# Instabilities in the bottom boundary layer reduce boundary layer arrest, allowing cross-isobath spread of downwave flows and ventilating the boundary layer

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#### Abstract

An along-isobath current in stratified waters leads to a bottom boundary layer. In models with no alongshore variation, crossisobath density transport in this bottom boundary layer reduce the velocity in the bottom boundary layer via thermal wind, and thus the bottom friction experienced by the current above the boundary layer – this is bottom-boundary-layer arrest.

If, however, alongshore variation of the flow is allowed, the bottom boundary layer is baroclinically unstable. We show with high resolution numerical models that these instabilities reduce this arrest and allow bottom friction to decelerate the flow above the bottom boundary layer when the flow is in the Kelvin wave direction (so that the bottom Ekman transport is downwelling). Both the arrest of the bottom boundary layer and the release from this arrest are asymmetric; the friction experienced by flows in the direction of Kelvin-wave propagation (downwave) is much greater than flows in the opposite direction.

The strength of the near bottom currents, and thus the magnitude of bottom friction, is found to be governed by the destruction of potential vorticity near the bottom balanced by the offshore along-isopycnal transport of this anomalous potential vorticity. A simple model of this process is created and used to quantify the magnitude of this effect and the resulting reduction of arrest of the bottom boundary layer. It is shown that the instabilities allow along-isobath flows to spread across isobaths and move boluses of weakly stratified bottom water into the stratified interior.

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2	cross-isobath spread of downwave flows and ventilating the boundary layer.			
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9	2/4/2021			
10	Key points:			
11	1. Instabilities in the bottom boundary layer prevent the arrest of the bottom boundary layer for flows in the Kelvin wave direction			
12	<ol> <li>The prevention of bottom boundary layer arrest allows along-isobath flows to cross-isobaths, and</li> </ol>			
14 15	allows slope flows to more effectively drive shelf flows. Instabilities in the bottom boundary layer transport water from the bottom boundary layer into the			
15 16	stratified interior along isopycnals for flows in the Kelvin wave direction.			
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26 If, however, alongshore variation of the flow is allowed, the bottom boundary layer is baroclinically 27 unstable. We show with high resolution numerical models that these instabilities reduce this arrest and 28 allow bottom friction to decelerate the flow above the bottom boundary layer when the flow is in the 29 Kelvin wave direction (so that the bottom Ekman transport is downwelling). Both the arrest of the bottom 30 boundary layer and the release from this arrest are asymmetric; the friction experienced by flows in the

31 direction of Kelvin-wave propagation (downwave) is much greater than flows in the opposite direction.

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34 isopycnal transport of this anomalous potential vorticity. A simple model of this process is created and

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36 layer. It is shown that the instabilities allow along-isobath flows to spread across isobaths and move

37 boluses of weakly stratified bottom water into the stratified interior.

38 **Plain Language Summary:** It has long been thought that alongshore shore flows over the shelf and slope 39 will, over time, change the cross-shelf distribution of salt and temperature near the bottom such that the 40 flow at the bottom is small. This reduction in near-bottom flow reduces the friction between the 41 alongshore flows and the bottom, in theory allowing the water to flow forever along the coast. This work 42 shows, instead, that this cessation of flow near the bottom does not happen. Building on existing 43 understanding of how the near bottom flow can break up into eddies, this work quantifies how much 44 friction the alongshore flow feels, and finds it is greater than had been understood when the flow has the 45 coast on its right in the Northern Hemisphere, and on its left in the southern Hemisphere. The greater 46 friction, and the eddies created near the bottom, allow the deep ocean flows near the coastal ocean to 47 drive alongshore flow on the shallow coastal ocean, and also take fluid from near the bottom and inject it into the deep ocean. Both of these effects couple the deep ocean to the shelf. 48

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50 Keywords: Bottom boundary layer, Baroclinic Instabilities, Boundary Layer Arrest

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- 52

#### 53 1 Introduction

54 Garrett et al. (1993) and the references therein describe how an alongshore uniform along-isobath flow

55 over a sloping bottom in a rotating stratified fluid leads to cross-isobath density transport. This transport

56 would set up a cross-isobath buoyancy gradient which would lead to vanishing flows at the bottom –

57 bottom boundary layer (BBL) arrest. The arrest of the near-bottom flow would eliminate the effect of

bottom friction on flows higher in the water column, and Chapman (2002) found that the deceleration of

an unforced along-isobath flow would be halted by the boundary layer arrest. Chapman and Lentz (1997)

60 found that the bottom-boundary arrest would prevent an along-slope current flowing in the direction of

61 Kelvin wave propagation (hereafter downwave) from spreading indefinitely across the shelf. Brink (2012)

62 found that the effects of stratification and BBL arrest on limiting cross-isobath spread of alongshore flows

63 became important as the slope Burger number exceeds roughly 0.2. BBL arrest prevents slope flows from

64 driving shelf flows by preventing the frictional dissipation of relative vorticity in the overlying flows, a

65 mechanism which allows flows to cross isobaths (e.g. Csanady, 1978).

66 These prior results did not focus on instabilities of the sloping isopycnals in the BBL, though the presence

of isopycnal slope indicates the presence of available potential energy which could be released by an

68 instability (Stone, 1970). Sloping isopycnals which intersect the bottom can be baroclinically unstable

69 (Blumsack & Gierasch, 1972), and this instability persists in the presence of strong frictional effects (K.

70 Brink & Cherian, 2013), and even be driven by them (McWilliams, 2016). Recent work has explicitly

examined the linear stability of an arrested BBL, and found that it is unstable, with instability fast

72 compared to the arrest scale for currents downwave directions, and slower growing instabilities for flows

in the upwave sense (Wenegrat et al., 2018; Wenegrat & Thomas, 2020). Flows in the downwave sense

read to downwelling in the BBL, upwave flows to upwelling.

A pair of models is used to understand how the development of finite amplitude instabilities in the BBL

can greatly reduce the arrest of the BBL. In particular, the magnitude of the near bottom velocity (and

thus bottom friction) that persists despite the arrest mechanisms is estimated as a function of slope Burger

number bottom, friction, stratification and alongshore flow strength. The results of this work is used to

restimate how finite amplitude instability in the BBL can increase coupling between slope and shelf

80 currents, at least for downwave slope currents (Gula et al. (2016) discusses upwave). The focus is on BBL

arrest and instability within and closely above the BBL; sub-mesoscale motions caused by the separation

82 of the BBL from the bottom at topographic variation is not examined (e.g. Gula et al., 2014; Molemaker

83 et al., 2014). The same BBL instability mechanism that reduces arrest also ventilates the BBL, pushing

84 boluses of weakly stratified bottom water into the stratified interior.

#### **2 Methods: The primitive equation numerical model**

86 To understand the role of stratification, friction and instability processes in decelerating an along-isobath

87 flow over a sloping bottom, the deceleration of an initially barotropic current in a stratified, alongshore

88 periodic ocean is examined. The fundamental approach is similar to Chapman (2002), but with a model

89 domain that is both much more highly resolved and allows alongshore variation in the flow.

The deceleration of the flow by the bottom friction is caused by bottom stress. Because the prior literature
uses both linear and quadratic drag laws, and because depending on the system being modeled, results for
both a quadratic drag law

93

$$\vec{t}_{bot} = C_d \rho_0 \, \vec{u}_{bot} \cdot |\vec{u}_{bot}| \tag{1}$$

94 and a linear drag law

95

108

$$\vec{\tau}_{bot} = r\rho_0 \vec{u}_{bot} \tag{2}$$

96 will be given.  $\vec{u}_{bot}$  is the velocity in the numerical model grid-point nearest to the bottom.

97 The deceleration of the alongshore current is examined with the Regional Ocean Modeling System

98 (ROMS, Moore et al., 2011). The domain is 200 km across the shelf and 400 km along the shelf, with

99 (except where specified) an alongshore resolution of 500 m. Some parameter sets are re-run with no100 alongshore variation.

In the development below, it is found that the bottom slope is an important parameter; to make scaling easier it is kept constant in the middle portion of the domain. The decay timescale of the alongshore flow is proportional to 1/depth, and to reduce the effect of the cross-isobath divergence of the cross-isobath flux of momentum we wish to have a relatively weak cross-shelf gradient in the alongshelf flow. To do this, and to roughly model the continental slope, the depth of the ocean is given by a hyperbolic tangent (for a nearly constant slope over the middle part of the domain) and is offset by a large constant (to reduce the factional depth change):

$$H(x) = H_0 + \alpha W \tanh\left(\frac{x - x_{pivot}}{W}\right)$$
(3)

109 where  $H_0$  is the depth offset,  $\alpha$  is the bottom slope, W is the width of sloping region, and  $x_{pivot}$  is the 110 location where the depth is always equal to  $H_0$  (Figure 1). For model runs presented in this paper,  $H_0=600$ 111 m, W=30 km and  $x_{pivot}=100$  km. The bottom slope is varied by run, and an example of the geometry is 112 shown in figure 1. There are walls at x=0 and 200 km.

