# Frequency Dependent Mantle Viscoelasticity via the Complex Viscosity: cases from Antarctica

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**Abstract**

Studies of glacial isostatic adjustment (GIA) often use paleoshorelines and present-day deformation to constrain the viscosity of the mantle and the thickness of the elastic lithosphere. However, different studies focused on similar locations have resulted in different estimates of these physical properties. We argue that these different estimates infer apparent viscosities and apparent lithospheric elastic thicknesses, dependent on the timescale of deformation. We use recently derived relationships between these frequency dependent apparent quantities and the underlying thermodynamic conditions to produce predictions of mantle viscosity and lithospheric thickness across a broad spectrum of geophysical timescales for two Antarctic locations (Amundsen Sea and the Antarctic Peninsula). Our predictions are constrained by input from seismic tomography, require the self-consistent consideration of elastic, viscous, and transient rheological behavior and also include non-linear steady state viscosity, which have been determined by several laboratories. We demonstrate that these frequency dependent predictions of lithospheric thickness and apparent viscosity display a significant range and that they align to first order with estimates from GIA studies on different timescales. We suggest that observational studies could move towards a framework of determining the frequency dependence of apparent quantities – rather than single, frequency independent values of viscosity – to gain deeper insight into the rheological behavior of Earth.

# Frequency Dependent Mantle Viscoelasticity via the Complex Viscosity: cases from Antarctica

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**Key Points:**

- Differing estimates of viscosity and plate thicknesses are observed from various loading processes across Antarctica
- Using new theory and laboratory laws, frequency dependent viscosity and plate thickness are predicted for Antarctic sites
- Our results indicate that these predictions of viscosity and plate thickness can significantly contribute to the observed discrepancies

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## Abstract

Studies of glacial isostatic adjustment (GIA) often use paleoshorelines and present-day deformation to constrain the viscosity of the mantle and the thickness of the elastic lithosphere. However, different studies focused on similar locations have resulted in different estimates of these physical properties. We argue that these different estimates infer apparent viscosities and apparent lithospheric elastic thicknesses, dependent on the timescale of deformation. We use recently derived relationships between these frequency dependent apparent quantities and the underlying thermodynamic conditions to produce predictions of mantle viscosity and lithospheric thickness across a broad spectrum of geophysical timescales for two Antarctic locations (Amundsen Sea and the Antarctic Peninsula). Our predictions are constrained by input from seismic tomography, require the self-consistent consideration of elastic, viscous, and transient rheological behavior and also include non-linear steady state viscosity, which have been determined by several laboratories. We demonstrate that these frequency dependent predictions of lithospheric thickness and apparent viscosity display a significant range and that they align to first order with estimates from GIA studies on different timescales. We suggest that observational studies could move towards a framework of determining the frequency dependence of apparent quantities – rather than single, frequency independent values of viscosity – to gain deeper insight into the rheological behavior of Earth.



## Plain Language Summary

The viscoelastic structure of the solid Earth has important consequences for ice-melting events, and other processes that involve shifting mass on Earth's surface. As mass moves on Earth's surface, the Earth subsides or rebounds, where the degree and time-scale of these responses depend on Earth's viscosity. Inferences of Earth's viscosity often consider a single viscosity that does not take into account the time-scale effects of how slowly or quickly mass is exchanged on Earth's surface (e.g., ice sheet collapse compared with slow melt spanning thousands of years). Using a new theoretical framework and applying it to cases in Antarctica (in particular, the Antarctic Peninsula and Amundsen Sea Embayment), we demonstrate that such time-scale factors should be considered for future studies.


## 1 Introduction

The growth and decay of ice sheets over the Pleistocene represent large variations in Earth's climate system and induce significant deformation of the solid Earth. This deformation (including associated changes to Earth's gravity field and rotation), in response to the redistribution of ice and ocean mass, is known as "glacial isostatic adjustment" (GIA). Areas that were formerly covered by or close to major ice sheets during the last glacial period, such as North America and Antarctica, continue to experience the highest rates of GIA-related deformation, even though ice has retreated partially or entirely (e.g., Sella et al., 2007). Pertinent to understanding this solid Earth deformation, and, as a consequence, related climatological feedbacks (e.g., Larour et al., 2019; Pan et al., 2021) is knowledge of the planet's viscoelastic structure.

Constraining Earth's viscoelastic structure has been the focus of many GIA, geodynamic, seismic, and mineral/rock physics studies. In GIA studies, viscoelastic deformation is often assumed to be linear with one viscous timescale (Maxwell rheology), linear with multiple viscous timescales (transient, e.g., Burger's rheology), or a combination of linear and non-linear deformation (e.g., composite rheology)(see Fig. 2). This means that deformation can be timescale- or stress-dependent. In all cases, viscosity is additionally dependent on the thermodynamic state variables (most importantly temperature, but also composition, grain size, melt fraction, or water content), which causes it to vary radially and laterally within the mantle (e.g. Kaufmann et al., 2005; Hay et al., 2017; Nield et al., 2018; Powell et al., 2020).

Transient behavior in form of a Burger's rheology has been considered in some GIA studies (e.g., Caron et al., 2017) in which observations are used to constrain the two distinct viscosities (Fig. 2). In addition, comparisons between model predictions including steady-state non-linear mechanical effects and observations of deglacial sea level have also been explored in GIA (e.g., Wu, 1992; van der Wal et al., 2010, 2013; Huang et al., 2019). In this study we aim to investigate the role of more complex rheologies by simultaneously exploring the effects of frequency dependent and non-linear rheology on GIA in Antarctica.

We use a new theoretical framework (Lau & Holtzman, 2019) to predict continuous frequency- and stress-dependent rheological parameters relevant for GIA for two regions beneath Antarctica: Amundsen Sea Embayment "ASE" and the Antarctic Peninsula "AP", Fig. 1(C). The upper mantle of two locations have been recently constrained by seismic structural models (Lloyd et al., 2019; An et al., 2015) and each location has experienced a variety of ice melt processes that span a range of timescales readily available to compare to our forward predictions. Additionally, because these are locations of rapid present-day ice melt at which GIA might help to stabilize the ice sheet, understanding the rheology is of particular importance. Lastly, focusing on distinct locations rather than larger regions allows us to minimize – to some extent – the effect of lateral variability in viscoelastic structure on the observed GIA response.

The variation of GIA responses to the wide variety of mass perturbations is well studied (e.g., Haskell, 1935; Peltier & Andrews, 1976; Nakada & Lambeck, 1989; Spada, 2017; Whitehouse, 2018) and analyses are often cast in terms of two controlling Earth parameters: mantle viscosity and the elastic thickness of the lithosphere, $z_{\mathrm{LAB}}$. We will hone in on mantle viscosity down to 400 km depth, which we refer to as the "uppermost upper mantle", i.e., $\eta_{\mathrm{UMM}}$. Fig. 1(A,B) displays estimates of these two parameters from regionally overlapping geophysical studies that observed GIA responses to, e.g., recent rapid ice sheet collapses (Barletta et al., 2018) and deglacial ice melt (Wolstencroft et al., 2015) (see caption for all references). Since our aim is to understand frequency dependent behaviour, we broadly subdivide these inferences into the timescale of observations from which they were derived (shorter centennial and longer millennial timescale loading processes "short" and "long", and, for $z_{\mathrm{LAB}}$ estimates, seismic structural models). We note that, at short timescales, accurate knowledge of viscoelastic properties is important for predicting the rebound process, with important implications for future ice and sea level change (Barletta et al., 2018; Pan et al., 2021).

Figs 1(A,B) reveal inconsistencies between the inferred values for these parameters for each site. While the differences between the two sites may be explained by varying thermodynamic state, the different measurements should in principle agree between each other at the same site. This apparent discrepancy can arise from two main sources: (i) differences in what depth and lateral range of viscosity these processes sample (van der Wal et al., 2015; Crawford et al., 2018); and (ii) differences in the nature – more specifically, the stress and time dependence – of the GIA response to mass perturbations. In this study, we do not aim to uniquely resolve these apparent discrepancies. Instead, we provide an in-depth investigation of (ii), to appreciate the extent to which these processes *could* contribute to the apparent discrepancy in the inferred viscoelastic structure.

The frequency dependent behavior of viscosity has long been demonstrated within experimental studies of the mechanical behavior of rock: The endmember elastic (and anharmonic) and viscous properties of rock have been well characterized in the experimental and theoretical rock physics community (e.g., Stixrude & Lithgow-Bertelloni, 2005; Hirth & Kohlstedt, 2003). In between these endmembers is the anelastic or transient regime, which is currently being explored by several laboratories working on olivine

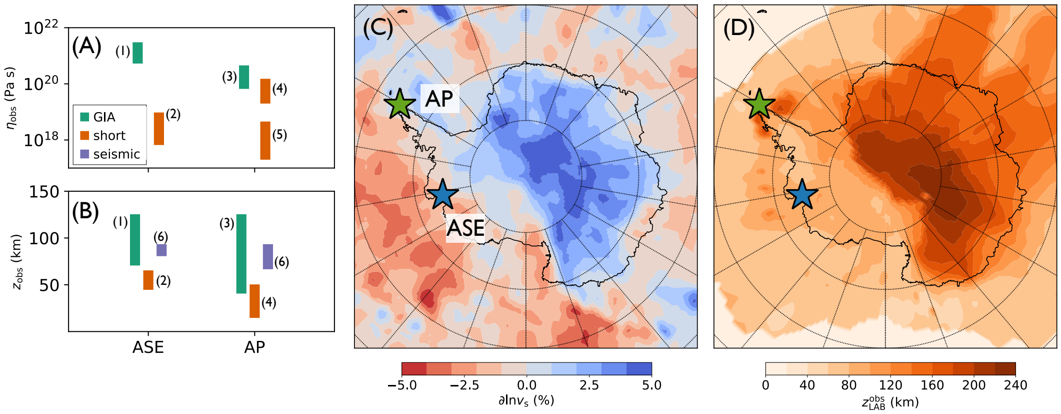


**Figure 1. Estimates and locations of upper mantle viscosity and plate thickness.** A compilation of (A) upper mantle viscosity, $\eta_{\rm obs}$, and (B) lithospheric thickness, $z_{\rm obs}$, estimates across the Amundsen Sea Embayment (ASE) and the Antarctic Peninsula (AP). References labeled are (1) Whitehouse et al. (2012); (2) Barletta et al. (2018); (3) Wolstencroft et al. (2015); (4) Ivins et al. (2011); (5) Samrat et al. (2020); and (6) An et al. (2015). (C) The seismic tomography model of Lloyd et al. (2019) at 225 km depth. Locations of ASE (102.667 °W; 64.833 °S); and AP (60.409 °W; 64.476 °S) are marked. (D) The seismically derived estimates of lithospheric thickness from An et al. (2015). Note, the lithospheric thickness estimates are based on the tomography model of An et al. (2015) and thus are not necessarily in agreement with ANT-20 (Lloyd et al., 2019).

samples (e.g., Jackson & Faul, 2010; Sundberg & Cooper, 2010; Faul & Jackson, 2015) and analogue materials (Takei, 2017). This work is building towards a clearer picture of the grain-scale processes that govern macroscopic transient creep, including both diffusion- and dislocation- (both linear and non-linear) related grain boundary and intracrystalling deformation mechanisms (e.g., Karato & Wu, 1993; Hansen et al., 2019). (See Havlin et al. (2021) for a summary of these processes.) From these microphysical mechanisms, distinct frequency-dependent rheological properties of rock can be predicted.

