Cross-sectional Variables of Alluvial Channels

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Abstract

In the process of a fluvial evolution, the water discharge, sediment charge and stream energy expenditure dominant the channel patterns of a river. Given water and sediment, an alluvial channel is self-organizing, adjust to achieve a stable equilibrium state, and form a characteristic channel geometry (channel width, depth and slope). In the equilibrium condition which is also said to be in regime or graded, the flux of the water and sediment from the watershed should be equal to the flux of the downstream channel(s). By studying bed load transportation and stream power conversion on a steady and uniform stream, we suggest two characteristic parameters that are energy conversion length and regime transportation length of sediment. The regime equation and equations of fractal features are set here. All cross-section variables (stream width, depth and velocity) of a regime stream, who theoretically derived under the equilibrium of sediment transportation and the conversion of stream power, are exclusively determined by the two lengths and the water discharge.

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Key words: morphodynamics; fluvial process; regime equation; bedload transport; stream power conversion

Key Points:

1) The cross-section dimension of an alluvial stream is determined by the water discharge and regime transportation length of sediment.

2) The stream width is determined by the water discharge and the energy conversion length.

3) The stream power expenditure (the energy conversion length) is relevant to Kolmogorov's microeddies in turbulence.

Abstract

In the process of a fluvial evolution, the water discharge, sediment charge and stream energy expenditure dominant the channel patterns of a river. Given water and sediment, an alluvial channel is self-organizing, adjust to achieve a stable equilibrium state, and form a characteristic channel geometry (channel width, depth and slope). In the equilibrium condition which is also said to be in regime or graded, the flux of the water and sediment from the watershed should be equal to the flux of the downstream channel(s). By studying bed load transportation and stream power conversion on a steady and uniform stream, we suggest two characteristic parameters that are energy conversion length and regime transportation length of sediment. The regime equation and equations of fractal features are set here. All cross-section variables (stream width, depth and velocity) of a regime stream, who theoretically derived under the equilibrium

of sediment transportation and the conversion of stream power, are exclusively determined by the two lengths and the water discharge.

Plain Language Summary

The dynamic characteristics of every river on the Earth are different. In order to grasp the principles of river evolution and changing processes, People have done a lot of exploration, but so far, no ideal results have been obtained. The disciplines of sediment transportation, stream energy conversion and channel fractal features are utilized, here, as constrain equations to study the crass-sectional variables of river evolution. We suggest two characteristic length that are the regime transportation length and the energy conversion length from stream power to turbulence energy. The bigger value of the regime transportation length means that the land of the basin is difficult to erode and the less sediment load is required for channel to transport. And the bigger value of the energy conversion length requires smaller bed area per unit stream reach (stream width *B*) for converting stream power to turbulence energy. In the paradigm of Newtonian mechanics, the kinetic relationships among variables of a channel cross-section are theoretically given. These relationships can help us to understand the self-sculpting behavior of fluvial processes.

1 Introduction

A river is a product of the interaction between its water flow and channel bed, which is achieved by means of sediment scouring and depositing on the bed. Sediment materials are sometimes as a component that takes part in the water flow or sometimes a component of the loose bed. The water flow acts on and shapes the loose bed. Meanwhile, the bed constrains the flow and affects the flowing structure. The water flow and the loose boundary are interdependent, inter-played, and mutually organized, and are always in the process of changing and adjusting. As long as the conditions of discharge and imposed sediment remain undischarged, there is an over-all tendency for the stream to approach a grade condition by self-adjustment. A stream in equilibrium, or graded, is also said to be in regime. Therefore, a regime stream is the outcomes of the self-organization and evolution according to certain dynamic principles. But there is no certainty that any stream ever has attained such an equilibrium condition in the past, and there are none at present.

However, for engineering purpose, there is extremely importance that the natural tendencies of the individual stream should be well understood the hydrologic and geomorphic reaction which it may develop when disturbed by human activities. The study on the basic principles of the self-organization of a fluvial process has a long history, and many hypotheses are proposed, of which some representatives are the uniform distribution of energy dissipation and minimum total work in the system (Leopold, 1964), minimum variance (Langbein, 1964), minimum channel bed activity (Dou, 1964), minimum unit stream power (Yang , 1971), critical initiative condition (Li, 1976; Parker, 1978), minimum function of channel system(Chang, 1979), maximum energy dissipation rate (Huang, 1981), maximum sediment transport rate (White, 1982), maximum resistance coefficient (Davis, 1983) and so on. All these hypotheses derived a series of semi-theoretical and semi-empirical formulas, which cannot be proved rigorously and theoretically and may lead to conclusions incompatible with some observations. Therefore, the study of the principles of self-adjustment of the fluvial process is still a difficult task in river dynamics.

The geometric shape of a channel formed by self-organization is mainly determined by the factors such as water discharge, the rate of sediment supply and the debris size (Naito and Parker, 2019). For the case where the channel flow's intensity is greater than the sediment starting condition and less than the suspending condition, only the bed load exists. When the flow intensity is greater than the suspending condition of sediment, the rate of bed load and the rate of suspended load increase and decrease synchronously. So the bed load transport is accompanied all time with the fluvial process. Especially in the upstream part of a river basin, the transport of sediment is mainly bed load. The overall patterns of a river are principal components of the pool - riffle - bar unit and modes of bar development. In a fluvial process, the bar evolution is the essential connection between the episodic nature of bed material transport and the production of river morphology (Church and Ferguson, 2015). The bed load continuously takes part in the process of the river evolution and inherently deforms channel patterns laterally, such as straight, meandered or braided channels. And the channel morphology is an inevitable outcome of following the principles of self-organization, which is mainly trimmed and shaped by the bed load (Dodov and Foufoula-Georgiou, 2005).

Based on the above understanding, the study on the cross-sectional variables of river channels is implemented in combining the channel hydraulics of sediment transportation and stream power expenditure and fractal features of channels.

2 Bed load transport rate of an alluvial river

2.1 Effective shear stress of sediment transport

The stream power of an alluvial channel is usually consumed in three aspects, which overcomes the form resistance of the channel, the shear stress of transporting sediment and the shear stress acting on the stationary particles on the channel bed namely. The shear stress of the water flow acts on sediment particles is called the effective shear stress in river dynamics. Einstein (1950) proposes that the effective shear stress τ_b' of water flow obeys the logarithmic law of turbulence

$$\frac{u}{u_*} = 2.5\ln\left(30.2\frac{\chi y}{k_s}\right) \tag{2-1}$$

where u is the velocity of water flow; u_* is the shear velocity; k_s is the roughness element dimension of river bed; χ is the parameter for transition smooth-rough (Einstein gives the calculation curve); y is the vertical distance from the bed.

For the convenience of calculation, Fan (1992, 1995) gives the fitting equations of the calculation curve of χ as

$$\begin{cases} \dfrac{k_s}{\delta} \le 0.2, \quad \chi = 3.5\dfrac{k_s}{\delta}; \\ 0.2 \le \dfrac{k_s}{\delta} \le 0.431, \quad \chi = 0.755\ln\left(\dfrac{k_s}{\delta}\right) + 1.923; \\ 0.431 \le \dfrac{k_s}{\delta} \le 2.0, \quad \chi = 1.613\cos\left(0.767\ln\dfrac{k_s}{\delta}\right); \\ \dfrac{k_s}{\delta} \ge 2.0, \quad \chi = 1.5\left(\dfrac{k_s}{\delta}\right)^{-2} + 1.0 \end{cases} \tag{2-2}$$

where $\delta = 11.6\dfrac{\nu}{u_*}$ is the nominal thickness of the boundary layer, ν is the kinematic viscosity coefficient of water.

The roughness of the bed surface k_s in Eq. (2-1) is twice the particle size d of the sediment on a river bed which is the median particle size of bed load material d_{50} (Jeremy et al, 2015). Averaging Eq. (2-1) along the water depth for the wide-channel approximation, yields

$$\frac{U}{\sqrt{gHJ}} = 2.5\ln\frac{11\chi H}{2d} \tag{2-3}$$

where U is the average velocity, H is the average water depth, J is the slope of water flow, and g is the gravity acceleration.

According to Einstein's (1950) recommendation, the formula for calculating the effective hydraulic radius R_b' of the effective shear stress τ_b' can be obtained by replacing H with R_b'. This becomes

$$\frac{U}{\sqrt{gR_b'J}} = 2.5\ln\frac{11\chi R_b'}{2d} \tag{2-4}$$

In order to express simply, the effective shear stress τ_b' is written in dimensionless form θ_b'. Consequently,

$$\theta_b' = \frac{\tau_b'}{(\gamma_s - \gamma)d} = \frac{u_*'^2}{\frac{\gamma_s - \gamma}{\gamma} gd} \tag{2-5}$$

where γ_s is the specific gravity of sediment particles; γ is the specific gravity of water; u_*' is effective shear velocity .

The relationships among effective shear stress τ_b', effective shear velocity u_*', and effective hydraulic radius R_b' are $u_*' = \sqrt{\tau_b' / \rho} = \sqrt{gR_b'J}$, in which ρ is the density of the liquid.

