## Some lava flows may not have been as thick as they appear

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### Abstract

Individual lava flows in flood basalt provinces are composed of sheet  $p\bar{a}hoehoe$  lobes and the 10-100 m thick lobes are thought to form by inflation. Quantifying the emplacement history of these lobes can help infer the magnitude and temporal dynamics of these prehistoric eruptions. Here we use a phase-field model to describe solidification and re-melting of sequentially-emplaced lava flows to explore additional processes that may lead to thick flows. We calibrate model parameters using field measurements at Makaopuhi lava lake. We vary the thickness of individual flows and the time interval between eruptions to study the interplay between thermal evolution, flow thickness and emplacement frequency. Our theoretical analysis shows that, if the time between emplacement is sufficiently short, reheating and re-melting may merge sequentially emplaced flows — making flows appear thicker than they actually were. Our results suggest that fused flows could be another mechanism that creates apparently thick lava flows.

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### Key Points:

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- Lava flows can heat and melt underlying flows if the flows are hot enough;
  - Superimposed lava flows can merge if erupted in close enough succession;
  - Macroscopic structures may not reflect the original flow thicknesses.

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- theoretical analysis shows that, if the time between emplacement is sufficiently short,
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- thicker than they actually were. Our results suggest that fused flows could be another
- <sup>25</sup> mechanism that creates apparently thick lava flows.

### <sup>26</sup> Plain Language Summary

The observation of thick basaltic lava flows has long been explained by inflation. Here we explore an additional mechanism that could explain the formation of thick lava flows, where a sequence of thinner lobes that are emplaced on top of each other could fuse into one large flow. Our theoretical analysis suggests the formation of a thick flow by merging can occur if the flows are emplaced relatively close to each other in time.

### 32 1 Introduction

Continental flood basalt (CFB) province eruptions contain the largest  $(> 1,000 \text{ km}^3,$ 33 Bryan and Ernst (2008); Self et al. (2014)) and longest ( $\sim 1000$  km; Self et al. (2008)) 34 lava flows. Since CFBs are frequently coeval with severe environmental perturbations 35 including mass extinctions, ocean anoxic events and hyperthermal events (Clapham & 36 Renne, 2019), understanding the physical process and time-scale of flow field emplace-37 ment would help quantify the release of volcanic gases that have environmental impacts 38 (e.g.,  $CO_2$ ,  $SO_2$ ). Despite decades of work, however, the tempo of CFB eruptions remains 39 poorly quantified. 40

CFB lava flow fields are composed of 5 - 100 m thick dominantly pāhoehoe lobes 41 (Self et al., 1998). Given the general lack of large lava tubes in CFBs (Kale et al., 2020; 42 Self et al., 1998), the primary process hypothesized for creating thick flows is the for-43 mation of pāhoehoe lobes by inflation. If the quasi-continuous magma flux into individ-44 ual lava lobes is sufficient, the solidifying surface crust can continuously rise due to in-45 creasing pressure (Hon et al., 1994a; Hoblitt et al., 2012). If the lateral magma pressure 46 is large enough, the flow can propagate laterally by sporadic breakouts (Hon et al., 1994a; 47 Kauahikaua et al., 1998). This process has been observed in modern meter-scale Icelandic 48 and Hawaiian lobes (Self et al., 1998). In addition, the lobe structures in CFB flows have 49 similar internal characteristics as Hawaiian inflated lobes (Vye-Brown et al., 2013). The 50 maximal final inflated lobe thickness in Hawaiian flows, however, is only 10 - 15 m (Kauahikaua 51 et al., 1998), which is smaller than many CFB flows (up to 80-100 m, Puffer et al. (2018); 52 Self et al. (2021)). Furthermore, lava flow inflation has been shown to require pulsating 53 eruptive conditions that may not always be possible (Rader et al., 2017). Thus, a fun-54 damental question remains: how do CFB flows become so thick? 55

In this study, we explore an additional process that can lead to apparently thick flows, in which the final flow is an amalgamation of numerous smaller lobes, piled on top of each other quickly enough to remelt the intervening solidified crust (Basu et al., 2012, 2013). In Section 2, we describe a phase-field model for lava flow cooling. We then simulate solidification of a single flow and two sequentially emplaced flows using the model in one dimension. In Section 3, we outline three distinct regimes characterized by inter<sup>62</sup> lobe cooling. We finally compare our results with observations to assess whether remelt-

<sup>63</sup> ing can help explain the thick CFB flows. Our results are used to put lower bounds on

how quickly CFB flow fields were emplaced in order to preserve multiple lobes within
 a single flow field.

### <sup>66</sup> 2 A phase-field model of lava solidification

The phase-field framework is a mathematical approach to describe systems out of 67 thermodynamic equilibrium (Anderson et al., 1998). It was first introduced in the con-68 text of solidification processes and phase transitions of pure or multi-component mate-69 rials (Cahn & Hilliard, 1958; Boettinger et al., 2002). The framework allows us to evolve 70 the solidification front as part of the solution to the system of partial differential equa-71 tions, avoiding the need for explicit treatment and tracking of the moving interface as 72 is traditional done in the Stefan problem (Anderson et al., 1998). Here, we consider a 73 simplified model of lava solidification where we track the binary solidification of lava through 74 a phase variable, denoted  $\phi$  ( $\phi = 1$  for melt and  $\phi = 0$  for solid phase), and the cor-75 responding temperature, denoted T. In a phase-field framework, the evolution of  $\phi$  and 76 T can be described with the following system of coupled, nonlinear partial differential 77 equations: 78