The broad goal of this study is to quantify and test the role of frequency and stress-dependent viscoelastic deformation in the two specific study regions (Fig. 1). In Section 2 we briefly summarize relevant experimental and theoretical background and present the framework introduced by Lau and Holtzman (2019) which will serve as a basis for our interpretation. These constitutive laws require thermodynamic conditions as input, and we use seismic observations to constrain this (Section 3). With the thermodynamic conditions of each region determined, we predict their complex viscosities, $\eta^*(\omega; z)$ (which is a measure of viscous dissipation, introduced in Section 2), spanning geophysically relevant frequencies. With this approach we treat viscoelastic rheology in a manner akin to mapping attenuation as a function of frequency, $Q^{-1}(\omega)$, as studied within the fields of seismology and Earth tides (Shito et al., 2004; Benjamin et al., 2006; Lekić et al., 2009; Lau & Faul, 2019). In order to compare our $\eta^*(\omega)$ predictions to results from prior GIA studies we first determine the frequency content of the time-domain GIA data (Fig. 1) and then investigate to what extent the predicted values of both $\eta_{\rm UMM}$ and $z_{\rm LAB}$ as a function of frequency can contribute to the variation obtained in the observationally driven estimates.

## 2 The full spectrum of viscoelastic deformation

In the following we first describe phenomenological models that are typically used to describe viscoelastic behavior noting where each regime of deformation lies in relation to these models (Section 2.1). We then follow with how we can use the complex viscosity to derive two frequency dependent parameters relevant to GIA dynamics, the apparent viscosity and lithospheric thickness (Section 3.1.3). This ultimately requires knowledge of the mechanical properties of rock across all frequencies of interest (hereafter, the "full spectrum" mechanical properties).

To do this we will rely on a recently released open-source software library, the "Very Broadband Rheology calculator" (VBRc), developed by several of the co-authors here (Havlin et al., 2021). This software takes as input the thermodynamic conditions and other state variables, $\mathcal{S}$ – where $\mathcal{S} = [T, P, \phi, g, \sigma, X]$ (temperature, pressure, melt fraction, grain size, stress state, and composition, respectively) – and determines the full spectrum mechanical properties for a selection of constitutive laws. In Section 2.3 we briefly describe this process.

In Section 2.4 we describe our current understanding on the broadband characteristics of deformation from the experimental community and how these micro-scale descriptions are applied to geophysical applications. We subdivide these into their deformational regimes (elastic, viscous, transient, and non-linear) and also describe the specific constitutive laws the VBRc implements for each.

### 2.1 Summary of Phenomenological Models

In order to incorporate frequency-dependent properties predicted by experiments into geophysical forward predictions, the constitutive laws are described by phenomenological parameterizations via combinations of springs (described by modulus $M_i$) and viscous dashpots (described by viscosity $\eta_i$) that, in isolation, characterize elastic and viscous behavior, respectively. The simplest arrangement is a single spring-dashpot pair in series, known as the Maxwell model (see Fig. 2). This is the most common viscoelastic model considered in GIA studies since a convenient semi-analytical solution may be determined (e.g., Peltier & Andrews, 1976; Nakada & Lambeck, 1989; Mitrovica & Milne, 2003).

Fig. 2 partitions the different flavors of phenomenological models in their different frequency limits. All models have two endmembers in common: The elastic regime occurs at infinite frequency ($f \to \infty$) where no energy is dissipated and deformation is both instantaneous and fully recoverable. Any quantity corresponding to this endmember, e.g., the modulus, $M$, will be specified as $M_\infty$. The opposite endmember refers to the viscous or steady-state regime, when $f \to 0$, wherein deformation is fully dissipative and no longer recoverable. Quantities, like $M$, associated with this frequency limit will be denoted $M_0$.

In between these endmembers, there exists the transient creep regime. Within transient creep, anelastic processes may occur, where deformation is fully recoverable, though time dependent. Such behavior may be mimicked by Kelvin-Voigt pairs (one spring and one dashpot in parallel). Such recovery reflects microscopic restoring forces (Section 2.4.3). However, irrevseribile material transport also accompanies transient creep. (If transient creep does indeed occur during GIA, gravity provides a macroscopic restoring force, analogous to elasticity at the grain scale) Additional transient elements can be added to the Maxwell model to account for anelastic effects, giving rise to more complex models such as the Andrade model (Sundberg & Cooper, 2010), the Extended Burgers model (Faul & Jackson, 2015), and relaxation function fitting approaches (McCarthy et al., 2011; Yamauchi & Takei, 2016). In addition, complexity can arise from non-linear (stress-dependent) effects (Hirth & Kohlstedt, 2003; Hansen et al., 2011).

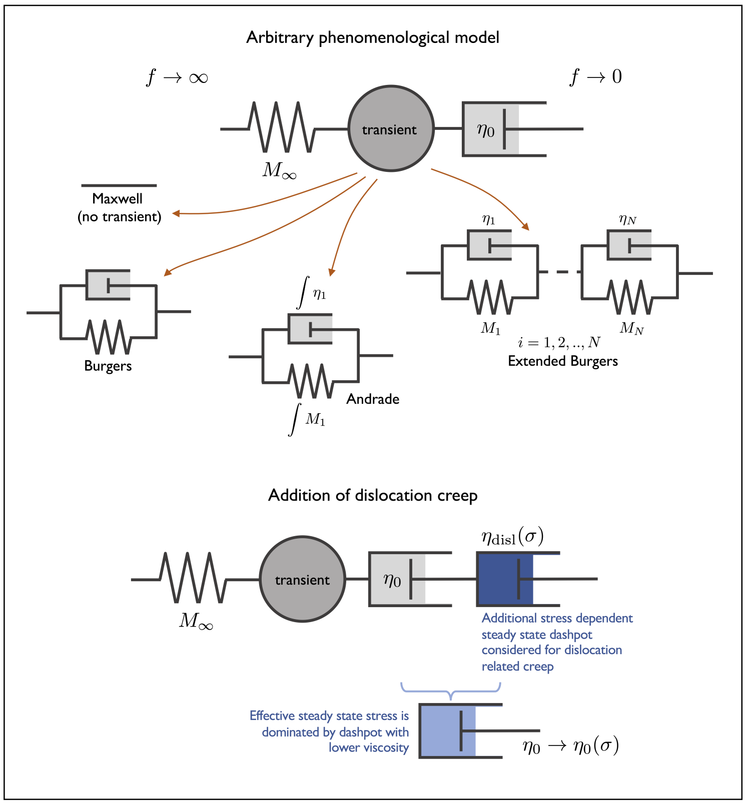


**Figure 2. Phenomenological Viscoelastic Models.** Depiction of 1-D phenomenological viscoelastic models. The dark gray circle symbolically represents any combination of springs and dashpots that mimic transient deformation. Replacing the circle with any of the components linked by an arrow will form the commonly adopted models labelled. With the addition of steady state dislocation creep, the viscosity of the steady state dashpot, $\eta_0$, becomes stress, $\sigma$, dependent.

From these phenomenological models several measures of dissipation may be determined depending on the arrangement of the discrete springs, $M_i$, and dashpots, $\eta_i$, including the dissipation (or attenuation), $Q^{-1}(\omega)$, the complex modulus, $M^*(\omega)$, and its inverse, the complex compliance, $J^*(\omega)$. The latter two have their time-domain equivalents, $M(t)$ and $J(t)$ (Nowick & Berry, 1972). The latter is known as the creep function. We will also introduce the complex viscosity, $\eta^*(\omega)$, which is distinct from the discrete dashpots within any given phenomenological model.

### 2.2 Complex Viscosity

Lau and Holtzman (2019) explored the use of complex viscosity, $\eta^*(\omega)$, in geophysical applications involving viscoelasticity, building on its use more commonly within the materials science literature (e.g., Gunasekaran & Ak, 2002). To motivate its application, we begin with the more familiar (and closely related) complex modulus, $M^*(\omega)$. $\mathrm{Re}[M^*(\omega)]$ reduces with decreasing frequency, causing the dispersion of seismic waves, while $\mathrm{Im}[M^*(\omega)]$ captures the dissipative effects. Within seismology, dispersion manifests as the reduction of seismic wave-speeds at lower frequencies (e.g., Kanamori & Anderson, 1977). Similarly, attenuation, $Q^{-1}$, where

$$Q^{-1}(\omega) \equiv \mathrm{Im}[M^*(\omega)]/\mathrm{Re}[M^*(\omega)], \tag{1}$$

increases with lower frequency across the seismic band (e.g., Shito et al., 2004; Lekić et al., 2009) and the tidal band (Benjamin et al., 2006; Lau & Holtzman, 2019). In Fig. 3(A) we show a schematic figure of how these parameters are sampled by seismic waves of different frequency. *No specific phenomenological model* is assumed, only the measurements of $Q^{-1}(\omega)$ and $v(\omega)$ at their given frequency are determined, such that a frequency trend may be mapped out.

Lau and Holtzman (2019) extended this analogy to GIA by using a more appropriate parameter, $\eta^*(\omega)$. The relationship between $\eta^*(\omega)$ and $M^*(\omega)$ is

$$\eta^*(\omega) = i\frac{M^*(\omega)}{\omega} \tag{2}$$

and upon inspection, one can see that $\eta^*$ has the same units as viscosity, where the real and imaginary parts, and their physical meaning, have now been switched relative to $M^*$. No matter the arrangement of springs and dashpots, analagous to $M^*(\omega)$, a continuous function of $\eta^*(\omega)$ across frequency may be derived.

Just as the dispersion of wave-speed captures $\mathrm{Re}[M(\omega)]$ (or more specifically, its square-root), we argue that estimated viscosities determined by different observations are sampling $\|\eta^*(\omega)\|$ at their respective frequency bands (Fig. 3B), where we have plotted $\|\eta^*(\omega)\|$, the *apparent* viscosity (Section 3.1.3) of a Maxwell viscoelastic model. For clarification, while the Maxwell model has a single viscous dashpot, since it also has a frequency dependent complex modulus, eq. 2 shows that even the Maxwell model will have a frequency dependent $\eta^*(\omega)$. The quantity $\|\eta^*(\omega)\|$ may be interpreted as an indication of the degree of viscous dissipation at a given frequency (Lau & Holtzman, 2019). By considering GIA inferred $\eta^*(\omega)$ at a given frequency of observation, the link to laboratory-based constitutive laws is clear. In contrast, discrete springs and dashpots obscure this connection.