113 At the shallow (x=0 km) and offshore (x=200 km) boundaries, there is a 20km wide region in which the

density structure is relaxed back to the initial structure; this damps the effect of Ekman pumping at the

boundaries, preventing the formation of jets which, when they become unstable, can contaminate the

interior portion of the model domain which are of interest. The timescale of decay is 0.25 day at theboundary and increases to infinity 20km from the boundary.

The initial state of the model is a uniform stratification  $N_{\theta}$  and a spatially uniform alongshore geostrophic 118 119 velocity in balance with an alongshore surface pressure gradient. There is no surface wind or buoyancy 120 forcing, so the model spins down from this initial condition. 45 vertical levels are used, and to preserve 121 resolution in the BBL, the following vertical coordinate parameters are chosen: V<sub>transform</sub>=2, V<sub>stretching</sub>=4, 122  $\theta_s = 2.0, \theta_b = 4.0$  and T<sub>cline</sub>=300 m (Shchepetkin & McWilliams, 2005). The base case bottom friction is  $r=5 \times 10^{-4} m s^{-1}$  for linear friction and  $C_d = 1.0 \times 10^{-3}$  for a quadratic drag law. The horizontal 123 friction is biharmonic along s-coordinate surfaces with a magnitude of  $2.38 \times 10^5 \ m^4 \ s^{-1}$ . Halving the 124 125 horizontal friction has no effects on the results presented here. When the horizontal resolution is changed, 126 the horizontal friction is also changed to keep the damping timescale for  $2\Delta x$  features constant. Vertical 127 mixing is parameterized by the  $k - \epsilon$  Generic Length Scale Mixing scheme (Warner et al., 2005) with the default ROMS parameters except with the background turbulent kinetic energy changed to match Warner 128 et al. (2005) (KMIN= $10^{-10}$  kg m<sup>-1</sup> s<sup>-2</sup> in the model) to provide reasonable background diffusivity at large 129 Richardson number and to prevent rapid dissipation of background stratification. 130

#### 131 **3 Results: The model spindown**

The spindown of the alongshore currents is examined in a series of model runs with varying parameters;
guided by the existing literature on BBL arrest (K.H. Brink, 2012; D. C. Chapman, 2002), the spindown
at the center of the slope of an initially spatially uniform velocity of 30 cm s<sup>-1</sup> is shown (Figure 2) for
flows in the upwave and downwave direction for several values of the Slope Burger number

136 
$$S = \frac{N_0}{f} \frac{\partial H}{\partial x}$$
(4)

137 In these runs, the initial stratification  $N_0 = 5 \times 10^{-3} \text{ s}^{-1}$  and  $f=10^{-4} \text{ s}^{-1}$  and *S* is varied by altering the 138 maximum bottom slope  $\partial H/\partial x$ . In the two-dimensional runs (the dashed lines of Figure 2) the 139 deceleration ceases for larger values of S, and the deceleration is noticeably less for the upwave then 140 downwave flows. These results are consistent with the boundary layer arrest literature cited above.

In the model runs with alongshore variation (the solid lines in the figure), the deceleration does not come to a halt in the downwave case. With alongshore variation, the dependence on S is complex. Deceleration is less with greater S as would be expected, but the difference between the alongshore varying and alongshore uniform runs also increases with greater S. In all cases the downwave flows decelerate faster than the upwave flows. Because these results indicate that the effect of allowing alongshore variation is more pronounced on the deceleration of the downwave flow, the focus below is on downwave flows.

In alongshore averaged fields (Figure 1) for the S=0.5 runs, the differences between the runs with and 147 148 without alongshore variation are clear. Five days into the model run, a thick and well mixed BBL develops where downwelling bottom Ekman flows moves less-dense water under more dense water; the 149 150 scale of the BBL is consistent with the existing literature (Garrett et al., 1993; Trowbridge & Lentz, 1991). There is a cross-shelf instability in the BBL for the first few days consistent with the symmetric 151 instability discussed by Allen and Newberger (1998); but it quickly disappears when instabilities with 152 alongshore variation grow. The suppression of symmetric instability by subsequent baroclinic instability 153 154 has been seen in similar systems (e.g. K. H. Brink, 2015; Haine & Marshall, 1998). Within a few days there is alongshore variation in the alongshore velocity field in the BBL (Figure 1 bottom panels). By day 155 156 20, the deceleration of the along-isobath flows is noticeably different between the two- and three-157 dimensional cases, and the BBL has restratified in the latter case. By day 55 the instabilities are still bottom trapped but have extended horizontally offshore. The eddy processes in the downwave case are 158 159 notably similar to the downwelling relaxation described in Brink (2015), and the restratification of the 160 BBL is essentially similar to that described in Callies (2018).

To illustrate the evolution of the BBL instabilities, and their offshore progress with time, Figure 3 shows the evolution of a passive tracer in the S=0.5 case. The passive tracer is initialized so that its value is equal to the isobath depth it is over – so a tracer with a value of 630 started over the 630m isobath. The instability starts with small amplitude and a relatively small alongshore lengthscale. Over time, the alongshore length scale and offshore extent of the instability grows. In the bottom panels of Figure 1 the alongshore variability is seen to be largely trapped to the BBL.

# 167 **3.1 Bottom boundary layer instabilities and the stratified interior**

The instabilities restratify the BBL (e.g. Wenegrat et al., 2018). This restratification fundamentally alters 168 169 the penetration of water near the bottom into the interior by reducing the small-scale turbulent diapycnal mixing of water (e.g. Perlin et al., 2018). At the same time, the eddies driving the restratification transport 170 water horizontally away from the bottom, allowing communication of water properties from the near 171 172 bottom into the geostrophic interior. In Figure 3, BBL eddies can be seen moving the tracer away from the boundary. As one would expect from work with dense water overflows (Yankovsky & Legg, 2018), 173 174 localized surface cooling (David C. Chapman & Gawarkiewicz, 1997; Visbeck et al., 1996) and work on parameterization of mesoscale eddy fluxes (Gent & McWilliams, 1990 and the vast literature which grew 175 176 from it), this eddy mixing away from the BBL tends to mix water more along isopycnal surfaces than 177 across them. This is significantly different then the mixing that would be expected in a weakly stratified turbulent BBL, which mixes water across density classes in the turbulent BBL but also keeps the BBL 178

isolated from the stratified waters above by the lack of turbulent mixing across the stratification above theBBL.

181 To examine the processes that mix water from the BBL into the stratified interior, a simulation of the

182 linear friction base case, S=0.5, is conducted starting from day 15 of the model integration, after eddies

have developed. Tracer is inserted into the model within 50m of the bottom between the 4.75 and 5.75°C

isotherms (in this constant salinity model, isotherms are isopycnals). In Figure 4, the evolution of the

185 tracer is shown over 45 days. As seen in other studies, the large majority of the mixing of tracer away

186 from the BBL is along isopycnals, with little diapycnal mixing. The eddies pull the tracer 40km into the

interior in 45 days.

188 When the near bottom flow is downwave, dissipation and mixing near the bottom produces water with

189 greatly reduced Ertel Potential Vorticity (hereafter PV), and when the flow is upwave, upslope advection

190 of density and diapycnal processes generates stratified water with larger PV (Benthuysen & Thomas,

191 2012). The eddies generated by the BBL instabilities then move the anamolous PV into the interior – in

192 Figure 5A the value of the PV on the 5.25°C isotherm is shown for the same time as the tracer in Figure

193 *4*A; the tracer contours are overlain on the PV. Most of the anomalous PV has an anomalously low

194 magnitude because the bottom flow is largely downwave. The contours of the tracer inserted at the

bottom match the low-PV anomaly well, indicating that the weakly stratified low-PV water was produced

196 near the bottom and then carried into the interior.

197 The low PV anomaly can be seen to be limited to the region near the slope where the isopycnals have bent198 downward to intersect the slope, reducing the stratification (Figure 5B). The eddies have a Rossby

199 number less than 1 and the low-PV is associate with lower stratification. To lower the stratification, the

200 isopycnals which are horizontal in the interior must be tilted downward to intersect the slope, increasing

201 the vertical spacing between the isopycnals. This downward tilt of the isopycnals sets up horizontal

202 density gradients which, through thermal wind, reduce the near bottom alongshore velocity and reduces

203 (though does not eliminate) the bottom drag on the overlying alongshore flow. In the development below,

the BBL is defined as the region of low PV near the bottom.

#### **3.2 A reduced-physics model of the extent of BBL arrest for downwave flows**

206 Given the results above, it is hypothesized that the along-isopycnal mixing of PV by geostrophic eddies

away from the bottom, and the subsequent restratification and reduction of horizontal density gradients in

the BBL, is the mechanism reducing the BBL arrest. To test this, a "reduced physics" model is created

- which in which the evolution of PV near the sloping bottom is governed by the anomalous PV generated
- at the bottom and the transport of that PV anomaly by geostrophic eddies along isopycnals into the

interior. If the reduced-physics model compares well to the solutions in the full primitive-equation model,it will support the hypothesis.

