A superposition algorithm to construct efficient pseudo-random waveform for frequency-domain controlled-source electromagnetic method

Yang Yang[1], Jishan He[2], and Shucai Li[1]

[1]Shandong University
[2]Central South University

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Abstract

The controlled-source electromagnetic method (CSEM) has been widely used for geophysical surveys, such as controlled-source audio-frequency magnetotellurics (CSAMT) for mineral explorations. Many types of signals are hired for different CSEM applications, such as square wave,2n sequence pseudo-random waveform, binary symmetric wave, and other specific waveforms for MCSEM. In frequency domain CSEM exploration, it is often to change the frequency of transmitting signals to obtain more information from the subsurface, which is sometimes time-consuming. Under such circumstances, we propose a novel method for adaptively constructing a wide-band, high-frequency-density pseudo-random signal. Based on this method, we successfully combine all interested frequencies into one waveform, with the energy of the interested frequencies uniformly or almost uniformly distributed. Benefit on that, for most frequency domain CSEM cases, exploration can be conducted by only one signal. This new kind of signal can significantly improve exploration efficiency compared with the past transmitter waveforms.

A superposition algorithm to construct efficient pseudo-random waveform for frequency-domain controlled-source electromagnetic method

Yang Yang*†, Jishan He‡, Shucai Li*†

* Geotechnical and Structural Engineering Research Center, Shandong University
† School of Civil Engineering, Shandong University
‡ School of Geoscience and Info-physics, Central South University

Key Points:

- A novel method is proposed to adaptively design efficient transmitter waveforms meeting different needs for frequency-domain CSEM.
- All interested frequencies are successfully combined into a pseudo random waveform, with the energy of these frequencies evenly distributed.
- Interested frequencies of optimal waveform can be log non-uniformly distributed, making exploration more cost-effective.

Corresponding author: Jishan He, Email:hejishan@mail.csu.edu.cn

Abstract

[The controlled-source electromagnetic method (CSEM) has been widely used for geophysical surveys, such as controlled-source audio-frequency magnetotellurics (CSAMT) for mineral explorations. Many types of signals are hired for different CSEM applications, such as square wave, 2^n sequence pseudo-random waveform, binary symmetric wave, and other specific waveforms for MCSEM. In frequency domain CSEM exploration, it is often to change the frequency of transmitting signals to obtain more information from the subsurface, which is sometimes time-consuming. Under such circumstances, we propose a novel method for adaptively constructing a wide-band, high-frequency-density pseudo-random signal. Based on this method, we successfully combine all interested frequencies into one waveform, with the energy of the interested frequencies uniformly or almost uniformly distributed. Benefit on that, for most frequency domain CSEM cases, exploration can be conducted by only one signal. This new kind of signal can significantly improve exploration efficiency compared with the past transmitter waveforms.]

Plain Language Summary

[Frequency domain CSEM has been widely used to obtain subsurface electrical information at different scales or depths by sending different frequencies signal. In past, many instruments conduct exploration by sweeping frequencies, that is, only one frequency square wave or sinusoidal signal was sent at one time. The advantage of this transmission mode is that the signal energy is relatively strong, and it is easier to obtain high signal-to-noise ratio signals. The disadvantage is the efficiency is relatively low. Scientists have designed composite waveforms to improve the exploration efficiency, but these designed waveforms often contain only a few efficient frequency components, and the signal transmission type still needs to be changed during the exploration. Especially when the cost of signal transmission is relatively high, it will increase the cost of exploration and reduce efficiency. Under such circumstances, we propose a method to merge all the frequencies of interest into one signal with their amplitude almost the same, so that only one signal is needed to conduct the whole exploration. Since signals of different frequencies are collected simultaneously, it can bring a lot of convenience in the later signal processing, and much easier to obtain geophysical data with a high signal-to-noise ratio.]

1 Introduction

The frequency-domain controlled-source electromagnetic method(CSEM) method has been widely used in mineral resources, geological hazard exploration, and oil-gas exploration. In frequency domain electromagnetic exploration, the frequency response of ground is embedded in the received signal, and different frequencies involve different electromagnetic energy penetration characters. That is the basis for the frequency domain sounding method. By certain geophysical inverse algorithms, it is possible to obtain detail geophysical information from the subsurface. A electric dipole is usually hired as an artificial signal source, such as controlled-source audio-frequency magnetotellurics (CSAMT)(Goldstein & Strangway, 1975)(Jishan, 1991), and Marine CSEM (Constable & Srnka, 2007).

