

# Errors in Positioning of Borehole Measurements and How They Influence Seismic Inversion

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## Abstract

Inversion of seismic data using information from horizontal wells is often hampered by cumulative well- location errors. These errors can be significant and propagate to the well-log measurements in the borehole. To achieve a proper data integration and arrive at correct uncertainty estimates, we formulate the problem in a fully probabilistic framework and present a numerical approach for improving subsurface imaging using uncertain well-log data and their uncertain locations, as well as uncertain seismic data. The result is improved model error quantification in the seismic inversion process.

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## ABSTRACT

Inversion of seismic data using information from horizontal wells is often hampered by cumulative well-location errors. These errors can be significant and propagate to the well-log measurements in the borehole. To achieve a proper data integration and arrive at correct uncertainty estimates, we formulate the problem in a fully probabilistic framework and present a numerical approach for improving subsurface imaging using uncertain well-log data and their uncertain locations, as well as uncertain seismic data. The result is improved model error quantification in the seismic inversion process.

**Keywords:** subsurface imaging, seismic inversion, uncertainty quantification, natural resources, computational modelling, applied geophysics.

## INTRODUCTION

Quantifying uncertainties in subsurface imaging generated from theoretical models, geological analogs, or field measurements, possibly in areas of difficult access, offers great challenges. Geophysical error quantification methods are well-known and described in, e.g., Tarantola (2005), but in application of these methods, measurement location data is mostly considered to be free from errors. In most cases the assumption of having precise location of measured data is enough to create a convenient and approximate model, but in certain cases where we need to rely on remote interpretations of indirect measurements and at different scales, errors in the location coordinates of the acquired data may become significant. Hence, depending on the problem, location errors may appear as hidden measurement errors, having consequences for our interpretations, ranging from mild to severe.

Obtaining precise locations data is in many cases difficult and, in some cases, impossible. This can be seen in a wide range of different applications where physical access is difficult, such as drilling

operations, satellites positioning and planetary exploration.

Uncertainty quantification in connection with imaging of the subsurface has long been a topic of study due to its importance in how accurate and reliable our models can be. Computational methods for probabilistic seismic inversion using linearized methods (*Buland and Omre, 2003; Hansen et al, 2006*) or Markov chain Monte Carlo (MCMC) methods (*Mosegaard and Tarantola, 1995; Sambridge and Mosegaard, 2002*) have been used, such as in studies carried out by *Dehan Zhu and Richard Gibson (2018)* and *Georgia K. Stuart, Susan E. Minkoff, and Felipe Pereira (2019)*. In such studies, there is a need of keeping several parameters of the problem fixed to reduce the computation cost. Location parameters are often amongst the fixed numbers, and, as we shall see, this may have important consequences for the outcome of the analysis.

In a study carried out by *Winkler (2017)*, the uncertainties in wellbore locations were incorporated and he formulated a probabilistic inverse problem using Bayesian networks. Other

studies also developed methods of assessing the uncertainties in resistivity well-logs and trajectory of the wells such as *Kullawan et al. (2014)*. In *Eidsvik and Hokstad (2006)*, seismic data were used in the form of VSP travel times to estimate the well positions, the earth model and the seismic velocities.

Unfortunately, joint uncertainty analysis of seismic data, wellbore data and position data is still a poorly developed field. Following the numerical approach presented by *Fernandes and Mosegaard (2021)* where the uncertainty in well locations were cumulatively propagated throughout the trajectory of the well, we present an integrated seismic inversion formulation. In this fully probabilistic study, the approach of *Tarantola and Valette (1982)* and *Tarantola (2005)* is used in a case where positions of the physical data are also accounted for as data. The model parameters to be determined are subsurface acoustic impedances and well trajectory coordinates at which seismic data are constrained to well-log data.

In our formulation, we consider errors in the seismic data, well data and wellbore locations. The cumulated errors in the positions on the well trace are considered 3-dimensional, i.e., have three spatial coordinates – depth and 2D horizontal location. The challenge in this method is to correctly integrate uncertainties of the seismic data and all other (dependent) sources of data. Mislocation of a well can interfere with the seismic inversion and introduces errors in subsequent subsurface interpretations. In our study, simulations of the variability of subsurface structure were generated, from which uncertainty of the model could be calculated.

