The Equation of Fibonacci Numbers

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Abstract

There are different numerical models, such as the transmission-line matrix model or partially uniform knee model used to predict Schumann radiation. This report introduces a new idea, and reasoning to the previously stated idea of locating Schumann resonances on a single particle's radiation pattern using a Golden ratio and their Octave, triad relationship. In addition, this different prediction method for Schumann resonances derived from the first principle fundamental physics combining both particle radiation patterns and the mathematical concept of the golden ratio spiral that expands at the rate of the golden ratio. The idea of golden ratio spiral allows locating Schumann resonant frequencies on particle's radiation patterns. The Octaves allows us to predict the magnitude of other Schumann resonances on the radiation pattern of a single accelerated charged particle conveniently by knowing the value of the initial Schumann resonant frequency. In addition, it also allows us to find and match Schumann resonances that are on the same radiation lobe. Furthermore, it is important to find Schumann octaves as they propagate in the same direction and have a higher likelihood of wave interference. Method of Triads together with Octaves helps to predict magnitude and direction of Schumann resonant points without needing to refer to a radiation pattern plot. As the golden ratio seems to be part of the Schumann resonances, it is helpful in understanding to know why this is the case. The main method used in the reasoning of the existence of golden ratio in Schumann resonances is the eigenfrequency modes, sqrt{n(n+1)} in the spherical harmonic model. It has been found that eigenfrequency modes have two a start off points, n_0 = 0 or n_0 = frac{\sqrt{5}-1}{2} where the non-zero one is exactly the golden ratio. This allows to extend the existing eigenfrequency modes to \sqrt{(n_0+n)^2+(n_o+n)} in order to explain why golden ratio exist within Schumann resonances. As Fibonacci numbers increase, ratio of two consecutive Fibonacci number approaches to the value of Golden ratio. New equation describing the value of Fibonacci number can be used to re-write eigen-frequency orders of Schumann resonances.

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Key Points:

- New equation of Fibonacci Numbers derived from the Pascal's Triangle.
- Equation provides exact value of any Fibonacci number without having to write the series.
- Eigen-frequencies of the Schumann resonances can be written in Fibonacci numbers.

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Abstract

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1 Introduction

Schumann resonances are extremely low-frequency waves that bounce back and forth between the ground and the ionosphere of the earth. Schumann resonances originate mostly from lightning discharges. However, a contribution can also be from outer space. Schumann resonances were first predicted by Schumann in 1952 (Schumann, 01 Feb. 1952) and experimentally observed in 1960 (Balser & Wagner, 1960). In addition, Schumann resonances can be predicted, with numerical methods such as the partially uniform knee model (Pechony & Price, 2004) or with the Transmission Line Matrix model (Morente et al., 2003). Recently, Golden ratio, Golden ratio spiral, and rectangle all were combined and introduced to be capable of finding the magnitudes and locating Schumann resonances on a single particle radiation pattern (Yucemoz, 2020). The Golden ratio spiral is quite an important method, as it enables to know the location of Schumann resonant frequencies on a radiation pattern of a single charged particle that is consists of many frequencies from low to ionizing part of the spectrum. Furthermore, as an expansion to the idea of locating Schumann resonances using the Golden ratio spiral, the method of electromagnetic octaves was introduced. Octaves exist in standing transverse waves and sound waves in the form of music discovered by the Pythagoras using the Pythagorean ratios (Crocker, 1964). One octave between the two waves is double frequency apart from each other, but they sound the same (Schellenberg & Trehub, 1994). In terms of an accelerated relativistic particle, radiation is emitted in the form of a forward-backward radiation pattern. This radiation pattern consists of lobes that are different from each other due to physical Bremsstrahlung and Doppler asymmetries (Yucemoz & Füllekrug, 2020). These lobes are closed loops, and they are bound to the charged particle. The standing transverse octave waves method predicts only the values of Schumann resonant frequencies that are located on the same radiation lobe as the input Schumann frequency point.

