

Changes in Crack Shape and Saturation in Laboratory-Induced Seismicity by Water Infiltration in the Transversely Isotropic Case with Vertical Cracks

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Abstract

Open cracks and cavities play important roles in fluid transport. Underground water penetration induces microcrack activity, which can lead to rock failure and earthquake. Fluids in cracks can affect earthquake generation mechanisms through physical and physicochemical effects. Methods for characterizing the crack shape and water saturation of underground rock are needed for many scientific and industrial applications. The ability to estimate the status of cracks by using readily observable data such as elastic-wave velocities would be beneficial. We have demonstrated a laboratory method for estimating the crack status inside a cylindrical rock sample based on a vertically cracked transversely isotropic solid model by using measured P- and S-wave velocities and porosity derived from strain data. During injection of water to induce failure of a stressed rock sample, the crack aspect ratio changed from 1/400 to 1/160 and the degree of water saturation increased from 0 to 0.6. This laboratory-derived method can be applied to well-planned observations in field experiments. The in situ monitoring of cracks in rock is useful for industrial and scientific applications such as the sequestration of carbon dioxide and other waste, induced seismicity, and measuring the regional stress field.

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2 **Changes in Crack Shape and Saturation in Laboratory-Induced Seismicity by Water**

3 **Infiltration in the Transversely Isotropic Case with Vertical Cracks**

4

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10 Short title: Changes in Crack Shape and Saturation in Laboratory-Induced Seismicity

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14

15 **Summary**

16 Open cracks and cavities play important roles in fluid transport. Underground water penetration
17 induces microcrack activity, which can lead to rock failure and earthquake. Fluids in cracks can
18 affect earthquake generation mechanisms through physical and physicochemical effects.
19 Methods for characterizing the crack shape and water saturation of underground rock are needed
20 for many scientific and industrial applications. The ability to estimate the status of cracks by
21 using readily observable data such as elastic-wave velocities would be beneficial. We have
22 demonstrated a laboratory method for estimating the crack status inside a cylindrical rock sample
23 based on a vertically cracked transversely isotropic solid model by using measured P- and S-
24 wave velocities and porosity derived from strain data. During injection of water to induce failure
25 of a stressed rock sample, the crack aspect ratio changed from 1/400 to 1/160 and the degree of
26 water saturation increased from 0 to 0.6. This laboratory-derived method can be applied to well-
27 planned observations in field experiments. The in situ monitoring of cracks in rock is useful for
28 industrial and scientific applications such as the sequestration of carbon dioxide and other waste,
29 induced seismicity, and measuring the regional stress field.

30

31

32 **Key words**

33 Fracture and flow; Geomechanics; Hydrogeophysics; Acoustic properties; Induced seismicity

34

35 **1. Introduction**

36 Pore pressure change and fluid migration are known to cause rock deformation and failure (e.g.,
37 Healy et al., 1968; Lei et al., 2008; Ohtake, 1987; Prioul et al., 2000; Raleigh et al., 1976).

38 Because open cracks and cavities play important roles in fluid transport (Caine et al., 1996;
39 Rutqvist et al., 2008), the evolution of microcracking in the presence of underground fluids and
40 crustal stresses is a critical factor in geothermal energy extraction (e.g., Fehler, 1989), carbon
41 dioxide capture and storage (e.g., Baines & Worden, 2004; Hangx et al., 2010), waste disposal,
42 and induced seismicity (Ellsworth et al., 2013; Schultz et al., 2020). Methods to measure the
43 volume and shape of cavities and cracks would be of great assistance in planning industrial and
44 scientific applications, including characterization of regional stress fields and sequestration of
45 carbon dioxide and other waste. Methods are particularly needed for in situ monitoring
46 microcrack evolution at depths of around 1 km.

47

48 In this research on microcrack activity caused by hydrological effects inside rock samples, we
49 conducted laboratory studies with the aim of constructing a basic model for these underground
50 processes. This paper describes an in situ monitoring method for estimating the crack shape and
51 degree of water saturation in rock samples from measured P- and S-wave velocities and porosity
52 changes.

53

54 Field experiments on water-induced seismicity have been conducted for scientific and industrial
55 purposes. Experiments conducted in Matsushiro, Japan (Ohtake, 1987), and Rangely, Colorado,
56 USA (Raleigh et al., 1976), have revealed relationships between water injection and induced
57 microearthquakes. Water-injection experiments conducted in deep boreholes, such as the

58 German Continental Deep Drilling Program (KTB), have revealed characteristics of induced
59 seismicity (Jost et al., 1998; Zoback & Harjes, 1997). Other studies have examined
60 microearthquakes induced by water injection to infer seismic mechanisms and movements of
61 fluids (e.g., Eyre et al., 2019, 2020; Lei et al., 2008; Prioul et al., 2000; Schultz et al., 2017,
62 2018; Wang et al., 2020). Such studies yield information on the relationship between water
63 injection and initiation of microcracking; however, it is hard to obtain sufficient information
64 about crack shape and degree of water saturation for modeling underground crack and fracture
65 systems. Because of the difficulty of determining crustal stresses underground and distributing
66 observation stations optimally, it is difficult to construct basic models using field studies.

67

68 To investigate the shape of microcracks induced in rock samples, we studied hydromechanical
69 effects on the complex processes that control rock failure in the laboratory. Laboratory studies
70 enable us to tightly control conditions and precisely measure data such as sample deformation
71 and velocity changes in P and S waves. Laboratory studies are useful for constructing physical
72 models for basic mechanisms that take place in long-term geological processes (Benson et al.,
73 2008; Burlini et al., 2009; Masuda, 2013; Masuda et al., 2012). For example, Kranz et al. (1990),
74 Lockner and Byerlee (1977), Masuda et al. (1990), Lockner et al. (1991), and Scholz (1968)
75 investigated microfracturing through acoustic emission (AE) inside rock samples and developed
76 techniques for analyzing AE. The relationship between water migration and induced
77 microfractures also has been investigated in the laboratory (e.g., Masuda et al., 1990, 1993;
78 Stanchits et al., 2011).

79

80 In this study, we developed a procedure for estimating crack status inside a rock sample based on
81 a vertically cracked transversely isotropic solid model. We estimated two crack characteristics,
82 crack shape and the degree of water saturation, and their changes during water migration into a
83 granitic rock subjected to confining pressure and differential stress.

84

85 **2. Sample and Methods**

86 A cylinder (50 mm in diameter and 100 mm in length) of medium-grained Inada granite with an
87 average grain size of 5 to 6 mm was used for the experiment. A differential stress of 370 MPa,
88 which corresponds to about 70% of fracture strength, was applied to the rock sample in the axial
89 direction at a constant rate of 0.06 MPa/s under 30 MPa confining pressure and was held
90 constant throughout the experiment. When the primary creep stage and AE caused by the initial
91 loading had ceased, we injected distilled water into the bottom end of the sample at a constant
92 pressure of 25 MPa until macroscopic fracture occurred. Figure 1 shows the stress conditions and
93 the number of AE events as a function of time.

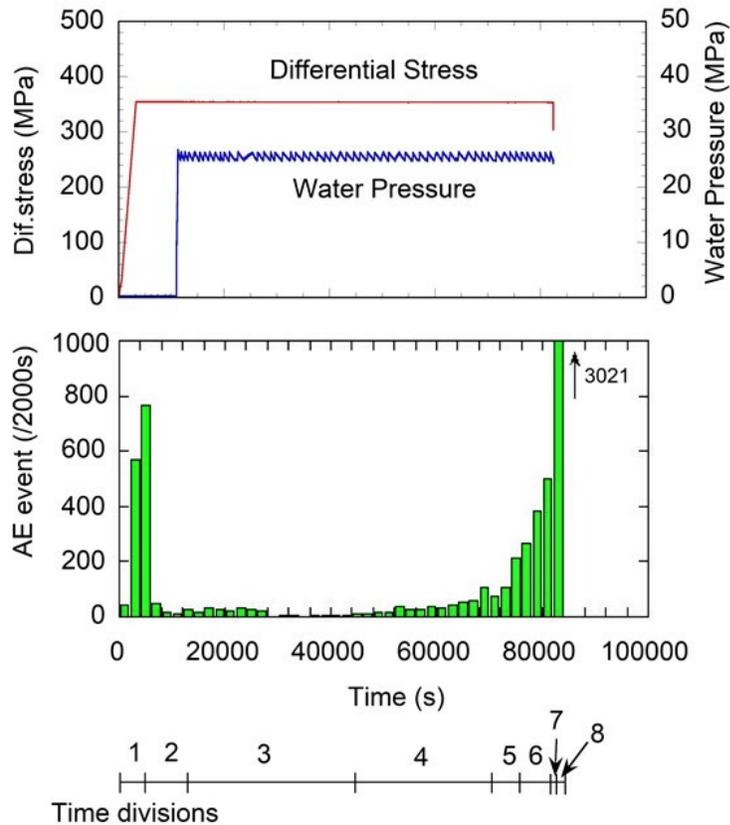


Figure 1

94

95 **Figure 1.** Changes in differential stress, water pressure at the injection site, and number of
 96 acoustic emission (AE) events as a function of time. Confining pressure was 30 MPa. Numbers
 97 at the bottom of the figure show time divisions for the plot of AE locations in Figure 8.

98

99

100 During water migration, P- and S-wave velocities, which propagated along five paths parallel to
 101 the top and bottom surfaces of the sample, were measured. Strains of the sample surface and AE
 102 were monitored and recorded. The locations of instrumentation on the surface of the rock sample
 103 are shown in Figure 2. Axial and circumferential strains were measured using six pairs of strain
 104 gauges at the midpoint of the sample's length as indicated by the large blue circles in Figure 2.

105 Piezoelectric transducers (PZTs) with 2-MHz resonant frequency were attached at the 18 places
106 indicated by the small red circles in Figure 2 and inside the top and the bottom end-pieces. At 10
107 of these locations (1–5 and 10–14 in Figure 2), we attached transducers for P waves, vertical S
108 waves (S_v), and horizontal S waves (S_H), in which the subscript specifies the direction of
109 vibration. With the pulse transmission method, P-wave and horizontal and vertical S-wave
110 velocities across the rock sample at five locations were measured (Figure 3). Because low-
111 porosity aggregate was considered, the effect of porosity on the density of the sample could be
112 ignored in calculating velocities (Anderson et al., 1974). In this study, the sample rock material
113 was considered to be homogeneous because the wavelength of the wave velocity was longer than
114 the scale length of heterogeneity in the rock. The sampling rate of the digital recording system
115 was 50 ns. AE signals were also recorded with all 18 P-wave transducers shown in Figure 2, plus
116 the two transducers on the top and bottom of the rock sample. AE hypocenters were determined
117 by the automatic arrival time and hypocenter determination method of Lei et al. (2004).