This paper presents a complete approach for seismic inversion following up from the study of *Fernandes and Mosegaard (2021)* on uncertain spatial location data. Using a Monte Carlo formulation to compute model realizations and to assess their variability (uncertainty) we aim at providing an improved basis for more realistic interpretations of geological structures.

## THEORETICAL FORMULATION

Our mathematical formulation is rooted in a probabilistic approach (*Bayes, 1763*) and based on the formalism of *Tarantola and Valette (1982)*, adapted to the combined geosteering and seismic inverse problem. For unknown elastic- and well-position parameters  $\mathbf{x}$  the solution to the inverse problem is given by the posterior probability distribution (we ignore normalization constants here and in the following):

$$\sigma(\mathbf{x}) = \rho_x(\mathbf{x})L_b(\mathbf{x})$$

where  $\rho_x(\mathbf{x})$  is the prior probability distribution of the unknowns  $\mathbf{x}$ , and  $L_d(\mathbf{x})$  is a likelihood function measuring the fit between the combination of observed seismic- and well-position data  $\mathbf{b}_{\text{obs}}$  and the corresponding computed data  $\mathbf{b}(\mathbf{x})$ . Our forward relation is:

$$\begin{pmatrix} \mathbf{d} \\ \mathbf{c} \end{pmatrix} \equiv \mathbf{b} = g_x(\mathbf{x}) \equiv g_{m,r}(\mathbf{m}, \mathbf{r}) = \begin{pmatrix} g_m(\mathbf{m}) \\ g_r(\mathbf{r}) \end{pmatrix} \\ = \begin{pmatrix} \mathbf{d}(\mathbf{m}) \\ \mathbf{c}(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{d}(\mathbf{m}) \\ \mathbf{r} \end{pmatrix}$$

where  $\mathbf{d}(\mathbf{m})$  is the seismic data,  $\mathbf{c}$  is the observed well location coordinates (positions),  $\mathbf{m}$  is the physical subsurface parameters (in our case acoustic impedances), and  $\mathbf{r}$  are the true well positions.  $g_{m,r}$  is the function that maps true parameters into observed data.

Our likelihood function can be expressed (*Tarantola and Valette, 1982*):

$$L_{d,c}(\mathbf{m}, \mathbf{r}) = \rho_{d,c}(g_{m,r}(\mathbf{m}, \mathbf{r})) .$$

where  $\rho_{d,c}$  is the joint prior/noise distribution of our seismic data and well position data. Since the noise on the seismic data and the well-position data are statistically independent, we have

$$\rho_{d,c}(\mathbf{d}, \mathbf{c}) = \rho_d(\mathbf{d})\rho_c(\mathbf{c})$$

and this gives us the following expression for the joint posterior:

$$\begin{aligned} \sigma_{m,r}(\mathbf{m}, \mathbf{r}) &= \rho_{m,r}(\mathbf{m}, \mathbf{r})L_{d,c}(\mathbf{m}, \mathbf{r}) \\ &= \rho_{m,r}(\mathbf{m}, \mathbf{r})L_d(\mathbf{m})L_c(\mathbf{r}) . \end{aligned}$$

Considering that the prior on the subsurface parameters are only available *conditioned* on the well position, namely as  $\rho_{m|r}(\mathbf{m}|\mathbf{r})$ , we write  $\rho_{m,r}(\mathbf{m}, \mathbf{r}) = \rho_{m|r}(\mathbf{m}|\mathbf{r})\rho_r(\mathbf{r})$ , and get:

$$\sigma_{m,r}(\mathbf{m}, \mathbf{r}) = L_d(\mathbf{m})L_c(\mathbf{r})\rho_{m|r}(\mathbf{m}|\mathbf{r})\rho_r(\mathbf{r}).$$