### 2.3 The Very Broadband Rheology calculator (VBRc)

The Very Broadband Rheology Calculator (VBRc) is an open source software package recently developed by Havlin et al. (2021, available at https://vbr-calc.github.io/vbr/). Given a set of thermodynamic state variables, $\mathcal{S}$, it will predict the full spectrum mechanical properties for composition, $X$, of olivine (90% forsterite) assuming certain constitutive laws. By "full spectrum", we mean the elastic, anelastic, and viscous regimes

Figure 3. Mapping out dissipation parameters with geophysical observations. (A) A schematic depiction of seismic wave-speed reduction (top panel) with lowering frequency due to dispersion (or Re[$M(\omega)$]) and the increase of attenuation, $Q^{-1}(\omega)$ (bottom panel). Boxes denote how these trends are sampled by seismic data at different frequencies. (B) The analog to (A) but how GIA processes may sample apparent viscosity $\tilde{\eta}(\omega)$. Note, "lake rebound" refers to how the drainage of lakes can produce isosatic adjustment (e.g., the rapid drainage of Lake Bonneville ($\sim$14 ky BP) over $\sim$500 y; Austermann et al., 2020).

are denoted $v_s^{obs}$ associated with $\omega_{obs}$. We extract $z_{LAB}^{obs}$ (at $\omega = \omega_{obs}$) from the lithospheric thickness models with the same horizontal averaging. For the uncertainty in these constraints, $\sigma_{obs}$, we have assumed $\pm 0.05$ km/s for the wave speed measurements and $\pm 5$ km for the LAB depth.

#### *3.1.2 Bayesian Inference of Thermodynamic State*

To determine the subsurface structure at each location down to 400 km depth, we consider depth dependent variations in thermodynamic and mechanical parameters only (localized radial stratification) and characterize this structure as a 1-D plate model. To produce a single plate model we solve the transient 1-D heat equation with variable thermal conductivity (as in Xu et al., 2004) and heat capacity (as in Berman & Aranovich, 1996) across a conductive plate to steady state using the finite difference forward model provided in the VBRc library. The solution depends on two input parameters: the potential temperature, $T_P$, and the thickness of the conductive lid, $z_{LID}$. Beneath the plate, the mantle temperature profile follows the adiabat.

We acknowledge that this is a simplification of the subsurface structure, but the smoothness of the $v_s$ profiles (Fig. 6, subpanels ii-iii) suggests that such an approximation is reasonable and the advantage is that we can fully characterize the thermal structure with the two input parameters $T_P$ and $z_{LID}$. Our aim is not to produce a detailed subsurface model, but to identify broad characteristics that might differ between our two locations. Many plate models are produced, which encompass $T_P = [1000, 1050, ..., 1800]$ °C and $z_{LID} = [50, 60, ...250]$ km. These alone characterize the temperature profile (i.e., $T$ and $P$ dependence) of the subsurface. In order to translate these into mechanical properties using the VBRc constitutive laws, we additionally provide uniform priors for grain size and melt fraction over the ranges $g = [0.001, 0.004, ..., 0.03]$ m and $\phi = [0.0, 0.005, .., 0.03]$, respectively. The result is a large suite of plate models which are converted to mechanical properties, like $v_s$ and $\tilde{\eta}_{UMM}$ across the relevant frequency bands.

The Bayesian inference follows in two stages. The first is to find $\mathcal{S}$ for the depth range (gray shaded bar, Fig. 6A,B) across which we extract $v_s^{obs}$. This constraint will be compared to forward predictions of the plate temperature profiles at the same depth, narrowing down the range of plausible plate profiles. The second is to use $z_{LAB}^{obs}$ to constrain $z_{lid}$ from the subset of remaining plate models.

Given $v_s^{obs}$, associated with the depth range, we determine posterior distributions of the state variables $\phi$, $T$, and $g$ (melt fraction, temperature, and grain size). The statement of our inference is

$$P(\mathcal{S}|v_s^{obs}) = \frac{P(v_s^{obs}|\mathcal{S})P(\mathcal{S})}{P(v_s^{obs})} \tag{6}$$

where $P(\mathcal{S}|v_s^{obs})$ is the posterior probability that represents how well constrained the state variables of interest are given $v_s^{obs}$, $P(v_s^{obs}|\mathcal{S})$ is the likelihood that describes how likely $v_s^{obs}$ is given $\mathcal{S}$, $P(v_s^{obs})$ is the probability of observing the measurement itself (used here as a normalizing factor), and $P(\mathcal{S})$ is the prior distribution of the state variables. $P(v_s^{obs}|\mathcal{S})$ is determined from a $\chi^2$-misfit with which we construct a Gaussian likelihood matrix where

$$P(v_s^{obs}|\mathcal{S}) = \frac{1}{\sqrt{2\pi\sigma_{obs}^2}} \exp\left(\frac{\chi^2}{2}\right), \tag{7}$$

and

$$\chi^2 = \frac{[v_s^{obs} - v_s(\omega_{obs}; \mathcal{S})]^2}{\sigma_{obs}^2}. \tag{8}$$

Here, $\sigma_{obs}$ is the uncertainty in the observation. To construct $P(v_s^{obs}|\mathcal{S})$ we use the VBRc to calculate $v_s(\omega_{obs}; \mathcal{S})$ from our plate model suite. For our prior distribution of the state variables we assume a uniform probability across all state variables (where the search range is noted above) and treated as independent of each other for simplicity.

To determine the lithospheric thickness we use the same Bayesian inference framework but the observation is now $z_{\mathrm{LAB}}^{\mathrm{obs}}$. With many definitions for the LAB, $z_{\mathrm{LAB}}^{\mathrm{obs}}$ may not be equivalent to $z_{\mathrm{LID}}$. Such definitions include the depth at which the largest negative value of $\partial v/\partial z$ occurs (Hopper & Fischer, 2018) or the maximum depth for which $v_{\mathrm{s}}$ anomalies are consistently greater than 2% (Conrad & Lithgow-Bertelloni, 2006). See Lau et al. (2020) for further discussion. However, the lithospheric thickness model of An et al. (2015) define $z_{\mathrm{LAB}}^{\mathrm{obs}}$ as the intersection of the base of the conductive lid, and so in this case, this definition is precisely $z_{\mathrm{LID}}$. The misfit in this case is calculated as:

$$\chi^2 = \frac{[z_{\mathrm{LAB}}^{\mathrm{obs}} - z_{\mathrm{LID}}(\omega_{\mathrm{obs}}; \mathcal{S})]^2}{\sigma_{\mathrm{obs}}^2}. \tag{9}$$

For our analysis below, we will select the profile of $\mathcal{S}(z)$ that maximizes $P(z_{\mathrm{LAB}}^{\mathrm{obs}}|\mathcal{S})$.

#### *3.1.3 Determination of Apparent Viscosity and Apparent Lithospheric Thickness*

With $\mathcal{S}(z)$ determined, the VBRc is used, with $\mathcal{S}(z)$, to output the full-spectrum profiles of $M^*(\omega)$, $Q^*(\omega)$, and $\eta^*(\omega)$ at each location. Using the latter, we introduce two simple parameters: the apparent viscosity $\tilde{\eta}(\omega)$, which is the absolute value of the complex viscosity and the apparent lithospheric thickness, $\tilde{z}_{\mathrm{LAB}}$. For $\tilde{z}_{\mathrm{LAB}}$, within any column of mantle rock, we may determine the Maxwell time as a function of depth ($\tau_{\mathrm{M}}(z)$), since this is the ratio of two depth-dependent material properties, $\eta_0(z)$ to $M_\infty(z)$. If we are interested in $\tilde{z}_{\mathrm{LAB}}$ at a given frequency, we find the depth at which $\tau_{\mathrm{M}}$ is equivalent to this frequency. This essentially marks the transition from elastic to viscous behavior for this frequency. This definition breaks down if the frequency is higher than any Maxwell frequency in the mantle, where we propose that $\tilde{z}_{\mathrm{LAB}}$ becomes essentially frequency independent. At these high frequencies the notion of a plate itself becomes unclear. We note that this is just one definition of many that exists for the LAB, though this definition highlights the frequency dependence of $\tilde{z}_{\mathrm{LAB}}$.

As is common in GIA studies that consider non-linear rheologies (e.g., van der Wal et al., 2010), we reinterpret the reported values of "$\eta$" and "$z_{\mathrm{LAB}}$" presented in Fig. 1 as *apparent* quantities, (where apparent lithospheric thicknesses are explored in greater detail in Lau et al. (2020) and we only briefly describe the method by which we determine $\tilde{z}_{\mathrm{LAB}}$ below).

### 3.2 Task 2: Determining Frequency Content of Observed Inferences of Uppermost Upper Mantle Viscosities and Lithospheric Thicknesses

Observationally derived estimates of lithospheric thickness and viscosity are generally obtained by combining knowledge of past load changes (ice sheets or lakes) with observations of deformation (GPS or reconstructions of paleo-water level) assuming a viscoelastic model. The estimates we have used for comparison against our VBR-driven profiles are all parameters reported from studies which applied geodetic constraints, and thus are subject to a range of simplifying assumptions. The estimates of both $z_{\mathrm{LAB}}$ and $\eta_{\mathrm{UMM}}$ have been compiled from the following investigations for each region (listed in order of decreasing timescale): For Amundsen Sea Embayment, we include the results from Whitehouse et al. (2012), who used relative sea level data to study the effects of long term GIA across the entire Antarctica region (and select their reported values associated with the ASE) and from Barletta et al. (2018), who used GPS derived measures of decadal-scale rebound at the Amundsen Sea Embayment. For the Antarctic Peninsula, decadal (Nield et al., 2014; Samrat et al., 2020) and centennial (Ivins et al., 2011; Wolstencroft et al., 2015) responses to ice mass change, once more measured by GPS, were used to derive viscosity estimates.

We will also investigate $z_{\mathrm{LAB}}$ estimates derived from gravity-based data of Chen et al. (2018). These provide effective elastic thickness estimates of the Antarctic plate and should be interpreted as the thickness of a completely elastic lid overlying an inviscid mantle (where $\tilde{\eta}_{\mathrm{UMM}} \to 0$). Such a result would be relevant at zero frequency. For a more detailed discussion on this matter, see Lau et al. (2020).

In order to compare our predictions of $\tilde{\eta}_{\mathrm{UMM}}(\omega)$ and $\tilde{z}_{\mathrm{LAB}}(\omega)$ to the observations presented in Fig. 1, we need to determine their appropriate frequency content. In all the cases, the data points fit were uplift rates extracted from a time series of GPS data. These single uplift rates were used to determine $z_{\mathrm{LAB}}$ and $\eta$ and capture a whole range of frequencies embedded in not only the loading history (here, approximated by $\tau_{\mathrm{dur}}$, the duration over which the loading/unloading event in question occurred) but also the delay in measuring the response (here, approximated by $\tau_{\mathrm{del}}$, the time delay between the end of the event and when the observation of solid Earth deformation was made). For example, in Barletta et al. (2018), one of their analyses involved investigating the change in ice loss between 2002-2014 and GPS measurements were made throughout this time. In this case, $\tau_{\mathrm{dur}} = 12$ years and $\tau_{\mathrm{del}} = 0$ years. These designations of $\tau_{\mathrm{del}}$ and $\tau_{\mathrm{dur}}$ – tabulated and summarized for each observation in Table A1 – are approximate and our aim is to cover a wide enough frequency band to provide the most conservative estimate possible.

We will use these two important timescales to determine an appropriate frequency band, $[f_{\mathrm{low}}, f_{\mathrm{upp}}]$. For $f_{\mathrm{upp}}$ we know that, given the timescale of any process, the highest possible frequency must be bound by $\tau_{\mathrm{dur}}^{-1}$. The Fourier transform of a linear trend is dominated by these low frequencies and longer values of $\tau_{\mathrm{del}}$ will result in any relatively high frequency response to diminish. But just how low is this bound? We will determine this lower bound by applying an empirical relationship based on replicating the loading history and measurements of deformation.

