# A Doppler direction finding method using only a single base antenna array

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November 24, 2022

**Abstract**

By difference treatment of the rate of change of the radial distance, the interchange relationship between the Doppler shift and the path difference is obtained, so that the Doppler shift can be used to obtain the path difference in an equivalent way. On this basis, by using the linear solution of the double-base linear array and constructing a virtual double-base array, a Doppler direction finding method using only the single-base array is obtained. Because the transformation method based on frequency shift and path difference avoids the direct comparison of phase, and the equivalence of transformation is mainly related to the accuracy of frequency shift measurement, the new method is likely to lay a theoretical foundation for the application of Doppler direction finding method in higher frequency bands.

# A Doppler direction finding method using only a single base antenna array

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**Key Points:**

- A Doppler direction finding method using only the single-base array.
- The result of this paper is an engineering application of linear solution of double basis linear array.
- By using the interchange relationship between frequency shift and range difference, Doppler frequency shift is converted to path difference, thus Doppler direction finding based on one-dimensional array is realized.

## Abstract

By difference treatment of the rate of change of the radial distance, the interchange relationship between the Doppler shift and the path difference is obtained, so that the Doppler shift can be used to obtain the path difference in an equivalent way. On this basis, by using the linear solution of the double-base linear array and constructing a virtual double-base array, a Doppler direction finding method using only the single-base array is obtained. Because the transformation method based on frequency shift and path difference avoids the direct comparison of phase, and the equivalence of transformation is mainly related to the accuracy of frequency shift measurement, the new method is likely to lay a theoretical foundation for the application of Doppler direction finding method in higher frequency bands.

## 1 Introduction

In many direction-finding systems, Doppler direction-finding technology has the advantages of no ambiguity, high accuracy, no spacing error, small polarization error, high sensitivity, ability to measure elevation angle, and to resist wavefront distortion. However, the direction finding method based on rotating motion to obtain Doppler frequency shift seems not applicable to the airborne platform[1]. One of the author's existing research achievements[2] is to obtain a direction finding method based on Doppler frequency difference measurement by using the rate of change of direction cosine[3], on the basis of expressing the signal's incident sinusoidal angle as a function related to Doppler frequency difference, angular velocity, wavelength and baseline length, the unknown wavelength and angular velocity are eliminated by using two orthogonal arrays. Compared with the classical Doppler direction finding method based on antenna motion, the orthogonal frequency difference direction finding method greatly simplifies the antenna system, but it still needs to be arranged on a two-dimensional plane.

In this paper, as an engineering application of linear solutions of double-basis linear arrays, based on the positioning theory of double basis path difference[4], the author presents a passive Doppler direction finding method using only one-dimensional single basis receiving array by constructing a virtual double basis array by using the interchange relationship between Doppler frequency shift and path difference.

## 2 Linear solutions of one-dimensional equidistant double basis linear arrays

For the one-dimensional double-base equidistant linear array shown in Fig. 1, two adjacent path differences:

$$\Delta r_{12} = r_1 - r_2 \tag{1}$$

$$\Delta r_{23} = r_2 - r_3 \tag{2}$$

If the midpoint of the whole array is taken as the origin of coordinates, the following two auxiliary geometric equations can be listed by the law of cosines:

$$r_1^2 = r_2^2 + d^2 - 2r_2 d\cos\left(90^0 + \theta_2\right) \\ = r_2^2 + d^2 + 2r_2 d\sin\theta_2 \tag{3}$$

$$r_3^2 = r_2^2 + d^2 - 2r_2 d\cos\left(90^0 - \theta_2\right) \\ = r_2^2 + d^2 - 2r_2 d\sin\theta_2 \tag{4}$$

Since $x = r_2 \sin\theta_2$, the auxiliary geometric equation can be rewritten as:

$$r_1^2 = r_2^2 + d^2 + 2d \cdot x \tag{5}$$

$$r_3^2 = r_2^2 + d^2 - 2d \cdot x \tag{6}$$

Where: $d$ is the length of a single baseline; $x$ the abscissa of the Cartesian coordinate system.

