

A Dynamic Socio-hydrological Model of The Irrigation Efficiency Paradox

Ansir Ilyas¹

¹Lahore University of Management Sciences

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Abstract

Improving irrigation efficiency (IE) is conventionally perceived as a water-conserving practice in the agriculture sector. The common understanding is that increased on-farm IE leads to an increase in water availability at the basin. However, in the recent past, many instances have been reported where increasing on-farm IE failed to increase water availability at the basin scale. This phenomenon is commonly known as the ‘Irrigation Efficiency Paradox (IEP)’. In this paper, we present a dynamic systems model of the IEP. Our model combines a simple mass-balance description of the water flows with the rent-seeking behavior of consumers. Through the socio-hydrological model, we arrive at a parametric characterization of the IEP, which is given by three attributes: the maximum short-term benefit enjoyed after improvement in IE, the time duration after which the paradox occurs, and the escalation of the paradox once it occurs. We find that the paradox in basins with lower evaporation and higher recharge is more pronounced, the policy implications of which are in contrast to the common narrative that seeks to reduce evaporation and increase recharge. We also use our findings along with global data sets to identify regions that are most susceptible to the IEP. We argue that much caution must be practiced while introducing efficient irrigation technologies in the identified regions so as to avoid paradoxical effects to as much extent as possible. We also discuss the seemingly counter-intuitive role of evaporation and recharge properties of the basin and how it ties in with contemporary policy narratives.

A Dynamic Socio-hydrological Model of The Irrigation Efficiency Paradox

Ansir Ilyas^{1,2,3}, Talha Manzoor¹, and Abubakr Muhammad^{1,2,3}

¹Center for Water Informatics and Technology at Lahore University of Management Sciences (LUMS),

Lahore, Pakistan

²NCRA Agricultural Robotics Lab at LUMS, Lahore, Pakistan

³Department of Electrical Engineering at LUMS, Lahore, Pakistan

Key Points:

- A dynamical model of the irrigation efficiency paradox is developed by coupling the on-farm water flows with farmers' rent-seeking behavior
- The model predicts that the paradox is more pronounced in basins with low evaporation and high recharge
- Larger improvements in irrigation efficiency increase short term benefits but lead to a faster escalating paradox

Corresponding author: Ansir Ilyas, ansir.ilyas@lums.edu.pk

Abstract

Improving irrigation efficiency (IE) is conventionally perceived as a water-conserving practice in the agriculture sector. The common understanding is that increased on-farm IE leads to an increase in water availability at the basin. However, in the recent past, many instances have been reported where increasing on-farm IE failed to increase water availability at the basin scale. This phenomenon is commonly known as the ‘Irrigation Efficiency Paradox (IEP)’. In this paper, we present a dynamic systems model of the IEP. Our model combines a simple mass-balance description of the water flows with the rent-seeking behavior of consumers. Through the socio-hydrological model, we arrive at a parametric characterization of the IEP, which is given by three attributes: the maximum short-term benefit enjoyed after improvement in IE, the time duration after which the paradox occurs, and the escalation of the paradox once it occurs. We find that the paradox in basins with lower evaporation and higher recharge is more pronounced, the policy implications of which are in contrast to the common narrative that seeks to reduce evaporation and increase recharge. We also use our findings along with global data sets to identify regions that are most susceptible to the IEP. We argue that much caution must be practiced while introducing efficient irrigation technologies in the identified regions so as to avoid paradoxical effects to as much extent as possible. We also discuss the seemingly counter-intuitive role of evaporation and recharge properties of the basin and how it ties in with contemporary policy narratives.

1 Introduction

Irrigation accounts for roughly 70% of the total extractions of global freshwater and as such is the world’s largest water-consuming sector (Grafton et al., 2018; Perez-Blanco et al., 2019). At the same time, however, it suffers from very low productivity of water use (Perez-Blanco et al., 2019). Dwindling freshwater resources combined with rapid population growth and the necessity of extensive economic development have exerted immense pressure on irrigation systems to enhance their water productivity (Igor, 1993; Whiting, 2020). The irrigation system consists of both on-farm and off-farm components. Both components carry the huge potential to save water, which may be diverted and utilized elsewhere in the basin. In this study, we concern ourselves with technological interventions in on-farm irrigation systems, made with the intention of increasing water availability at the basin level.

The notion of ‘irrigation efficiency’ serves as a measure of the extent to which water diverted from a reservoir is delivered to the farm field and contributes beneficially to crop growth (Israelsen et al., 1950). Many technological interventions aim to save water by increasing the irrigation efficiency of the on-farm irrigation system. Such interventions include the deployment of drip or sprinkler irrigation technologies, laser leveling, and watercourse lining, to name a few (Pérez-Blanco et al., 2020). While decision and policymakers have supported the proliferation of irrigation-efficient technologies much enthusiastically, studies have shown that in many cases, the deployment of such technologies failed to result in the expected water savings at the basin-scale (Grafton et al., 2018; Lankford, 2013; Perry, 2007). This phenomenon where an increase in irrigation efficiency at the farm scale fails to increase water availability at the basin-level is known as the Irrigation Efficiency Paradox (IEP) (Grafton et al., 2018).

The primary phenomenon that brings about the IE paradox can be attributed to irrigator behavior. Once an improvement in on-farm irrigation efficiency is brought about, the response of a rational profit-maximizing farmer can be seen in Figure 1. The efficiency improvement leads to a reduction in farm water use, which the farmer perceives as an opportunity to use the saved water for an additional economic benefit (in the remainder of the paper, we refer to this phenomenon as ‘rent-seeking behavior’). In irrigation systems, the outcome of such behavior appears in the form of expansion of the irrigated

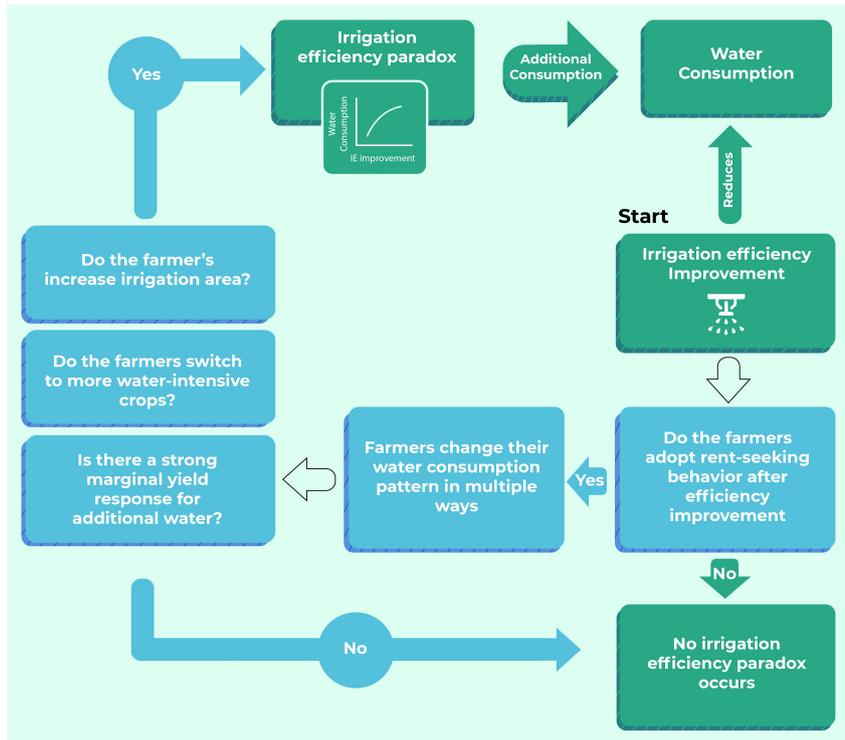


Figure 1. The IEP will not occur unless the efficiency improvement affects the consumer’s perception of the spare water available at the farm endowment. The concept of the figure is inspired from (Paul et al., 2019). Blue color represents the consumer’s behavior and the factors that strongly influence paradox, whereas green color represents the hydrology at the farm level.

66 area, a switch to more water-intensive crops or increased irrigation withdrawals if there
 67 is a marginal yield response for additional water. All of these outcomes result in increased
 68 water withdrawals after the improvement in efficiency occurs and have been observed
 69 in many irrigation efficiency programs over the globe (Ward & Pulido-Velazquez, 2008;
 70 Dumont et al., 2013; Scott et al., 2014; Berbel et al., 2015; Sears et al., 2018; Pfeiffer &
 71 Lin, 2014). This phenomenon where the expected saving from an increase in efficiency
 72 is reduced due to behavioral or other systemic factors is a classical concept in the eco-
 73 nomics literature called the Rebound Effect (Chitnis et al., 2014). A particular instance
 74 of the rebound effect is one where the rebound is so large that it completely offsets the
 75 potential saving from the efficiency improvement, thus resulting in even greater consump-
 76 tion than before. This effect is known as Jevon’s Paradox. The paradox was first reported
 77 by its namesake (Jevons, 1865) in the context of coal consumption and has been observed
 78 in many sectors in which the energy sector is especially prominent (Chitnis et al., 2014;
 79 Murray, 2013). However, recent studies have reported the occurrence of the paradox in
 80 irrigation efficiency programs as well (Grafton et al., 2018; Lankford et al., 2020; Berbel
 81 et al., 2018).