$$\tau \partial_t \phi + \nabla \cdot \left( -\omega_\phi^2 \nabla \phi \right) = -g'(\phi) - \frac{L}{H} \frac{(T - T_m)}{T_m} P'(\phi) , \qquad (1)$$

$$\partial_t T + \nabla \cdot (-\alpha \nabla T) = \frac{L}{c_p} h'(\phi) \,\partial_t \phi, \qquad (2)$$

where  $T_m$  is the melting temperature of the lava,  $\alpha = k\rho^{-1}c_p^{-1}$  is the thermal diffusion coefficient (k thermal conductivity,  $\rho$  density,  $c_p$  specific heat),  $\omega_{\phi}$  characterizes the length 79 80 of the interfacial transition zone,  $\tau$  characterizes the time scale of solidification across 81 the interface, and H is the energy barrier. The above equations are completed with the 82 following auxiliary functions:  $g(\phi) = \phi^2 (1 - \phi)^2$ ;  $P(\phi) = (3 - 2\phi)\phi^2$ ;  $h(\phi) = P(\phi)$ 83 (Provatas & Elder, 2010). To obtain parameter values of the model that accurately char-84 acterize solidification dynamics of basaltic lava, we adopt typical values of thermal prop-85 erties of basaltic melt (Patrick et al., 2004). The phase-field modeling parameters  $\tau$  and 86  $w_{\phi}^2$  are derived in Text S1 (in the supporting information) using the approach adopted 87 from Kim and Kim (2005) and then calibrated based on field data collected from Makaop-88 uhi lava Lake (Wright & Okamura, 1977; Wright et al., 1972; Wright & Marsh, 2016), 89 as shown in Figure S1. Table 1 summarizes the parameter values used in our study. 90

We use the phase-field model and parameters to perform two types of simulations 91 of basaltic lava solidification. We first simulate solidification of a single lava lobe of thick-92 ness h to obtain the total time  $t_h$  it takes to reach complete solidification ( $\phi = 0$  ev-93 erywhere). The results are used to design the second set of simulations, where we sim-94 ulate sequential emplacement of two lava lobes of equal thickness h, separated by a time 95 period of  $t_{\rm emp}$ . We consider h from 0.1m to 20m to explore the behaviors of both thin 96 pāhoehoe lobes (< 1m), as seen in recent Kilauea eruptions, and thick lobes ( $\gg$  1m), 97 as seen in Columbia River Basalt Group (CRBG) and other Continental Flood Basalts 98 (Self et al., 2021). For the sequential emplacement simulations, we explore nine different emplacement intervals for each thickness. All the simulations are performed in one 100 dimension. The domain initially consists of a substrate that is  $4 \times h$  thick with a uni-101 form ground temperature of  $T_g = 20^{\circ}C$ . The total domain grows dynamically as lava 102 lobes are emplaced at temperature  $T_0 = 1200^{\circ}C$ : 103

$$\phi(t=0) = \begin{cases} 1 & z \in [0,4h) \\ 0 & z \in [4h,5h], \end{cases} \quad T(t=0) = \begin{cases} T_g & z \in [0,4h) \\ T_0 & z \in [4h,5h] \end{cases}$$

The bottom boundary condition is set to a constant temperature of  $T_g$  and always solid, assuming that the deep ground maintains a fixed temperature. The top boundary con-

	Definition	Unit	Values used
L	latent heat of fusion	$J/m^3$	$4 \times 10^5$
$c_p$	specific heat at constant pressure	$J/(m^3 \cdot K)$	$2.57  imes 10^6$
$\bar{k}$	thermal conductivity	$\mathrm{J}/(\mathrm{m}\cdot s\cdot\mathrm{K})$	$9.64  imes 10^{-1}$
$T_m$	melting temperature	$^{o}\mathrm{C}$	1070
au	characteristic time of solidification	s	$2.90 \times 10^6$
$\alpha$	thermal diffusivity	$m^2/s$	$3.75 \times 10^{-7}$
$w_{\phi}^2$	interfacial coefficient	m	$1.04 \times 10^{-1}$
$\sigma^{'}$	interfacial energy	$\mathrm{J/m^2}$	$5  imes 10^{-1}$
$\beta$	kinetic coefficient	$m Pa/K^2$	$5.6  imes 10^{-8}$
H	energy barrier	$J/m^{3}$	6.59
$h_c$	convective heat transfer coefficient of air	$W/(m^2 \cdot K)$	$2.62 \times 10^1$
$\sigma_s$	Stefan-Boltzmann constant	$W/(m^2 \cdot K^4)$	$5.67  imes 10^{-8}$
ε	emissivity of the lava surface		0.6

<b>Table 1.</b> Parameters used for the mode	able 1.	. Parameters u	used for	the mode	1.
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dition is set to lose heat due to black-body radiation, convection by a fixed wind speed and conduction. We also assume that a crust readily forms at the top of lava surface:

Bottom boundary : 
$$\phi = 1; \quad T = T_g;$$
 (3)

Top boundary : 
$$\phi = 1; \quad k \frac{\partial T}{\partial z} = -h_c \left(T - T_s\right) - \sigma_s \varepsilon \left(T^4 - T_s^4\right).$$
 (4)

Here,  $T_s = 30^{\circ}$ C is the surface air temperature,  $h_c$ ,  $\sigma_s$ , and  $\varepsilon$  are the convective heat transfer coefficient for air flow, Stefan-Boltzmann constant, and the emissivity of the lava surface, respectively. In practice,  $h_c$  depends on the wind speed and angle at which it travels with respect to the lava. However, considering that fluctuations in the external environment are on a much smaller timescale compared to the solidification timescale, we assume a constant  $h_c$  as shown in Table 1, which corresponds to a wind speed of roughly 2 m/s (Patrick et al., 2004).

