Relativistic runaway electron avalanches within complex thunderstorm electric field structures

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Abstract

Relativistic runaway electron avalanches (RREAs) are generally accepted as a source of thunderstorms gamma-ray radiation. Avalanches can multiply in the electric field via the relativistic feedback mechanism based on processes with gamma-rays and positrons. This paper shows that a non-uniform electric field geometry can lead to the new RREAs multiplication mechanism - “reactor feedback’, due to the exchange of high-energy particles between different accelerating regions within a thundercloud. A new method for the numerical simulation of RREA dynamics within heterogeneous electric field structures is proposed. The developed analytical description and the numerical simulation enables us to derive necessary conditions for TGF occurrence in the system with the reactor feedback
Observable properties of TGFs influenced by the proposed mechanism are discussed.

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Key Points:

- Heterogeneity of thunderstorm electric field can lead to the enhancement of energetic particle flux
- A new technique of modeling particle propagation in electric field is developed
- The model with nonuniform electric field fits the observed directional pattern of TGFs

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Abstract

Relativistic runaway electron avalanches (RREAs) are generally accepted as a source of thunderstorms gamma-ray radiation. Avalanches can multiply in the electric field via the relativistic feedback mechanism based on processes with gamma-rays and positrons. This paper shows that a non-uniform electric field geometry can lead to the new RREAs multiplication mechanism - "reactor feedback", due to the exchange of high-energy particles between different accelerating regions within a thundercloud. A new method for the numerical simulation of RREA dynamics within heterogeneous electric field structures is proposed. The developed analytical description and the numerical simulation enables us to derive necessary conditions for TGF occurrence in the system with the reactor feedback Observable properties of TGFs influenced by the proposed mechanism are discussed.

1 Introduction

High-energy radiation originating from thunderclouds can be registered by detectors on satellites and on the ground surface. Intense bursts of photons with energy 10 keV – 100 MeV lasting 0.1–5 ms are called terrestrial gamma-ray flashes (TGFs) and are usually observed from satellites (Fishman et al., 1994). Thunderstorms ground enhancements (TGEs) and gamma-glows can be observed under thunderclouds and have a duration up to several hours (Chilingarian, 2011; A. Gurevich et al., 2016; Torii et al., 2009). The gamma-radiation of thunderclouds is caused by bremsstrahlung of runaway electrons, which accelerate and multiply in the electric field, forming relativistic runaway electron avalanches (RREAs) (A. Gurevich et al., 1992; J. Dwyer et al., 2012). Numerical estimations show that 10^4–10^{13} RREAs, about 10^6 runaway electrons in each one, are required to cause a TGF observable from space (J. R. Dwyer & Cummer, 2013; A. V. Gurevich & Zybin, 2001; Khamitov & Nozik, 2020). There are two models of TGF production discussed up to day. The lightning leader model assumes that avalanches emitting gamma-rays originate from thermal electrons accelerated in the strong local electric field of the lightning leader tip (Moss et al., 2006). The relativistic feedback model firstly introduced in (J. R. Dwyer, 2003) considers the multiplication of avalanches and can lead to the self-sustaining development of RREAs: generation of a large number of avalanches even without an external source of high-energy particles (J. R. Dwyer, 2007).

The relativistic feedback model describes the creation of new avalanches by positrons or energetic photons of the initial avalanche in the region with above-critical electric field. A new avalanche can be created by a particle that moves towards the start on an initial avalanche. It should be noted that all the particles, including gamma-photons, are radiated mainly along with the avalanche development. Thus, the efficiency of the relativistic feedback mechanism is limited by the probability for a positron or gamma-ray to obtain the speed in the direction reverse to the movement of the avalanche. The efficiency of creation of new avalanches can be higher if it does not require a reversal of the particle movement. To discuss this possibility, let us consider the electric field structure, which is nonuniform on a scale greater than the avalanche length. In this case, particles emitted by the initial avalanche can reach regions with the direction of the electric field different from that in the region of the initial avalanche. Thus, the change of direction required for a particle to create a RREA will be smaller than that in the uniform field. For this reason, the initial avalanche can create more new avalanches. Moreover, each of the new avalanches emits particles mainly along itself, and some of them can reach the region of the initial avalanche, enhancing it. The described processes lead to the creation of new avalanches and amplification of the initial one, and hereinafter are referred to as the "reactor model". The new kind of feedback occurring in the uniform electric field is called "reactor feedback".

This paper presents the numerical simulation and the analytical description of the reactor model. The spatial distribution and the time dependence of gamma-ray flux are calculated. The conditions for TGF occurrence within a reactor thundercloud are derived. In the 5 section predictions of the model are compared with observation data and conclusions of other modeling studies. Question of the electric cloud structure and applicability of the model of the reactor structure is addressed.

2 Random reactor model

The reactor model describes the interaction of avalanches developing in regions of the strong (above-critical) electric field, which are further called "cells". The "reactor feedback" can occur in a thundercloud with a complex electric structure, consisting of several cells with different directions of the electric field. Figure 3 illustrates the interaction of cells in the reactor structure. Let a seed electron form a RREA within one of the cells. The RREA produces gamma-rays via bremsstrahlung. On thunderstorm altitudes the mean free path of gamma-rays is about several hundred meters or more (400 m for 1 MeV gamma on 10 km altitude (M.J. Berger & Olsen, 2010)), so gamma-photons can move through regions with the under-critical field, and reach another cell and produce RREAs in it. A new RREA, similarly to the initial one, radiates gamma-rays, which can generate RREAs in other cells of the thundercloud. The closer the direction of the field is to the direction to the other cell, the greater the probability of creating a new avalanche in the initial cell by the radiation of secondary avalanches. By the described way, the complexity of the electric field structure can lead to self-sustainable RREA multiplication due to the exchange of high-energy particles between cells. In other words, RREA in different strong field regions can amplify each other. A great number of RREAs developed under the influence of the reactor feedback can be sufficient for the production of TGF (Zelenyi et al., 2019).

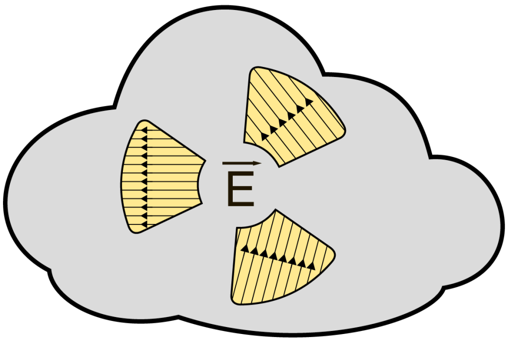

Figure 1. The scheme of the electric field distribution in a cloud, within the model of the completely random reactor. Yellow regions are "cells" with the quasi-uniform field sufficient for the RREA development. The electric field outside cells is under-critical.