In hydraulic calculations, the average water depth H of a channel is commonly used, and does not appear in Eq. (2-4). But the slope J of water flow appears in the equation which is not easy to determine and brings inconvenience to calculate the effective shear stress. Alternatively, in order to utilize the average water depth H in calculating the dimensionless effective shear stress θ_b', let Eq.(2-4) be changed into

$$\sqrt{\frac{U^2}{\frac{\gamma_s - \gamma}{\gamma} gd} \frac{1}{\theta_b'}} = 2.5\ln\left(\frac{11\chi}{2}\frac{\gamma_s - \gamma}{\gamma}\frac{\theta_b'}{J}\right) \tag{2-6}$$

From Eq.(2-2), the smooth-rough transition parameter χ is deduced as a function of $\frac{\omega d}{\nu}\sqrt{\theta_b'}$

$$\chi \sim \frac{k_s}{\delta} = \frac{2}{11.6}\frac{u_*'d}{\nu} = \frac{2}{11.6}\frac{\sqrt{\frac{\gamma_s-\gamma}{\gamma}gd}\,d}{\nu}\frac{u_*'}{\sqrt{\frac{\gamma_s-\gamma}{\gamma}gd}} \sim \frac{\omega d}{\nu}\sqrt{\theta_b'} \tag{2-7}$$

where ω is the terminal settling velocity of a single sediment particle in still clear water. For the medium sediment particle, the settling velocity is suggested to be calculated by the formula of Zhang (1961), which is

$$\omega = \sqrt{\left(13.95\frac{\nu}{d}\right)^2 + 1.09\frac{\gamma_s-\gamma}{\gamma}gd} - 13.95\frac{\nu}{d} \tag{2-8}$$

In accordance with Eq.(2-6) and Eq.(2-7), the dimensionless effective shear stress θ_b' is a function of $\frac{\omega d}{\nu}$, $\frac{U^2}{\frac{\gamma_s-\gamma}{\gamma}gd}$ and the slope J.

The well-known Manning formula reads

$$U = \frac{1}{n_m}R_b^{2/3}J^{1/2} \tag{2-9}$$

where n_m is Manning's roughness value; R_b (in meters, m) is the hydraulic radius that is about equal to the average depth H ($R_b = H$) for wide channel approximation; U is in meters per second, m/s.

Eq. (2-9) can be changed into

$$J = \frac{n_m^2U^2}{R_b^{4/3}} = \frac{n_m^2}{\nu^{4/3}}\left(\frac{\gamma_s-\gamma}{\gamma}gd\right)^{10/3}\left(\frac{U^2}{\frac{\gamma_s-\gamma}{\gamma}gd}\right)^{10/3} \Bigg/ \left(\frac{UR_b}{\nu}\right)^{4/3} \tag{2-10}$$

As Manning's roughness coefficient n_m is a function of the particle size d of the sediment on bed-surface in alluvial channel, it is considered that the term

$\frac{n_m^{\ 2}}{\nu^{4/3}}\left(\frac{\gamma_s-\gamma}{\gamma}gd\right)^{10/3}\Big/\frac{\gamma_s-\gamma}{\gamma}$ is a function of $\frac{\omega d}{\nu}$. Thus, being deduced from Eq. (2-6), Eq. (2-7) and Eq. (2-10), the dimensionless effective shear stress θ_b' is a function of $\frac{\omega d}{\nu}$, $\frac{U^2}{\frac{\gamma_s-\gamma}{\gamma}gd}$ and $\left(\frac{U^2}{\frac{\gamma_s-\gamma}{\gamma}gd}\right)^{1.25}\Big/\sqrt{\frac{UR_b}{\nu}}$. The symbol $\frac{\omega d}{\nu}$ represents the dimensionless size of the sediment particle, $\frac{U^2}{\frac{\gamma_s-\gamma}{\gamma}gd}$ the moving capacity of the sediment particles and $\frac{UR_b}{\nu}$ the strength of the flow turbulence. And the empirical expression(Fan, 1992 and 1995) is simply given

$$\theta_b' = f'\left(\frac{\omega d}{\nu}\right)\cdot F_b^n \tag{2-11}$$

where *n* is the exponent that is given as

$$n=\begin{cases}0.7; & \frac{\omega d}{\nu}<1\\ 0.8; & \frac{\omega d}{\nu}\geq 1\end{cases} \tag{2-12}$$

the function $f'\left(\frac{\omega d}{\nu}\right)$ is given as following

$$f'\left(\frac{\omega d}{\nu}\right)=\begin{cases}0.145\left(\frac{\omega d}{\nu}+0.01\right)^{0.152}; & \frac{\omega d}{\nu}<15\\ 0.048\left(\frac{\omega d}{\nu}-8.8\right)^{0.38}; & \frac{\omega d}{\nu}\geq 15\end{cases} \tag{2-13}$$

and F_b is the comprehensive capacity parameter for the alluvial channel flow, which is

$$F_b = \left(\frac{U^2}{\frac{\gamma_s - \gamma}{\gamma} gd} \right)^{1.5} \Bigg/ \sqrt{\frac{UR_b}{\nu}} \tag{2-14}$$

The curve of Eq. (2-11) has two parts that belong to smooth wall and rough wall (Fan, 2017) of water flow respectively, which is disconsecutive at $\frac{\omega d}{\nu} = 15$.

2.2. The average velocity of bed load particles

According to Newton's law, the following relationship can be presented for a design moving discrete particle with velocity v_s

$$\rho_s \frac{\pi}{6} d^3 \frac{dv_s}{dt} = F_D - F_f \tag{2-16}$$

where F_D is the tractive force acting on the moving discrete particle; F_f is the frictional force for resisting the movement of the particle.

The tractive force can be expressed as

$$F_D = \frac{C_D}{2} \frac{\pi}{4} d^2 \rho \left(u - v_s \right)^2 \tag{2-17}$$

where C_D is drag force coefficient C_D =1.2 (Cheng, 1997) for the stationary particles on bed and C_D =0.82 for the moving particles (Zhang G., et al., 2016).

And the frictional force is

$$F_f = C_f \left(\gamma_s - \gamma \right) \frac{\pi}{6} d^3 \tag{2-18}$$

where C_f is dynamic friction factor of submerged sediment particles $C_f = 0.5$ (Engelund and Fredsoe 1976).

Combining Eqs. (2-16), (2-17) and (2-18), we get

$$\frac{dv_s}{dt} = \frac{C_f g}{V_0^2} \frac{\gamma_s - \gamma}{\gamma} \left[\left(u - v_s \right)^2 - V_0^2 \right] \tag{2-19}$$

where $V_0 = \sqrt{\frac{4C_f}{3C_D} \frac{\gamma_s - \gamma}{\gamma} gd} = 0.9\sqrt{\frac{\gamma_s - \gamma}{\gamma} gd}$.

It is observed on channel bed that sediment particles take action suddenly for moving or stopping, which means the acceleration time of a particle is negligible compared with its entire movement time. Let T is the movement period of the particle, getting

$$\frac{1}{T}\int_0^T \frac{dv_s}{dt} dt \approx 0 \tag{2-20}$$

Take Eq. (2-19) in time average and combine Eq. (2-20), yielding

$$v_s = u - V_0 \tag{2-21}$$

Since the thickness of the bed load layer is 2d (Einstein, 1950), the average velocity of bed load particles is expressed as

$$V_s = \frac{1}{2d}\int_0^{2d} v_s dy \tag{2-22}$$

If Eqs. (2-1), (2-21) and (2-22) are combined, the average velocity of bed load particles is obtained

$$V_s = \varphi_0 u'_* - V_0 \tag{2-23}$$

where $\varphi_0 = 2.5\ln\left(11\chi\right)$ is the wall law coefficient of water flow.

2.3. The number of bed load particles

Following R. A. Bagnold's bed load mechanics (1966), the bed load layer is a water-sediment mixture flowing over a gravity bed. The effective shear stress τ'_b of sediment transport is equal to two parts, viz.

$$\tau_b' = \tau_c' + T_s \tag{2-24}$$

where τ_c' is the critical shear stress; T_s is the summing tractive force of flow in the 2*d* thickness of bed load layer.

If there are *N* sediment particles in quantity moving on the channel bed per unit area, the summing tractive force T_s, acting on the moving discrete particles *N* , gives similar to Eq.(2-17)

$$T_s = N\frac{C_D}{2}\frac{\pi}{4}d^2\rho\left(u - v_s\right)^2 \tag{2-25}$$

Substituting of Eq. (2-21) in Eq. (2-25), yields

$$T_s = NC_f\frac{\pi}{6}\left(\gamma_s - \gamma\right)d^3 \tag{2-26}$$

Substituting of Eq. (2-26) in Eq. (2-24), the number of sediment particles *N* is obtained

$$N = \frac{1}{C_f}\frac{\tau_b' - \tau_c'}{\frac{\pi}{6}\left(\gamma_s - \gamma\right)d^3} \tag{2-27}$$

2.4. Bed load transport of channel flow

The amount of bed load transport per unit time is called the rate of bed load transport and represented by the symbol "G_B" in kg/s or T/s. And the rate of bed load transport per unit width of a channel is commonly expressed in the symbol "g_b" in kg/s/m or T/s/m. For the sediment particles per unit bed area *N* (in 1/m^2) along flowing direction with average velocity V_s, the bed load transport rate per unit width should be

$$g_b = \rho_s\frac{\pi}{6}d^3NV_s \tag{2-28}$$

where ρ_s is the density of the liquid in T/m^3.