For this empirical relationship, we replicate the loading histories of our observations, $\sigma(t)$, apply this stress load to a given material of compliance $J$ and calculate the deformation, i.e., the strain, $\varepsilon(t)$. The compliance will be that associated with our upper mantle structures beneath ASE and AP (trends shown in Figs 6v,vi). Our generic stress history is depicted in Fig. 5(A).

There are two ways in which we may determine $\varepsilon(t)$. We could perform this calculation in the Fourier domain and multiply the stress history with $J^*(\omega)$ and take the inverse Fourier transform to produce the time domain response. This seems ideal since from experimental results, as output by the VBRc, we readily know $J^*(\omega)$. However, several factors complicate this approach: the frequency range in which we sample both $\sigma^*(\omega)$ and $J^*(\omega)$ are naturally entirely different (where the former is linear, whereas the latter is logarithmic) and inverting the Fourier transform with either of these options presents some limitations (see, e.g., Haines & Jones, 1988).

Instead, we choose to convolve the stress history with $J(t)$. That is,

$$\varepsilon(t) = \int_{-\infty}^{t} J(t-t')\mathrm{d}\sigma(t'). \tag{10}$$

Given that only $J^*(\omega)$ is determined by the VBRc and not $J(t)$, we obtain $J(t)$ by fitting $J^*(\omega)$ to that of an Andrade viscoelastic model, $J^*_{\mathrm{An}}(\omega)$ (Fig. 2). We choose the Andrade model since it can capture the full spectrum of viscoelastic (elastic, transient, viscous) behavior with few model parameters (Cooper, 2002) where expressions for $J^*_{\mathrm{An}}(\omega)$ and $J_{\mathrm{An}}(t)$ are given by

$$J_{\mathrm{An}}(t) = \frac{1 + t/\tau_{\mathrm{M}}}{M_\infty} + \beta t^n \tag{11}$$

and

$$J^*_{\mathrm{An}}(\omega) = J_\infty + \beta\Gamma(1+n)\omega^{-n}\cos\left(\frac{n\pi}{2}\right) - i\beta\Gamma(1+n)\omega^{-n}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{\eta_0\omega} \tag{12}$$

(Faul & Jackson, 2015). Here, $\Gamma$ is the standard gamma function of order $n$, where typical values of $n$ are 1/3 (Cooper, 2002). For each of our regions, $\tau_{\mathrm{M}}$, $J_\infty$, $M_\infty$, $\eta_0$, and $J^*(\omega)$ are provided by the VBRc. $\beta$ determines the strength of the anelastic contribution and thus once knowledge of $\beta$ is acquired, calculating $J_{\mathrm{An}}(t)$ is trivial.

We solve for the best fitting $\beta$ via a grid search and the resulting $\tilde{\eta}(\omega)$ values for each region are shown in Fig. 5(B). The solid lines are reproduced from Fig. 6 (subpanels v). With the simple time domain expression for $J_{\mathrm{An}}(t)$ we can readily perform the convolution in eq. 10.

We apply the linearly increasing load for a period of $\tau_{\mathrm{dur}}$ depicted in Fig. 5(A) and perform many tests across the range ($10^{-2} \leq \tau_{\mathrm{dur}} \leq 10^6$) years, effectively varying the stress rate, and convolving this with the best-fitting $J_{\mathrm{An}}(t)$ expressions (Fig. 5B). In order to emulate measurements made by the unloading/loading scenarios in our dataset, we determine the resulting strain rate, $\dot{\varepsilon}$, from Eq. 10. We calculate $\dot{\varepsilon}$ values at various values of $\tau_{\mathrm{del}}$ across the range ($10^{-4}\tau_{\mathrm{dur}} \leq \tau_{\mathrm{del}} \leq 10^4\tau_{\mathrm{dur}}$) years, a range that covers all scenarios. Using the strain rate, we estimate the viscosity assuming a *Maxwell viscoelastic model* for which $\eta_{\mathrm{est}}$ can be calculated as (for the simple 1-D case)

$$\eta_{\mathrm{est}} = \sigma \left( \dot{\varepsilon} - \frac{\dot{\sigma}}{M_\infty} \right)^{-1}. \tag{13}$$

As argued previously, these viscosity estimates are capturing the apparent viscosity of the underlying Andrade model, i.e., $\tilde{\eta}_{\mathrm{An}}$, at its respective frequency. We can therefore estimate $f_{\mathrm{low}}$ as the frequency for which $\tilde{\eta}_{\mathrm{An}}$ is equivalent to $\eta_{\mathrm{est}}$. For each observation, we show the frequency band dictated by $\tau_{\mathrm{dur}}$ and $\tau_{\mathrm{del}}$ in Figs 5(C,D).

# 4 Results and Discussion

## 4.1 Resulting Subsurface Structure

In our main result (Fig. 6) we show the predictions of the anelastic model of Yamauchi and Takei (2016) (the "Pre-melt Maxwell-scaled" model, PM) and we discuss the subsurface structure in Section 4.1.1. In Section 4.1.2 we show the differences in both the thermodynamic constraints and the frequency dependent mechanical parameters that arise if we repeat the VBRc calculations with two other anelastic models, the Extended Burgers (EXB) model of Jackson and Faul (2010) and the Maxwell Frequency Scaling (MXF) model of McCarthy et al. (2011) (a predecessor of the PM model).

### *4.1.1 Results from Pre-melt (PM) Model*

The resulting steady state viscosity (colored lines) and temperature (black lines) profiles are shown in Fig. 6, subpanels iv. The resulting $T_P$ for ASE and AP are 1800 °C and 1700 °C, respectively. These are above the temperatures for average mantle, but somewhat in line with other estimates of sublithospheric mantle temperature. For example, the temperature model of An et al. (2015) beneath ASE at the depth range we constrain is in the similar range. However, they too converted seismic wave-speeds to temperature so any comparison most likely reflects the choice of conversion. An et al. (2015) used the empirical scalings determined by Goes et al. (2000), which estimated temperature-composition-seismic wave-speed relations using seismic wave-speed data across Europe and laboratory results.

The steady-state viscosity profiles (Fig. 6, subpanels iv) of each region show the structure most relevant to mantle convection timescales. The steady state viscosity structure is approximately $10^{20}$ Pa s and $3 \times 10^{21}$ Pa s averaged across our domain. These results are in accord with results from O'Donnell et al. (2017) who once more used seismic wave-speeds to estimate the thermodynamic structure, but incorporated laboratory are denoted $v_s^{obs}$ associated with $\omega_{obs}$. We extract $z_{LAB}^{obs}$ (at $\omega = \omega_{obs}$) from the lithospheric thickness models with the same horizontal averaging. For the uncertainty in these constraints, $\sigma_{obs}$, we have assumed $\pm 0.05$ km/s for the wave speed measurements and $\pm 5$ km for the LAB depth.

#### *3.1.2 Bayesian Inference of Thermodynamic State*

To determine the subsurface structure at each location down to 400 km depth, we consider depth dependent variations in thermodynamic and mechanical parameters only (localized radial stratification) and characterize this structure as a 1-D plate model. To produce a single plate model we solve the transient 1-D heat equation with variable thermal conductivity (as in Xu et al., 2004) and heat capacity (as in Berman & Aranovich, 1996) across a conductive plate to steady state using the finite difference forward model provided in the VBRc library. The solution depends on two input parameters: the potential temperature, $T_P$, and the thickness of the conductive lid, $z_{LID}$. Beneath the plate, the mantle temperature profile follows the adiabat.

We acknowledge that this is a simplification of the subsurface structure, but the smoothness of the $v_s$ profiles (Fig. 6, subpanels ii-iii) suggests that such an approximation is reasonable and the advantage is that we can fully characterize the thermal structure with the two input parameters $T_P$ and $z_{LID}$. Our aim is not to produce a detailed subsurface model, but to identify broad characteristics that might differ between our two locations. Many plate models are produced, which encompass $T_P = [1000, 1050, ..., 1800]$ °C and $z_{LID} = [50, 60, ...250]$ km. These alone characterize the temperature profile (i.e., $T$ and $P$ dependence) of the subsurface. In order to translate these into mechanical properties using the VBRc constitutive laws, we additionally provide uniform priors for grain size and melt fraction over the ranges $g = [0.001, 0.004, ..., 0.03]$ m and $\phi = [0.0, 0.005, .., 0.03]$, respectively. The result is a large suite of plate models which are converted to mechanical properties, like $v_s$ and $\tilde{\eta}_{UMM}$ across the relevant frequency bands.

The Bayesian inference follows in two stages. The first is to find $\mathcal{S}$ for the depth range (gray shaded bar, Fig. 6A,B) across which we extract $v_s^{obs}$. This constraint will be compared to forward predictions of the plate temperature profiles at the same depth, narrowing down the range of plausible plate profiles. The second is to use $z_{LAB}^{obs}$ to constrain $z_{lid}$ from the subset of remaining plate models.

Given $v_s^{obs}$, associated with the depth range, we determine posterior distributions of the state variables $\phi$, $T$, and $g$ (melt fraction, temperature, and grain size). The statement of our inference is

$$P(\mathcal{S}|v_s^{obs}) = \frac{P(v_s^{obs}|\mathcal{S})P(\mathcal{S})}{P(v_s^{obs})} \tag{6}$$

where $P(\mathcal{S}|v_s^{obs})$ is the posterior probability that represents how well constrained the state variables of interest are given $v_s^{obs}$, $P(v_s^{obs}|\mathcal{S})$ is the likelihood that describes how likely $v_s^{obs}$ is given $\mathcal{S}$, $P(v_s^{obs})$ is the probability of observing the measurement itself (used here as a normalizing factor), and $P(\mathcal{S})$ is the prior distribution of the state variables. $P(v_s^{obs}|\mathcal{S})$ is determined from a $\chi^2$-misfit with which we construct a Gaussian likelihood matrix where

$$P(v_s^{obs}|\mathcal{S}) = \frac{1}{\sqrt{2\pi\sigma_{obs}^2}} \exp\left(\frac{\chi^2}{2}\right), \tag{7}$$

and

$$\chi^2 = \frac{[v_s^{obs} - v_s(\omega_{obs}; \mathcal{S})]^2}{\sigma_{obs}^2}. \tag{8}$$

Here, $\sigma_{obs}$ is the uncertainty in the observation. To construct $P(v_s^{obs}|\mathcal{S})$ we use the VBRc to calculate $v_s(\omega_{obs}; \mathcal{S})$ from our plate model suite. For our prior distribution of the state variables we assume a uniform probability across all state variables (where the search range is noted above) and treated as independent of each other for simplicity.

