Estimation of the mode conversion effect on the determination of southern boundary for the 100 MeV electron precipitations

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Key Points: * The influence of the ultraenergetic relativistic electron precipitations on mode conversion in the terrestrial waveguide was analysed. * New statement of a mode conversion problem was used. * The effect of a normal wave conversion may be neglected in the problem of southern boundary determination, if the precipitations are not powerful. Abstract Previously the parameters of sporadic D s layer of electric conductivity caused by ultraenergetic relativistic electron (URE) precipitations were determined by us due to indirect electromagnetic method. Then we determined the southern boundaries of these precipitations in the frames of the following supposition: the effect of a normal wave reflection and its conversion into other normal waves on the boundary between disturbed and undisturbed parts of a radio path might have been ignored. Now we show by accurate simulation that it was true for strong and moderate URE precipitations. For the powerful disturbances of the same nature (16 events for 11 years) the effect is significant. In order to obtain these results, we had to change a traditional statement of problem about a normal wave conversion in the terrestrial waveguide and to solve numerically this new problem with an abrupt change of inhomogeneous properties on a transverse surface of the waveguide. We did not divide a near-Earth waveguide at two artificial altitude parts: with vacuum and with inhomogeneous conducting medium along the transverse coordinate. Formally the waveguide was considered by us as inhomogeneous from its top to its bottom.

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- New statement of a mode conversion problem was used.
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1 Introduction

A relatively long phenomenon of ultraenergetic relativistic electron (URE) precipitations with abnormally high intensity into polar atmosphere was stated by the help of indirect very low frequency (VLF) method (Remenets & Beloglazov, 1985, 2013) more than 30 years ago. During these years the monitoring of the fluxes of ~ 100 MeV electrons in the near cosmos did not appear although the fluxes of highly-energetic relativistic electrons (HRE) with energy from ~ 1 to ~ 10 MeV were measured about the same 30 years (Callis & et al., 1991; Pesnell & et al., 1999) and the fluxes of GeV electrons were monitored more than 10 years (Andriani & et al., 2017). The window (gap) between these measurements was partially overlapped by the ground VLF measurements, which we have indicated above and (Remenets & Shishaev, 2019).

The VLF waves generated by a transmitter ground based and propagating in a terrestrial waveguide between two conducting mediums (which are the ground and the bottom ionosphere) are sensitive to the time dependence of electric conductivity of this bottom and to its dynamics being caused either due to the ionization of neutrals by the precipitating electrons or by hard electromagnetic radiation. At the same time the ultraenergetic relativistic electrons (URE′s) are the sources of very significant bremsstrahlung radiation in the atmosphere (Hayakawa, 1969; Remenets & Beloglazov, 2013), if the density of corpuscular flux is abnormally high (Remenets & Beloglazov, 2013).

It is known that the electrons with energy ~ 100 MeV do not penetrate mainly into the atmosphere deeper than the altitude with the pressure about 5 kPa ($50g/cm^2$), that is, deeper than ~ 40 km. Therefore, significant electric conductivity, registered by VLF-method at the altitudes 30 km and lower (Beloglazov & Remenets, 2005) can be caused only by the bremsstrahlung X – and gamma rays. These rays created a sporadic D_s - layer, which manifested itself in both existing numerical solutions of the inverse VLF problem (Remenets, 1997; Beloglazov & Remenets, 2005; Remenets & Beloglazov, 2013). The pointed solutions were based on a theory of VLF wave propagation in near-Earth waveguide, that is, they were based on the Maxwell equation consequences for a physics model of waveguide (Gyunninen & Zabavina, 1966; Makarov & et al., 1993). At one of the pointed solutions of the inverse problem the ray (″hop″) theory (with the Watson-Fock diffraction wave and two rays reflecting from a ″bottom″ of atmosphere ionized) (Remenets, 1997; Bondarenko & Remenets, 2001; Remenets & Beloglazov, 2013; Remenets & Astafiev, 2015, 2016) were used. At another version of invers problem the normal waves (modes) for the waveguide were used (Remenets, 1994; Beloglazov & et al., 1999). Both types of inverse problem solution complement

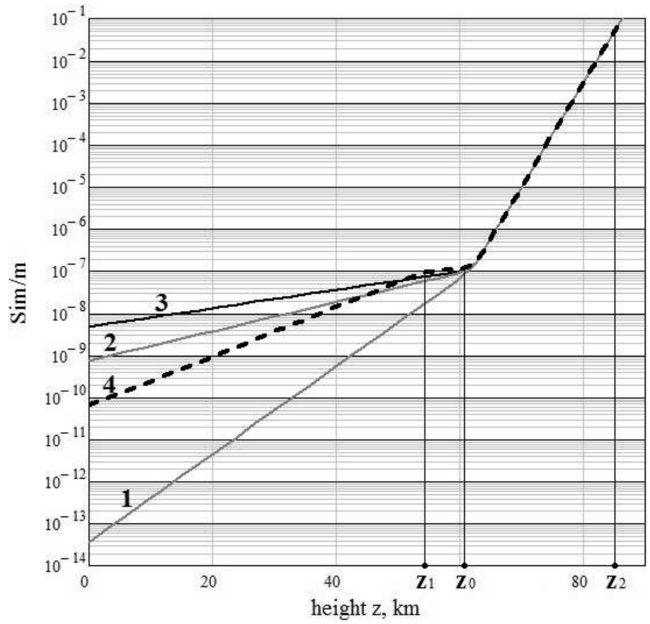

Figure 1. The effective profiles of electric conductivity for undisturbed

each other and give practically the same values of effective heights (Remenets, 1997; Beloglazov & Remenets, 2005).

In order to model the D_s-layer electric conductivity as function of altitude z we used in the last pointed works an approximation of its profile with following two free parameters: the breadth $z_1 \div z_0$ of its homogeneous conductivity and the increment β of its exponential dependence at altitudes below z_1, Figure 1. The upper part of ionosphere higher than $z_0 \sim 60$ km was considered undisturbed by the URE precipitations, and was approximated by exponential dependence on z for an electron concentration profile $N_e(z)$ (Beloglazov & Zabavina, 1982a, 1982b) due to the following: the energy of URE′s is so high and the atmosphere density at the altitudes higher z_0 km is so relatively small, that the ionization and bremsstrahlung processes are relatively small there. The logarithm of schematic σ function (a dotted curve with number 4 for Figure 1) has two ′knees′ at altitudes z_0 and z_1. Additional explanation to the curves of the Table 1 is the following: The effective profiles of electric conductivity for undisturbed (σ^I – curve 1) and disturbed conditions (σ^{II}_{str} – curve 2, σ^{II}_{pow} – curve 3) are presented. The bottom indexes of electric conductivity ″*str*″ and ″*pow*″ correspond to the maximums of a strong and a powerful VLF disturbances (Bondarenko & Remenets, 2001; Beloglazov & Remenets, 2005). Undisturbed auroral profile σ^I was taken from the work (Beloglazov & Zabavina, 1982a).

