On the Burridge and Knopoff Model and the Theoretical Seismicity

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Abstract

In the mid-sixties of the 20$^{\text{th}}$ century, a seismicity one-dimensional model, based on a spring system with friction, was presented by the two authors mentioned in the title. Through this model was studied the viscosity behaviour involved in the stratums mutual displacement. However, in the analysis pertinent to the occasion, seeing the fig.13 ibid it can be observed an error related to the viscosity function (low speeds') limits, which determines its "performance" during the time - lapse of the stratums breaking. From the height of the present knowledge, using the Elementary Catastrophes Theory developed in later times by René Thom and Zeeman, we have reached certain conclusions which, although do not refute the results of said article, they can significantly improve some aspects of the model and in general of the earthquakes physics.

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Abstract

In the mid-sixties of the 20^{th} century, a seismicity one-dimensional model, based on a spring system with friction, was presented by the two authors mentioned in the title. Through this model was studied the viscosity behaviour involved in the stratums mutual displacement. However, in the analysis pertinent to the occasion, seeing the fig.13 ibid it can be observed an error related to the viscosity function (low speeds') limits, which determines its "performance" during the time - lapse of the stratums breaking. From the height of the present knowledge, using the Elementary Catastrophes Theory developed in later times by René Thom and Zeeman, we have reached certain conclusions which, although do not refute the results of said article, they can significantly improve some aspects of the model and in general of the earthquakes physics.

1 Introduction

Starting in the first half of the 20^{th} century, the data coming from seismic observations lead to the (although loin) possibility of arriving at earthquakes forecasts based on a better understanding of their mechanisms. In the article by Burridge, R., & Knopoff, L. (1967) a study of telluric phenomena through simulations and experiments is proposed, beginning from an introduced there one-dimensional model. After this first attempt to analyze the details of the mechanisms of earthquakes, the results of other investigations have come to light. The report presented by Dahl, P. R. (1968) is a reference that is cited by different authors when considering a friction model; however, this work does not delve into the dependence in relation to relative velocity and neither in the behavior that occurs at critical points. In the article by Haessig Jr, D. A., & Friedland, B. (1991), the friction model and its dependence on relative velocity are presented, however, even in its approach there is no clarity when considering the critical points. De Sousa Vieira, M. (1999) assumes the earthquake model from a different perspective that considers Chaos theory, nevertheless, there is no type of consideration regarding the friction dependence on the relative velocity. In the article by Carlson, J. M., & Langer, J. S. (1989) taking into account the characteristics presented in the Burridge, R., & Knopoff, L. (1967) model, the dependence of the relative velocity with respect to the friction force leads to similar considerations that leave aside the critical points. Later, Karnopp, D. (1985) improves the experiments and simulations made by the previous authors, reaching results that are in some sense inconclusive. However, it should be noted that their conclusions are based on an empirical approach and that is why it is difficult to agree or rule out the validity of the different reasonings.

A later article by Galvanetto, U. (2002) reconsiders the characteristic form of friction but the motion equation leaves aside the physical phenomenon around critical points. More recently Awrejcewicz, J., & Olejnik, P. (2005) reviewed the differents friction models, including the one presented in Burridge, R., & Knopoff, L. (1967), however, the discontinuity around the critical points are neglected.

In these circumstances, taking into account the high complexity estimating any type of viscosity, we have opted for the application of a tool of a completely theoretical nature. Our intention is to use the Elementary Catastrophes Theory in order to obtain rigorously the friction behaviour within the Newton's viscosity segment of the relative velocity domain where, in our modest opinion, unfortunately, an important part of more objective information has remained lacking or lost. All this came to the surface when we during the analysis of the fig.13 of the first "founding" article have detected a non-compliance error at the limits of the resistance function values. The catastrophes theory is part of the (semi-)qualitative theory of the nonlinear complex systems (Thom, R. (1975), Mathematical Models of Morphogenesis, (1974)). There it is considered how minuscule (even infinitesimal) external perturbations change the behavior of a specific system (no matter its nature) within certain critical circumstances (Zeeman, E. C. (1976)).

The strong conviction that the Catastrophes Theory in this case is applicable, is based on an epistemological point of view, similar to the justification used when it is about the Statistical Mathematics handling i.e., when one has to deal with large numbers. In others words we agree to take that, the mathematics, in general, reflects the objective laws of the nature and therefore, if we have a situation where a certain classification theorem allows only a strongly determinate set of options, the last is also valid in the "real life" which in the present context is the geological and geophysical sciences.