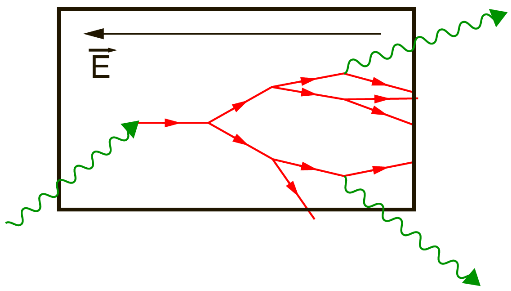

Figure 2. The electron avalanche created by a gamma-photon produces new gamma-photons in the strong field region, one of the cells of the reactor structure.

Fig.2 shows the diagram of gamma-ray multiplication in the strong field region. High energy photon interacts with air via Compton scattering, photo-effect of electron-positron pair production, leading to the production of the high energy electron, which might produce a RREA. The RREA emits gamma-rays, leading to the multiplication of the initial high-energy photon. The electric field outside the cell is under-critical, so the ener-

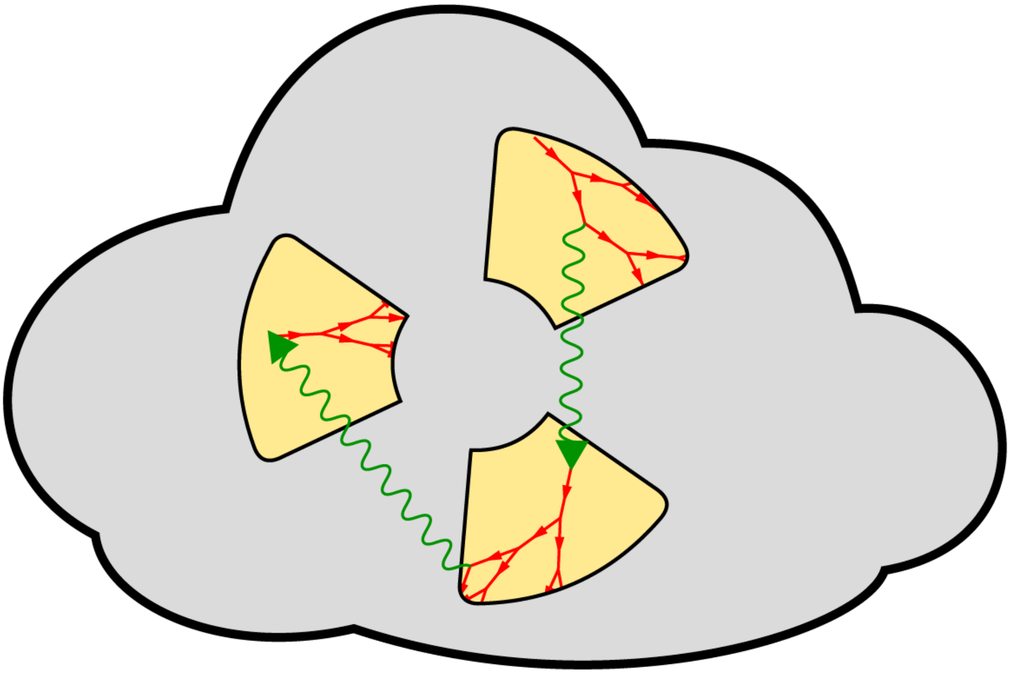

Figure 3. The dynamics of relativistic runaway electron avalanches in complex thunderstorm electric field structures.

getic electrons are quickly absorbed by the air. If an energetic electron reaches another cell, it can initiate a RREA, similarly to a gamma-photon.

The reactor feedback can be conveniently discussed within the electric field of the structure hereinafter called "completely random" (Zelenyi et al., 2019), which consists of a huge number of cells with different directions of the electric field, Figure 1. The multi-cell random structure exhibits a chain reaction of gamma-ray interactions with cells. The described high-energy particle dynamics brings to mind the behavior of neutrons in a nuclear reactor. For this reason, the concept of exchange of relativistic particles between strong field regions is called the "reactor model".

3 Simulation

The movement of runaway electrons is defined by the electric field, while bremsstrahlung gamma-rays can move through the cloud uninfluenced by the electrical structure. Consequently, RREAs dynamics within a thundercloud can be described as RREAs developing in a region with the strong quasi-uniform electric field and energetic particles propagating between strong field regions and initiating RREAs in it. For this reason, behavior of RREAs in the complex electric field structure can be conveniently modeled in two stages: microscopic (RREA development in strong field regions, simulated using GEANT4) and macroscopic (propagation of particles between regions of RREA development, described by the original model). The approach presented below requires rather less computational time than straightforward modeling.

3.1 Microscopic simulation

The microscopic Monte Carlo modeling describes the development of a RREA within a cell, calculates cross-sections of high-energy particle interactions. The microscopic modeling is carried out for different values of the initial speed of the electron for calculating energy, momentum, and spatial distributions of resulting particles. Figure 4 presents the dependence of gamma-ray attenuation length and the mean free path before the production of runaway electrons on the air density, obtained by GEANT4 simulation. It turned out that the vast majority of the electrons produced by gamma-rays have critical energy. For this reason, the dependence of the length of runaway electrons production on the electric field is negligible.

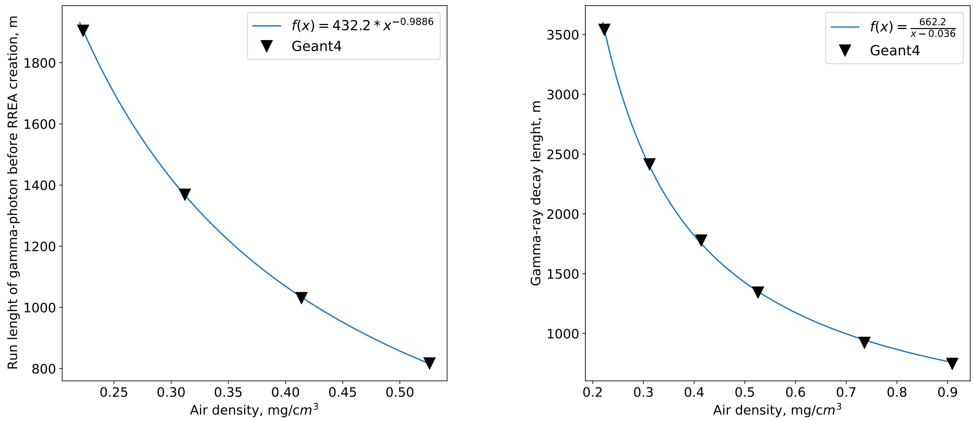

Figure 4. The results of modeling gamma-rays using GEANT4 (black triangles) and approximation (blue curves). Gamma-ray energy is 7 MeV, cell length is 4 km. Characteristic gamma-ray decay length depending on the air density (left) and characteristic length of runaway electron production by gamma-rays, depending on the air density (right).