With such kind of approximation for a sporadic D_s-layer the inverse problems were solved in the pointed works above. The output parameters of a solution were the z_1 and β, and their values are reproduced for several time moments of the URE precipitations on 29 September 1989 (after night), 21 and 22 January 2002 (polar night) for the Table 1 and Table 2.

The approximation parameters, z_1 and β for profile $N_e(z)$ (with the profile of effective electron collisions having been fixed), were found due to the procedure of minimization of a discrepancy-function for 3 frequencies between the experimental and calculated amplitude and phase variations of signals caused by an URE precipitation. It is necessary to note that when one tries to satisfy the experimental VLF data (amplitude and phase variations for 3 frequencies) for the auroral radio path S_1 (Aldra-

Table 1. Results of the inverse VLF problem solutions [a]

UT[b]	0600	0730	0900
$\frac{(A_1)_d}{(A_1)_c}$	0.7 ± 0.1 (0.9)	0.5 ± 0.1 (0.6)	0.7 ± 0.1 (0.9)
$\varphi_{1c} - \varphi_{1d}$, μs	3.6 ± 0.5 (4.3)	8.6 ± 1.5 (8.1)	5.2 ± 0.5 (5.0)
$\frac{(A_2)_d}{(A_2)_c}$	0.7 ± 0.1 (1.0)	0.6 ± 0.2 (0.7)	0.7 ± 0.1 (0.8)
$\varphi_{2c} - \varphi_{2d}$, μs	2.0 ± 0.5 (1.7)	0.6 ± 1.5 (7.5)	4.4 ± 0.5 (6.1)
$\frac{(A_3)_d}{(A_3)_c}$	0.6 ± 0.1 (0.7)	0.6 ± 0.1 (0.6)	0.7 ± 0.1 (0.7)
$\varphi_{3c} - \varphi_{3c}$, μs	2.0 ± 0.5 (3.4)	4.5 ± 1.5 (5.6)	4.0 ± 0.5 (3.6)
z_1, km	60 ± 1	58 ± 2	60 ± 1
β, 1/km	-0.01 ± 0.01	-0.04 ± 0.01	-0.02 ± 0.01
h', km	56 ± 1	50 ± 2	54 ± 1
h'', km	55 ± 1	46 ± 2	54 ± 1
h''', km	50 ± 3	43 ± 2	52 ± 2
h, km	57-60	48-49	55-56

[b] - Time moment of the disturbance. [a]- The URE precipitation took a place at 0400 – 1000 UT interval. The values $(A_j)_c$ and $(\varphi_j)_c$ were referred to 0400 UT. The values $(A_j)_c$ and $(\varphi_j)_c$ were referred to 0400 UT. The computed values are given in brackets for comparison with the experimental ones.

Table 2. Results of the inverse VLF problem solutions [b]

Date [a]	21 Jan. 1992	21 Jan. 1992	21 Jan. 1992	21 Jan. 1992
	2250	2250	0740	0740
	70	75	70	75
$\frac{(A_1)_d}{(A_1)_c}$	0.04 ± 0.04 (0.09)	0.04 ± 0.04 (0.14)	0.08 ± 0.04 (0.09)	0.08 ± 0.04 (0.15)
$\varphi_{1c} - \varphi_{1d}$, μs	12 ± 1 (15)	12 ± 1 (13)	13 ± 1 (15)	13 ± 1 (13)
$\frac{(A_2)_d}{(A_2)_c}$	0.10 ± 0.03 (0.07)	0.10 ± 0.03 (0.12)	0.09 ± 0.03 (0.07)	0.09 ± 0.03 (0.13)
$\varphi_{2c} - \varphi_{2d}$, μs	20 ± 1 (18)	20 ± 1 (19)	18 ± 1 (18)	18 ± 1 (19)
$\frac{(A_3)_d}{(A_3)_c}$	0.17 ± 0.07 (0.07)	0.17 ± 0.07 (0.19)	0.11 ± 0.04 (0.07)	0.11 ± 0.04 (0.20)
$\varphi_{3c} - \varphi_{3d}$, μs	23 ± 1 (21)	23 ± 1 (24)	22 ± 1 (21)	22 ± 1 (23)
z_1, km	66	67	66	68
β, 1/km	-0.09	-0.07	-0.09	-0.07
h', km	30	36	30	37

[a] - The time of the disturbance maximum, UT; z_0, km. [b] - The computed values are given in the brackets for comparison with the experimental ones.

Apatity) with length $\sim$ 900 km (in the cases of URE precipitation disturbances) with the help of *monotonous* exponential electron concentration profile $N_e(z)$, it turns out that it is impossible to do it correctly (reliably): the amplitude data demand for themselves an $N_e(z)$ with its effective height at 20 km lower than the effective height of an electron concentration profile suitable for the phase data. It was possible to overpass this contradiction only by an adoption that for satisfying the amplitude and phase data simultaneously it was necessary to use not monotonic *effective* profile N_e, the corresponding profile of electric conductivity having been monotonic (Figure 1). This qualitative discrepancy between the $\sigma(z)$ and $N_e(z)$ profiles is caused by the point that electric conductivity below 50 km is determined by the electrons, positive and negative ions, so $N_e(z)$ is a profile of the **effective electron concentration**.

Now we return to the Table 1 and Table 2 for which a comparison of experimental and theoretical magnitude values is represented. The profile of electron collision frequency (with the atoms of air) was accepted for these calculations as it follows: $\nu_{eff} = 0.87\ 10^7 \exp b(z - 70km)$ 1/s with $b = -0.14$ 1/km . Theoretical values of the field magnitudes (the digits in the brackets) were calculated using the values z_1 and β for the σ-profile of kind number 4, Figure 1. The description of the table parameters and magnitudes is the following:

an index $j = 1, 2, 3$ is a number of frequencies used in the experiment (10.2, 12.1, 13.6 kHz correspondingly);

$(A_j)_c$ – a value of amplitude in undisturbed (calm) conditions; $(A_j)_d$ – a value of disturbed amplitude at UT moment pointed in first lines of the Tables 1 and 2;

$(\varphi_j)_c$ – a value of phase in undisturbed conditions; $(\varphi_j)_d$ – a value of disturbed phase at UT moment pointed in first line.