We perform the numerical simulations with a 4th-order centered difference discretiza-115 tion in space to properly resolve the phase boundary. We use the AB4-AM4 predictor-corrector 116 method (Atkinson, 1988; Zlatev, 1985) to integrate in time, which allows us to increase 117 time step size while ensuring accuracy for the highly-resolved grid. Because our simu-118 lations need to capture temporal dynamics that span from the order of seconds (initial 119 cooling) to years, we also implement adaptive time-stepping as monitored with Milne's 120 device (Atkinson, 1988; Zlatev, 1985; Fujii, 1991) (see also Text S2). We use Ralston's 121 4th-order Runge-Kutta method (Ralston, 1962) to predict the first four time steps af-122 ter each change in time step size. The spatial grid size we use  $\Delta x$  roughly scales with 123 h, which balances computational efficiency with numerical precision (see Text S3). In 124 the following section, we describe the results from these numerical studies and discuss 125 their implications for understanding emplacement dynamics of thin and thick lava flows. 126

### 127 **3 Results**

<sup>128</sup> We perform a total of 153 simulations that explore 17 different lobe thickness  $(0.1m \le h \le 20m)$  and nine different emplacement intervals (in months, unless noted otherwise) <sup>130</sup> for each h. Based on these simulations, we have identified three distinct regimes of inter-<sup>131</sup> lobe solidification. These regimes can be delineated based on the ratio between  $t_{\rm emp}$  and <sup>132</sup> the conductive time scale (approximated by  $h^2/\alpha$ ). Below, we describe each regime in <sup>133</sup> detail with examples for the case of h = 10m lava flows in Figure 1.

- In sequence  $(t_{\rm emp} > 0.06h^2/\alpha)$ : The first lava lobe completely solidifies before the sec-134 ond lobe is emplaced (Figure 1, right). The temporal cooling dynamics of both 135 flows are similar and the bottom flow does not remelt. 136 In parallel  $(0.01h^2/\alpha < t_{emp} < 0.06h^2/\alpha)$ : As indicated by the narrowing of both black 137 contours in the top plot and the decreasing melt thickness in the lower plot with 138 time, both lava lobes solidify for overlapping time, but the interface between them 139 does not remelt (Figure 1, middle). Because the bottom flow is hot, the collective 140 cooling of both flows is slower than *in sequence* flows, as indicated by the decrease 141 in slope in Figure 1 (bottom middle). 142 **Fused flow**  $(0 < t_{emp} < 0.01h^2/\alpha)$ : After emplacement, the solidified portion of the 143 lower lava lobe eventually remelts completely, and then both lobes combine to form 144 one large lobe which solidifies as one. For early times, there are four solid-melt 145 interfaces that correspond to the simultaneous solidification of two independent 146 lobes. However, the two interior interfaces disappear at some point, marking the 147 melting and merging of the two lobes. The remelting event is also clear when we 148 track the total melt thickness over time (Figure 1 bottom). After the arrival of 149 the second lobe (indicated by red dot), the total melt thickness increases slightly 150
- 151at some point, corresponding to the remelting that caused a reduction in solid frac-152tion. Despite a monotonic loss of entropy over time after the second flow arrives,153the remelting can occur as some sensible heat is converted into latent heat. In the154other two regimes, the melt thickness never increases after the arrival of the sec-155ond lobe.



Figure 1. Emplacement of two 10m-thick lava slabs where the second slab is emplaced after 8.5 days (left), 2 months (middle) and 3 years (right). Top: Evolution of the temperature field over time. The white line marks the ground and the dark line marks the solid-liquid boundary as defined by  $\phi = 0.5$ . The ground portion extends between 0-40 meters (only half of the ground is shown here). Bottom: the corresponding solidified fraction of the total emplaced lava over time. The red dot marks the arrival of the second slab.

We compile the results from all the simulations into a regime diagram in Figure 2, which shows the combined control of individual flow thickness and emplacement intervals on the inter-lobe solidification during sequential emplacement. We map the three regions of inter-lobe solidification, separated by two boundaries extrapolated from our results:  $t_{\rm emp} = 0.01h^2/\alpha$  and  $t_{\rm emp} = 0.06h^2/\alpha$ . These regimes and the boundaries that define them are universal for both thin and thick lobes.



Figure 2. Regime diagram of two-lobe emplacement dynamics for different flow thickness and emplacement intervals, focusing on the dynamics for thin lobes (left) and thick lobes (right). The black dots mark the parameters we have simulated using our model. The bottom four panels illustrate examples of lava flow of various thickness that appear to have been emplaced *in parallel* or *in sequence* as suggested by their distinct inter-lobe boundaries. These examples are also marked in the regime diagrams, where the vertical position of the marker corresponds to the minimum emplacement interval predicted by our model (e.g.  $t_{\rm emp} = 0.01h^2/\alpha$ ). In particular, the polygonal marker corresponds to ~10cm thin lobes as seen in Kupaianaha flow field that are predicted to be emplaced at least ~4 minutes apart; the square marker corresponds to ~0.5m thin lobes as seen in Elephanta Caves, and are predicted to be emplaced at least ~2 hours apart; the circular marker corresponds to ~8m thick lobes as seen in Rajahmundry Traps (Fendley et al., 2020a), that are predicted to be emplaced at least ~20 days apart; the star-shaped marker corresponds to ~20m thick lobes as seen in CRBG that are predicted to be emplaced at least ~4 months apart.