213 In the following derivation, it is assumed that all isopycnals that exist in the low-PV region above where

the modeled isopycnal intersects the bottom also intersect the bottom where conditions (e.g. bottom slope,

alongshore currents) are similar. This allows the further assumption that the cross-slope gradients in

- stratification, boundary layer thickness, horizontal density gradients and velocity are weak. This is
- 217 defensible in the systems shown above, where the bottom slopes in the region of interest are nearly
- constant, the initial stratification is uniform, and the fractional depth changes are limited by the relatively
- 219 deep water modeled. It limits the applicability of this model in cases where the region of high
- stratification is relatively thin, or where the bottom slope changes quickly across the shelf. In particular,
- the model cannot be directly applied to the shelf-break front intersecting the bottom near the shelf-break.

In this model, the along-isopycnal transport of PV is caused by eddies created by BBL instabilities. The flux of the PV q has been successfully modeled as a diffusive process with a diffusivity  $A_h$  that scales as the product of the length scale  $L_{eddy}$  of the eddy, a velocity scale  $V_{eddy}$  and a constant efficiency factor C<sub>e</sub>

225 (e.g. David C. Chapman & Gawarkiewicz, 1997; Spall, 2011; Visbeck et al., 1997):

 $A_h = C_e L_{eddy} V_{eddy} \tag{5}$ 

227 and

228

$$\frac{\partial q}{\partial t} = A_h \frac{\partial^2 q}{\partial x^2} \tag{6}$$

The model equation is written in terms of the cross-shelf distance x and q is evaluated on an isopycnal; because the vertical excursion of the isopycnals is small relative to the horizontal length scales, there is no important difference between the along-isopycnal distance in the two-dimensional model and the crossshelf coordinate x. The initial condition of our model system is uniform alongshore flow and stratification  $N_0^2$ , so the initial PV (and the PV in the interior) is

- 234  $q = \omega_a \cdot \nabla b = (f\hat{\boldsymbol{z}} + \nabla \times \boldsymbol{u}) \cdot (\nabla b) = fN_0^2$ (7)
- 235 where *b* is the buoyancy anomaly  $b = -g(\rho \rho_0)/\rho_0$ .

The lengthscale of the eddies doing the mixing is assumed to scale as the internal radius of deformation inthe BBL (e.g. Spall, 2011; Wenegrat et al., 2018):

238 
$$L_{eddy} = \frac{N_{LPV}\delta_{LPV}}{f}$$
(8)

239 where  $N_{LPV}$  is the buoyancy frequency in the low-PV region near the bottom, and  $\delta_{LPV}$  is the vertical

extent of that region. The eddy velocity  $V_{eddy}$  is assumed to scale with the average geostrophic alongshore

velocity of the BBL. The geostrophic alongshore velocity is in the BBL,  $V_{LPV}$ , is the overlying current V

reduced by thermal-wind shear in the BBL:

243 
$$V_{LPV} = V + (z + H - \delta_{LPV}) \frac{1}{f} \frac{\partial b}{\partial x}$$
(9)

Both  $L_{eddy}$  and  $V_{LPV}$  depend on the horizontal and vertical buoyancy gradients in the BBL. The horizontal density gradient can be estimated from the vertical buoyancy gradient in the low-PV region  $N_{LPV}^2$ . Above the low-PV region, the PV remains the initial PV, and the stratification remains the initial stratification  $N_0^2$ . The buoyancy in both these regions is then (assuming *b*=0 at *z*=0 and a water depth *H*)

248 
$$b = N_0^2 z \text{ for } z \ge -H + \delta_{LPV}$$
(10)

249 
$$b = N_{LPV}^2 z + C \text{ for } z \le -H + \delta_{LPV}$$
(11)

250 Setting the constant C so that the buoyancy b is continuous at the top of the low-PV region and taking the 251 cross-shelf derivative of b gives the cross-shelf gradient in buoyancy in the bottom low-PV region:

252 
$$\frac{\partial b}{\partial x} = (N_0^2 - N_{LPV}^2) \left( -\frac{\partial H}{\partial x} - \frac{\partial \delta_{LPV}}{\partial x} \right)$$
(12)

253 Because of the assumption that the dynamics of the low-PV region is locally uniform in the cross-shelf 254 direction,  $\frac{\partial \delta_{LPV}}{\partial x}$  is assumed to be zero.

255 To estimate  $N_{LPV}^2$ ,  $q_{LPV}$  is written assuming balanced flow (e.g. Holmes et al., 2014) so that:

256 
$$q_{LPV} \approx f N_{LPV}^2 \left\{ 1 + \frac{1}{f} \frac{\partial V_{LPV}}{\partial x} - \frac{1}{N^2} \left( \frac{\partial V_{LPV}}{\partial z} \right)^2 \right\}$$
(13)

Equations (9), (12) and (13), along with the assumption that cross-shelf conditions are roughly constant (  $\frac{\partial^2 b}{\partial x^2} = 0$  and  $\frac{\partial \delta_{LPV}}{\partial x} = 0$ ) can be used to find

259 
$$N_{LPV}^{2} = \left(\frac{\frac{q_{LPV}}{f} + \frac{N_{0}^{4}}{f^{2}}\left(\frac{\partial H}{\partial x}\right)^{2}}{1 + \frac{N_{0}^{2}}{f^{2}}\left(\frac{\partial H}{\partial x}\right)^{2}}\right) = \left(\frac{\frac{q_{LPV}}{f} + N_{0}^{2}S^{2}}{1 + S^{2}}\right)$$
(14)

260 Where S is the slope Burger number given the initial stratification from (4). From these, the average

velocity in the BBL,  $V_{eddy}$ , and the geostrophic velocity at the bottom from which drag is calculated  $V_{bot}$ can be written:

263 
$$V_{bot} = V + \frac{\delta_{LPV}}{f^2} (fN_0^2 - q_{LPV}) \frac{\partial H}{\partial x}$$
(15)

264 
$$V_{eddy} = V + \frac{\delta_{LPV}}{2f^2} (fN_0^2 - q_{LPV}) \frac{\partial H}{\partial x}$$
(16)

Note that *V* is negative for downwave flows for this bathymetry. *V*, the current above the BBL, can be estimated by assuming that the depth averaged velocity  $\langle V \rangle$  is decelerated by the bottom drag

267 
$$\frac{\partial \langle V \rangle}{\partial t} = -\frac{1}{H} \frac{\tau_{bot}^{y}}{\rho_{0}}$$
(17)

and correcting for the difference between the depth averaged velocity and the velocity above the BBLcaused by the velocity deficit in the BBL:

270 
$$V = \langle V \rangle + \frac{\delta_{LPV}}{2f^2H} (fN_0^2 - q_{LPV}) \frac{\partial H}{\partial x}$$
(18)

This treatment of *V* neglects the cross-shelf advection of alongshelf momentum; this is discussed in thesupplementary information.

- 273 PV is conserved in the interior; the anomalous PV near the bottom is caused by the flux of low-PV from 274 the boundary where the isopycnal being modeled intersects the bottom. This flux is created by the viscous 275 dissipation of alongshore momentum, and is well described by Benthuysen and Thomas (2012) and 276 Wenegrat and Thomas (2020). A derivation closely following their work is given in the supplementary 277 information. The flux of PV normal to the boundary is (where  $\theta$  is the angle of the bottom from
- horizontal and  $\tau_{bot}^{y}$  is the alongshore bottom stress calculated from  $V_{bot}$  and a drag law):

279 
$$J^{n} = \hat{k} \cdot \nabla b \times \boldsymbol{F} \approx N_{LPV}^{2} \theta \frac{\partial}{\partial z} (\rho_{0}^{-1} \tau^{y}) \approx N_{LPV}^{2} \theta \frac{\tau_{bot}^{y}}{\delta \rho_{0}} = N_{LPV}^{2} \theta \frac{\tau_{bot}^{y}}{\delta_{LPV} \rho_{0}}.$$
 (19)

280 The vertical length scale  $\delta$  has in other work (e.g. Wenegrat & Thomas, 2020) been set to the vertical 281 scale of the low-PV layer, which is called  $\delta_{LPV}$ . The justification for this is unclear, both in the prior 282 literature and to this author. This assumption would appear inconsistent with the assumption that the PV 283 dynamics are primarily along-isopycnal, unless there is divergence of the flux of momentum carried by 284 the eddies along isopycnals across the low-PV region. The justification for using  $\delta_{LPV}$  remains an open 285 question; however, it is seen below that (19) correctly predicts the flux of PV from the bottom.