2 The total ("all-inclusive") friction formula

In the Burridge, R., & Knopoff, L. (1967) article it is used the viscosity function $F(v)$ that includes all the effects of the between stratum resistance (although immediately below fig. 13 in the text a symbol F^* is inserted apart from F). Following the concepts of this article, we will split the function's graph by three branches (the symbols $-\infty$ and ∞ signify its natural finite limits)

$$F(v) = \begin{cases} F_-(v) \\ f(v) \\ F_+(v). \end{cases} \tag{1}$$

Which are as follows: the left part that matches the expression used in the mentioned article (with its argument between $-\infty$ and $-H$, see our fig.1):

$$F_-(v) = \frac{B}{1 - A(v + H)} - Ev, \tag{2}$$

the function $f(v) = f(v;\alpha) = v^3 - \alpha v$ that represents the simplest, so-called fold catastrophe (Bröcker, T. (1978)) and which velocity domain is $[-H, H]$ being α a positive number and finally, the right part, which also matches the expression used in the cited article (of course, its argument belongs to the interval locked between H and ∞, see again our fig.1)

$$F_+(v) = -\frac{B}{1 + A(v - H)} - Ev. \tag{3}$$

Here A, B, E, and H are certain positive constants which values can be found empirically. In that, is important underline the same branches dimensions of F: speed cubed. Further, always when we refer to the two figures in this article we will use the adjective "our", without mentioning anything else - this are figures in the Burridge, R., & Knopoff, L. (1967) article while, the graphics, coming from other sources, will be cited normally.

3 The Catastrophe Theory correction

According to the top graph of the fig.11 it is presented the simple law of friction without seismic radiation, with instability at zero velocity where there is an "instantaneous" and directly jump from B to $-B$. As this is unrealistic, the bottom part of the same figure presents a situation where the jump is already indirect: at $v = 0$ the friction first goes up (to B), then goes down (to $-B$), and, finally, it rises to a certain value. But this is even less acceptable than the above. Let us remember this behaviour to compare it later with the borderline case results to which it leads our proposal.

In the above mentioned Burridge, R., & Knopoff, L. (1967) article (and in other ones too), one can read about the potential energy: "...when all the springs are stretched so that the masses are all on the verge of instability...any trigger applied to the system will cause a large shock to take place..." which means, that there, there are all the conditions to be applied the Elementary Catastrophes Theory - a sudden response during the little by little continuous perturbations increasing. We are going to take advantage

96 of the well-known principle of simplicity by using the simplest known elementary catastrophe – the fold catastrophe (Bröcker, T. (1978)). Graphically, after all the three branches of the total resistant function (1) have been assembled, we obtain the following figure:

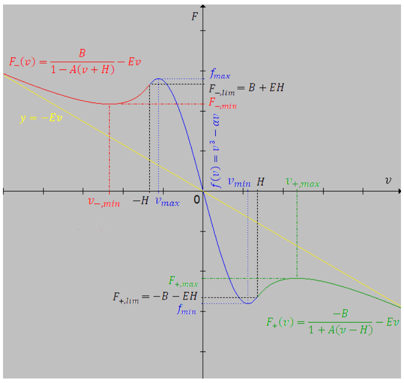

Figure 1. Example of a total resistant function graph , with $A = 1$, $B = 2$, $E = 0.5$, $H = 1.3247$, $\alpha = 3.76449027$.

In order to achieve the analytical construction let us sew the three branches of $F(v)$ at its two joining points $-H$ and H, see the equation (1), (2), and (3):

$$F_-(H) = f(-H), \quad f(H) = F_+(H). \tag{4}$$

And after to executing the calculations we obtain:

$$B + EH = -H^3 + \alpha H \Leftrightarrow H^3 - \alpha H = -B - EH. \tag{5}$$

Because in Classical Physics it is settled that all the auxiliary expressions with physical sense are differentiable at least once time, we iron the two joints of the graph (see our fig.1):

$$F_-^{'}(-H) = f^{'}(-H), \quad f^{'}(H) = F_+^{'}(H), \tag{6}$$

and this gives us respectively:

$$BA - E = 3H^2 - \alpha \Leftrightarrow 3H^2 - \alpha = BA - E. \tag{7}$$

From the equation (7), we can express directly our unique co-variable i.e. the perturbation $\alpha = 3H^2 + E - BA$. Despite, it easy to see, that, the total resistance function $F = F(v)$ has a continuous first derivative, it cannot be polished more, however, it is not necessary either according to the well-known theorem of existence and unicity of the ordinary differential equations.