### <sup>162</sup> 4 Discussion

A body of literature commonly assumes that even the thickest (> 40m) CFB flows 163 were formed by flow inflation (Self et al., 1996, 1998; Anderson et al., 1999; Rader et al., 164 2017) based on the observations of Hawaiian lava flows (Hon et al., 1994b). However, 165 our theoretical analysis suggests that thick (30-40 m total height) flows could also arise 166 by fusing of flows if eruption intervals are shorter than a month or two. Fusing would 167 remove structures that identify the crusts of the two lobes. However, some relics of the 168 originally distinct flow may remain, such as compositional differences (Vye-Brown et al., 169 2013; Reidel, 2005) and possibly structures indicative of fused flow crusts such as vesicle-170 rich horizons and multiple entablature zones (Figure 3). 171

One potential example of a CFB flow that may have formed as a fused flow is the 172  $\sim$  70m thick Cohassett Flow from the CRFB. The flow is a member of the Grande Ronde 173 Basalt and is a member of the Sentinel Bluffs Member lava flows in the Pascoe Basin (e.g., 174 McMillan et al., 1989; Reidel, 2005, see Figure 3A for a map of outcrops and drill core 175 data). As shown in the annotated picture in Figure 3B, the Cohassett has a multi-tiered 176 structure with alternating entablatures and colonnades, as well as a 6.5m thick internal 177 vesicular zone (IVZ,  $\sim 20$  m from the flow top, Figure 3B,C,D) with many  $\sim 1$  cm di-178 ameter vesicles (McMillan et al., 1989; Tomkeieff, 1940). The Cohassett flow exhibits one 179 of the most striking geochemical variations amongst the Grande Ronde flows. The flow 180 has an approximate vertical bilateral symmetry geochemically centered just under the 181 IVZ, as seen from the data across sections more than 50 km apart (Figure 3). Using char-182 acteristic patterns in TiO<sub>2</sub>, P<sub>2</sub>O<sub>5</sub> (and other major and trace elements), Reidel (2005) 183 defined four distinct compositional types within the flow - California Creek, Airway Heights, 184 Stember Creek, and Spokane Falls. Typically, these compositional types are separated 185 by a vesicular horizon. For example, a horizon  $\sim 13-15$  m from flow top separates mas-186 sive basalt of the California Creek composition from the Airway Heights composition. 187 Similarly, the Airway Heights and Stember Creek transition is characterized physically 188 by a series of large vugs. The IVZ acts as the contact between the Spokane Falls and the 189 Stember Creek compositional types (Figure 3B,C,D). Finally, a vesicular horizon  $\sim 40$ 190 m from flow top defines the transition from the Spokane Falls back to the Stember Creek 191 compositional types. Interestingly, the subsequent compositional type changes from Stem-192 ber Creek to California Creek/Airway Heights lack clear vesicular horizons (Figure 3). 193

Corresponding spatially with these geochemical changes, the Cohassett flow also 194 exhibits systematic changes in plagioclase abundance and fine-grained fraction (groundmass, 195 Figure 3C based on data from Reidel, 2006). In particular, the flow part comprising the 196 IVZ and the Spokane Falls composition member has a fine fraction much more indica-197 tive of a flow top rather than the flow interior. Thus, this flow interior was potentially 198 emplaced rapidly and cooled faster than a continuously inflating flow lobe interior (McMillan 199 et al., 1989; Philpotts & Philpotts, 2005). The IVZ-entablature-colonnade sequence in 200 the Spokane Falls lava further supports the conclusion that the cooling rates in this part 201 of the flow were more akin to a flow top (DeGraff et al., 1989; Forbes et al., 2014). Even 202 on an overall flow scale, the textural data for Cohassett flow are inconsistent with the 203 slow cooling expected for a  $\sim 70$  m flow. The plagioclase crystal size does not signifi-204 cantly change throughout the flow, unlike the case for a slowly cooling ponded lava lake 205 (Philpotts & Philpotts, 2005; Cashman & Marsh, 1988). 206

Previously, Reidel (2005) proposed that the Cohassett flow formed by the combi-207 nation of different sheet flows (for each compositional type), each sourced from a differ-208 ent magma reservoir and eruptive vent. These individual flows sequentially intruded into 209 the Cohassett flow as flow lobes and inflated it to its final height. One potential chal-210 lenge for this model is to explain the abrupt shift to distinct compositional types along 211 with sharp vesicle horizons (Figure 3B, B1-B2) without any signs of magma mixing or 212 shear instabilities despite intrusion and transport within the Cohassett flow for 10s of 213 km. Alternatively, S. Self & Th. Thordarson (personal comm., see also Vye-Brown et 214 al. (2013)) proposed that the Cohassett flow was formed by semi-continuous inflation with 215 changing magma compositions in the magmatic system feeding the eruption. Philpotts 216 and Philpotts (2005) proposed that crystal-mush compaction in an inflated sheet lobe 217 can also partially explain the observed geochemical variation. We propose a third alter-218 native, building upon the original idea proposed by Reidel (2005). We posit that the Co-219 hassett flow is an example of a *fused* flow with multiple flow lobes having different com-220 positions. Suppose the Cohassett was close to the boundary between the fused and in-221 parallel flow types (Figure 2). In that case, the presence of separating vesicle horizons 222 as well as high fine-grained size fraction, especially for Spokane Falls type, can be ex-223 plained. Within this scenario, each constituent  $\sim 10-20$  m lobe would have to be emplaced 224 within a few months of the previous lobe. However, more detailed modeling work specif-225

ically focused on the Cohassett as well as textural analysis (Cashman & Marsh, 1988;