286 The PV flux in (19) is normal to the bottom, but the model in (6) is written in the cross-shelf coordinate.

- 287 To correct for this rotation, the flux into (6) must be divided by the bottom slope. At small slope angles,  $\theta$
- is approximately equal to the bottom slope, and the boundary condition on (6) at the slope is

289 
$$A_h \frac{\partial q}{\partial x} = \frac{J^n}{\frac{\partial H}{\partial x}} \approx N^2 \frac{\tau_{bot}^y}{\delta \rho_0}$$
(20)

To close the model, the thickness of the low-PV region  $\delta_{LPV}$  and the average PV in this region,  $q_{LPV}$  are needed. The height of the low-PV layer near the bottom,  $\delta_{LPV}$  is the along-isopycnal lengthscale of the low-PV anomaly  $L_{LPV}$  multiplied by the bottom slope

293 
$$\delta_{LPV} = L_{LPV} \frac{\partial H}{\partial x}$$
(21)

294 because the top-most isopycnal in the low-PV layer at a point  $x_0$  has, by assumption, the same  $L_{LPV}$  and 295 the same cross-slope change in depth as the isopycnal originating at  $x_o$ , since it depends on the PV flux 296 along that isopycnal where it intercepts the bottom. By assumption, that flux is the same as at  $x_0$ . The size 297 of the near-bottom low-PV region  $L_{LPV}$  is found from the solution to the eddy-flux model (6). It is the extent of the near-bottom region where the PV is less than  $1 - \epsilon$  of the interior PV  $f N_0^2$ . The model 298 results are not sensitive to the exact value of  $\epsilon$ . The results below are for  $\epsilon = 0.1$ , but minimal changes 299 300 are seen for 0.05. The mean potential vorticity of the low-PV region  $q_{LPV}$  is the average PV from the slope 301 boundary of the model to  $L_{LPV}$  offshore. These definitions of  $L_{LPV}$  and  $\delta_{LPV}$  are computationally robust and efficient, but their non-linearity and non-algebraic form are the chief impediment to an analytic 302 303 treatment of this model.

This completes the simplified model for the evolution of the low-PV layer. This model is solved with a small Python code included in the supplementary information, and can be run on a laptop.

## 306 **3.3** Evaluating the reduced-physics model of BBL arrest for downwave flows

307 The reduced physics model described above is compared to the full solution of the ROMS numerical

308 circulation model over the 600m isobath (the "full" model), which is always at the center of the slope.

309 This ensures that the assumption that conditions do not vary locally across-isobaths is met. Both models

are run for S from 0.1 to 1.8, initial currents of 0.15 m/s to 0.6 m/s, linear friction of 5 and  $2.5 \times 10^{-4}$  m/s or

guadratic drag of 0.5, 1.0 and  $2.0 \times 10^{-3}$ , and initial values of N of 5 and  $10 \times 10^{-3} \text{ s}^{-1}$ ; the details are given in

table 1. In the simple model, there is a single free parameter, the eddy mixing efficiency  $C_e$ . This is set to

313 0.06, which is the value that gives the minimum sum of squared differences between the ratio of bottom

velocity to surface velocity in the two models in the quadratic drag case. This value is about 4 times

315 greater than the value found for high Richardson number eddies in the surface ocean (Visbeck et al.,

**316** 1997).

The results from the full numerical circulation model and the simple model for the quadratic drag law are compared after a 30 day model integration (figure 6). They compare well (in the supplementary

- information an animation is given showing the time evolution of this figure over 60 days; the results are
- 320 very similar over all these times). Panels A and B show the surface and bottom velocities scaled by the
- 321 initial velocity. In both more positive values indicate either greater BBL shutdown or reduced drag
- 322 coefficient, and there is good agreement between the models. Panel C compares the estimate of the
- 323 thickness of the low-PV layer, and the comparison is similarly good.
- Panel D of figure 6 is the most dynamically important of the results shown; it is the ratio of the bottom velocity to the surface velocity,  $\gamma$ , and is a measure of the BBL arrest. Smaller values indicate greater arrest, with 0 being full arrest and 1 being no arrest. The agreement of the two models is good, with a correlation of 0.9 and a clustering around the 1:1 line. There is a systematic error where smaller initial alongshore currents lead the simple model to under-predict the extent of arrest when compared to the full circulation-model (e.g. for the S=0.5 (red) points, follow the sequences  $\bigstar$ ,  $\bigstar$  and  $\checkmark$  or  $\bigtriangledown$ ,  $\bigstar$  and  $\blacktriangleleft$  for initial alongshore velocities of 0.15, 0.3 and 0.6 m/s, respectively, at different drag values). One reason
- for this is that the BBL eddy mixing is assumed to be instantly fully developed; but in the primitive-
- equation model it takes time for the instability to develop, with weaker flows taking more time to develop
- eddies. But overall, the agreement is good.
- 334 The results for the linear bottom friction are similar, though with a tendency for the simple model to
- overpredict the amount of BBL arrest when the bottom velocity is weak (Figure 7B and C). In part this is
- because there is greater frictional decay of the alongshore flow for the linear drag, since the spin down
- timescale does not increase as the currents weaken with linear friction, as it does for the quadratic drag
- law (note that the effective friction coefficient r in the quadratic drag case is  $Cd|\vec{v}|$ ). This effect is
- especially pronounced over the flatter portions of the model domain on either side of the slope (figure 1),
- 340 leading to stronger currents over the slope with much weaker flows on either side. This leads to a weak
- instability and a meandering in the current over the slope which extends throughout the water column,
- 342 leading to near bottom alongshore flow and bottom drag. This meander is seen as a weak deviation of the
- alongshore flow from the mean alongshore flow even above the low-PV region in the bottom row of
- figure 1. This is a mechanism distinct from that discussed here.
- 345 The close correspondence of the results between the reduced-physics model and the primitive equation
- 346 numerical circulation model confirms the relevance of the reduced-physics model; the dynamics of partial
- BBL arrest and evolution can, for downwave flows, be understood as the creation of anomalous PV at the
- 348 bottom and its fluxing into the interior by eddies that arise in the unstable BBL.
- 349 **3.4** The parameter dependence of the reduced physics model

350 The reduced physics model can now be used to examine the dynamics of partial BBL arrest of downwave 351 flows. The focus is on a BBL arrest parameter  $\gamma$ , and on the quadratic drag law, as this drag law makes the 352 results more directly comparable to recent work on BBL shutdown (e.g. K. H. Brink & Lentz, 2009).

- Begin by assuming that the water depth above the BBL is very great, so that the alongshore flow is not
- decelerated by the bottom drag; this is an assumption commonly made in studying BBL dynamics
- 355 (Benthuysen & Thomas, 2012; e.g. K. H. Brink & Lentz, 2009; MacCready & Rhines, 1993). This
- assumption removes a timescale from the analysis, as it makes the timescale of the decay of the
- 357 alongshore flow infinite. The simple model then depends on a small number of quantities: the drag
- 358 coefficient  $C_d$ ; the un-varying alongshore velocity above the BBL, V; the near-bottom geostrophic
- velocity  $V_{bot}$ ; the initial stratification  $N_0$ ; the Coriolis parameter f; the bottom slope  $\frac{\partial H}{\partial r}$ ; and the time since
- the model has started with depth-uniform velocity and depth-uniform stratification, t. These seven
- quantities in two units, length and time, would suggest from Buckingham-P theory that this system is
  governed by 5 non-dimensional parameters. However, earlier results (e.g. K. H. Brink & Lentz, 2009 and

363 citations therein) suggest that some of these non-dimensional parameters can be combined to reduce the 364 dimensionality of the system.  $C_d$  and  $\frac{\partial H}{\partial x}$  are combined with the other parameters to make the scaled drag 365  $C_d N_0 / f$  and the slope Burger number  $S = \frac{N}{f} \frac{\partial H}{\partial x}$ . The remaining non-dimensional parameters are N/f, the

- **366** BBL arrest parameter  $\gamma = V_{bot}/V$ , and the nondimensional time *ft*.
- 367 I have not been able write the reduced-physics model analytically, because  $\delta_{LPV}$  is calculated numerically from the solution to (6), and so it cannot yet be rigorously shown that  $\gamma$  can be written as a function of S. 368 N/f, tf, and  $C_d N_0/f$ . However, numerical solutions to the reduced-physics model find that for infinite water 369 depth,  $\gamma$  is constant for any oceanographically realistic set of S, N/f, tf, and  $C_d N_0/f$  even as the individual 370 371 dimensional parameters (e.g. N and f) are varied. This is shown in figure 8, where in the infinite depth case  $\gamma$  is shown for S=1.0, N/f=50.0, tf=42.3, and C<sub>d</sub>N<sub>0</sub>/f=0.15 as the quantities f, N, and C<sub>d</sub> are varied by 372 373 an order of magnitude;  $\gamma$  is constant even as the dimensional parameters vary. Even in the finite depth 374 case, where H=600m,  $\gamma$  varies by 10% or less as the dimensional parameters are varied by an order of magnitude. The infinite depth solution remains close to the finite depth solution when the timescale of the 375 current spindown  $H/(\gamma C_d V)$  is long compared to the timescale of evolution of the reduced-physics model. 376
- 377 These results strongly suggest that the non-dimensional parameters S, N/f, tf, and  $C_d N_0/f$  are sufficient to
- understand the BBL arrest in the limit of a steady overlying flow, and are a good guide to understanding
- 379 the magnitude of BBL arrest in finite depth cases.  $\gamma$  is shown as a function S, tf, and C<sub>d</sub>N<sub>0</sub>/f in panels B-E
- 380 of figure 8, with greater  $\gamma$  indicating less arrest.