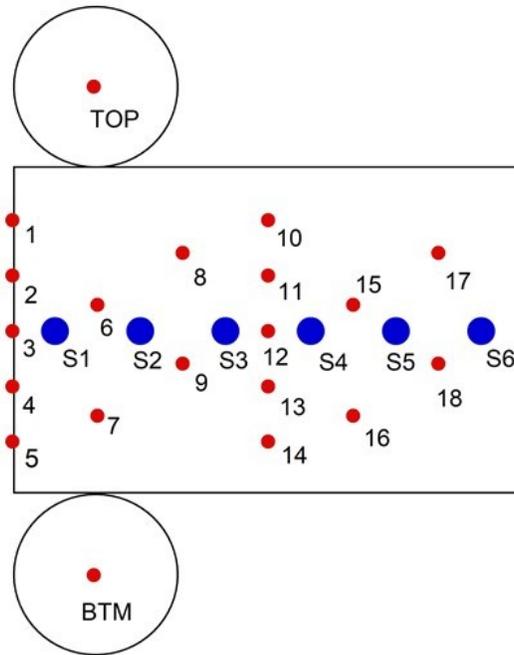


Figure 2

118

119 **Figure 2.** Locations of piezoelectric transducers and strain gauges on a rock sample. Schematic
 120 map of the cylindrical surface of the sample. Large blue circles (S1 to S6) indicate the locations
 121 of pairs of cross-strain gauges that monitored surface strains. Small red circles (1 to 18, TOP,
 122 BTM) indicate the locations of piezoelectric transducers.

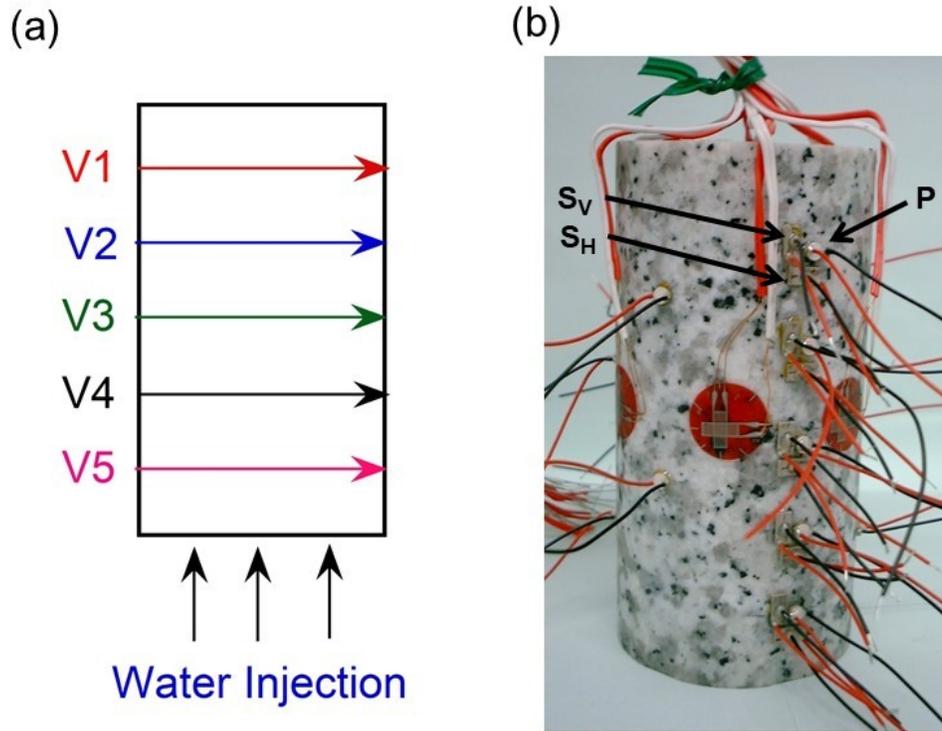


Figure 3

123

124 **Figure 3.** Paths of velocity measurements across the rock sample. (a) For P- and S-wave velocity
 125 measurements, elastic pulses were initiated from transducers V1 through V5 (at locations 1
 126 through 5 in Figure 2) and received by transducers on the other side (at locations 10 through 14
 127 in Figure 2). Water was injected uniformly from the bottom surface of the rock sample. (b)
 128 Photograph of the instrumented rock sample showing placement of transducers for detection of
 129 three types of elastic waves (locations 1–5 and 10–14 in Figure 2).

130

131

132 After the experiment, X-ray computer tomography (CT) images of the rock sample were created.

133 Images were made in the plane perpendicular to the sample axis at 1-mm intervals, then

134 combined into a three-dimensional model displaying the internal structure of the sample,
135 including the shapes and locations of the fracture planes.

136

137 **3. Results**

138 **3.1. P-wave velocity**

139 The measured P-wave velocities in the rock sample are shown in Figure 4 as a function of time.

140 During the initial loading stage, the P-wave velocity decreased due to the opening of new

141 microcracks. After water injection began, the P-wave velocity increased on each measurement

142 path in sequence from 5 to 1 as the water reached it (Figure 4) because open pores were partially

143 filled with water, a phenomenon well documented in the literature (e.g., Budiansky & O'Connell,

144 1976; O'Connell & Budiansky, 1974). The P-wave velocity then gradually decreased due to

145 undersaturation as the rate of cracking exceeded the rate of fluid flow, similar to the pattern

146 documented by Masuda et al. (1990, 1993).

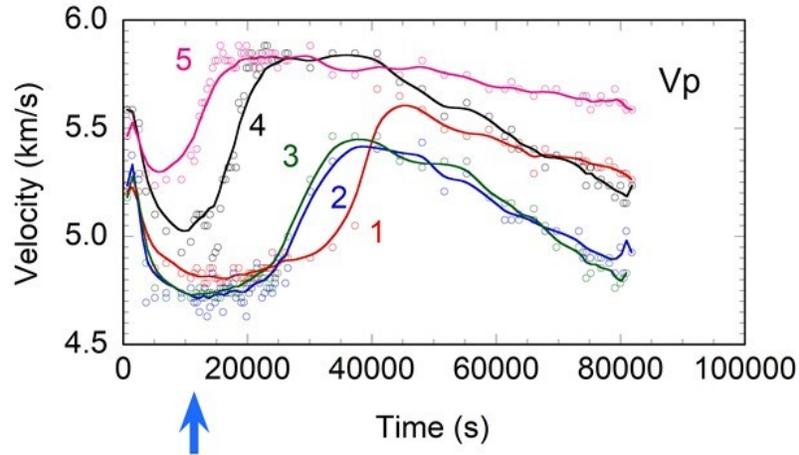


Figure 4

147

148 **Figure 4.** P-wave velocity for five transects of the rock sample (locations shown in Figure 3a) as
 149 a function of time. The blue arrow indicates the time when water injection started (modified from
 150 Masuda et al., 2013).

151

152

153 3.2. S-wave velocity

154 The velocities of the vertical and horizontal S waves are shown in Figure 5. Some transducers
 155 failed and yielded no data for the vertical S waves in measurement path 4 and for the horizontal
 156 S waves in paths 1, 3, and 4. At the times when the injected water front reached the measurement

157 paths, as estimated from the P-wave velocity changes, the S-wave velocities increased slightly
 158 and then decreased gradually.

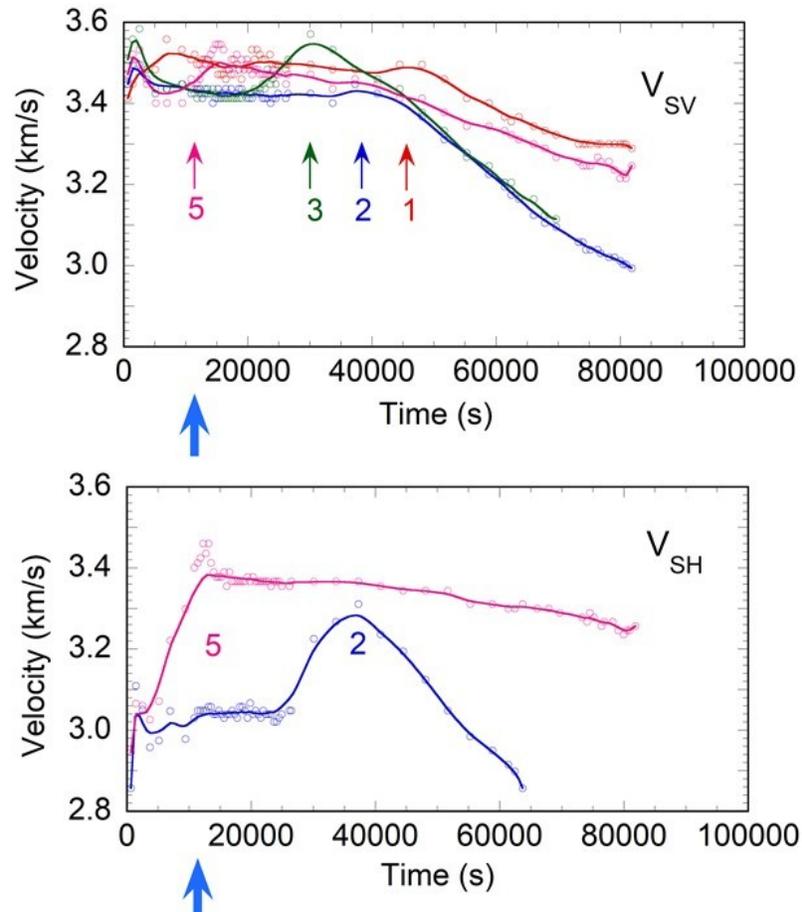


Figure 5

159
 160 **Figure 5.** S-wave velocity with (a) vertical vibration S_V and (b) horizontal vibration S_H for four
 161 transects of the rock sample (locations shown in Figure 3a) as a function of time. The blue arrow
 162 indicates the time when water injection started. The numbered arrows show the estimated time
 163 when the water front reached the corresponding measurement path. Velocity data for several
 164 transects were not available.

165

166

167 The changes in V_p/V_s (where V_p and V_s are the P- and S-wave velocities, respectively) for
 168 vertical and horizontal S waves are plotted in Figure 6. This ratio tends to be somewhat higher in
 169 the absence of cracks, and saturating the cracks leads to an increase in V_p/V_s as described by
 170 Paterson and Wong (2005).

