Quantifying Intrinsic and Extrinsic Contributions to Seismic Anisotropy in Tomographic Models

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Abstract

Seismic anisotropy in the Earth's mantle inferred from seismic observations is usually interpreted either in terms of intrinsic anisotropy due to Crystallographic Preferred Orientation (CPO) of minerals, or extrinsic anisotropy due to rock-scale Shape Preferred Orientation (SPO). The coexistence of both contributions misconstrues the origins of seismic anisotropy observed in seismic tomography models. It is thus essential to discriminate CPO from SPO. Homogenization/upscaling theory provides means to achieve this goal. This theory enables to compute the effective elastic properties of a heterogeneous medium, as seen by long-period waves. In this work, we investigate the effects of upscaling an intrinsically anisotropic and highly heterogeneous Earth's mantle. We show analytically in 1-D that the full effective radial anisotropy ξ * is approximately the product of the effective intrinsic radial anisotropy ξ * CPO and the extrinsic radial anisotropy ξ * SPO : ξ * [?] ξ * CPO x ξ * SPO. This law is verified numerically in the case of a 2-D marble cake model of the mantle with a binary composition, and in the presence of CPO obtained from a micro-mechanical model of olivine deformation. We compute the long-wavelength effective equivalent of this mantle model using the 3-D non-periodic elastic homogenization technique. Our numerical findings predict that for wavelenghts smaller than the scale of deformation patterns, tomography may overestimate the true anisotropy (i.e. intrinsic anisotropy due to CPO) due to significant SPO-induced extrinsic anisotropy. However, at wavelenghts larger than deformation patterns, intrinsic anisotropy is always underestimated in tomographic models due to the spatial averaging of the preferred orientation of anisotropic minerals. Thus, we show that it is imperative to homogenize a CPO evolution model first before drawing comparisons with tomographic models. As a demonstration, we use our composite law with a homogenized CPO model of a plate-driven flow underneath a mid-ocean ridge, to estimate the SPO contibution to an existing tomographic model of radial anisotropy.

Quantifying Intrinsic and Extrinsic Contributions to Seismic Anisotropy in Tomographic Models

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Key Points:

- We propose a theoretical expression that relates the observed seismic anisotropy to its intrinsic and extrinsic contributions.
- For wavelengths longer than the scale of deformation patterns in the mantle, we show that observed anisotropy in tomographic models underestimates intrinsic anisotropy, due to the spatial averaging of intrinsic anisotropy over long wavelengths.
- For wavelengths shorter than the scale of deformation patterns, observed anisotropy overestimates intrinsic anisotropy due to the presence of extrinsic anisotropy.

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Abstract

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$$\xi^* \approx \xi^*_{\mathrm{CPO}} \times \xi^*_{\mathrm{SPO}}.$$

This law is verified numerically in the case of a 2-D marble cake model of the mantle with a binary composition, and in the presence of CPO obtained from a micro-mechanical model of olivine deformation. We compute the long-wavelength effective equivalent of this mantle model using the 3-D non-periodic elastic homogenization technique. Our numerical findings predict that for wavelenghts smaller than the scale of deformation patterns, tomography may over-estimate the true anisotropy (*i.e.* intrinsic anisotropy due to CPO) due to significant SPO-induced extrinsic anisotropy. However, at wavelenghts larger than deformation patterns, intrinsic anisotropy is always underestimated in tomographic models due to the spatial averaging of the preferred orientation of anisotropic minerals. Thus, we show that it is imperative to homogenize a CPO evolution model first before drawing comparisons with tomographic models. As a demonstration, we use our composite law with a homogenized CPO model of a plate-driven flow underneath a mid-ocean ridge, to estimate the SPO contibution to an existing tomographic model of radial anisotropy.

1 Introduction

Seismic anisotropy in the Earth's mantle originates from various processes and can be observed at different spatial scales (Hansen et al., 2021). At the mineral scale, crystallographic preferred orientation (CPO) of anisotropic mantle minerals due to progressive shearing over time produces large-scale intrinsic anisotropy (Nicolas & Christensen, 1987; Maupin & Park, 2015). On the other hand, rock-scale shape preferred orientation (SPO) such as layered heterogeneous materials, seismic discontinuities, preferentially-oriented cracks or conduits containing fluid intrusions unresolved by long period seismic waves are mapped as large-scale extrinsic anisotropy (Backus, 1962; Crampin & Booth, 1985).

Although these two mechanisms are completely different, a medium may be either (or both) intrinsically anisotropic and extrinsically anisotropic at a given scale, depending on the minimum wavelength of the observed wavefield used (Maupin et al., 2007; Wang et al., 2013; Fichtner et al., 2013; Bodin et al., 2015). Backus (1962) showed that a horizontally-layered isotropic medium is equivalent to a homogeneous radially anisotropic medium with a vertical axis of symmetry when sampled by seismic waves whose wavelength is much longer than the thickness of layers. This urged seismologists to interpret tomographic models separately depending on the type of data used (*i.e.*, different data-types sample different length scales). Scattering studies use high frequency body waves and interpret small-scale isotropic heterogeneities in terms of phase changes (e.g. Tauzin & Ricard, 2014) or chemical stratification (e.g. Tauzin et al., 2016). On the other hand, long period surface waves with typical wavelengths of the order 10^2 km retrieve a smooth anisotropic mantle with scales consistent with convective flow (e.g. Debayle & Ricard, 2013; Bodin et al., 2015; Maupin & Park, 2015). Surface waves however lack the resolving power to recover sharp seismic discontinuities and instead, map these as long wavelength radial anisotropy (Backus, 1962; Capdeville et al., 2013). Anisotropic structures retrieved from tomography may therefore be a combination of apparent extrinsic anisotropy due to SPO and deformation-induced intrinsic anisotropy. The ambiguity on the origin of observed anisotropy (i.e. whether a material is intrinsically anisotropic or strongly heterogeneous) may mislead seismologists in interpreting the structural origin of seismic anisotropy observed in tomographic images.

1.1 Intrinsic anisotropy due to Crystallographic Preferred Orientation

Intrinsic anisotropy results from the preferred alignment of anisotropic crystals in an aggregate when subjected to a macroscopic deformation. In the mantle, single crystal olivine exhibits orthorhombicity, and hence suffers variations in fast and slow

P- and S-wave velocities up to 20 % (Kumazawa & Anderson, 1969). When olivine and pyroxene form a polycrystalline aggregate and are subsequently deformed in the mantle flow, their CPO can be described in terms of a hexagonally symmetric medium (e.g. J. Montagner & Nataf, 1988).

Observations of large-scale anisotropy in tomographic models appear to be ubiquitous in regions associated with strong deformation, and have often been interpreted in terms of convective flow (McKenzie, 1979). For instance, tomographic imaging has revealed the presence of positive radial anisotropy (*i.e.*, horizontally propagating $SH-$waves traveling faster than $SV-$waves) of about 4% in the upper ~ 250 km of the mantle and has been interpreted as lateral flow (refer to Long and Becker (2010) for a comprehensive review). Long wavelength seismic anisotropy is also prevalent in the transition zone (e.g. Trampert & van Heijst, 2002; Wookey & Kendall, 2004) although its origin is still highly debated (Chen & Brudzinski, 2003; Chang & Ferreira, 2019; Sturgeon et al., 2019). Probing deeper depths, the lower mantle appears to be isotropic (e.g. Meade et al., 1995) barring the D" layer where enough evidence have shown it to be anisotropic (e.g. Kendall & Silver, 1998; McNamara et al., 2002; Panning & Romanowicz, 2006).

Since CPO maps the deformation patterns, CPO may deviate from the flow direction. This is because the deformation patterns relate not to the velocity field itself, but to the velocity gradient. Moreover, CPO is not instantaneous, but depends on the history of the deformation. As a result, regions with short deformation trajectories such as beneath mid-ocean ridges appear to have under-developed CPO, and would lag behind the direction of shear deformation (É. Kaminski & Ribe, 2002).

Based on laboratory experiments of simple shear, the fast axis of olivine tends to align parallel to the long axis of the finite strain ellipsoid (FSE) at low strains due to plastic deformation (Zhang & Karato, 1995). At larger strains, dynamic recrystallization facilitates the alignment of the olivine fast axis towards the direction of shear (Zhang & Karato, 1995; Bystricky et al., 2000). Mechanical models of CPO evolution, coupled with geodynamic flow modeling have been developed to replicate these results and have been extrapolated at scales consistent with mantle deformation patterns. Among these is the viscoplastic self-consistent (VPSC) model which is used to explain the mechanical response of polycrystals to plastic deformation (Tommasi et al., 2000). Such tools however are computationally expensive, especially when applied to 3-D and non-steady state flows (Lev & Hager, 2008). Another well-received method is the D-Rex model, that utilizes a simple kinematic approach (E. Kaminski et al., 2004). The predicted CPO is then converted to an elastic medium in which seismic waves can propagate, and may explain anisotropic signatures observed in seismic data recorded at the surface.

1.2 Extrinsic anisotropy due to Shape Preferred Orientation

Extrinsic anisotropy is observed under two conditions: (1) when the scale of the heterogeneities is much smaller than the minimum wavelength of the observed wavefield, and (2) when the contrast between seismic wave velocities (*i.e.* the amplitude of heterogneities) is large.

One of the known configurations at which extrinsic anisotropy is produced is rock-scale shape preferred orientation (SPO). In the Earth's mantle, rock-scale SPO can be the result of igneous differentiation, or more generally of the stirring of chemical heterogeneities by tectonic or convective deformation (e.g. Faccenda et al., 2019). Since magmatically differentiated oceanic lithosphere is composed of a basaltic crustal layer blanketed by a depleted harzburgitic mantle (Allègre & Turcotte, 1986), mantle structure is often modeled in terms of a mechanical mixture of these two end-member compositions (e.g. Hofmann, 1988; Xu et al., 2008; Ballmer et al., 2015).

Large-scale thermal convection in the mantle triggers the constant injection of oceanic lithosphere into the mantle (Coltice & Ricard, 1999). It then mechanically stirs with the surrounding mantle and experiences a series of stretching and thinning due to the normal and shear strains associated with convection (Allègre & Turcotte, 1986). This led Allègre and Turcotte (1986) to develop a geodynamic model of the mantle that would depict marble cake-like patterns. In their model, the layering may be erased either by dissolution processes when the stripes become thin enough that chemical diffusion becomes efficient, or by mantle reprocessing at mid-ocean ridges. Assuming that the mixing preserves the physical properties of the two-end members with depth and over geological time scales, such processes may explain rock-scale seismic heterogeneities observed in the mantle in agreement with the spectrum of isotropic anomalies obesrved along ridges (Agranier et al., 2005; Xu et al., 2008; Stixrude & Jeanloz, 2015).

In this paper, we extend the work of Alder et al. (2017) by estimating the long-wavelength effective equivalent of a marble cake mantle as hypothesized by Allègre and Turcotte (1986), but in the presence of intrinsic anisotropy. Our aim is to quantify the level of effective anisotropy resulting from elastic homogenization, that is, the relegated version of the true Earth as seen by long-wavelength seismic tomography. Section 2 is a brief overview of the homogenization theory and provides a definition of some terms and notations to guide the reader throughout the paper. Section 3 shows 1-D analytical expressions for homogenization and highlights a composite law that separates intrinsic and extrinsic anisotropy for a layered and anisotropic media. Here, we demonstrate that the effective anisotropy varies with the square of isotropic heterogeneities, as well as with the square of anisotropic heterogneities, plus a cross term related to their coupling. In section 4, we build a 2-D media analogous to the marble cake model where we consider a mechanical mixture of two end-member compositions. We follow this by introducing intrinsic anisotropy due to mantle deformation associated with convection patterns consistent with the marble cake model. We compute the long-wavelength effective equivalent of the 2-D models using the Fast-Fourier Homogenization algorithm (Capdeville et al., 2015). Section 5 presents the results of the previous section: one of the major findings is that in the absence of isotropic heterogeneities, intrinsic anisotropy is always underestimated upon homogenization due to the spatial averaging of the preferred orientation of the anisotropic minerals. We also verify numerically that the composite law derived in section 3 can be extended to 2-D media. Finally in section 6, we apply the composite law to infer the extrinsic component of anisotropy from a tomographic model of the upper-mantle beneath a mid-ocean ridge with the help of a homogenized CPO model.