Giuliani et al., 2020, e.g., stratigraphic crystal size distributions to estimate cooling rates)

would be needed to ascertain which of the proposed models is correct and if Cohassett

is indeed a *fused* flow.

It is similarly difficult to distinguish between in parallel and in sequence flows based 230 on field volcanological observations alone without detailed textural analysis. One poten-231 tial distinguishing feature may be the 2D shape of the bottom flow lobe in a *in paral*-232 lel flow since it will be visco-elastically deformed by the load from the overlying flow lobe 233 234 (Abbott & Richards, 2020). One consequence of this would be formation of squeeze-up structures at flow lobe edges seen in some CFB flow edges (e.g., Dole et al., 2020; Fend-235 ley et al., 2020b, for the Western Ghats and the Rajahmundry Trap flows in the Dec-236 can CFB respectively). 237

### <sup>238</sup> 5 Conclusion

We provide the theoretical lower bound on emplacement interval that distinguishes a *fused flow* from non-merged flows. For instance, a distinct boundary between two lobes of 10 cm each suggests that they were emplaced at least 4 minutes apart  $(t_{\rm emp} > 0.01h^2/\alpha \approx$ 4 minutes). The same calculation for two 20 m thick lobes suggests that the emplacement interval is at least 4 months if a distinct boundary is present between the two lobes. We also show the effectiveness of using phase-field models and high-order numerical schemes in simulating lava solidification problems with drastically varying timescales.



Figure 3. Stratigraphic sections for multiple Cohassett flow outcrops and cores in the Pasco Basin, Columbia River Basalts. (A) Regional Map showing the location of the sections plotted in the figure (red points) and other drill cores with similar stratigraphy (blue stars). (B) Internal stratigraphy of the Cohassett flow in the Sentinel Gap outcrop with zoom in pictures (B1, B2) showing the sharp vesicularity transitions at the Internal vesicular Zone (IVZ)  $\sim 20$  from the flow top (modified from McMillan et al. 1989). Colm e - Columnar, Ent - Entablature, and Col - Colonnade. The right panels show the vescicle parosity and geochemical variations in the DC-16 borehole. Panels (C) and (D) show stratigraphic section with geochemical and textural variations in the Cohassett flow in the RR1-6 Core and DC-6 cores respectively (Data from Reidel 2005). We also show the assigned compositional types to parts of the Cohassett flow by Reidel 2005

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some extra movies) can be found in the folder movies. 255

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# Supporting Information for "Some lava flows may not have been as thick as they appear"

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### Introduction

In Text S1, we describe how we derived or calibrated the phase-field parameters  $\tau$ ,  $\omega_{\phi}^2$ , and H from either known quantities or the interface width d, which numerically acts somewhat like a viscosity/smoothing term. Figure S1 also gives relevant details as to how

we calibrated d. In Text S2, we define and explain the relative  $L^2$  error, which is used in Figure S2. In Text S3, we describe the choice of grid size  $\Delta x$  for each simulation. Figures S2-S7 describe and showcase several compelling yet somewhat tangential properties we observed in our simulations, as well as detailed descriptions about how qualitative aspects of the lava solidification dynamics change with the emplacement time interval  $t_{\rm emp}$  and lobe height h. Finally, we provide captions for three movies which correspond to the three cases featured in Figure 1.

### Text S1. Formulation of parameters

Rewriting the parameters in (Kim & Kim, 2005) in terms of the parameters in our model, we have that  $\omega = H$ ,  $M_{\phi} = M$ ,  $\varepsilon = \varepsilon_{\phi}$ ,  $D_T = \alpha$ ,  $\Delta H_m = L$ ,  $g(\phi) = \phi^2 (1 - \phi)^2$ ,  $f_c = 0$ , and  $f_{\phi}(\phi, T) = \frac{(T - T_m)L}{T_m} P'(\phi)$ , where  $P(\phi) = (3 - 2\phi) \phi^2$ . Then, from here, we go through the same derivations in (Kim & Kim, 2005) to derive the interface width  $d = 2\xi$ and the interface energy  $\sigma$ .