Combining Eqs. (2-23), (2-27) and (2-28), we obtain the rate of bed load transport per unit width in dimensionless form which is quite similar to many other sediment transport functions (Meyer-Peter and Mueller, 1948; Engelund and Fredsoe, 1976; Whipple and Tucker, 1999)

$$\Phi_b = 2(\theta_b' - \theta_c')(\varphi_0 \sqrt{\theta_b'} - 0.9) \tag{2-29}$$

where $\Phi_b = \dfrac{g_b}{\rho_s d \cdot \sqrt{\dfrac{\gamma_s - \gamma}{\gamma} gd}}$ is the dimensionless bed load transport rate per unit width, $\theta_c' = \dfrac{\tau_c'}{(\gamma_s - \gamma)d} = 0.045$ is the dimensionless critical shear stress (Jeremy et al., 2015), the symbol φ_0 is the same physical descriptor in Eq. (2-23).

3 Features of steady uniform stream with equilibrium sediment transportation

3.1. The geometry dimension of a stream cross-section

If a channel has a movable boundary, however, then the stream width and depth can change, along with the establishment of the channel slope. The adjusted cross-section dimension will largely depend upon the ability of the flow to transport its sediment charge.

For the bed load transportation of the stream, the sediment particles are big enough that generally $\dfrac{\omega d}{\nu} \geq 1$ is tenable, so n = 0.8 in Eq. (2-11), namely

$$\theta_b' = f'\left(\frac{\omega d}{\nu}\right) \cdot F_b^{0.8} \tag{3-1}$$

When the effective dimensionless shear stress θ_b' is much larger than the dimensionless critical shear stress $\theta_b' \geq \theta_{\mathrm{c}}'$ in Eq. (2-29), as a reasonable approximation, the non-dimensional bed load rate is simplified as

$$\Phi_b \approx 2\varphi_0 \theta_b'^{1.5} \tag{3-2}$$

The relationship between the bed load transport rate per unit width g_b and the rate of bed load transport G_B of the stream is

$$g_b = \frac{G_B}{B} \tag{3-3}$$

where B is stream width.

The relationship between water discharge Q and the variables of a cross-section (width, depth and velocity) is

$$Q = BHU \tag{3-4}$$

where H is the average depth and U the average velocity of the cross-section.

For the alluvial channel, the cross-sections are generally wider which are $\frac{B}{H} > 10$. The hydraulic radius R_b is approximately equal to the average depth H of the stream. Substituting of Eq. (3-1) in Eq. (3-2) with combining Eqs. (2-14), (3-3), and (3-4), yields

$$\frac{\sqrt{g}BH^{1.8}}{Qd^{0.3}}\left(\frac{G_B}{\gamma_s Q}\right)^{1/2} = \left(\frac{2\varphi_0\gamma}{\gamma_s - \gamma}\right)^{1/2}\left[f'\left(\frac{\omega d}{\nu}\right)\right]^{3/4} \Bigg/ \left(\frac{\sqrt{\frac{\gamma_s - \gamma}{\gamma} gd \cdot d}}{\gamma}\right)^{0.3} \tag{3-5}$$

When the particles of bed load materials are not very fine sand, for $\frac{\omega d}{\nu} \geq 15$, we obtain

$$\left[f'\left(\frac{\omega d}{\nu}\right)\right]^{3/4} \Bigg/ \left(\frac{\sqrt{\frac{\gamma_s - \gamma}{\gamma} gd \cdot d}}{\gamma}\right)^{0.3} \approx 0.38 \tag{3-6}$$

For the full rough bed, the Einstein's parameter for transition smooth-rough is $\chi = 1$. The wall law coefficient is $\varphi_0 = 2.5\ln(11) = 6$. Therefore, Eq. (3-5) can be approximately written as

$$\frac{\sqrt{g}BH^{1.8}}{Qd^{0.3}}\left(\frac{G_B}{\gamma_s Q}\right)^{1/2} = 1.0 \tag{3-7}$$

From Eq. (3-7), the geometric dimension of the stream can be obtained with response to the water discharge, sediment discharge and the debris size

$$BH^{1.8} = \left(\frac{\gamma_s Q}{G_B} d^{0.6}\right)^{1/2} \frac{Q}{\sqrt{g}} \tag{3-8}$$

The left side of Eq. (3-8) is the cross-section dimension $BH^{1.8}$ of the stream. And the water discharge at left of the right side of the equation presents the scale of the stream. The physical meaning of Eq. (3-8) is that the cross-sectional dimension $BH^{1.8}$ of the stream should meet the need of the conveyance of the water in term $\frac{Q}{\sqrt{g}}$ and keep the flow ability for transporting sediment concentration $\frac{G_B}{\gamma_s Q}$ with the debris size *d* that are delivered from its watershed.

As erosion occurs in a river basin, sediment enters its channel(s). A process of sediment transportation by a channel is the process that the channel adjusts its self to achieve a balance between sediment supply and sediment transportation. The charge ratio $\frac{\gamma_s Q}{G_B}$ of water to sediment in Eq. (3-8) is the volume of runoff required to produce unit volume of sediment from the basin and is also the discharge required to transport the same (unit) volume of sediment to the downstream for the equilibrium of sediment transport. The bigger is the charge ratio $\frac{\gamma_s Q}{G_B}$ and the debris size *d*, the stronger the resistance to land erosion of the basin. Therefore, the term $\frac{\gamma_s Q}{G_B} d^{0.6}$ in Eq. (3-8) indicates the synthesized resistance to land erosion.

It is realized that parameterization of sediment transport must be an integral part of any rational description of river regime and morphodynamics (Church and Ferguson, 2015). And the term $\frac{\gamma_s Q}{G_B} d^{0.6}$ in Eq. (3-8) is just such an integral part, thus, we suggest a synthesized parameter l_d in the unit of length

$$l_d = \left(\frac{\gamma_s Q}{G_B} \right)^{5/3} d \tag{3-9}$$

And Eq. (3-7) becomes

$$BH^{1.8} = l_d^{\ 0.3} \frac{Q}{\sqrt{g}} \tag{3-10}$$

From the perspective of alluvial dynamics, Eq. (3-9) and Eq. (3-10) state that the channel automatically adjusts its cross-section geometries to equilibrium conditions to fit the transportation of the water discharge and sediment charge accompanied by its debris size. Therefore, the synthesized parameter l_d is called regime transportation length of sediment. And Eq. (3-10) is the regime equation for steady uniform streams of alluvial channels.

3.2. Energy conversion of the stream

Practically, all problems involve non-uniform (or varied) flow in fluvial processes. However, in order to simplifying researching, the alluvial channel problems of a given reach can be treated in terms of an approximate solution based on an assumption of steady uniform flow. The uniform flow has an equilibrium between gravitational and frictional forces, that is, the gravitational component in the direction of flow must balance the frictional resistance.

The frictional force is basic to the energy dissipation process in the turbulence field of a channel flow. It makes the irrecoverable conversion of stream power into turbulence energy, and finally into heat energy. In turbulence field, the players of the turbulence energy transferring and dissipating are eddies with different scales. In the continuous cycle of turbulent activities in the channel flow, large momentum bodies with small-size eddies in the form of vortex clusters promote a series of bursting of big eddies from the channel bed during the processes of sweeping the bed surface, and make the transfer of momentum. The big eddies later split themselves into smaller eddies step by step with energy consumption and momentum transfer. Very small-size eddies diffuse and dissolve in the whole turbulence fluid finally. This cycling movement of the vortex clusters and eddies provides the required energy for the turbulence field, maintains the various functions of the channel flow, and ensures the transportation of the water discharge and sediment charge with its composition.

Stream power quantifies the rate of potential energy expended by stream flow on the bed and banks and dominates the transportation of sediment (Bagnold, 1966;

Whipple & Tucker, 1999; Eaton & Church, 2011).The stream power per unit stream length γQJ is converting into turbulence energy at the wetted boundary of the channel. The surface of the wetted boundary is the platform where the stream power is turning into turbulence energy. For the alluvial channel flow, the bigger is the energy value γQJ, the larger the wetted boundary area needed per unit stream length, or the longer the wetted perimeter needed.

The shear stress τ_b is

$$\tau_b = \rho u_*^2 = \rho g R_b J \tag{3-11}$$

For wide channel approximation, the wetted perimeter is approximated to equal to the channel width *B*. The following expression can read

$$R_b = H \tag{3-12}$$

Eq. (3-4) can be changed as

$$\gamma QJ = \tau_b UB \tag{3-13}$$

The physical meaning of Eq. (3-13) is that the stream power γQJ is converted into turbulent energy $\tau_b UB$.

For a reach in unit stream length, discharge *Q* is the only independent variable. But the other variables of Eq. (3-13) are all dependent variables. Therefore, the channel width *B* can be considered as a function of the water discharge *Q* in mathematical logic. It is given by Leopold & Maddock (1953) as following

$$B = \alpha_1 Q^{\beta_1} \tag{3-14}$$

where α_1, who can be considered to be independent of channel width B in the point of mathematical logic, is a coefficient related to the stream energy transferring; β_1 is an exponent.

3.3. The fractal features of alluvial channels

It has been noticed for a long time that the channel network of a river basin is similar to the branches of botanical trees (Mandelbrot, 1983). In fact, the morphology of channel networks is similar to that of tree roots which are all confluence growth and have fractal characteristics. Based on the fractal relationship between area and diameter, Mandelbrot(1983) introduces diameter exponent Δ, which is expressed as

$$d_t^{\Delta} = d_{t1}^{\Delta} + d_{t2}^{\Delta} \tag{3-15}$$

where d_t is the diameter of botanical trees, and the exponent is $\Delta=2$.