2 Elastic homogenization

Seismic tomography is only able to recover a smooth representation of the real Earth due to the limited frequency band of seismic data. This smooth average, however, is not just a simple spatial average but is produced from highly non-linear upscaling relations. In the context of wave propagation, such upscaling relations, also known as elastic homogenization, remove seismic heterogeneities whose scales are much

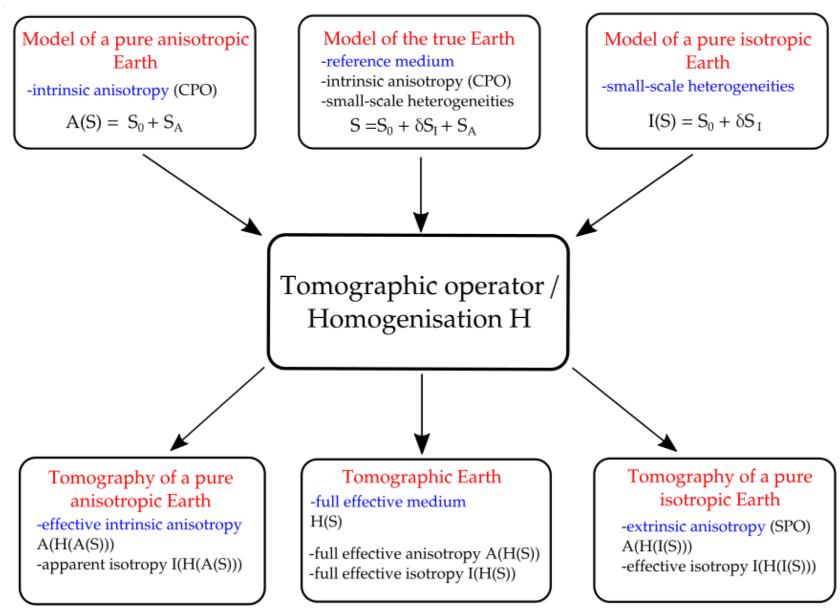

Figure 1. Homogenization of different Earth models and their respective outputs. The true Earth mantle (top middle box) is described by an average isotropic model $\mathbf{S_0}$, isotropic heterogeneities, $\delta\mathbf{S_I}$ and intrinsic anisotropy $\mathbf{S_A}$, the sum of which being the elastic model $\mathbf{S}$ that tomography tries to recover. However, tomographic methods have only access to a homogenized model $\mathcal{H}(\mathbf{S})$ (or full effective medium). This model has both isotropic components symbolized by $\mathcal{I}(\mathcal{H}(\mathbf{S}))$ and anisotropic components, $\mathcal{A}(\mathcal{H}(\mathbf{S}))$. The goal of this paper is to quantify the differences between $\mathcal{A}(\mathcal{H}(\mathbf{S}))$ and $\mathcal{A}(\mathbf{S})$, $\mathcal{I}(\mathcal{H}(\mathbf{S}))$ and $\mathcal{I}(\mathbf{S})$. Numerically we can also discuss how an anisotropic model without isotropic heterogeneities (boxes on the left) can be recovered and if the tomographic inversion can lead to apparent isotropic heterogeneities. Reciprocally (boxes on the right), one can quantify how much a pure isotropic model is recovered by the tomographic inversion and what is the level of extrinsic anisotropy (SPO) that can be estimated.

smaller than the minimum wavelength of the observed wavefield, and instead replace them with effective properties.

Hereafter, what we refer to as the *reference medium* $\mathbf{S}(\mathbf{r})$ is an elastic model of the real Earth varying in space $\mathbf{r}$ that accounts for both intrinsic anisotropy due to CPO and small-scale isotropic heterogeneities that resemble marble cake-like patterns. This reference medium can be treated as a sum of several decompositions resulting from a cascade of orthogonal projections (Browaeys & Chevrot, 2004). One can then express $\mathbf{S}(\mathbf{r})$ in terms of an isotropic tensor $\mathbf{S_I}(\mathbf{r})$ plus an intrinsically anisotropic component $\mathbf{S_A}(\mathbf{r})$ related to CPO:

$$\mathbf{S}(\mathbf{r}) = \mathbf{S_I}(\mathbf{r}) + \mathbf{S_A}(\mathbf{r}), \tag{1}$$

where $\mathbf{S_I}(\mathbf{r})$ can be decomposed further into:

$$\mathbf{S_I}(\mathbf{r}) = \mathbf{S_0} + \delta\mathbf{S_I}(\mathbf{r}). \tag{2}$$

Here, $\mathbf{S_0}$ is an isotropic tensor uniform in space, and $\delta\mathbf{S_I}(\mathbf{r})$ is a deviation from $\mathbf{S_0}$ related to small-scale isotropic heterogeneities. The reference medium becomes:

$$\mathbf{S}(\mathbf{r}) = \mathbf{S_0} + \delta\mathbf{S_I}(\mathbf{r}) + \mathbf{S_A}(\mathbf{r}). \tag{3}$$

For convenience, let us introduce an operator $\mathcal{I}$ that extracts the isotropic component from $\mathbf{S}$, and an operator $\mathcal{A}$ that extracts the anisotropic component from $\mathbf{S}$:

$$\begin{aligned} \mathcal{I}(\mathbf{S}(\mathbf{r})) &= \mathbf{S_I}(\mathbf{r}) \\ \mathcal{A}(\mathbf{S}(\mathbf{r})) &= \mathbf{S_0} + \mathbf{S_A}(\mathbf{r}). \end{aligned} \tag{4}$$

These notations will be used heavily in the rest of the text to denote the isotropic and anisotropic components of an elastic medium. Radial anisotropy, in particular, quantifies the level of anisotropy when the medium is averaged azimuthally (J.-P. Montagner, 2007; Maupin et al., 2007). In such a vertically transverse isotropic medium (VTI), the level of radial anisotropy is given by $(V_{SH}/V_{SV})^2$, where V_{SV} is the velocity of vertically traveling S waves or horizontally traveling S waves with vertical polarization, and V_{SH} is the velocity of horizontally traveling S waves with horizontal polarization. The *intrinsic radial anisotropy* associated with $\mathcal{A}(\mathbf{S})$ (*i.e.* due to the component $\mathbf{S_A}$) will be denoted by ξ_{CPO}.

In the event where long-period waves sample this reference medium, small-scale heterogeneities are seen only through their effective properties. Computing these effective properties is designated by a mathematical operator $\mathcal{H}$ called *upscaling* or *homogenization.* Setting aside the imperfections of inversion algorithms and data coverage, performing seismic tomography can be viewed as applying the operator $\mathcal{H}$ that homogenizes $\mathbf{S}$. The seismic tomography model/long-wavelength effective medium of $\mathbf{S}$ is then $\mathcal{H}(\mathbf{S}) = \mathcal{H}(\mathbf{S_0} + \delta\mathbf{S_I} + \mathbf{S_A})$ which we now refer to as the *full effective medium.* The anisotropic component of the full effective medium given by $\mathcal{A}(\mathcal{H}(\mathbf{S}))$ will be referred hereafter as the *full effective anisotropy* and its isotropic component $\mathcal{I}(\mathcal{H}(\mathbf{S}))$ is the *full effective isotropy.* We will symbolize the *full effective radial anisotropy* corresponding to $\mathcal{A}(\mathcal{H}(\mathbf{S}))$ with ξ^*.

On the other hand, the homogenized counterpart of a pure anisotropic Earth (*i.e.*, a model where only the anisotropic component varies spatially) is $\mathcal{H}(\mathcal{A}(\mathbf{S})) = \mathcal{H}(\mathbf{S_0} + \mathbf{S_A})$ where $\mathcal{A}(\mathcal{H}(\mathcal{A}(\mathbf{S})))$ is the *effective intrinsic anisotropy.* The *effective intrinsic radial anisotropy* corresponding to $\mathcal{A}(\mathcal{H}(\mathcal{A}(\mathbf{S})))$ will then be designated as ξ^*_{CPO}. Note that due to the non-linearity of $\mathcal{H}$, homogenization creates apparent isotropic heterogeneities in the elastic tensor $\mathcal{I}(\mathcal{H}(\mathcal{A}(\mathbf{S})))$ as a byproduct, albeit of low amplitude.

Finally, the tomographic counterpart of a pure isotropic Earth (*i.e.*, a model where the isotropic component varies spatially, and the anisotropic component is zero) is $\mathcal{H}(\mathcal{I}(\mathbf{S})) = \mathcal{H}(\mathbf{S_0} + \delta\mathbf{S_I})$ where the non-negligible apparent anisotropic component due to SPO $\mathcal{A}(\mathcal{H}(\mathcal{I}(\mathbf{S})))$ is called *extrinsic anisotropy.* Here, *extrinsic radial anisotropy* will be denoted by ξ^*_{SPO} (Refer to Figure 1 for a comprehensive summary).

3 Analytical expressions in the 1-D case

3.1 Backus homogenization

A vertically transverse isotropic (VTI) medium is an elastic medium with hexagonal symmetry and vertical symmetry axis. It can be described by five elastic parameters A, C, F, L, and N, also known as the Love parameters (Love, 1906). Supposing that axis 3 is the symmetry axis, the local $\mathbf{S}$ for a VTI solid can be expressed in Mandel notation as:

$$\mathbf{S} = \begin{pmatrix} A & A-2N & F & 0 & 0 & 0 \\ A-2N & A & F & 0 & 0 & 0 \\ F & F & C & 0 & 0 & 0 \\ 0 & 0 & 0 & 2L & 0 & 0 \\ 0 & 0 & 0 & 0 & 2L & 0 \\ 0 & 0 & 0 & 0 & 0 & 2N \end{pmatrix}, \tag{5}$$

and the level of shear wave radial anisotropy can be written as:

$$\xi = \frac{N}{L}. \tag{6}$$

Backus (1962) explicitly showed analytical upscaling relations for seismic waves propagating in a 1-D stratified medium. For a 1-D layered medium where each layer is a VTI medium, the long-wavelength effective medium is also a VTI medium. The effective equivalent of the elastic constants, for instance, N and L concerning the shear wave velocities are given by an arithmetic mean and a harmonic mean, respectively:

$$N^* = \langle N \rangle \,, \tag{7}$$

$$L^* = \langle 1/L \rangle^{-1} \,, \tag{8}$$

where $\langle . \rangle$ refers to the spatial average over a wavelength of any periodic function (in this case, N and $1/L$), and * denotes a long wavelength effective property. The effective density ρ^* is simply the arithmetic mean of the local density ρ:

$$\rho^* = \langle \rho \rangle \,. \tag{9}$$

The effective shear wave radial anisotropy ξ^* is essentially the ratio between the effective equivalents of N and L:

$$\xi^* = \frac{N^*}{L^*} = \langle N \rangle \langle 1/L \rangle \,. \tag{10}$$

In this way, for a 1-D fine-scale medium where each layer is isotropic ($N = L$), the long-wavelength effective medium is transversely isotropic, and the level of extrinic radial anisotropy is given by $\langle N \rangle \langle 1/N \rangle$ (Alder et al., 2017).