Consider a partially-solidified lava system at equilibrium where we have a 1D interface between solid  $\phi = 1$  at x = d and liquid  $\phi = 0$  at x = 0. Since this system is at equilibrium and we assume the equal temperature condition for pure substances,  $\partial_t \phi = \partial_t T = 0$  and  $T = T_m$ , such that equation (1) from the main paper (the PDE for the phase,  $\phi$ ) can be integrated for the equilibrium phase-field profile  $\phi_0(x)$ .

$$\omega_{\phi}^{2}\partial_{x}^{2}\phi_{0} - g'(\phi_{0}) - \frac{L}{H}\frac{(T_{m} - T_{m})}{T_{m}}P'(\phi_{0}) = 0 \Rightarrow \omega_{\phi}^{2}\partial_{x}\phi_{0}\partial_{x}^{2}\phi_{0} - \partial_{x}\phi_{0}g'(\phi_{0}) = 0$$
$$\Rightarrow \frac{d}{dx}\left[\frac{1}{2}\omega_{\phi}^{2}\left(\partial_{x}\phi_{0}\right)^{2} - g\left(\phi_{0}\right)\right] = 0 \Rightarrow \frac{1}{2}\omega_{\phi}^{2}\left(\partial_{x}\phi_{0}\right)^{2} - g\left(\phi_{0}\right) = \text{const.}, \tag{1}$$

where we assume that  $\omega_{\phi}$  is a constant and const. =  $\frac{1}{2}\omega_{\phi}^2 (\partial_x \phi_0(x_0))^2 - g(\phi_0(x_0))$  at some reference position  $x_0$ . Finally, without loss of generality, we can let  $x_0 = 0$  and put a

Dirichlet boundary condition here (we would expect one anyways if the lava is fully liquid there), such that  $g(\phi_0(x_0)) = g(\phi_0(0)) = g(0) = 0$  and  $\partial_x \phi_0(x_0) = \partial_x \phi_0(0) = 0$ . Hence, const. = 0 in (1), in which case we can integrate (1):

$$\frac{1}{2}\omega_{\phi}^{2}\left(\partial_{x}\phi_{0}\right)^{2} = g\left(\phi_{0}\right) \Rightarrow \frac{\partial\phi_{0}}{\partial x} = \sqrt{\frac{2}{\omega_{\phi}^{2}}g\left(\phi_{0}\right)}$$
$$\Rightarrow d = \int_{\phi_{a}}^{\phi_{b}} \frac{d\phi_{0}}{\sqrt{\frac{2}{\omega_{\phi}^{2}}g\left(\phi_{0}\right)}} = \frac{\omega_{\phi}}{\sqrt{2}}\int_{\phi_{a}}^{\phi_{b}} \frac{d\phi_{0}}{|\phi_{0}|\left|1-\phi_{0}\right|}.$$
(2)

As in Kim and Kim, we use  $\phi_a = 0.1$  and  $\phi_b = 0.9$  to integrate (2), from which we get that

$$d = \omega_{\phi} 2\sqrt{2} \ln 3, \tag{3}$$

which is essentially the same result derived in (Kim & Kim, 2005).

Next, to obtain the interface energy, we again repeat the steps in (Kim & Kim, 2005) by considering an equilibrium system with a cylindrical solid in liquid matrix while maintaining a diffuse interface between them. This gives us the following:

$$\sigma = \varepsilon_{\phi}^{2} \int_{-\infty}^{\infty} \left(\frac{d\phi_{0}}{dr}\right)^{2} dr = \sqrt{2}\varepsilon_{\phi} \int_{0}^{1} \sqrt{Hg\left(\phi_{0}\right)} d\phi_{0}$$
$$= \sqrt{2}\varepsilon_{\phi} \sqrt{H} \int_{0}^{1} |\phi_{0}| \left|1 - \phi_{0}\right| d\phi_{0} = \frac{\varepsilon_{\phi}}{3} \sqrt{\frac{H}{2}}.$$
(4)

Making necessary assumptions in the thin interface limit, equation (22) from (Kim & Kim, 2005) gives us that

$$J = \int_{0}^{1} \frac{h_{p}(\phi) \left[1 - h_{d}(\phi)\right]}{\sqrt{g(\phi)}} d\phi$$
$$= \int_{0}^{1} \frac{\phi^{3} \left(6\phi^{2} - 15\phi + 10\right) \left[1 - \phi^{3} \left(6\phi^{2} - 15\phi + 10\right)\right]}{|\phi| \left|1 - \phi\right|} d\phi = \frac{209}{420}.$$
 (5)

Thus, using (5), equation (21) in Kim and Kim implies that

$$\beta = \frac{1}{3\sqrt{2}} \frac{T_m \sqrt{H}}{\varepsilon_{\phi} LM} - \frac{L}{\alpha c_p} \frac{\varepsilon_{\phi}}{\sqrt{2H}} J = \frac{1}{3\sqrt{2}} \frac{T_m \sqrt{H}}{\varepsilon_{\phi} LM} - \frac{209}{420} \frac{L}{\alpha c_p} \frac{\varepsilon_{\phi}}{\sqrt{2H}}.$$
(6)  
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Our only unknown parameter is  $\omega_{\phi} \sim d$ , which we have to adjust as we run simulations to match known solidification data, but once given  $\omega_{\phi}$  and d, we can derive H,  $\tau$ , and  $\varepsilon_{\phi}$ . Therefore, our parameter search is only *one-dimensional*, since once we choose a value of d or  $\omega_{\phi}$ , all other parameters can be immediately determined.