A high-energy particle interacting with a cell produces a seed electron that can initiate a RREA. The momentum direction of a generated seed electron is random, thus, in general, this electron has to turn in the direction against the cell electric field to produce a RREA. Consequently, one of the crucial parameters of the energetic particle is the probability of a reversal of the generated electron. In this paper, the GEANT4 simulation was carried out to calculate the reversal probability depending on the parameters of the electric field structure. Seed electrons were launched from the middle of the cell with fixed energy and momentum direction. The resulting RREA was investigated using the detector modeled at the edge of the cell. If the seed electron produces a RREA then it has reversed, otherwise, it was absorbed and it did not have any further impact on RREAs dynamics within the thundercloud. In this study, the electron reversal probability was calculated as the number of reversed seed electrons divided by the total number of launched seed electrons.

The electron reversal probability depends on the electric field in the cell, air density, seed electron energy, and momentum direction. The calculated dependences of average parameters of energetic particles on the electric field and air density are used in the macroscopic simulation described in the next subsection. To obtain the average reversal probability, the probability of electron reversal calculated using Geant4 was convoluted with seed electron energy spectrum and momentum direction distribution. The spectrum of seed electrons is defined by the RREAs spectrum. It is known from previous works that RREAs spectrum relatively slightly depends on the electric field value and air density (Babich, 2020). Moreover, seed electrons producing a RREA in one cell are usually emitted by the RREA developing in other cells. For this reason, we apply the approximation of the similar spectrum of seed electron for all parameters of the electrical structure. The probability of a seed electron to produce a RREA was modeled for an isotropic source of 1 MeV seed electrons, Figures 5 and **??**. In the case of the under-critical electric field, RREAs can not develop, which means that the probability of RREA generation is 0. For the electric field higher than the critical value the probability is close to 1. The characteristic spatial scale of electron reversal is below 2 meters for 1 MeV electron, which is much less than the typical size of the cell.

$$P\left(\frac{E}{\rho}\right) = \begin{cases} \frac{1}{2} \cdot \left(1 + erf\left(3.0378\frac{E}{\rho} - 0.0074\right)\right) & , 3.0378\frac{E}{\rho} - 0.0074 \leq 0 \\ \frac{1}{2} \cdot \left(1 + \frac{3.0378\frac{E}{\rho} - 0.0074}{1 + \left(3.0378\frac{E}{\rho} - 0.0074\right)}\right) & , 3.0378\frac{E}{\rho} - 0.0074 \geq 0 \end{cases} \tag{1}$$

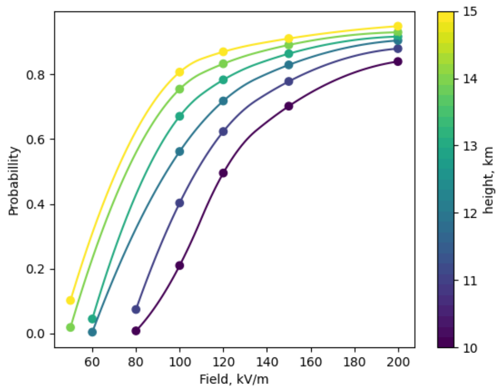

Figure 5. Probability for a high energy seed electron to produce a RREA depending on the electric field value: the GEANT4 modeling results (dots) and the quadratic interpolation (lines).

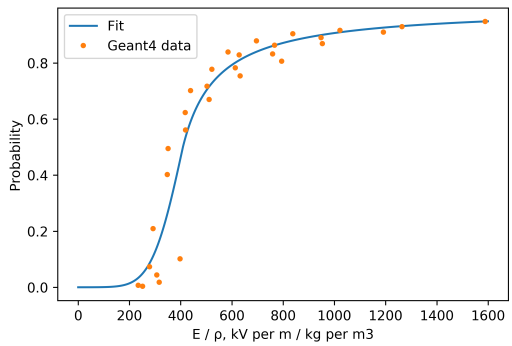

Figure 6. Probability for a high energy seed electron to produce a RREA depending on the ratio of the electric field strength and the air density ($\frac{E}{\rho}$): the fit with sigmoid function 1 for the GEANT4 simulation for the 1 MeV electron.

3.2 Macroscopic simulation

Contrary to the microscopic simulation carried out using GEANT4, the macroscopic modeling operates with averaged parameters of energetic particles and does not take into account individual events of particle interaction. The interaction of cells is caused mainly by high-energy photons because their movement is not influenced by the electric field and the interaction with air is rather smaller than that for electrons. The impact of the runaway electron transport between cells can be neglected. For this reason, the performed macroscopic modeling characterizes the propagation of high-energy photons between strong field regions within the thundercloud.

The macroscopic model is implemented in Kotlin (Nozik, 2019). The source code and distributions of the macroscopic model implemented in Kotlin (Nozik, 2019) are available in (altavir, 2020). Simulation describes two types of particles: runaway electrons (with energy above Gurevich critical energy for given altitude and electric field) and photons (with the energy above the energy of runaway electron) capable of creating runaway electrons via the photo-ionization process. Each particle is characterized by the origin point. The movement of the particle is described by the velocity vector and energy.

Within a macroscopic simulation, cells can be implemented in two different ways. The first way is to divide the thundercloud volume into cells before the simulation run. The second way is to generate cells on the run: in this case, the start of the cell is defined as the point of a RREA production. The second option is implemented in the modeling described below.

The macroscopic simulation is based on the following assumptions:

- A photon moves in the same direction until the interaction. Distance between the origin point and the interaction point is described by the exponential dependence with mean free path calculated in microscopic modeling.
- A photon produces the electron with the same energy and direction. In other words, we assume that all electron production is caused by photo-effect. Our calculations show that the assumption does not strongly affect the general modeling results, though for the typical parameters energetic electron production via the Compton effect usually dominates.
- The direction of the electric field in each point is random.
- Bremsstrahlung photons of the RREA are generated at a fixed distance (the avalanche length) in the direction of the electric field at the point of the RREA origin. All generated photons have the same energy and move alongside the electric field in the RREA origin. The number of bremsstrahlung photons generated by the RREA follows the Poisson distribution with a given average which is called the local multiplication factor.