82 Here we cast the problem of the IEP into the relatively new paradigm of socio-hydrology
 83 (Sivapalan et al., 2012). While the aforementioned approaches to modeling focus either
 84 on the hydrological or social components of the IEP, it is becoming increasingly clear that
 85 none of these components act in isolation. Rather, both social and hydrological compo-
 86 nents co-evolve with one another to produce the effects that socio-hydrologists aim to
 87 understand. Even though the perspective of socio-hydrology is relatively new, it has trig-

gered considerable activity even beyond initial illustrations, and clarification of definitions (Pande & Sivapalan, 2017). The key strength of the framework is its inclusion of human agency (psychology, economics, technology, norms) as something endogenous to the system. Thus, while there exists a profound limit on our ability to predict human behavior, socio-hydrology is providing new insights into the dynamic relationship between humans and water systems (Hall, 2019). The efficiency paradox is one such relationship that has been portrayed (Sivapalan et al., 2014) as a problem that can be resolved only within a framework that encompasses the two-way coupling between human and water systems. In this study, we first formulate a conceptual model of the IEP whose hydrological component is compartmentalized according to standard water accounting terminology. The dynamics of the hydrological component are based on simple water-balance as followed in (Grafton et al., 2018). A social component is conceptualized and coupled with the hydrological dynamics of the basin. The dynamics of the social component are based on the phenomenon of rent-seeking on the part of the irrigating farmers. The on-farm irrigation is driven by the behavior of the farmers and co-evolves along with the stock of recoverable water at the basin scale.

In this paper, we combine a basin-scale hydrological model based on simple mass-balance with the social dynamics of rent-seeking behavior of the irrigators. The terminology that we adopt is similar to the one used in (Grafton et al., 2018) where the authors demonstrate the IEP through a static one-shot instance of a mass-balance hydrological model of a basin. They show that an increase in crop water use at the farm-level is bound to result in a decrease in the stock of recoverable water at the basin scale. However, the behavior of the irrigators that leads towards increased water use at the farm is not included in the model and the increase in demand is taken to be exogenous. Several economic studies have modeled this effect as a profit maximization problem from the farmer viewpoint to determine conditions under which an increase in irrigation efficiency may lead to decreased consumption (see (Pfeiffer & Lin, 2014; Huffaker & Whitelsey, 2003; Caswell & Zilberman, 1986) and included references). Such models interpret the irrigation efficiency problem entirely from an economic perspective without an appropriate representation of the basin hydrology. Another body of work is focused on simulation models that use calibrated data to predict the effects of adopting advanced irrigation technologies (Smith, 2011; Dukes, 2012). While such models are well-suited for large-scale studies that integrate data over multiple domains, they lack the rigor that analytical models provide for uncovering qualitative relationships between pivotal variables of the system under investigation.

After presenting the socio-hydrological model of the irrigation system, we give a simple definition of the irrigation efficiency paradox. An irrigation efficiency paradox occurs if the stock of recoverable water after an improvement in efficiency is reduced in comparison to a scenario where no improvement in efficiency is made at all. We find that the difference in the stock of recoverable water for both scenarios follows a fixed pattern, which we use to give a novel characterization of the paradox. More specifically, we observe that after the efficiency improvement is made, a short-lived period of benefit immediately follows. After this period, as the farmers' rent-seeking behavior kicks in, water consumption increases, resulting in an overall reduction in recoverable water compared to before the efficiency improvement, hence the paradox. We thus use three parameters to characterize the IEP: the maximum benefit gained before the paradox occurs, the time at which the paradox starts, and the intensity of the paradox after it occurs. We determine the effect of the basin attributes, namely the evaporation and recovery coefficients, on the paradox characteristics through detailed numerical simulations. In the end, we apply the results to a global data-set to identify regions that are most vulnerable to the efficiency paradox.

139 2 Methodology

140 This section demonstrates our modeling approach to captures the IE paradox. We
 141 first present the water accounting terminology to classify flows in the water-balance for
 142 our model. Based on the accounting terminology and the hydrology of the on-farm flows,
 143 we present the hydrological sub-model. The social sub-model is also presented that cap-
 144 tures the rent-seeking behavior of the irrigators. Afterward, we present the coupled socio-
 145 hydrological model, followed by a simple transformation to reduce the parameter space.
 146 To demonstrate the on-farm implementation of the model, we construct a hypothetical
 147 farm field called WITFarm. In the end, we present the characteristics of the IE para-
 148 dox.

149 2.1 Water Accounting Terminology

150 In the following sections of the paper, we adopt the standard water accounting ter-
 151 minology as proposed by the International Commission on Irrigation and Drainage (Perry,
 152 2007) to classify flows in the water-balance for our model. A pictorial representation of
 153 the classification can be seen in Figure 2. It has been argued (Perry, 2011) that unam-
 154 biguous terminology is essential for water accounting both within and across sectors so
 155 as to avoid misleading interpretation of statistics and to more effectively trace the ef-
 156 fect of increasing water productivity in agriculture. Only a portion of water applied to
 157 the farm is taken up by the plants. While the remaining fraction may be considered as
 158 wastage from the farmers’ point of view, that water may become available for use in other
 159 areas of the basin, either by seeping to recharge the aquifer or by re-entering the river
 160 system in the form of return flows (Paul et al., 2019; Grafton et al., 2018; Berbel & Ma-
 161 teos, 2014; Perry, 2011).

162 We now define the adopted terminology. ‘Water withdrawals’ are meant to con-
 163 stitute any amount of water removed from surface or groundwater bodies for use in a
 164 particular activity, which in this case is irrigation. Once a withdrawal is made, it is then
 165 distributed as follows,

- 166 1. The consumed fraction: the amount of water that is converted into water vapor
 167 through plant transpiration or evaporation from wet soil. This fraction is further
 168 divided as:
 - 169 (a) Beneficial consumption: the water actually consumed by the crop or the wa-
 170 ter converted to water vapor through crop transpiration.
 - 171 (b) Non-beneficial consumption: water converted to water vapor by means other
 172 than crop transpiration, e.g., the water evaporated from wet soil or transpired
 173 by weed.

174 The non-consumed fraction: the return flows, which consist of:

- 175 (a) Recoverable return flows: water that either flows back to the river system or
 176 recharges the usable groundwater aquifer.
- 177 (b) Non-recoverable return flows: water lost for further use, e.g., water seeping into
 178 a salt sink.

179 In terms of the above terminology, an intervention results in water conservation at the
 180 basin-scale only if the quantity of recoverable return flows is increased. From the dis-
 181 tribution just discussed, this is possible only if there is a corresponding decrease in the
 182 aggregate of beneficial consumption (crop transpiration), non-beneficial consumption (evap-
 183 oration and weed transpiration), and non-recoverable return flows (percolation to salt
 184 sink). Advanced on-farm irrigation technologies target a reduction in non-beneficial con-
 185 sumption at the farm to produce a proportionate increase in recoverable return flows (Pérez-
 186 Blanco et al., 2020). Recoverable return flows contribute to local groundwater recharge
 187 and streamflow generation, which can subsequently be withdrawn by farmers in follow-

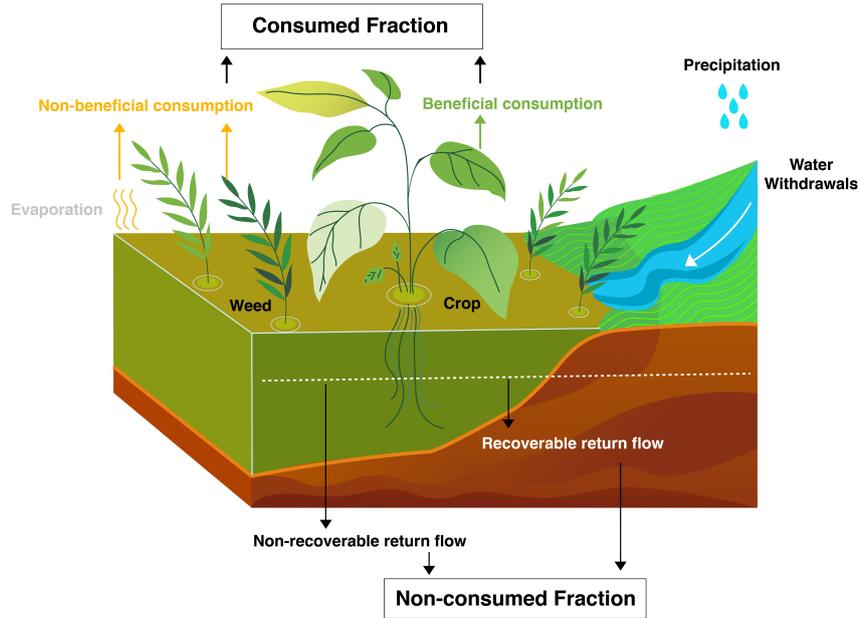


Figure 2. The standard terminology of water basin accounting of agriculture water withdrawals.

188 ing seasons as an alternative to surface water deliveries, thus providing a short-term stor-
 189 age buffer resilient to droughts (Ward & Pulido-Velazquez, 2008; Niswonger et al., 2017).
 190 These flows also help regulate soil quality and offset seasonal variations in the water ta-
 191 ble (Qureshi et al., 2008). Therefore, an accounting of recoverable return flows is con-
 192 siderably important in assessing the impact of any intervention meant to increase wa-
 193 ter availability at the basin-scale.

194 2.2 The Hydrological Sub-model

195 Here we present a dynamical system model of an on-farm irrigation system based
 196 on hydrological processes through a system of linear differential equations. We model
 197 the water flow at an irrigated farm based on the Law of Conservation of Mass: the in-
 198 flow to any point equals the outflow. The causal loop diagram (CLD) in Figure 3, shows
 199 the distribution of water inflows and outflows at a farm. We follow the system dynam-
 200 ics approach (Sterman, 2001) in which the boxes represent the stock (volume), and the
 201 valves represent the flow (volume per unit time) of water.