3.2 An analytical expression to quantify CPO and SPO in a 1-D layered media

Let us consider an intrinsically anisotropic (CPO component) and finely-layered (SPO component) VTI medium. Similar as to how we defined our reference medium in section 2, we regard the elastic parameters N and L as the sum of an isotropic component defined by the shear moduli μ, and local anisotropic perturbations N_A and L_A, respectively:

$$N(z) = \mu(z) + N_A(z), \tag{11}$$

$$L(z) = \mu(z) + L_A(z). \tag{12}$$

The isotropic equivalent of μ can be computed as (J.-P. Montagner, 2007; Maupin et al., 2007):

$$\mu = \frac{1}{15}(C + A - 2F + 6L + 5N), \tag{13}$$

where A and C are elastic parameters concerning $P-$waves, and F relates to the so-called 'ellipticity'(*i.e.*, the velocity along the direction interposing fast and slow velocities). Assuming no $P-$wave anisotropy and setting F to unity, one can simplify equation (13) to:

$$\mu = \frac{1}{3}(2L + N). \tag{14}$$

Knowing equations (6) and (14), one can re-write N and L in terms of μ and $\xi_{\mathrm{CPO}} = N/L$ giving:

$$N = \xi_{\mathrm{CPO}} \frac{3\mu}{2 + \xi_{\mathrm{CPO}}}, \tag{15}$$

$$L = \frac{3\mu}{2 + \xi_{\mathrm{CPO}}}. \tag{16}$$

It is now straightforward to determine the anisotropic components N_A and L_A by equating equations (11) and (15). For the sole purpose of segregating the isotropic and anisotropic components, the forms of N and L consistent with equation (1) is therefore:

$$N(z) = \mu(z) + \left(\xi_{\mathrm{CPO}}(z) \frac{3\mu(z)}{2 + \xi_{\mathrm{CPO}}(z)} - \mu(z) \right), \tag{17}$$

$$L(z) = \mu(z) + \left(\frac{3\mu(z)}{2 + \xi_{\mathrm{CPO}}(z)} - \mu(z) \right), \tag{18}$$

where any variable as a function of z implies variations in space.

To calculate the long-wavelength effective equivalent of such a medium, let us first write the parameters μ and ξ_{CPO} as:

$$\mu(z) = \langle \mu \rangle + \delta\mu(z), \tag{19}$$

$$\xi_{\mathrm{CPO}}(z) = \langle \xi_{\mathrm{CPO}} \rangle + \delta\xi_{\mathrm{CPO}}(z), \tag{20}$$

where $\langle\mu\rangle$ and $\langle\xi_{\mathrm{CPO}}\rangle$ are the spatially-averaged counterparts. $\delta\mu$ and $\delta\xi_{\mathrm{CPO}}$ are small-scale heterogeneities in the shear modulus and intrinsic radial anisotropy, respectively. They verify $\langle\delta\mu\rangle$ and $\langle\delta\xi_{\mathrm{CPO}}\rangle = 0$. The long-wavelength effective equivalents N^* and $1/L^*$ are:

$$N^* = \langle N\rangle = \left\langle \xi_{\mathrm{CPO}} \frac{3\mu}{2+\xi_{\mathrm{CPO}}} \right\rangle = \left\langle (\langle\xi_{\mathrm{CPO}}\rangle + \delta\xi_{\mathrm{CPO}}) \frac{3(\langle\mu\rangle + \delta\mu)}{2+\langle\xi_{\mathrm{CPO}}\rangle + \delta\xi_{\mathrm{CPO}}} \right\rangle, \tag{21}$$

$$1/L^* = \langle 1/L\rangle = \left\langle \frac{2+\xi_{\mathrm{CPO}}}{3\mu} \right\rangle = \left\langle \frac{2+\langle\xi_{\mathrm{CPO}}\rangle + \delta\xi_{\mathrm{CPO}}}{3(\langle\mu\rangle + \delta\mu)} \right\rangle. \tag{22}$$

We can simplify equations (21) and (22) by assuming a weak contrast in the shear modulus $\delta\mu \ll \langle\mu\rangle$. Using a second-order Taylor expansion, we get:

$$N^* \approx \frac{3\langle\mu\rangle}{2+\langle\xi_{\mathrm{CPO}}\rangle} \left(\langle\xi_{\mathrm{CPO}}\rangle - \frac{2}{(2+\langle\xi_{\mathrm{CPO}}\rangle)^2} \langle\delta\xi_{\mathrm{CPO}}^2\rangle + \frac{2}{\langle\mu\rangle(2+\langle\xi_{\mathrm{CPO}}\rangle)} \langle\delta\mu\cdot\delta\xi_{\mathrm{CPO}}\rangle \right), \tag{23}$$

$$1/L^* \approx \frac{2+\langle\xi_{\mathrm{CPO}}\rangle}{3\langle\mu\rangle} \left(1 + \frac{1}{\langle\mu\rangle^2}\langle\delta\mu^2\rangle - \frac{1}{\langle\mu\rangle(2+\langle\xi_{\mathrm{CPO}}\rangle)} \langle\delta\mu\cdot\delta\xi_{\mathrm{CPO}}\rangle \right). \tag{24}$$

Using equation (10), we multiply equations (23) and (24) and neglect terms higher than order two to obtain the full effective radial anisotropy ξ^* due to both fine-layering and intrinsic anisotropy:

$$\xi^* \approx \langle\xi_{\mathrm{CPO}}\rangle - \frac{2}{(2+\langle\xi_{\mathrm{CPO}}\rangle)^2}\langle\delta\xi_{\mathrm{CPO}}^2\rangle + \frac{\langle\xi_{\mathrm{CPO}}\rangle}{\langle\mu\rangle^2}\langle\delta\mu^2\rangle + \frac{2-\langle\xi_{\mathrm{CPO}}\rangle}{\langle\mu\rangle(2+\langle\xi_{\mathrm{CPO}}\rangle)}\langle\delta\mu\cdot\delta\xi_{\mathrm{CPO}}\rangle. \tag{25}$$

Equation (25) explicitly shows the separate effects of the small-scales in the isotropic component and in the intrinsically anisotropic component onto the effective anisotropy as 'seen' by long-period seismic waves. Assuming the medium to be devoid of intrinsic anisotropy (*i.e.*, $\xi_{\mathrm{CPO}} = 1$ and $\delta\xi_{\mathrm{CPO}} = 0$), the full effective radial anisotropy ξ^* directly relates to the level of heterogeneities in the shear moduli. Here, $\langle\delta\mu^2\rangle$ refers to the variance of small-scale heterogeneities of the shear modulus $\delta\mu$. It can be interpreted as the extrinsic radial anisotropy ξ^*_{SPO} due to the seismically unresolved small-scale isotropic heterogeneities. It varies as the square of the heterogeneities, in agreement with the result of Alder et al. (2017).

On the other hand, when the isotropic component is uniform (*i.e.*, $\delta\mu = 0$), ξ^* also varies with the square of heterogeneities, but now in intrinsic anisotropy. This can be interpreted as the effective intrinsic radial anisotropy ξ^*_{CPO}, i.e. the intrinsic anisotropy that gets smoothed out as a result of upscaling. Interestingly, its overall effect is to reduce the level of intrinsic anisotropy as indicated by the minus sign in front of the second term. In the absence of small-scale isotropic heterogeneities, we anticipate anisotropy to be always underestimated by tomography.

Finally, equation (25) suggests the existence of a cross-term $\langle \delta\mu \cdot \delta\xi_{\mathrm{CPO}} \rangle$ due to the spatial correlation between intrinsic anisotropy and shear modulus. Supposing spatial variations in both components are significant such as at major seismic discontinuities, the correlation term should influence the anisotropy mapped in tomographic models. Nevertheless, it is propounded that this correlation term is usually negligible and hence may need not be accounted for (Bakulin, 2003).

Similarly, the effective Voigt-averaged shear modulus μ^* is given by:

$$\mu^* = \frac{2L^* + N^*}{3}. \tag{26}$$

Plugging equations (23) and (24) into equation (26), we get:

$$\mu^* = \langle \mu \rangle - \frac{2}{\langle \mu \rangle (2 + \langle \xi_{\mathrm{CPO}} \rangle)} \langle \delta\mu^2 \rangle - \frac{2\langle \mu \rangle}{(2 + \langle \xi_{\mathrm{CPO}} \rangle)^3} \langle \delta\xi_{\mathrm{CPO}}^2 \rangle + \frac{4}{(2 + \langle \xi_{\mathrm{CPO}} \rangle)^2} \langle \delta\mu \cdot \delta\xi_{\mathrm{CPO}} \rangle. \tag{27}$$

Ignoring intrinsic anisotropy (*i.e.*, $\xi_{\mathrm{CPO}} = 1$ and $\delta\xi_{\mathrm{CPO}} = 0$), the effective shear modulus μ^* is always smaller than its spatially-averaged version $\langle \mu \rangle$. Such a result is logical in the 1-D case. Here, radial anisotropy induced by fine-layering is always positive (equation (25)) thereby having $N^* > L^*$. Since L 'counts' twice and N once in its isotropic average, its long-wavelength effective equivalent μ^* is always slower than $\langle \mu \rangle$. Contrastingly, if one neglects isotropic heterogeneities and only consider variations in intrinsic anisotropy, homogenization also results in the underestimation of the shear modulus. One would predict that homogenization leads to the creation of apparent isotropic heterogeneities due to small-scale variations in CPO. Lastly and as expected, the cross term recurs due to the spatial correlation between the shear modulus and intrinsic anisotropy.

We acknowledge that the homogenized expressions given by equations (25) and (27) in terms of the isotropic shear modulus μ may not be particularly convenient for seismologists. In practice, we observe spatial distributions in V_S and not in μ. If one assumes that density is uniform, then $\delta\mu/\mu$ can be simply replaced by $2\delta V_S/V_S$. On the other hand, if one assumes that density increases with V_S, one could also establish long-wavelength effective expressions for V_S in the same manner as μ using simple empirical relations for density such as that of Tkalčić et al. (2006).

In the Earth's asthenosphere, we do not expect velocity variations (*i.e.* $\Delta V_S/(V_{S1}+V_{S2})$) oftentimes to be larger than 5% (e.g. Xu et al., 2008; Stixrude & Jeanloz, 2015). This roughly translates to ∼10% heterogeneities in the shear modulus assuming constant density ρ. To perform a numerical estimate, let us examine a stack of planar layers with alternating shear moduli values determined by $\pm\ \Delta\mu/2$ (Figure 2a middle panel) corresponding to shear wave velocity variations $\Delta V_S/(V_{S1}+V_{S2})$ of about 10%. The 1-D depth profiles depict periodic variations with layers of equal thicknesses of 20 km. Positive intrinsic radial anisotropy ($\xi = 1.2$) is prescribed in the even layers, whereas the odd layers are isotropic ($\xi = 1$) (Figure 2a right panel). After upscaling over a wavelength much larger than 20 km, the resulting profiles for N^* and L^* are homogeneous, and simply given by their arithmetic and harmonic means, respectively (Figure 2a left panel). Once the long-wavelength effective N^* and L^* are acquired, we can compute the full effective radial anisotropy ξ^* through equation (10) (solid red line in Figure 2a right panel), and the effective shear modulus μ^* through equation (26) (solid red line in Figure 2a middle panel). Figure 2b illustrates a different scenario where ξ only exists in the odd layers (Figure 2b right panel). In essence when the shear modulus and intrinsic anisotropy are uncorrelated, the homogenized parameters μ^* and ξ^* should be the same regardless. However, a slight offset in μ^* and ξ^* of Figure 2b with respect to Figure 2a can be observed which is exclusively attributed to this cross term as hinted by equations (25) and (27). Strictly speaking, the reduction in the amplitude of the effective properties arises from the switch in signs in the cross term from positive to negative $\langle \delta\mu \cdot \delta\xi \rangle$, implying that in the second scenario, the shear modulus and intrinsic anisotropy are now anti-correlated.