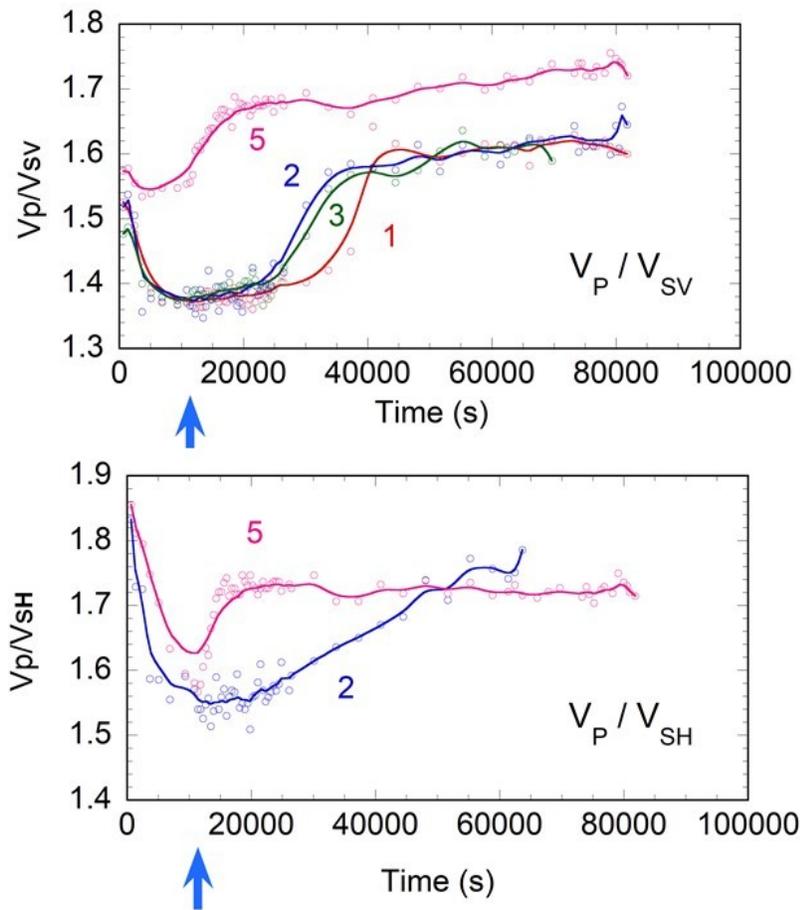


Figure 6

171

172 **Figure 6.** V_p/V_s ratios for S_v and S_h waves for transects of the rock sample (locations shown in

173 Figure 3a) as a function of time.

174

175 3.3 Strain

176 The average axial, circumferential, and volumetric strains measured at the midpoint of the

177 sample are shown in Figure 7. The strain gauges for axial strain at point S4 (Figure 2) failed, so

178 no data were available. The plotted axial strain data were averaged over the remaining five
 179 locations. The circumferential strain shown is the average data for all six locations of the strain
 180 gauges. The volumetric strain was calculated as the axial strain plus two times the
 181 circumferential strain. After a time duration of 60,000 s, the circumferential strains measured at
 182 locations S3 and S4 rapidly increased and the strain gauges at these locations failed.

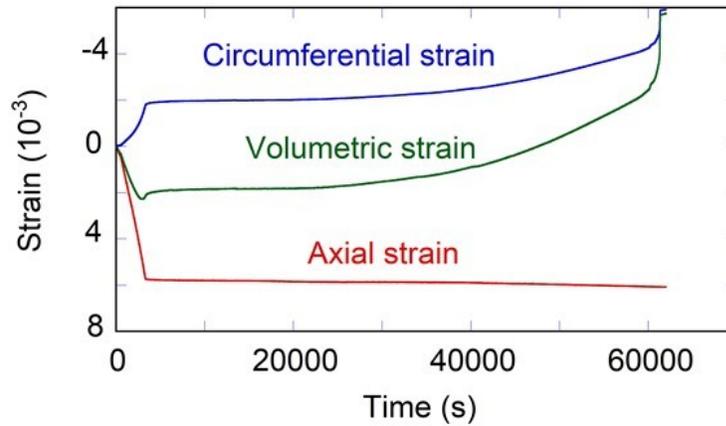


Figure 7

183

184 **Figure 7.** The average axial, circumferential, and volumetric strains as a function of
 185 experimental duration.

186

187

188

189 **3.4. AE hypocenter distribution**

190 AE hypocenters were calculated by automated detection of P-wave arrivals. Figure 8 shows
191 stereographic projections of AE hypocenter distributions for the eight time periods shown in
192 Figure 1. The estimated location error for most AE events was less than 2 mm (e.g., Lei et al.,
193 2004; Schubnel et al., 2003). Clustering of AE events was not observed except just before the
194 final fracture (periods 7 and 8). In the initial loading stage (period 1), AE was distributed evenly
195 throughout the sample, suggesting that the loading was achieved uniformly. Before the start of
196 water injection (period 2) and just afterward (period 3), AE activity decreased. In periods 3 and
197 4, AE persisted in the middle of the sample and expanded upward, and in periods 5 and 6 AE
198 activity increased. Just before the fracture (periods 7 and 8), AE clustered in the lower part at a
199 location corresponding to the final fracture surface.

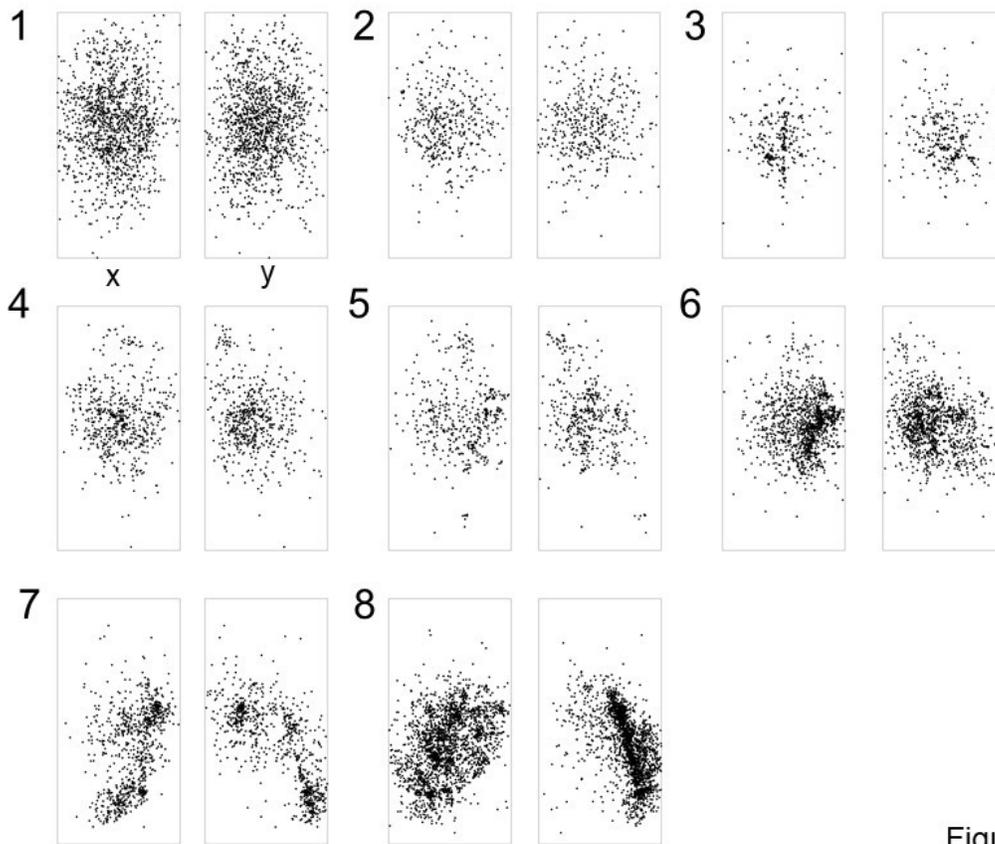


Figure 8

200

201 **Figure 8.** The vertical cross sections of the rock sample showing AE event locations during the
 202 eight time periods indicated in Figure 1.

203

204

205 3.5. X-ray CT imagery

206 After the experiment, we compiled 2D X-ray CT-images perpendicular to the sample axis, each
 207 representing 1 mm in thickness, at 1-mm intervals along the sample axis (99 total). The 3D
 208 image shown in Figure 9 was constructed from these 2D images. Figure 10 shows vertical cross
 209 sections derived from the 3D image. Three fracture surfaces (F1, F2, and F3) were recognized.

210 The location and shape of surface F1 correspond to the fracture surface indicated by the AE

211 hypocenter distributions (Figure 8). Fracture surface F2 formed at the edge of the sample,
 212 parallel to the sample surface. Fracture surface F3 was a small planar feature derived from the
 213 main rupture surface F1. The AE hypocenter distributions shown in Figure 8 indicate that rupture
 214 surface F1 formed and grew throughout the experiment whereas surfaces F2 and F3 formed just
 215 before the final fracture. According to microstructural studies by Moore and Lockner (1995),
 216 Paterson and Wong (2005), and Wong (1982), as the onset of marked localization of
 217 microcracking is approached, microcracks continue to be predominantly of axial orientation, but
 218 an increasing proportion are of inclined orientation or shear character.

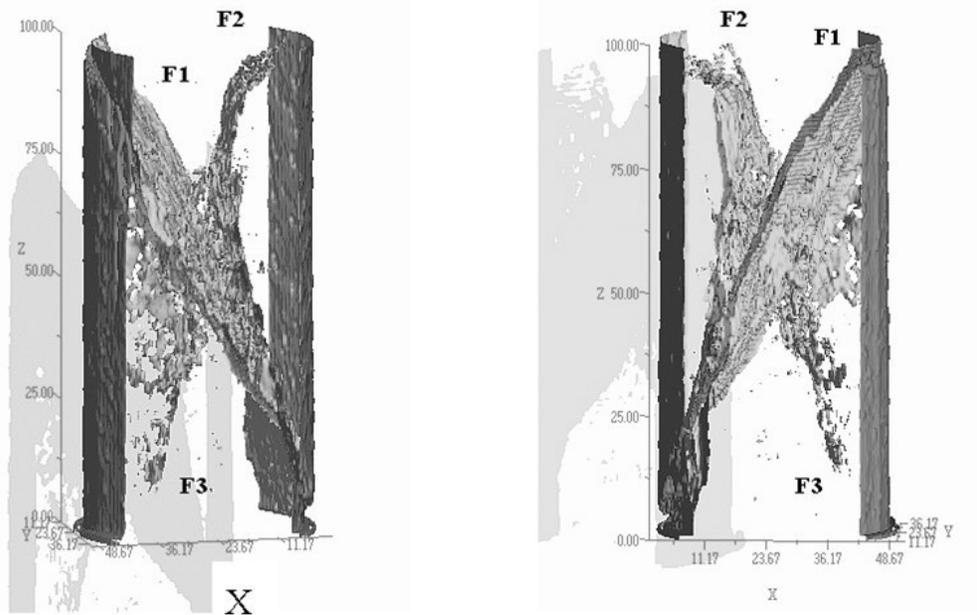


Figure 9

219
 220 **Figure 9.** 3D CT image of the fracture planes (F1 through F3) of the sample from two
 221 viewpoints. The X-direction indicated in the left-hand image is the same as shown in Figure 10.