### The Subsurface Prior

Our conditional prior on the model parameters  $\rho_{m|r}(\mathbf{m}|\mathbf{r})$  is given as follows: We use Cartesian coordinates  $(x, y, z)$  to describe positions in the subsurface, and our acoustic impedance is represented by a positive real function  $m(x, y, z)$  over the space. We now chose a family of orthonormal base functions (kernels)  $\varphi_1(x, y, z)$ ,  $\varphi_3(x, y, z)$ , ... and parameters  $m_1, m_2, m_3, \dots$  such that  $m(x, y, z)$  can be approximated as

$$m(x, y, z) \approx \sum_{n=1}^M m_n \varphi_n(x, y, z).$$

for sufficiently large  $M$ . The coefficients  $m_n$  are our model parameters, the components of  $\mathbf{m}$ . We now define  $\rho_{m|r}(\mathbf{m}|\mathbf{r})$  as a Gaussian over the  $\mathbf{m}$ -space with zero mean and covariance  $\mathbf{C}_\rho$ , conditioned on the linear subspace

$$\mathcal{W} = \left\{ \mathbf{m} \left| \sum_{n=1}^M m_n \varphi_n(\mathbf{r}_k) \right. \right. \\ \left. \left. \begin{aligned} &= m(\mathbf{r}_k) \text{ for } k \\ &= 1, \dots, K \end{aligned} \right. \right\} \quad (1)$$

where  $\mathbf{r}_k$  is the position of the  $k$ 'th the well point. This means that realizations of  $\rho_{m|r}(\mathbf{m}|\mathbf{r})$  are weighted sums of our kernels, all fitting the given values at the well points. Details of, how to compute realizations of  $\rho_{m|r}(\mathbf{m}|\mathbf{r})$ , can be found in *Appendix I*.

### The Well Location Prior

In this study we assume that the prior information about the well positions  $\rho_r(\mathbf{r})$  is uniform (constant),

and hence that all information about the well trace is obtained from drilling data and their estimated position uncertainties. All this information is given by  $L_c(\mathbf{r})$ . This leads to

$$\sigma_{m,r}(\mathbf{m}, \mathbf{r}) = L_d(\mathbf{m})L_c(\mathbf{r})\rho_{m|r}(\mathbf{m}|\mathbf{r}).$$

### The Seismic Likelihood

The Seismic Likelihood Function is given by:

$$L_d(\mathbf{m}) = \rho_d(g_m(\mathbf{m}))$$

and in this study, we assume that the seismic noise is Gaussian, leading to:

$$L_d(\mathbf{m}) = K \cdot \exp \left( -\frac{1}{2} (\mathbf{d}_{obs} - g_m(\mathbf{m}))^T \mathbf{C}_n^{-1} (\mathbf{d}_{obs} - g_m(\mathbf{m})) \right)$$

where  $\mathbf{C}_n$  is the covariance matrix of the seismic noise, and  $K$  is a normalization constant.

### The Well Location Likelihood

Following *Fernandes and Mosegaard (2021)*, if the well position measurements are  $\mathbf{c} = (\mathbf{c}_1 \dots, \mathbf{c}_K)$ , where  $\mathbf{c}_i$  is the location of the  $i$ 'th measurement starting from the surface, the uncertainty of  $\mathbf{c}$  is

$$\rho_c(\mathbf{c}) = \prod_{i=1}^{K-1} \rho_c(\mathbf{c}_{i+1}|\mathbf{c}_i)$$

expressing the accumulation of uncertainty, since the position and uncertainty of point  $i + 1$  depends on position and uncertainty of the point  $i$ .

The well-position likelihood function is then

$$L_c(\mathbf{r}) = \prod_{i=1}^{K-1} \rho_c(\mathbf{h}_{i+1}(\mathbf{r}_i))$$

where  $\mathbf{r}_i$  and  $\mathbf{r}_{i+1}$  are connected through  $\mathbf{r}_{i+1} = \mathbf{h}_{i+1}(\mathbf{r}_i)$ .

## NUMERICAL METHOD

We follow the extended Metropolis sampling strategy outlined in *Mosegaard and Tarantola (1995)*. A flow diagram of the algorithm is shown in Figure 1. The characteristic feature of this method is that an algorithm sampling the prior (when run independently) is used to randomly propose perturbations of the current model  $\mathbf{x} = (\mathbf{m}, \mathbf{r})$ . Accepting the proposed models using a likelihood-ratio acceptance probability will then ensure correct (asymptotic) sampling the posterior distribution.