To validate the second-order approximation, we also show ξ^* and μ^* using equations (25) and (27) respectively (dashed blue lines in Figures 2a and 2b middle and right panels). Clearly by applying equation (25), the intrinsic component (first term) contributes the most to the effective anisotropy with $1 - \langle \xi_{\text{CPO}} \rangle = 0.1$ wherein its spatial variations' overall effect is to tone-down the amplitude of anisotropy by an amount of $\sim -10^{-3}$ (second term). This is followed by the SPO component (third term) which is responsible for the amplification of anisotropy ($\sim +10^{-2}$). Lastly, the cross term provides the least contribution ($\sim \pm 10^{-3}$) and therefore can reasonably be ignored in most cases. The $\pm$ sign denotes that it may increase or decrease anisotropy depending on the coupling pattern between the shear modulus and intrinsic anisotropy.

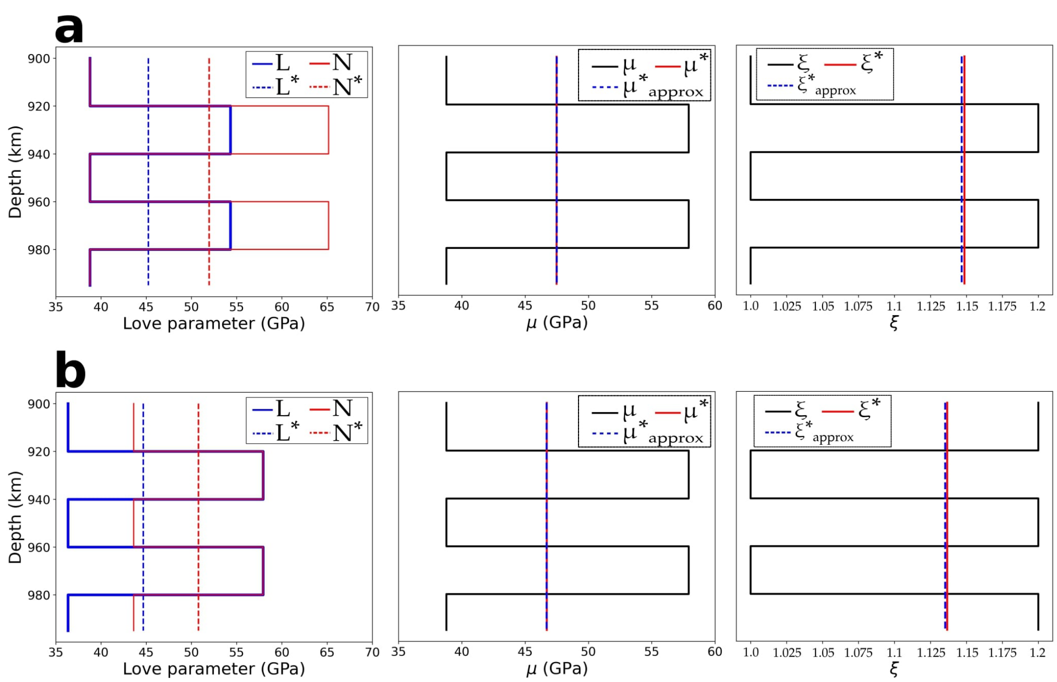

Figure 2. 1-D binary and periodic media with 10% isotropic heterogeneities in terms of $\Delta V_S/(V_{S1} + V_{S2})$ prescribed across: (a) even layers, and (b) odd layers. Upon homogenization, the resulting profiles are homogeneous (variables denoted by (*)). The dashed blue lines at the middle (μ^*_{approx}) and right panels (ξ^*_{approx}) correspond to the predicted long-wavelength effective equivalents using the second order approximations from equations (27) and (25), respectively. The difference in the homogenized shear moduli and radial anisotropy between (a) and (b) is attributed to the cross term as implied by equation (25). Since the medium is periodic, it is enough to only display a portion of the medium.

3.3 Composite law for radial anisotropy

In this section, we show how the total effective anisotropy can be expressed in terms of its intrinsic and extrinsic contributions. For that, we investigate two special cases: (1) a purely isotropic 1-D layered medium, (2) a purely anisotropic 1-D medium (*i.e.*, no spatial variations in isotropic component), and find equivalent expressions for extrinsic radial anisotropy ξ^*_{SPO} and effective intrinsic radial anisotropy ξ^*_{CPO}. By doing so, we elicit a simple composite law related to equation (25) that can be extrapolated to 2-D and 3-D media.

In the case of an isotropic medium with spatially-varying shear modulus, the radial anisotropy is entirely due to SPO. Equation (25) reforms into:

$$\xi^*_{\mathrm{SPO}} \approx 1 + \frac{1}{\langle\mu\rangle^2}\langle\delta\mu^2\rangle. \tag{28}$$

On the other hand, a purely anisotropic medium without spatial variations in the shear modulus leads to an effective intrinsic anisotropy:

$$\xi^*_{\mathrm{CPO}} \approx \langle\xi_{\mathrm{CPO}}\rangle - \frac{2}{(2+\langle\xi_{\mathrm{CPO}}\rangle)^2}\langle\delta\xi^2_{\mathrm{CPO}}\rangle. \tag{29}$$

By taking the product between equations (28) and (29), neglecting terms higher than order two, one has simply:

$$\xi^*_{\mathrm{CPO}} \times \xi^*_{\mathrm{SPO}} \approx \langle\xi_{\mathrm{CPO}}\rangle - \frac{2}{(2+\langle\xi_{\mathrm{CPO}}\rangle)^2}\langle\delta\xi^2_{\mathrm{CPO}}\rangle + \frac{\langle\xi_{\mathrm{CPO}}\rangle}{\langle\mu\rangle^2}\langle\delta\mu^2\rangle, \tag{30}$$

which is approximately equal to ξ^* in equation (25) but without the cross term. Therefore, ignoring spatial correlations between intrinsic anisotropy and shear modulus, the full effective radial anisotropy can be quantified through the following composite law:

$$\xi^* \approx \xi^*_{\mathrm{CPO}} \times \xi^*_{\mathrm{SPO}}. \tag{31}$$

In practice, ξ^* can be estimated from a tomographic inversion (Debayle & Kennett, 2000; Plomerová et al., 2002; Gung et al., 2003; Nettles & Dziewoński, 2008a; Fichtner et al., 2010). Seismologists often compare ξ^* with the intrinsic radial anisotropy ξ_{CPO} computed from a geodynamically-based CPO model (Becker et al., 2003, 2006; Ferreira et al., 2019; Sturgeon et al., 2019). The comparison should be done instead with an effective model ξ^*_{CPO}, which is difficult to estimate without access to any elastic homogenization tools. Furthermore, equation (25) suggests that there is a non-negligible extrinsic component of anisotropy due to the unresolved small-scale isotropic heterogeneities. While it is difficult to rigorously establish analytical solutions in the case of a 2-D/3-D complex media, following the logic above, we hypothesize that the mismatch often observed between homogenized CPO models and tomographic models is the extrinsic radial anisotropy ξ^*_{SPO}.

4 Methods for 2-D media

4.1 Homogenization in 2-D and in 3-D media

The classic homogenization method of Backus is only applicable in 1-D to media exhibiting spatial periodicity. The true Earth, however, is a complex 3-D and multi-scale medium. To alleviate this problem and quantify effective elastic properties in a mantle-like medium, we rely on the non-periodic elastic homogenization technique developed by Capdeville and Marigo (2007); Capdeville et al. (2010); Guillot et al. (2010); Capdeville et al. (2015). Originally, this method has been developed as a pre-processing step enabling one to solve the elastostatic wave equation using a simple mesh, speeding up the computations for wave propagation in complex media. It has also been used to improve the convergence and computational cost of full waveform inversion (Capdeville & Métivier, 2018; Hedjazian et al., 2021).

In the non-periodic case, the homogenization is not performed with respect to the periodicity of the medium, but with respect to the minimum wavelength present in the wavefield. The assumption that this minimum wavelength λ_{min} exists is required for non-periodic medium with no scale separation such as the true Earth. Scales smaller than λ_{min} are seen by the wavefield only through their effective properties. To separate the small and the large scales, we need to define a threshold wavelength λ_h, called the homogenization wavelength. λ_h is a user-defined parameter, and all scales smaller than λ_h are homogenized. Numerical examples suggest that, for all natural media, homogenization with a value $\lambda_h = 0.5\lambda_{\mathrm{min}}$ is sufficient to accurately reproduce the complete wavefield (Capdeville et al., 2010). Hence, this value is chosen in the rest of the present study. Computing the effective properties of an elastic medium with homogenization wavelength λ_h requires to solve an elastostatic problem numerically. To do this, we use the 3-D Fast-Fourier Homogenization algorithm developed by Capdeville et al. (2015).

Assuming a good data coverage, Capdeville and Métivier (2018) numerically verified that for complex elastic media, a seismic tomography model and the homogenized model are in agreement at spatial wavelengths higher than λ_h. Hence, homogenization can be viewed as a first-order tomographic operator. We will consider the homogenized model as the best image one could get from seismic tomography. This can be translated to:

$$\mathbf{S}^* = \mathcal{H}(\mathbf{S}) \tag{32}$$

where $\mathcal{H}$ is the tomographic operator, $\mathbf{S}$ is the reference medium, and the homogenized model $\mathbf{S}^*$ is the full effective medium (*i.e.*, the best recovered image as seen by a wavefield of a given minimum wavelength λ_{min} and assuming perfect data coverage). In this paper, we apply this 'tomographic operator' to a 2-D composite medium by upscaling the marble cake model in the presence of deformation-induced anisotropy.

4.2 Isotropic heterogeneities in a 2-D mechanically-mixed mantle

To define our 2-D incompressible flow model imitating mantle convection, we use a stream function similar to that of Alder et al. (2017):

$$\Psi(x,z,t) = \sin(a\pi z)\big[\sin(b\pi x) + \alpha(t)\sin((b+1)\pi x) + \beta(t)\sin((b+2)\pi x)\big] \tag{33}$$

where $\alpha(t)$ and $\beta(t)$ are sinusoidal functions of time that introduces chaotic mixing. The variables a and b relate to the number of distinguishable convection cells and are chosen arbitrarily. The form of the function Ψ ensures free-slip boundary conditions. Finally, the resulting velocity field is scaled using a reference value of 1 cm·yr^{-1}.

We replicate the marble cake patterns by deforming a circular anomaly at the center of the box using our prescribed flow field. To do this, control points that define the contour of the anomaly are advected using fourth-order Runge Kutta methods with variable time-stepping (Press et al., 1992). To achieve a final configuration for the anomaly, we define an advection mixing time T_{SPO}. Figure 3 illustrates the evolution of the pattern when subjected to the flow field defined in equation (33). Setting $a = 1$, $b = 2$, and $T_{\mathrm{SPO}} = 75$ My, we have a mechanically-mixed medium with two characteristic convection cells.