The pointed magnitudes (without the brackets) are the input measured ones for an inverse problem. The parameter values of the tables (in the brackets) below the input (measured) amplitude and phase data are the output values of the same magnitudes calculated in the frames of inverse problem solution:

z_1 and β; h' – an effective height of a waveguide with a nonmonotonic $N_e(z)$ profile, acquired by using the amplitude and phase data (6 magnitudes); h'' – an effective height of a waveguide with a nonmonotonic $N_e(z)$-profile, gotten by using phase data (3 magnitudes); h''' – an effective height of a waveguide with a nonmonotonic $N_e(z)$-profile, gotten by using phase data (3 magnitudes); h – an effective height of a waveguide, gotten for the same time moments by another VLF inverse problem solution (Remenets & Beloglazov, 1985, 1992); this method is a self-consistent one, that is, it does not need the input geophysical data, and is based on the experimental VLF data and the theory of wave propagation completely.

Concluding the description and discussion of the Table 1 and Table 2 we ought to point out that the effective heights with primes for a given $N_e(z)$-profile were calculated by a special procedure (Galyuk & Ivanov, 1978; Remenets & Beloglazov, 2013; Remenets & Astafiev, 2019): 1-st step – the value of an impedance function is calculated for any altitude at which the electric conductivity is negligible by integration of nonlinear equation for impedance function from z_2 top to down; 2-d step – the gotten value is used as the initial value for integration of the impedance function equation for empty medium in opposite direction until a height h' for which the impedance function becomes real (Galyuk & Ivanov, 1978). Such acquired height is called an effective one due to the statement that the phase path of a propagating mode in the real terrestrial waveguide is the same as in a model air waveguide with h' height. Therefore, the tables represented and described is a part ofelectromagnetic proof of sporadic D_s-layer existence. Now we pass to a new calculation problem.

According to the z_1 values of Table 1 (which is for daytime conditions) it can be seen that these values differ negligibly from z_0. Therefore, we use below an approximation with one parameter – an incremment β for the σ profile, and the equality $z_1 \equiv z_0$ being adopted (Bondarenko & Remenets, 2001). Therefore, we use at present publication one ″knee″ approximation of the D_s-layers, the curves $1 - 3$ for Figure 1.

It is not important in our present estimation of mode conversion either day or polar night features has the top part of a used profile, because the atmosphere pressure and the depth of URE precipitation may be considered not depending on the times of day.

2 Physical and mathematical problems

In the works (Remenets & Astafiev, 2015, 2016) we have determined the southern boundaries of URE precipitations which had been registered (about 300 events for 1982 – 1992 years with 16 powerful ones) by a VLF-method for 10 16 kHz. The method was based on continuous ground-based measurements (by the scientists of the Polar Geophysical Institute – RAS, Apatity, Murmansk reg., Russian Federation) of amplitude and phase disturbances for several VLF-signals for two radio paths: one path S_1 with 10 14 kHz monochromatic signals was completely auroral and the second path S_2 (United Kingdom – Kola Peninsula) with 16 kHz working frequency was partly auroral. The part of the atmosphere which is higher than 61° of magnetic latitude is electrically disturbed during an URE precipitation (UREP). This boundary is higher at several degrees than the analogic boundary for the protons with energy 0.1 – 0.4 GeV, figure 39 of the work (Andriani & et al., 2017). Due to an URE precipitation the profile of electric conductivity is changing below a certain altitude (z_0). Under a regular ionosphere D-layer a sporadic D_s-layer appears due to the bremsstrahlung X-ray radiation which is generated by the precipitating electrons. Therefore, the radio path S_2 becomes significantly inhomogeneous along its length, and an abrupt discontinuity at transverse (relative to the wave guide) surface appears. The breaking of radio path homogeneity we model by an abrupt hop of electric properties at a distance D from a receiver, and the properties for both sides from this boundary are being homogeneous along the path but different. The pointed inhomogeneity of radio path is the cause of conversion of a normal wave of terrestrial wave guide at other normal waves. Having certain quantitative estimations about a significance of this effect on the base of our predecessor calculations for other types of inhomogeneity (such as an abrupt change of electric properties of ground, an abrupt one-step change of waveguide effective height) (Bahar & Wait, 1965; Wait, 1968; Wait & Spies, 1968; Pappert & Morfitt, 1972; Smith, 1977; Osadchy & Remenets, 1979; Pappert & Ferguson, 1986) we ignored the pointed effect in the publications (Remenets & Astafiev, 2015, 2016). Nevertheless, here we have to justify our previous neglect by taking into account the effect of conversion of a normal wave in other ones and to point for what type of URE precipitation (moderate, strong or powerful) it is necessary to take into account this effect while a boundary latitude calculation. For this purpose, we would change the traditional problem statement, but before doing it let us give short comments of the works devoted to the mode conversion in a terrestrial waveguide taking into account that the anisotropic property of low ionosphere is absolutely negligible for us. We analyse here only the URE precipitations, which create additional ionization below 60 km. At the same time, we were induced to use the day events because of the fact, that only at this long radio path S_2 conditions we had *one* normal wave achieving the boundary of discontinuity.

For a day the effective height h of a waveguide is falling down from 60 to 50, 40 and even 30 km during the UREPs. One ought to take into account that this height is always present inside the altitude part of ionosphere which determines a mode reflection to ground. Among the works of pointed type (Bahar & Wait, 1965; Wait, 1968; Wait & Spies, 1968) there are the following ones: a work, in which an abrupt change of top waveguide boundary with given impedance is introduced, the works with the models of the day-night transition in a VLF waveguide, and cylindrical model of the wave guide instead of spherical one being used. These types of models do not suit us because of the following issues:

1) the eigenfunctions of day-side do not orthogonal in the night-side section and vice versa due to the different widths of the left and right sections, therefore this item generates an uncontrolled error;

2) the named eigenfunctions are accurately orthogonal only if the impedance of a top waveguide boundary is constant, that is, the impedance does not depend on the eigenvalue. But the last item works sufficiently good only at high altitudes, where the electric conductivity is sufficiently high. This item works badly if the bottom boundary is chosen for an altitude, where the electric conductivity is low.