222 The right-hand image is shown from the opposite side of the left-hand image. The offset gray
 223 shadowy images are visual aids.

224

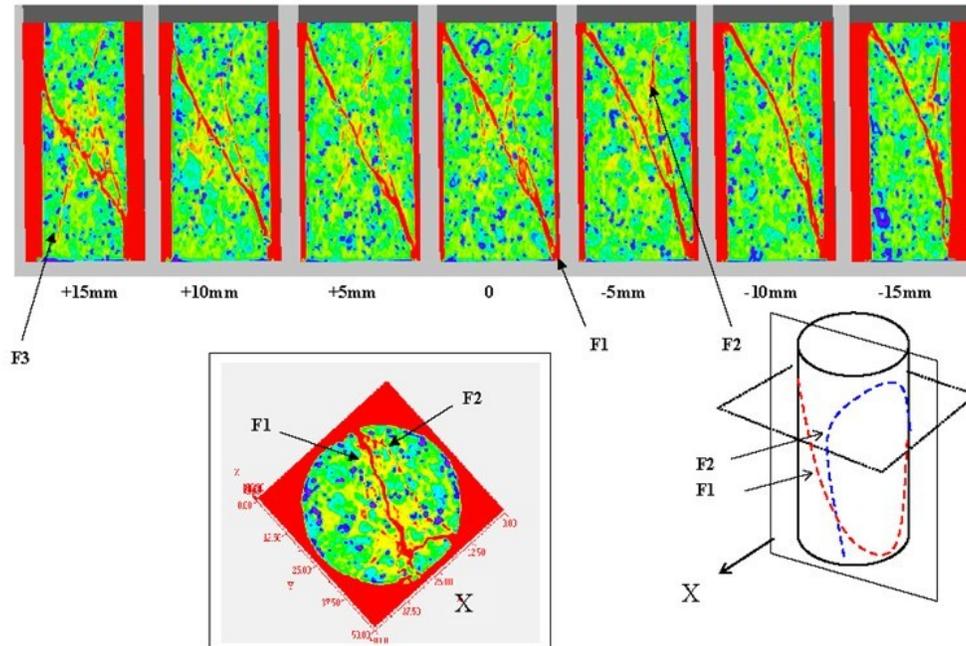


Figure 10

225

226 **Figure 10.** Horizontal CT cross section of the sample (65 mm from the sample base) after the
 227 experiment and vertical 2D images derived from 3D CT model at intervals of 5 mm from the
 228 center of the sample. Red color indicates low density. Progression from red to yellow, green, and
 229 blue colors indicates increasing density. The X-direction indicated in the left-hand image is the
 230 same as shown in Figure 9.

231

232

233 4. Discussion

234 4.1 Cracked solid model

235 We estimated crack parameters such as aspect ratio, the degree of water saturation, and their
236 changes as a function of time. In the evaluation of these parameters, a cracked solid model (e.g.,
237 Crampin, 1978, 1984; Hudson, 1981; Nishizawa & Masuda, 1991; Soga et al., 1978) that
238 approximates the rock as an elastic solid containing cavities representing pore space was applied.
239 Because cavities are more compliant than solid material, they have the effect of reducing the
240 elastic stiffness of the rock (e.g., Avseth et al., 2005; Mavko et al., 2009; Meglis et al., 1996).

241

242 In this study, we assumed a transversely isotropic medium in which vertical cracks of ellipsoidal,
243 penny-shaped geometry distributed with the orientations of the crack normals randomly
244 distributed in the plane perpendicular to the maximum stress as described in the Supporting
245 Information (Figures S1 and S2) (e.g., Fossen, 2010; Scholz, 2002). Transverse isotropy is
246 considered to be realistic when we consider seismic wave propagation in the Earth's crust. If this
247 is the case, the velocities of S waves that propagate horizontally would be affected differently, as
248 is seen in Figure 11; this effect is referred to as shear wave splitting (e.g., Anderson et al., 1974;
249 Paterson & Wong, 2005). Figure 11 shows that the velocities of S waves with horizontal
250 vibration are reduced more than those with vertical vibration for the data measured on both paths
251 2 and 5. The differences between V_{S_V} and V_{S_H} are larger for the data measured on path 2. This is
252 because path 2 is near the center of the sample, whereas path 5 is close to the end of the sample
253 where the crack volume is smaller than it is in the center. All of these observations support the
254 use of a vertically cracked transversely isotropic model in this study.

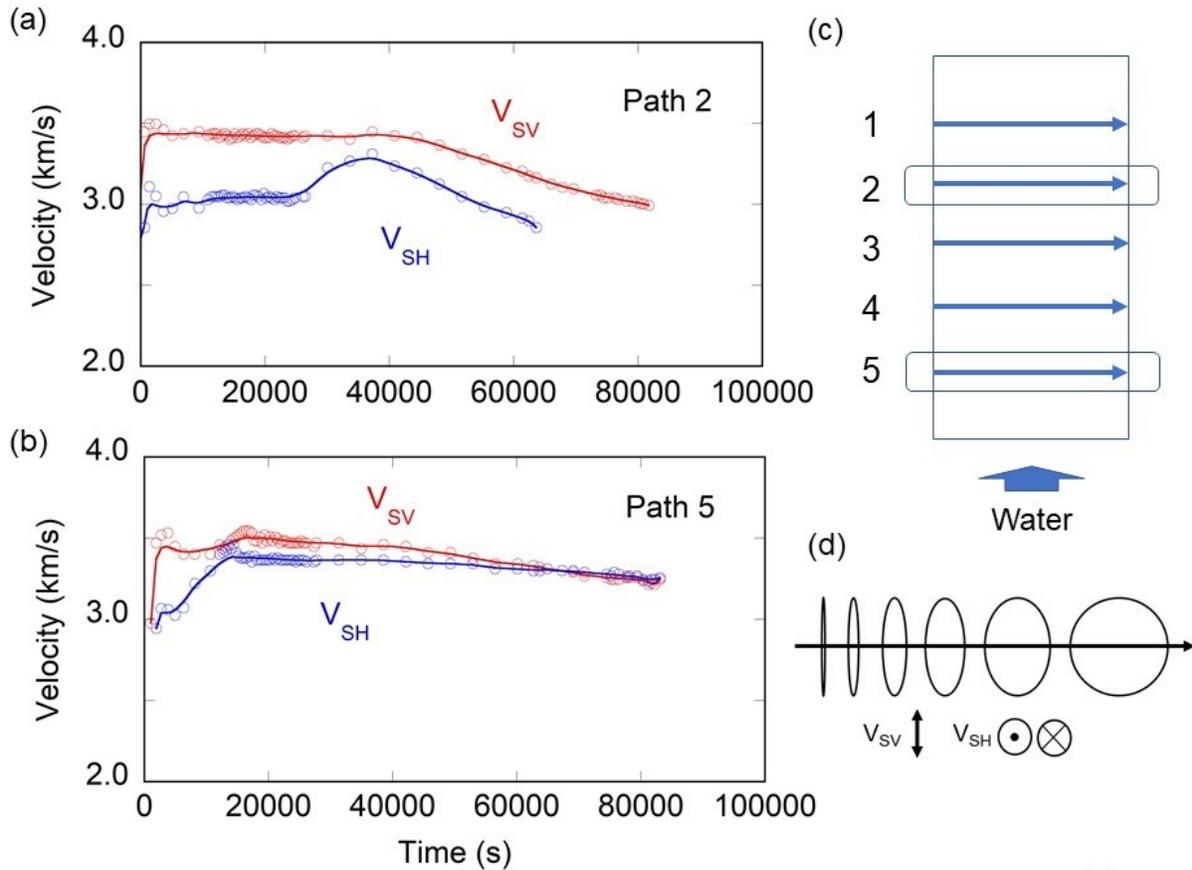


Figure 11

255

256 **Figure 11.** S-wave velocity with vertical vibration S_V and horizontal vibration S_H for (a) path 2
 257 and (b) path 5. (c) Paths of the velocity measurements. (d) The wave propagation direction (thick
 258 blue arrow) and side view of the crack distribution in the transversely isotropic model with
 259 vertical cracks. The directions of vibration of the S_V and S_H waves are shown. Shear wave
 260 splitting was observed, supporting the assumption of transversely isotropic symmetry with
 261 vertical crack distribution.

262

263

264 The change in elastic wave velocities during the experiment was attributed to open cracks in the
 265 rock sample. The effect of cracks on the elastic properties of solids depends on various factors

266 such as the shape, number, and orientation of the cracks. When very thin spheroidal cracks are
 267 randomly distributed in a solid, O'Connell and Budiansky (1974) showed that the effect of
 268 cracks on velocity is well described by the crack density parameter,

269

$$270 \quad \varepsilon = N \langle a^3 \rangle = \frac{3}{4\pi} \frac{\phi}{\alpha}, \quad (1)$$

271

272 where $\langle a \rangle$ is the mean major axis of the crack ellipsoid; N is the number of cracks per unit
 273 volume of the solid; ϕ is the volume of cracks per unit volume of the solid, which can be written
 274 as

275

$$276 \quad \phi = \frac{4}{3} \pi \langle a^2 c \rangle N; \quad (2)$$

277

278 and α is the aspect ratio of a very thin spheroidal crack ($a = b \gg c$), $\alpha = c/a$. According to
 279 Hudson (1981) and Soga et al. (1978), the effect of cracks on velocity, in terms of the ratio of
 280 velocities with and without cracks, is proportional to the crack density parameter ε at small
 281 values of ε ,

282

$$283 \quad (V/V_0)^2 = 1 - p_i \varepsilon, \quad (3)$$

284

285 where V_0 and V are the elastic wave velocities of the rocks without and with cracks, respectively.

286 The constants p_i can be calculated for P waves and two kinds of S waves under dry and saturated

287 states (Table 1). Details regarding how to calculate the coefficients listed in Table 1 are
 288 described in the Supporting Information.

289

290 **Table 1.** The right sides of equation (3) with the constants p_i for dry and wet states. Details
 291 regarding the determination of the constants p_i are described in the Supporting Information.