202 We now introduce the notation of variables used in the model (a comprehensive
 203 list of the variables along with units and mathematical ranges is given in Table A1 of
 204 the Appendix). $I(\tau)$ represents inflow the rate of water inflow to the farm from either
 205 the surface deliveries or the groundwater withdrawals at time τ . The accumulation of
 206 total inflows is called farm endowment. As discussed previously in Section 2.1, all flows
 207 into the farm eventually contribute to the following stocks: Transpiration by the crops
 208 (represented by $x_T(\tau)$), Evaporation and water transpired by weed (represented by $x_e(\tau)$),
 209 Non-recoverable water (represented by $x_{nr}(\tau)$), and the Recoverable water (represented
 210 by $x_r(\tau)$). The rate at which the water is applied to the farm for irrigation is represented
 211 by $r_I(\tau)$. The water applied to the farm is further divided into a consumed fraction and
 212 return flows, which eventually goes to non consumed fractions (Grafton et al., 2018) out-
 213 lined in Figure 3. $x_T(\tau)$ and $x_e(\tau)$ collectively constitute the consumed fraction, $x_r(\tau)$
 214 and $x_{nr}(\tau)$ collectively constitute the non-consumed fraction. Furthermore, in the con-

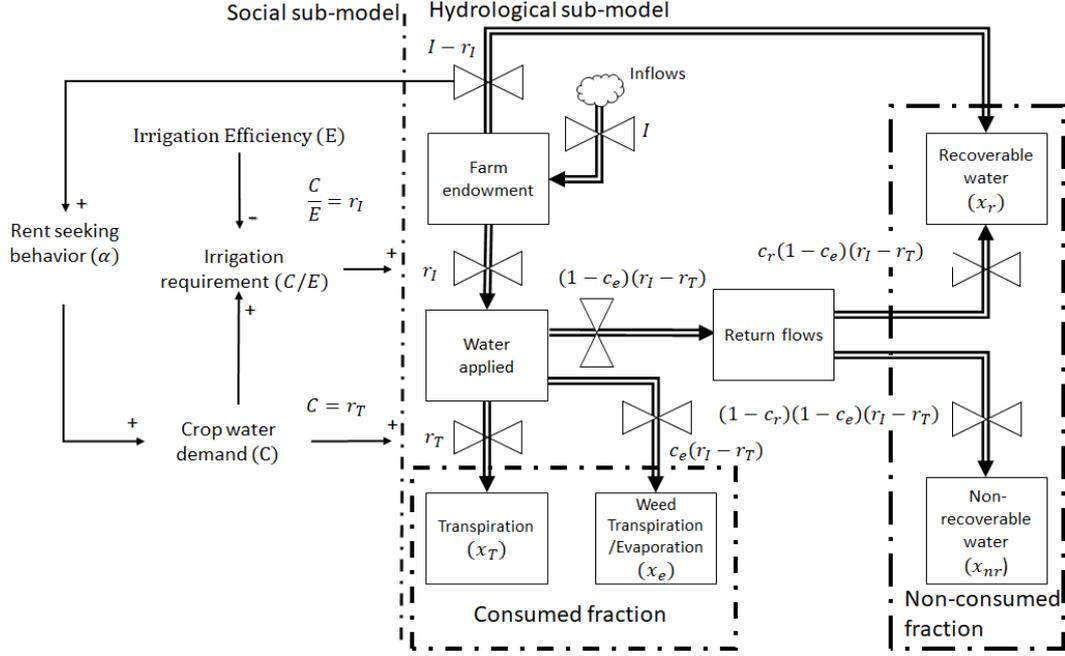


Figure 3. Causal loop diagram showing dynamics of hydrological and social sub-model for the irrigation efficiency paradox.

215 summed fraction, the beneficial consumption includes the crop transpiration, the rate at
 216 which crops consume water. We call $r_T(\tau)$ the rate of beneficial consumption. There-
 217 fore the dynamics of the crop transpiration can be written as follows,

$$218 \quad \dot{x}_T(\tau) = r_T(\tau), \quad (1)$$

219 where the overdot represents the derivative with respect to τ . The non-beneficial con-
 220 sumption in a consumed fraction is the total amount of water consumed by weed and
 221 evaporated from wet soil, this can be represented mathematically as follows,

$$222 \quad \dot{x}_e(\tau) = c_e(r_I(\tau) - r_T(\tau)), \quad (2)$$

223 where c_e is the evaporation coefficient (Marek & Straub, 2001). The evaporation coef-
 224 ficient represents the effectiveness of the process, which converts liquid water into wa-
 225 ter vapor. It is defined as the ratio of the evaporated water to the total water applied.
 226 c_e varies from zero to one (see Table A1), its value close to zero represents a smaller frac-
 227 tion of water, whereas its value close to one represents a larger fraction of water that goes
 228 into vaporization.

229 At the farm, the water left after the consumption goes to return flows. The return
 230 flows are then divided into recoverable and non-recoverable water. This water division
 231 depends on the recovery coefficient c_r . This represents the ability of the irrigation sys-
 232 tem to recharge the aquifer. It is defined as the ratio of recharged water to the total wa-
 233 ter applied. c_r varies from zero to one, its value close to one represents a larger frac-
 234 tion of water, whereas its value close to zero represents a smaller fraction of water that goes
 235 to recharge the aquifer. c_e and c_r are taken to be the basin's physical characteristics, and
 236 they depend upon the geographical location, weather, climate and soil conditions, and
 237 vapor pressure, to name a few.

238 We assume that the spare water (Lankford et al., 2020), i.e., the water left from
 239 the farm endowment $I(\tau) - r_T(\tau)$ directly goes to the recoverable fraction. Therefore

240 the change in the stock of recoverable water $x_r(\tau)$ can be written mathematically as fol-
 241 lows,

$$242 \quad \dot{x}_r(\tau) = I(\tau) - (1 - c_r(1 - c_e)(1 - E)) \frac{r_T(\tau)}{E}. \quad (3)$$

244 Furthermore, from Figure 3 all non-evaporated and non-recovered water at the farm goes
 245 to the non-recoverable stock. Therefore the change in the stock of non-recoverable wa-
 246 ter $x_{nr}(\tau)$ can be written mathematically as follows,

$$247 \quad \dot{x}_{nr}(\tau) = (1 - c_e)(1 - c_r)(r_I(\tau) - r_T(\tau)). \quad (4)$$

248 The accounting paradigm underlying all assumptions and the flow contributions is pre-
 249 sented in Figure 4, which demonstrates that the total water in the system is always con-
 250 served and that the inflow is divided to crop transpiration $x_T(\tau)$, to evaporation and weed
 251 transpiration x_e , to aquifer recharge $x_r(\tau)$, and to a non-recoverable stock $x_{nr}(\tau)$ (Perez-
 252 Blanco et al., 2019). Equations 1 - 4 collectively represent the overall dynamics of the
 hydrological sub-model.

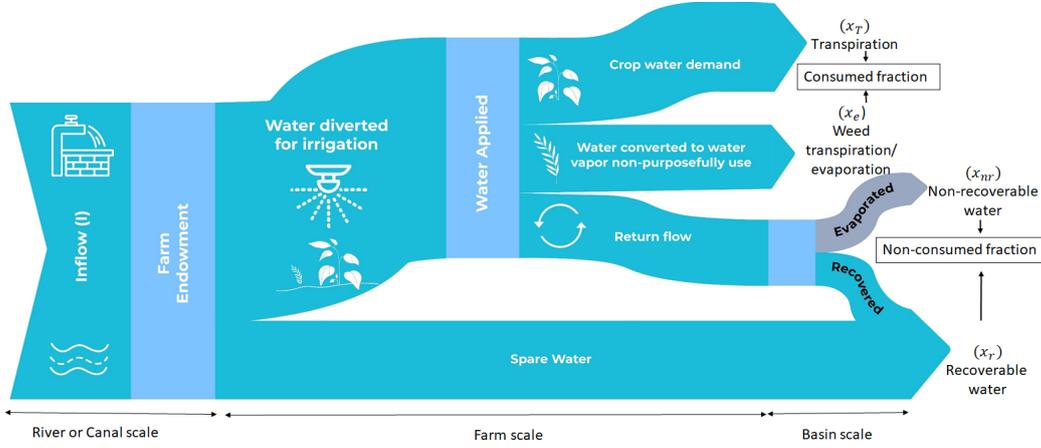


Figure 4. Water flow balance for irrigation water withdrawals used in our dynamical modeling framework.

253

254 2.3 The Social Sub-model

255 In the social sub-model, we model the rent-seeking behavior (explained in Section
 256 1) of the farmers towards the spare water ($I(\tau) - r_I(\tau)$) from the farm endowment, which
 257 is not used for irrigation. We represent the rate of crop water demand by $C(\tau)$ which
 258 is directly influenced by the rent-seeking behavior of the farmers as shown in Figure 3.
 259 We assume that the farmers see the spare water as a lost opportunity. Afterward, they
 260 increase their water consumption patterns in multiple ways as explained in Section 1 to
 261 gain maximum economic benefit. This change in consumption pattern increases the crop
 262 water demand. This effect can be represented mathematically as follows,

$$263 \quad \dot{C}(\tau) = \alpha (I(\tau) - r_I(\tau)), \quad (5)$$

265 where α is the sensitivity of the farmer's consumption to the quantity of spare water,
 266 in short, we called it the rent-seeking sensitivity. Furthermore, we assume that all farm-
 267 ers are rent-seekers (Renger & Wolf, 2000), therefore, we consider α to only take on posi-
 268 tive values. A higher value of α represents a farmer who is more sensitive, whereas its

269 value close to zero represents a farmer who is less sensitive in reallocating the spare wa-
 270 ter in order to gain maximum economic benefit.