3) In the works (Pappert & Morfitt, 1972; Pappert & Ferguson, 1986) the plain waveguide model was used. This type of modelling does not suit us too, because the height gain functions for a plain waveguide and a spherical waveguide are significantly different for relatively high frequencies.

4) In the work (Galeys, 1964) the author used the plain model too taking into account the sphericity effect either by a certain approximation or using a cylindrical waveguide (Galeys, 1968, 1969) according to the work (Wait, 1964).

The pointed mathematical differences are relatively subtle notions, but it is necessary to pay main attention that in all mentioned works the heterogeneities were lateral (sidelong) ones: either an abrupt change of the boundary surface impedance or a change of the height of an empty waveguide (modelling of a transition from day to night). Therefore, it is necessary *in our cases of URE precipitations* to make another statement of the problem about a normal wave conversion *on an abrupt radial change (in spherical system of coordinates) of the medium properties* without artificial and weakly controlled dividing the transverse conducting medium at two following parts: the bottom part which is empty medium (vacuum) and top conducting part which is not involved in the mode conversion calculations directly.

In our case the middle altitudes of the northern part of the waveguide atmosphere are disturbed by an URE precipitation and an appearance of a sporadic D_s-layer. Therefore, we work with a transverse radial inhomogeneity which is homogeneous along a disturbed part of the radio path in our simulation. The problem with such two-dimension inhomogeneity will be solved in this paper. To-day, when the simulation possibilities are much greater than $50-60$ years ago it is not necessary to pass from spherical geometry to the cylindrical or plane ones for simplicity$'$s sake. We have not met any rigorous solution of the problem relative to our one represented here. We do not discuss the cases when the quasi-optic approximations may be used as in a fiber-optics analysis.

Therefore, we are solving a problem about the effect of normal wave conversion due to an abrupt discontinuity of the transverse electric properties of a terrestrial waveguide in the following model. The ideal spherical model of the Earth with homogeneous impedance boundary conditions for its surface and with its radius equal to R = 6370 km is surrounded by two ball sectors of isotropic ionosphere each of which is determined by its profile of electron concentration, either $N_e^I(r)$ or $N_e^{II}(r)$, and a mutual profile of electron collision frequency with neutrals $\nu_{eff}(r)$. These profiles determine the profiles of electric conductivity $\sigma^I(r)$ or $\sigma^{II}(r)$ for the VLF wave propagation. According to the magneto-electric theory of cold isotropic plasma the last pointed profiles are proportional to the ratios $N_e(r)/\nu_{eff}(r)$. The top part of the model (higher than altitude z_0) has spherical symmetry due to our choice $N_e^I(r) = N_e^{II}(r) \sim \exp(\beta r)$ where increment $\beta = 0.411/km$. For bottom part of inner $''$frustrum$''$ the similar equality is the following: $N_e^I(r) \sim \exp(\beta_I r)$ with $\beta = 0.11/km$. The bottom part of external spherical sector has the similar altitude dependence with increment $\beta_{str} = -0.061/km$ for strong URE precipitation and $\beta_{pow} = -0.091/km$ for powerful URE precipitation. Taking into account that increment for $\nu_{eff}(r)$ in our case is $b = -0.14$ one gets the inclines of the semi-strait lines for Figure 1.

The first inner undisturbed cone sector containing on its axis a source of the normal radio waves (a transmitter) is a truncated cone bounded at its bottom by ground. The cone is characterized by a value of angle θ_{irr} in the spherical coordinate system r, θ, φ (with its beginning in the centre of Earth), and by a distance, which is equal to Rθ_{irr}, from a ground based source to the boundary of discontinuity along the ground surface. The second space ball sector (disturbed) is the space characterized by $\theta > \theta_{irr}$.

Our purpose is to determine the reflection coefficient and conversion coefficients for a given normal wave penetrating through the boundary of abrupt medium changing at $\theta = \theta_{irr}$. In our case it is fist vertically polarized normal wave (TM_0) generated by a vertical electric dipole. The inner cone is characterized by undisturbed auroral ionosphere, and the outer sector is characterized by the auroral ionosphere disturbed by an URE precipitation at the altitudes below a certain altitude value z_0.

The waveguide properties are determined by a profile of electric conductivity which is increasing exponentially at radial infinity: $\sigma(x) = Im(\varepsilon'(x))\omega$,

$k = \omega\sqrt{\varepsilon'\mu_0} = \omega\sqrt{\varepsilon'_m\varepsilon_0\mu_0} = k_0\sqrt{\varepsilon'_m}$,

$\varepsilon'_m(x) = 1 - \frac{X(x)}{1+jZ(x)}$, $X(x) = \frac{\omega_p^2(x)}{\omega^2}$, $Z(x) = \frac{\nu_{eff}(x)}{\omega}$, where $j = \sqrt{-1}$, $\omega_p(x)$ - the plasma frequency, $x = k_0 r$ is the dimensionless radial coordinate; k_0 is a wave number for free space; $\varepsilon' = \varepsilon'_m \cdot \varepsilon_0$.

The electric conductivity is approximated by two exponential functions for which its logarithm is a function with an $'$knee$'$ at the altitude $z_0 = r_0-$R, (Beloglazov & Zabavina, 1982a, 1982b). This approximation corresponds to the auroral undisturbed or moderately disturbed low ionosphere, Figure 1 with the curve 1 for $\sigma^I(z)$, where z is an altitude. At a certain distance from the transmitter (S_2- D, km; $D = R\theta_{irr}$) a new profile of effective electric conductivity which models a sporadic D_s-layer (for a certain time of disturbance) with the help of significantly other elbow function: either $\sigma^{II}_{str}(z)$ – curve 2 or $\sigma^{II}_{pow}(z)$ – curve 3 (Bondarenko & Remenets, 2001), see Figure 1. Due to the pointed models of conductivity profiles the normal waves exist and penetrate until altitude z_2 at which the impedance boundary condition is used. Therefore, we come to a problem of mode conversion at a transverse boundary of two spherical jointed waveguides with equal width z_2, inhomogeneous radially inside and with sufficiently accurate impedance boundary conditions at the bottom $r = R$ and top $r = R + z_2$ waveguide boundaries.