If each of the fibers that constitute botanical trees has the same cross-section area a_f at every stage of its height, the total numbers of the fibers are n_f. And each area of tributaries is

$$A_t = \sum_{i=1}^{n_f} a_f\,, \qquad A_{t1} = \sum_{i=1}^{n_{f1}} a_f\,, \qquad A_{t2} = \sum_{i=1}^{n_{f2}} a_f$$

where A_t is the cross-section area of botanical trees; n_{f1} and n_{f2} are the numbers of n_f in the tributaries A_{t1} and A_{t2} respectively.

As $n_f = n_{f1} + n_{f2}$, thus the area relationship of botanical trees is

$$A_t = A_{t1} + A_{t2} \tag{3-16}$$

where A_t is the cross-section area of botanical trees.

Therefore

$$\frac{\pi}{4} d_t^2 = \frac{\pi}{4} d_{t1}^2 + \frac{\pi}{4} d_{t2}^2$$

And

$$d_t^2 = d_{t1}^2 + d_{t2}^2 \tag{3-17}$$

Thus, the exponent $\Delta=2$ in Eq. (3-17) indicates that the area of every fiber along the tree trunk is unchanged.

Studying on the fractal features of the Mississippi River in the United States, Mandelbrot considers that the bankfull widths of the tributaries are also in accordance with Eq. (3-17), which the exponent Δ=2 is obtained for the widths of the channels of Mississippi River, which is the same as the diameter exponent of botanical trees.

For a steady uniform stream network in accordance with Eq. (3-17), it is considered that the streams of the network have the fractal feature in the following form (Mandelbrot,1983)

$$B^{\Delta} = \sum_{i=1}^{n} B_i^{\Delta} \tag{3-18}$$

where $i = 1, 2, ..., n$; n is the number of tributaries.

Let's simplify the stream network what two channels meet only, and marked by "1" and "2" for each. The water discharge of channel "1" is Q_1, and of channel "2" Q_2. The water discharge of the main channel downstream the confluence of the channel "1" and "2" is Q. According to the law of continuity and Eq. (3-18), yields

$$\begin{cases} Q = Q_1 + Q_2 \\ B^{\Delta} = B_1^{\Delta} + B_2^{\Delta} \end{cases} \tag{3-19}$$

If Eq. (3-19) does hold unconditionally, there must be

$$\frac{B^{\Delta}}{Q} = \frac{B_1^{\Delta}}{Q_1} = \frac{B_2^{\Delta}}{Q_2} = P_{\varepsilon} \tag{3-20}$$

where P_{ε} is a proportional parameter. Since channel "1" or "2" is arbitrary tributary, this parameter P_{ε} is independent of channel width B and flow discharge Q in the point of mathematical logic.

Therefore

$$B^{\Delta} = P_{\varepsilon} Q \tag{3-21}$$

This is another form of Eq. (3-14), thus, the proportional parameter P_{ε} should be a related to the energy conversion of the stream because it represents the parameter α_1 of Eq. (3-14), which implies that the fractal features of Eq. (3-18) and Eq. (3-20) are dominated by the energy conversion from stream power into turbulence energy per unit time.

Combining Eq. (3-10) of sediment transportation and Eq. (3-20) of the fractal feature, yields

$$\frac{B^{\Delta\text{-}1} l_d^{\ 0.3}}{H^{1.8}} = \frac{B_1^{\Delta\text{-}1} l_{d1}^{\ 0.3}}{H_1^{\ 1.8}} = \frac{B_2^{\ \Delta\text{-}1} l_{d2}^{\ 0.3}}{H_2^{\ 1.8}} = \sqrt{g} P_\varepsilon \tag{3-22}$$

Thus

$$\frac{B^{\Delta\text{-}1}}{H^{1.8}} = \frac{\sqrt{g} P_\varepsilon}{l_d^{\ 0.3}} \tag{3-23}$$

This fractal feature of Eq. (3-23) implies the embodiment of morphodynamic paradigms because the fractal feature of a cross-section is produced by the energy expenditure and sediment transportation of the stream. It shows the consistency of natural phenomenon and mechanism.

Due to the diversity of geological and geomorphological conditions and the hydrometeorological environment, channel networks in large river basins have formed various patterns. Only vary small catchments with a single river, the river may have a single stable form. Regardless of whether it is a large or a small river basin, rainfall and soil erosion in the catchments provide the river(s) with incoming water and sediment and the sediment composition. In order to match such conditions of the incoming water and sediment and sediment composition, the channels in the basins evolve into corresponding channel morphology for transporting the corresponding water and sediment. The geometric shape of the channel cross-section should be affected not only by sediment transportation as Eq. (3-10) showing, but also by stream energy expenditure as Eq. (3-14) stating. Thus, substituting Eq. (3-14) into Eq. (3-10), yields

$$H^{1.8} = \frac{l_d^{\ 0.3}}{\alpha_1 \sqrt{g}} Q^{1-\beta_1} \tag{3-24}$$

Dividing Eq. (3-14) by Eq. (3-24), yields

$$\frac{B}{H^{1.8}} = \frac{\alpha_1^{\ 2} \sqrt{g}}{l_d^{\ 0.3}} Q^{2\beta_1 - 1} \tag{3-25}$$

Here, the term $\frac{B}{H^{1.8}}$ that reflects the cross-section shape of a channel is a symbol not related to B because all factors at the right side of Eq.(3-25) are not related to B where α_1 and β_1 have been mentioned in Eq.(3-14).

Because the regime transportation length l_d of sediment is not related to channel width B, all factors at the right side of Eq.(3-23) are irrelevant to channel width B. Because Eq. (3-21) is another form of Eq. (3-14), Eq. (3-23) and Eq. (3-25) should be the same. So there must be

$$\begin{cases} \Delta - 1 = 1 \\ 2\beta_1 - 1 = 0 \\ P_\varepsilon = \alpha_1^{\ 2} \end{cases} \tag{3-26}$$

In order to simply state the characteristics of the coefficients α_1 and P_ε on energy converting from stream energy to turbulence energy, we suggest an factor l_ε in the dimension of length for the expression of α_1 and P_ε that can be written in harmonious form of dimension

$$\alpha_1 = \sqrt{P_\varepsilon} = \left(\frac{1}{g l_\varepsilon} \right)^{1/4} \tag{3-27}$$

where l_ε is named energy conversion length.

The energy conversion length l_ε that is another integral part of the description of river regime and morphodynamics, is also a parameterization of the stream power expenditure, which is influenced by many factors, such as the shape of a channel, the debris size on bed surface and the channel slope.

Combining Eq. (3-25), Eq. (3-26) and Eq. (3-27), yielding

$$\frac{B}{H^{1.8}} = \frac{1}{l_\varepsilon^{\ 0.5}} \frac{1}{l_d^{\ 0.3}} \tag{3-28}$$

If the channel boundary is consisted of sedimentation mainly, Eq. (3-28) presents that the channel shape $\frac{B}{H^{1.8}}$ is determined by two factors, one is the transportation length l_d, the other is the energy conversion length l_ε.

Substituting Eq. (3-26) into Eq. (3-18), the width relationship between a channel stream and the tributaries is obtained

$$B^2 = \sum_{i=1}^{n} B_i^2 \tag{3-29}$$

Eq. (3-29) states two meanings. One shows the fractal relation of an alluvial channel network. The another presents dynamic relation of the bed area required per unit stream length for transferring stream power into turbulence energy.

Substituting Eq. (3-26) and Eq. (3-27) into the fractal formula of Eq. (3-21), yields

$$B = \left(\frac{1}{l_\varepsilon}\right)^{1/4} \left(\frac{Q}{\sqrt{g}}\right)^{1/2} \tag{3-30}$$

Under the aforementioned recognition, Eq. (3-14) presents the channel bed area per unit stream length for transferring stream power γQJ to turbulence energy. By combining the stream power conversion formula Eq. (3-14) with Eq. (3-26) and Eq. (3-27), the resulting equation is Eq. (3-30), too. The same outcomes of the two methods of stream power conversion and fractal analyzing reveal the consistency between physical mechanism and natural phenomena.

For an equilibrium alluvial channel, or a channel graded, as the value *B* of stream width is the bed area per unit stream length, Eq. (3-30) implies this area is required for transferring stream power γQJ to turbulence energy. The powers of the two terms $\frac{1}{l_\varepsilon}$ and $\frac{Q}{\sqrt{g}}$ are 1/4 and 1/2 respectively. And the bigger value of the energy conversion length l_ε requires smaller place area per unit stream reach (stream width *B*) for converting stream power to turbulence energy. In order to clarify the physical meaning, Eq. (3-30) is call energy conversion equation.

3.4. Cross-sectional hydraulic variables and relations

For a loose boundary channel, the ability of the flow to transport its sediment charge will largely dependent upon the flow velocity or/and slope. The adjusted flow velocity and slope are the outcomes of the stream power conversion and the equilibrium sediment transportation of a stream during fluvial processes.