The multiplication factor is the ratio of the number of particles in one generation to that of the previous generation. The local multiplication factor is the mean number of gamma-photons generated by one gamma-photon in one multiplication process. The described simplifications give the model an important advantage: the opportunity to characterize the system dynamics using only two parameters — the size of the modeling region (the size of the cloud, which is considered cubic) and the local multiplication factor. The avalanche length has a small effect on the simulation results. The local multiplication factor describes many parameters including the angle between the electron velocity and the electric field in the strong field region (for large angles, the electron "dies" without starting the avalanche) and the actual distribution of the field inside the cell.

Figure 7 illustrates the rate of production of high energy photons in the completely random reactor model for different values of the multiplication factor. A lifetime of one generation is the time of photon propagation before its interaction, it can be estimated as cell length plus gamma-ray free path length divided by the speed of light, which gives about 1 μs. Figure 7(a) demonstrates the dramatic increase of the number of gamma-photons on the time scale of TGF. Figure 7(b) is obtained for the electric field structure of less size (1200 m instead of 1250 m), which leads to a decrease of multiplication factor down to 1. As a result, the system exhibits a TGE-like mode with approximately constant energetic particle flux.

4 Analytical completely random reactor model

The developed analytical model of the avalanche dynamics is based on the following assumptions:

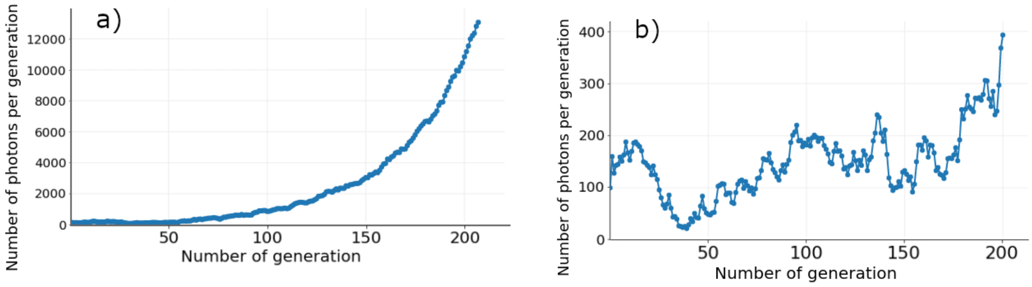

Figure 7. The dependence of the number of gamma-rays on the gamma-ray generation number, calculated using the macroscopic simulation (altavir, 2020). (Cell length is 300 meters, mean free path of photons is set to 100 meters, the initial number of high energy photons is 100.) (a) a TGF-like mode with the rapid increase of the number of gamma-rays: multiplication factor is 1.5 (cloud size is 1250 meters). (b) a mode similar to a gamma-ray glow or TGE: a long-duration flux of approximately constant intensity. The multiplication factor is close to 1 (cloud size is 1200 meters).

- The electric field is completely random at any given point, which makes gamma-ray local multiplication isotropic.
- The electric field outside cells is under-critical.
- The critical electric field and the air density in the cloud is uniform.
- All gamma-rays have the same energy determined by bremsstrahlung of RREAs.
- Gamma-photon emitted by the RREA is generated in the point of interaction of the initial gamma-photon leading to the production of this RREA.
- The energetic photon can leave the system in two ways: by escaping the thundercloud or by losing energy via the production of a runaway electron.
- The system is axially symmetrical. The simulated volume (the thundercloud) is a cylinder with a height H and a radius R.

With the assumptions above, the dynamics of gamma-rays in the thundercloud can be described by the reactor diffusion equation:

$$D\Delta n(t,r,z) - c\Sigma n(t,r,z) + \nu c\Sigma n(t,r,z) = \frac{\partial n(t,r,z)}{\partial t} \tag{2}$$

($n(\vec{r},t,z)$ is gamma-ray concentration, $D = \frac{c\lambda}{3}$ — diffusion coefficient, λ — mean free path length for gamma-rays, $\Sigma = \frac{1}{\lambda_{\gamma\to e^-}}$ — mean macroscopic cross-section of runaway electron production by a gamma-photon, ν — local multiplication factor. All the mentioned parameters are defined by the structure of the electric field and by air density.

The term $-c\Sigma n$ is responsible for gamma-ray extinction via the production of runaway electrons. The term $\nu c\Sigma$ is responsible for gamma-ray production via RREA bremsstrahlung. The creation of a RREA takes a considerable amount of energy from the photon, which is absorbed shortly afterward. For this reason, we use the assumption that macroscopic cross-sections of gamma extinction and gamma multiplication are equal, as two parameters describe the same process. Strictly speaking, the cross-section of gamma-ray multiplication is a little bit higher than that of the extinction because one gamma might be energetic enough to produce more than one RREA.

The Laplace operator for the system with the axial symmetry is written as follows:

$$\Delta_2 + \frac{\partial^2}{\partial^2 z} \tag{3}$$

The departure of particles from the cloud is described by the following boundary condition:

$$n(t,r,z)|_{r=R} = 0, \tag{4}$$
$$n(t,r,z)|_{z=0,H} = 0 \tag{5}$$

Let us present an eigenfunction as the product of the spatial and the temporal parts:

$$n(r,z,t) = N_{km}(t)n_{km}(r,z) \tag{6}$$

Taking into account the boundary conditions, n_{km} are taken as eigenfunctions of the Laplace operator:

$$n_{km}(r,z) = J_k(\frac{a_k r}{R})sin(\frac{(m+1)\pi z}{H}) \tag{7}$$

Here a_k are zeros of Bessel functions. The temporal part of the solution is described by the following equation:

$$N_{km}(t)\left(\frac{3(\nu-1)}{\lambda\lambda_{\gamma\to e^-}} - (\frac{a_k}{R})^2 - (\frac{(m+1)\pi}{H})^2\right) = \frac{3}{\lambda c}\frac{dA_{km}}{dt} \tag{8}$$

For simplicity, the initial condition is chosen as follows:

$$N_{km}|_{t=0} = N_0 = const, \tag{9}$$

which leads to the following solution:

$$n(r,z,t) = N_0 \cdot \sum_{k,m=0}^{\inf} J_k\left(\frac{a_k \cdot r}{R}\right) sin\left(\frac{(m+1)\pi z}{h}\right) e^{\varepsilon_{km} t}, \tag{10}$$
$$\varepsilon_{km} = \frac{\lambda c}{3}\left(\frac{3(\nu-1)}{\lambda\lambda_{\gamma\to e^-}} - \left(\frac{a_k}{R}\right)^2 - \left(\frac{(m+1)\pi}{H}\right)^2\right) \tag{11}$$