This statement of the simulation problem may seem to be far away from the real situation because: (i) In reality there is a second axis of symmetry connected with the geomagnetic field, which determines a circular zone of very energetic solar proton (100 MeV) (Dmitriev & et al., 2010) and URE precipitations (Remenets & Astafiev, 2015); therefore, a normal wave propagating from England to Kola Peninsula is falling on the boundary of irregularity not normally, see figure 1 in the work (Remenets & Astafiev, 2016). (ii) The real radius of boundary curvature is centred at South magnetic pole, the centre of model cone boundary is placed in England and its radius on earth has value $R\theta_{irr} \simeq 2000$ km.

The first item we ignore because our purpose is to get an estimate of the effect. The second item is insignificant for the problem because according to the Maxwell equations the propagation of electromagnetic waves is characterized by a local principle, which is formulated with the help of the Fresnel zones. They are plotted on the base of the transmitter and receiver points in space which are the focuses of the ellipsoids enveloping the points. The width of a zone is about square root of a product $S_2\lambda \simeq 200$ km, where the first multiplier is the distance between the source in England and the receiver on the Kola Peninsula ($S_2 \simeq 2500$ km), and λ is a wavelength for 16 kHz.

New computational problem is as follows. In the first cone section of the spherical model a TM_0 normal wave with a given complex amplitude is propagating from the transmitter to the waveguide joint which is described above. It is necessary to find the complex amplitudes of the followings: of a normal wave TM_0 penetrating and reflecting, of other normal waves generated by the junction boundary in both cone sections. The last normal waves are propagation from the boundary to the transmitter at first section and are propagating from the boundary to the receiver at the second section.

In order to calculate the pointed amplitudes, it is necessary to demand the continuity of complex E_r and H_φ components at the cone boundary. The radial parts of normal waves of a fixed cone section are orthogonal to each other in the complex Hilbert space and this property is sufficient for answering the main question of the investigation: is it necessary to take into consideration the conversion of the normal waves when one determines the southern boundary of URE precipitation?

3 Components of electromagnetic waves in the radially inhomogeneous

This section is not original one, but it is a setup text relative to the next analytical original section and corresponding definitions.

The complex amplitudes $\overrightarrow{E}$ and $\overrightarrow{H}$ of the electromagnetic field in the radial inhomogeneous, electrically conducting medium with the central symmetry, satisfy to the system of Maxwell equations. The time dependence of a source signal and the fields is excepted in the form $exp(-j\omega t)$, where j is an image unit:

$$rot\overrightarrow{E} = j\omega\mu_0\overrightarrow{H}, \tag{1}$$

$$rot\overrightarrow{H} = -j\omega\varepsilon'\overrightarrow{E}, \tag{2}$$

where $\varepsilon'(r) = \varepsilon(r) + j(\frac{\sigma(r)}{\omega})$.

The fields $\overrightarrow{E}$ and $\overrightarrow{H}$ are expressed due to the Hertz vector $\overrightarrow{\Pi}$:

$$\overrightarrow{H} = -j\omega rot\overrightarrow{\Pi}, \tag{3}$$

$$\overrightarrow{E} = \frac{1}{\varepsilon'}rot(rot\overrightarrow{\Pi}), \tag{4}$$

if the vector $\overrightarrow{\Pi}$ satisfies to the equation

$$rot\{\frac{1}{\varepsilon'}rot(rot\overrightarrow{\Pi})\} - \omega^2\mu_0 rot\overrightarrow{\Pi} = 0, \omega^2\mu_0 rot\overrightarrow{\Pi} = 0, \tag{5}$$

The last equation may be transformed to the following:

$$rot(rot\overrightarrow{\Pi}) - k^2\overrightarrow{\Pi} - k^2\Phi = 0, \tag{6}$$

where $k(r) = \omega\sqrt{\epsilon'(r)\mu_0} = k_0\sqrt{\varepsilon'_m(r)}$; $\varepsilon'_m(r) = \varepsilon'(r)/\varepsilon_0$, and Φ is an any smooth function. The magnitude $\varepsilon'_m(r)$ is characterized here by the central symmetry. In this case the Herz vector has only one component Π_r which satisfies to the following equation

$$\frac{1}{r^2\sin\theta}[\frac{d}{d\theta}(\sin\theta\frac{d\Pi_r}{d\theta})] + \varepsilon'_m\frac{d}{dr}(\frac{1}{\varepsilon'_m}\frac{d\Pi_r}{dr}) + \varepsilon'_m k_0^2\Pi_r = 0, \tag{7}$$

and the transverse components of the electromagnetic field are expressed by the following relations:

$$H_\varphi = \frac{j\omega}{r}\frac{d\Pi_r}{d\theta}, \tag{8}$$

$$E_r = \frac{1}{\varepsilon'}(\varepsilon'_m\frac{d}{dr}(\frac{1}{\varepsilon'_m}\frac{d\Pi_r}{dr}) + \varepsilon'_m k_0^2\Pi_r). \tag{9}$$

Correspondingly, the component E_r is expressed with the help of the H_φ component:

$$E_r = \frac{i}{\varepsilon' r\omega \sin\theta} \cdot \frac{\partial(\sin\theta \cdot H_\varphi)}{\partial\theta}. \tag{10}$$

According to the Maxwell equations (1) and (2) the equation for the H_φ component is gotten:

$$rot[\frac{1}{\varepsilon'} rot H_\varphi] = k_0^2 \varepsilon'_m H_\varphi, \tag{11}$$

that is,

$$\frac{1}{r^2} \cdot \frac{d}{d\theta}(\frac{1}{\sin\theta} \cdot \frac{d(\sin\theta \cdot H_\varphi)}{d\theta}) + \frac{\varepsilon'_m}{r} \frac{d}{dr}(\frac{1}{\varepsilon'_m} \frac{d(r \cdot H_\varphi)}{dr}) + \varepsilon'_m k_0^2 H_\varphi = 0. \tag{12}$$

The equation (7) may be represented as a sum of the angular and radial differential operations relative to the $\Pi_r(r,\theta)$, $L_\theta \Pi_r + L_r \Pi_r = 0$, where

$$L_\theta = \frac{1}{\sin\theta}[\frac{d}{d\theta}(\sin\theta \frac{d}{d\theta})], \tag{13}$$

$$L_r = r^2 \varepsilon'_m \frac{d}{dr}(\frac{1}{\varepsilon'_m} \frac{d}{dr}) + r^2 \varepsilon'_m k_0^2. \tag{14}$$