At this point, if the path difference (1) and (2) corresponding to two adjacent baselines are substituted into the geometric auxiliary equations (5) and (6), the following binary first order system can be obtained after the transposition:

$$2d \cdot x - 2\Delta r_{12} r_2 = -d^2 + \Delta r_{12}^2 \tag{7}$$

$$2d \cdot x - 2\Delta r_{23} r_2 = d^2 - \Delta r_{23}^2 \tag{8}$$

The transverse distance of the target can be directly solved from it:

$$x = \frac{\left(d^2 - \Delta r_{12}^2\right)\Delta r_{23} + \left(d^2 - \Delta r_{23}^2\right)\Delta r_{12}}{2d\left(\Delta r_{12} - \Delta r_{23}\right)} \tag{9}$$

And the radial distance of the target:

$$r_2 = \frac{2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2}{2\left(\Delta r_{12} - \Delta r_{23}\right)} \tag{10}$$

Thus, the arrival angle of the target can be obtained:

$$\sin\theta_2 = \frac{x}{r_2} = \frac{\left(d^2 - \Delta r_{12}^2\right)\Delta r_{23} + \left(d^2 - \Delta r_{23}^2\right)\Delta r_{12}}{d\left(2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2\right)} \tag{11}$$

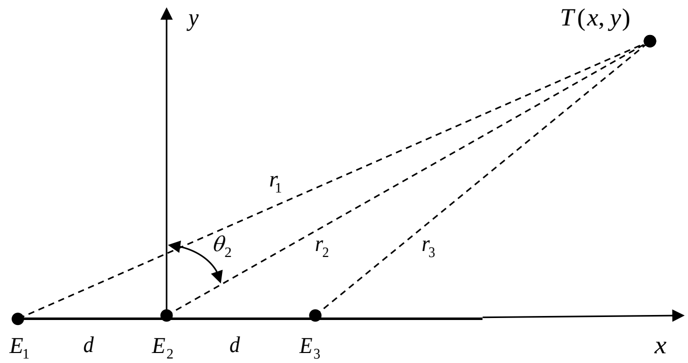


Figure 1: One-dimensional double-base array

## 3 Differential processing of radial velocity

As shown in Figure 2, assuming that Doppler receiver *R* is installed on the moving platform to detect stationary or slow-moving target *T* on the ground, the Doppler frequency shift received by the receiving array is as follows:

$$\lambda f_d = v\cos\beta \tag{12}$$

Where: $f_d$ is Doppler frequency shift; $\lambda$ the wavelength; $v$ the moving speed of the moving platform; $\beta$ the front angle.

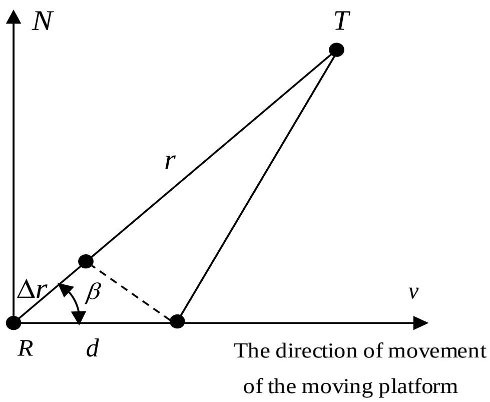


Figure 2: Doppler frequency shift detection of moving single station

According to the relationship between the rate of change of radial distance and the radial velocity, and between the radial velocity and the Doppler frequency shift, the relationship between the Doppler frequency shift and the rate of change of radial distance can be obtained

$$\frac{\partial r(t)}{\partial t} = v_r = v\cos\beta = \lambda f_d \tag{13}$$

Assuming that the change of time is short, the difference calculation method can be used to convert the differential of distance to time into

$$\frac{\partial r(t)}{\partial t} \approx \frac{\Delta r}{\Delta t} \tag{14}$$

Among them, $\Delta r$ is the path difference. $\Delta t$ is the time difference used to move the mobile platform from position 1 to position 2.