381 The least arrest ( $\gamma$  close to 1) is seen where S or the non-dimensional drag  $C_d N_0 / f$  are small. This is

- 382 consistent with the timescale needed for arrest in a downwelling-favorable (downwave) flow, which
- becomes infinite as either of these parameters becomes small. Figure 8 panels B) to E) include a line
- marking where the non-dimensional time *tf* is equal to the equivalent time scale for BBL arrest given in
- 385 Brink and Lentz (2009) for a system with no alongshore variation. Above and to the right of this line the
- 386 Brink and Lentz timescale is less than the time the model has been run (though it should be noted that
- 387 instabilities modify the solution significantly well before the Brink and Lentz timescale). The most arrest
- 388 occurs when both S and the non-dimensional drag  $C_d N_0 / f$  become larger, with bottom currents less then
- 389 20% of the overlying currents as *S* exceeds  $\approx 1.5$  and *C*<sub>d</sub>*N*<sub>0</sub>/*f* exceeds  $\approx 0.15$ . In all cases, increased non-
- 390 dimensional bottom slope and stratification (S) or increased non-dimensional bottom drag decrease  $\gamma$  and
- thus reduce the effect of bottom drag on the overlying flow. At the same time, the instabilities prevent
- BBL arrest and allow bottom drag long after Brink and Lentz would predict complete BBL arrest. The
- 393 code of the reduced-physics model used to produce figure 8 are provided in the supplementary materials.

#### 394 4 Discussion

395 A reduced-physics model of the evolution of the bottom boundary layer was created, based on a simple 396 eddy mixing parameterization of the effect of the eddies created in the BBL and the observation that 397 eddies flux PV anomalies created at the bottom boundary along isopycnals away from the boundary. This model compares well with a fully primitive-equation numerical model of a downwave flow over a 398 399 slopping bottom, strongly supporting the ideas that these limited dynamics are sufficient to capture effects 400 of stratification on the bottom drag of a downwave flow in a strongly stratified ocean over a sloping 401 bottom. These specific results are limited to the case where the stratification at the bottom does not very 402 greatly in the cross-shelf direction.

# 403 4.1 Implications for numerical modeling of slope systems

The instability processes involved can be simulated by a standard hydrostatic numerical ocean model, such as the ROMS model used above. However, to do this, the resolution of the model must be sufficient to resolve these instabilities, and this resolution is finer than typical in most large-scale or regional models of coastal systems.

- 408 Figure 9 shows the decay of along-isobath surface currents for S=0.5 and linear bottom friction for
- 409 horizontal model resolutions of 250m to 2km, along with the solutions for flows without alongshore
- 410 variation. For the coarse 2km resolution, the runs with alongshore variation and instability are similar to
- 411 those with no alongshore variation and experience significant BBL arrest; it should be noted that even this
- 412 2km resolution exceeds or matches that of most existing regional models. It is only as resolution becomes

- finer that the model solution converges, and the solution has both BBL instability and enhanced drag onthe alongshore flow.
- 415 The failure of the relatively coarse resolution models to accurate simulate the deceleration of the
- 416 alongslope currents in models that do not resolve the instability processes in the BBL, and fact that the
- 417 resolution of these failing models is comparable or finer than many models of the shelf/slope sea,
- 418 suggests that efforts to parameterize the effects of BBL instabilities are necessary.

#### 419 **4.2** Cross-slope coupling of currents

- 420 In the linear, low Rossby number limit, for a flow along the slope to cross-isobaths and drive flows on the 421 shelf something must cause the depth-averaged potential vorticity of flow, f/H, to change. In the arrested 422 topographic wave (ATW) limit, an along-isobath jet can spread across isobaths because of the frictional dissipation of relative vorticity (Csanady, 1978). In the ATW limit, slope flows drive shelf flows, and 423 424 these shelf flows increase in the downwave direction as the current spreads across the shelf. Chapman and Lentz (1997), hereafter CL97, included stratification and bottom boundary layer arrest into a similar 425 analysis on an *f*-plane. CL97, like ATW, assumes alongshore lengthscales are greater than cross-shelf 426 scales and does not allow instabilities. CL97 found that boundary layer arrest, by shutting down the flow 427 428 at the bottom and thus bottom friction, prevented the jet from widening or dissipating. This left it to flow 429 unchanged along isobaths. In their limit, a slope jet in stratified slope waters remains mostly confined to 430 the slope, and does not drive large shelf flows. The contrast between CL97 and ATW leads to the 431 hypothesis that when instabilities reduce BBL arrest, they will allow a downwave slope jet to spread
- 432 across isobaths as it progresses downwave.
- 433 To test this hypothesis, an idealized numerical model run was created with ROMS, with 100km wide
- 434 shelf with a bathymetric-slope of  $10^{-3}$  and a 200km wide slope with a bathymetric slope of  $10^{-2}$ . The
- 435 stratification was such that S=0.5 over the slope, and the system was driven by a jet flowing in from the
- 436 northern boundary between the shelfbreak and 75km farther offshore with a velocity of  $0.60 \text{ m s}^{-1}$  (Figure
- 437 10). The domain has an alongshore extent of 1200km, or roughly the along-coast distance from the
- 438 Laurentian Channel to Cape Cod or from the northern part of the Gulf of Maine to Cape Hatteras.
- 439 Because of this latitudinal extent, the model is run on a  $\beta$ -plane, with  $f=10^{-4}$  s<sup>-1</sup> and  $\beta=1.6 \times 10^{-11}$  s<sup>-1</sup>m<sup>-1</sup>.
- 440 With the same domain and forcing, the model of CL97 was implemented, along with the ATW model. To
- 441 match CL97, the linear bottom friction was  $5.0 \times 10^{-4}$  m s<sup>-1</sup>.
- 442 The results of these models are shown 1000km downwave of the inflow in Figure 10. A significant
- fraction of the jet has moved both offshore to deeper isobaths and onto the shelf in all cases. In all but the
- 444 ATW model, along-slope flow increases downwave because the flow remains somewhat along lines of

445 constant f/H, and as the flow moves equatorward, these lines converge shoreward because of  $\beta$ ,

- 446 accelerating the flow. In the CL97 solution, there is much less broadening of the jet; because of BBL
- 447 arrest, the flow ceases to spread across lines of constant f/H. The effects of the reduced BBL arrest in the
- 448 ROMS model which allows BBL instabilities is evident on the shelf, where the flow is 50% to 100%
- greater than would be expected from CL97. The model with BBL instabilities is significantly closer to the
- 450 ATW solution because the instabilities allow bottom drag on the along-isobath flow.
- 451 These results are consistent with the changes in jet dynamics expected with stronger bottom friction. This 452 suggests that the instability-driven reduction of BBL arrest and associated bottom friction seen in Figure 453 10 are important to include in efforts to understand the dynamics of downwave jets along continental 454 slope, such as the Labrador current and its equatorward extension and the Oyashio. In a recent study of the effect of gyre scale circulation on shelf flows, the strength of the bottom friction was found to be a 455 456 key control on the ability of gyre-scale circulation to change coastal sea level (Wise et al., 2018). This 457 suggests that BBL instability increase the influence of the gyre scale circulation on coastal sea level when 458 the western boundary current is downwave, as is true in the sub-polar gyres and the equatorward
- 459 extension of their western boundary currents.

## 460 4.3 Observing BBL non-arrest

Observations of boundary layer arrest on the shelf are sparse; of BBL instabilities more so. As seen in 461 Figure 1, once the BBL becomes unstable, it becomes restratified even in the presence of along-isobath 462 463 downwave flows which would be expected to continuously destroy the stratification . While the alongisobath current is flowing downwave, instabilities would be continuously fueled by the downslope Ekman 464 transport's injection of potential energy into the density field - and would create enhanced variability in 465 the along-isobath flow in the low-PV region relative to the more stratified waters above (e.g. Figure 1, 466 bottom panels). Attempting to estimate bottom boundary layer dissipation of energy in the ocean Sen et 467 468 al. (2008) and Wright et al. (2012) examined their current data for evidence of BBL shutdown and failed to find any evidence for it. Wright et al. (2012) found greater currents near the bottom than above. These 469 470 are consistent with the predictions above – but neither effort controlled for the slope Burger number S. 471 Callies (2018) studied the restratification of abyssal BBL by instabilities, compared his predictions to 472 observed BBL stratification in the South Atlantic Ridge, and found good agreement.