271 The next part of the social sub-model connects the crop water demand $C(\tau)$ to the
 272 quantity of water applied for irrigation $r_I(\tau)$. This depends on the efficiency of the on-
 273 farm irrigation system. In the model, the IE, as defined in Section 1, is represented by
 274 a non-dimensional fraction E , which varies from zero (absolutely inefficient irrigation sys-
 275 tem) to one (absolutely efficient irrigation system). As shown in Figure 3, the irrigation
 276 water requirement is defined in the model as the ratio of crop water demand rate $C(\tau)$
 277 to the irrigation efficiency E , i.e. $C(\tau)/E$. We assume that the rate $r_I(\tau)$ at which the
 278 farmer diverts water for irrigation is exactly enough to supply water to the crops at rate
 279 $C(\tau)$, this can be given by $r_I(\tau) = C(\tau)/E$. This relationship represents that the farm-
 280 ers also consider the IE of the farm when they divert the water for irrigation, with high
 281 IE, they divert less water and vice versa. Furthermore, the crop water demand rate is
 282 equal to the rate at which the water is transpired by the crop, i.e., $C(\tau) = r_T(\tau)$ or $r_T(\tau) =$
 283 $r_I(\tau)E$. Using these relationships (5) can be written as follow,

$$284 \quad \dot{r}_T(\tau) = \frac{\alpha}{E}(EI(\tau) - r_T(\tau)). \quad (6)$$

286 Equation 6 represents the overall dynamics of the social sub-model. Next, we combine
 287 the hydrological sub-model with the social sub-model to form the coupled socio-hydrological
 288 model.

289 2.4 Coupled Socio-Hydrological Model

290 Together Equations 1 - 4 describe the overall dynamics of the hydrological sub-model
 291 and Equation 6 describes the dynamics of the social sub-model. Now we combine the
 292 hydrological sub-model based on the on-farm water flows with the social sub-model based
 293 on the rent-seeking behavior of the farmers. The coupled socio-hydrological model is given
 294 as follows,

$$295 \quad \begin{aligned} \dot{x}_T(\tau) &= r_T(\tau), \\ \dot{x}_e(\tau) &= c_e(1 - E)\frac{r_T(\tau)}{E}, \\ \dot{x}_r(\tau) &= I(\tau) - (1 - c_r(1 - c_e)(1 - E))\frac{r_T(\tau)}{E}, \\ \dot{x}_{nr}(\tau) &= (1 - c_e)(1 - c_r)(1 - E)\frac{r_T(\tau)}{E}, \\ \dot{r}_T(\tau) &= \frac{\alpha}{E}(EI(\tau) - r_T(\tau)), \end{aligned} \quad (7)$$

296 where $r_I(\tau)$ has been replaced with $r_T(\tau)/E$ based on the discussion in Section 2.3. The
 297 initial conditions, units, and all other mathematical details of the variables and param-
 298 eters are given in Table A1 of the Appendix. The coupled model given by (7) describes
 299 the overall dynamics of the on-farm irrigation system, which is shown in Figure 3. Mov-
 300 ing forward, we consider the following subsystem of equations, which give the dynam-
 301 ics of the crop transpiration rate and the stock of recoverable water as,
 302

$$303 \quad \begin{aligned} \dot{r}_T(\tau) &= \frac{\alpha}{E}(EI(\tau) - r_T(\tau)), \\ \dot{x}_r(\tau) &= I(\tau) - (1 - c_r(1 - c_e)(1 - E))\frac{r_T(\tau)}{E}. \end{aligned} \quad (8)$$

305 It is important to note here that while (8) captures only part of (7), the two variables
 306 $r_T(\tau)$ and $x_r(\tau)$ adequately capture the phenomenon of the IE paradox as we discuss
 307 in the following sections. Moreover, the water balance represented in Figure 3 can be used
 308 along with the initial conditions to capture the other stocks of (7) if required. In the next
 309 section, we present a transformation that reduces the parameter space and simplifies the
 310 analysis that follows.

311 2.5 Non-dimensionalized Model

312 In the preceding sections, we have defined the coupled socio-hydrological model,
313 which will be used later on to define the IE paradox. We now undertake some transfor-
314 mations, which reduces the overall dimensionality of the parameter space for (7).

315 Let $y(\tau)$ be the crop transpiration rate relative to the inflow $I(\tau)$ and $x(\tau)$ be the
316 recoverable stock of water relative to the volume $I(\tau)/\alpha$, i.e., $y(\tau) = r_T(\tau)/I(\tau)$ and
317 $x(\tau) = x_r(\tau)\alpha/I(\tau)$. Next define $t = \alpha\tau$ as the new, non-dimensional time. We now
318 arrive at the non-dimensional model as follow, (see Section A1 for complete derivation)

$$\begin{aligned} \dot{y}(t) &= 1 - \frac{y(t)}{E}, \\ \dot{x}(t) &= 1 - \left(1 - \beta\left(\frac{1}{E} - 1\right)\right)y(t), \end{aligned} \quad (9)$$

321 where the overdot represents the derivative with respect to t . Here $\beta = c_r(1 - c_e)$ and
322 E are only the parameters of the model and both vary from zero to one. We call β the
323 Physical Coefficient, which captures the physical characteristics of the basin. The value
324 of β close to zero represents an irrigation system with high evaporation and low recov-
325 ery, whereas its value close to one represents an irrigation system with high recovery and
326 low evaporation. All subsequent analysis in this paper will be carried out on model (9).

327 2.6 Occurrence of the Irrigation Efficiency Paradox in the Model

328 In order to define the IE paradox, we first describe how we represent an improve-
329 ment in IE in our model. Assume a farm with irrigation efficiency E_1 situated in a basin
330 with physical coefficient β . By solving (9) (see Section A2 for derivation), with zero ini-
331 tial conditions, we get the progression of the contribution of the farm to the stock of re-
332 coverable water as,

$$333 \quad x_1(t) = E_1(1 - \beta(1 - E_1))(1 - e^{-\frac{t}{E_1}}) + \beta(1 - E_1)t, \quad (10)$$

335 where e represents Euler's number, now assume another farm named WITFarm (the name
336 WITFarm is based on The Center for Water Informatics & Technology (WIT)) with ir-
337 rigation efficiency E_2 , which is also situated in the same basin with physical coefficient
338 β . We further assume that the progressive owner of WITFarm upgrades the irrigation
339 system to a technology that is more efficient than the farm with irrigation efficiency E_1 ,
340 so that $E_2 > E_1$. The contribution of WITFarm to the stock of recoverable water is
341 given as,

$$342 \quad x_2(t) = E_2(1 - \beta(1 - E_2))(1 - e^{-\frac{t}{E_2}}) + \beta(1 - E_2)t. \quad (11)$$

344 Now we define the IE paradox in the context of our model. Consider the difference be-
345 tween the stock of recoverable water, $\Delta x(t) = x_1(t) - x_2(t)$, which represents a com-
346 parison between the contribution to recoverable water stock from both farms. In order
347 to ensure a fair comparison, the initial conditions and parameters (apart from the IE's)
348 are assumed to be equal for both scenarios. Thus the comparison for both scenarios is
349 made *ceteris paribus*. We consider an IE paradox to occur if an increase in IE results
350 in a decrease in the quantity of recoverable water. Thus a negative value of $\Delta x(t)$ rep-
351 represents that WITFarm contributes more to the recoverable water stock with an efficient
352 irrigation system, which indicates a non-paradoxical outcome. On the other hand, a pos-
353 itive value of $\Delta x(t)$ represents the contribution of the farm with efficiency E_1 to recov-
354 erable water is higher than WITFarm, which indicates a paradox. Next, we present the
355 characteristics of the IE paradox by qualitatively analyzing the graphical form of $\Delta x(t)$.
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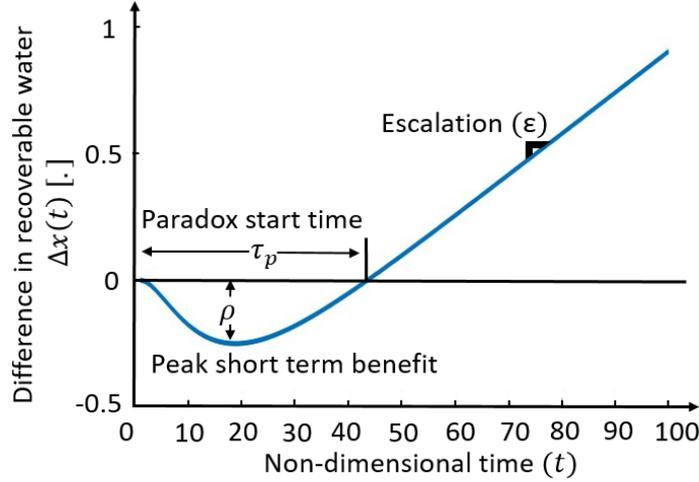


Figure 5. The difference in recoverable water, i.e., $\Delta x(t) = x_1(t) - x_2(t)$, where $x_1(t)$ and $x_2(t)$ are the stocks of recoverable water of an ordinary farm is operating at IE = 35%, and WIT-Farm is operating at IE = 100% respectively. In this case, we assumed both farms are situated in the same basin. Therefore, the value of $\beta = 0.25$ is taken to be equal in both farms. The negative value of $\Delta x(t)$ indicates the non-paradoxical outcome, whereas its positive value indicates a paradox.