Multiplying the equation (7) by H and subtracting it from the equation (5) we get:

$$-2H^3 = -B - BAH. \tag{8}$$

That is, for H we have the following cubic equation:

$$2H^3 - BAH - B = 0. \tag{9}$$

Its unique real solution according to the Cardano procedure is:

$$H = \frac{1}{2}\left[\sqrt[3]{2B\left(1+\sqrt[2]{1-\frac{2BA^3}{27}}\right)} + \sqrt[3]{2B\left(1-\sqrt[2]{1-\frac{2BA^3}{27}}\right)}\right] > 0. \tag{10}$$

The other two roots are conjugated complex numbers and that is why we are going to abandon them. It is obvious that there are limitations on A and B (the discriminants of a real square root are non-negative):

$$BA^3 \leq \frac{27}{2}. \tag{11}$$

The above cubic equation allows us to deduce that in this case, the parameter α will always be positive; in fact $B > 0$ then from eq.(9) $BA - 2H^2 < 0$ and hence $3H^2 > BA$. Using the existence of first derivatives we can easily find the coordinates of the four extremities of our curve. These are

$$v_{\substack{-,min \\ +,max}} = \mp\left(H - \frac{1}{A} + \sqrt{\frac{B}{AE}}\right), \quad F_{\substack{-,min \\ +,max}} = F\left(v_{\substack{-,min \\ +,max}}\right) = \pm\left[E\left(H - \frac{1}{A}\right) + 2\sqrt{\frac{BE}{A}}\right], \tag{12}$$

and for the "global" extremities (watch carefully our figure 1):

$$v_{\substack{max \\ min}} = \mp\sqrt{\frac{\alpha}{3}}, \quad F_{\substack{max \\ min}} = F\left(v_{\substack{max \\ min}}\right) = f\left(v_{\substack{max \\ min}}\right) = \pm 2\sqrt{\left(\frac{\alpha}{3}\right)^3}. \tag{13}$$

A look at the fig. 11 of the analyzed article, leads us to the following speculation around the error that we observed in the fig.13 ibid. If on our fig.1 we carry out a horizontal compression eliminating thus the velocity interval presented by the fold catastrophe, we will obtain the following graph (see fig.2):

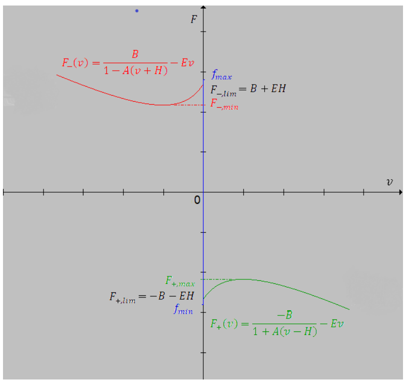

Figure 2. The borderline case of our fig.1 (the argument here is unimportant).

The one that explains the not elegant appearance of the vertical in $v = 0$ segment between f_{max} and $F_{-,lim}$ (as well as the one between $F_{+,lim}$ and f_{min}) in other authors. So in the diploid form of the graph (fig.13) the maximal value (at low speeds) is B (the error) and not $B+EH$; in all probability, it is (implicitly) admitted by the authors that our f_{max} (they do not introduce it but, there must be one like this) coincide with $B+EH$ of the fig.11 and for some reason must be removed.

Here is the place to find out what happens in the bifurcation values surroundings $\alpha = 0$. The point is that, because the catastrophe function behaviour is local, the branches F_- and F_+ are not always presented by the formulae (2) and (3). Actually, the eq.(11) indicates the region of its validity. Anyway, the respective curve of the total resistance "distributes" itself along the abscissa.

4 Conclusions

The advantage of our approach consists of the possibility to consider the real behavior of the seismic viscosity around the critical points without resorting to any kind of (particular) modeling. It is very significant that, if we do a velocity (the last one is relative) inversion in the freehand graphs of the Karnopp, D. (1985) article, namely a) and b), we will see exactly a caricature of our graphs, 2 and 1 respectively, obtained based on a rigorous theory.

In this order of ideas, we dare to say that maybe the time has come to redo Burridge and Knopoff's experiment, but this time using the latest in 21st-century technology: multichannel NATIONAL INSTRUMENTS interfaces with the powerful software LabVIEW RT. This would allows to carry out an appropriate acquisition of data in real-time which could later be "delayed in time" and through a computer to compare our graph with the experiment in the low speeds domain.

However, we think the best benefit that could be obtained through this approach consists of the real possibility of predicting earthquakes based on the real time - behavior - analysis of the friction between $v_{-,min}$ and v_{max} as well as between v_{min} and $v_{+,max}$.

Acknowledgments

To Talivaldis Strautkalns for the objective review of this article, on the other hand, it is worth noting that no data were used in this study.

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