According to the continuity formula Eq. (3-4), the equilibrium sediment transportation Eq. (3-10) can be changed into

$$U=\left(\frac{H}{l_d}\right)^{0.3}\sqrt{gH} \tag{3-31}$$

For a steady uniform and equilibrium sediment transportation stream, Eq. (3-31) gives the relation between the average velocity and the average water depth of the cross section of the stream.

Along with the establishment of a channel slope (graded channel), the stream width and depth can change on the loose channel bed. Then, the velocity of flow depends not only on discharge but also on the adjusted width and depth, and each of which also depends on discharge. Substituting Eq. (3-28) into Eq. (3-30), the relationship between water depth and flow discharge is yielding

$$H=l_d^{1/6}l_\varepsilon^{5/36}\left(\frac{Q}{\sqrt{g}}\right)^{5/18} \tag{3-32}$$

The area formula is

$$A=BH \tag{3-33}$$

where A is the cross-section area of the stream.

Combining Eq. (3-30), Eq. (3-32) and Eq. (3-33), the resulting equation is

$$A=\frac{l_d^{1/6}}{l_\varepsilon^{1/9}}\left(\frac{Q}{\sqrt{g}}\right)^{7/9} \tag{3-34}$$

Substituting Eq. (3-30) and Eq. (3-32) into Eq. (3-4), the velocity U is obtained

$$U = \frac{l_\varepsilon^{1/9}}{l_d^{1/6}} \sqrt{g} \left(\frac{Q}{\sqrt{g}} \right)^{2/9} \tag{3-35}$$

Eq. (3-30), Eq. (3-32), Eq. (3-34) and Eq. (3-35) reveal that the crass-section variables (width *B*, depth *H*, area *A* and average velocity *U*) of a stream have a fixed exponential relationship with the water discharge as L. B. Leopold and T. Jr. Maddock (1953) state, of which stream width *B* is the only factor not impinged by the sediment transportation. The water depth *H* in Eq. (3-32), the cross-section area *A* in Eq. (3-34) and the stream velocity *U* in Eq. (3-35) are influenced by the energy conversion length and regime transportation length.

The ratio of width to depth is a parameter frequently used in river engineering. Dividing Eq. (3-30) by Eq. (3-32), yields

$$\frac{B}{H} = \left(\frac{1}{l_d} \right)^{1/6} \left(\frac{1}{l_\varepsilon} \right)^{7/18} \left(\frac{Q}{\sqrt{g}} \right)^{2/9} \tag{3-36}$$

Besides the regime transportation length l_d of a given river basin and the energy conversion length l_ε, Eq. (3-36) states the width-to-depth ratio $\frac{B}{H}$ of alluvial stream directly proportional to the water discharge Q whose power is 2/9. Thus, the ratio of width to depth increases as the stream's discharge and abundance of the sediment discharge increases and as the size of bed material and channel slope (be proportional to the energy conversion length l_ε) decreases.

For simplifying the expression of a channel cross-section and taking the advantage of Eq. (3-28), we suggest a cross-section indicator η

$$\eta = \frac{B^{5/9}}{H} = \left(\frac{1}{l_d} \right)^{1/6} \left(\frac{1}{l_\varepsilon} \right)^{5/18} \tag{3-37}$$

The physical meaning of Eq. (3-37) is that a stream in equilibrium (graded, or in regime) is as one which has developed just the right cross-section shape to fit the two parameterization factors of sediment transport l_d delivered from watershed and the stream power conversion l_ε. This cross-section indicator η whose dimension is the reciprocal of length power 4/9, is a function of the regime transportation length l_d and the energy conversion length l_ε and not related to water discharge and more convenient for engineering practice.

4. Hydraulic geometry of an alluvial channel

4.1. The equilibrium state about bankfull characteristics

In a natural river, discharge supplied by runoff to a river channel fluctuates considerably over time and space, so does the sediment transportation. The key consequence of the elaboration of river morphodynamics over the past studies has been explicit recognition that river morphology is the consequence of sediment transport in the river (Church and Ferguson, 2015). The river organizes its bankfull characteristics toward an equilibrium state. And the bankfull variables of the river are considerable stable and imply the spatiotemporal equilibrium of bankfull channel characteristics. This means the bankfull characteristics of a channel presents a regime stream. In this equilibrium condition, the river is able to transport sediment at precisely the rate that is supplied to the reach without causing overall bed aggradation or degradation (Naito and Parker 2019). This physical implication of bankfull characteristics should adhere to the regime equation of Eq. (3-10).

Let B_{bf}, H_{bf} and Q_{bf} are bankfull width, depth and discharge respectively, therefore, the bankfull relation of a stream can read according to Eq. (3-10)

$$B_{bf} H_{bf}{}^{1.8} = l_d{}^{0.3} \frac{Q_{bf}}{\sqrt{g}} \tag{4-1}$$

where B_{bf}, H_{bf} and Q_{bf} is the bankfull width, depth and water discharge of the stream.

If an alluvial channel is graded, or in regime conditions, the rate of sediment transported by the channel is equal to the rate of sediment eroded from the drainage area. Therefore, the term $\frac{\gamma_s Q}{G_B}$ of Eq. (3-9) is also equal to the annual volume ratio of the water to sediment charge. Thus

$$l_d = \left(\frac{\gamma_s Q}{G_B}\right)^{5/3} d = \left(\frac{T_{VW}}{T_{VS}}\right)^{5/3} d_{50T} = \frac{d_{50T}}{S_{Vm}{}^{5/3}} \tag{4-2}$$

where, T_{VW} is the annual volume of water charge; T_{VS} is the annual volume of gross sediment charge; d_{50T} is the medium diameter of the total sediment charge. S_{Vm} is the mean concentration of the gross sediment to the water charge.

The reciprocal of the symbol $\frac{T_{VW}}{T_{VS}}$ in Eq. (4-2) also indicates the annual sediment concentration S_{Vm}. Using Eq. (4-2) to calculate the regime transportation length l_d of Eq. (4-1), thus, the sediment charge should be the gross sediment supplied from upstream, no matter whether the sediment transportation is suspended load material and/or bed load material. Eq. (4-1) is the regime equation of alluvial channels at the bankfull stage, which are set the particular values of bankfull discharge and corresponding bankfull channel geometry.

4.2. Cross-sectional bankfull geometry and relations

Bankfull geometry refers to the channel width and depth at that discharge, as well as down-channel slope. Because bankfull discharge and bankfull channel geometry describe fundamental features of a river, a tool for predicting the change in bankfull characteristics (i.e., bankfull discharge and bankfull geometry) has a wide range of uses in engineering practice and river restoration.

Of the four variables (the slope J, width B, depth H and velocity U), only three are independent. Consequently, three independent equations are necessary to describe the uniform (or in regime) flow in an erodible channel. The three equations can be expressed in various forms. The simplest ones, perhaps, are the cross-sectional variables of bankfull stage which are set up by Leopold, L. B., and Maddock, T., Jr. (1953), where the stream width B, water depth H and flow velocity U of a stream are expressed as a function of water discharge Q at the bankfull level

$$\begin{cases} B_{bf} = \alpha_1 Q_{bf}{}^{\beta_1} \\ H_{bf} = \alpha_2 Q_{bf}{}^{\beta_2} \\ U_{bf} = \alpha_3 Q_{bf}{}^{\beta_3} \end{cases} \tag{4-3}$$

where α_1, α_2 and α_3 are coefficients; β_1, β_2 and β_3 are exponents; U_{bf} is the bankfull velocity of the water flow.

The first equation of the three is Eq. (3-14) rewritten at graded condition. Thus, Eq. (3-30) reads

$$B_{bf}=\left(\frac{1}{l_\varepsilon}\right)^{1/4}\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{1/2} \tag{4-4}$$

In fluvial processes, Eq. (4-4) states that the stream width at the bankfull level is inversely proportional to the energy conversion length l_ε and directly proportional to the water discharge Q_{bf}, but irrelevant to sediment discharge.

In accordance with Eq. (3-32), Eq. (3-34), Eq. (3-35), Eq. (3-36) and Eq. (3-37), the other bankfull variables read

$$H_{bf}=l_d^{1/6}l_\varepsilon^{5/36}\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{5/18} \tag{4-5}$$

$$A_{bf}=\frac{l_d^{1/6}}{l_\varepsilon^{1/9}}\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{7/9} \tag{4-6}$$

$$U_{bf}=\frac{l_\varepsilon^{1/9}}{l_d^{1/6}}\sqrt{g}\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{2/9} \tag{4-7}$$

$$\frac{B_{bf}}{H_{bf}}=\left(\frac{1}{l_d}\right)^{1/6}\left(\frac{1}{l_\varepsilon}\right)^{7/18}\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{2/9} \tag{4-8}$$

$$\eta=\frac{B_{bf}^{5/9}}{H_{bf}}=\left(\frac{1}{l_d}\right)^{1/6}\left(\frac{1}{l_\varepsilon}\right)^{5/18} \tag{4-9}$$

Eq. (4-9) that determines the cross-section shape is the only morphodynamic relation which is not related to the water discharge in the symbol $\frac{Q}{\sqrt{g}}$ evidently. Except Eq. (4-4) and the cross-section indicator η Eq. (4-9), the other bankfull variables are exclusively determined by the three parameters that are the energy conversion length l_ε, the regime transportation length l_d and the water discharge in the symbol of $\frac{Q}{\sqrt{g}}$.