	V_p	V_{SV}	V_{SH}
Dry	$1 - \frac{71}{21} \varepsilon$	$1 - \frac{8}{7} \varepsilon$	$1 - \frac{15}{7} \varepsilon$
Wet	$1 - \frac{8}{21} \varepsilon$	$1 - \frac{8}{7} \varepsilon$	$1 - \frac{8}{7} \varepsilon$

292

293

294 In partially saturated cases, P- and S-wave velocities can be interpolated from the dry and wet
 295 (saturated) velocities by using the degree of water saturation parameter ξ , which ranges from 0
 296 (dry) to 1.0 (saturated). In this study, it is assumed that elastic constants in the partially saturated
 297 state can be expressed as the weighted average of the elastic constants of dry and saturated states
 298 (Voigt's average). Then the velocity V of a partially saturated state can be written as

299

$$300 \quad V^2 = \xi V_w^2 + (1 - \xi) V_d^2, \quad (4)$$

301

302 where V_w and V_d are the velocities for the totally saturated and totally dry cases, respectively.

303 From equations (1) through (4), equation (3) can be rewritten as

304

$$305 \quad 1 - \left(\frac{V_{p,s}}{V_0} \right)^2 = p_i \frac{3}{4\pi} \frac{\varphi}{\alpha} \quad (5)$$

306

307 Here V_p and V_s are the P- and S-wave velocities for rocks that include cracks. The total crack
 308 volume ratio ϕ is calculated by using the measured surface strains of the rock sample as shown in
 309 the following section. Thus, given a set of α and ξ values, we can calculate curves of $1 - (V/V_0)^2$
 310 versus ϕ for P and S waves. By comparing the calculated curves to the measured data shown in
 311 Figure 12, we estimate α and ξ and their variation with time by fitting for each measured
 312 experimental data set.

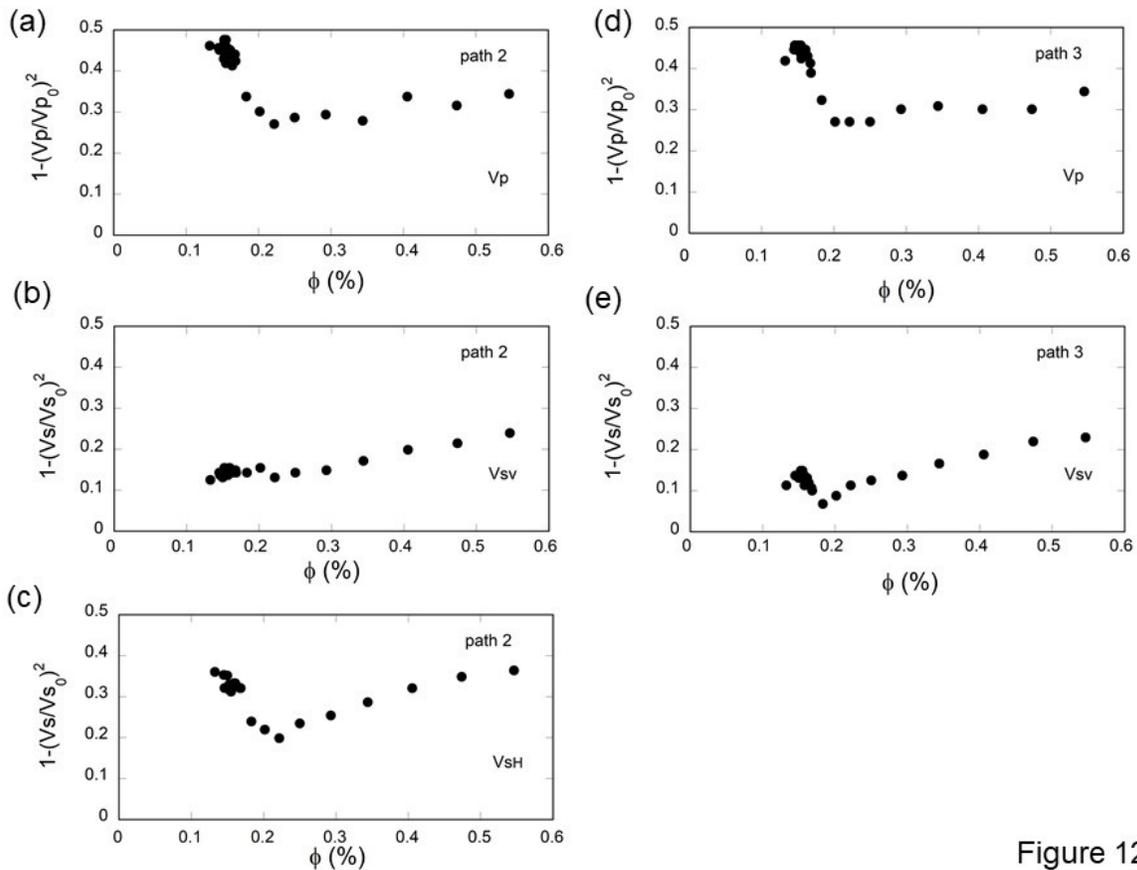


Figure 12

313

314 **Figure 12.** Measured data plotted for ϕ vs. $1 - (V/V_0)^2$. (a) V_p , (b) V_{sv} , (c) V_{sH} for path 2, and

315 (d) V_p , (e) V_{sv} for path 3.

316

317

318 4.2. Change in crack shape and degree of water saturation

319 Figure 13 shows the P-wave velocity change $1 - (V_p/V_{p0})^2$ as a function of ϕ , the volume of
320 cracks per unit volume of the solid. We calculated ϕ based on strain data measured at the center
321 of the rock sample (Figures 2 and 7). We first calculated the volumetric strain ε_v from the
322 averages of the axial strain ε_z and circumferential strain ε_θ as $\varepsilon_v = \varepsilon_z + 2\varepsilon_\theta$. The stress–strain
323 curve was linear in the early stage of loading except for the initial stage. Taking the linear part of
324 the volumetric strain to represent the elastic volumetric strain, we fitted the ε_v versus stress line
325 in the range from 1/9 to 1/3 of the failure stress by a straight line using the least-squares method.
326 This stress range was used to avoid the effects of initial cracks at low stress levels and new
327 cracks at high stress levels. We then calculated the dilatant strain ε_{dv} from the observed
328 volumetric strain by subtracting the elastic volumetric strain predicted by the straight line (e.g.,
329 Brace et al., 1966; Paterson & Wong, 2005). We then used the calculated dilatant strain to
330 represent ϕ . Here we used the P- and S-wave velocity data measured along paths V2 and V3 at
331 the midpoint of the sample (Figure 3). Figure 13 shows that the curve of $\alpha = 1/400$ and $\xi = 0$ is
332 best fitted to the data indicated by the arrow. Thus we estimated that the aspect ratio of the
333 cracks before water injection (in the dry state, $\xi = 0$) was 1/400.

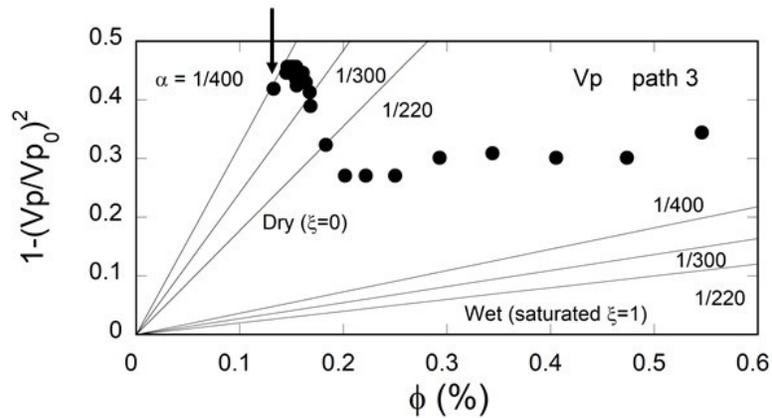


Figure 13

334

335 **Figure 13.** Procedure for estimating the pair of values for the crack aspect ratio and water
 336 saturation (α , ξ) for the case of $\xi = 0$. Values of $1 - (V_p/V_{p0})^2$ as a function of crack volume ϕ are
 337 plotted for V_p measured on path 3. Curves are shown for three values of α and endpoint values
 338 (0 and 1) of ξ . The data point indicated by the arrow, which was measured at the beginning of the
 339 velocity measurement, is nearly on the curve for the (α , ξ) pair (1/400, 0). Curves for the
 340 saturated case ($\xi = 1$) are shown as references.

341

342

343 To estimate α and ξ simultaneously after water injection for the case of $\xi > 0$, we need more than
 344 two kinds of data, such as V_p , V_{sv} , or V_{sh} . The best fitted set of (α , ξ) to the measured data was

345 estimated by the grid search method using equations (4) and (5) with the constants p_i listed in
 346 Table 1. For example, Figure 14 shows the curves of $\alpha = 1/160$ with $\xi = 0, 0.2, 0.4, 0.6, 0.8,$ and
 347 1.0 for V_p and $\xi = 0.6$ with $\alpha = 1/200, 1/180, 1/160, 1/140, 1/120,$ and $1/100$ for V_{sv} for path 3.
 348 For the last data point plotted, around $\phi = 0.55$, our best fitted set of (α, ξ) was $\alpha = 1/160$ and $\xi =$
 349 0.6.

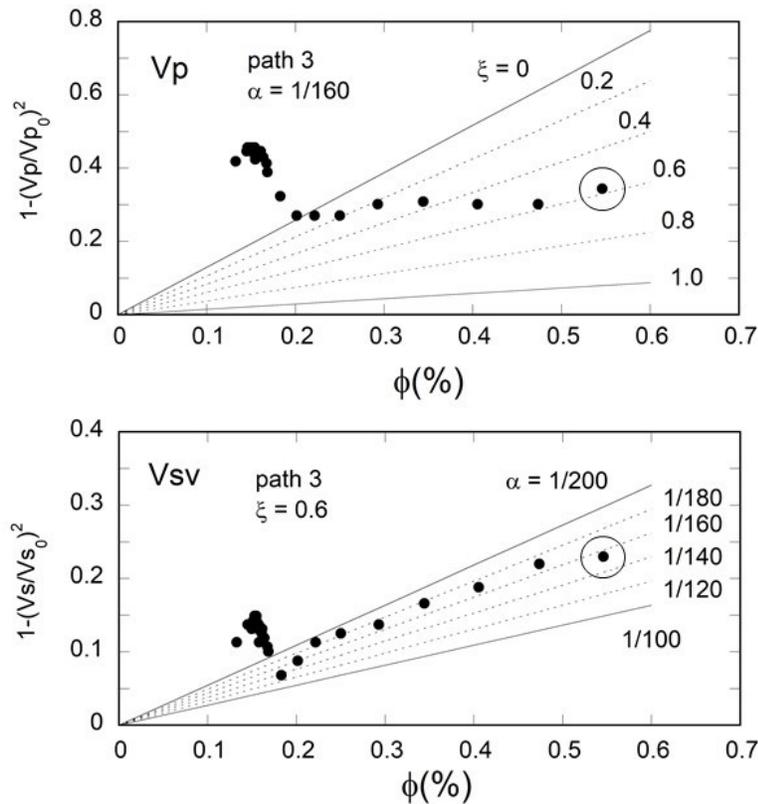


Figure 14

350

351 **Figure 14.** $1-(V/V_0)^2$ as a function of ϕ for V_p with curves for $\alpha = 1/160$ and six values of ξ , and
 352 for V_{sv} with curves for $\xi = 0.6$ and six values of α . V_p and V_{sv} data were used to simultaneously
 353 estimate a pair of values for the crack aspect ratio and water saturation (α, ξ) . The last data point
 354 indicated by the circle, at about $\phi = 0.55$, was best fit by the set of $\alpha = 1/160$ and $\xi = 0.6$.