The eigenvalues λ_n of the radial operator L_r, which is defined on the set of functions attenuating in the conducting plasma medium against the altitude and satisfying to the impedance boundary conditions on the ground surface. These values are defined by the following equality:

$$L_r \Pi_r = \lambda_n \Pi_r,$$

and, as it is $\Pi_r(r,\theta) = U(r) \cdot P(\theta)$, then:

$$L_r U(r) = \lambda_n U(r). \tag{15}$$

The last equation is transformed into the differential Riccati equation (17) of first degree, if to introduce the impedance function:

$$u(r) = \frac{\frac{dU(r)}{dr}}{\varepsilon'_m(r) \cdot U(r)}. \tag{16}$$

Then

$$\frac{du(r)}{dr} + \varepsilon'_m(r) \cdot u(r)^2 = -k_0^2 + \frac{\lambda}{\varepsilon'_m(r) \cdot r^2}. \tag{17}$$

An eigenvalue λ_n is connected with the index of cylindrical functions in the cases of homogeneous medium with $\varepsilon_m = 1$ by the following relation: $\lambda_n = \nu_n^2 - 1/4$. The same parameter ν_n defines the asymptotic angular dependence of a normal wave field as follows: $P(\theta) \sim \exp(j\nu_n\theta)$.

Inclusion of the computer integration of this Riccati equation (17) from a top waveguide boundary (z_2) to the ground (for which the impedance boundary condition is given to us) into an iteration process relative to the parameter ν produces an eigenvalue ν_n and corresponding eigenfunction $U(r,\nu_n)$ according to the (15). According to the (9) the following relation has place for an eigenfunction with number n:

$$Er,n = \frac{1}{\varepsilon'_m(r) \cdot r^2} L_r \Pi_n(r,\theta) \simeq \frac{1}{\varepsilon'_m(r) \cdot r^2} \nu_n^2 \Pi_n(r,\theta)$$

$$=$$

$$\frac{1}{\varepsilon_m'(r)\cdot r^2}\nu_n^2 U_n(r)P_n(\theta).(18)$$

According to the same relation (17) and to the asymptotic relation $P_n(\theta)\sim \exp(j\nu_n\theta)$ one gets the value of the singular magnetic component of the TM_n normal wave with number n:

$$H_{\varphi,n}=\frac{j\omega}{r}\cdot\frac{d\Pi_{r,n}(r,\theta)}{d\theta}=\frac{j\omega}{r}\cdot U_n(r)\cdot\frac{dP_n(\theta)}{d\theta}=-\nu_n\frac{\omega}{r}\cdot U_n(r)\cdot P_n(\theta). \tag{19}$$

4 The system of equations generated by the demand of continuity of the transverse components of the TM electromagnetic field on the cone boundary of two mediums with different radial properties

According to the electromagnetic law the transverse components E_r (18) and H_φ (19) must be continuous on the boundary surface of two different mediums, that is on the truncated cone with $\theta_{irr}=(S_2-D)/R$ in our case. In the following we shall consider the conversion of one normal wave, n= 1, into other ones. This normal wave (which according to (Makarov & et al., 1993) is TM_0 normal wave) with a given amplitude is propagating from the cone with index I into to the outer medium with index II. The radial function $U_1(r)$ of its complex amplitude E_r is normalized to value 1 on spherical ground surface boundary ($x=k_0R$, R is the radius of Earth) inside the cone. In this cone the electromagnetic field is the sum of the falling wave (E_1,H_1), reflected wave (R_1E_{-1}, R_1H_{-1}) and the sums of excited normal waves with numbers $n>1$: $\sum_2^M \mathrm{R}_nE_{-n}$, $\sum_2^M \mathrm{R}_nH_{-n}$. Then we have:

$\mathrm{E}_I=E_1+\sum_{n=2}^M R_nE_{-n}$,

$$H_I=H_1+\sum_{n=2}^{M}R_nH_{-n}. \tag{20}$$

In the outer medium with number (II) the field is the sum of the excited normal waves which are propagating from the boundary:

$\mathrm{E}_{II}=\sum_{n=1}^M T_nE_n$,

$$H_{II}=\sum_{n=1}^{M}T_nH_n, \tag{21}$$

where the component indexes r and φ are omitted, and T_n are the complex amplitudes of the passed wave with number $n=1$ and the excited ones. Demanding the continuity of the full field components on the transverse boundary and taking into consideration that ν_n for a wave propagating in positive direction differs from the wave propagating in opposite direction by a sign ($\nu_{-n}=-\nu_n$) we have the following relations for the radial eigenfunctions and the conversion coefficients on the cone boundary:

$$\frac{{\nu_1^I}^2}{\varepsilon_{m,I}(x)}U_1^I(x)+\sum_{n=1}^{M}R_n\frac{{\nu_n^I}^2}{\varepsilon_{m,I}(x)}U_n^I(x)=\sum_{n=1}^{M}T_n\frac{{\nu_n^{II}}^2}{\varepsilon_{m,II}(x)}U_n^{II}(x), \tag{22}$$

$$\nu_1^I\cdot U_1^I(x)-\sum_{n=1}^{M}\nu_n^I\cdot R_nU_n^I(x)=\sum_{n=1}^{M}\nu_n^{II}\cdot T_nU_n^{II}(x), \tag{23}$$

where the integer M is used instead of infinite value. For a value choice $M=3$ the equality (23) for full H_φ component and the equality (22) for full E_r component may be rewritten in the form (24) and (25) correspondingly:

Table 3. The values of complex reflection and transition coefficients R_j and T_j for two models of inhomogeneity junction: with the conductivity profiles $\sigma^{II}_{str}(z)$ and $\sigma^{II}_{pow}(z)$, Figure 1.[c]

Sporadic D_s layer	$\sigma^{II}_{str}(z)$	$\sigma^{II}_{pow}(z)$
R_1	- 0.0005 + j0.0008	- 0.001 + j0.003
T_1	+ 0.9223 + j0.1095	+ 0.839 + j0.188
R_2	+ 0.0010 - j0.0013	+ 0.004 - j0.005
T_2	+ 0.0872 - j0.1216	+ 0.179 - j0.174
R_3	- 0.0004 + j0.0006	- 0.001 + j0.001
T_3	- 0.0168 - j0.0150	- 0.019 - j0.006