An infinite feedback occurs when at least one of the terms in 11 has $\varepsilon_{km} > 0$. The higher k and m, the lower ε_{km}. Consequently, if ε_{00} is slightly more than 0 then other terms decreases over time. Taking into account that the thundercloud becomes discharged earlier than the second term of the sequence starts to grow only the first term determines the gamma-ray dynamics:

$$n(r,z,t) = N_0 \cdot J_0\left(\frac{a_0 \cdot r}{R}\right) sin\left(\frac{\pi z}{H}\right) e^{\varepsilon t}, \tag{12}$$
$$\varepsilon = \frac{\lambda c}{3}\left(\frac{3(\nu-1)}{\lambda\lambda_{\gamma\to e^-}} - \left(\frac{2.405}{a}\right)^2 - \left(\frac{\pi}{h}\right)^2\right) \tag{13}$$

$a_0 = 2.405$. ε is called the “global multiplication factor”: if $\varepsilon > 0$ then the number of gamma-rays produced by the reactor-like thunderstorm grows exponentially, in other words, the reactor system explodes. Thus, the criterion of the reactor explosion is as follows:

$$\frac{\lambda c}{3}\left(\frac{3(\nu-1)}{\lambda\lambda_{\gamma\to e^-}} - \left(\frac{a_0}{R}\right)^2 - \left(\frac{\pi}{h}\right)^2\right) > 0 \tag{14}$$

The criterion of reactor explosion not only depends on the local properties of the electrical structure characterized by the local multiplication factor ν. Whether there is a gamma-ray explosion or not depends on the size of the thundercloud as well. The larger the reactor, the smaller the value of the electric field is required for the explosion. It should be noted that for the spatially infinite thundercloud ($R = \infty$, $H = \infty$) the criterion of the explosion takes the form $\nu > 1$.

Gamma-ray flux generated by the random reactor thundercloud can be simply derived from the formula $\Phi = D\nabla n$. As TGFs are observed mostly from the top and from the bottom of thunderstorms, we consider the case of observation close to the zenith or nadir, then the flux is as follows:

$$|\Phi(r,t)|\Big|_{z=0,H} = \frac{\lambda c}{3}\frac{\partial n(r,z,t)}{\partial z}\Big|_{z=0,H} = \frac{\pi\lambda c}{3H} N_0 \cdot J_0\left(\frac{2.405 \cdot r}{R}\right) e^{\varepsilon t} \tag{15}$$

The equation 15 describes the exponential growth of the flux typical of the beginning of TGF and characterizes the dependence of flux on the radius from the axis of the system.

4.1 Local multiplication factor

The local multiplication factor is the number of gamma-photons produced by the initial gamma-photon on the current stage of the RREA development. The assumption of the arbitrary direction of the electric field in each point of the storm means that the electric field consists of multiple occasionally-directed cells. Let the value of the electric field within a cell be E, air density — ρ, cell length — L. These parameters determine the local multiplication factor, which can be described analytically in the following way. Let gamma-ray produce a runaway electron with random momentum direction at the beginning of a cell. Let the probability of RREA formation be equal to P. This probability includes electron reversal so that it moves in the direction opposite to the electric field direction and RREA formation after reversal. In this study, the probability of reversal is calculated using GEANT4. $\lambda_{e^-\to\gamma}$ is the mean path of a runaway electron before production of the energetic photon which is able to produce a runaway electron avalanche. The RREA e-folding length can be described as following, (J. R. Dwyer, 2007):

$$\lambda_{RREA} = \frac{7300\ keV}{E - \frac{\rho}{\rho_0} \cdot 276\ \frac{kV}{m}} \tag{16}$$

Here ρ_0 is the air density under normal conditions. If a gamma-ray produces a RREA at the beginning of the cell with probability P, then number of gamma-rays radiated by this avalanche can be found from the following equation:

$$dN_\gamma(z) = \frac{dz}{\lambda_{e^-\to\gamma}} \cdot P \cdot e^{\frac{z}{\lambda_{RREA}}} \tag{17}$$

Consequently, the RREA during all the development produces the following number of gamma-rays:

$$N_\gamma(L) = P \cdot \frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}} \cdot \left(e^{\frac{L}{\lambda_{RREA}}} - 1\right) \tag{18}$$

In the completely random reactor model a gamma-photon can interact with air in the cell at any point. Therefore local multiplication factor should be found as follows:

$$\nu = \int_0^L \frac{dl}{L} N_\gamma(l) \tag{19}$$

Thus the local multiplication factor is defined according to Formula 20:

$$\nu = \frac{P}{L}\frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}} \left(\lambda_{RREA} e^{\frac{l}{\lambda_{RREA}}} - \lambda_{RREA} - L\right) \tag{20}$$

4.2 Local multiplication factor with electron transport between cells

In the previous section, it was assumed that cells of the completely random structure exchange only gamma-rays with each other. In this section, we take into account the exchange of runaway electrons between cells. The RREA development in a cell results in the following number of runaway electrons:

$$P \int_0^L \frac{dl}{L} e^{\frac{l}{\lambda_{RREA}}} = P\frac{\lambda_{RREA}}{L}\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right) \tag{21}$$

and gamma-rays:

$$\frac{P}{L}\frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}} \left(\lambda_{RREA} e^{\frac{l}{\lambda_{RREA}}} - \lambda_{RREA} - L\right) \tag{22}$$

A runaway electron can enter the neighboring cell both along the field and against the field. If the runaway electron enters the cell along the electric field, it decelerates and does not produce gamma-rays. On the contrary, entering the cell against the electric field accelerates the electron, allowing the RREA creation. In the completely random case, the probability of the electron acceleration in the cell is 0.5. Let us assume that the probability of RREA creation in the cell by a runaway electron is $\tilde{P}$. Therefore, on average, $0.5{\cdot}\tilde{P}$ of transported runaway electrons form a new avalanche, which influences the local multiplication factor as follows. Runaway electrons reaching another (second) cell can form RREAs at the beginning of the second cell. That leads to the following number of runaway electrons at the end of the second cell:

$$0.5\tilde{P}P\frac{\lambda_{RREA}}{L}\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right) \cdot e^{\frac{L}{\lambda_{RREA}}} \tag{23}$$

RREAs developed in the second cell radiate gamma-rays:

$$0.5\tilde{P}P\frac{\lambda_{RREA}}{L}\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right) \cdot \frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}}\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right) \tag{24}$$