Using equations  $\omega_{\phi} = \varepsilon_{\phi}/\sqrt{H}$  and  $\tau = 1/(HM)$  from (Provatas & Elder, 2010) along with (3), (4), and (6) above, we can rewrite all unmeasured parameters in terms of measurable quantities and d as follows:

$$\omega_{\phi} = \frac{\sqrt{2}}{4\ln 3}d, \quad H = 12\ln 3\frac{\sigma}{d}, \quad \tau = \frac{1}{8\ln^2 3}\frac{d^2L}{\sigma T_m}\left(\beta + \frac{209}{1680\ln 3}\frac{dL}{\alpha c_p}\right). \tag{7}$$

Using the sample parameters from Table 1 in the main paper, the last two equations in (7) become

$$H \approx \frac{6.592}{d} \text{ J m}^{-2}, \quad \tau \approx 2.899 \times 10^6 d^3 \text{ s m}^{-3} + 3.454 \times 10^{-6} d^2 \text{ s m}^{-2}.$$
 (8)

Even if the solid-liquid interface width were microscopic, i.e.,  $d \sim 10^{-9}$  m, the second equation in (8) would still imply that  $3.454 \times 10^{-6} d^2$  s m<sup>-2</sup>  $\ll 2.899 \times 10^6 d^3$  s m<sup>-3</sup>, since in that case,  $\frac{3.454 \times 10^{-6} d^2 \text{ s m}^{-2}}{2.899 \times 10^6 d^3 \text{ s m}^{-3}} \sim 10^{-3}$ . Thus, we can further make the simplification  $\tau \approx 2.899 \times 10^6 d^3$  s m<sup>-3</sup>, and in general, for parameters similar to basalt lava, the third equation in (7) can be simplified to

$$\tau = \frac{209}{13440 \ln^3 3} \frac{d^3 L^2}{\alpha c_p \sigma T_m}.$$
(9)

Finally, by the above considerations and equations, we can also derive the following informative scaling properties:

$$\omega_{\phi} \sim d, \quad H \sim \frac{\sigma}{d}, \quad \tau \sim \frac{d^3 L^2}{\alpha c_p \sigma T_m}, \quad M \sim \frac{\alpha c_p T_m}{d^2 L^2}, \quad \varepsilon_{\phi} \sim \sqrt{\sigma} \sqrt{d}.$$
 (10)

The scaling relationships in (10) provide a physical interpretation of these variables, as well as simple sanity-checks of the validity of the assumptions we made for a given choice of parameters. And as mentioned in (Kim & Kim, 2005), (10) should in theory hold as long as  $d \ll \alpha/V$  and  $d \ll R$ , where V and R are the velocity of the solidification front and the local radius of curvature for the solid-liquid interface, respectively.

### Text S2. Definition of relative $L^2$ error

Say we have n data points in space. Suppose that f(x) is our exact function and  $\hat{f}(x)$  is an approximation for f. Then, the exact  $L^2$  error on [0, L] would be

$$e = \left[\int_{0}^{L} \left(f(x) - \widehat{f}(x)\right)^{2} dx\right]^{1/2}$$

However, given that  $\hat{f}$  lives on a grid with *n* points and spatial intervals of size  $\Delta x$ , we have to approximate *e* as follows, using a Riemann sum:

$$e \approx \left[\Delta x \sum_{i=1}^{n} \left( f(x_i) - \widehat{f}(x_i) \right)^2 \right]^{1/2},$$

where  $x_n = L$  and  $x_1 = \Delta x$ . Finally, to compute the relative  $L^2$  error,  $e_{rel}$ , we divide e by the  $L^2$  norm of f, i.e.,

$$e_{\rm rel} \approx \frac{\left[\Delta x \sum_{i=1}^{n} \left(f(x_i) - \widehat{f}(x_i)\right)^2\right]^{1/2}}{\left[\Delta x \sum_{i=1}^{n} f(x_i)^2\right]^{1/2}} = \left[\frac{\sum_{i=1}^{n} \left(f(x_i) - \widehat{f}(x_i)\right)^2}{\sum_{i=1}^{n} f(x_i)^2}\right]^{1/2}.$$

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### Text S3. Spatial grid size $\Delta x$

The spatial grid size we use is

$$\Delta x = 10^{-3} \min\left\{ \min\left(\sqrt{10h}\right), 10 \right\}, \tag{11}$$

where nint is the function which rounds its argument to the nearest integer. Intuitively, we can think of Equation 11 as interpolating  $\Delta x$  from  $10^{-3}$  (for h = 0.1, 0.2) to  $10^{-2}$  (for h = 10, 15, 20) using the square root function, except rounding each value to the third decimal place for simplicity's sake. That way,  $\Delta x$  roughly scales with h, which balances computational efficiency with numerical precision.

Figure S2 Caption. Using nonlinear least squares, we fit the solidification data for a *single* lobe cooling by itself to the function

$$t_h = \frac{h^2}{\alpha} \left[ \frac{A}{h^B} + C \exp\left(-Dh\right) + E \right]$$
(12)

which heuristically models the nonlinear trend after the conventional cooling estimate  $t_h \sim h^2$  derived from solving the Stefan problem. The best fit parameters we find are  $A \approx 0.0110$ ,  $B \approx 0.2294$ ,  $C \approx 0.3346$ ,  $D \approx 24.8922$ , and  $E \approx 0.0320$ . With these parameters, the relative  $L^2$  error (as defined in Text S2) between  $t_h \alpha/h^2$  as fitted above and the actual data is  $\approx 9.8237 \times 10^{-3} < 1\%$ , which shows very respectable agreement. Hence, (12) could be a starting point for modeling  $t_h$  with more general physical parameters and initial conditions.