[c] The undisturbed conductivity profile was the $\sigma^I(z)$ one, Figure 1.

$$\nu_1^I \left[U_1^I(x) - R_1\, U_1^I(x) - \frac{\nu_2^I}{\nu_1^I}\, R_2\, U_2^I(x) - \frac{\nu_3^I}{\nu_1^I}\, R_3\, U_3^I(x)\right] =$$
$$\nu_1^{II} \left[T_1\, U_1^{II}(x) + \frac{\nu_2^{II}}{\nu_1^{II}}\, T_2\, U_2^{II}(x) + \frac{\nu_3^{II}}{\nu_1^{II}}\, T_3\, U_3^{II}(x)\right], \quad (24)$$

5 Determination of the mode conversion coefficients

By multiplying consequently these last equalities by the eigenfunctions of the not self-adjoint operator of conjugate boundary problem for 2-nd section and by calculating the scalar products according their definition for a not self-adjoint operator (Titchmarsh, 1946; Keldish, 1951; Wait, 1968; Wait & Spies, 1968; ?, ?; Krasnushkin & Baibulatov, 1968; ?, ?; Pappert & Smith, 1972) $< U_n * U_m > \equiv \int_{x=k_0R}^{x=k_0(R+z_2)} U_n(x)U_m(x)dx$, which is not equal to zero for $n = m$, one gets the following 6 algebraic relations corresponding to 3 normal waves, used in our numerical calculations, $n = 1, 2, 3$:

$$\frac{1}{\varepsilon_m^I(x)} \left[(\nu_1^I)^2\, U_1^I(x) + R_1\, (\nu_1^I)^2\, U_1^I(x) + R_2\, (\nu_2^I)^2\, U_2^I(x) + R_3\, (\nu_3^I)^2\, U_3^I(x)\right] =$$
$$\frac{1}{\varepsilon_m^{II}(x)} \left[T_1\, (\nu_1^{II})^2\, U_1^{II}(x) + T_2\, (\nu_2^{II})^2\, U_2^{II}(x) + T_3\, (\nu_3^{II})^2\, U_3^{II}(x)\right], \quad (25)$$

where i is a number of the $U_i^{II}(x)$ eigenfunction used for multiplication, and the N_i^{II} is its norm. We used the following normalization: $U_i^{I,II}(r = R) = 1$.

According to these 6 equations three reflection coefficients R_n^I and three transition coefficients T_n^{II} were determined for abrupt transition in the waveguide from its part with $\varepsilon_m^I(x)$ to the part with $\varepsilon_m^{II}(x)$ for which the functions $\sigma^I(z)$, $(\sigma^{II}_{str}(z))$ and $(\sigma^{II}_{pow}(z))$ of Figure 1 correspond. These coefficients are represented for Table 3.

The data of last column (for the powerful disturbance (PwD)) for this table were used for getting the relative comparison of the altitude distributions of the terms (of their real and image parts) of the sum at (24), and this comparison is represented for

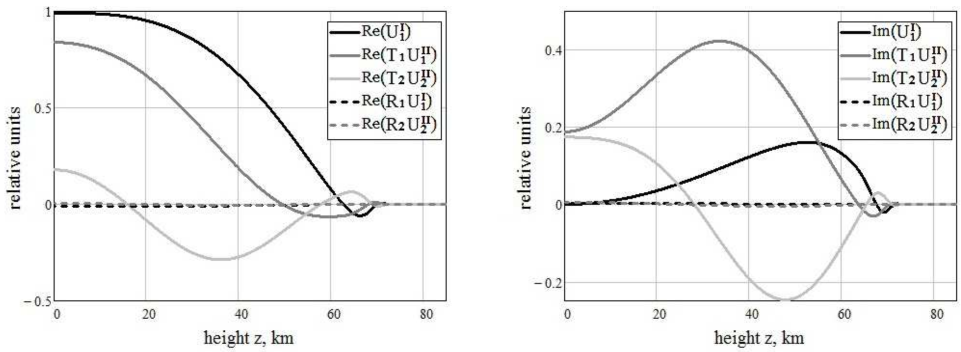

Figure 2. Comparison of the altitude distributions for the complex amplitudes of the converted normal waves in the waveguide by the longitudinal D_s heterogeneity. The source of excitation is a normal wave TM_0 ($U_1^I(x)$ normalized to 1 at $x = k_0R$) propagating to the cone boundary. The left and right parts of the panel are the real and imaginary parts of the magnitudes.

Figure 2. This comparison is sufficiently correct due to the fact that in our frequency case the first eigenvalues ν_n differ from each other at 4-th – 3-d digits.

Therefore, the Figure 2 gives the relative comparison of the complex amplitude against altitude of the TM_0 normal wave falling on two-dimensional inhomogeneity (with the variables r and θ) caused by an URE precipitation

(i) with the amplitudes of the TM_0 transmitted $T_1U_1^{II}$ and reflected $R_1U_1^I$,

(ii) with the TM_1 generated by the boundary of inhomogeneity with one of its $T_2U_2^{II}$ part propagating to a receiver and the other $R_2U_2^I$ one of it propagating to the VLF source and

(iii) with the TM_2 normal wave generated by the inhomogeneity with its $T_3U_3^{II}$ part propagating to a receiver and the other its part $R_3U_3^I$ propagating to the VLF source.

The Table 3 and Figure 2 demonstrate at the same time that (i) the reflection effect for the TM_0 normal wave is negligible and that (ii) the conversion effect of TM_0 normal wave into the TM_2 normal wave (with number $n = 3$) is negligible too relative to the errors of measurements, and, consequently, our analysis usage of only 3 normal waves is justified. The (i) result does not contradict with analog result of our author predecessors, when they analysed the mode conversion effect for the day-night transition at the VLF radio paths.

6 Discussion

Before to continue the discussion of our results we have to remind that our definitions of powerful, strong and moderate disturbances are based at the depth of an *abnormal* interference minimums (against a disturbance) for the signal amplitudes ($10 \div 14$ kHz), for the distance ~ 900 km from a transmitter to a receiver and for a completely an auroral radio path (by Beloglazov, M. I. and Shishaev V. A., PGI KSC AC, Murmansk reg., Apatity, Russian Federation). For undisturbed day time conditions at middle latitudes the normal interference amplitude minimum is equal to ~ 600 km (16 kHz; Hollingworth, 1924). The interference abnormality took place due to (i) the sphericity of Earth and due to (ii) abnormally low altitude values of the bottom layer of electric conductivity. The last abnormality appears due to the URE precipitations. The sphericity of Earth excludes the strait ray of seeing from a transmitter at a receiver point. Instead of it the Watson-Fock diffraction wave works. The phase velocity of the last is less than the light velocity in free space.