Similarly, the number of energetic particles in the third cell will differ from that in the second cell by the factor $0.5\tilde{P}e^{\frac{L}{\lambda_{RREA}}}$. Therefore, local multiplication factor influenced by runaway electron transport can be calculated as follows:

$$\nu = \frac{P}{L}\frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}}\left(\lambda_{RREA}e^{\frac{l}{\lambda_{RREA}}} - \lambda_{RREA} - L\right) + 0.5\tilde{P}P\frac{\lambda_{RREA}}{L}\cdot$$
$$\cdot\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right)\frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}}\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right)\cdot\sum_{0}^{+\infty}0.5\tilde{P}e^{\frac{L}{\lambda_{RREA}}} \tag{25}$$

To consider a finite thundercloud we should limit the number of terms in the sum to $\approx \frac{L}{R}$, where R is a characteristic size of the thunderstorm. In what follows, for simplicity, the infinite sum is calculated. For the case $0.5\tilde{P}e^{\frac{L}{\lambda_{RREA}}} < 1$ the local multiplication factor gets the following form:

$$\nu = \frac{P}{L}\frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}}\left(\lambda_{RREA}e^{\frac{l}{\lambda_{RREA}}} - \lambda_{RREA} - L\right) +$$
$$+\frac{\lambda_{RREA}}{\lambda_{e^- \to \gamma}}\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right)\cdot\frac{0.5\tilde{P}P\frac{\lambda_{RREA}}{L}\left(e^{\frac{L}{\lambda_{RREA}}} - 1\right)}{1 - 0.5\tilde{P}e^{\frac{L}{\lambda_{RREA}}}} \tag{26}$$

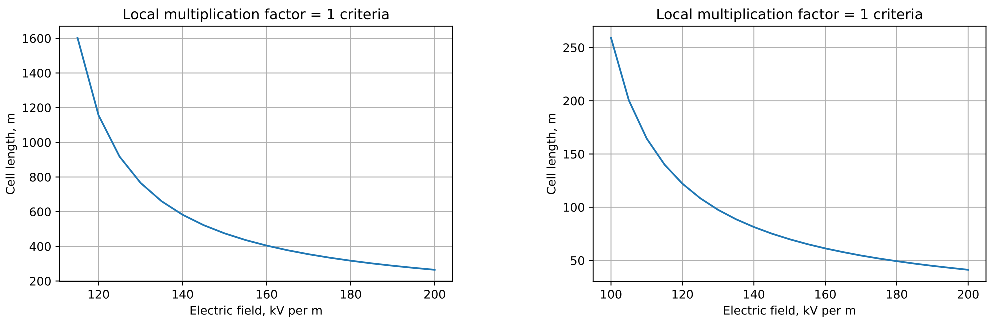

Figure 8. The diagram of the rate of multiplication of gamma-photons within the completely random reactor model. The interaction of cells is ensured by gamma-photons propagation between cells (left plot, Formula 20) and propagation of gamma-photons and runaway electrons (right plot, Formula 26). On the curve the local multiplication factor $\nu = 1$. Above the curve $\nu > 1$ and gamma-rays multiply, under the curve $\nu < 1$ and avalanche fades.

Figure 8 presents the criteria of the enhancement of the energetic flux in a cloud calculated for the altitude 10 km. The criterion is derived from the condition for the local multiplication factor: $\nu > 1$. It could be seen from the comparison of Figure 8(right) to Figure 8(left), that the condition of the generation of gamma flash is rather achievable for the case with the transport of runaway electrons between cells.

The proposed analytical is convenient for predicting the system behavior before the detailed simulation and provides a physical explanation for qualitative relations. Formula 14 might be used as a necessary condition for infinite feedback in reactor-like systems. The local multiplication factor should be estimated via formula 20 for solely gamma-ray exchange between cells and via formula 26 for a reactor system with the exchange of gamma-photons and runaway electrons between cells. The crucial parameters of the reactor system are the electric field strength, cell length, and air density, which affect local relativistic runaway electron dynamics, which influences local gamma-ray multiplication. Air density and thunderstorm size affect macroscopic gamma-ray dynamics, its transport between cells. It should be noted that the dependence of the system behavior on the thunderstorm size is significant for the thunderstorm size less than 1.5 km (Figure 7), while for a larger system the size-depending term becomes negligible, Formula 14.

5 Discussion

The paper analyzes the dynamics of RREAs in the thundercloud with the complex electric field distribution, demonstrating the impact of the new kind of positive feedback in the development of RREAs — "reactor feedback". The proposed reactor model can describe both short intensive gamma-ray bursts like TGFs and long-scale particle fluxes like TGEs and gamma-glows, depending on the intensity of the interaction of the strong field regions in the cloud, Figure 7.

The proposed "cell" concept can be considered as a next step on the way of description of the RREA development in real thunderclouds, preceded by the model with uniform electric field widely used in numerical modeling (J. R. Dwyer, 2007; Skeltved et al., 2014; Chilingarian et al., 2018). The system of cells can be used as a more accurate model of any electric field structure which creates RREAs.

RREA dynamics in the cylindrical electric field of a lightning leader nonuniform field are analyzed in (Kutsyk et al., 2011; Babich, 2020). The system considered in (Kutsyk et al., 2011; Babich, 2020) demonstrates the feedback effect of RREA amplification influenced by the system geometry, similarly to the present study. The cylindrical structure of the electric field can be considered as the reactor structure consisting of thin radial cells with the electric field directed to the axis of the cylinder. The RREA developing in radial direction emit bremsstrahlung towards the axis and in this way amplifies RREAs in the opposite cells. The results of (Kutsyk et al., 2011; Babich, 2020) supports the idea that the heterogeneity of thunderstorm electric field might lead to feedback processes in RREA dynamics, enhancing fluxes of relativistic particles in a thunderstorm. We would like to note that an arbitrary heterogeneity of the electric field can enhance the feedback because the radiation of the initial avalanche would easily reach other strong field regions.

The reactor model can be conveniently applied to study real clouds within the main widely used models of the cloud charge distribution. The cloud electrical structure is often described as a "classical tripole" or a "dipole", though more complicated multi-layer geometries are discussed as well (Williams, 1989; Ette & Olaofe, 1982; Rust & Marshall, 1996). The widely used layered models regardless of the number of charge layers include the system of two regions with the quasi-uniform critical electric field of opposite direction. This system experiences the reactor feedback because runaway electrons accelerated in one cell move towards the cell with the opposite direction of the electric field. Other simple geometries exhibiting the reactor feedback are "cylindrical" and "spherical" discussed in (Kutsyk et al., 2011). All mentioned geometries might lead to infinite feedback in RREA dynamics, while the electric field strength and cell length required for reactor explosion depend on the parameters of the charge structure. The modeling results shown in Figure **??** enable estimating the size of "cells" sufficient for infinite feedback being in range 50–500 m. The reactor model demonstrates the multiplication of avalanches if the cell is larger than the avalanche e-folding length. Investigations of the electrical structure of clouds, including direct measurements, indicate its heterogeneity. The results of the balloon- and aircraft-based measurements in thunderclouds show that the scale of heterogeneity of the electric field can lie within the estimated range of infinite feedback: 50–500 m (Marshall et al., 1995; Marshall & Stolzenburg, 1998; Stolzenburg & Marshall, 2008). For this reason, we assume that the proposed mechanism of the reactor feedback can be important for the RREA development in real clouds.