The square wave is frequently used as the transmitter waveform. Compared with a sine wave, square wave and pseudo-random waveform signals are easier to implement in hardware, especially when a large current is needed. So CSEM waveforms are usually either binary or ternary signals. In this kind of wave, most of the transmitted energy is in the main frequency and first harmonic, although sometimes more harmonics can be used, it is still not convenient for frequency-domain EM exploration. If we want to get 40 or more frequencies underground information from 0.1Hz to 10000Hz in CSAMT, the transmitted signals will be changed many times for different frequency information, which is quite time-consuming and inconvenient.

In the field of radar communications, a variety of waveforms are designed to meet the needs of different situations(Bell, 1993)(Yang & Blum, 2007)(Sturm & Wiesbeck, 2011)(Aubry et al., 2014). However, due to the different principles of communication and geophysical prospecting, many methods are not suitable for geophysical prospecting signals designing. According to the characteristics of geophysical prospecting methods, many waveforms based on different methods are designed and created, in which energy is distributed at more frequencies, such as binary symmetric waveform by an analytical method (David et al., 2011), 2^n sequence signal based on closed addition in a three-element (Jishan, 2010), pseudo-random binary sequence (PRBS)(Ziolkowski et al., 2011), square waveform shaping by Monte Carlo approach (Mittet & Schaugpettersen, 2007) and specialized waveforms for MCSEM (Constable & Cox, 1996)(Lu & Srnka, 2009). All these waveforms have quite a good spectrum property. In these signals, it is not necessary to continuously change the sending frequency, but it still needs to be changed especially when there are a lot of frequencies of interest.

Under such circumstances, we developed a novel method to design more adaptive and complicated waveforms to obtain more interested frequencies at one time. By taking advantage of this method, we successfully combine all interested frequencies into one waveform. It simultaneously broadcasts dozens of frequencies at one time, in which amplitudes of interested frequencies are almost homogeneous distributed. Benefit on this development, only one signal is needed to conduct explorations for different geophysical purposes without waveform changing, which also brings convenience for data processing and denoising(Yang et al., 2018). Furthermore, different frequency-density can be customized in different frequency bands for specialized exploration purposes.

2 Method

2.1 Principle

The transmitter waveform for frequency-domain CSEM is first generated based on computer calculation, simulation, and coding, then sent by the Insulated Gate Bipolar Transistor (IGBT)(Kuang & Williams, 2000)(Khanna, 2003) and electric generator for high current. In this article, we will focus on the simulation part of creating an optimal waveform. First, we consider a target waveform that has been created, where there are decades of main frequencies, i.e. those interested frequencies, meanwhile there also exist a lot of harmonics as well. To evaluate this waveform for CSEM, we set up a criterion for those interested frequencies, which is called the smallest relative root mean square error (RRMSE) to construct an optimal signal with the energy of those interested frequencies evenly distributed. The way to calculate RRMSE is shown in equation 1.

$$RRMSE = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{n}(x_i - \bar{x})^2}}{\bar{x}} \tag{1}$$

In which, $RRMSE$ is related to the relative root mean square error, x_i is the amplitude of the ith main frequency, $\bar{x}$ is the average amplitude of all main frequencies. This parameter will be used as the only criterion to evaluate the quality of waveform characteristics. The smaller the parameter value is, the more uniform the main frequency energy is distributed. For every designed signal, we will calculate its RRMSE to judge whether it is qualified or not.

In the first step, we collected all interested frequencies to construct a set of sinusoidal signals, which contains three parameters for each sinusoidal signal, as shown in equation 2.

$$S(t) = A\sin(2\pi ft + \phi) \tag{2}$$

In which A is amplitude, f is interested frequency, ϕ is phase, $S(t)$ is signal value. Secondly, we put sinusoidal signals to square waves in the following way. If signal bigger than 0, then put signal to A, and if smaller than 0, put it to $-A$. At those zero-value locations, put them to A or $-A$ by index parity. The corresponding rectangular wave as shown in figure 1(a), in which there are only A and $-A$. For computation convenience, we will put A as a constant 100 in this paper. Furthermore, with phase changing, we can get different square waves at 1Hz. Figure 1(b) shows an example of a sinusoidal signal with 1Hz with phase at 90 degrees and its corresponding square wave.

In this way, we can construct a series of periodic square wave signals with frequencies respectively at 1, 2, 4, 8, 16, 32, and 64Hz with phase at 0 degrees for example. In this method, frequencies construct for a superimposed signal should comply with the law of mutual multiples of 2^n. Figure 1(c) and 1(d) shows 2Hz and 64Hz square wave respectably. Add up all these square waves to get a superimposed signal as one basic unit. The add-up result is shown in figure 2(a). Benefit on the correlation between frequencies, there is no frequency interference in this superimposed signalthe basic unit, since there will be only odd harmonics for periodic square wave and the base frequency of square waves is incremented by a multiple of 2. All amplitudes of main frequencies and harmonics are held as original in the monochromatic square wave.