For a moving single station, when the moving distance of the platform is $d$, the time difference experienced by the formation path difference is:

$$\Delta t = \frac{d}{v} \tag{15}$$

Thus, the expression of path difference based on Doppler frequency shift measurement and independent of time difference measurement is obtained

$$\Delta r = \frac{\lambda d}{v} f_d \tag{16}$$

# 4 The direction finding solution with virtual arrays

## 4.1 Composition of virtual array

To facilitate understanding, the formation of a double-base array is demonstrated using a moving single-station trajectory. Assume that a moving single station moves in a straight line, as shown in Figure 3, from position 1, through position 2, to position 3, and assume that the lengths of the two distances traveled are equal. If its motion trajectory is regarded as a double-base equidistant line receiving array, two adjacent path differences is $\Delta r_{12}$ and $\Delta r_{23}$.

In fact, a single moving station can detect the Doppler shift for three consecutive times, but only two adjacent Doppler shift measurements are needed to obtain two adjacent path difference. From the physical definition, Doppler shift is the projection of the motion velocity of the detection platform in the radial direction. Thus, the Doppler shift computed from $\Delta r_{12}$ appears to be $f_{d1}$. At the same time, the preliminary simulation results show that the final direction finding results are the same using the Doppler shift at the first two positions and the Doppler shift at the last two positions. Therefore, we will directly use the Doppler shift values at the first two positions of the motion trajectory. Corresponding to a double-base array, two adjacent path difference equation given by frequency shift measurement is

$$\Delta r_{12} = \frac{\lambda d}{v} f_{d1} \tag{17}$$

$$\Delta r_{23} = \frac{\lambda d}{v} f_{d2} \tag{18}$$

From the expression of path difference based on Doppler frequency shift measurement and the geometric model shown in Figure 3, it can be seen that the two adjacent path difference equations needed to construct a double-base array can be obtained by using only one single-base receiving array in practical application. Therefore, the double-base linear array involved in the actual direction finding calculation is actually a virtual array.

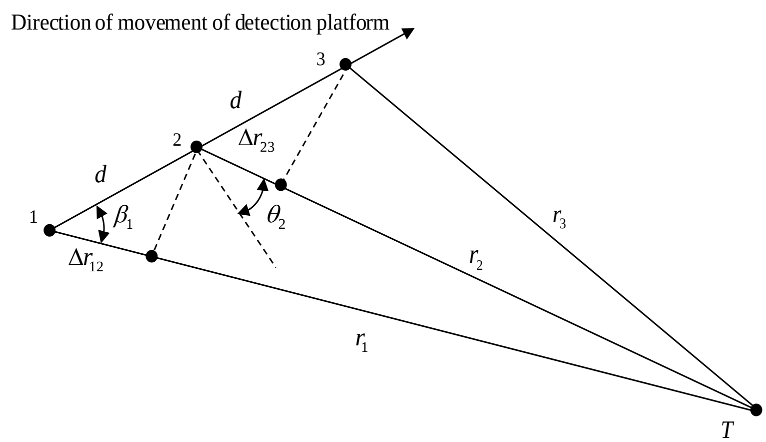


Figure 3: Formation of virtual double-base arrays

## 4.2 Simulation calculation of direction finding solution

Based on the foregoing analysis, it is only necessary to set a single base array on the moving single station platform, and arrange the antenna and Doppler frequency shift receiver at both ends of the array respectively. By measuring Doppler frequency shift and using the relationship between frequency shift and path difference, the equations of two adjacent path difference required to construct a dual base array can be obtained.

On this basis, the target arrival angle of the midpoint of the virtual array can be obtained directly by using double-base path difference direction finding formula (11)

$$\sin\theta_2 = \frac{\left(d^2 - \Delta r_{12}^2\right)\Delta r_{23} + \left(d^2 - \Delta r_{23}^2\right)\Delta r_{12}}{d\left(2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2\right)} \tag{11}$$

In the process of simulation calculation, the front angle $\beta_1$ at the starting point 1 of the detection platform, the radial distance $r_1$ from the starting point 1 to the target, the flight distance $d$ and flight speed $v$ of the detection platform, and the wavelength $\lambda$ of the detected signal are preset first.