To determine the lithospheric thickness we use the same Bayesian inference framework but the observation is now $z_{\mathrm{LAB}}^{\mathrm{obs}}$. With many definitions for the LAB, $z_{\mathrm{LAB}}^{\mathrm{obs}}$ may not be equivalent to $z_{\mathrm{LID}}$. Such definitions include the depth at which the largest negative value of $\partial v/\partial z$ occurs (Hopper & Fischer, 2018) or the maximum depth for which $v_{\mathrm{s}}$ anomalies are consistently greater than 2% (Conrad & Lithgow-Bertelloni, 2006). See Lau et al. (2020) for further discussion. However, the lithospheric thickness model of An et al. (2015) define $z_{\mathrm{LAB}}^{\mathrm{obs}}$ as the intersection of the base of the conductive lid, and so in this case, this definition is precisely $z_{\mathrm{LID}}$. The misfit in this case is calculated as:

$$\chi^2 = \frac{[z_{\mathrm{LAB}}^{\mathrm{obs}} - z_{\mathrm{LID}}(\omega_{\mathrm{obs}}; \mathcal{S})]^2}{\sigma_{\mathrm{obs}}^2}. \tag{9}$$

For our analysis below, we will select the profile of $\mathcal{S}(z)$ that maximizes $P(z_{\mathrm{LAB}}^{\mathrm{obs}}|\mathcal{S})$.

#### *3.1.3 Determination of Apparent Viscosity and Apparent Lithospheric Thickness*

With $\mathcal{S}(z)$ determined, the VBRc is used, with $\mathcal{S}(z)$, to output the full-spectrum profiles of $M^*(\omega)$, $Q^*(\omega)$, and $\eta^*(\omega)$ at each location. Using the latter, we introduce two simple parameters: the apparent viscosity $\tilde{\eta}(\omega)$, which is the absolute value of the complex viscosity and the apparent lithospheric thickness, $\tilde{z}_{\mathrm{LAB}}$. For $\tilde{z}_{\mathrm{LAB}}$, within any column of mantle rock, we may determine the Maxwell time as a function of depth ($\tau_{\mathrm{M}}(z)$), since this is the ratio of two depth-dependent material properties, $\eta_0(z)$ to $M_\infty(z)$. If we are interested in $\tilde{z}_{\mathrm{LAB}}$ at a given frequency, we find the depth at which $\tau_{\mathrm{M}}$ is equivalent to this frequency. This essentially marks the transition from elastic to viscous behavior for this frequency. This definition breaks down if the frequency is higher than any Maxwell frequency in the mantle, where we propose that $\tilde{z}_{\mathrm{LAB}}$ becomes essentially frequency independent. At these high frequencies the notion of a plate itself becomes unclear. We note that this is just one definition of many that exists for the LAB, though this definition highlights the frequency dependence of $\tilde{z}_{\mathrm{LAB}}$.

As is common in GIA studies that consider non-linear rheologies (e.g., van der Wal et al., 2010), we reinterpret the reported values of "$\eta$" and "$z_{\mathrm{LAB}}$" presented in Fig. 1 as *apparent* quantities, (where apparent lithospheric thicknesses are explored in greater detail in Lau et al. (2020) and we only briefly describe the method by which we determine $\tilde{z}_{\mathrm{LAB}}$ below).

### 3.2 Task 2: Determining Frequency Content of Observed Inferences of Uppermost Upper Mantle Viscosities and Lithospheric Thicknesses

Observationally derived estimates of lithospheric thickness and viscosity are generally obtained by combining knowledge of past load changes (ice sheets or lakes) with observations of deformation (GPS or reconstructions of paleo-water level) assuming a viscoelastic model. The estimates we have used for comparison against our VBR-driven profiles are all parameters reported from studies which applied geodetic constraints, and thus are subject to a range of simplifying assumptions. The estimates of both $z_{\mathrm{LAB}}$ and $\eta_{\mathrm{UMM}}$ have been compiled from the following investigations for each region (listed in order of decreasing timescale): For Amundsen Sea Embayment, we include the results from Whitehouse et al. (2012), who used relative sea level data to study the effects of long term GIA across the entire Antarctica region (and select their reported values associated with the ASE) and from Barletta et al. (2018), who used GPS derived measures of decadal-scale rebound at the Amundsen Sea Embayment. For the Antarctic Peninsula, decadal (Nield et al., 2014; Samrat et al., 2020) and centennial (Ivins et al., 2011; Wolstencroft et al., 2015) responses to ice mass change, once more measured by GPS, were used to derive viscosity estimates.

We will also investigate $z_{\text{LAB}}$ estimates derived from gravity-based data of Chen et al. (2018). These provide effective elastic thickness estimates of the Antarctic plate and should be interpreted as the thickness of a completely elastic lid overlying an inviscid mantle (where $\tilde{\eta}_{\text{UMM}} \to 0$). Such a result would be relevant at zero frequency. For a more detailed discussion on this matter, see Lau et al. (2020).

In order to compare our predictions of $\tilde{\eta}_{\text{UMM}}(\omega)$ and $\tilde{z}_{\text{LAB}}(\omega)$ to the observations presented in Fig. 1, we need to determine their appropriate frequency content. In all the cases, the data points fit were uplift rates extracted from a time series of GPS data. These single uplift rates were used to determine $z_{\text{LAB}}$ and $\eta$ and capture a whole range of frequencies embedded in not only the loading history (here, approximated by $\tau_{\text{dur}}$, the duration over which the loading/unloading event in question occurred) but also the delay in measuring the response (here, approximated by $\tau_{\text{del}}$, the time delay between the end of the event and when the observation of solid Earth deformation was made). For example, in Barletta et al. (2018), one of their analyses involved investigating the change in ice loss between 2002-2014 and GPS measurements were made throughout this time. In this case, $\tau_{\text{dur}} = 12$ years and $\tau_{\text{del}} = 0$ years. These designations of $\tau_{\text{del}}$ and $\tau_{\text{dur}}$ – tabulated and summarized for each observation in Table A1 – are approximate and our aim is to cover a wide enough frequency band to provide the most conservative estimate possible.

We will use these two important timescales to determine an appropriate frequency band, $[f_{\text{low}}, f_{\text{upp}}]$. For $f_{\text{upp}}$ we know that, given the timescale of any process, the highest possible frequency must be bound by $\tau_{\text{dur}}^{-1}$. The Fourier transform of a linear trend is dominated by these low frequencies and longer values of $\tau_{\text{del}}$ will result in any relatively high frequency response to diminish. But just how low is this bound? We will determine this lower bound by applying an empirical relationship based on replicating the loading history and measurements of deformation.

For this empirical relationship, we replicate the loading histories of our observations, $\sigma(t)$, apply this stress load to a given material of compliance $J$ and calculate the deformation, i.e., the strain, $\varepsilon(t)$. The compliance will be that associated with our upper mantle structures beneath ASE and AP (trends shown in Figs 6v,vi). Our generic stress history is depicted in Fig. 5(A).

There are two ways in which we may determine $\varepsilon(t)$. We could perform this calculation in the Fourier domain and multiply the stress history with $J^*(\omega)$ and take the inverse Fourier transform to produce the time domain response. This seems ideal since from experimental results, as output by the VBRc, we readily know $J^*(\omega)$. However, several factors complicate this approach: the frequency range in which we sample both $\sigma^*(\omega)$ and $J^*(\omega)$ are naturally entirely different (where the former is linear, whereas the latter is logarithmic) and inverting the Fourier transform with either of these options presents some limitations (see, e.g., Haines & Jones, 1988).

Instead, we choose to convolve the stress history with $J(t)$. That is,

$$\varepsilon(t) = \int_{-\infty}^{t} J(t-t')\mathrm{d}\sigma(t'). \tag{10}$$

Given that only $J^*(\omega)$ is determined by the VBRc and not $J(t)$, we obtain $J(t)$ by fitting $J^*(\omega)$ to that of an Andrade viscoelastic model, $J^*_{\text{An}}(\omega)$ (Fig. 2). We choose the Andrade model since it can capture the full spectrum of viscoelastic (elastic, transient, viscous) behavior with few model parameters (Cooper, 2002) where expressions for $J^*_{\text{An}}(\omega)$ and $J_{\text{An}}(t)$ are given by

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$$\eta_{\mathrm{est}} = \sigma \left( \dot{\varepsilon} - \frac{\dot{\sigma}}{M_\infty} \right)^{-1}. \tag{13}$$

As argued previously, these viscosity estimates are capturing the apparent viscosity of the underlying Andrade model, i.e., $\tilde{\eta}_{\mathrm{An}}$, at its respective frequency. We can therefore estimate $f_{\mathrm{low}}$ as the frequency for which $\tilde{\eta}_{\mathrm{An}}$ is equivalent to $\eta_{\mathrm{est}}$. For each observation, we show the frequency band dictated by $\tau_{\mathrm{dur}}$ and $\tau_{\mathrm{del}}$ in Figs 5(C,D).

## 4 Results and Discussion

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#### *4.1.1 Results from Pre-melt (PM) Model*

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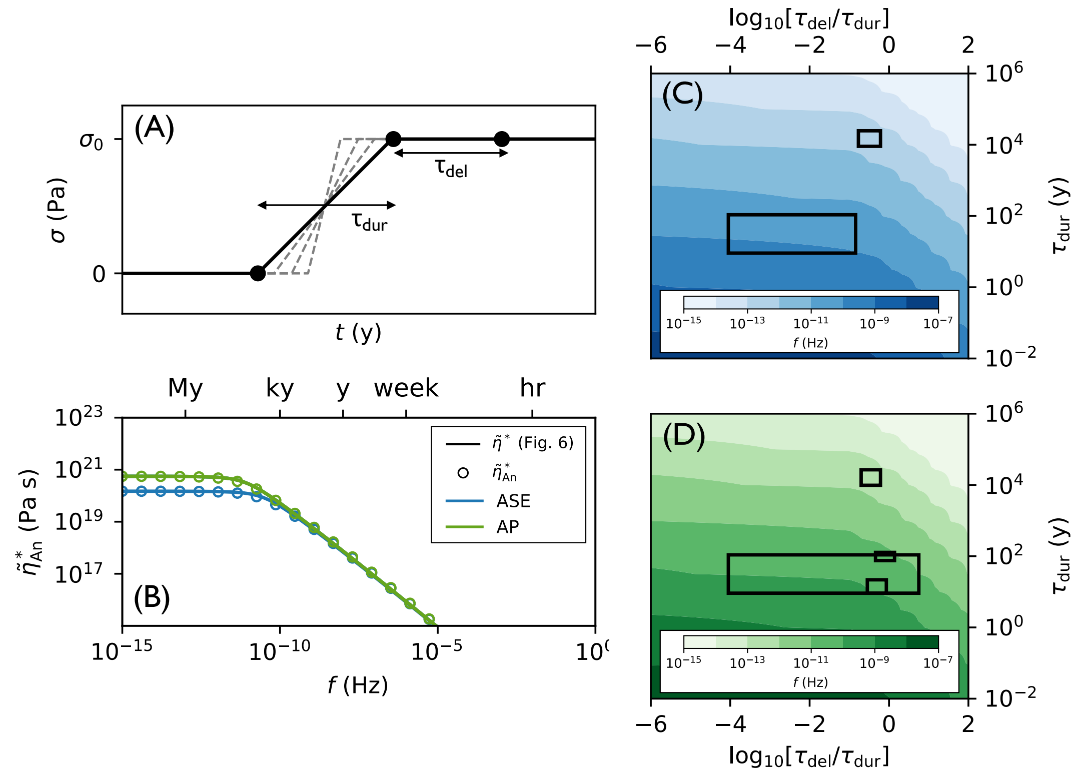


**Figure 5. Converting time domain observations to the frequency domain.** (A) Schematic depiction of applied stress history, $\sigma(t)$, labelling the two relevant timescales, $\tau_{\mathrm{dur}}$ and $\tau_{\mathrm{del}}$. (B) The apparent viscosities for each region of the 1-D Andrade model that $\sigma(t)$ is applied to. The solid lines are reproduced from Fig. 6(subpanels v) and circles are the result of finding the best fitting Andrade parameters for each region. (C,D) Contours mark the frequency for which the estimated $\tilde{\eta}_{\mathrm{est}}$ (assuming a Maxwell) is equivalent to $\tilde{\eta}_{\mathrm{An}}$ (i.e., where $\tilde{\eta}_{\mathrm{est}} = \tilde{\eta}_{\mathrm{An}}(f_{\mathrm{An}})$ for ASE (C) and AP (D)). The boxes are the associated $\tau_{\mathrm{dur}}$ and $\tau_{\mathrm{del}}$ ranges for each observation we include (see Fig. 1).