355

356

357 Figure 15 shows the estimated crack aspect ratio α and degree of water saturation ξ as a
 358 function of time. The aspect ratio changed from 1/400 to about 1/160 during the deformation as a
 359 result of water injection. Water saturation in the middle of the sample increased from 0 to 0.6.
 360

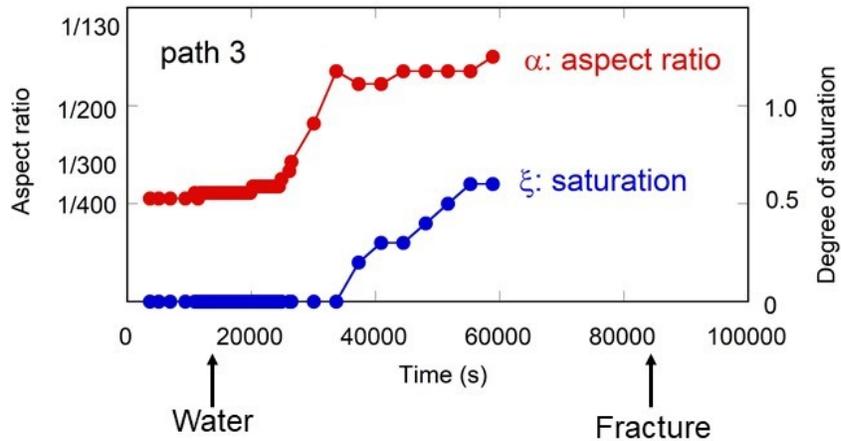


Figure 15

361

362 **Figure 15.** Changes in the aspect ratio (red) and water saturation (blue) at the midpoint of the
 363 sample as a function of time. The arrows labeled “Water” and “Fracture” mark the times when
 364 water injection started and when the rock sample fractured, respectively.

365

366 **5. Conclusions**

367 We demonstrated an in situ monitoring method for estimating crack shape and degree of water
368 saturation from measured P- and S-wave velocities and porosity changes in the laboratory. We
369 fractured an instrumented rock sample by injecting it with water under well-controlled
370 differential stress and confining pressure conditions. We estimated the crack aspect ratio and the
371 degree of water saturation by applying a cracked solid model to our experimental data on P- and
372 S-wave velocities and crack density. We observed that (1) the aspect ratio α of dry cracks before
373 water injection was 1/400, (2) during the water migration the aspect ratio changed from 1/400 to
374 1/160, and (3) the degree of water saturation ξ increased from 0 to 0.6. The monitoring methods
375 described in this study may be useful in the estimation of microcracking at depth. Reliable
376 monitoring methods for detecting crack characteristics and their time variation will aid in
377 planning industrial and scientific applications including measurement of regional stress fields,
378 induced seismicity, and sequestration of carbon dioxide and other waste.

379

380 **Acknowledgments and Data Availability**

381 T. Maruyama of the University of Tsukuba contributed to the experimental study. O. Nishizawa
382 and X. Lei of the Geological Survey of Japan contributed to the data manipulation.

383

384 The data underlying this article are available in the article and in its online supplementary
385 material.

386

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533 **Appendix A: Elastic constants of rock material for the case of transversely isotropic**
 534 **symmetry along the x-3 axis (z-axis) with randomly distributed vertical cracks**

535

536 Here we describe the method of calculation of elastic constants for the case in which plane
 537 normals of cracks are randomly distributed in directions perpendicular to the x-3 axis (z-axis).

538 We also show that the ratio of the elastic constant of rock material that includes cracks to that of
 539 the matrix or the square of velocity ratio is expressed as $(V/V_0)^2 = 1 - p_i \varepsilon$, where V and V_0 are
 540 the elastic-wave velocities with and without cracks, respectively, and ε is the crack density
 541 parameter defined by

542

$$543 \quad \varepsilon = \frac{3\varnothing}{4\pi\alpha}, \quad (\text{A1})$$

544

545 where \varnothing is the porosity and $\alpha = c/a$ is the aspect ratio of the crack ($a = b \gg c$). In addition, we
 546 derive the coefficients p_i .

547

548 The focus of this study is on a transversely isotropic medium with the x-3 axis (z-axis) as the
 549 axis of symmetry and with a vertical crack distribution in which the plane normals of the cracks
 550 are randomly distributed in horizontal directions (directions parallel to the x-1,2 plane, x-y
 551 plane). The right-handed rectangular coordinate system is used in this study (Figure S1a).

552

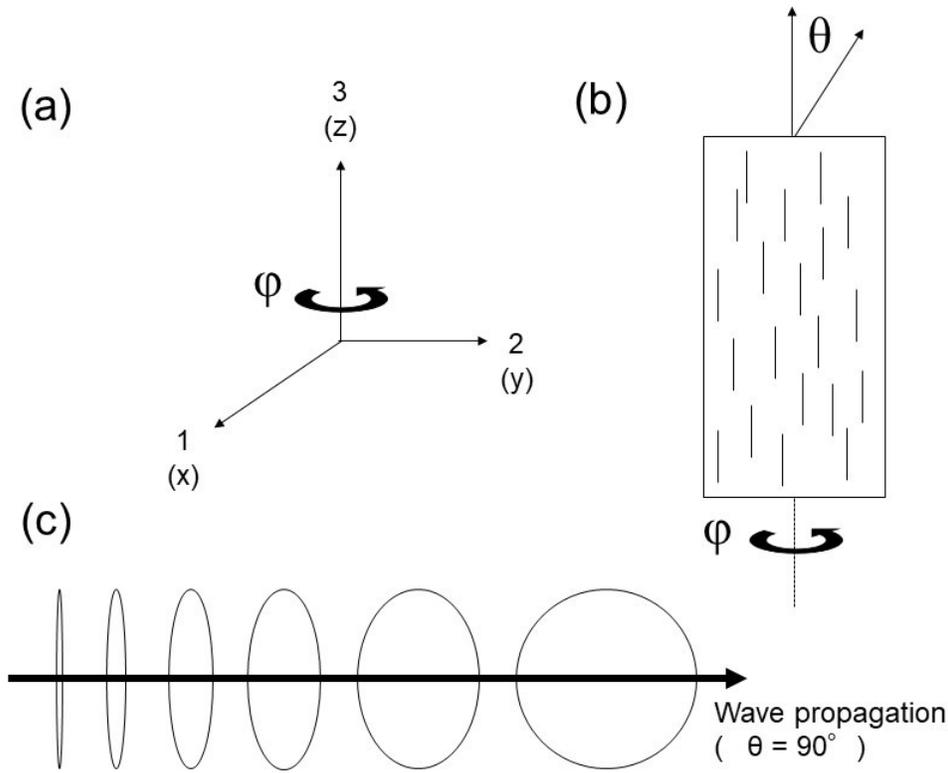


Figure S1

553

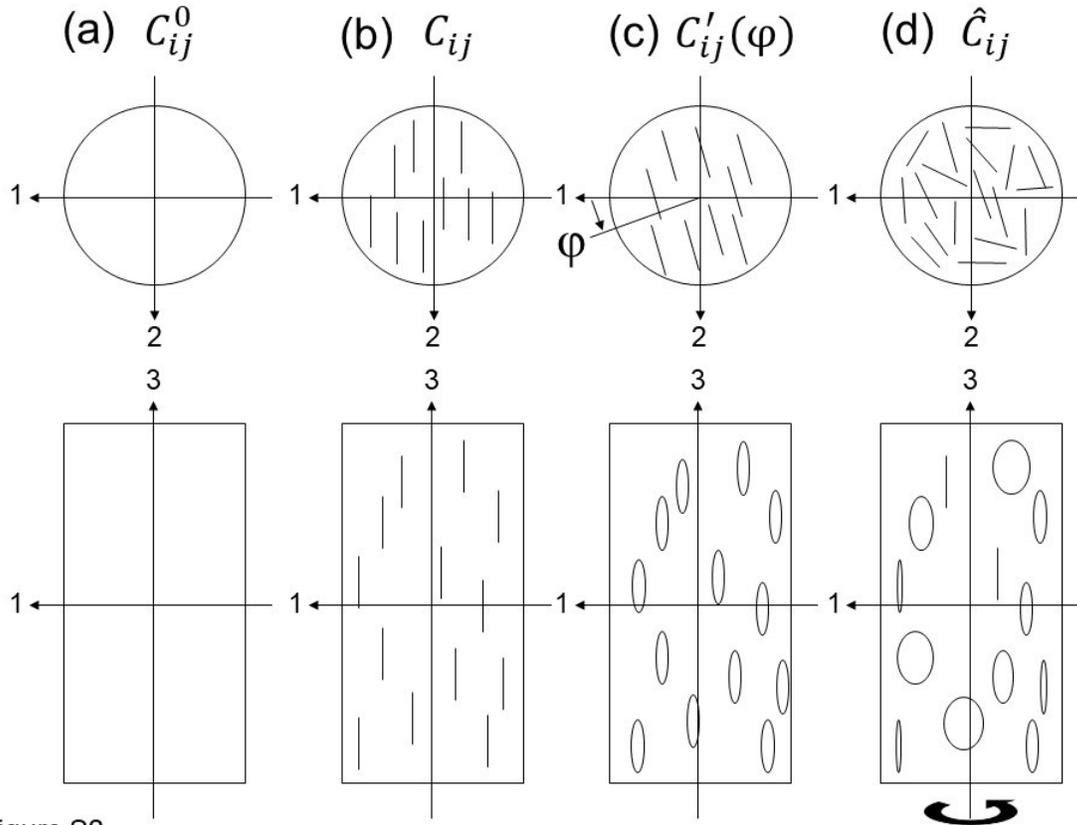
554 **Figure S1.** The basic assumptions in this study: (a) the coordinate system, (b) vertical cross
 555 section of transversely isotropic rock with vertical cracks, and (c) side view of the direction of
 556 wave propagation and crack distribution.