Then the target arrival angle $\theta_2$ at the midpoint of the virtual array and other geometric parameters are calculated by using trigonometric function relations. According to the definition of Doppler frequency shift (12), the value of Doppler frequency shift is calculated. Then the path difference of the double-base array is given by the relation between frequency shift and path difference.

On this basis, the front angle changes linearly within the scope of $0^0 < \beta_1 < 90^0$, and the direction finding value obtained by the virtual two bases array is compared with the theoretical value obtained by using trigonometric function.

Fig. 4 shows the relative calculation errors of direction finding solutions with different baseline lengths, from which it can be seen that the calculation accuracy is inversely proportional to the baseline length.

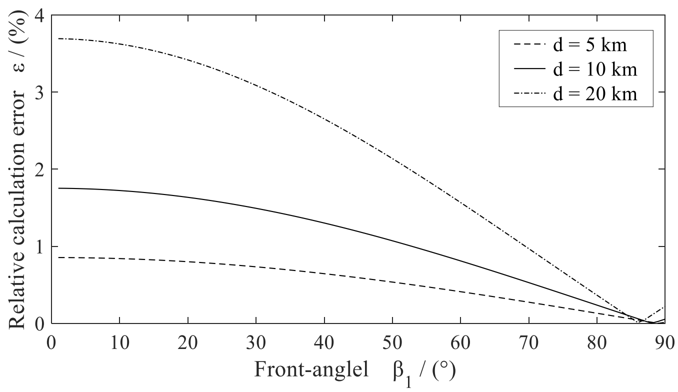


Fig. 4 Direction finding accuracy

The parameters taken in the simulation calculation are: the radial distance of the target $r_1 = 300\text{km}$, the baseline length of the single base array $d = 10m$, the flight speed of the detection platform $v = 300m/s$, and the wavelength of the detection signal $\lambda = 0.3m$.

Meanwhile, the simulation results show that the relative calculation error is inversely proportional to the radial distance. The simulation results also show that the values of flight speed and wavelength have little influence on the analysis of relative calculation error.

## 5 Direction finding error

5.1 Total differential method

Relative ranging error is analyzed by total differential method. First set

$$\sin\theta_2 = \frac{R}{d \cdot S}$$

$$R = \left(d^2 - \Delta r_{12}^2\right)\Delta r_{23} + \left(d^2 - \Delta r_{23}^2\right)\Delta r_{12}$$

$$S = \left(2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2\right)$$

(1) Direction finding errors resulting from frequency shift $f_{di}$ measurements $(i = 1,2)$

$$\frac{\partial\theta_2}{\partial f_{di}} = \frac{1}{d\cos\theta_2 S^2}\left(S\frac{\partial R}{\partial f_{di}} - R\frac{\partial S}{\partial f_{di}}\right)$$

$$\frac{\partial R}{\partial f_{di}} = -2\Delta r_{12}\frac{\partial \Delta r_{12}}{\partial f_{di}}\Delta r_{23} + \left(d^2 - \Delta r_{12}^2\right)\frac{\partial \Delta r_{23}}{\partial f_{di}} - 2\Delta r_{23}\frac{\partial \Delta r_{23}}{\partial f_{di}}\Delta r_{12} + \left(d^2 - \Delta r_{23}^2\right)\frac{\partial \Delta r_{12}}{\partial f_{di}}$$

$$\frac{\partial S}{\partial f_{di}} = -2\Delta r_{12}\frac{\partial \Delta r_{12}}{\partial f_{di}} - 2\Delta r_{23}\frac{\partial \Delta r_{23}}{\partial f_{di}}$$