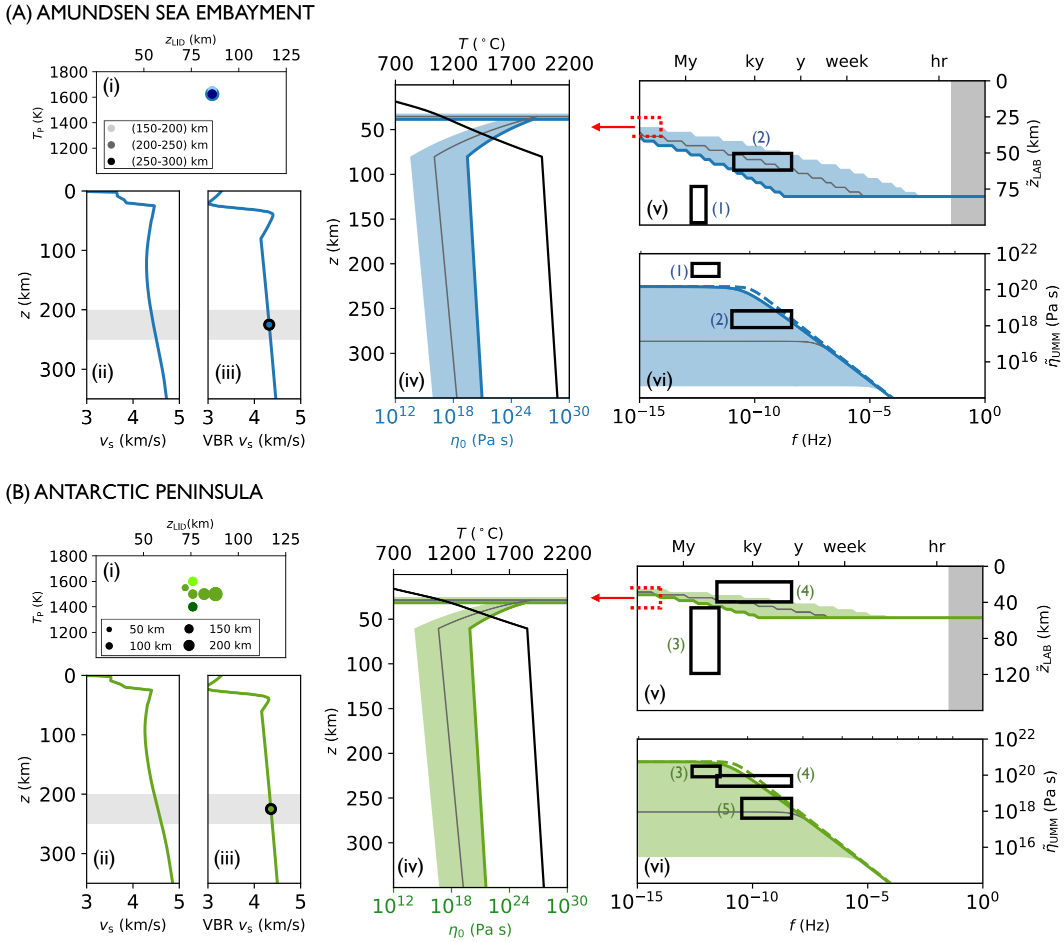


**Figure 6. Resulting Thermodynamic and Mechanical Properties for Case Regions.** For each region (A) and (B), subpanel (i) shows best-fit potential temperature-plate thickness pairs for differing horizontal and vertical averaging length-scales. The legend in A(i) corresponds to depth and in B(i) to horizontal length-scales and both legends are associated with the two subpanels (i). Subpanel (ii) shows the shear wave-speed, $v_s$, profiles at select regions in Fig. 1(C,D) (averaged over a lateral radius of 50 km) of the tomography model of Lloyd et al. (2019). The gray shaded region marks the depth window across which we used $v_s$ to constrain uppermost upper mantle temperature, where circles mark the resulting $v_s$ from the fitting procedure. (We tested other depth windows shown in panel (i).) Subpanels (iii-vi) are generated by the VBR fitting procedure (see Fig. 4, where (iii) shows the $v_s$ profile of best fitting plate model, while (iv) shows the associated temperature profile (solid black line). The following corresponding mechanical properties are: (iv) the steady state viscosity $\eta_0$ (solid colored line) where horizontal lines mark the base of the lithosphere; (v) the apparent lithosphere-asthenosphere boundary, $\tilde{z}_{\mathrm{LAB}}$, and (vi) the apparent uppermost upper mantle viscosity, $\tilde{\eta}_{\mathrm{UMM}}$, as a function of frequency, $f$, averaged across the depth of the domain (base of lithosphere to 400 km). In addition, the effect of steady state dislocation creep on these parameters across a stress range ($0 \leq \sigma \leq 10$ MPa) in denoted by the shaded regions (iii-vi). The solid colored and gray lines coincide with $\sigma = 0$ MPa and 1 MPa, respectively. The black boxes are the observationally derived estimates from Fig. 1. References are labeled: (1) Whitehouse et al. (2012); (2) Barletta et al. (2018); (3) Wolstencroft et al. (2015); (4) Ivins et al. (2011); (5) Samrat et al. (2020); The vertical gray shaded region spans the seismic band within which the thermodynamic conditions were constrained. The dashed red open box in subpanel (v) denotes an estimate of effective elastic thickness of Chen et al. (2018). In (vi) the dashed lines are the predictions of $\tilde{\eta}_{\mathrm{UMM}}$ with equivalent Maxwell model ($\eta_0$ and $M_\infty$) without transient elements.

scalings of Hirth and Kohlstedt (2003), Hansen et al. (2011), and Ohuchi et al. (2015) (for dislocation processes). Given the similarity of chosen flow laws, this is to be expected.

Turning now to frequency dependence of $\tilde{\eta}_{\mathrm{UMM}}$, we see that this parameter can vary significantly for different frequencies (Fig. 6 (v) and (vi) colored solid lines). In both locations, the predicted $\tilde{\eta}_{\mathrm{UUM}}$ decreases as a function of increasing frequency, which would result in higher frequency processes deforming a less viscous mantle.

The horizontal-colored line in Fig. 6, subpanels iv, shows the depth at which $\tilde{z}_{\mathrm{LAB}}$ occurs at the zero-frequency limit. This can be thought of as the true thickness of the plate – which, by many definitions within the literature, is the base of the top thermal boundary layer of mantle convection (Fischer et al., 2010). However, as Lau et al. (2020) argue, many studies infer *apparent plate thickness* at the frequency of the unlodaing process (colored bold lines, subpanels v). For example, changes in seismic velocity gradients, seismic anisotropy measurements, receiver functions and attenuation data (e.g. Hopper & Fischer, 2018; Mancinelli et al., 2017) have been used to infer $\tilde{z}_{\mathrm{LAB}}$ at frequencies of $\sim (0.01-0.1)$ Hz. This inference of $\tilde{z}_{\mathrm{LAB}}$ lies towards the far right of subpanel v (gray bar). The physical relationship between the seismic LAB and the convective LAB is discussed in more detail in Lau et al. (2020). As can be seen, moving towards lower frequency, $\tilde{z}_{\mathrm{LAB}}$ relaxes to significantly shallower depths as the asthenosphere beneath becomes increasingly viscous, impinging on the rigid plate above. At high frequency, we find that $\tilde{z}_{\mathrm{LAB}}$ is at a maximum as hotter conditions (i.e., deeper depths) must be reached for the elastic-to-viscous transition to occur. These panels demonstrate that one cannot assume that LAB values inferred on seismic timescales are appropriate for processes acting on convection or GIA timescales. An analogous discussion based on the Effective Elastic Thickness of plates at different timescales can be found in Watts et al. (2013).

While lateral variations in thermodynamics and the potential variability in the sampled subsurface of each observation can contribute to the variability in $z_{\mathrm{LAB}}$ and $\eta_{\mathrm{UMM}}$ within each region (including artefacts introduced as a consequence of inferring localized radially stratified (1-D) versus laterally variable (3-D) viscosity structures, see e.g., van der Wal et al., 2015; Lau et al., 2017; Crawford et al., 2018; Powell et al., 2020), our comparison between these observation-driven estimates and VBRc determined predictions of frequency dependent parameters illustrates that such processes are not only important to consider, but can possibly explain some of the variation in GIA-based estimates. We therefore suggest moving towards a framework that aims to map the continuous frequency trend of $\tilde{\eta}(\omega)$ through observations occurring at different frequencies – akin to mapping out the frequency dependence of $Q^{-1}(\omega)$ in seismic studies (Fig. 3). If a complete trend may be mapped, more concrete inferences about the underlying viscoelastic model may be made. However, unlike the seismic application, quantifying the frequency content for any given time-domain GIA process is not trivial and while we propose one approach here (Section 3.2), we argue that further work is required to better understand this relationship.

#### *4.1.2 Sensitivity of Thermodynamic State to other Experimental Constitutive Laws*

The anelastic constitutive model adopted has a strong influence on the determined thermodynamic state. For instance, the PM model results in much lower UMM temperatures than both the MXF and EXB models, but the difference is somewhat more pronounced between PM and EXB (compare subpanels (iii) in Fig. 7). As previously mentioned, this is likely due to the premelting effects included in PM which incorporates significant weakening behavior as temperatures approach the solidus. As such, to account for any dispersion effects of $v_{\mathrm{s}}$ at $f_{\mathrm{obs}}$, this additional weakening means that temperatures need not be so elevated as in the two models without premelting mechanisms.