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2.7 Characteristics of the IE Paradox

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A plot of the difference in recoverable water $\Delta x(t) = x_1 - x_2$ against time for a selected set of parameter values is shown in Figure 5 (the plot of x_1 and x_2 is also shown in Figure A1). As previously discussed, a positive value of $\Delta x(t)$ indicates a paradox. We observe that whenever a paradox appears, it is preceded by an apparent uplift in recoverable water indicated by a negative value of $\Delta x(t)$. This uplift represents the short-term benefits of the adoption of an improved irrigation system. The graph of $\Delta x(t)$ shows that this benefit exists for a short interval of time, after which the paradox starts. We call this time interval the paradox start time τ_p . We further observe that the short term benefit reaches a maximum value before the advent of the paradox. We call the magnitude of this maximum value the peak short term benefit ρ . The final parameter is the escalation ε of the paradox, which is basically the slope of $\Delta x(t)$ after the paradox occurs. The escalation indicates how fast the rebound accumulates after the short-term benefits vanish. With the combination of the three paradox's parameters, τ_p , ρ , and ε , the paradox is characterized completely.

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We consider the effect of an on-farm IE improvement on the stock of recoverable water at the basin scale. The deceptive uplift in the basin's recoverable water level and the eventual occurrence of the paradox can be explained by examining the dynamics of the model in Figure 3. An increase in irrigation efficiency decreases the water used on-farm for irrigation, which leads to an increase in the stock of recoverable water. This effect occurs in the short term benefits duration. However, the water that is saved by the increased efficiency activates the rent-seeking behavior of the farmer, who then utilizes the saved water to generate additional economic benefit by the means described in Section 1. This leads to even more consumption of water, the dynamics of which are described by Equation 6.

382 In the simulations presented in the next section, we explore the effect of varying
 383 the different parameters of the socio-hydrological model on the paradox characteristics
 384 just described.

385 **3 Results**

386 In Section 2.6 we defined the occurrence of the IE paradox in our model, based on
 387 the comparison of two irrigation systems having different efficiencies, i.e., E_1 and E_2 . In
 388 Section 2.7 we characterized the IE paradox as a combination of the paradox param-
 389 eters, τ_p , ρ and ε . The value of these parameters depends on the physical coefficient β and
 390 the irrigation efficiencies E_1 and E_2 . To explore the sensitivity of these parameters to
 391 the aforementioned variables, we present exhaustive simulations over the range of β and
 392 IE's. First, we present the effect of τ_p , ρ , and ε on the nature of the paradox. Finally,
 393 we apply our modeling framework over global data sets to distinguish the global regions
 394 where the IE paradox is more or less articulated.

395 **3.1 Sensitivity of the Paradox to Physical & Technological Attributes**

396 Here we investigate the effect of the physical parameters on the IE paradox's ex-
 397 tent through exhaustive simulations over the parameters' range. Before moving on, it
 398 is essential to reflect upon the role of τ_p , ρ , and ε on the nature of the paradox. τ_p and
 399 ε are straightforward to interpret. A higher value for the paradox start time (the dura-
 400 tion of the benefits) τ_p is beneficial because it delays the paradox's occurrence by extend-
 401 ing the period over which a net-increment in recoverable water is enjoyed. Next, as men-
 402 tioned earlier, ε indicates how fastly the rebound accumulates after the short-term ben-
 403 efits vanish. A lower value for ε is beneficial because it attenuates the paradox, there-
 404 fore, it is desirable. In contrast, the peak short-term benefit ρ carries diverse interpre-
 405 tations. As it represents the maximum benefit gained through the improvement in ir-
 406 rigation efficiency, it would seem that a higher value is more desirable from the perspec-
 407 tive of a central planner. However, whether or not a higher value for ρ is overall ben-
 408 efiticial or detrimental is a subjective matter and depends upon the policy maker's inten-
 409 tion. The short term benefits duration may be interpreted as a window during which ei-
 410 ther the technology must further be improved or other system parameters are contin-
 411 uously influenced to extend the uplift duration. Suppose the planning horizon is less than
 412 or equal to this duration. In that case, a higher value for ρ is beneficial since it repre-
 413 sents the maximum benefit obtained from the increment in efficiency (the conclusions
 414 that follow are interpreted in light of this perspective).

415 Figures 6 shows the results of simulating the values of τ_p , ρ , and ε by varying the
 416 parameters given in (9) over their range, which is shown in Table A1 of the Appendix.
 417 Our findings are discussed below.

418 **3.1.1 Basins with High Recovery and Low Evaporation are Most Effected** 419 **by the Paradox**

420 From the definition of the physical coefficient β , an increase in β reflects an increase
 421 in recovery and/or a decrease in evaporation. In the simulations presented in Figure 6,
 422 we observe that an increase in β (see panels from left ($\beta = 0.1$) to right ($\beta = 0.9$) of
 423 Figure 6), is associated with a decrease in the paradox start time τ_p (see Figure 6a), a
 424 decrease in the peak short term benefit ρ (see Figure 6b), and an increase in the esca-
 425 lation ε (see Figure 6c) of the paradox. This can also be seen in the context of our model.
 426 For example, from the expression (A9) of difference in recoverable water stock for the
 427 two IE systems, one may investigate that an increase in β , reduces the contribution of
 428 the exponential terms and increases the contribution of other terms involved in the ex-
 429 pressions, which reduce ρ , τ_p , and increase ε . In Figure 6c after inspecting the panels

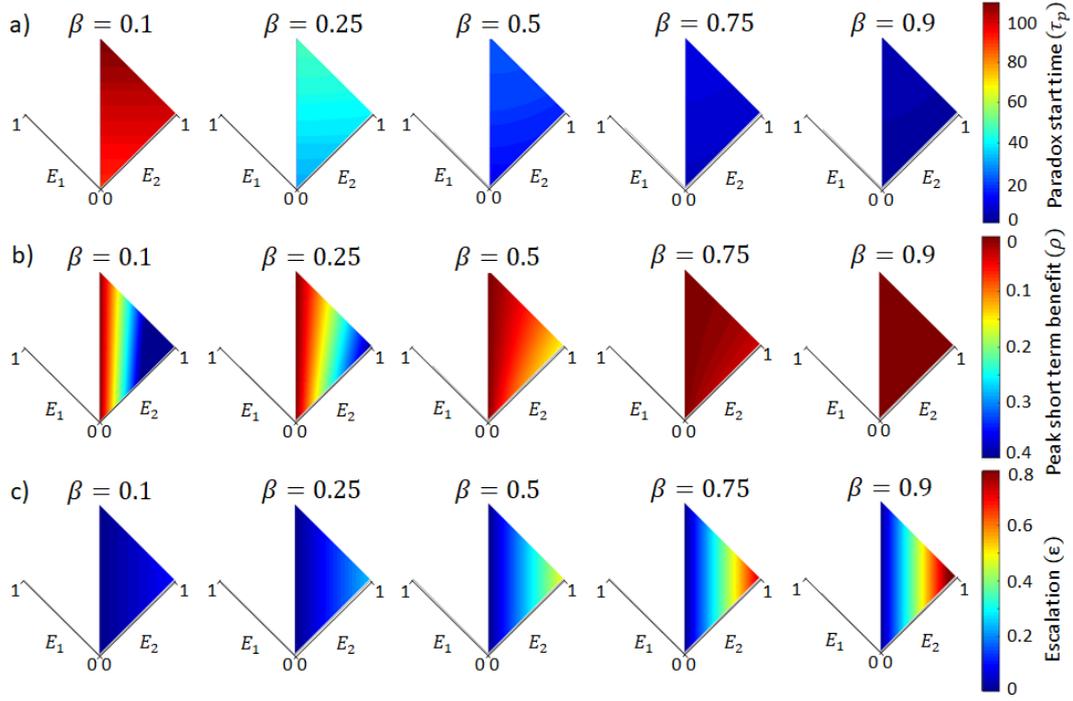


Figure 6. The detailed numerical simulation shows the evolution of the paradox parameters, depicted in Figure 5. In this figure, a) the paradox start time τ_p , b) the magnitude of peak short term benefit ρ , and c) the paradox’s escalation. Here we vary IEs over their possible ranges and vary β for different values within its range. By definition E_2 must always be greater than E_1 , therefore we plot only the region where $E_2 > E_1$. For completeness, the E_1 and E_2 axes have also been shown in each plot.

430 from left to right (increasing value of β), we observe that an increase in ε , which depict
 431 an effect that is also observable from analytical expression (A10). However, as discussed
 432 in Section 3.1, a lower value for τ_p , a higher value for ε , and a lower magnitude of ρ col-
 433 lectively characterize a more pronounced instance of the paradox. Therefore, to atten-
 434 uate the paradox, we need to reduce β , which is possible with a decrease in the recov-
 435 ery and/or increase in the evaporation. It is important to note here that this result does
 436 not imply that a decrease in recovery or increase in evaporation will reduce the stock of
 437 recoverable water for any given basin. Indeed as the flows in Figure 1 illustrates, it is
 438 the opposite, i.e., an increase in c_r or decrease in c_e increase the rate of inflow to the re-
 439 coverable water stock. Rather, the result presented here only applies to the case of an
 440 efficiency improvement and conveys that after the improvement is made, the paradox
 441 is expected to be more intense in areas with the properties discussed above.