$$\frac{\partial \Delta r_{12}}{\partial f_{d1}} = \frac{\lambda d}{v}$$

$$\frac{\partial \Delta r_{23}}{\partial f_{d1}} = 0$$

$$\frac{\partial \Delta r_{12}}{\partial f_{d2}} = 0$$

$$\frac{\partial \Delta r_{23}}{\partial f_{d2}} = \frac{\lambda d}{v}$$

(2) Range error caused by baseline length measurement

$$\frac{\partial \theta_2}{\partial d} = \frac{1}{d^2 \cos\theta_2 S^2}\left(dS\frac{\partial R}{\partial d} - RS - Rd\frac{\partial S}{\partial d}\right)$$

$$\frac{\partial R}{\partial d} = \left(2d - 2\Delta r_{12}\frac{\partial \Delta r_{12}}{\partial d}\right)\Delta r_{23} + \left(d^2 - \Delta r_{12}^2\right)\frac{\partial \Delta r_{23}}{\partial d} + \left(2d - 2\Delta r_{23}\frac{\partial \Delta r_{23}}{\partial d}\right)\Delta r_{12} + \left(d^2 - \Delta r_{23}^2\right)\frac{\partial \Delta r_{12}}{\partial d}$$

$$\frac{\partial S}{\partial d} = 4d - 2\Delta r_{12}\frac{\partial \Delta r_{12}}{\partial d} - 2\Delta r_{23}\frac{\partial \Delta r_{23}}{\partial d}$$

$$\frac{\partial \Delta r_{12}}{\partial d} = \frac{\lambda}{v} f_{d1}$$

$$\frac{\partial \Delta r_{23}}{\partial d} = \frac{\lambda}{v} f_{d2}$$

(3) Ranging error caused by flight speed

$$\frac{\partial \theta_2}{\partial v} = \frac{1}{d\cos\theta_2 S^2}\left(S\frac{\partial R}{\partial v} - R\frac{\partial S}{\partial v}\right)$$

$$\frac{\partial R}{\partial v} = \left(-2\Delta r_{12}\frac{\partial \Delta r_{12}}{\partial v}\right)\Delta r_{23} + \left(d^2 - \Delta r_{12}^2\right)\frac{\partial \Delta r_{23}}{\partial v} + \left(-2\Delta r_{23}\frac{\partial \Delta r_{23}}{\partial v}\right)\Delta r_{12} + \left(d^2 - \Delta r_{23}^2\right)\frac{\partial \Delta r_{12}}{\partial v}$$

$$\frac{\partial S}{\partial v} = -2\Delta r_{12}\frac{\partial \Delta r_{12}}{\partial v} - 2\Delta r_{23}\frac{\partial \Delta r_{23}}{\partial v}$$

$$\frac{\partial \Delta r_{12}}{\partial v} = -\frac{d\lambda}{v^2} f_{d1}$$

$$\frac{\partial \Delta r_{23}}{\partial v} = -\frac{d\lambda}{v^2} f_{d2}$$

5.2 Basic calculation formula

When the error of each observation quantity is zero mean, independent of each other, absolute direction finding error

$$\sigma_\theta = \sqrt{\sum_{i=1}^{2}\left(\frac{\partial \theta_2}{\partial f_{di}}\sigma_f\right)^2 + \left(\frac{\partial \theta_2}{\partial d}\sigma_d\right)^2 + \left(\frac{\partial \theta_2}{\partial v}\sigma_v\right)^2} \quad (19)$$

Where: $\sigma_f$ , $\sigma_d$ and $\sigma_v$ are respectively the root mean square errors of measurement errors of frequency shift, baseline length and flight speed.

The geometrical parameters and Doppler shifts are set in the same way as in the simulation calculation. By making the front angle change linearly in the range of $0^0 < \beta_1 < 90^0$, the Doppler frequency shift and path difference are calculated, and the direction finding error is finally obtained.

5.3 Direction finding errors under different geometry and motion parameters

The calculation shows that the change of the radial distance has no effect on the direction finding error.

Unless otherwise specified, the geometrical and motion parameters selected in the calculation are: radial distance of the target $r_1 = 300\text{km}$, baseline length $d = 10\text{m}$, motion velocity of the detection platform $v = 300m/s$, and wavelength of the detection signal $\lambda = 0.3m$.