- 473 A more easily observed indicator of BBL instabilities and arrest escape would be an increase in the
- 474 frequency of boluses low PV, weakly stratified water in the otherwise stratified waters near the slope
- 475 when the near-bottom flows are downwave, even while the BBL does not create a thick (e.g. comparable
- to the predictions of Chapman and Lentz (1997)) bottom mixed layer (e.g. figures *1* and 5; see also
- 477 Wenegrat et al. (2018)). For example, the slope Burger number of coastal Peru at about 15°S during the

478 late summer of 1977 was roughly 1.6. As part of the Coastal Ecosystem Upwelling Analysis program in

479 coastal Peru from March to mid-May of 1977 (Kenneth H. Brink et al., 1978, 1979) two moorings were

480 placed in the water column at a depth 121m (12km from the coast) and in 86m (4km from the coast) at

481 about 15°S. The near bottom flows showed 7 to 12 day periods of downwave (negative/poleward) flow

482 alternating with periods of upwave flows of shorter duration (2-5 days). The data are low-pass filtered

with the pl33 filter (Beardsley et al., 1985) with a half-power point of 46 hours (Figure 11).

In CTD surveys taken along the mooring line, regions of well mixed water can be found away from the
bottom or surface boundary layers; an example from the 19<sup>th</sup> of March 1977 is given in Figure 11, where
a regions is marked as well mixed if there is a less than 0.01°C change in the vertical. These well mixed

487 patches were smaller than the horizontal resolution of the CTD sections (roughly 6km), most were less

than 10m thick, and nearly all less than 20m thick. Most of the well mixed patches were in the lower half

489 of the water column. Unfortunately, the temporal resolution of the CTD data was insufficient to compare490 to the currents.

The temperature sensors on the moorings were widely spaced; for the mooring at 121m depth, the sensors
were at 115, 100, 80, 59m and shallower (Figure 11). In an analysis (not shown) of stratification
estimated between the bottom two instruments of each mooring, a strong association was found between
downwave flows and relatively "well-mixed" bottom boundary layers as estimated by the relatively

widely spaced current meters (between 115 and 100m in 121m depth, and 78 and 67m in 86m depth; a

496 "mixed layer" is defined as a temperature difference  $<0.02^{\circ}$ C; this is the strictest criterion which makes

497 sense given the accuracy of the Anderaa current meters (K.H. Brink et al., 1978)). This is as would be
498 expected for downwelling flows. However, interestingly, the "well-mixed" periods were intermittent even

499 near the bottom during strong downwelling favorable flows. Over a two-day period the maximum
500 frequency of time the water was well-mixed was less than 30% of the time at the shallow mooring, and

likewise at the deeper mooring. This intermittency might be caused by the large vertical spacing of the
sensors and their distance from the bottom. Or they might be due, as suggested by the modeling above, to
the eddy-restratification of the bottom boundary layer.

Away from the bottom, in the next higher pairs of temperature sensors on both moorings, there is evidence of boluses of "well-mixed" water passing between the sensors. 1 percent of the time at the 86m depth mooring, between 46 and 67m, the water is "well-mixed." 6 percent of the time the water is "wellmixed" between the 80 and 100m sensors at the 121m mooring (Figure 11). These "well-mixed" periods were intermittent. Even over a one-day window, the peak frequencies of "well-mixed" water was about 30-50%. To compare the frequency of the mixed layer to the alongshore current, the alongshore velocity (filtered with a two-day average) at the second sensor above the bottom is correlated to the daily mixed 511 layer frequency. Because the distribution of mixed layer frequency is very non-Gaussian, Kendall's  $\tau$ , a

512 non-parametric correlation coefficient, is used and the confidence calculation is adjusted for the large

number of ties in the data (Daniel, 2000). The correlation between the alongshore velocity and the

frequency of well-mixed water between 80 and 100m is -0.27, and -0.38 between 100m and 115m, both

significant at the 95% level. The sign of the correlation is consistent with downwave flows leading to

boluses of well mixed waters leaving the bottom boundary layer. The relatively low correlations and

517 intermittency of mixed layers is consistent with kilometer scale filaments and boluses of relatively well

518 mixed (low PV) waters of kilometer-scale horizontal extent surrounded by stratified waters being

advected through the moorings by an alongshore velocity of  $O(0.1 \text{ to } 0.2) \text{ m s}^{-1}$  when the flow (averaged

520 over eddies) is downwave.

# 521 Conclusions:

The BBL arrest suggested by Garret et al. (1993), Trowbridge and Lentz (1991), Chapman and Lentz 522 523 (1997) and others is found to be reduced by baroclinic instabilities in the BBL when the flow is downwave. The downwave flow drives diabatic mixing which destroys PV in the bottom boundary layer 524 525 (Benthuysen & Thomas, 2012), creating horizontal density gradients which become baroclinically 526 unstable. The resulting eddy mixing stirs the low-PV water offshore, restoring stratification in the BBL 527 and reducing the BBL arrest. Where boundary layer arrest is broken down by instabilities in the boundary 528 layer, the overlying geostrophic flow remains frictionally coupled to the bottom. The resulting Ekman 529 pumping can both slow the alongshore flows and allow them to spread across isobaths. This mechanism can enhance the coupling between downwave slope flows and the shelf, and lead to stronger flows on the 530 shelf in the same direction as the slope flows and coupling shelves to the adjacent gyre scale circulation 531 532 (Wise et al., 2018). The eddies created by the BBL instabilities mix water from the BBL nearly 533 horizontally into the stratified interior, a potentially important mechanism for coupling the sediment/water interface into the stratified interior. Given that large scale ocean models currently have resolutions that 534 fail to capture the effects of BBL instabilities (e.g. Figure 9), some parameterization of their dynamics, 535

based on theory and validated by observations, is essential to understanding their broader impact.

A simple model has been created, and is provided in the supplementary information, which captures these
processes and accurately predicts the extent of BBL layer arrest and the spread of low-PV water into the
stratified interior.

540 The strength of the near bottom flows relative to overlying flow, and thus the strength of the drag on the 541 overlying flows, is a function of the non-dimensional parameters  $S = \frac{N}{f} \frac{\partial H}{\partial x}$ , *N/f, tf,* and *C\_dN\_0/f* in the limit of 542 an infinitely deep ocean or an overlying flow of fixed strength. In the limit of finite depth and a flow

- which decelerates due to drag, these results are still approximately true. The near bottom flow is greater 543
- 544 (the arrest in the BBL is less) when the slope Burger number S is less or the non-dimensional drag  $C_d N_0 / f$
- is less; the sensitivity to the other parameters is much less, for reasonable oceanographic values. These 545

546 results suggest that BBL instability and reduced BBL arrest should be broadly important - for instance on

- 547 the slope adjacent to the Mid-Atlantic Bight where the shelf and slope flows are downwave (Robinson &
- Brink, 2006), and the slope Burger numbers are moderate (Figure 12). 548

The direct observation of these relatively small scale and near bottom instabilities is challenging – but the 549

- predictions of near bottom instabilities, boluses of weakly stratified water horizontally adjacent to the 550
- 551 BBL and stratified BBL's during downwave flow are consistent with observations (e.g. Stahr & Sanford,
- 1999 and citations within and above). 552

553 The description above of the impact of BBL instability on the stratification and flow near the bottom is 554 incomplete; much remains to be done. Most importantly will be to express the simple model presented 555 above in a more analytic formulation in order to more mechanistically understand the relation between the 556 governing parameters and the velocity in the BBL. In particular, the reason the vertical length scale of 557 stress-divergence in (19) can be expressed as the thickness of the low-PV region remains unclear. The simple model created above also assumes that the stratification and bottom slope are slowly changing in 558

the cross-shelf direction, so that over a horizontal length scale  $\delta_{LPV} \left(\frac{\partial H}{\partial x}\right)^{-1}$  the alongshore flow and 559 stratification at the bottom remain relatively constant. This is not applicable where the thermocline 560 561 intersects the bottom, leading to a thin region of enhanced S (e.g. the shelf/slope fronts seen in Figure 12.)

562 Nonetheless, the instabilities of the BBL have been shown to have impacts well outside of the bottom 563 boundary layer, and are important in the coupling of the overlying stratified waters to the BBL, both for momentum and the injection of BBL water into the stratified interior. These small scale dynamics are 564 565 usually missing in our larger scale models, be they conceptual, idealized or numerical.

