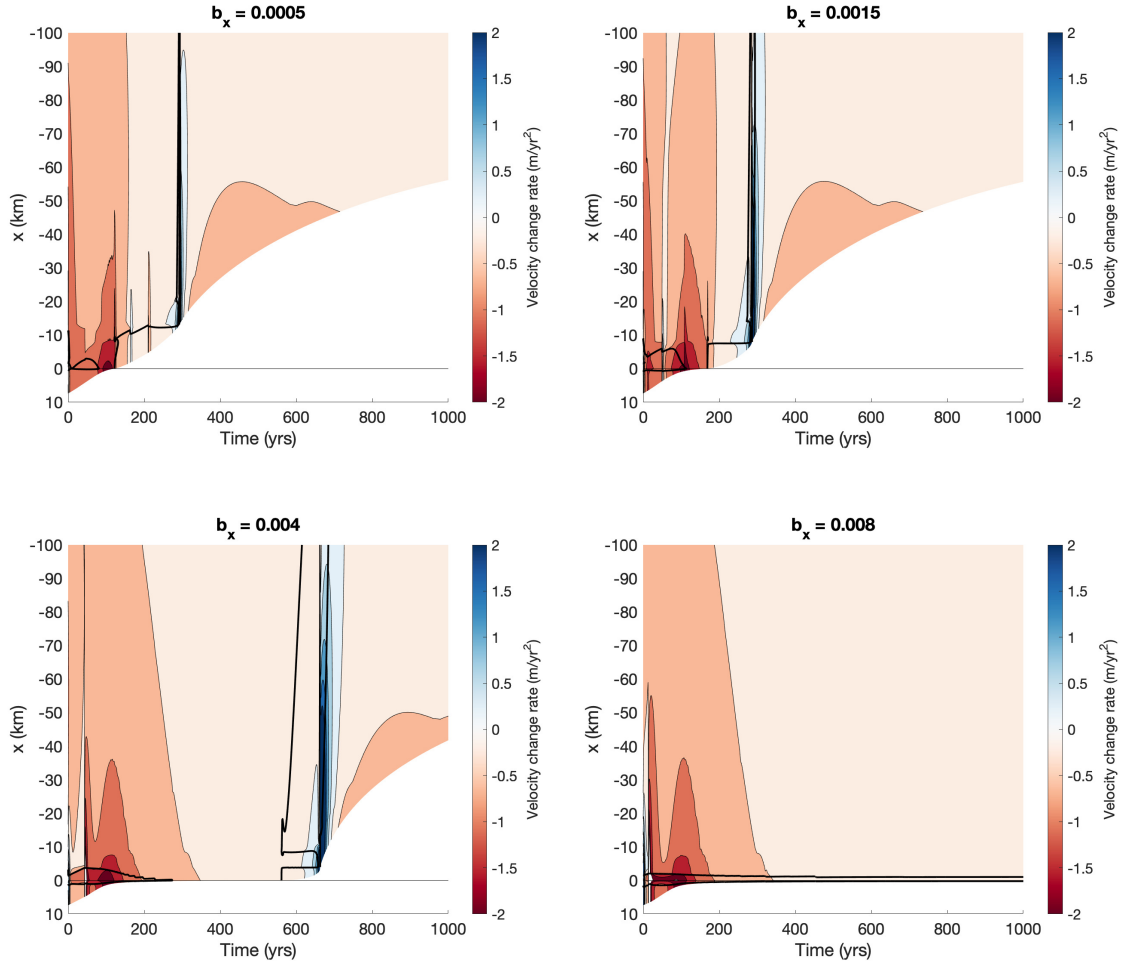


**Figure S5.** Comparison between simulated transient persistence and retreat over bed peaks with different upstream bed slope. Simulations are the transient response to a trend in SMB over the first 200 years of the simulation. The total change in SMB is the same as in the simulations plotted in Figure 4. The idealized bed topographies here correspond to the three steepest bed peaks plotted in Fig. 2a. It can be noted that though the timing of retreat onset is slightly delay (presumably due to the slower forcing), the qualitative behavior of the transient persistence at bed peaks is unchanged.