The presented consideration of the random reactor model provides new opportunities for diagnostics of TGF and TGE mechanism. The crucial property of the RREA development is its gamma-ray radiation pattern. The conventional RREA mechanism in the uniform electric field leads to bremsstrahlung in a narrow cone directed backward to the electric field (J. R. Dwyer, 2008). In the random reactor model, the electric field in each cell might have any direction, thus the pattern can be wide-angled or even quasi-isotropic, Formula 11. The thundercloud with the reactor structure might radiate gamma-rays up, down, and, possibly, sideways with approximately the same brightness, depending on the electric field geometry. The analysis of the angular distribution of observed TGFs leads to the conclusion that TGF sources have a wider angular distribution than directed one (Hazelton et al., 2009; Gjesteland et al., 2011). However, a wide gamma-ray emission angle implies much more relativistic particles within thunderstorms during TGFs than with directed radiation to fit observable from space gamma-ray fluxes. In a reactor-like thunderstorm infinite feedback is achieved via interaction between different parts of the storm, allowing the creation of a great number of relativistic particles: in the runaway electron avalanche mechanism RREAs are developed in the strong field region, while in the reactor model all the cloud is engaged in the RREA production and several strong field regions amplify RREAs within each other.

The reactor mechanism can produce a TGF or a TGE depending on the electrical structure of the cloud, which defines the global multiplication factor 11. The feedback effect can lead to the auto-tuning of the charge distribution. increasing the discharging for higher values of the electric field and slowing the discharging as the electrical field strength decreases. A nearby lightning flash usually terminates a TGE or gamma-glow (Chilingarian et al., 2017; Wada et al., 2019). The reactor model provides a new possible relation between a lightning flash and a RREA. Namely, a lightning flash can decrease the electric field below the critical value in some part of a cloud, while the field in other regions would remain sufficient for the RREAs development. In other words, some strong field regions will be destroyed and some will remain, making possible the flux continuing after a lightning discharge in the cloud. The described effect might lead to multi-pulse TGF or TGF afterglow if the global multiplication factor falls below zero after the lightning discharge. What is more, a charge transition caused by a lightning discharge might increase the heterogeneity of the electric field in the cloud, leading to TGF or gamma-glow initiation via the reactor feedback. The described possibility is a mechanism of energetic flux production by a lightning discharge, different from the lighting leader model. The local increase of the electric field in the reactor model may explain the TGE-like intensification of energetic flux following a gamma-glow reported in (Wada et al., 2019).

We assume that the RREAs can demonstrate the reactor-like behavior in a wide variety of heterogeneous electric field structures, as far as the only necessary condition is that the bremsstrahlung of one avalanche reaches the cell where other avalanches develop. Therefore, the investigation of the thunderstorm electric field structure is crucial for understanding the physics of the RREAs and their gamma-emission.

6 Conclusions

In this paper, a new feedback mechanism for relativistic runaway electron avalanches dynamics is proposed. The "reactor" feedback arises in complex thunderstorm electric field structures due to high energy particles exchange between different strong field regions in a thundercloud. The analysis of the completely random reactor model shows that the feedback can cause the self-sustaining development of relativistic electron avalanches, which can lead to an energetic particle flux of long duration, similar to a gamma-glow or TGE. Moreover, the presented mechanism with more intense feedback can produce a TGF. Based on the analytical consideration and modeling results we show that strong field regions of size 50–500 m with different field direction are required for the reactor feedback. The distinguishing observable feature of the reactor mechanism is a wide-angle direction diagram of the resulting gamma-radiation, which is in accordance with measurements reported in (Hazelton et al., 2009; Gjesteland et al., 2011). We assume that the RREAs can demonstrate the reactor-like behavior in a wide variety of heterogeneous electric field structures, as far as the only necessary condition is that the bremsstrahlung of one avalanche reaches the cell where other avalanches develop. Therefore, the investigation of the thunderstorm electric field structure is crucial for understanding the physics of the RREAs and their gamma-emission.

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Appendix A: Gamma-radiation dynamics in the model with relativistic feedback

Let us consider the dynamics of RREAs with relativistic feedback and external source of seed particles. We aim to calculate the dependence of the particle flux on time for the cell of length L with the flux of seed electrons I_{SE}. The feedback coefficient γ is the number of RREAs produced by one avalanche. The increase in the number of avalanches in case of zero flux of seed particles is provided only by feedback:

$$dN_{RREA} = N_{RREA}(0) \cdot (\gamma - 1) \cdot \frac{c}{2L} dt \tag{27}$$

where $N_{RREA}(0)$ is the number of runaway electron avalanches in the cell at the initial moment. $\frac{2L}{c}$ is the duration of one feedback cycle, equal to twice the time of photon propagation through the cell. The factor $(\gamma - 1)$ means that $(\gamma - 1)$ new RREAs are born in one cycle of feedback. If $\gamma = 1$, then the avalanches are self-sustaining: $N_{RREA} = const$. The solution to 27 is:

$$N_{RREA}(t) = N_{RREA}(0) \cdot e^{\frac{c}{2L}(\gamma-1)t} \tag{28}$$

For $\gamma = 1$ all RREAs developed in the cell remain there. Consequently, in case of the flux of seed electrons I_{SE}, the accumulation of avalanches occurs as follows:

$$dN_{RREA} = I_{SE} \cdot S \cdot dt \tag{29}$$

where S is the area of the cell perpendicular to the field direction. Thus, under the considered conditions, the number of avalanches grows linearly with time:

$$N_{RREA}(t) = I_{SE} \cdot S \cdot t \tag{30}$$

In the presence of feedback and seed particles the number of avalanches takes the following form:

$$dN_{RREA} = N_{RREA} \cdot (\gamma - 1) \cdot \frac{c}{2L} dt + I_{SE} \cdot S \cdot dt \tag{31}$$

By replacing $\alpha = N_{RREA} + \frac{2L}{c(\gamma-1)} S I_{SE}$, the equation is reduced to an equation with separable variables, the solution of which with initial condition $N_{RREA}(0) = 0$ is as follows:

$$N_{RREA}(t) = N_{RREA}(0) e^{\frac{c}{2L}(\gamma-1)t} + I_{SE} S \frac{2L}{(\gamma-1)c}(e^{\frac{c}{2L}(\gamma-1)t} - 1) \tag{32}$$