4.3. Tentative testing by observed data

4.3.1. The fractal relation

All hydraulic variables fore-derived are based on the equilibrium of sediment transport and stream power conversion or the fractal features of streams. The equilibrium conditions of sediment transportation are presented by Eq. (4-1) that shows the cross-section dimension of a bankfull channel. And energy conversion relation Eq. (4-4) implies the fractal relation of channels which is derivative from the original fractal Eq. (3-29). Therefore, the two equations that is relevant to the regime transportation length l_d and the energy conversion length l_ε respectively should be verified by observed data at least.

As limited by the data we obtained, only tentative tests can be made. The data set of gravel-bed rivers in Colorado (E. D. Andrews, 1984) and gravel-bedded reaches in the northern Rocky Mountains of USA are presented by E. R. Mueller and J. Pitlick (2014). The second data set is a large gravel bed river of USA reported by J. Pitlick and R. Cress (2002), a nearly contiguous alluvial segment of the Colorado River between approximately Rulison, Colorado, and Moab, Utah. The third data set (Table 1) is sand-bedded reaches of the Songhua River Basin located in the black soil area of Northeast China.

Table 1 *Bankfull values of the main and tributaries of the Songhua River Basin*

River	Station	Q_{bf} (m^3/s)	A_{bf} (m^2)	B_{bf} (m)	H_{bf} (m)	$d_{50bs}{}^{a}$ (mm)	J $\times 10^{-4}$	Note
Nenjiang	Jiangqiao	2049	1194	224	5.32	1.25	0.3	Main
Nenjiang	Dalai	1624	935	169	5.53	2.5	0.2	Main
Songhua River	Xiadaiji	2998	1699	272	6.24	1.1	0.14	Main
Songhua River	Haerb	3614	2215	427	5.18	0.25	0.5	Main
Songhua No.2	Fuyu	601	436	175	2.49	0.4	0.2	Tributary

Lalinhe	Caijiagou	516	361	148	2.44	0.7	0.3	Tributary
Yinmahe	Simajia	176	111	32	3.45	0.8	2	Tributary

[a] d_{50bs} is the medium size of debris on bed surface.

Utilizing E. R. Mueller and J. Pitlick's data to checkout Eq. (4-4), only the data of single-thread channels is selected. Fig.1 shows that Eq. (4-4) fits the three data sets forementioned when the energy conversion length is 0.244mm. The abscissa presents the symbol $\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{1/2}$ and the ordinate the bankfull width B_{bf}, i.e., x- and y-axis are $\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{1/2}$ and B_{bf} respectively, where all variables are in meter and second. The equation of the straight line fitted the three data sets is

$$B_{bf} = 8\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{1/2} \tag{4-10}$$

Therefore, the average value of the energy conversion length is

$$l_{\varepsilon} = \left(\frac{1}{8}\right)^{4} = 2.44\times10^{-4}\,\mathrm{m} \tag{4-11}$$

This value of the energy conversion length has an amazing coincidence with the size of microeddies (Kolmogorov's microscale, the resolved length scale). The size values of microeddies in a compound channel are varied within the ranges of 0.08 to 0.48 mm, which are observed by Adam Kozioł (2015).

According to theoretical and experimental investigations (Batchelor,1959; Jong et al. 2009; Buschmann and Gad-el-Hak 2010; Ahmad and Huang, 2014; Krieger, Sinai and Nowak 2020) about Kolmogorov's microscale, the cascade proceeds to smaller and smaller scales until the Reynolds number is small enough for dissipation to be effective. Noting the fact that the Kolmogorov Reynolds number of small eddies is 1, it is estimated that the minimum value of the energy conversion length should be $l_{\varepsilon} > 5.0\times10^{-6}\,\mathrm{m}$.

The fitted line of the three data sets, however, is close to Lacey's (1929) formula in meter and second, viz

$$B_{bf} = 8.56\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{1/2}$$
$$l_\varepsilon = \left(\frac{1}{8.56}\right)^4 = 1.86\times10^{-4}\text{m} \tag{4-12}$$

Mueller and Pitlick (2013) also gives the same exponent for single-thread gravel bed streams and rivers as flowing

$$B_{bf} = 12.22\left(\frac{Q_{bf}}{\sqrt{g}}\right)^{1/2}$$
$$l_\varepsilon = \left(\frac{1}{12.22}\right)^4 = 4.49\times10^{-5}\text{m} \tag{4-13}$$

The testing about Eq. (4-4), although tentatively, confirms that the exponent of the fractal Eq. (3-14) is 1/2, and shows that the energy conversion length of Eq. (3-30) is not a constant which needs further investigation how it is relevant to the Kolmogorov’s microeddies in the field of channel turbulence.

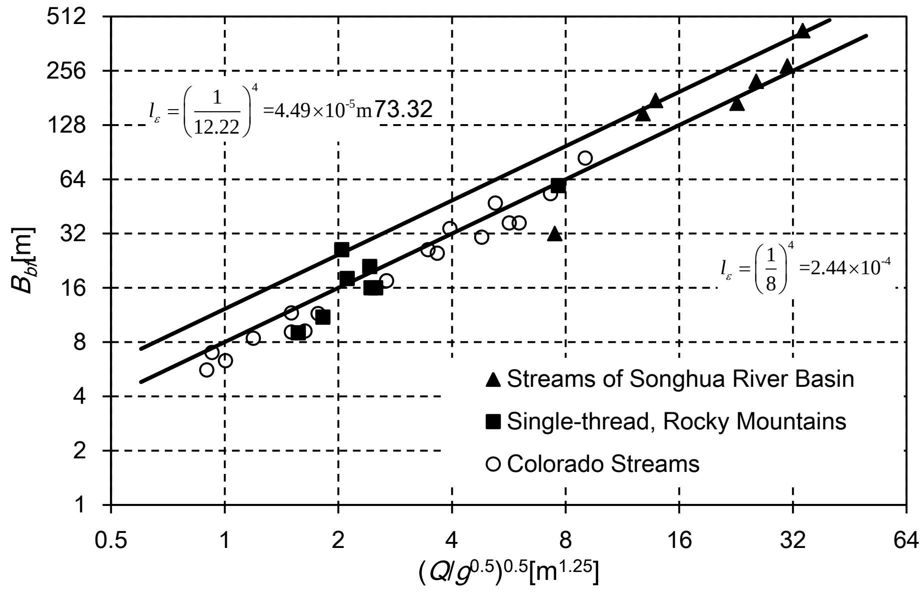

Fig.1 Reverse seeking the average value of the energy conversion length

4.3.2. Equilibrium sediment transportation and bankfull cross-section dimension

Although differences in sediment supply between single-thread and braided channel types provide a long-recognized pattern discrimination (Mueller and Pitlick, 2014), all channel types should consistent with the sediment transportation principle of Eq. (4-1) because of $\frac{1}{\sqrt{g}}\sum_{i=1}^{n} l_{di}{}^{0.3} Q_{bfi} = \sum_{i=1}^{n} B_{bfi} H_{bfi}{}^{1.8} = \overline{H}_{bf}{}^{1.8} \sum_{i=1}^{n} B_{bfi}$ and $\frac{1}{\sqrt{g}}\sum_{i=1}^{n} l_{di}{}^{0.3} Q_{bfi} = \frac{\overline{l}_d{}^{0.3}}{\sqrt{g}} \sum_{i=1}^{n} Q_{bfi} = \overline{l}_d{}^{0.3} \frac{Q_{bf}}{\sqrt{g}}$, where $\overline{H}_{bf}$ is the mean bankfull depth of the braided reach and $\overline{l}_d$ is the mean value of the streams; the width term $\sum_{i=1}^{n} B_{bfi}$ refers to the entire braid plain for the braided reach; *n* is the stream numbers of the braided reach，viz

$$\overline{H}_{bf}{}^{1.8} \sum_{i=1}^{n} B_{bfi} = \overline{l}_d{}^{0.3} \frac{Q_{bf}}{\sqrt{g}} \tag{4-14}$$

This expression of Eq. (4-1) for all channel types is demonstrated by the data set of E. R. Mueller and J. Pitlick (2014) and E. D. Andrews (1984) as shown in Fig.1. The x-axis is $\overline{H}_{bf}{}^{1.8} \sum_{i=1}^{n} B_{bfi}$ for multi-channels or $B_{bf} H_{bf}{}^{1.8}$ for single-thread, and y-axis $\frac{Q_{bf}}{\sqrt{g}}$. The mean value $\overline{l}_d$ of the streams in the northern Rocky Mountains is $\overline{l}_d = 1.0\mathrm{m}$.