442 **3.1.2 Higher Advancement in IE Increases Short Term Benefit but Leads**
 443 **to a Faster Escalating Paradox**

444 Figure 6 allows us to observe the paradox characteristics’ dependence on the nature of
 445 the IEs improvement. We observe that an improvement in IE (when we move hori-
 446 zontally in the direction of E_2 , in all panels of Figure 6, we observe a decrease in E_1 and
 447 increase in E_2 , which collectively increases the magnitude of $E_2 - E_1$), leads to a rise
 448 in ρ (Figure 6b) and ε (Figure 6c). In contrast, the improvement in IE does not directly
 449 influence τ_p , because after inspecting Figure 6a, we observe that the value of ρ is hori-
 450 zontally constant (see first panel ($\beta = 0.1$) of Figure 6a, in which the horizontal con-

tours are more observable comparatively) along with the increasing magnitude of $E_2 - E_1$, which illustrates that once the paradox occurs, the decrease in existing efficiency E_1 in conjunction with an increase in improved efficiency E_2 will produce no effect on it. On the other hand, from the same figure, τ_p is increasing vertically with the constant magnitude of $E_2 - E_1$ (vertically, E_1 and E_2 both increasing with the same aggregate), which illustrate that if the initial efficiency E_1 is higher, the same improvement in E_2 leads to a delayed occurrence of paradox (a higher value for τ_p).

All of the foregoing arguments show that the paradox occurs later in a basin with high prior IE. In contrast, in the basin operating with low initial efficiency E_1 , the paradox occurs sooner associated with high peak short term benefit ρ and slow escalation ε . However, if the improvement in IE is temporary, then the higher value for ρ is beneficial at the basin scale, but in the long-run, the paradoxical effect will surely outweigh the short term benefit, and the improvement in the IEs escalates the paradoxical outcome, which is not desirable. It can be explained in a way that an increase in efficiency results in savings in water. These savings trigger further activities that consume the saved water. The more increase there is in efficiency, the more savings there are to trigger even more consuming activities.

Indications of the proportionality between the extent of efficiency improvement and the magnitude of the resulting paradox can be observed from the common narrative in rebound literature (Grafton et al., 2018; Pérez-Blanco et al., 2020; Pfeiffer & Lin, 2014; Ward & Pulido-Velazquez, 2008; Berbel et al., 2018; Camacho Poyato et al., 2011). Relevant literature on rebound effects (the energy sector is most examined) supports the conclusion that technological rebound effects are more pronounced in underdeveloped and developing communities as compared to developed ones. The reason for this is that in developed communities, the technology in question is already so prevalent that even after improvement in efficiency, there is not much potential for intensification. However, this margin is much more in underdeveloped and developing communities. The authors in (Chitnis et al., 2014) carry out a comparative study of energy-efficient improvements in household consumption in the UK and observe that the rebound effect is most extensive in households with low income. A consistent conclusion is also drawn in (Murray, 2013) where the author studies rebound effects of efficiency improvements in electricity and fuel consumption technologies in Australia. Our model predicts that the same effect can be expected for IE enhancing technologies as well.

3.2 Application to Global Data Sets

We now interpret our findings to distinguish the world regions where the irrigation efficiency paradox is more or less articulated. Using the ArcGIS software (*ArcGIS*, 2020) we produce the global maps required for the IE paradox analysis. In order to compute the parameters of paradox represented in Section (2.7), our model given in (9) required the pixel-wise value of irrigation efficiency (IE) and the physical coefficient β . For the comparison to identify the paradox as discussed in Section 2.6, we assume that the improved irrigation efficiency $E_2 = 100\%$, and E_1 is the actual values on the map. The country-wise aggregated irrigation efficiency E_1 map is shown in Figure 7a, and the data is taken from (Jägermeyr, 2017).

The physical coefficient $\beta = c_r(1 - c_e)$ can be calculated by assessing the magnitude of evaporation and recovery coefficients c_e and c_r respectively. As discussed previously, c_e is the ratio of evaporated water to the total incident water. Therefore, we first find the global evaporation potential, the data set is taken from (*GLEAM*, 2019). Afterward, we consider the total incident water as the sum of irrigation water applied and the amount of precipitation at the same point on the map, (the irrigation water applied and the precipitation data are taken from (*AQUASTAT*, 1993-2017) and (Worldclim, 2020) respectively. Next, we compute the ratio of evaporated water to the total incident

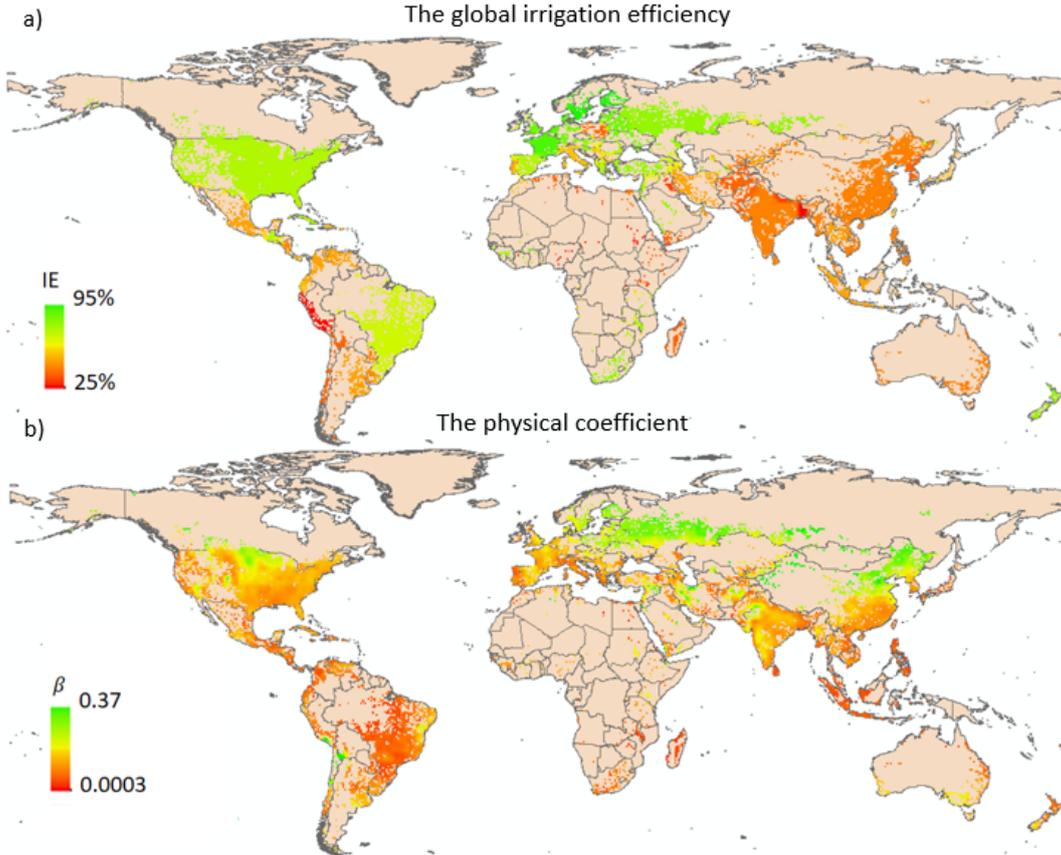


Figure 7. The maps of global data sets: (a) global irrigation efficiency and the data is taken from (Jägermeyr, 2017), (b) physical coefficient $\beta = c_r(1 - c_e)$.

502 water, which provides c_e . Similarly, we find the recovery coefficient c_r , which is defined
 503 as the ratio of recharged water to the total water applied (see Section 2.2). In order to
 504 find c_r we assessed the groundwater recharge potential by using the data set from (Mohan
 505 et al., 2018). In this case, the total water applied is equal to the difference between the
 506 total water applied for irrigation and the water goes in evaporation. After computing
 507 the required parameters globally, we identify the IE paradox characteristics explained
 508 in section 2.7. After incorporating the parameters in the model (9) and the model out-
 509 puts are shown in Figures 8, Our findings from this figure are described below.

510 Figure 8a shows the paradox start time τ_p (as previously discussed, the time indi-
 511 cated here is the non-dimensional time). The color in red represents the regions where
 512 paradox starts quickly. If we compare these regions with the IE map (Figure 7a), we ob-
 513 serve that in these regions, the IE is relatively high, for example, in North Europe. How-
 514 ever, in northeast China, the IE is at a moderate level (40%), but the paradox can oc-
 515 cur quickly. For the reason that the physical coefficient β (Figure 7b) are high in north-
 516 east China, this reflects the fact that the IE and β collectively affect the paradox occur-
 517 rence. In Figure 6a we concluded that the low value of β delays the paradox. This can
 518 be seen in Figure 7b, for example, in Malaysia, Singapore, and other neighboring regions,
 519 the value of β is low, and in the same regions, the paradox start time is high compar-
 520 atively. This result is in line with the arguments we build in Section 3.