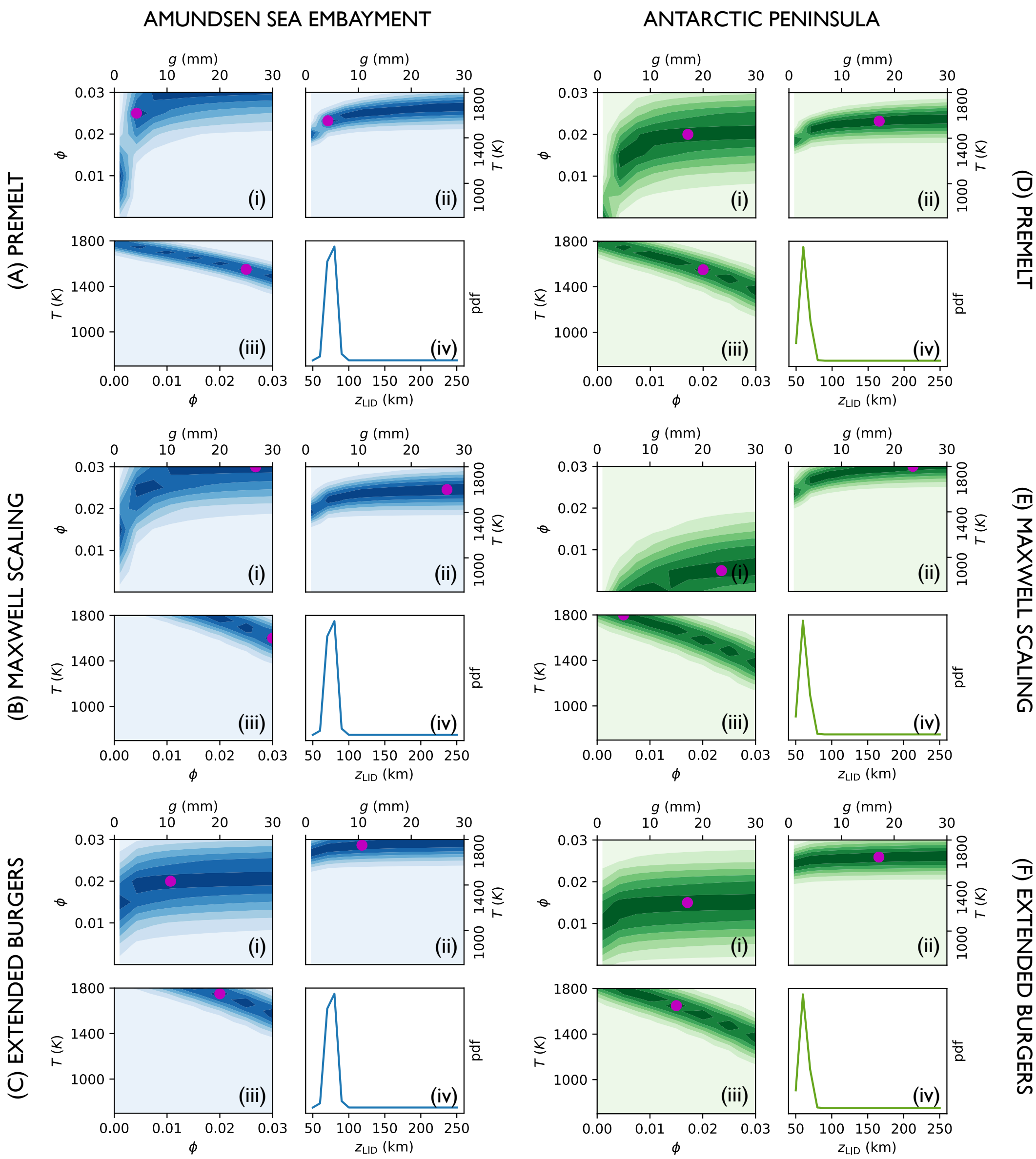


**Figure 7. Joint Posterior Probability Distributions of Thermodynamic Conditions.** For ASE (left panels) and AP (right panels), each row represents the anelastic model used. Subpanels (i-iii) display the joint posterior probability distributions for fitting $v_s$ between 200-250 km depth (shaded bars in Fig. 5A.ii,B.ii) between temperature, $T$, grain size, $g$, and melt fraction, $\phi$, as described in Section 3.1.2. The more intense colors denote increasing likelihood. The green dot is the maximum posterior probability. Subpanels (iv) display the posterior probability distribution of the thickness of the conductive lid, $z_{\mathrm{lid}}$.

Trade-offs between grain size and the other parameters are also significantly different between PM/MXF and EXB. The former two have a grain size exponent of 3, related to the direct incorporation of the steady state diffusion creep viscosity; thus the region of high likelihood shows significantly more curvature than that of EXB, which has a grain size exponent of 1.2. See subpanels (i-ii). For PM and MXF, if grain size was a known quantity, determining $T$ and $\phi$ becomes much more tightly constrained. The importance of the role of grain size (and subgrain size) in dissipation mechanism remains an open question with many implications for the estimation of melt fraction, $\phi$. All models show negative covariance between $T$ and $\phi$, which is to be expected. Havlin et al. (2021) demonstrated that placing a strong prior has a significant effect on the temperature estimates, but because we are focused here on the viscosity trade-offs, not the temperature inference, we leave this level of complexity for future work.

While we have used a completely forward modeling approach adopting standard constants as published for each anelastic constitutive models, an alternative method was used by Richards et al. (2020); Austermann et al. (2021). Using independent constraints on the thermodynamic structure of the upper mantle, they adopted the PM parameterization but calibrated several constants to given seismic observations. Interesting questions arise in regard to the lab-to-earth time scaling and to the degree of spatial homogenization between a laboratory-based grain-scale observation versus the macroscropic observation made as a seismic wave passes through mantle rock. This point is further elaborated on in the next section. The union of these two approaches, however, may provide a new path forward towards further constraining rheological laws.

### 4.2 Predicted and Observed Frequency Dependencies: grain-scale processes and their manifestation in GIA

In Fig. 6 we have also placed the apparent viscosities and plate thicknesses obtained from observational estimates (Fig. 1) in order to compare them against our modeled predictions. Across all regions some of the observations of $\tilde{\eta}_{\mathrm{UMM}}$ (subpanels vi) fit the predicted values well (colored solid lines) and some fall within the shaded regions, which we discuss in Section 4.2.1. A slightly less clear picture is seen with $\tilde{z}_{\mathrm{LAB}}$ (subpanels v), where several observations do not align with the predictions (solid colored lines). We note that $\tilde{z}_{\mathrm{LAB}}$ observations and predictions are particularly mismatched for AP. Here, we point to the inherent variability of this region and note that a greater degree of scatter exists for $T_{\mathrm{P}}$ and $z_{\mathrm{LID}}$ when choosing different lateral and depth averaging length-scales (compare Fig. 6B.i with 6A.i). As such, the lateral structure could be significantly contributing to the differences in observed $\tilde{z}_{\mathrm{LAB}}$ values, which is not captured by our simplistic local stratified model. In particular, it is important to consider that the continental lithosphere is comprised of a narrow peninsula rather than a large continental interior and the observations we selected span the stretch of the entire Peninsula. In addition, slightly beneath the depth range we consider, there is a subducting slab that may also affect GIA (Lloyd et al., 2019).

The non-loading related observations of effective elastic lithospheric thickness (Chen et al., 2018) appear as dashed red open boxes in Fig. 6, subpanel (v). The arrow attached denotes that such thicknesses are to be interpreted as the thickness are associated with the $\omega = 0$ endmember. As can be seen, these results are largely consistent with our predictions. This suggests that the steady state predictions of $z_{\mathrm{LAB}}$ from the viscous portion of the constitutive laws are relatively well calibrated with the Earth scale.

#### *4.2.1 Diffusion and Dislocation Creep*

What might we infer from these results about the activation of certain deformation mechanisms at the grain-scale?

The solid lines in Fig. 6 mark rheological parameters for which the underlying mechanism is diffusion creep. In GIA, the most commonly adopted viscoelastic model used to phenomenologically describe such creep is the Maxwell model (as is the case with all the observations included here). Now we assess the degree to which transient creep may contribute to the trend in $\tilde{\eta}(\omega)$ captured by our selected observations.

The solid colored lines in subpanels vi capture the trend of Pre-melt Maxwell-scaled model of Yamauchi and Takei (2016). The dashed colored lines are identical but with no transient creep component (i.e., no HTB), representing a Maxwell model with the same $\eta_0$ and $M_\infty$ values. The departure from $\tilde{\eta}$ between the Pre-melt and its Maxwell-only equivalent clearly occurs across a frequency band spanned by most of the processes we consider. While these discrepancies in $\tilde{\eta}$ seem slight in the figures, the differences between these two models can be significant, e.g., a factor of $\sim$ 2.3 at $10^{-10}$ Hz and $\sim$ 2.6 at $10^{-12}$ Hz for ASE and AP, respectively. These factors are emphasized in the plots of normalized $\tilde{\eta}$ in Lau et al. (2020). Our data here cannot distinguish the difference between the two trends of $\tilde{\eta}_{\mathrm{UMM}}$ but future studies may be designed to identify the degree of HTB transient creep and explore additional dissipative mechanisms in such deformation. An additional point to note is that while for the $\tilde{\eta}$ parameter, differences between the inclusion of the full anelastic model versus the Maxwell-equivalent is most prominent across the GIA frequency band, for $Q^{-1}(\omega)$, these differences are significant in the seismic band. These differences will likely affect the way our thermodynamic setting is inferred (see Section 3.1).

Transient diffusion creep rate is linear in stress and relevant at low levels of both strain and stress, where deformation probes the microstructure but does not modify it. Processes like GIA and seismic wave propagation are characterized by small strains ($\sim 10^{-5}$) and low stresses ($\sim$kPa), such that it is possible that transient diffusion creep dominates any transient response, and likely that grain sizes are not altered by those processes.

Towards estimating the potential effects of dislocation creep in the wide-band responses considered here, we incorporate steady state dislocation creep into the our results in Fig. 6 (without considering the transient role of dislocations). In subpanels (iv-vi) of Fig 6, shaded regions reflect the effect of steady state dislocation creep (Hirth & Kohlstedt, 2003) and encompass variations in $\eta_0$, $\tilde{\eta}_{\mathrm{UMM}}$, and $\tilde{z}_{\mathrm{LAB}}$ for stresses ranging ($0 \leq \sigma \leq 10$) MPa (where bold colored lines and fine gray lines coincide with 0 and 1 MPa, respectively). As shown by these regions, macroscopically, there is a reduction in the value of all parameters (Fig. 6) and the transition of $\tilde{z}_{\mathrm{LAB}}$ is shifted to higher frequency as the effective Maxwell time is now reduced. We note that, in certain parts of the plate, GIA processes reach such levels of stress (in Fig. A1 we show the deviatoric stress beneath the ice sheet over a representative GIA cycle). Since several of the observations fall within these shaded regions, it is possible that non-linear deformation is occurring to explain these estimates of $\tilde{\eta}$ and $\tilde{z}_{\mathrm{LAB}}$. These steady state effects across Antarctica have been shown to be significant (though without the consideration of transient creep) by, e.g., van der Wal et al. (2015), and for the lithosphere, by Nield et al. (2018).

#### *4.2.2 Can Observations Distinguish Between Experimental Constitutive Laws?*

Using the maximum likelihood thermodynamic parameters in Fig. 7 (pink circles), predictions of $\tilde{\eta}_{\mathrm{UMM}}(\omega)$ and $\tilde{z}_{\mathrm{LAB}}(\omega)$ using each anelastic model PM, EXB, and MXF were made. Fig. 8 shows the result (where the PM result is identical to that in Fig. 6). The frequency axis focuses on the lower frequency half of the results shown in Fig. 6. Unlike the thermodynamic parameters which show a large variation between the three anelastic constitutive laws, what is predicted at the macroscale is largely similar.