557

558

559 First, based on the method of Hudson (1981), we calculated C_{ij} for a material that includes
 560 vertical cracks that are plane normal along the x-1 axis (x-axis) (Figure S1b). Next, we took the
 561 rotational average of C_{ij} around the x-3 axis (z-axis), resulting in \hat{C}_{ij} , which were shown to be
 562 transversely isotropic with the x-3 axis (z-axis) by using a method similar to that of Nishizawa
 563 and Masuda (1991).

564



565

Figure S2

566 **Figure S2.** Procedure for calculating the elastic constants in the transversely isotropic rock with567 vertical cracks: (a) C_{ij}^0 elastic constants of the rock matrix; (b) C_{ij} elastic constants for the rock

568 material with vertical cracks for which the plane-normal direction is along the x-1 axis (x-axis);

569 (c) $C'_{ij}(\varphi)$ elastic constants for rock material with vertical cracks for which the angle between570 the plane-normal direction and the x-1 axis (x-axis) is φ ; and (d) \hat{C}_{ij} elastic constants for rock

571 material with transversely isotropic symmetry along the x-3 axis (z-axis) with vertical cracks

572 with random values of φ .

573

574

575 **1. C_{ij}^0 elastic constants of the isotropic rock matrix (Figure S2a)**

576

577 In this study, we use abbreviated 2-index Voigt notation to express elastic constants such as C_{ij}
 578 instead of 4-index notation for the fourth-rank tensor c_{ijkl} . We assume that the matrix of rock
 579 without cracks or inclusions is isotropic with two independent constants:

580

$$581 \quad C_{ij}^0 = \begin{pmatrix} C_{11}^0 & C_{12}^0 & C_{12}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{11}^0 & C_{12}^0 & 0 & 0 & 0 \\ C_{12}^0 & C_{12}^0 & C_{11}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44}^0 \end{pmatrix} = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix} \quad (A2)$$

582

$$583 \quad C_{12}^0 = C_{11}^0 - 2C_{44}^0 \quad (A3)$$

584

585 The relationships between the elements C_{ij}^0 and Lamé's parameters λ and μ of isotropic linear
 586 elasticity are

587

$$588 \quad C_{11}^0 = \lambda + 2\mu, C_{12}^0 = \lambda, C_{44}^0 = \mu. \quad (A4)$$

589

590

591 **2. C_{ij} elastic constants for rock material with cracks that are plane normal along the x-1**
 592 **axis (x-axis) (Figure S2b)**

593

594 Hudson (1981) modeled fractured rock as an elastic solid with thin, penny-shaped ellipsoidal
 595 cracks or inclusions. The effective moduli C_{ij} are given as

596

$$597 \quad C_{ij} = C_{ij}^0 + C_{ij}^1, \quad (\text{A5})$$

598

599 where C_{ij}^0 are the isotropic background moduli and C_{ij}^1 are the first-order corrections. For the
 600 case in which the vertical cracks have crack normals along the x-1 axis (x-axis), the axis of
 601 symmetry of the material lies along the x-1 axis (x-axis), which has hexagonal symmetry with
 602 five independent constants as

603

$$604 \quad C_{ij}^1 = \begin{pmatrix} C_{11}^1 & C_{12}^1 & C_{12}^1 & 0 & 0 & 0 \\ C_{12}^1 & C_{22}^1 & C_{23}^1 & 0 & 0 & 0 \\ C_{12}^1 & C_{23}^1 & C_{22}^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^1 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55}^1 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55}^1 \end{pmatrix}, C_{44}^1 = \frac{1}{2}(C_{22}^1 - C_{23}^1). \quad (\text{A6})$$

605

606 The following correction terms are given by Schön (2011, Table 6.15) for the case in which the
 607 crack normals are aligned along the x-1 axis (x-axis), including the vertical cracks:

608

$$609 \quad C_{11}^1 = \frac{-(\lambda + 2\mu)^2}{\mu} \varepsilon U_3 \quad (\text{A7})$$

610

$$611 \quad C_{13}^1 = \frac{-\lambda(\lambda + 2\mu)^2}{\mu} \varepsilon U_3 \quad (\text{A8})$$

612

$$613 \quad C_{33}^1 = \frac{-\lambda^2}{\mu} \varepsilon U_3 \quad (\text{A9})$$

614

$$615 \quad C_{44}^1 = 0 \quad (\text{A10})$$

616

$$617 \quad C_{66}^1 = -\mu \varepsilon U_1 \quad (\text{A11})$$

618

619 in which the correction terms C_{ij}^1 are negative; thus, the elastic properties decrease with
 620 fracturing. U_1 and U_3 depend on the crack conditions (Mavko et al., 2009; Schon 2011).

621 For dry cracks,

622

$$623 \quad U_1 = \frac{16(\lambda + 2\mu)}{3(3\lambda + 4\mu)}; U_3 = \frac{4(\lambda + 2\mu)}{3(\lambda + \mu)}. \quad (\text{A12})$$

624

625 For wet cracks, Hudson's expressions for infinitely thin fluid-filled cracks are

626

$$627 \quad U_1 = \frac{16(\lambda + 2\mu)}{3(3\lambda + 4\mu)}; U_3 = 0. \quad (\text{A13})$$

628

629 Therefore, for the dry case, C_{ij} are

630

$$631 \quad C_{11} = C_{11}^0 + C_{11}^1 = (\lambda + 2\mu)(1 - 6\varepsilon) \quad (\text{A14})$$

632

$$633 \quad C_{13} = C_{13}^0 + C_{13}^1 = \lambda(1 - 6\varepsilon) \quad (\text{A15})$$

634

$$635 \quad C_{33} = C_{33}^0 + C_{33}^1 = (\lambda + 2\mu) \left(1 - \frac{2}{3}\varepsilon\right) \quad (\text{A16})$$

636

$$637 \quad C_{44} = C_{44}^0 + C_{44}^1 = \mu \quad (\text{A17})$$

638

$$639 \quad C_{66} = C_{66}^0 + C_{66}^1 = \mu \left(1 - \frac{16}{7}\varepsilon\right). \quad (\text{A18})$$

640

641 For the wet case,

642

$$643 \quad C_{11} = C_{11}^0 + C_{11}^1 = \lambda + 2\mu \quad (\text{A19})$$

644

$$645 \quad C_{13} = C_{13}^0 + C_{13}^1 = \lambda \quad (\text{A20})$$

646

$$647 \quad C_{33} = C_{33}^0 + C_{33}^1 = \lambda + 2\mu \quad (\text{A21})$$

648

$$649 \quad C_{44} = C_{44}^0 + C_{44}^1 = \mu \quad (\text{A22})$$

650

$$651 \quad C_{66} = C_{66}^0 + C_{66}^1 = \mu \left(1 - \frac{16}{7}\varepsilon\right). \quad (\text{A23})$$

652

653 C_{ij} has hexagonal symmetry with the x-1 axis (x-axis) expressed with five independent moduli.

654

655

656 **3. $C'_{ij}(\varphi)$ elastic constants for rock material with vertical cracks that have an angle φ**
 657 **between the plane-normal direction and the x-1 axis (x-axis) (Figure S2c)**

658

659 When we rotate C_{ij} around the x-3 axis (z-axis) by an angle of φ from the x-1 axis (x axis), C_{ij} is
 660 a function of φ , as expressed by $C'_{ij}(\varphi)$.

661

662 Regarding coordinate transformations, the elastic compliances c_{ijkl} are, in general, fourth-rank
 663 tensors and hence transform according to

664

$$665 \quad c'_{ijkl} = \beta_{ip} \beta_{jq} \beta_{kr} \beta_{ls} c_{pqrs}, \quad (\text{A24})$$

666

667 where c'_{ijkl} and c_{pqrs} are the elastic compliances after and before the coordinate transformation,
 668 respectively. For rotation around the x-3 axis (z-axis), β_{ij} is the following matrix element

669

$$670 \quad \begin{pmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (\text{A25})$$

671

672 In this study, we use the abbreviated 2-index Voigt notation C_{ij} instead of c'_{ijkl} and c_{ijkl} . Although
 673 an elastic constant looks like a second-rank tensor (C_{ij}) with this notation, it is indeed a fourth-
 674 rank tensor; when one performs a coordinate transformation, one must go back to the full
 675 notation and follow the transformation rules for a fourth-rank tensor. The usual tensor
 676 transformation law is no longer valid. However, the change of coordinates for C_{ij} is more
 677 efficiently performed with the 6×6 Bond Transformation Matrices, \mathbf{M} (Mavko et al., 2009). The

678 advantage of the Bond method for transforming compliances is that it can be applied directly to
 679 the elastic constants given in 2-index notation, as expressed as follows:

680

$$681 \quad [C'] = [M][C][M]^T \quad (\text{A26})$$

682

683 **M**

$$684 \quad \mathbf{M} = \begin{bmatrix} \beta_{11}^2 & \beta_{12}^2 & \beta_{13}^2 & 2\beta_{12}\beta_{13} & 2\beta_{13}\beta_{11} & 2\beta_{11}\beta_{12} \\ \beta_{21}^2 & \beta_{22}^2 & \beta_{23}^2 & 2\beta_{22}\beta_{23} & 2\beta_{23}\beta_{21} & 2\beta_{21}\beta_{22} \\ \beta_{31}^2 & \beta_{32}^2 & \beta_{33}^2 & 2\beta_{32}\beta_{33} & 2\beta_{33}\beta_{31} & 2\beta_{31}\beta_{32} \\ \beta_{21}\beta_{31} & \beta_{22}\beta_{32} & \beta_{23}\beta_{33} & \beta_{22}\beta_{33} + \beta_{23}\beta_{32} & \beta_{21}\beta_{33} + \beta_{23}\beta_{31} & \beta_{22}\beta_{31} + \beta_{21}\beta_{32} \\ \beta_{31}\beta_{11} & \beta_{32}\beta_{12} & \beta_{33}\beta_{13} & \beta_{12}\beta_{33} + \beta_{13}\beta_{32} & \beta_{11}\beta_{33} + \beta_{13}\beta_{31} & \beta_{11}\beta_{32} + \beta_{12}\beta_{31} \\ \beta_{11}\beta_{21} & \beta_{12}\beta_{22} & \beta_{13}\beta_{23} & \beta_{22}\beta_{13} + \beta_{12}\beta_{23} & \beta_{11}\beta_{23} + \beta_{13}\beta_{21} & \beta_{22}\beta_{11} + \beta_{12}\beta_{21} \end{bmatrix}$$

685 (A27)