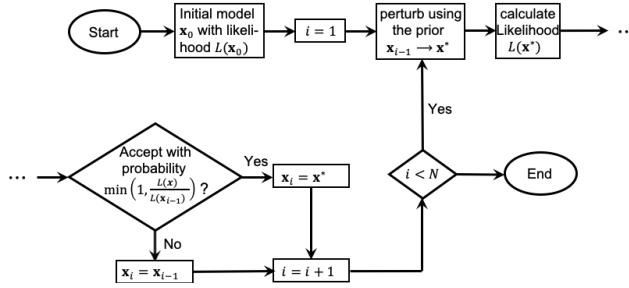


Figure 1 - Flow diagram of the general MCMC algorithm used in this study.

The challenges in our implementation come from the interdependencies between the well data, the well location measurements, and the seismic data. As explained above, well location measurements at one point during the drilling are always conditioned on the (uncertain) location of the previous, shallower point. The seismic data, on the other hand, depend on the (unknown) subsurface parameters. In the well, the subsurface parameters are known with a higher precision than in the surroundings, but the location of the well points are uncertain.

To satisfy all these interrelated, soft constraints, we proceed in the following way (see Figure 2):

In each iteration, choose between perturbing the location of a single well point  $\mathbf{r}^{(i-1)} \rightarrow \mathbf{r}^*$  (with probability  $\alpha$ ), or perturbing the subsurface model  $\mathbf{m}_{i-1} \rightarrow \mathbf{m}^*$  (with probability  $1 - \alpha$ ).

If a well point is chosen for perturbation:

1. Perturb the well trace  $\mathbf{r}^{(i-1)} \rightarrow \mathbf{r}^*$  by changing the location of a random well point using one step of a random walk in the  $(x, y, z)$ -space. In

this case we use a spatially isotropic Gaussian perturbation, centered at the well point. This walk will, if unimpeded, sample the constant prior  $\rho_r(\mathbf{r})$ .

2. Compute the acceptance probability

$$P_{acc} = \min\left(1, \frac{P(\mathbf{n} | \mathbf{r}^*)P(\mathbf{r}^* | \mathbf{p})}{P(\mathbf{n} | \mathbf{r}^{(i-1)})P(\mathbf{r}^{(i-1)} | \mathbf{p})}\right)$$

where  $\mathbf{p}$  is the location of the previous point, and  $\mathbf{n}$  is the location of the next point (see Figure 2).

3. Generate a random number  $u \in [0, 1]$
4. If  $u < P_{acc}$ :
  - a. Accept the perturbed well point location:  $\mathbf{r}^{(i)} = \mathbf{r}^*$
  - b. Adjust the subsurface model parameters  $\mathbf{m} \rightarrow \mathbf{m} + \Delta\mathbf{m}$  to fit the impedance in the new well point location. The adjustment is a linear combination of the kernel functions and is designed to have minimal norm  $\|\Delta\mathbf{m}\|$ . See *Appendix 1* for details.
5. Otherwise reject  $\mathbf{r}^*$  and set  $\mathbf{r}^{(i)} = \mathbf{r}^{(i-1)}$ .

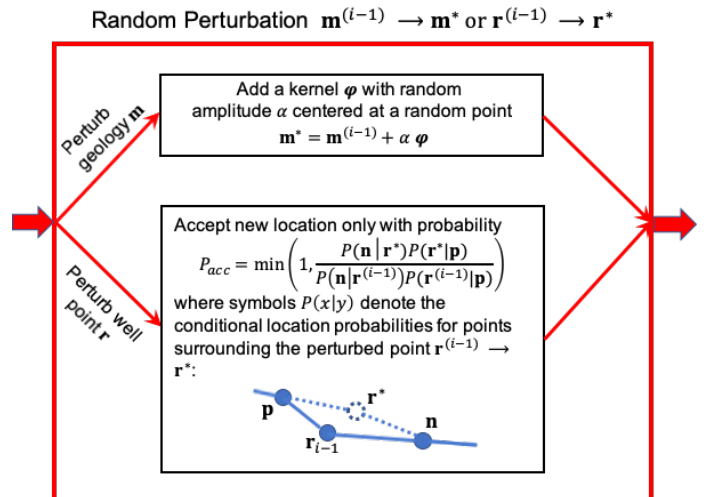


Figure 2 - Flow diagram of random perturbations of unknown parameters in our algorithm.  $\mathbf{x} = (\mathbf{m}, \mathbf{r})$  is the joint set of unknowns, where  $\mathbf{m}$  and  $\mathbf{r}$  are the subsurface parameters and the well location parameters, respectively.