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- 572

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680

# 698 Table 1: Key to model run symbols.

Color	Slope Burger #	f
•	S=0.1	10 <sup>-4</sup>
٠	S=0.3	10 <sup>-4</sup>
•	S=0.5	10 <sup>-4</sup>
•	S=0.7	10 <sup>-4</sup>
•	S=0.9	10 <sup>-4</sup>
٠	S=1.2	$0.5 \times 10^{-4}$
•	S=1.5	$0.5 \times 10^{-4}$
•	S=1.8	$0.5 \times 10^{-4}$

Marker	Initial Velocity	Drag Coefficient	Initial Buoyancy Frequency		
•	0.15 ms <sup>-1</sup>	5e-04	5e-03 s <sup>-1</sup>		
*	0.30 ms <sup>-1</sup>	5e-04	5e-03 s <sup>-1</sup>		
•	0.60 ms <sup>-1</sup>	5e-04	5e-03 s <sup>-1</sup>		
٠	0.15 ms <sup>-1</sup>	1e-03	5e-03 s <sup>-1</sup>		
*	0.30 ms <sup>-1</sup>	1e-03	5e-03 s <sup>-1</sup>		
•	0.60 ms <sup>-1</sup>	1e-03	5e-03 s <sup>-1</sup>		
*	0.15 ms <sup>-1</sup>	2e-03	5e-03 s <sup>-1</sup>		
×	0.30 ms <sup>-1</sup>	2e-03	5e-03 s <sup>-1</sup>		
*	0.60 ms <sup>-1</sup>	2e-03	5e-03 s <sup>-1</sup>		

Marker	Initial Velocity	Bottom Friction r	Initial Buoyancy Frequency
•	0.15 ms <sup>-1</sup>	2.5e-04 ms <sup>-1</sup>	5e-03 s <sup>-1</sup>
*	0.15 ms <sup>-1</sup>	2.5e-04 ms <sup>-1</sup>	1e-02 s <sup>-1</sup>
•	0.30 ms <sup>-1</sup>	2.5e-04 ms <sup>-1</sup>	5e-03 s <sup>-1</sup>
×	0.30 ms <sup>-1</sup>	2.5e-04 ms <sup>-1</sup>	1e-02 s <sup>-1</sup>
•	0.60 ms <sup>-1</sup>	2.5e-04 ms <sup>-1</sup>	5e-03 s <sup>-1</sup>
*	$0.60 \ ms^{-1}$	2.5e-04 ms <sup>-1</sup>	1e-02 s <sup>-1</sup>
•	$0.15 \ ms^{-1}$	5.0e-04 ms <sup>-1</sup>	5e-03 s <sup>-1</sup>
•	$0.15 \ ms^{-1}$	5.0e-04 ms <sup>-1</sup>	1e-02 s <sup>-1</sup>
*	0.30 ms <sup>-1</sup>	5.0e-04 ms <sup>-1</sup>	5e-03 s <sup>-1</sup>
¤	0.30 ms <sup>-1</sup>	5.0e-04 ms <sup>-1</sup>	1e-02 s <sup>-1</sup>
•	0.60 ms <sup>-1</sup>	5.0e-04 ms <sup>-1</sup>	5e-03 s <sup>-1</sup>
•	0.60 ms <sup>-1</sup>	5.0e-04 ms <sup>-1</sup>	1e-02 s <sup>-1</sup>

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700 The parameters used in the full numerical model runs, both with linear and quadratic bottom drag, along

with a key to the symbols and colors used in the figures to represent these model runs.





Figure 1: Top row: the alongshore velocity (colors) and isopycnals (red) for days 5, 20 and 55 of the case
with no alongshore variation and an initial downwave-ward flow. Middle row: the alongshore flow and
isopycnals for the same days in the case with alongshore variation. Bottom row: the standard deviation
from the alongshore mean of the alongshore velocity (v') for the case with alongshore variation.



Figure 2: The surface velocity 100km offshore over sixty days for 12 model runs which all start with an alongshore velocity of 0.3 m/s. The dashed lines are for a model with no alongshore variation. The solid lines are for model runs which allow alongshore variation. There are models for a Slope Burger number of 0.0, 0.1, 0.5 and 0.9, and the initial velocity is oriented in either the upwave (positive, poleward) or downwave (negative, equatorward). For these runs, N=5 × 10<sup>-3</sup> s<sup>-1</sup>, r= 5 × 10<sup>-4</sup> m s<sup>-1</sup> and f =1 × 10<sup>-4</sup> s<sup>-1</sup>, and S is altered by altering the bathymetry.

716



Figure 3: figTracer: The model run is initialized with a passive tracer whose value is equal to the depth
of the isobath over which it starts; its initial value does not vary with either depth or along-shelf position.
The tracer is shown at 600m depth on day 5, 20, and 50 for initial flows of 30 cm/s in the downwave

721 The tracer is shown at 600m depth on day 5, 20, and 50 for initial flows of 30 cm/s in the downwave 722 direction. N= $0.005 \text{ s}^{-1}$ , f= $10^{-4} \text{ s}^{-1}$ , r= $5 \times 10^{-4} \text{ m s}^{-1}$  and S=0.5 in both cases.

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Figure 5: A) The potential vorticity computed on the 5.25°C isotherm scaled by the initial potential

vorticity  $f N_0^2$  for the same time and model run as the tracer field shown in figure 4. The grey contours are

the tracer field shown in figure 4A; the red line is the position of the slice shown in figure 4B. **B**) The

alongshore averaged potential vorticity scaled by the initial potential vorticity (colors) overlain by the

737 isotherms (black, with 1°C interval). The red isotherm is the 5.25°C isotherm.

738



Figure 6: A) The surface velocity scaled by the initial velocity after 30 days for the simple model (the "estimate" on the vertical axis) versus the complete numerical model (the "truth", on the horizontal axis) in day 30 of runs made with quadratic bottom friction. B) A comparison of the velocity at the bottom scaled by the initial velocity; otherwise as (A). C) A comparison of the thickness of the low-PV layer. D) A comparison of  $\gamma$ , the ratio of the bottom velocity to the surface velocity. A smaller gamma indicates a greater bottom boundary layer arrest. The correlations in the title are Pearson's R, and all are significant (P>0.05). A key to the symbols and colors is given in Table 1.

747







Figure 8: A) The ratio of surface to bottom velocity  $\gamma$  at 30 days for the non-dimensional parameters 754 S=1.0, N/f=50.0,  $C_dN/f=0.15$  and a model run 41.3 inertial periods. Each line represents either N, f, or  $C_d$ 755 varying by a factor 0.2 to 2.0 while keeping the non-dimensional parameters constant; the dashed line is 756 757 for a model run with a depth of 600m, the solid for a model run of effectively infinite depth. Panels B) 758 through E) are  $\gamma$  as a function of S and  $C_d N/f$  for 15, 30, 45 and 60 inertial periods. In all, N/f=50.0. The 759 red line indicates where the bottom arrest timescale of Brink and Lentz (2009) matches the modeled time; the arrest time scale is smaller than the modeled time above and to the right of the line. The symbols on 760 the plots correspond to the parameters of the full numerical model runs made with a quadratic drag law, 761 762 and match the symbols in table 1 and figures 7 and 6.



Figure 9: The vertical mean alongshore current 100km offshore over the 600m isobath for models with

different resolutions, S=0.5 and initial currents of 0.3 m/s with a linear friction of  $5 \times 10^{-4}$  m s<sup>-1</sup>. The model

resolution is indicated as  $\Delta x$  for 500m,  $\Delta x/2$  for 250m,  $2\Delta x$  for 1km, and  $4\Delta x$  for 2km resolution. The

dashed line indicate the solutions with no alongshore variation; these have no resolution dependence.



Figure 10: A) The alongshore velocity at the northern boundary (inflow velocity) and 1000km downwave for an inflow of 60 cm/s over the upper slope. Shown are the depth averaged velocities for the numerical model, the Chapman & Lentz (1997) solution with boundary layer arrest, and the unstratified Arrested

775 Topographic wave solution (ATW). B) Same as A, but with the vertical axis enlarged to show the slope

flows. C) The bathymetry.





Figure 11: A) A cross-shelf temperature section from the Coastal Upwelling Ecosystem Analysis Program at 15°S in Peru on March 19, 1977. Red stars indicate "well-mixed" (as defined in the text) portions of the water column. Blue lines are stars are moorings. B) the lowpass filtered alongshore current at 100m depth on the 121m isobath (roughly 10km offshore). C) The temperature at that mooring at 80 and 100m depth. The darker lines are low passed, the faded lines are the raw hourly data. Blue stars indicate where the water is "well-mixed." D) The frequency within a day of weakly stratified water detected between the 80







Figure 12: figBurgerNumber: The slope Burger number calculated from the Northern Hemisphere 

World Ocean Atlas hydrography for summer and bottom bathymetry from the STRM15 data set, smoothed by a 7km filter for the Mid-Atlantic Bight shelf/slope systems. The red lines indicate the 200

and 2000m isobaths.