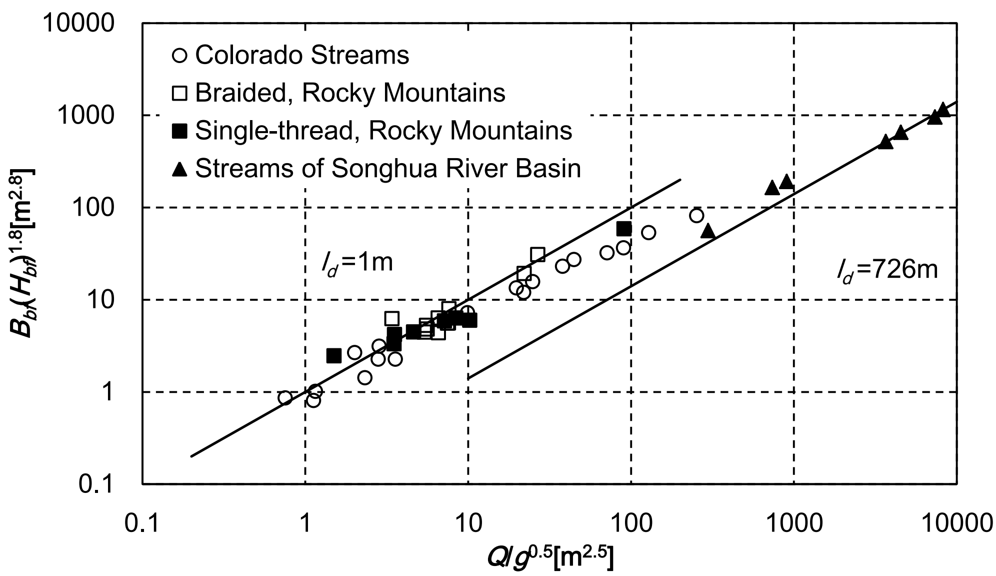

Figure 2. The cross-section dimension and discharge of bankfull stage

Fig.2 presents the data sets of the Colorado streams and the Songhua River Basin, too. With an annual runoff of 734.7 billion cubic meters and a drainage area of 565,800 square kilometers, the Songhua River Basin has 16 tributaries whose drainage area are exceeding 10,000 square kilometers. The summary information for the main and three tributaries of the Songhua River is presented in Table 2. Unlike the energy conversion length that can be obtained by reverse seeking, the regime transportation length l_d can be calculated according observed data by Eq. (4-2). The calculating outcomes are listed in Table 2, where the mean value of the length l_d of the Songhua River basin is 726m shown in Fig.2.

Table 2 *Summary information for the main and tributaries of the Songhua River*

River	Station	Drainage Area, km^2	Mean Annual Discharge, m^3/s	Annual Sediment Load, t/yr	S_{Vm} $\times 10^{-5}$	d_{50T} (mm)	$l_d{}^{b}$ (m)
Nenjiang	Jiangqiao	162,569	347	329×10^4	11.34	0.048	181
Nenjiang	Dalai	221,715	642	268×10^4	4.98	0.046	682
Songhua River	Xiadaiji	363,923	1151	339×10^4	3.52	0.038	1005

Songhua River	Haerb	389,769	1395	634×10^4	5.44	0.037	474
Songhua No.2	Fuyu	71,783	528	111×10^4	2.52	0.034	1565
Lalinhe	Caijiagou	18,339	99.70	28×10^4	3.38	0.041	1160
Yinmahe	Simajia	7,573	19.63	71.5×10^4	43.59	0.035	14

b l_d is calculated according to Eq. (4-2).

The testing of Fig.2 about Eq. (4-1) indicates that the bankfull relation between the dimension of channel cross-section and the water discharge are the outcomes of the fluvial processes in the equilibrium of sediment transport.

5. Discussion

A stream of bankfull stage should adhere to the mechanical principle of Eq. (3-8). Therefore, the term $\frac{\gamma_s Q}{G_B} d^{0.6}$ in Eq. (3-8) can be written as

$$l_d = \frac{\gamma_s Q}{G_B} d^{0.6} = \frac{d_{50bf}{}^{0.6}}{S_{Vbf}} \tag{6-1}$$

where S_{Vbf} is the volume concentration of the total sediment transportation at the benkfull lever; d_{50bf} is the medium diameter of the total sediment.

Because the bankfull characteristics of a channel presents a regime stream, Eq. (4-10) should include condition of Eq. (6-1), thus

$$l_d = \frac{d_{50T}}{S_{Vm}{}^{5/3}} = \frac{d_{50bf}}{S_{Vbf}{}^{5/3}} \tag{6-2}$$

Thus, a dynamic expression is obtained

$$\frac{d_{50bf}}{d_{50T}} = \left(\frac{S_{Vbf}}{S_{Vm}}\right)^{5/3} \tag{6-3}$$

Because the alluvial channel patterns are determined by sediment transportation, it may make sense to introduce an indicator related channel patterns according to Eq. (6-3)

$$I_{cp} \propto \left(\frac{S_{Vbf}}{S_{Vm}} \right)^{5/3} = \frac{d_{50bf}}{d_{50T}} \tag{6-4}$$

where I_{cp} is the indicator of channel patterns.

For the convenience of engineering practice, we suggest the indicator in the following form

$$I_{cp} = \frac{d_{50bs}}{d_{50fp}} \tag{6-5}$$

where d_{50bs} is the median grain size of bed surface; d_{50fp} is the median diameter of floodplain material.

In future study, therefore, we are going to utilize this indicator to deal with the patterns (single-thread, meandering and braided) of channels and to research the energy conversion length with Kolmogorov’s microeddies theory, how the patterns are relevant to stream power expenditure and channel slope.

6. Conclusion

The two governing equations of fluvial processes are Eq. (3-10) and Eq. (3-30), which state the equilibrium of sediment transportation and stream power conversion.

The regime equation Eq. (3-10) unveils the mechanical relation between the water discharge Q and the cross-section dimension $BH^{1.8}$ of the alluvial channel flow at the bankfull level.

The fractal features of Eq. (3-28) and Eq. (3-29) are the outcomes of the alluvial evolution that is consistent with the mechanism of sediment transportation and stream power conversion.

The tentative tests demonstrate three aspects: 1) the exponent 1/2 of the energy conversion equation Eq. (3-30) is suitable to the bankfull condition Eq. (4-4) and is also in general agreement with the observations (Fig.1); 2) Eq. (3-10) of the equilibrium sediment transporting does reflect the dynamic relation between the cross-section dimension of a channel and the water discharge at the channel bankfull level Eq. (4-1); 3) the bankfull stage of the channel implies such an equilibrium condition that the annual value of the transportation length (Fig.2) fits its cross-section variables (channel width, depth and discharge).

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References

Ackers P., W. R. White (1973). Sediment transport: new approach and analysis. *Journal of Hydraulic Division, American Society of Civil Engineers*, 99(11):2041- 2060.

Ackers P. (1992). Canal and river regime in theory and practice. 1929 to 1992. 1992 Gerald Lacey memorial lecture, British Section, *Ice Proceedings Water Maritime & Energy*, 96, 167– 178, London, UK.

Adam Kozioł (2015). Scales of Turbulent Eddies in a Compound Channel, *Acta Geophysica*, 63(2), 514-532. DOI: 10.2478/S11600-014-0247-0

Ahmad I.,Y. Huang, Z., Lu (2014). Detrended analysis of Reynolds stress in a decaying turbulent flow in a wind tunnel with active grids, *Journal of Hydrodynamics*. 26(1),122-128.DOI: 10.1016/S1001-6058(14)60014-7

Andrews, E. D. (1984) Bed-material entrainment and hydraulic geometry of gravel-bed rivers in Colorado, Geological Society of America Bulletin , Vol. 95, 371-378.

Bagnold, R. A. (1966), An approach to the sediment transport problem from general physics. US Geological Survey Professional Paper, No. 422-1, Washington, D. C.

Bagnold R. A. (1977). Bedload transport by natural rivers. *Water Resources Research*, 13(2): 303 - 312.

Batchelor, G. K. (1959). Small-scale variation of convected quantities like temperature in turbulent fluid part 1. general discussion and the case of small conductivity. *Journal of Fluid Mechanics*, 5, 113-133 .

Buschmann M.H., M. Gad-el-Hak (2010). Kolmogorov scaling of turbulent flow in the vicinity of the wall. *Physica D*, 239:1288 – 1295.

Carbonneau, P., S. Lane, and N. E. Bergeron (2004). Catchment - scale mapping of surface grain size in gravel bed rivers using airborne digital imagery. *Water Resources Research*, 40. W07202. https://doi:10.1029/2003WR002759

Carling P.A., Cao Z., Holland M. J., Ervie D.A. and Babayan Koopaei K. (2002) Turbulent flow across a natural compound channel. *Water Resources Research*, AGU. 38, 6-1- 6-11.

Carson, M. A. (1984). The meandering - braided river threshold: A reappraisal. *Journal of Hydrology*, 73, 315–334.

Chang, H. H. (1979). Minimum Stream Power and River Channel Patterns. *Journal of Hydrology*, 41, 303–327.

Cheng, N. S. (1997). A simplified settling velocity formula for sediment particle. *Journal of Hydraulic Engineering*, 123: 149–152.

Church, M., M. A. Hassan, and J. F. Wolcott (1998). Stabilizing self-organized structures in gravel-bed stream channels: Field and experimental observations. *Water Resources Research*, 34, 3169–3179.

Church, M. (2006). Bed material transport and the morphology of alluvial river channels. *Annual Review of Earth and Planetary Sciences*, 34, 325–354.

Church M, and R. I. Ferguson (2015). Morphodynamics: Rivers beyond steady state. *Water Resources Research*, 51(4), 1883–1897.

Coulthard, T. J. (2005). Effect of vegetation on braided stream pattern and dynamics. *Water Resources Research*, 41, W04003. https://doi:10.1029/2004/WR003201

Dade, W. B., and P. F. Friend (1998). Grain-size, sediment-transport regime, and channel slope in alluvial rivers. *Journal of Geology*, 106, 661–675.

Davis T. R. H., and Sutherland A. J. (1983). Extremal hypotheses for river behavior. *Water Resources Research*, 19(1), 141–148.