There is an alternative approach to the same problem. Let $I_{SE}Sd\tau$ of seed particles arrive at the cell at the moment τ. Then by the time t they will multiply due to relativistic feedback, and their number will become equal to $dN_{RREA} = I_{SE}Se^{\frac{c}{2L}(\gamma-1)(t-\tau)}d\tau$. Integration of this expression over τ leads to the Formula 32 describing the number of avalanches in a cell. Provided that one RREA produce $N_{particles\ from\ RREA}$ particles (for example, high energy photons or runaway electrons) during one feedback cycle, the total number of particles of these type depends on time as follows:

$$N_{particles\ total}(t) = N_{particles\ from\ RREA} \cdot N_{RREA}(t) \tag{33}$$

The same formalism can be applied to the analytical completely random reactor model. We define the global multiplication factor $\varepsilon = \gamma - 1$. Then the increase in the concentration of high energy photons in the point (r, z) at the moment t is following:

$$dn(r,z,t) = \frac{\partial n_{cosmic}}{\partial t}dt + n(r,z,t)\cdot \varepsilon dt \tag{34}$$

The solution of 34 satisfying the initial condition $n(r, z, 0) \equiv n_0$ is:

$$n(r,z,t) = n_0 e^{\varepsilon t} + \frac{\frac{\partial n_{cosmic}}{\partial t}}{\varepsilon}\cdot\left(e^{\varepsilon t} - 1\right) \tag{35}$$

The presented consideration leads to the following conclusion, common for all models of feedback in the dynamics of RREAs. In case of $\gamma = 1$ the flux grows linearly, the avalanches are self-sustaining. The linear increas of the number of RREAs $N_{RREA}(t) = I_{SE}St$ can be obtained from Formula 32 by Taylor expansion in the small parameter $(\gamma - 1)$. Therefore, even at $\gamma = 1$, TGF can be generated by the feedback mechanism. For $\gamma > 1$ gamma-ray flux increases exponentially in time. $\gamma < 1$ leads to the exponential decay of the flux with the asymptotic constant value, which is higher than RREAs radiation without feedback by a factor $\frac{1}{1-\gamma}$. For $(\gamma < 1)$ the factor $\left(e^{\frac{c}{2L}(\gamma-1)t} - 1\right)$ decreases in time (this factor is negative, and $(\gamma - 1)$ in the denominator of Formula 32 is also negative, therefore, the total number of avalanches is positive). The resulting dynamics of the number of RREAs is an exponential growth gradually turning into a constant value. The greater the γ, the greater the final constant flux. Thus, strong feedback is not required to describe gamma-glows and TGE. Finally, if the initial gamma-ray flux is high, for example, just after TGF peak, and $(\gamma < 1)$, then the flux will decay exponentially. This fact might explain TGF afterglows within the framework of models of RREAs dynamics with feedback (relativistic feedback model or reactor model), Figure 9.

7 Appendix B: Microscopic simulation

The modeling of RREA evolution is carried out using GEANT4 toolkit in two stages. In the first stage, a mean free path of a gamma-photon is calculated. A rectangular volume with air is modeled, at the end of which the detector is located. A 7 MeV gamma-photon is launched in the direction of the detector. Increasing the distance to the detector, we find the mean free path of the gamma-photon. The results of modeling for different values of the air density are presented in Figure 4. The mean free path does not depend on the magnitude of the electric field, since gamma-photons do not interact with the electric field directly. The second stage of modeling provides information on the characteristic run length of the gamma-photon before the generation of an electron with critical energy. A gamma-photons is launched in a rectangular air cell with a 4 km length. As the particle moves in the cell, secondary particles are generated. Information on secondary electrons is registered at the moment of birth and then they are taken out of consideration in order to get rid of their influence on the simulation results. After receiv-

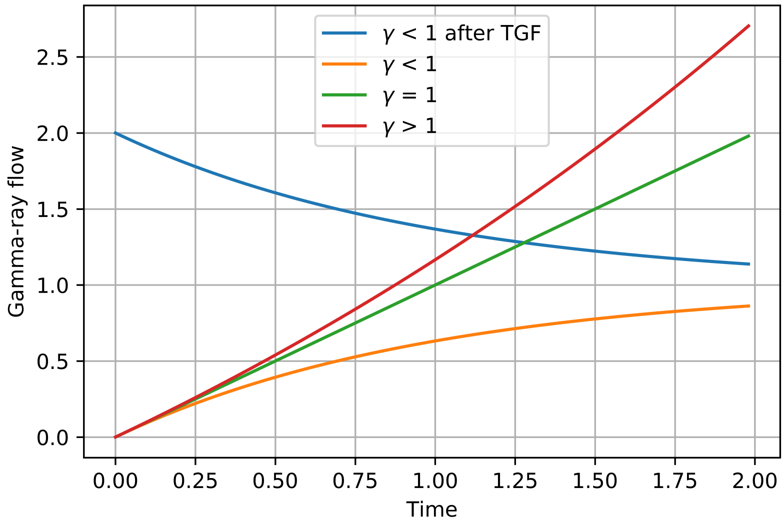

Figure 9. Dynamics of the gamma-ray flux within the reactor feedback model in the presence of an external constant source of seed particles, approximation of modeling results, Formula 32. γ is the average ratio of the number of particles in the next generation of feedback to the number of particles in the current generation.

ing information about the created electrons, one can filter out particles whith under-critical energy for the corresponding electric field strength. The filtered data may be approximated as follows: $dN_{e^-}(z) = N_\gamma(0)e^{-\frac{z}{\lambda_\gamma}}\frac{dz}{\lambda_{e^-}}$, λ_γ — gamma flow attenuation length, $N_\gamma(0)$ — initial number of gamma, λ_{e^-} — mean free path of gamma governing the production of a RREA, $N_{e^-(z)}$ — number of electrons with the energy above the threshold.

8 Appendix C: macroscopic simulation

The developed macroscopic simulation does not directly track the time, instead each particle is characterized by a number of its generation, which is increased by one for each particle created by the considered one. The lifespan of one generation is the time for a relativistic particle to travel back and forth through a cell 50–150 m long, which is about 1 μs. All particles in one generation are computed in parallel with automatic scaling on the number of processor cores present in the system. The computation of the generation is done lazily, which means that the next generation is computed only when it is requested. The described approach allows to automatically stop the simulation when the number of particles in the simulation exceeds the given threshold, leading to a significant optimization of modeling of the exponential process.