Using the last panel of Figure 3, the binary system is defined by assigning a reference S-wave velocity value $V_{S_2} = 4.52$ km·s^{-1} to the yellow region, and $V_{S_1} =$

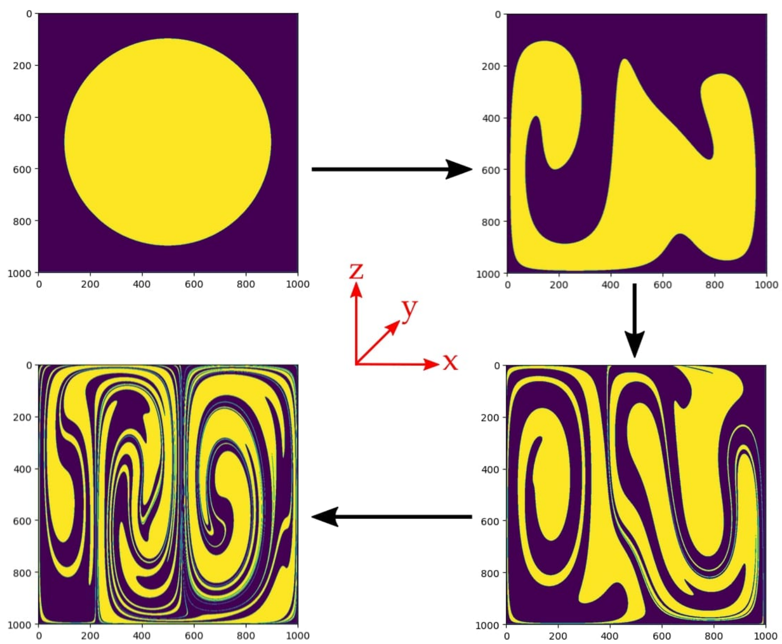

Figure 3. Initially a circle, the anomaly is deformed progressively until the medium reaches a stage resembling marble cake-like patterns.

3.7 km$\cdot$s^{-1} to the purple region so that the level of isotropic heterogeneities is given by $100\% \times (V_{S_1} - V_{S_2})/(V_{S_1} + V_{S_2})) = 10\%$. $P-$wave velocities are computed by imposing a constant ratio $V_P/V_S = 1.7$ (Obrebski et al., 2010). Following the work of Tkalčić et al. (2006), we compute the density ρ using the empirical relation $\rho = 2.35 + 0.036(V_P - 3)^2$. These values are used to define the local isotropic tensor $\mathbf{S_I}$ in equation (1).

4.3 Modeling of Crystallographic Preferred Orientation

Crystallographic preferred orientation of mantle minerals forms due to strain accumulation over time. The velocity gradient tensor field of the marble cake model $\boldsymbol{\nabla}\mathbf{u}$ is derived from the stream function Ψ described previously. We then model CPO evolution of olivine aggregates using `D-Rex`, a program which calculates strain-induced CPO by plastic deformation, and dynamic recrystallization (E. Kaminski et al., 2004). The activities of olivine slip systems are chosen to correspond to dry mantle conditions, while other parameters are taken as in the reference `D-Rex` model. A time scale for CPO evolution T_{CPO} is defined to control the level of intrinsic anisotropy.

In our numerical experiments, we compute CPO everywhere irrespective of the actual mineralogical phase. We scale the elastic tensor derived from `D-Rex` so that its isotropic component is identical to the binary system derived in Section 4.2. The reference medium can be constructed from equation (1) where $\mathbf{S_I}$ now relates to the small-scale isotropic heterogeneities in the mechanically-mixed mantle, and $\mathbf{S_A}$ is the intrinsically anisotropic component computed with `D-Rex`.

4.4 Quantifying the level of anisotropy

In this section, we define two ways to quantify the level of seismic anisotropy for any given elastic tensor $\mathbf{S}$. The first one is radial anisotropy. We project the elastic tensor in terms of an azimuthally-averaged vertically transverse isotropic (VTI) medium to obtain a tensor described as in equation (5). Here, the parameters L and N can be computed from $\mathbf{S}$ as follows (J.-P. Montagner & Nataf, 1986):

$$L = \frac{1}{2}(S_{44} + S_{55}) \tag{34}$$

$$N = \frac{1}{8}(S_{11} + S_{22}) - \frac{1}{4}S_{12} + \frac{1}{2}S_{66}. \tag{35}$$

The level of radial anisotropy is then given by equation (6).

Another convenient way to quantify anisotropy is to compute the percentage of total anisotropy by taking the L2-norm fraction of the anisotropic part of the elastic tensor with respect to the isotropic part. This quantity is called the anisotropy index and is given by:

$$\text{anisotropy index} = \frac{||\mathbf{S} - \mathbf{S_I}||}{||\mathbf{S_I}||}. \tag{36}$$

5 Elastic homogenization of a 2-D mechanically-mixed mantle in the presence of CPO

Figure 4 displays some seismic properties of the reference medium $\mathbf{S}$ before and after homogenization. The left panels are the true structures, whereas the middle and right panels are the structures equating to the full effective medium $\mathcal{H}(\mathbf{S})$ at homogenization wavelengths λ_h of 200 km and 500 km, respectively. The first row depicts the shear wave velocities. The anisotropy characterized by its radial component, and by its norm fraction are depicted in the second row and in the third row, respectively. Each pixel initially contains an isotropic part derived from the marble cake model with a mixing time for advection T_{SPO} ~75 My , and an anisotropic part computed from a

CPO model with a time scale for CPO evolution of $T_{\rm CPO}$ ∼40 My corresponding to a moderately developed crystal fabric. Hereafter, we run our simultaion in a box of size 1000 × 1000 km, define $\lambda_{\rm max} = 1000$ km as the length scale of our simulations.

Several glaring features can be observed such as the presence of positive radial anisotropy ($\xi > 1$) at the top and bottom boundaries where flow is sub-horizontal, and likewise negative ($\xi < 1$) at regions where the flow is sub-vertical. As expected, homogenization results in the smoothing of the structures with the level of smoothing modulated by λ_h. However, homogenization is not just a simple spatial average but a product of highly non-linear upscaling relations. With increasing homogenization wavelengths, the full effective medium becomes devoid of anisotropy in some areas.

After decomposing $\mathbf{S}$ into an isotropic tensor $\mathcal{I}(\mathbf{S})$ and an anisotropic tensor $\mathcal{A}(\mathbf{S})$ through equations (2) and (4), one can also homogenize and analyze each component separately, i.e. $\mathcal{H}(\mathcal{I}(\mathbf{S}))$ and $\mathcal{H}(\mathcal{A}(\mathbf{S}))$. Figure 5 shows the level of effective radial anisotropy of these two separate components after homogenization. The top panels recreate the results of Alder et al. (2017). Indeed, homogenizing the fine-layered isotropic medium produces extrinsic radial anisotropy $\xi^*_{\rm SPO}$ (*i.e.*, radial anisotropy of model $\mathcal{H}(\mathcal{I}(\mathbf{S}))$). Notice that the patterns of effective intrinsic radial anisotropy and extrinsic radial anisotropy maps are roughly similar. For example, they both induce a positive radial anisotropy $\xi > 1$ in the horizontal layers: the stretched heterogeneities that induce SPO become elongated along the direction of the maximum principal strain rate that also controls the CPO.

Figure 6 depicts the apparent isotropic heterogeneities created upon homogenization of $\mathcal{A}(\mathbf{S})$. It produces maximum velocity perturbations of about 0.25 % at $\lambda_h = 200$ km and 0.2 % at $\lambda_h = 500$ km. It appears to be a small effect, especially considering the large and sharp variations of intrinsic anisotropy in our CPO model.

To better illustrate the behaviour of different contributions to anisotropy, we plot in Figure 7 the amplitude of radial anisotropy (in terms of its standard deviation over the entire 2-D model domain) against the wavelength of homogenization λ_h. In the following cases, the intrinsic anisotropy component of the reference medium $\mathbf{S}$ is computed for a CPO developing over increasing duration $T_{\rm CPO}$ of 5, 40, or 75 Myr. Several point can be noted in Figure 7:

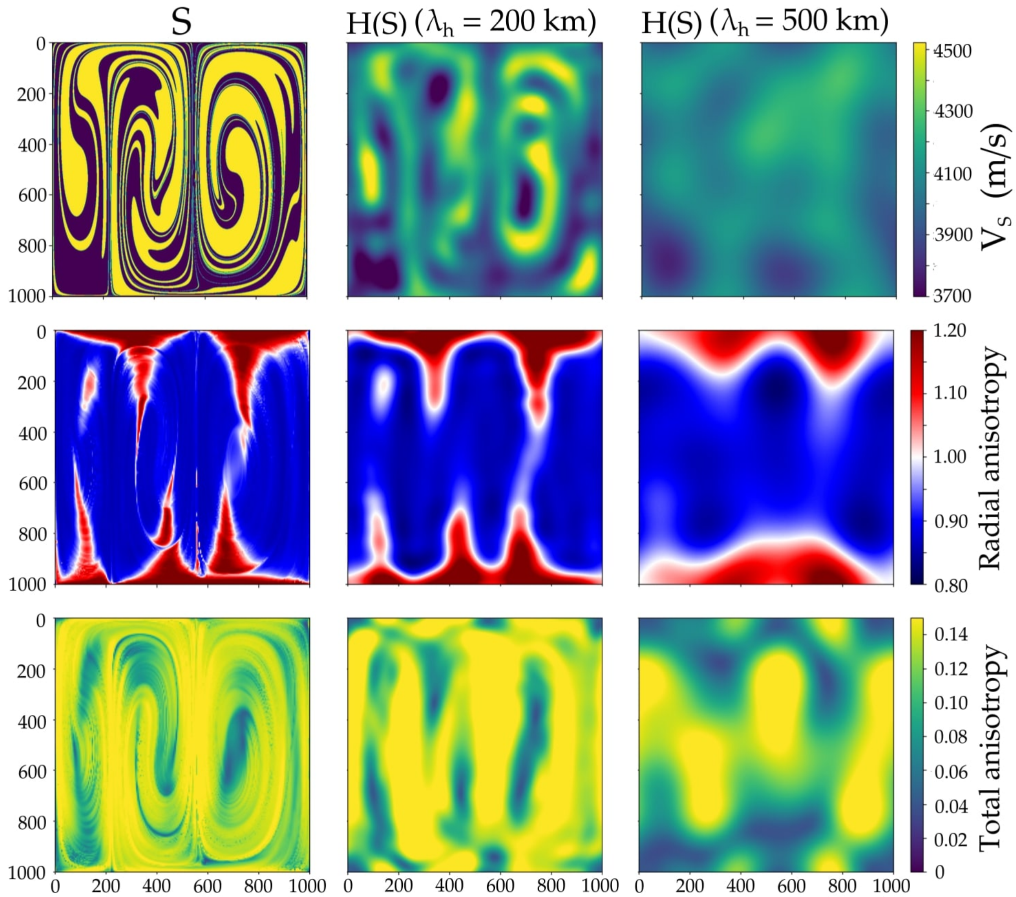

Figure 4. Seismic properties of the reference medium **S** before and after homogenization. The model dimensions are 1000 km × 1000 km × 1000 km, with cell sizes of 5 km × 5 km × 5 km. Here, each pixel contains an **S** which consists of small-scale isotropic heterogeneities and an intrinsically anisotropic perturbation computed with D-Rex (E. Kaminski et al., 2004). The present-day marble cake patterns correspond to a mixing time for advection $T_{\text{SPO}} \sim 75$ My, whereas the time scale for CPO evolution is $T_{\text{CPO}} \sim 40$ My. We homogenized **S** using the Fast-Fourier homogenization algorithm of Capdeville et al. (2015). (From left to right) First row: V_s models derived from **S**, $\mathcal{H}(\mathbf{S})$ at $\lambda_h = 200$ km, and $\mathcal{H}(\mathbf{S})$ at $\lambda_h = 500$ km. Second row: ξ_{CPO}, ξ^* at $\lambda_h = 200$ km, and ξ^* at $\lambda_h = 500$ km. Last row: Total anisotropy in terms of the norm fraction of **S**, $\mathcal{H}(\mathbf{S})$ at $\lambda_h = 200$ km, and $\mathcal{H}(\mathbf{S})$ at $\lambda_h = 500$ km. Elastic homogenization can be viewed as the best possible model reconstructed by seismic tomography assuming perfect ray-path coverage.