The root mean square error of the measurement error of each observed quantity is $\sigma_f = 50Hz$, $\sigma_v = 1m/s$, $\sigma_d = 0.1m$.

(1) Baseline length

Figure 5 shows, in an extreme way, the direction finding errors at different baseline lengths. The calculation results show that the change of baseline length has little effect on the direction finding error.

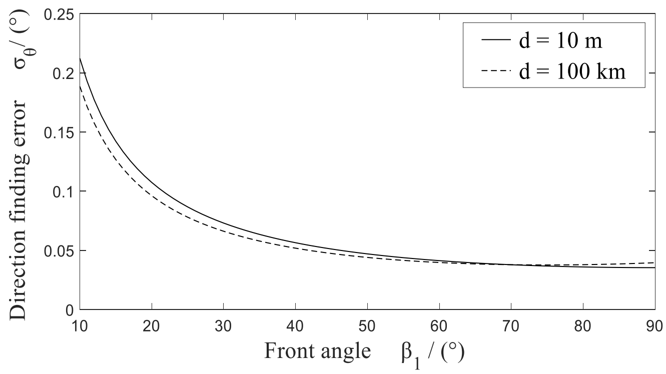


Figure 5 Direction finding errors at different baseline lengths

(2) Movement speed

Figure 6 shows the direction finding errors at different motion velocities. Obviously, increasing the motion speed of the detection platform is beneficial to improve the accuracy of direction finding.

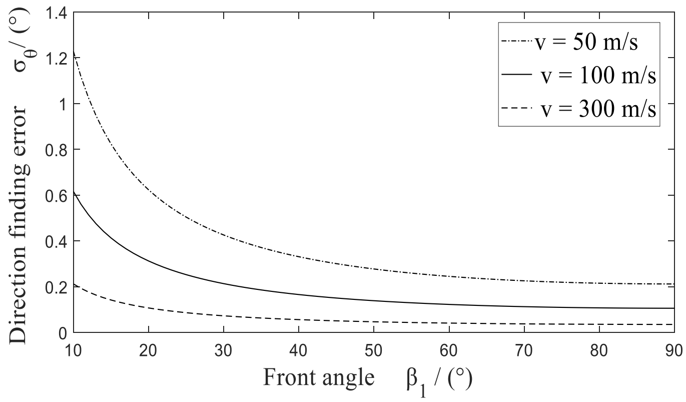


Figure 6 Direction finding error at different velocity

(3) The signal wavelength

Figure 7 shows the direction finding errors at different signal wavelengths. The results show that the shorter the wavelength, the better the accuracy of direction finding.

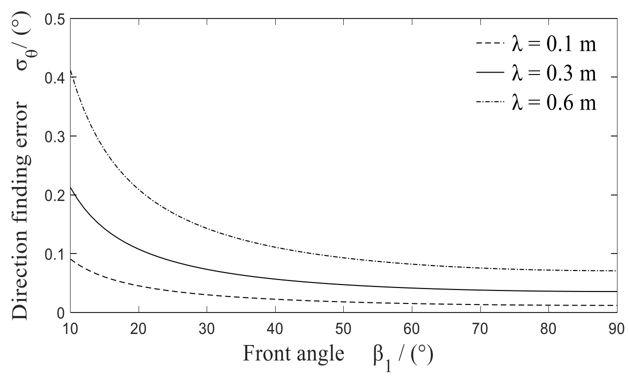


Figure 7 Direction finding error at different wavelengths

5.4 Influence of root mean square errors on direction finding errors

If there is no special explanation, the values of each geometric or kinematic parameter, as well as the root mean square errors of the measurement error of each observation quantity are the same as those in the previous section.

(1) Baseline length

In fact, for a long baseline, the root mean square error of the baseline length measurement error may be relatively large. Figure 8 shows the direction finding error when the root mean square error of the measurement error of different baseline lengths is taken when the baseline is 100 meters long.