521 Figure 8b shows the peak short term benefit ρ . We have categorized the benefits
 522 in terms of the peak magnitude (high represents the more negative value). We observe

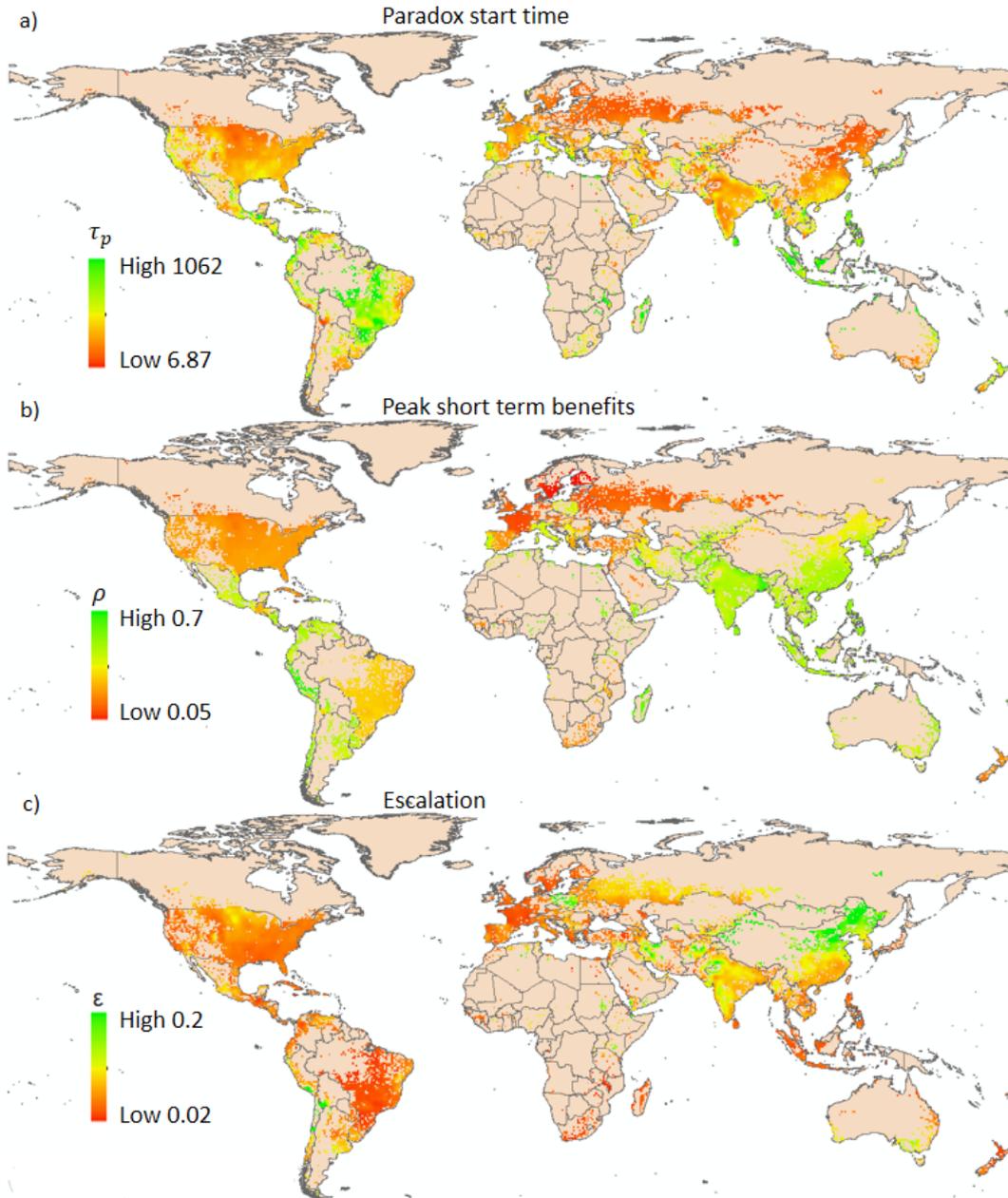


Figure 8. The global paradox parameters of the IEP. a) shows the paradox start time τ_p , b) shows the peak short term benefits ρ , and c) shows the escalation of the paradox ε .

523 that the high value of ρ occurs where the IE is previously at a low level. For example,
 524 in Nepal, Bangladesh, Iraq, Sudan, Nigeria, and Peru, the IE (see Figure 7a for IE val-
 525 ues) is at a low level, and the peak short term benefits are high. Moreover, in North Eu-
 526 rope (especially Finland and Sweden), where the IE already at high (90%), IE's improve-
 527 ment is not significant; therefore, we observe a low magnitude of ρ . As we already dis-
 528 cussed in Section 3.1, whether a high value of ρ is overall beneficial or not is a subjec-
 529 tive matter and depends upon the intention of the policymaker. On the other hand, β
 530 also influences the magnitude of ρ . A high value of β reduces the magnitude of ρ in most
 531 of the regions, which can also be observed in Figure 6b.

532 Figure 8c shows the escalation of the paradox ε . The green color represents the re-
 533 gions where paradox escalates relatively faster, including northeast and northwest China,
 534 Mongolia, Poland, the western part of India, the eastern part of Pakistan, the central
 535 part of Afghanistan, western Iran, South Peru, Bolivia, and Sudan. If we compare with
 536 the IE map (Figure 7a), we observe that the prior IE are relatively low, and after com-
 537 paring with β , we find that β is relatively high in all mentioned regions. These obser-
 538 vations reflect that whenever the paradox occurs in the regions with low prior IE and
 539 high recovery (high β means high recovery), it escalates fastly. From the low prior IE
 540 point of view, Nepal and Bangladesh, and some other neighboring regions also have low
 541 prior IE, but ε is not high. The reason is that β is relatively low in these regions. It pro-
 542 vides evidence that the low prior IE is not only escalating the paradox, a high value of
 543 β is also required. In light of the above findings, one may identify the regions vulner-
 544 able to the IE paradox.

545 4 Discussion and Conclusions

546 In this study, we have presented a socio-hydrological model of on-farm irrigation
 547 that couples the farm's water flow dynamics with the rent-seeking behavior of the irri-
 548 gating farmers. The model, given by a system of linear differential equations, incorpo-
 549 rates the efficiency of the on-farm irrigation system and physical characteristics describ-
 550 ing the water balance of the farmland. During the formulation of the water balance, it
 551 has been assumed that all of the water available for irrigation is lumped into the single
 552 variable of farm endowment from which farmers may divert water for irrigation at will.
 553 This does not cover irrigation inputs that enter through rainfall. While precipitation may
 554 be incorporated as a separate input to the water applied for irrigation (see Figure 3),
 555 we do not include it in the current study as it simplifies the analysis and does not pro-
 556 foundly affect our conclusions regarding the IE paradox. In the water balance, we also
 557 assume that all water not extracted from the farm endowment contributes entirely to-
 558 wards the stock of recoverable water. While this holds reasonably well for groundwater,
 559 it does not necessarily hold true for surface water deliveries, which may be lost to evap-
 560 oration, percolate to unrecoverable sinks, or simply flow down into the sea (Staatz, 1989).
 561 Another simplifying assumption is the fixed tendency of the farmers to completely uti-
 562 lize the water flowing into the farm towards generating maximum economic benefit (rent-
 563 seeking behavior). However, real-world farmers may exhibit cooperative (rent-free) be-
 564 havior, in which case the rent-seeking sensitivity could take on negative values (Kimball,
 565 1988; Staatz, 1989). We leave it for a future study to consider the impact of mixed farmer
 566 populations on the model and the resulting paradox.

567 The nature of the model allows a quantifiable comparison of the contribution of
 568 two irrigation fields, operating at different irrigation efficiencies, to the stock of recov-
 569 erable water at basin-scale. The difference in the contribution of the two farms is used
 570 to define the IE paradox, with a paradox defined to occur if the farm with higher IE con-
 571 tributes less than the farm with lower IE. We qualitatively characterize the paradox through
 572 three parameters: The paradox start time, the peak short-term benefit and the paradox
 573 escalation. An exhaustive numerical simulation has been conducted to study the sensi-
 574 tivity of these paradox parameters to the other parameters of the model. Our finding
 575 that larger improvements in IE, while yielding higher benefits in the short-term, leads
 576 to a faster escalating paradox is intuitive and also consistent with what has been observed
 577 in real-world programs of efficiency improvement in other applications (Murray, 2013;
 578 Chitnis et al., 2014). Our second finding is that more articulated paradoxes are expected
 579 to occur in basins with lower recovery and higher evaporation. Although this finding may
 580 seem to promote the decrease of recovery (for instance, by lining canals) and increase
 581 in evaporation (for instance, by discouraging canopy cover on streams), we must empha-
 582 size here that this is not the conclusion that should be drawn and our model certainly
 583 does not support it. In fact, the flow of Figure 3 illustrates that for fixed on-farm effi-

584 ciency, water availability is actually increased with a rise in recovery or a fall in evap-
 585 oration. The result referred to above holds only when an improvement in irrigation ef-
 586 ficiency is made. Therefore this should not at all be perceived as a prescription to de-
 587 crease recovery or increase evaporation in the basin. Rather our results only suggest that
 588 caution must be practiced while upgrading irrigation efficiency in regions with such char-
 589 acteristics since the subsequent paradox is expected to be relatively more pronounced
 590 as compared to other regions.

591 Finally, we have applied the results of our model to global datasets to highlight re-
 592 gions with prominent paradox characteristics. For instance, we find that in regions with
 593 efficient irrigation technologies already in place, i.e., most of northern Europe, USA, Canada,
 594 Russia, and other regions (Figure 7a), an even further increase in efficiency is not im-
 595 mediately expected to lead to a paradox (Figure 8a). On the other hand, in basins with
 596 relatively inefficient irrigation systems, for instance, Nepal, Bangladesh, Iraq, Sudan, Nige-
 597 ria, and Peru, to name a few, an upgrade in efficiency is predicted to lead to higher short
 598 term benefits (Figure 8b) followed by a faster escalating paradox (Figure 8c). We de-
 599 liberately avoid combining the three paradox parameters in a single measure of paradox
 600 intensity since all parameters hold different implications for policy-making and must be
 601 interpreted in light of the relevant context independently from one another.