If a subsurface parameter (not coinciding with a well point) is chosen for perturbation:

1. Perturb the model  $\mathbf{m}^{(i-1)} \rightarrow \mathbf{m}^*$  by changing a random model parameter with a Gaussian random number with zero mean and variance  $\sigma_{mod}^2$ .
2. Compute the resulting change in the impedance model, under the constraint that it remains unchanged at all the well points.
3. Compute the acceptance probability

$$P_{acc} = \min \left( 1, \frac{L_d(\mathbf{m}^*)}{L_d(\mathbf{m}^{(i-1)})} \right)$$

4. Generate random number  $u \in [0,1]$
5. If  $u < P_{acc}$ : Accept the perturbed model:  $\mathbf{m}^{(i)} = \mathbf{m}^*$
6. Otherwise reject  $\mathbf{m}^*$  and repeat  $\mathbf{m}^{(i-1)}$ :  $\mathbf{m}^{(i)} = \mathbf{m}^{(i-1)}$ .

## RESULTS

We use an acoustic impedance model of size  $8 \text{ km} \times 8 \text{ km}$  geographic area, with a maximum well depth of 2 km (*Figure 1*). The acoustic impedance model considered in this case was taken from the North Sea F3 Demo 2016 training v6 dataset, Offshore Netherlands (<https://terranubis.com/datainfo/F3-Demo-2016>).

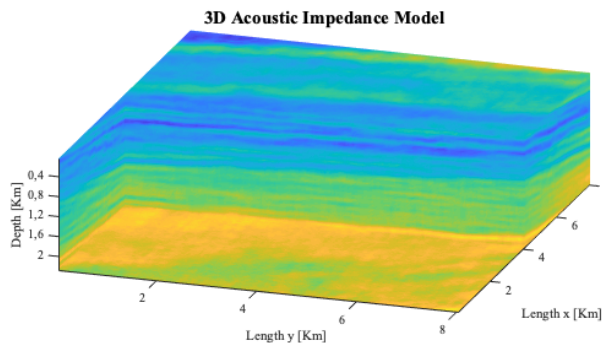


Figure 3 – Acoustic impedance model.

From our impedance volume we estimate 3D kernel/base functions to represent the statistical variability of the impedance in space. We assume that the impedance is a spatially homogeneous Gaussian process, leading to kernel/covariance functions that are everywhere the same. Under this assumption, the impedance can be viewed as a convolution of 3D white noise with the kernel, allowing a (zero phase) kernel (*Figure 4*) to be estimated from its 3D wavenumber spectrum.

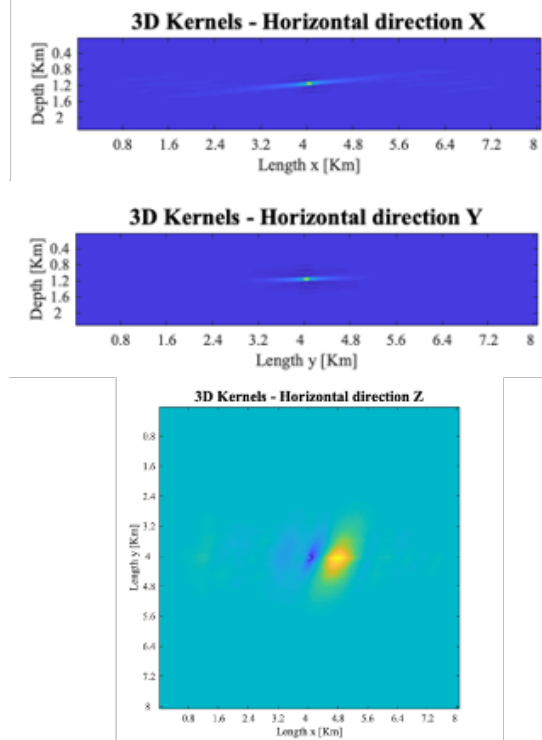


Figure 4 - Kernel on the x-depth plane (up), on the y-depth plane (middle) and on the x-y plane (bottom).

Synthetic seismic data were computed by convolving the reflectivity with a 40 Hz Ricker wavelet within the range of the area of study (*Figure 5*). For the well-log, we simulated 50 points along a well trajectory where geological data are being collected during the drilling operations. Errors in the wellbore locations are considered to be cumulative and each increment introduces a Gaussian error.