By using the term "strong nonlinearity" in Figure S2, we are referring to how there is a qualitative difference in the curve for small enough lobe sizes. This difference is best explained by the quick decay of the exponential term in our curve fit: For h not too large, the exponential term quickly disappears and the trend becomes primarily dominated by

the power law term. Hence, motivated by how the relative  $L^2$  error between our bestfit curve and numerical solution is just under 1%, we heuristically have drawn the line between the "strongly nonlinear" and "weakly nonlinear" regions by indicating where the relative error between the fit with and without the exponential term falls below 1%. That point is at roughly h = 0.26344, after which the exponential term contributes an error which is below 1% and decreases further as h increases.

We label these two regions in Figure S2 to give a rough estimate of where the usual  $t_h \sim h^2$  scaling relationships are relatively valid, and show how for small enough lobe sizes, deviations from this trend begin to dominate significantly. The physical interpretation of these regions is as follows: As we work with smaller and smaller lobes, the nonlinear effects of convection cooling and radiative heat loss at the lava surface begin to dominate the time it takes for a lobe of that size to cool. The usual Stefan problem formulation often ignores these nonlinear effects in the boundary condition at the lava-air interface, but based off of our results here, we suggest that these will contribute a non-negligible effect to the solution when the lobe size is too small.

Notes for Figures S3-S7. For Figures S3-S7, we will consider the trends between different lobe thicknesses once we weight the emplacement time by  $t_h$ . For every dimensionless plot, the stars (\*) represent merged cases, the crosses (×) represent in parallel cases, and the pluses (+) represent in sequence cases.

Movie S1. Under the folder movies in the GitHub data repository, emplacementresults\_10\_10\_843K\_406 is a movie showing the solidification dynamics for the fused flow case shown in Figure 1.

Movie S2. Under the folder movies, emplacementresults\_10\_10\_843K\_3250hours.mp4 is a movie showing the solidification dynamics for the in parallel case shown in Figure 1. Movie S3. Under the folder movies, emplacementresults\_10\_10\_843K\_26000hours.mp4 is a movie showing the solidification dynamics for the in sequence case shown in Figure 1.

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Figure S1. The open red circles track how our simulated solidified crust thickness vs. time varies with the interface width d, while the stars mark known field data measured from Makaopuhi lava lake (Wright & Okamura, 1977; Wright et al., 1972; Wright & Marsh, 2016). All other physical parameters that we used are given in Table 1, with initial conditions consisting of a lava lake of arbitrarily large depth and initial temperature given according to (Wright & Okamura, 1977; Wright et al., 1972; Wright & Marsh, 2016). For the four cases we tested above, we observed virtually the same, consistent agreement between our simulation and the measured data. Hence, for simplicity's sake, we took d = 1 in our simulations, whence our values for  $\tau$ ,  $\omega_{\phi}^2$ , and H in Table 1 follow.



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Figure S3. (Compare with Figure S4.) The dimensionless log-log plot above shows the solidification time including the time between emplacement,  $t_{\text{solidification}}$ , as a function of the emplacement time interval,  $t_{\text{emp}}$ , with both axes scaled by  $t_h$ . Note in particular that the graph at any lobe size has a minimum near or slightly below  $t_{\text{emp}} = t_h$ . This minimum reflects some optimal balance between the emplacement time and the thermal/phase interaction between the two lobes which minimizes the solidification time across the domain. This optimal balance lies within the in parallel region.



Figure S4. The above plot highlights an alternate interpretation of the solidification time in which we neglect the time between emplacements. On either plot, we note that as  $t_{\rm emp} \rightarrow 0$ ,  $t_{\rm solidification} \rightarrow 4t_h$ . This reflects how, since  $t_h \sim h^2$ ,  $t_{2h} \sim (2h)^2 = 4h^2$ . Meanwhile, in comparison to Figure S3, this plot better demonstrates how as  $t_{\rm emp} \rightarrow \infty$ ,  $t_{\rm solidification} \rightarrow t_h + t_{\rm emp}$ .



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Figure S5. The above plot only considers the time for the first lobe to solidify vs.  $t_{emp}/t_h$ , thereby highlighting the thermal influence of the upper lobe upon how the lower lobe solidifies relative to  $t_h$ . As expected,  $t_{solidification} \rightarrow t_h$  when  $t_{emp} \rightarrow \infty$ . Physically, we can interpret this result as follows: If the lower lobe has fully solidified before the upper lobe is emplaced, then the upper lobe will have no influence on the solidification of the lower lobe.



Figure S6. This plot indicates the height, scaled to the lobe size, at which solidification completed across the entire two-lobe system vs.  $t_{\rm emp}/t_h$ . This variable is significant because horizontal fractures often form wherever solidification completes in a lava lobe, i.e., where two solidifying fronts meet. Note in particular that the smaller lobe sizes appear to have greater solidification heights in the merged and in sequence regions, while the opposite behavior is observed for the in parallel region. The quantitative differences in behavior across different lobe sizes appears to be greatest for the in parallel region, which we also see in Figure S7.



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**Figure S7.** This plot is the same as Figure S6, except that this plot measures solidification in the first lobe only, i.e., where the first lobe solidified. Note that for a given height, the graph appears to increase during the fused region, decrease sharply during the in parallel region, and then finally level out during the in sequence region. The trend in the in parallel region appears to be sharper the smaller the lobe size is, which indicates how the thermal influence of the upper lobe on the lower lobe increases as the lobe size decreases, assuming that the lobes do not just merge entirely. As with Figure S6, the greatest disparity in dynamics across different lobe sizes seems to be greatest for the in parallel region.