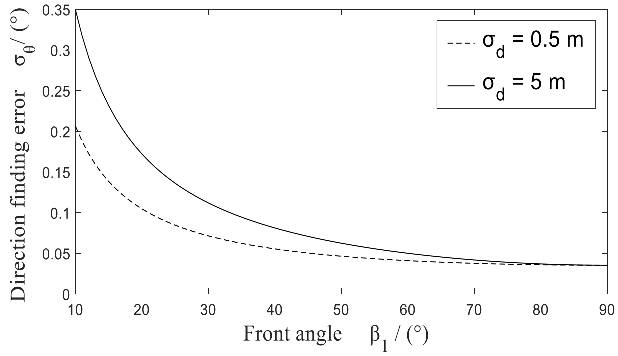


Figure 8 Direction finding errors with different root mean square errors of the baseline length measurement error

(2) Doppler shift

Figure 9 shows the direction finding errors at different root mean square errors of the Doppler shift measurement errors. The results show that reducing the measurement error of Doppler shift is helpful to improve the accuracy of direction finding.

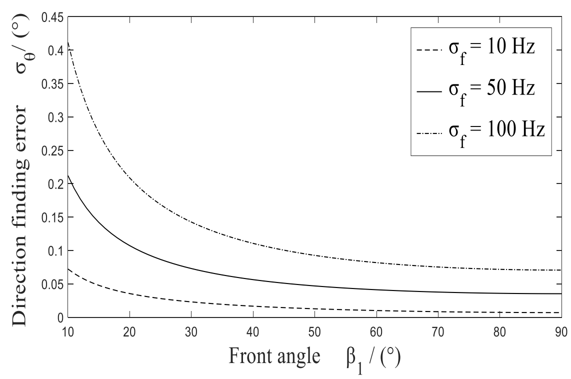


Figure 9 The direction finding error of different root mean square error of Doppler shift measurement error

(3) Motion speed

Figure 10 shows the direction finding error when the root mean square error of the measurement error of different platform motion velocity. The results show that reducing the measurement error of the platform velocity is helpful to improve the accuracy of direction finding.

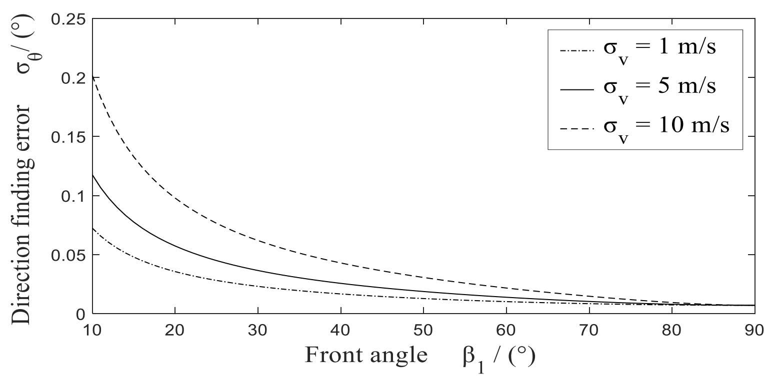


Figure 10 Direction finding error in different root mean square error of measurement error of platform velocity

## 6 Conclusions

The author first proposed the interchange relationship between frequency shift and path difference in research paper [5], which is actually one of the innovations of this research paper. Based on the interchange relationship between Doppler frequency shift and path difference, as well as the midpoint direction finding solution of a single basis, a passive ranging method suitable for a moving platform is presented in the literature [5], by constructing a virtual double-basis linear array. Based on the research ideas in literature [5], this paper further presents the Doppler direction finding method.

Taking moving single station as an example, this paper describes the basic principle of moving single station Doppler direction finding with virtual double base linear array. The analysis process and results can be immediately extended to Doppler direction finding of moving dual station. The Doppler direction finding method based on single base array proposed in this paper is more suitable for single moving station because the dimensions of the antenna only need to be extended in one dimension.

The existing Doppler direction finding equipment is mainly applicable to the lower frequency band, but the method based on frequency shift and path difference transformation in this paper avoids the direct comparison of phase, and the equivalence of transformation is mainly related to the accuracy of frequency shift measurement, which is likely to help improve the application frequency band of Doppler direction finding equipment.

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