This publication, as it seems to us, is a last one of our serious works (longer than 35 years) devoted to investigation of new geophysical phenomenon caused by the URE precipitations and having the Earth′s polar scale. We have theoretically analysed all what was possible to do with the indirect electromagnetic input VLF data including the estimation of the mode conversion on the boundary having been created by URE precipitation. This boundary is significantly more abrupt than day to night transition and exists significantly lower ($20 \div 60$ km) than this transition. Every new phenomenon must be investigated from all sides. One of the side investigated here has shown that at the maximum of a powerful VLF disturbance an effect of main normal wave TM_0 conversion ($T_1 \cdot U_1^{II}(z)$) for its amplitude **on ground** (z = 0) is $\approx 15\%$ and for its phase is ≈ 0.2 *rad.* The last value at 3 times greater than the phase measurement error in our case ($1\mu s \sim 0.06$ *rad.* for our working frequency 16 kHz). For a strong VLF disturbance (Table 3) the main TM_0 normal wave amplitude decrees at 0.9 times and the phase changes at 1.2 μs, id est., at 0.07 *rad.* The same changes of complex magnitude T_1 are comparable with the measurement errors of the experimental data used by us. Therefore, we have justified the correctness of our digital determination of the magnetic latitude of equatorward boundary of URE precipitation (Remenets & Astafiev, 2015), in which we have not used the powerful disturbances.

It ought to be noted that as the determined boundary of URE precipitations is an analysis product of a full disturbance (which is a function of time), then the mode conversion effect for the maximum of a disturbance cannot be extrapolated for all moments of the disturbance.

It seems that another normal wave TM_1 with $T_2 \cdot U_2^{II}(x)$ ought to be considered in analysis, but it is not so. The difference between the image parts of the eigenvalues for first two normal waves in the disturbed section (that is, the decrements for them) is so great that the second one which is excited by the first normal wave TM_0 at the inhomogeneity boundary does not change the value of the radio wave at a receiver point.

In addition to the pointed items it may be considered that in reality there is none abrupt change of the boundary properties. In reality a relatively smooth change with the space scale of electric property changes about one degree of latitude exists. Therefore, the represented values of the mode conversion are the above estimations for the real ones, and now we state more reliably the following:

(i) existence of a sporadic D_s- layer of electric conductivity appearing under the regular ionosphere D-layer has an electromagnetic proof;

(ii) that in our procedure of southern boundary determination in which only strong and moderate disturbances (StD′s and MdD′s) caused by the URE precipitations were used, the effect of normal wave conversion is negligible. In the cases of powerful disturbances, it is necessary to be careful in analysis.

At the same time, we may state that in the cases of the most powerful proton precipitations (such as on 16 February 1984 and 29 September 1989) for which the effective height fell down to 50 (Remenets & Beloglazov, 1992; Remenets & et al., 2020) and 45 km (Remenets & et al., 1989), one may expect similar qualitative results. But they should be quantitatively weaker, may be significantly, due to the absence of bremsstrahlung X-rays and the corresponding sporadic D$_s$-layer of the electric conductivity.

In addition to the above quantitative results important for the effects connecting with the ultraenergetic relativistic electron ($\sim$ 100 MeV) precipitations we have demonstrated in present paper an efficiency of new stating of a problem about mode (normal wave) conversion in a spherical waveguide and obtaining its solution. Such type of problems exists more than 50 years but only now it became possible to get the solution of the problem in more natural for spherical Earth model than the models used by our predecessors. In our problem statement the space above ground is not divided empirically on the air bottom part and on electrically active top part. After such dismemberment the previous authors solved the problem of mode conversion from one air part of waveguide to another air part with different effective hights, the input into conversion of the upper electrically active, different parts being ignored. The difference of the heights reduces to the uncontrolled errors when we use the orthogonal property of the eigenfunctions for different parts of a waveguide. In our case there are none such errors. We considered a mode transition from one transversely inhomogeneous medium to another transversely inhomogeneous medium with the corresponding quantitative results. We consider that our suggested and used approach to the mode conversion will be useful in the waveguides with natural or artificial transverse inhomogeneity the size of which is comparable with the effective height of a waveguide. Such situation appears not only in the cases of ultraenergetic relativistic electrons and protons comming from the Sun, in the cases of sudden ionospheric disturbances (due to the Sun X-ray flares), in the sinrise effect at radio path (because the detachment of electrons from the gaz molecules is passing much faster than the process of their attachment at sunset) but in the astrophysics cases too (Tanaka & et al., 2008), (Tanaka & et al., 2010). Very short (much less than one second) hard γ ray bursts from a certain space point illuminate the half of Earths atmosphere, create the corresponding D_s layer of electric conductivity in the middle and low atmosphere. The problem of transverse two-dimensional inhomogeneous boundary with the global scale appears too.

Relative success of our analysis is due to the fact that the spectra of radial (transvers) not self-adjoint operator for the problem of electromagnetic wave diffraction at a sphere is discrete (Fock, 1965).

We once more attract attention of the readers, that the ultraenergetic relativistic electrons being the cause of present investigation are the electron-killers with energy ~ 100 MeV which might be be much more dangerous for the telecommunication systems and electronics than the traditional relativistic electrons with energy $1 \div 10$ MeV.

Acknowledgments

As we explained in the Introduction, the present investigation is based on our previous solutions of inverse VLF problem. These solutions are based on the experimental data of the Polar Geophysical Institute, Apatity, Murmansk reg., Russian Federation, which are represented for figure 2 of (Remenets & Beloglazov, 1985); for table 2 of (Remenets, 1997); for figures 1 and 2 of (Remenets & Beloglazov, 1999); for figures 3, 4 of (Remenets & Beloglazov, 1992); for pages 240-261 of Application 6 to (Remenets, 2005); for figures 3, 4 and table 4 for (Beloglazov & Remenets, 2005); for figures 3 – 5 and tables 1, 2 of (Remenets & Beloglazov, 2013); for the figure and tables of (Remenets & Beloglazov, 2015); for the tables of publication (Remenets & Shishaev, 2019); for the tables of the work (Remenets & et al., 2020).

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