Dietrich, W. E., and J. D. Smith (1984). Bed load transport in a river meander. *Water Resources Research*, 20, 1355–1380.

Di Silvio, G., and M. Nones (2014). Morphodynamic reaction of a schematic river to sediment input changes: Analytical approaches. Geomorphology, 215, 74–82.

Dodov B, Foufoula-Georgiou E (2005) Fluvial processes and stream flow variability: interplay in the scale-frequency continuum and implications for scaling. *Water Resources Research* 41(5): W05005

Dou, G. (1964). Channel bed evolution of alluvial rivers and tidal estuaries in plain. *Journal of Hydraulic Engineering*, (2), 1–13 (In Chinese).

Eaton, B. C., & Church, M. (2011). A rational sediment transport scaling relation based on dimensionless stream power. *Earth Surface Processes and Landforms*, 36(7), 901– 910. https://doi.org/10.1002/esp.2120

Einstein, H. A. (1950). The bed-load function for sediment transportation in open channel flows, USDA Tech. Bull. 1026. Washington D.C.

Engelund, F., and J. Fredsoe (1976), A sediment transport model for straight alluvial channels, *Nordic Hydrology*, 7, 293– 306.

Fan, B. (1992). A Theoretical Study on Sediment Transport, 5th International Symposium on River Sedimentation, KARLSRUHE, FRG, 4.

Fan, B. (1995). Kinematic study on fluvial sediment. *Journal of Sediment Research*, (3), 72–78 (In Chinese).

Fan, B., et al. (2017). Theory and application of sediment transport and ice load in open channel flow (In Chinese). *China Water & Power Press (CWPP), Beijing*, 5.

Ferguson, R., and M. Church (2009). A critical perspective on 1 - D modeling of river processes: Gravel load and aggradation in lower Fraser River. *Water Resources Research*, 45, W11424. https://doi:10.1029/2009WR007740

Fiorentino M., et al. (1993). An entropy-based morphological analysis of river basin networks. *Water Resources Research*, 29(4), 1215–1224.

Gran, K., and C. Paola (2001). Riparian vegetation controls on braided stream dynamics. *Water Resources Research*, 37, 3275–3283.

Jähmlich S., L. C. Lund-Hansen, T. Leipe (2002). Enhanced settling velocities and vertical transport of particulate matter by aggregation in the benthic boundary layer. *Geografisk Tidskrift, Danish Journal of Geography* 102, 37–49.

Jeremy G. V., N. Domarad, M. Church, C. D. Rennie (2015). The gravel-sand transition: Sediment dynamics in a diffuse extension. *Journal of Geophysical Research: Earth Surface*, 10.1002/2014JF003328, 120, 6, 943-963.

Jong, J.D., . Cao, S. H. Woodward, J. P. L. C. Salazar, L. R. Collins and H. Meng. (2009) Dissipation Rate Estimation from PIV in Zero-Mean Isotropic Turbulence. *Experiments in Fluids*, 46, 499-515.

Habersack, H. M., and J. B. Laronne (2001). Bed load texture in an alpine gravel bed river. *Water Resources Research*, 37, 3359–3370.

Harvey, A. M. (2002). Effective time scales of coupling within fluvial systems. *Geomorphology*, 44, 175–201.

Howard, A. D. (1967). Drainage analysis in geological interpretation, A summation. *American Association of Petroleum Geologists Bulletin*, 51, 2246–2259.

Huang W. (1981). The law of maximum rate the energy dissipation on continuum dynamics. *Journal of Tsinghua University*, (1): 87–96 (In Chinese).

Kieran B. J. Dunne, Douglas J. Jerolmack. (2018). Evidence of, and a proposed explanation for, bimodal transport states in alluvial rivers. *Earth Surface Dynamics* 6:3, 583‑594, https://doi.org/10.5194/esurf-6-583-2018

Kleinhans, M. G., H. R. A. Jagers, E. Mosselman, and C. J. Sloff (2008). Bifurcation dynamics and avulsion duration in meandering rivers by one - dimensional and three - dimensional models. *Water Resources Research*, 44, W08454. https://doi:10.1029/2007WR005912

Krieger, M.S., Sinai, S. and, M.A.Nowak (2020). Turbulent coherent structures and early life below the Kolmogorov scale. *Nature Communications* 11, 2192 . https://doi.org/10.1038/s41467-020-15780-1

Lacey, G. (1929). Stable channels in alluvium. *Proceedings of the Institute of Civil Engineers*, London, 229: 259‑292.

Langbein, W. B. (1964). Geometry of river channels. *Journal of the Hydraulic Division, American Society of Civil Engineers*, 90(HY2), 301–312.

Leopold, L.B. and T. Maddock (1953). The hydraulic geometry of stream channels and some physiographic implications. *United States Geological Survey Professional Paper*, 252.Google Scholar

Leopold, L. B., M. G. Wolman and J. P. Miller (1964). Fluvial Processes in Geomorphology. W. M. Freeman and Co., 522 pages.

Li, R., D. D. Simons, and M. A. Stevens (1976). Morphology of cobble streams in small watersheds. *Journal of the Hydraulic Division, American Society of Civil Engineers*, 102(8), 1101–1118.

LI Y., J. DENG, and Z. SUN (2003). A study on river bed erosion downstream from the Tree Gorges Reservoir, *Journal of Basic Science and Engineering,* 11(3): 283-295.

Lom, A., J. S. Ribberink, and G. Parker (2008). Vertical sorting and the morphodynamics of bed form-dominated rivers: A sorting evolution model. *Journal of Geophysical Research*, 113, F01019. https://doi:10.1029/2006JF000618

Mandelbrot, B. B. (1983). The Fractal Geometry of Nature. San Francisco: Freeman, 156-163.

Meyer-Peter, E., and R. Mueller (1948), Formulas for bed-load transport, paper presented at 2nd Meeting, Int. Assoc. of Hydraul. Eng. and Res., Stockholm.

Millar, R. G. (2000). Influences of bank vegetation on alluvial channel patterns. *Water Resources Research*, 36, 1109–1118.

Mueller, E.R., and J. Pitlick. (2013). Sediment supply and channel morphology in mountain river systems: 1. Relative importance of lithology, topography, and climate . *Journal of Geophysical Research: Earth Surface*, 118, 2325‐2342. doi:10.1002/2013JF002843

Mueller, E.R. and J. Pitlick. (2014). Sediment supply and channel morphology in mountain river systems: 2. Single thread to braided transitions. *Journal of Geophysical Research: Earth Surface*, 119, 7, 1516-1541, doi:10.1002/2013JF002843.

Naito, K., & Parker, G. (2019). Can bankfull discharge and bankfull channel characteristics of an alluvial meandering river be cospecified from a flow duration curve?. *Journal of Geophysical Research: Earth Surface*, 124, 10, 2381-2401, https://doi.org/10.1029/2018JF004971.

Parker, G. (1978). Self-formed straight rivers with equilibrium banks and mobile bed: Part 2, The gravel rivers. *Fluid Mechanics*, 89(1), 127 – 146.

Pitlick J. and R. Cress (2002), Downstream changes in the channel geometry of a large gravel bed river. *Water Resources Research,* 38(10): 34-1-34-11. https://doi.org/10.1029/2001WR000898

Shi, C. and Zhang, D. (2003), Processes and mechanisms of dynamic channel adjustment to delta progradation: the case of the mouth channel of the Yellow River, China. *Earth Surface Processes and Landforms*, 28, 609-624.

Schumm, S. A. (1960). The shape of alluvial channels in relation to sediment type, US Geological Survey Professional Paper, 352B.

Shvidchenko A. B, G. Pender (2000). Flume study of the effect of relative depth on the incipient motion of coarse uniform sediments. *Water Resources Research,* 36(2): 619 - 628.

Slingerland, R., and N. D. Smith (2004). River avulsions and their deposits, A*nnual Review of Earth and Planetary Sciences*, 32, 257– 285.

Whipple, K. X., and Tucker, G. E. (1999). Dynamics of the stream - power river incision model: Implications for height limits of mountain ranges, landscape response timescales, and research needs. *Journal of Geophysical Research*, 104(B8), 17,661– 17,674.

Wilcock, P. R., and J. C. Crowe (2003). Surface-based transport model for mixed-size sediment. *Journal of Hydraulic Engineering*, 129, 120 – 128.

White, W. R., Bettes R., and Paris E. (1982). Analytical approach to river regime. *Journal of the Hydraulic Division, American Society of Civil Engineers,* 108(HY10), 1179 –1193.

Yager, E. M., J. W. Kirchner, and W. E. Dietrich (2007). Calculating bed load transport in steep boulder bed channels. *Water Resources Research*, 43(7), W07418. https://doi:10.1029/2006WR005432

Yen, B.C., (1992). Dimensionally homogeneous Manning’s formula. *Journal of Hydraulic Engineering*, 118 (9), 1326‑1332.

Yang, C.T. (1971). Potential Energy and Stream Morphology. *Water Resources Research*, 7(2), 312–322.

Zhang G., et al. (2016). Incipient velocity formula of non-cohesive uniform sediment based on relative exposure degree (In Chinese). *Journal of basic science and Engineering*. 24(4), 687-697.

Zhang R., et al. (1961). The dynamics of river (In Chinese). *China Industry Press, Beijing*.