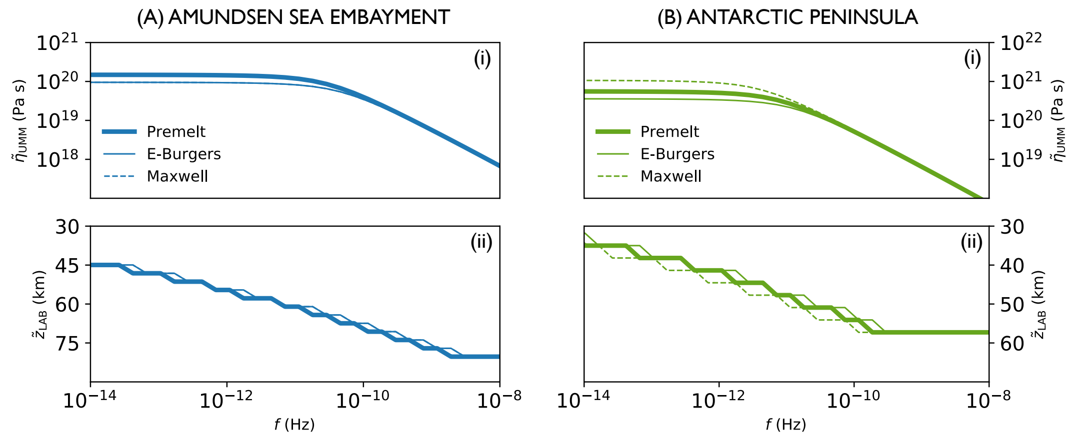


**Figure 8. Comparison of Apparent Viscosity and Lithospheric Thicknesses Between different Anelastic Models.** The frequency dependent predictions of $\tilde{\eta}_{\mathrm{UMM}}$ (i) and $\tilde{z}_{\mathrm{LAB}}$ (ii) for ASE (A) and AP (B) using the Pre-melt Maxwell-scaled model (Yamauchi & Takei, 2016), the Extended Burgers model (Jackson & Faul, 2010), and the Maxwell-scaled model (McCarthy et al., 2011). These were calculated using the maximum likelihood thermodynamic parameters in Fig. 7.

Indeed, within the uncertainty of the observations, it would be difficult to distinguish between the three models. Thus, determining which anelastic constitutive laws are more accurate may be difficult to constrain using macroscropic observations, since the trade-offs result in largely similar behavior across frequency. As suggested previously, between the approach of calibrating experimental parameterizations from macroscopic observations, like seismic data, it is clear that care must be taken and the conjunction of such an approach with the kind we have performed here provides a fruitful avenue of further investigation.

### 4.3 Implications for Ice Mass and Sea-level Change

Ice mass change, both past and present, span a wide frequency spectrum, and we show here that so too does the variation of the solid Earth's response to such perturbations. Based on our results, we suggest that by reinterpreting estimates of viscosity and plate thickness as *apparent viscosities* and *apparent plate thicknesses* and accounting for their frequency content may explain some of the variability in the values presented Fig. 1A,B. We also note that laboratory-based constitutive laws suggest that transient creep may play some role across the span of our observations (from rapid to $\sim$10 ky timescales).

Ultimately, ignoring deformation mechanisms acting across the wide frequency range of GIA processes may lead to mis-estimation of the sea-level response, whether those include rapid ice collapse, where studies typically invoke a purely elastic Earth (lithosphere *and* convecting mantle) (e.g., Gomez et al., 2010) or solid Earth responses modeled purely as Maxwell viscoelastic solids. For example, following our results shown in Fig. 6A, we predict that if the same amount of ice retreat in the Amundsen Sea area occurred over 1, 10, 100, and 1000 y, the uppermost upper mantle apparent viscosity would be $\sim 5\times 10^{17}$, $\sim 2\times 10^{18}$, $\sim 10^{19}$, and $\sim 7\times 10^{19}$ Pa s, respectively.

In not too dissimilar a region, Nield et al. (2016) demonstrated that, at the Siple coast, the importance of short term ice redistribution like the stagnation and reactivation of ice streams ($\sim$ 100 years) to crustal uplift, depends intimately on whether subsurface viscosity was below a certain threshold. On the Eastern side of the continent, King and Santamaría-Gómez (2016) explored the use of a Burgers rheology to explore the postseismic viscoelastic response. These studies represent other examples of the interactions between differing timescales of forcings and Earth's mechanical response and thus all these findings may have implications for the stabilizing effect of GIA on the Antarctic ice sheet, which affects predictions of future sea level change.

A further compounding factor on all timescales is that high frequency and high magnitude melt events will result in high strain rates, which may require the consideration of non-linear rheology that involves changes in the dislocation structure driven by these large external stresses. It is unclear how such extraneous stress regimes might alter GIA during these events, but one might expect that the apparent viscosity would be significantly reduced due to stress magnitude and the relatively high frequency of such intense melting events. So far non-linear rheologies have only been considered in isolation (Wu, 1992; van der Wal et al., 2010, 2013; Huang et al., 2019), but our preliminary calculations presented here show that these effects have repercussions across a wide frequency band.

## 5 Conclusions

The adoption of the Maxwell viscoelastic models derived from the early semi-analytical techniques (Wu & Peltier, 1982; Mitrovica & Milne, 2003) offered an elegant means to solve a complicated viscoelastic system and fit a whole range of sea-level and geodetic observations. The realization, however, that temperature effects alone result in lateral variations in viscosity that span orders of magnitude required a distinct departure from these semi-analytic techniques and a movement towards computationally demanding finite-element methods that continues today (e.g., Wu & van der Wal, 2003; Zhong et al., 2003; Latychev et al., 2005).

With a growing richness in datasets that capture increasingly subtle signals of ice melt, we believe the next level of complexity must be met. Our results outlined here have highlighted a potential pathway towards considering both thermodynamic variations within Earths subsurface and the nature of the forcing (both frequency and stress) for GIA-related processes. We have adopted two simple plate models for Western Antarctica and the Antarctic Peninsular and have self-consistently determined their full spectrum mechanical behavior. We have applied a means of assessing the frequency contents of observational results so that these observationally derived apparent quantities may be compared to prediction.

While our modeling of the subsurface structure is highly simplified, we have demonstrated that our current understanding of Earth deformation, derived from microphysical investigations that operate on timescales appropriate for the laboratory setting, shows significant promise in explaining some of the variability we observe on the planetary scale and across timescales that capture Earth's long and nuanced history. Looking to the future, we envision both the inclusion of viscoelastic models in GIA that move beyond the Maxwell model (e.g., Peltier et al., 1980; Chanard et al., 2018; Ivins et al., 2020), the determination of the frequency content within measurements of time-domain processes like GIA, and the search to map out the continuous function $\tilde{\eta}(\omega)$, rather than discrete values of $\eta_0$, to help improve predictions of cryosphere-solid Earth responses as rates of ice sheet melting and collapse increasingly occur on shorter timescales.

## Appendix A Stress Evolution during GIA

To demonstrate the stress levels that can be reached in the uppermost upper mantle, we perform a simple viscoelastic loading calculation adopting a Maxwell viscoelastic model. We use the formulation of Mitrovica and Milne (2003) though with simplifications: we assume longitudinal symmetry, no rotation, and that we only calculate solid earth deformation in response to ice growth (i.e., we do not consider the effects of the ocean); and thus, only focus on the vicinity of the ice sheet. The input parameters include the elastic and density profile of PREM (Dziewonski & Anderson, 1981). We impose a lithosphere of 200 km thickness and values of $\eta_0$ of $5\times10^{20}$ Pa s and $5\times10^{21}$ Pa s across the upper mantle (200 to 670 km depth) and lower mantle (670 to 2900 km depth), respectively. The growth of an ice sheet of 1,000 m over ~5,000 years results in stress levels within the lithosphere and asthenosphere that reach ~MPa – sufficient to induce changes in dislocation structure. The results are summarized in Fig. A1.

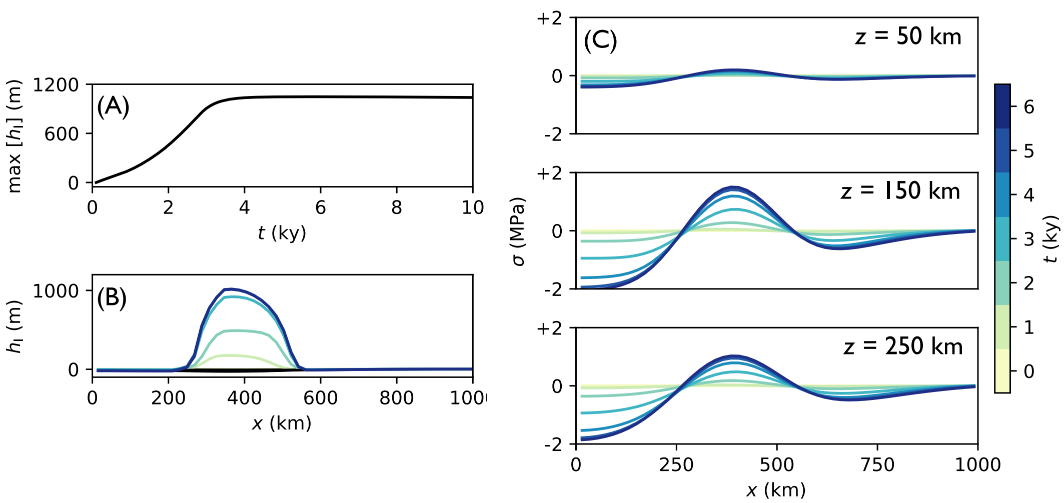


**Figure A1. Stress Evolution During GIA.** (A) Maximum ice height as a function of time. (B) Ice profiles as a function of distance from the North Pole ($x$). Each line represents the profile at a given time indicated by the colorbar. The solid black lines are the associated bedrock elevations. (c) The stress profiles at different depth, $z$, slices, as a function of $x$. Each line represents the stress profile at a given time indicated by the color bar.

## Appendix B Timescales Assigned to Observations

| Reference | Region | $\tau_{\text{dur}}$; $\tau_{\text{del}}$ | Comment |
|---|---|---|---|
| Whitehouse et al. (2012) | ASE | 15,000 y; 0 y | Investigation last glacial maximum with relative sea level data. |
| Barletta et al. (2018) | ASE | 102 y and 12 y; 0 y | Ice change in the Amundsen Bay region with two rates of ice loss between 1900-2002 and 2002-2014, measured from 2002 onwards. |
| Wolstencroft et al. (2015) | AP | 15,000 y 5,000 y | Last glacial maximum, 21,000 y BP, to 6,000 y BP with deformation measured by GPS today. |
| Ivins et al. (2011) | AP | 200 y and 80 y; 700 y and 70 y | Deformation from the Little Ice Age (1030 CE1300 CE) and modern ice mass change (18501930) recorded by GPS over a duration of 1993-2007. |
| Nield et al. (2014) | AP | 10 y; 7 y | Collapse of Larsen B ice shelf between 1993-2002, recorded by GPS stations from 2009. |
| Samrat et al. (2020) | AP | 15 y; 7 y | Ice mass loss of Larsen A and B ice shelves measured by GPS extended to up to 2018 from Nield et al. (2014). |

**Table B1.** Tabulation of all the assigned values of $\tau_{\text{dur}}$ and $\tau_{\text{del}}$ for each observation and the associated explanation. For each value of $\tau_{\text{dur}}$ and$\tau_{\text{del}}$ listed, we consider $\pm 20\%$ of these values also.

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