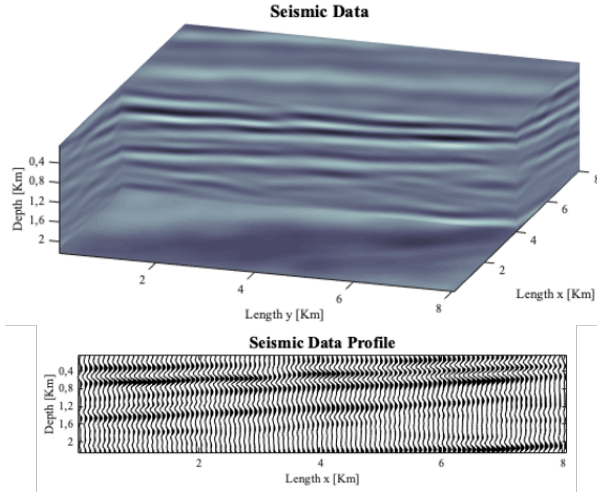


Figure 5 - 3D Seismic data volume (top) and seismic profile (bottom).

Samples from the posterior distribution can be seen on *Figure 6* alongside with the initial acoustic impedance model and the mean well trajectory. The variability of the impedance model and the well trace reflects their uncertainties in a way that is consistent with all a priori assumptions and uncertainties on location data and seismic data. These variabilities generate synthetic seismic data that all fit the (simulated) observed data 'within their error bars'.

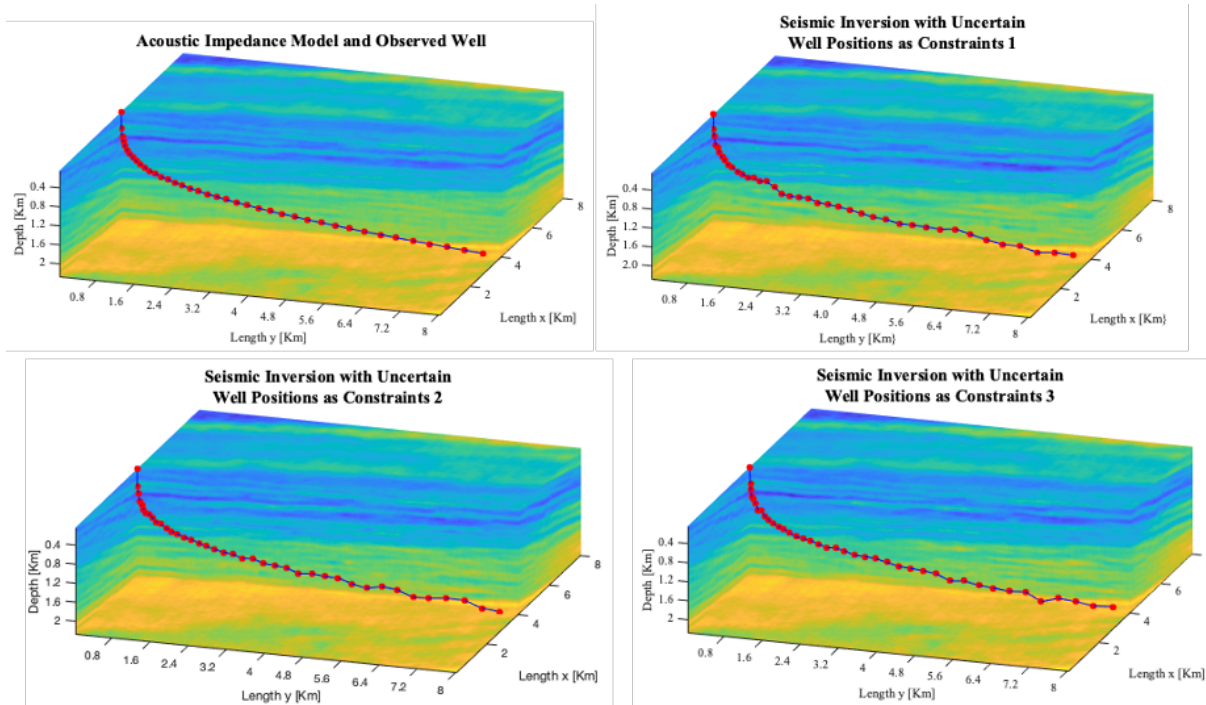


Figure 4 - Initial acoustic impedance model (top left) and 3 posterior realizations of the acoustic impedance using seismic data and uncertain well locations as constraints.