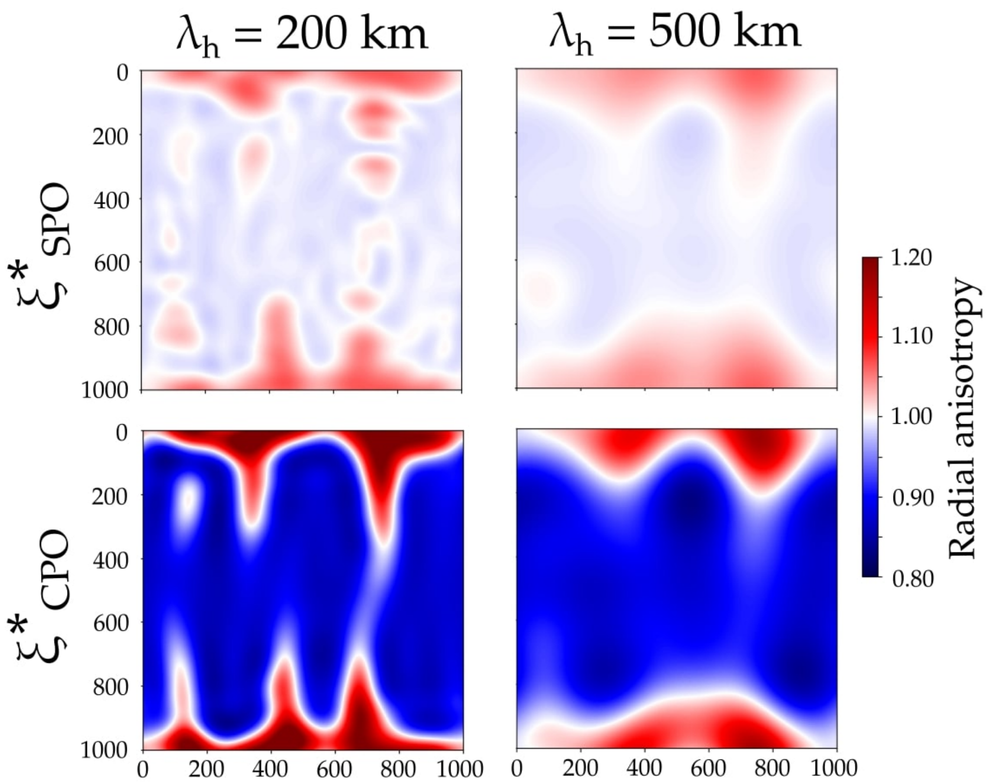

Figure 5. Extrinsic radial anisotropy ξ^*_{SPO} (*i.e.*, radial anisotropy of model $\mathcal{H}(\mathcal{I}(\mathbf{S}))$) (top panels) at two different wavelengths of homogenization λ_h. Here, $\xi^*_{\text{SPO}} > 1$ is now interpreted as horizontal layering whereas < 1 as vertical layering. The bottom panels show the effective intrinsic radial anisotropy ξ^*_{CPO} (*i.e.*, radial anisotropy of model $\mathcal{H}(\mathcal{A}(\mathbf{S}))$).

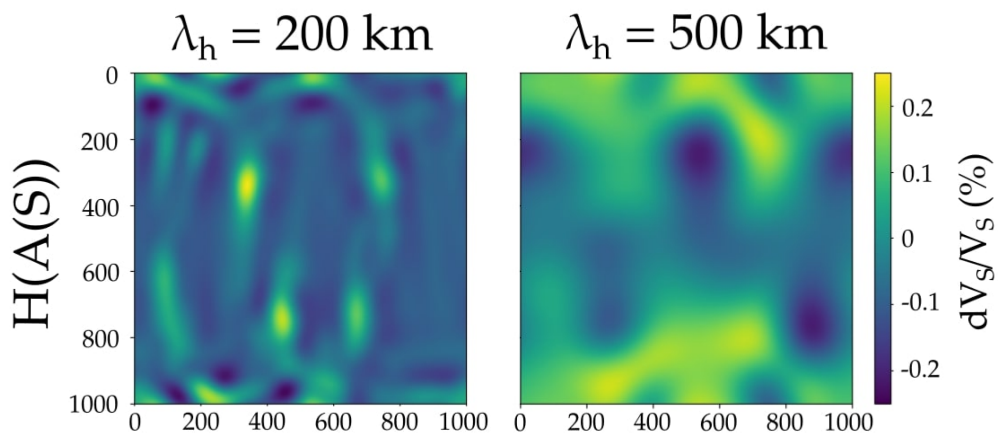

Figure 6. Apparent isotropic velocity perturbations with respect to a mean velocity V_S at two different wavelengths of homogenization λ_h. $\mathcal{H}(\mathcal{A}(\mathbf{S}))$ pertains to the homogenized model of a purely anisotropic medium. Even when placed in a very favorable scenario for intrinsic anisotropy, homogenizing a purely anisotropic medium produces a meager 0.25% artificial heterogeneities at $\lambda_h = 200$ km and 0.2% at $\lambda_h = 500$ km.

(i) The resulting intrinsic radial anisotropy ξ_{CPO} in terms of its standard deviation over the entire region (dashed lines) increases with T_{CPO}, although some saturation is observed (*i.e.*, the orientation of crystals depends mostly on their recent deformation, and lose the memory of the deformation they underwent too long ago).

(ii) The level of intrinsic anisotropy is diminished upon homogenization. ξ^*_{CPO} (hollow squares) is always lower than the reference value ξ_{CPO} (dashed lines), and diminishes with λ_h. This effect can be easily understood. For small λ_h, the wavelength of homogenization is small compared to the scale of deformation patterns (of order 100 km). At each point of the 2-D map, the direction of CPO is therefore locally constant over λ_h, which yields $\xi^*_{\mathrm{CPO}} \approx \xi_{\mathrm{CPO}}$. At larger scales, when λ_h increases compared to the scale of convection, this direction becomes likely random and CPO heterogeneities averaged over λ_h have different orientations: there is less of a preferential direction and the averaged level of CPO anisotropy is diminished.

(iii) On the contrary, the full effective radial anisotropy ξ^* at short wavelengths of homogenization λ_h is larger than ξ_{CPO}. This is in agreement with the analytical expression given by equation (25). This additional anisotropy is of course due to the existence of SPO (black circles) which reinforces the total level of effective anisotropy.

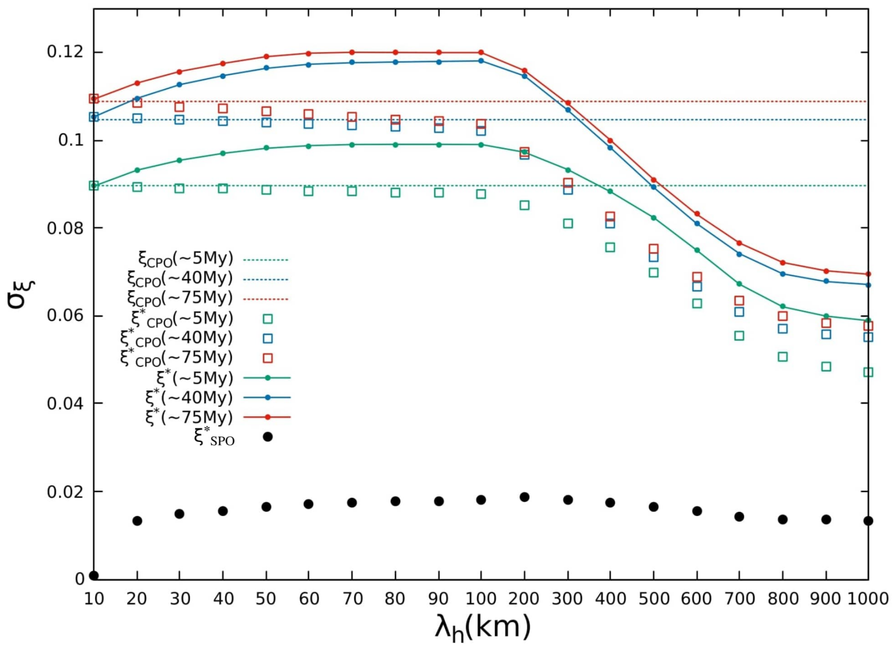

Figure 7. Effective radial anisotropy in terms of its standard deviation σ_ξ over the entire 2-D image, plotted as a function of homogenization length. The time scales indicated in million years pertain to the evolution history of CPO (a larger time scale leads to stronger CPO). Dashed lines represent the standard deviation of $\xi_{\rm CPO}$ in model $\mathbf{S}$ and serve as reference values. In this experiment, $\xi^*_{\rm SPO}$ of model $\mathcal{H}(\mathcal{I}(\mathbf{S}))$ (black circles) deemed to be five times smaller than $\xi^*_{\rm CPO}$ of model $\mathcal{H}(\mathcal{A}(\mathbf{S}))$ (hollow squares). Since SPO is mostly in-phase with CPO, the two anisotropic components add constructively giving the full effective radial anisotropy ξ^* (solid line-dots).

(iv) Both ξ^*_{CPO} and ξ^* converge toward ξ_{CPO} at infinitely short homogenization wavelengths. Only in this unrealistic case (*i.e.*, the perfect recording of the seismic wavefield up to infinitely short periods), would seismic tomography be able to map the true intrinsic anisotropy.

(v) Extrinsic radial anisotropy ξ^*_{SPO} here has an amplitude that is five times smaller than ξ^*_{CPO}. Such a result, of course, is specific to this numerical experiment, and that CPO is indeed stronger than SPO might not be always true. For instance, a longer mixing time would have resulted in a thinner and more complex layering that would have increased the SPO. We are unfortunately limited by the number of tracers necessary to describe the phase stirring which is exponentially increasing with time.

5.1 Verifying the composite law $\xi^* = \xi^*_{\mathrm{CPO}} \times \xi^*_{\mathrm{SPO}}$ in 2-D

In this section, we aim to verify numerically equation (31) in 2-D by plotting $\xi^*_{\mathrm{SPO}} \times \xi^*_{\mathrm{CPO}}$ against ξ^* for each pixel in our 2-D maps of radial anisotropy. Here again, the three quantities ξ^*_{SPO}, ξ^*_{CPO}, and ξ^* are respectively computed from $\mathcal{H}(\mathcal{I}(\mathbf{S}))$, $\mathcal{H}(\mathcal{A}(\mathbf{S}))$, and $\mathcal{H}(\mathbf{S})$. In this case, there is no spatial correlation between CPO and isotropic heterogeneities in the marble cake, and the effect of the cross-term should be mitigated. Figure 8b shows this for two different homogenization wavelengths λ_h. We can see that the relation holds exceptionally well even for large λ_h.

In practice however, tomographic models of ξ^* are interpreted in terms of intrinsic anisotropy, and directly compared with ξ_{CPO} computed from CPO models (Becker et al., 2003, 2006; Ferreira et al., 2019). We mimic this scenario by comparing $\xi^*_{\mathrm{SPO}} \times \xi_{\mathrm{CPO}}$ instead with ξ^* (Figure 8a). As it turns out, the relation only holds for small values of λ_h. At larger values of λ_h, the trend appears to be more dispersed as a consequence of the averaging process, losing its viability to some extent. In the absence of a homogenized CPO model, we project that this composite law would remain true in general under the condition that the minimum wavelength used in tomography is sufficiently small.