602 Appendix A

603 A1 The Non-dimensionalization of the Model

604 The transformations of the non-dimensionalization are shown in Section 9. Here
 605 we present the complete derivation. Let us rewrite the model given in (8) as follows,

$$606 \begin{aligned} \dot{r}_T(\tau) &= \frac{\alpha}{E}(EI(\tau) - r_T(\tau)), \\ \dot{x}_r(\tau) &= I(\tau) - (1 - c_r(1 - c_e)(1 - E))\frac{r_T(\tau)}{E}. \end{aligned} \quad (A1)$$

607 Let $y(\tau)$ be the crop transpiration rate relative to the inflow $I(\tau)$ at time τ , i.e. $y(\tau) =$
 608 $r_T(\tau)/I(\tau)$, since $r_T(\tau)$ and $I(\tau)$ both are the rates, therefore, the $r_T(\tau)$ in term of non-
 609 dimensional $y(\tau)$ can be written as follows,
 610

$$611 \dot{y}(\tau) = \alpha\left(1 - \frac{y(\tau)}{E}\right). \quad (A2)$$

612 Let $x(\tau)$ be the recoverable stock relative to the volume of water, i.e. $x(\tau) = x_r\alpha/I(\tau)$,
 613 where α is a time constant with units of inverse time, therefore, $\alpha/I(\tau)$ represents in-
 614 verse volume. The dynamics of $x(\tau)$ can be written as follows,
 615

$$616 \dot{x}(\tau) = \alpha\left(1 - \left(1 - \beta\left(\frac{1}{E} - 1\right)\right)y(\tau)\right). \quad (A3)$$

617 Next define $t = \alpha\tau$ as the new, non-dimensional time, then, $\frac{d}{d\tau} = \alpha\frac{d}{dt}$, thus with re-
 618 spect to the non-dimensional time t , the dynamics of (A2) and (A3) can be written as
 619 follows,
 620

$$621 \begin{aligned} \dot{y}(t) &= 1 - \frac{y(t)}{E}, \\ \dot{x}(t) &= 1 - \left(1 - \beta\left(\frac{1}{E} - 1\right)\right)y(t). \end{aligned} \quad (A4)$$

623 A2 Solution of the Model

624 The solution of the system (A4) can be computed by writing it in one second-order
 625 system as follows,

$$626 \ddot{x}(t) + \frac{1}{E}\dot{x}(t) = \beta\left(\frac{1}{E} - 1\right), \quad (A5)$$

627

628 where,

$$629 \quad \dot{x}(0) = 1 - \left(\frac{1}{E} - \beta\left(\frac{1}{E} - 1\right)\right)y(0). \quad 630$$

631 The solution of system (A5) can be written as follows,

$$632 \quad x(t) = x(0) + E(\dot{x}(0) - \beta(1 - E))(1 - e^{-\frac{t}{E}}) + \beta(1 - E)t. \quad (A6) \quad 633$$

634 If the initial conditions are zero then,

$$635 \quad x(t) = E(1 - \beta(1 - E))(1 - e^{-\frac{t}{E}}) + \beta(1 - E)t. \quad (A7) \quad 636$$

637 A3 The Comparison of Two Irrigation Systems

638 Equation A7 represents the recoverable water stock at time t . Next, we find the
639 stock of recoverable water for two irrigation systems with different IEs, i.e., E_1 and E_2 ,
640 with the same physical coefficient β , which can be written as follows,

$$641 \quad \begin{aligned} x_1(t) &= E_1(1 - \beta(1 - E_1))(1 - e^{-\frac{t}{E_1}}) + \beta(1 - E_1)t, \\ x_2(t) &= E_2(1 - \beta(1 - E_2))(1 - e^{-\frac{t}{E_2}}) + \beta(1 - E_2)t, \end{aligned} \quad (A8) \quad 642$$

where, $x_1(t)$ is the stock of recoverable water of an ordinary farm operating at irriga-

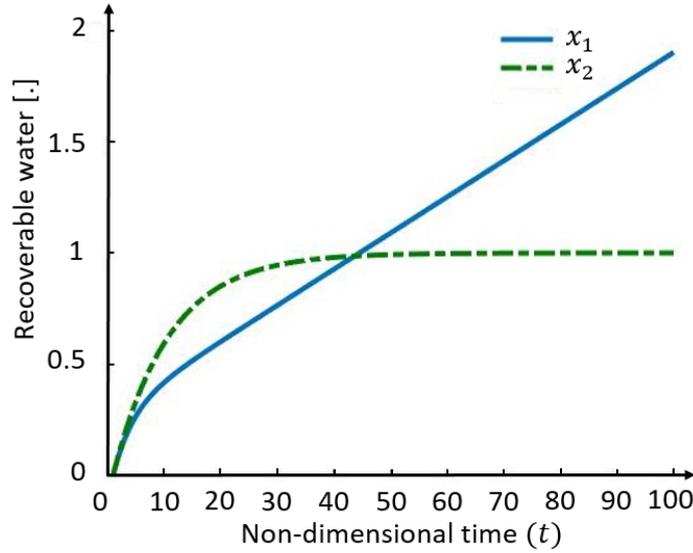


Figure A1. The stock of recoverable water of two irrigation farms operating at different IEs. x_1 is the stock of recoverable water of an ordinary farm operating at IE = 35% whereas x_2 is the stock of recoverable water of WITFarm operating at IE = 100%. In this case, we assumed both farms are situated at the same basin, therefore, the value of $\beta = 0.25$, is taken to be equal in both farms.

643 tion efficiency E_1 , whereas $x_2(t)$ is the stock of recoverable water of WITFarm operat-
644 ing at irrigation efficiency E_2 . Figure A1 shows the accumulation of recoverable water
645 stock of both irrigation farms. Next, by defining the change in recoverable water stock
646 by $\Delta x(t) = x_1(t) - x_2(t)$, this can be written as follows,
647

$$648 \quad \begin{aligned} \Delta x(t) &= E_1 - E_2 - \beta(E_1(1 - E_1) - E_2(1 - E_2)) + \beta(E_2 - E_1)t \\ &\quad - E_1(1 - \beta(1 - E_1))e^{-\frac{t}{E_1}} + E_2(1 - \beta(1 - E_2))e^{-\frac{t}{E_2}}. \end{aligned} \quad (A9) \quad 649$$

650 After inspecting and simulating the expression (A9), we observed that the constants and
 651 the exponential terms involve in the expression influenced the transient behavior of the
 652 system, whereas on the steady-state ($t \rightarrow \infty$) the term $\beta(E_2 - E_1)t$ drive the trajec-
 653 tory of the system. Therefore, we consider the escalation of the paradox is given by

$$654 \quad \varepsilon = \beta(E_2 - E_1). \quad (A10)$$

656 A4 Description of Variables & Parameters

657 Table A1 lists the variables and parameters of the dynamical system (7). Their units,
 658 initial conditions (ICs), ranges and the values used in simulations of Section (2.7, 3) also
 659 presented. The description of parameters, variables, and specific equations that the vari-
 ables and parameters refer to are also presented.

Table A1. Variables and parameters of the dynamical system 7 and the initial conditions (ICs) used in Section 2.7 and 3. Where $[L^3]$ represent the volume, $[LT^{-1}]$ represent the rate, and $[.]$ represent the non-dimensional units.

	Units	Description	Eq.	Range	ICs
x_T	$[L^3]$	beneficially consumption	1	$[0, \infty)$	0
x_e	$[L^3]$	evaporation and weed transpiration	2	$[0, \infty)$	0
x_{nr}	$[L^3]$	water goes to deep aquifer	4	$[0, \infty)$	0
x_r	$[L^3]$	recoverable water	3	$[0, \infty)$	0
α	$[LT^{-1}]$	rent-seeking parameter	5	$[0, \infty)$	0
E	$[.]$	irrigation efficiency	6,3	$[0, 1]$	35%
C	$[LT^{-1}]$	crop water demand	5	$[0, \infty)$	0
I	$[LT^{-1}]$	total inflows rate to a farm	5,6,3	$[0, \infty)$	0
r_I	$[LT^{-1}]$	water applied for irrigation rate	5	$[0, \infty)$	0
r_T	$[LT^{-1}]$	plant transpiration rate	6	$[0, \infty)$	0
c_e	$[.]$	evaporation coefficient	3	$[0, 1]$	-
c_r	$[.]$	recovery coefficient	3	$[0, 1]$	-
x_1	$[L^3]$	recoverable water for IE level E_1	10	$[0, \infty)$	0
x_2	$[L^3]$	recoverable water for IE level E_2	11	$[0, \infty)$	0
x	$[.]$	recoverable water relative to inflow	9, A4	$[0, \infty)$	0
y	$[.]$	beneficial consumption relative to inflow	9, A4	$[0, \infty)$	0
β	$[.]$	physical coefficient	9, A4	$[0, 1]$	0.25

660

661 A5 Global data sets

662 The IE data is taken from (Jägermeyr, 2017), the water withdrawals for the irri-
 663 gation sector is taken from (*AQUASTAT*, 1993-2017). The global precipitation, evap-
 664 oration, and groundwater recharge potential datasets set are taken from (Worldclim, 2020),
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