## DISCUSSION

Collecting and measuring data at locations that are difficult to access presents challenges to accuracy of the data. This paper presented a method to quantify the total errors in locations and measurements, incorporating them into seismic inversion, with the aim of giving a more reliable model of subsurface parameters and of the well trajectory.

Any time additional uncertainties are added to the solution of an inverse problem, the resulting solutions are more poorly resolved. This is also the case in this study. On the other hand, assuming that the borehole data were error-free would result in smaller apparent uncertainties in the inversion result, and this would introduce spurious artefacts: Assuming that well data are too precise (in location and rock property measurements) puts too much weight on these data, compared to the seismic data. As a result, actual (neglected) errors in borehole data will give rise to unreal geological structure and properties. In the method proposed in this paper we aim at a more proper evaluation of uncertainties, giving a correct weighting of all data.

Knowing that the measurements themselves can vary not only from errors inherently in the data collection processes but from the location accuracy, can change the way we deal with the problem. As a perspective, the growing position errors can be tackled from earlier stages, even on the fly, and interpretations made at more uncertain locations can be considered with more care.

## CONCLUSION

Seismic inversion with well-calibration was expanded to a more realistic and complete case by including uncertain borehole measurements and locations into the seismic inversion. This error quantification allowed us to provide a more reliable estimate of the uncertainties inherent in seismic inversion assisted with data from horizontal wells.

## APPENDIX 1

Assume that the subsurface model  $m(x, y, z)$ , with parameters  $\mathbf{m} = (m_1, m_2, \dots, m_M)$ , is fixed in  $K$  well point positions  $\mathbf{r} = (\mathbf{r}_1 \dots, \mathbf{r}_K)$ , and that we wish to perturb  $m(x, y, z)$  with the amount  $\delta m$  at a point  $\mathbf{r}_{K+1}$ , not coinciding with any of the points  $\mathbf{r}_1 \dots, \mathbf{r}_K$ . If the perturbed model is  $m'(x, y, z)$ , its parameters  $\mathbf{m}' = (m'_1, m'_2, \dots, m'_M)$  must satisfy

$$\sum_{n=1}^M (m'_n - m_n) \varphi_n(\mathbf{r}_{K+1}) = \delta m$$

and

$$\sum_{n=1}^M (m'_n - m_n) \varphi_n(\mathbf{r}_k) = 0 \quad \text{for } k = 1, \dots, K,$$

Defining a  $(K + 1) \times M$  matrix  $\mathbf{F}$  with components  $F_{ij} = \varphi_j(\mathbf{r}_i)$ , the above equation can be expressed:

$$\mathbf{F} \Delta \mathbf{m} = \mathbf{a}$$

where  $\Delta \mathbf{m} = \mathbf{m}' - \mathbf{m}$ , and where:

$$\mathbf{a} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \delta m \end{pmatrix}$$

is a vector with  $K + 1$  components. The system of equations (2) is, in general, underdetermined and has infinitely many solutions, but the least-squares solution for  $\Delta \mathbf{m}$  is:

$$\Delta \mathbf{m}_{\text{LS}} = \mathbf{F}^T (\mathbf{F} \mathbf{F}^T)^{-1} \mathbf{a}.$$

Knowing  $\mathbf{m}$  and  $\Delta \mathbf{m}_{\text{LS}}$  we can compute  $\mathbf{m}' = \mathbf{m} + \Delta \mathbf{m}_{\text{LS}}$ .

The above procedure can be used, both when perturbing a point that is not a well point, but also when a well point location is perturbed. In the latter case, the original position of the perturbed point is erased from  $\mathbf{r} = (\mathbf{r}_1 \dots, \mathbf{r}_K)$ , which is then reduced



to  $K - 1$  components, and at the new position of the perturbed point there is a change  $\delta \mathbf{m}$  in the model. In this point, the original model value is replaced by the value carried with the perturbed well point, and parameters of the surrounding model is updated to preserve the variability given by the kernel/covariance functions.

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