These Schumann points are known as octaves of the input Schumann values. Triads are an extension of octaves. They help predict and understand Schumann resonant pairs and where they are located on a relativistic radiation pattern without having to calculate Octave values. As the golden ratio seems to be part of the Schumann resonances, it is helpful in understanding to know why this is the case. The main method used in the reasoning of the existence of golden ratio in Schumann resonances is the eigenfrequency modes, $\sqrt{n(n+1)}$ in the spherical harmonic model. The simple form spherical cavity model relates Schumann frequency to the eigenfrequency modes, $\sqrt{n(n+1)}$ via $f_n = \frac{c}{2\pi R}\sqrt{n(n+1)}$. Where R is the radius of the planet, c is the speed of light, and n is the eigenfrequency mode order, $n = 1, 2, 3, ...$ This definition, excludes the ionosphere conductivity and height (Simões et al., 2012, equation 1). A more comprehensive spherical cavity model including ionosphere conductivity and height is given in (Simões et al., 2012, equation 2). In this contribution, it has been found that eigenfrequency modes have two a start off points, $n_0 = 0$ or $n_0 = \frac{\sqrt{5}-1}{2}$ where the non-zero one is exactly the golden ratio. This allows to extend the existing eigenfrequency modes to $\sqrt{(n_0+n)^2 + (n_o+n)}$ in order to explain why golden ratio exist within Schumann resonances. Hence, new spherical cavity model can be re-written and extended as, $f_n = \frac{c}{2\pi R}\sqrt{(n_0+n)^2 + (n_o+n)}$. Where, $n_0 = 0$ or $n_0 = \frac{\sqrt{5}-1}{2}$. The Golden ratio, $\frac{\sqrt{5}-1}{2}$ eigenfrequency offset, n_0 describes ionospheric changes. Fibonacci numbers can be predicted with the new equation presented in this contribution. Equation is derived considering the properties of a Pascal's triangle where sum of each diagonal line corresponds to a Fibonacci number. The new equation can be used to predict the higher order Schumann resonant frequencies in terms of Fibonacci numbers as they progress multiples of golden ratio, ϕ.

2 The New Equation of Fibonacci Numbers

$$N_{Fibonacci\, of\, nth\, Diagonal\, of\, Pascal's\, Triangle} = 1 + sin^2(2\pi + \frac{\pi}{2}n) + \sum_{\substack{y=1\\ c=1,2,3,4...}}^{y=n} A_{x^{n-c}_{n-y}} + B_{x^{n-c}_{n-y}} \tag{1}$$

Where,

n : Diagonal number.

$A_{x^{n-c}_{n-y}}$: Number in Pascal's triangle located at $n-y$ counting excluding the first row. The $n-c$ is the column location of the number counting from right hand side.

c : Continuous count starting from $c = 1$ for calculation of coefficient A, $c = 2$ for coefficient B. For any $y = n$, value of c keeps counting again for $y = n+1$ as $c = 3$ for coefficient of A, $c = 4$ for coefficient B.

n=0							1						
n=1						1		1					
n=2					1		2		1				
n=3				1		3		3		1			
n=4			1		4		6		4		1		
n=5		1		5		10		10		5		1	
n=6	1		6		15		20		15		6		1

In order to make formula independent from having to construct the Pascal's triangle, binomial theorem is incorporated.

$$N_{Fibonacci\,of\;nth\;Diagonal\;of\;Pascal's\;Triangle} = 2 + sin^2(2\pi + \frac{\pi}{2}n) + \sum_{\substack{y=1\\c=0\\D=1,2,3...(n-y-c-1)\\d=1,2,3...(n-y-c-2)\\c\leq n\\d\geq 0}}^{y=n} \frac{(n-y)D\left(\frac{n}{D}-\frac{y}{D}-1\right)}{(n-y-c)!} + \frac{(n-y)(n-y-d)}{(n-y-c-1)!} \tag{2}$$

3 Discussion & Conclusion

Calculating Fibonacci number for the $n = 8$ using the equation 2.

$N_{Fibonacci} = 2 + \frac{(8-1)(8-1-1)(8-1-2)(8-1-3)(8-1-4)(8-1-5)(8-1-6)}{(8-1-0)!} + \frac{(8-1)(8-1-1)(8-1-2)(8-1-3)(8-1-4)(8-1-5)}{(8-1-1-0)!} + \frac{(8-2)(8-2-1)(8-2-2)(8-2-3)(8-2-4)}{(8-2-1)!} + \frac{(8-2)(8-2-1)(8-2-2)(8-2-3))}{(8-2-1-1)!} + \frac{(8-3)(8-3-1)(8-3-2)}{(8-3-2)!} + \frac{(8-3)(8-3-1)}{(8-3-2-1)!} + \frac{(8-4)}{(8-4-3)!} = 55$

As demonstrated in the above example, 11^{th} number in the Fibonacci sequences corresponding to $n = 8$ is predicted successfully to be 55.

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