To test the effect of the rigidity-anisotropy cross-term, we consider another mantle model where CPO is only present in one of the two phases of the 2-D marble cake illustrated in Figure 3. In addition, we increase the percentage of isotropic heterogeneities in V_S to 15%. This increases the correlation between the shear modulus and

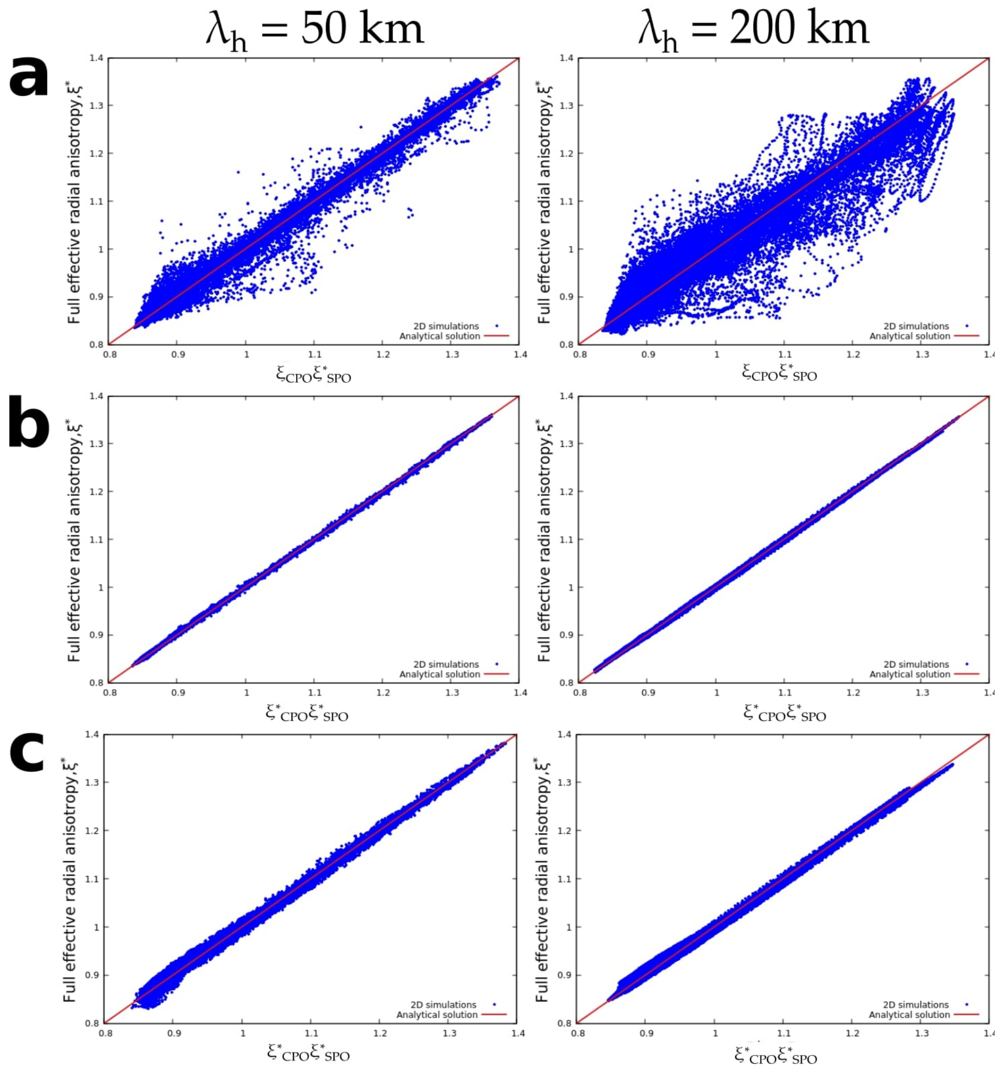

Figure 8. For each pixel i on our 2-D maps of ξ^*, ξ_{CPO}, ξ^*_{CPO}, and ξ^*_{SPO} Figure 8a: , we plot a cloud of points with coordinates: (a) $[\xi_{\mathrm{CPO}} \times \xi^*_{\mathrm{SPO}}(i), \xi^*(i)]$, (b) $[\xi^*_{\mathrm{CPO}} \times \xi^*_{\mathrm{SPO}}(i), \xi^*(i)]$ overlaying the analytical solution equation (31) (solid red lines). Our numerical simulations suggest that CPO models should be homogenized first before comparing with tomographic models. Figure 8c displays the effect of the cross term by increasing isotropic heterogeneities to 15% and by prescribing CPO only in the yellow phases of the marble cake model in Figure 3.

intrinsic anisotropy. Figure 8c displays the numerical solution at $\lambda_h = 50$ km and 200 km when CPO is computed in the yellow phases alone. Based on our analytical results, the points are much more spread-out than that of Figure 8b. In this scenario, CPO now varies sharply and in the same places as isotropic discontinuities (*i.e.*, $\delta\xi_{\mathrm{CPO}}$ terms in equation (25) are much larger), and as expected the cross-term is much more apparent. Nonetheless, this only produces small departures from the composite law (red line), implying that the predictions carried out by the composite law are robust.

5.2 Discussion

We investigated the effects of elastic homogenization to a specific class of fine-scale, marble cake-like models of the mantle in the presence of deformation-induced anisotropy. The homogenization procedure can be viewed as a tomographic operator applied to a reference elastic model (Capdeville et al., 2013).

We showed that the extrinsic radial anisotropy produced by fine-layering could reach up to 2% (see Figure 7) assuming 10% of isotropic heterogeneities. This anisotropy is much lower than the one induced by CPO where the effective intrinsic radial anisotropy could peak at nearly 11%. This result is however modulated by some parameters that regulate the level of effective anisotropy. For example, the layered filaments contrived from our marble cake models are of the order $10-100$ km whereas of those proposed by Allègre and Turcotte (1986) are much thinner and can stretch even further down to the centimeter scale, which would induce larger extrinsic anisotropy. Next is more of on the intrinsic aspect. Since CPO results from finite strain accumulation over time, the amplitude of intrinsic anisotropy increases with the time scale for CPO evolution T_{CPO}. Such presumptions may only be valid in regions where rock deformation varies over extended periods of time, although recrystallization and damage would limit the CPO that can be eventually accumulated (Ricard & Bercovici, 2009). Furthermore, we considered olivine of type-A crystal fabric as the solitary anisotropic mineral in our mantle models. Because of this, the intrinsic anisotropy produced from finite deformation should be seen as an upper bound. Inclusion of other anisotropic minerals such as pyroxene which make up a fraction in mantle periodotite (Maupin & Park, 2015) would change the net anisotropy. For instance, we anticipate that including a substantial amount of enstatite would dilute the amount of anisotropy (e.g. E. Kaminski et al., 2004). Therefore, whether CPO accounts for most of the bulk anisotropy observed in tomographic images remains inconclusive and needs further verification.

In light of the simulations conducted, we expect large-scale anisotropy to be only overestimated when CPO coexists with significant SPO as exemplified in our simulations. In the absence of SPO, homogenization can only decrease the strength of anisotropy. By accounting for both contributions, we showed that $\xi > 1$ is attributed to a combination of lateral flow and horizontal layering, and $\xi < 1$ is a combination of flow ascent and vertical layering. Indeed, the direction of shear not only dictates the preferred orientation of the anisotropic minerals, but also of the orientation of the folded strips that gives rise to fine-layering.

The repercussion of homogenizing intrinsic anisotropy alone amounts to the convection-scale averaging of the CPO as evidenced by our simulations. By definition, intrinsic anisotropy develops from the preferred alignment of the minerals at the crystal level. This explains why we can map-out the fast axis of anisotropy at an arbitrary location. When long period observations sample an intrinsically anisotropic medium, the wavefield spatially-averages these orientations. As a result, preferential orientations that are products of imbricated convection tend to appear more heterogeneous, thereby ostensibly losing intrinsic anisotropy upon homogenization. In contrast, spatially-coherent preferential orientations produced by simpler convection patterns are less susceptible to the dilution of intrinsic anisotropy.

The applicability of equation (31) in a 2-D complex media may be of interest to geodynamicists and tomographers alike. Not only does it permit one to directly quantify the discrepancy between the full effective radial anisotropy inferred from a tomographic model and the effective intrinsic radial anisotropy computed from a homogenized CPO model, it further solidifies the supposition that the mismatch is indeed a result of extrinsic radial anisotropy due to the seismically-unresolved small-scale isotropic heterogeneities. We have conducted several numerical experiments to prove that the composite law still holds exceptionally well even when the rigidity-intrinsic anisotropy cross term is amplified. Erroneous interpretation of a CPO model to explain tomographic observations may therefore be avoided with the help of homogenization.

The conclusions reached in this section are based on two assumptions: (1) We held the isotropic velocity contrast at a fixed value and assumed it to be representative of the entire mantle. In reality however, V_S variations between two-end member compositions generally decrease with depth (Xu et al., 2008; Stixrude & Jeanloz, 2015). This is not to mention the local presence of melt and water that contributes to the variations in wave velocities, and hence the strength of heterogeneities which completely alters the level of apparent anisotropy. (2) We disregarded the dependency of the elastic constants built from our mantle models on pressure P and temperature T. Future avenues one could take would be to incorporate $P-T$ dependence using empirical relations constrained from laboratory experiments. For instance, one may compute $P-T$ dependence using first-order corrections around a reference elastic tensor at ambient $P-T$ conditions (Estey & Douglas, 1986). The availability of self-consistent thermodynamic models based on free-energy minimization schemes (J. A. Connolly, 2005; J. Connolly, 2009) can also be employed in lieu of the simpler relations for more accurate predictions of seismic wave velocities in any given bulk composition (Stixrude & Lithgow-Bertelloni, 2011).

6 Separating SPO from CPO in tomographic models: Application to radial anisotropy beneath oceanic plates

6.1 Radial anisotropy beneath oceanic plates

Within the context of seismic tomography, surface waves offer the capability to image upper-mantle structure providing an in-depth view of large-scale anisotropy. Surface wave tomography images positive radial anisotropy underneath oceanic basins ($V_{SH} > V_{SV}$), characterized by a layer of strong signatures lying in between $\sim 80-200$ km depth, corresponding to the asthenosphere (e.g. J.-P. Montagner, 1985; Ekström & Dziewonski, 1998; Panning & Romanowicz, 2006; Nettles & Dziewoński, 2008b). The maximum positive vertical gradient of ξ^*, at ~ 80 km depth, independent of plate age, is a recurrent feature in these tomographic models. This has raised questions about the potential use of radial anisotropy as a marker of the lithosphere-asthenosphere boundary (LAB), which is expected on the contrary to deepen with plate age (Burgos et al., 2014; Beghein et al., 2019). The strong radial anisotropy in the asthenosphere is usually explained by geodynamic models including CPO evolution (Becker et al., 2006, 2008).

Across the oceanic lithosphere, plate-averaged radial anisotropy (*i.e.*, all points in the radial anisotropy models with the same plate age are averaged) displays modest levels of about 1−3%. Several models have been proposed to explain these observations. Hansen et al. (2016) and Hedjazian et al. (2017) suggest that CPO-related radial anisotropy developed below the ridge is subsequently frozen in the lithosphere, leading to an age-independent signature. It has also been proposed quasi-laminated melt structures, preserved during lithospheric thickening, can also explain this frozen-in signature of anisotropy (e.g. Auer et al., 2015; Debayle et al., 2020). Hence SPO may also be a potential explanatory mechanism, and a substantial fraction of the observed lithospheric anisotropy may be due to small-scale isotropic heterogeneities (Wang et al., 2013; Kennett & Furumura, 2015).

The corroboration of the composite law in a 2-D complex medium prompted us to assess the discrepancy between a tomographic model and a CPO model of upper-mantle radial anisotropy underneath a mid-ocean ridge. In our hypothesis, this should commensurate to the extrinsic radial anisotropy due to the unresolved small-scales in seismic velocities.