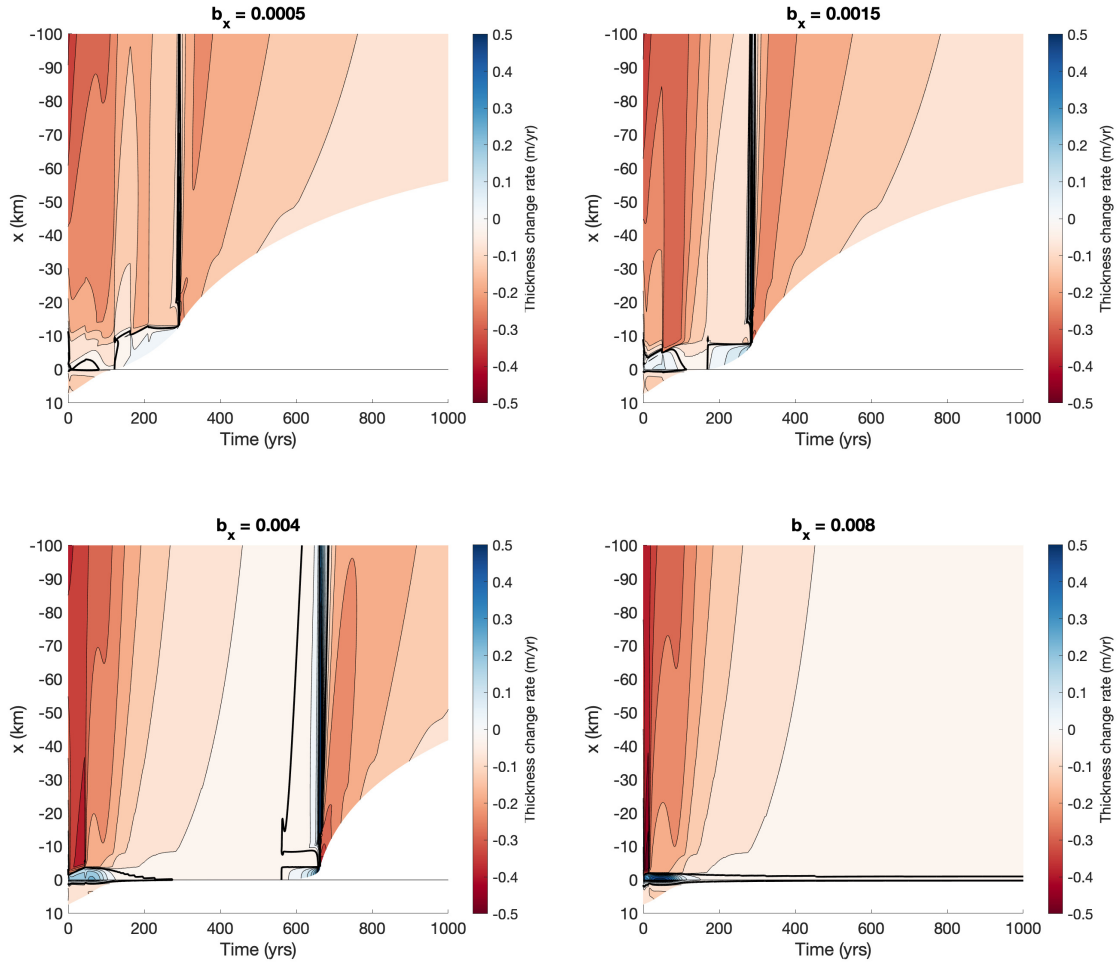
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**Figure S6.** Comparison between simulated transient persistence and retreat over bed peaks. Red line is the same simulation as the red line in Figure 4 in main text. Blue line is with bed peak smoothed over 1 km moving window. Multi-centennial persistence still occurs, though onset of rapid retreat is slightly early due to lower bed peak.



**Figure S7.** Rate of change of ice velocity in transient simulations plotted in Fig. 2 in main text. x-axis is time and y-axis is the along-stream distance relative to the terminus, where negative values are upstream.



**Figure S8.** Rate of change of thickness in transient simulations plotted in Figure 4 in main text. x-axis is time and y-axis is the along-stream distance relative to the terminus, where negative values are upstream.