686

687 Then, we obtain $C'_{ij}(\varphi)$ as

688

689 $C'_{11}(\varphi) = \cos^4 \varphi C_{11} + 2 \sin^2 \varphi \cos^2 \varphi C_{12} + \sin^4 \varphi C_{22} + 4 \sin^2 \varphi \cos^2 \varphi C_{55} \quad (\text{A28})$

690

691 $C'_{22}(\varphi) = \sin^4 \varphi C_{11} + 2 \sin^2 \varphi \cos^2 \varphi C_{12} + \cos^4 \varphi C_{22} + 4 \sin^2 \varphi \cos^2 \varphi C_{55} \quad (\text{A29})$

692

693 $C'_{12}(\varphi) = C'_{21}(\varphi) = \sin^2 \varphi \cos^2 \varphi C_{11} + (\sin^4 \varphi + \cos^4 \varphi) C_{12} + \sin^2 \varphi \sin^2 \varphi C_{22} - 4 \sin^2 \varphi \cos^2 \varphi C_{55}$

694 (A30)

695 $C'_{13}(\varphi) = C'_{31}(\varphi) = \cos^2 \varphi C_{12} + \sin^2 \varphi C_{23} \quad (\text{A31})$

696

$$697 \quad C'_{23}(\varphi) = C'_{32}(\varphi) = \sin^2 \varphi C_{12} + \cos^2 \varphi C_{23} \quad (\text{A32})$$

698

$$699 \quad C'_{33}(\varphi) = C_{22} \quad (\text{A33})$$

700

$$701 \quad C'_{44}(\varphi) = \cos^2 \varphi C_{44} + \sin^2 \varphi C_{55} \quad (\text{A34})$$

702

$$703 \quad C'_{55}(\varphi) = \sin^2 \varphi C_{44} + \cos^2 \varphi C_{55} \quad (\text{A35})$$

704

$$705 \quad C'_{66}(\varphi) = \sin^2 \varphi \cos^2 \varphi C_{11} - 2 \sin^2 \varphi \cos^2 \varphi C_{12} + \sin^2 \varphi \cos^2 \varphi C_{22} + (\cos^2 \varphi - \sin^2 \varphi)^2 C_{55}. \quad (\text{A36})$$

706

707 The following non-zero elements are zero in the next step 4, taking the rotational average:

708

$$709 \quad C'_{16}(\varphi), C'_{61}(\varphi), C'_{26}(\varphi), C'_{62}(\varphi), C'_{36}(\varphi), C'_{63}(\varphi), C'_{45}(\varphi), C'_{54}(\varphi).$$

710

711

712 **4. \hat{C}_{ij} elastic constants for rock material with transversely isotropic symmetry along the x-**

713 **3 axis (z-axis) and a vertical crack distribution (Figure S2d)**

714

715 We took the rotational average of C_{ij} around the x-3 axis (z-axis) to obtain \hat{C}_{ij} that showed

716 transversely isotropic symmetry along the x-3 axis (z-axis) in the case of a random vertical crack

717 distribution as follows:

718

$$719 \quad \hat{C}_{ij} = \frac{1}{2\pi} \int_0^{2\pi} C'_{ij}(\varphi) d\varphi, \quad (\text{A37})$$

720

721 which uses

722

$$723 \quad \frac{1}{2\pi} \int_0^{2\pi} \sin^4 \varphi d\varphi = \frac{3}{8}, \quad \frac{1}{2\pi} \int_0^{2\pi} \cos^4 \varphi d\varphi = \frac{3}{8}, \quad \frac{1}{2\pi} \int_0^{2\pi} \sin^2 \varphi \cos^2 \varphi d\varphi = \frac{1}{8},$$

$$724 \quad (\text{A38})$$

725

$$726 \quad \frac{1}{2\pi} \int_0^{2\pi} \sin^2 \varphi d\varphi = \frac{1}{2}, \quad \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \varphi d\varphi = \frac{1}{2}. \quad (\text{A39})$$

727

728 \hat{C}_{ij} shows hexagonal symmetry or transversely isotropic symmetry with the x-3 axis (z-axis) in

729 which there are five independent constants:

730

$$731 \quad \hat{C}_{11} = \frac{1}{2\pi} \int_0^{2\pi} C'_{11}(\varphi) d\varphi = \frac{3}{8}C_{11} + \frac{1}{4}C_{12} + \frac{3}{8}C_{22} + \frac{1}{2}C_{55} \quad (\text{A40})$$

732

$$733 \quad \hat{C}_{12} = \frac{1}{2\pi} \int_0^{2\pi} C'_{12}(\varphi) d\varphi = \frac{1}{8}C_{11} + \frac{3}{4}C_{12} + \frac{1}{8}C_{22} - \frac{1}{2}C_{55} \quad (\text{A41})$$

734

$$735 \quad \hat{C}_{13} = \frac{1}{2\pi} \int_0^{2\pi} C'_{13}(\varphi) d\varphi = \frac{1}{2}C_{12} + \frac{1}{2}C_{23} \quad (\text{A42})$$

736

$$737 \quad \hat{C}_{33} = C_{22} \quad (\text{A43})$$

738

$$739 \quad \hat{C}_{44} = \frac{1}{2\pi} \int_0^{2\pi} C'_{44}(\varphi) d\varphi = \frac{1}{2} C_{44} + \frac{1}{2} C_{55} \quad (\text{A44})$$

740

$$741 \quad \hat{C}_{66} = \frac{1}{2\pi} \int_0^{2\pi} C'_{66}(\varphi) d\varphi = \frac{1}{8} C_{11} - \frac{1}{4} C_{12} + \frac{1}{8} C_{22} + \frac{1}{2} C_{55} = \frac{1}{2} (\hat{C}_{11} - \hat{C}_{12}). \quad (\text{A45})$$

742

743 **5. Wave velocities which propagate in the horizontal directions**

744

745 In the material with transversely isotropic symmetry, there are three modes of wave propagation,

746 and their velocities are dependent on the angle θ between the axis of symmetry (in this case, x-3

747 axis or z-axis) and the direction of the wave vector:

748

$$749 \quad V_P = \sqrt{\frac{\hat{C}_{11} \sin^2 \theta + \hat{C}_{33} \cos^2 \theta + \hat{C}_{44} + A}{2\rho}} \quad (\text{A46})$$

750

$$751 \quad V_{SV} = \sqrt{\frac{\hat{C}_{11} \sin^2 \theta + \hat{C}_{33} \cos^2 \theta + \hat{C}_{44} - A}{2\rho}} \quad (\text{A47})$$

752

$$753 \quad V_{SH} = \sqrt{\frac{\hat{C}_{66} \sin^2 \theta + \hat{C}_{44} \cos^2 \theta}{2\rho}}, \quad (\text{A48})$$

754

$$755 \quad \text{where } A = \sqrt{[(\hat{C}_{11} - \hat{C}_{44}) \sin^2 \theta + (\hat{C}_{33} - \hat{C}_{44}) \cos^2 \theta]^2 + (\hat{C}_{13} + \hat{C}_{44})^2 \sin^2 2\theta}. \quad (\text{A49})$$

756

757 For $\theta = 90^\circ$, the relationship simplifies to $A = \hat{C}_{33} - \hat{C}_{44}$ and the wave velocity vectors that
 758 propagate perpendicular to the x-3 axis in horizontal directions (Figure S1c) are

759

$$760 \quad V_P = \sqrt{\frac{\hat{C}_{11}}{\rho}}, V_{SV} = \sqrt{\frac{\hat{C}_{44}}{\rho}}, V_{SH} = \sqrt{\frac{\hat{C}_{66}}{\rho}}, \quad (\text{A50})$$

761

762 where V_P , V_{SV} , and V_{SH} are the longitudinal-wave velocity, shear-wave velocity with vertical
 763 polarization, and shear-wave velocity with horizontal polarization, respectively.

764

765 We consider low-porosity aggregate and flat cracks, and have ignored the effect of porosity on
 766 the density of the composite (Anderson et al., 1974).

767 For the dry case, using $\lambda = \mu$,

768

$$769 \quad V_P^2 = \frac{\hat{C}_{11}}{\rho} = \frac{\lambda + 2\mu}{\rho} \left(1 - \frac{71}{21}\varepsilon\right) = V_{P0}^2 \left(1 - \frac{71}{21}\varepsilon\right) \quad (\text{A51})$$

770

$$771 \quad V_{SV}^2 = \frac{\hat{C}_{44}}{\rho} = \frac{\mu}{\rho} \left(1 - \frac{8}{7}\varepsilon\right) = V_{SV0}^2 \left(1 - \frac{8}{7}\varepsilon\right) \quad (\text{A52})$$

772

$$773 \quad V_{SH}^2 = \frac{\hat{C}_{66}}{\rho} = \frac{\mu}{\rho} \left(1 - \frac{15}{7}\varepsilon\right) = V_{SH0}^2 \left(1 - \frac{15}{7}\varepsilon\right), \quad (\text{A53})$$

774

775 where V with a subscript 0 are the velocities without cracks.

776

777 For the wet case,

778

$$779 \quad V_p^2 = \frac{\hat{C}_{11}}{\rho} = \frac{\lambda + 2\mu}{\rho} \left(1 - \frac{8}{21}\varepsilon\right) = V_{p0}^2 \left(1 - \frac{8}{21}\varepsilon\right) \quad (\text{A54})$$

780

$$781 \quad V_{sv}^2 = \frac{\hat{C}_{44}}{\rho} = \frac{\mu}{\rho} \left(1 - \frac{8}{7}\varepsilon\right) = V_{sv0}^2 \left(1 - \frac{8}{7}\varepsilon\right) \quad (\text{A55})$$

782

$$783 \quad V_{sh}^2 = \frac{\hat{C}_{66}}{\rho} = \frac{\mu}{\rho} \left(1 - \frac{8}{7}\varepsilon\right) = V_{sh0}^2 \left(1 - \frac{8}{7}\varepsilon\right). \quad (\text{A56})$$

784

785 The effect of cracks on velocity, in terms of the ratio of velocities with and without cracks, is

786 proportional to the crack density parameter ε at small values of ε :

787

$$788 \quad \left(\frac{V}{V_0}\right)^2 = 1 - p_i \varepsilon. \quad (\text{A57})$$

789

790

791 **Supporting Information**

792 Data1. Experimental conditions of applied stress and water pressure (Figure 1)

793 Data2. Number of AE events detected every 2000s (Figure 1)

794 Data3. P-wave velocity (Figure 4)

795 Data4. Sv-wave velocity (Figure 5)

796 Data5. Sh-wave velocity (Figure 5)

797 Data6. Strain data (Figure 7)

798 Data7. AE location data (Figure 8)