6.2 The tomographic model

In conjunction with the pre-existing global V_{SV} model of the upper-mantle constrained from Rayleigh wave data `DR2012` (Debayle & Ricard, 2012), we adopt the recent global V_{SH} model `CAM2016SH` of Ho et al. (2016) to acquire a plate-averaged 2-D profile of radial anisotropy associated with slow-spreading oceanic ridges.

The V_S models were reconstructed by independently inverting Love (for V_{SH} models) and Rayleigh (for V_{SV} models) waveforms up to the fifth overtone between the period range $50 - 250$ s using an extension of the automated waveform inversion approach of Debayle (1999). We refer the reader to Debayle and Ricard (2012) and Ho et al. (2016) for a more detailed description of the inversion procedure.

From the V_{SV} and V_{SH} models of the upper-mantle, we compute the tomographic counterpart of radial anisotropy using $\xi^* = (V_{SH}/V_{SV})^2$. Here, we acknowledge that ξ^* is not directly inferred from simultaneous inversions of Love and Rayleigh data but is a rudimentary estimate from the two $S-$wave velocity models that may conceivably have different qualities. We view the following exercise as only a proof-of-concept and therefore the results should be interpreted with caution.

The depth distribution of ξ^* spanning from $35 - 400$ km is shown in Figure 9 (top panel). Positive radial anisotropy values ($\xi^* > 1$) are confined in the upper $\sim$ 200 km of the model domain which is in close agreement with previous studies (e.g. J.-P. Montagner, 1985; Ekström & Dziewonski, 1998; Panning & Romanowicz, 2006). Although the origin of anisotropy imaged in the asthenosphere is well-understood purely in terms of CPO, anisotropy observed in the lithopshere may be a combination of CPO and SPO (Wang et al., 2013). Here our task is to invoke the composite law to isolate SPO from CPO in this tomographic model with the help of a homogenized CPO model.

6.3 The CPO model

In this section, we re-interpret the results of Hedjazian et al. (2017) where they examined radial anisotropy profiles predicted from CPO models produced by plate-driven flows underneath a mid-ocean ridge. From their work, we borrowed two CPO models that correspond to a fast-developing CPO and a slow-developing CPO. The rate is dictated by the dimensionless grain boundary mobility parameter M which controls the kinetics of grain growth (and hence, the degree of dynamic recrystallization) (E. Kaminski et al., 2004). In the first case, a value of $M = 125$ constrained from laboratory experiments (Nicolas et al., 1973; Zhang & Karato, 1995) corresponding to CPO produced from uniform deformation and initially-random CPO was imposed (E. Kaminski et al., 2004). Subsequently, the second case considers a case where $M = 10$ (*i.e.*, slower CPO evolution) which also reproduces experimental results but in the case of an initially developed CPO (Boneh et al., 2015). Hedjazian et al. (2017) compared the CPO anisotropy directly with tomographic models. They concluded that the patterns of radial anisotropy predicted with the slow CPO evolution were in better agreement with tomographic models. We homogenize the two CPO models and obtain their long-wavelength effective equivalent, and again appraise the resulting profiles in comparison with tomographic observations.

6.3.1 The intrinsic CPO mineralogical model

2-D surface-driven mantle flows were acquired using the code `Fluidity` (Davies et al., 2011). In both models, upper-mantle deformation is governed by a composite dislocation and diffusion creep rheology following the implementation of Garel et al.

(2014). `D-Rex` was used to model CPO evolution. A complete description of the methodology can be found in Hedjazian et al. (2017).

Figure 9 displays the intrinsic radial anisotropy profiles ξ_{CPO} belonging to the fast-evolving CPO with reference `D-Rex` values $M = 125$ (model A) and the slow-evolving CPO with $M = 10$ (model B). Model A predicts a layer with strong levels of intrinsic radial anisotropy of about 10% ($\xi_{\mathrm{CPO}} \approx 1.1$) at a depth of $\sim$ 80 km starting at approximately 20 My. At about the same depth, tomographic models yield approximately 5% radial anisotropy (e.g. Panning & Romanowicz, 2006; Nettles & Dziewoński, 2008b; Burgos et al., 2014). Hence, it has been argued that model A overpredicts the observed level of large-scale anisotropy in the upper-mantle (Hedjazian et al., 2017). On the contrary, model B predicts modest levels of intrinsic radial anisotropy, about 5% ($\xi_{\mathrm{CPO}} \approx 1.05$) across the oceanic lithosphere which is more consistent with tomographic observations.

6.3.2 The homogenized CPO model

Figure 9 now shows the effective intrinsic radial anisotropy profiles ξ^*_{CPO} of model A* and model B*. In both cases, the ensuing patterns of radial anisotropy are smoothed out as a result of homogenization. For instance, the apparent two-layered distribution of intrinsic radial anisotropy with depth (down to $\sim$ 250 km) in model A vanishes after homogenization. The depth profile of effective intrinsic radial anisotropy as a result contain one layer of radial anisotropy centered at $\sim$ 100 km depth, making it now compatible with tomographic models of the asthenosphere. Furthermore, it was implied that radial anisotropy predicted with typical laboratory-derived parameters exceeds tomographic observations. Here, we argue that it may also be the other way around. Due to finite-frequency effects and eventually limitations in resolution power, seismic tomography may underestimate the strength of intrinsic anisotropy, at least in the absence of small-scale isotropic heterogeneities. The level of radial anisotropy in models A and B are therefore larger than their homogenized/tomographic counterparts (models A* and B*). As opposed to common practice, the physical parameters used in CPO models of which are initially constrained by experimental data may need not be manually tuned, and perhaps that the action of varying such parameters to conform with tomographic observations deems unnecessary. We therefore conclude that direct visual comparison between a CPO model and a tomographic model could lead to wrong interpretations, and that homogenization is necessary to have correct interpretations of the CPO models.

6.4 Deriving an SPO model

The SPO models of Figure 9 (models C and D) can be estimated by using our composite law in equation (31). The extrinsic radial anisotropy is obtained by simply dividing the tomographic model of radial anisotropy by that of the homogenized CPO model:

$$\xi^*_{\mathrm{SPO}} = \frac{\xi^*}{\xi^*_{\mathrm{CPO}}}. \tag{37}$$

In this way, models C and D are obtained from models A* and B*, respectively.

Strong levels of positive extrinsic radial anisotropy near the ridge axis may be due to the inability of surface waves to register vertical flow because of its limited lateral resolution. Model D, associated with the slow-evolving CPO model B, is almost devoid of SPO. This is expected since model B was tailored to fit seismic tomography observations from CPO only. Based on our results, one should favor SPO model C that corresponds to a fast-evolving CPO model. It displays positive extrinsic radial anisotropy above 200 km depth. This is more consistent with the existence of lateral fine-scale structures at the base of the lithosphere (e.g. Auer et al., 2015; Kennett & Furumura, 2015).

7 Conclusion

Differentiating the relative contributions of CPO and SPO to the full effective medium is not a simple, straightforward process. The tomographic operator (here approximated by $\mathcal{H}$) acts as a smoothing operator, and its inverse is highly non-unique. It is therefore clearly impossible to separate the CPO and SPO contributions in a tomographic model. One of the most logical courses of action is to compare tomographic models of anisotropy with existing micro-mechanical models of CPO evolution (e.g. Becker et al., 2003, 2006; Ferreira et al., 2019). Here, we proposed a very simple composite law that directly relates the separate contributions of CPO and SPO to the full effective radial anisotropy ξ^* inferred from tomographic models:

$$\xi^* = \xi^*_{\mathrm{SPO}} \times \xi^*_{\mathrm{CPO}},$$

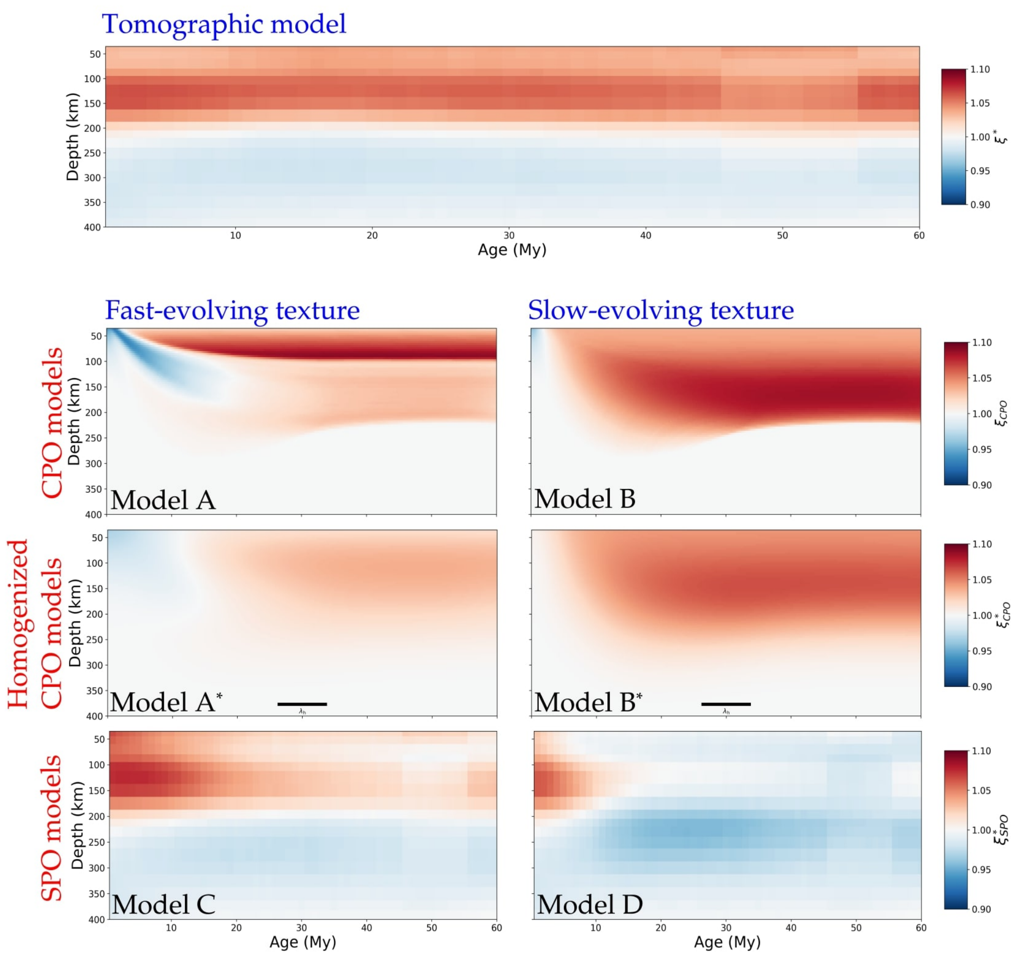

Figure 9. Plate-averaged radial anisotropy across the upper-mantle beneath oceanic basins with ages ranging between 0 and 80 Myrs obtained from a tomographic model (top panel), reference CPO models corresponding to fast and slow-evolving textures (models A and B), homogenized versions of model A (model A*) and of model B (model B*). Models C and D, respectively, are the extrinsic radial anisotropy profiles computed by dividing ξ^* of the tomographic model, by $\xi^*_{\rm CPO}$ of model A* and B*, using the composite law. Positive lithospheric radial anisotropy in model C implies the existence of horizontally-laminated structures. This is absent in model D which is expected since model B* is designed to fit observations.

which we have numerically verified using simple 2-D toy models of an intrinsically anisotropic and a heterogeneous mantle. Although our numerical experiments were mainly a proof-of-concept, comparing a CPO model directly to an existing tomographic model is unwarranted and we highly recommend homogenizing a CPO model as an intermediate step.

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