#### 795 Supplementary information on the reduced physics model: Evolution of alongshore velocity.

The along-isobath flow is initially balanced by a cross-shelf surface pressure gradient in a stratified ocean which starts with horizontal isopycnals. Given an alongshore periodic domain and the assumption that the relative change in water depth with time is small, the depth and alongshore integrated cross-shelf velocity u must be:

800 
$$\int_{alongshore} dy \int_{-H}^{0} u \, dz = 0 \quad . \tag{S1}$$

801 Using this, the alongshelf momentum equation

802 
$$\frac{\partial v}{\partial t} + \nabla \cdot (\vec{u} \cdot v) + fu = \frac{1}{\rho_0} \frac{\partial P}{\partial y} + \frac{\partial}{\partial z} A \frac{\partial v}{\partial z}$$
(S2)

averaged from the surface to the bottom and over the alongshore domain, under the assumption that thereis no along-shore wind-stress, results in

805 
$$\frac{\partial \langle V \rangle}{\partial t} + \frac{\partial}{\partial x} \langle u'v' \rangle = -\frac{1}{H} \langle \frac{\tau_y}{\rho_0} \rangle$$
(S3)

806 Where  $\langle V \rangle$  is the depth and alongshore averaged alongshore velocity,  $\langle \rangle$  a depth and alongshore 807 average, and *u*' and *v*' are the deviations of the cross- and along-shelf velocity from the depth and 808 alongshore averaged velocity and  $\tau_{bot}^{y}$  is the alongshore bottom stress.

To use (S3) to estimate  $\langle V \rangle$ , the divergence of the cross-shelf advection of alongshelf momentum in that equation  $-\frac{\partial}{\partial x} \ll u'v' \gg -$  must be neglected (this is the term analyzed in Lentz & Chapman, 2004 in a coastal upwelling system). In part for this reason, the modeling described above has been focused on relatively deep slope flows to reduce the magnitude of this advection term, and to ensure weak cross-shelf gradients as described above. In this deeper water there are smaller fractional changes in water depth, reduced cross-shelf gradients in alongshore velocity, and thus reduced cross-shelf divergence in the crossshelf momentum flux. The evolution of the alongshore velocity above the low-PV layer is then

816 
$$\frac{\partial \langle V \rangle}{\partial t} = -\frac{1}{H} \frac{\tau_{bot}^{y}}{\rho_{0}} .$$
 (S4)

817 To find the velocity in the region above the low-PV layer, this averaged velocity needs to be adjusted for 818 the velocity deficit in the low-PV layer caused by the thermal wind shear (9) over the depth of the low-819 PV layer  $\delta_{LPV}$  to get the velocity above the low-PV layer V:

820 
$$V = \langle V \rangle + \frac{\delta_{LPV}}{2f^2H} (fN_0^2 - q_{LPV}) \frac{\partial H}{\partial x}$$
(S5)

#### 821 Supplementary information on the reduced physics model: Bottom PV flux boundary condition.

The boundary condition at the slope is the flux J of potential vorticity out of the bottom boundary. J is composed of an advective component qu and frictional and diabatic components (Benthuysen & Thomas,

824 2012):

825 
$$\boldsymbol{J} = q\boldsymbol{u} + \nabla \boldsymbol{b} \times \boldsymbol{F} - \boldsymbol{D}\omega_a \quad , \tag{S6}$$

826 where F are the frictional terms of the momentum equation

827 
$$\frac{D\vec{u}}{Dt} + f\hat{k} \times \boldsymbol{u} = -\rho_0^{-1}\nabla P + \boldsymbol{F}$$
(S7)

828 and D is the diabatic term of the buoyancy equation

$$\frac{Db}{Dt} = D \quad . \tag{S8}$$

830 In the interior of the ocean and this model, diabatic and frictional processes are weak and the evolution of 831 PV is dominated by advection (e.g. Hallberg & Rhines, 2000; Haynes & McIntyre, 1987); here the advective fluxes are carried by the eddies, and are parameterized by the diffusive model (6). However, in 832 the turbulent BBL diabatic and frictional processes are sinks of PV, and where the isopycnals intersect the 833 bottom these processes must dominate, for there can be no advective fluxes out of the bottom. The flux 834 835 of PV normal to the bottom can be quantified following the existing literature (Benthuysen & Thomas, 836 2012; Taylor & Ferrari, 2010; Wenegrat & Thomas, 2020); the nomenclature here is most similar to that 837 of Benthuysen and Thomas (2012), but with x offshore and y alongshore. It is useful to use a coordinate 838 system  $\hat{x}$ ,  $\hat{y}$ ,  $\hat{z}$  where  $\hat{z}$  is perpendicular to the bottom,  $\hat{y}$  is parallel to the bottom and the coast, and  $\hat{x}$  is parallel to the bottom and perpendicular to the coast. The PV flux of interest is the flux parallel to the 839 840 isopycnals, and it is assumed that the alongshore-averaged isopycnals intersect the bottom 841 perpendicularly (in a fully two-dimensional system, this would be exact because there can be no diffusive 842 or advective flux into the bottom). The component of the PV flux due to diabatic processes,  $-D\omega_a$ normal to the bottom is then (where  $\hat{k}$  is the unit normal perpendicular to the bottom and K is a vertical 843 eddy diffusivity) 844

845 
$$-D\omega_a \cdot \widehat{k} \approx \left( f + \left( \frac{\partial v}{\partial \hat{x}} - \frac{\partial u}{\partial \hat{y}} \right) \right) \frac{\partial}{\partial \hat{z}} \left( K \frac{\partial b}{\partial \hat{z}} \right) \approx 0 \tag{S9}$$

because  $\frac{\partial b}{\partial \hat{z}}$ , is zero. The PV flux along the isopycnal due to viscosity is:

847 
$$\hat{k} \cdot \nabla b \times \mathbf{F} = \frac{\partial b}{\partial \hat{x}} F^{y} - \frac{\partial b}{\partial \hat{y}} F^{x}$$
(S10)

Since there is no net alongshore variation in the full model,  $\frac{\partial b}{\partial \hat{v}}$  is zero in the alongshore average. In the 848 limit of small bottom slope, the cross-shelf bottom-parallel buoyancy derivative  $\frac{\partial b}{\partial \hat{x}} = N_{LPV}^2 \theta$ , where  $\theta$  is 849 the bottom angle  $\tan^{-1} \frac{\partial H}{\partial x}$ ,  $\frac{dH}{dx}$  is the bottom slope and  $N_{LPV}^2$  is the buoyancy frequency  $\frac{\partial b}{\partial z}$  near the 850 bottom. The alongshore frictional term  $F^{\gamma}$  is then assumed to be dominated by the divergence of the 851 vertical turbulent mixing of momentum, so  $F^{y} = \frac{\partial}{\partial z} (\rho_{0} \tau^{y})$ . To estimate the magnitude of  $F^{y}$ , the 852 magnitude of  $\tau^{y}$  at the bottom,  $\tau_{hot}^{y}$  is estimated as the bottom-stress computed from the alongshore 853 854 average bottom velocity  $v_{bot}$ , and the length scale of the vertical derivative is called  $\delta$ , so that the PV flux 855 normal to the bottom is

856 
$$J^{n} = \hat{k} \cdot \nabla b \times \mathbf{F} \approx N^{2} \theta F^{y} = N_{LPV}^{2} \theta \frac{\tau_{bot}^{y}}{\delta \rho_{0}} = N_{LPV}^{2} \theta \frac{\tau_{bot}^{y}}{\delta_{LPV} \rho_{0}}.$$
 (S11)

The vertical length scale  $\delta$  has in other work (e.g. Wenegrat & Thomas, 2020) been set to the vertical scale of the low-PV layer, which is called  $\delta_{LPV}$ . The justification for this is unclear, both in the prior literature and to this author. This assumption would appear inconsistent with the assumption that the PV dynamics are primarily along-isopycnal, unless there is divergence of the flux of momentum carried by the eddies along isopycnals across the low-PV region. The justification for using  $\delta_{LPV}$  remains an open question; however, it is seen below that (*S*11) correctly predicts the flux of PV from the bottom.

The PV flux in (S11) is normal to the bottom, but the model in (6) is written in the cross-shelf coordinate, for the isopycnals that leave normal to the bottom quickly become quasi-horizontal (meters of vertical change over kilometers of across-shelf extent). To ensure that the total flux out of the bottom (the integral along the bottom of the bottom-normal flux) is consistent with the total flux into all the possible models along each isopycnal (the vertical integral of the flux in the cross-shelf direction for all isopycnals), the flux into (6) must be divided by the bottom slope. At small slope angles, the slope angle  $\theta$  is approximately equal to the bottom slope, and the boundary condition on (6) at the slope is

870 
$$A_h \frac{\partial q}{\partial x} = \frac{J^n}{\frac{\partial H}{\partial x}} \approx N^2 \frac{\tau_{bot}^y}{\delta \rho_0}$$
(S12)

## 871 Supplementary information: Animations

Figures 6 and 7 are provided as animations over time from 1 to 60 days in the files figure6\_animated.mp4and figure7\_animated.mp4.

#### 874 Supplementary information: Reduced physics model

875 The reduced physics model code, and the code that generates Figure 8, are included in

876 ReducedPhysicsModelCode.zip