Obviously, if we add an odd number of square waves of the same amplitude and different frequency together, there will be no zero in this superimposed signal. Then an algorithm is operated to put all values bigger than 100 to 100, values smaller than -100 to -100, which will be called “topping operation” in this paper. Then we get a new signal with only 100 and -100 exist, which is a pseudo-random signal as shown in figure 2(b). As shown in figure 2(c), its spectrum is quite suitable for frequency-domain EM exploration. In past, this kind of signal is coded based on an algorithm called closed addition in a three-element and has been called 7-frequency wave in 2^n sequence pseudo-random signal (Jishan, 2010), since there are 7 main frequencies in it and main frequencies are increased by a multiple 2. Now, we construct this kind of signal by applying the new method proposed in this paper, which gives us more flexibility to design more complicated waveforms.

The amplitudes of the main frequencies are almost the same, but not perfect. As we call a suffix ‘with phase at 0 degrees’ after 2^n sequence random signal, we introduce a new parameter, phase ϕ, into this kind of signal to modified it. By changing the phase ϕ constantly just like figure 1(b), we can calculate the related mean square error of main frequencies at different phases, for example using the $\frac{2\pi}{360}$ as interval, which means it has 360 possibilities for phase. Corresponding RRMSE for different phase is shown in figure 2(d). The index number of minimum point for RRMSE corresponds to 90 degrees. Then we set 90 degrees as the optimal phase to construct the superimposed signal and target pseudo-random signal as shown in figure 2(e). Target waveform and its spectrum are shown in figure 2(f), in which spectrum of interested frequencies and their harmonics are much more evenly distributed, comparing them in figure 2(c).

The workflow of the proposed method as shown in 3(a) mainly contains three steps, creating square wave construction, constructing a superposed signal, and “topping operation”. By applying the same method, we can construct a waveform with 13 main frequencies and spectrum as shown in figure 3(b) and (c). We call this waveform is L1-F13-1Hz-4096Hz signal, in which level 1 means there is only one superimposed signal component, 13 means 13 main frequencies exist and 1Hz-4096Hz means frequencies are from 1Hz to 4096Hz. Actually, with the number of main frequencies increasing, the average amplitude of the main frequencies does not decrease linearly. The reason is based on Parseval identity (Bracewell, 2002), that is the energy is positively related to the square sum of the amplitude, so the attenuation of the main frequency energy amplitude does not decrease linearly with the number of main frequencies increasing. In the following example, we will see a waveform with amplitude 100 have more than 40 main frequencies with an average amplitude of more than 15.

In computing simulation, RRMSE is calculated based on a discretized signal with a certain sampling frequency. Therefore, some high-frequency harmonic components will be aliased. However, if the sampling frequency is selected appropriately, the frequency where the aliasing occurs corresponds to low energy, and the calculation error caused by the calculation result can be ignored. Therefore, when doing computer simulations, considering the calculation efficiency and accuracy, we choose 16 times the highest frequency as the sampling frequency for calculation simulation.

2.2 High level Combined waveforms

Based on this method, we can design a high level of 2^n sequence pseudo-random signal by using more superimposed signals. As shown in figure 4(a), signal S1 in have 12 frequencies, including 1,2,4$\cdots$2048Hz, while S2 in figure 4(b) have 11 frequencies, including 3,6,12$\cdots$3072Hz. The fundamental frequency of S2 should be an odd multiple of that of S1. The number of main frequencies can be set according to actual needs. Then add S1 and S2 together to get a superposed signal (a new superimposed signal) and operate the "topping algorithm" which is done in level 1 pseudo-random signal designing. If the signal is bigger than 100, then put the value to 100, and if smaller than -100, put it to -100, then there will be 23 frequencies in this new signal. But if we add S1 and S2 together and operate directly, the spectrum will be not quite uniformly distributed. Phases of these two components need to be modified simultaneously to get good spectrum property.

Suppose signal S1 has a parameter ϕ_1 and signal S2 has a parameter ϕ_2, by using $\frac{2\pi}{36}$ as phase change unit, then we will have $1296 \quad (36^2)$ possibilities and calculate all RRMSE to choose the best phase combination. The RRMSE curve is shown in figure 4(c), in which there are 1296 values of RRMSE and the red dot is the best phase index location. Based on this best phase, we get the best L2-F23-1Hz-3072Hz signal as shown in figure 4(d), which means there are 2 superimposed signal component, 23 main frequencies, and 1Hz-3072Hz frequency range. The spectrum of main frequencies is evenly distributed. By this method, we can also design a much higher level and complicated signal as shown in figure 4(e), an L4-F41-1Hz-3072Hz pseudo-random signal for example.

Besides this frequency-domain log-uniformly distributed waveforms, we can also design log non-uniformly distributed signals, such as we give much more concern on a certain frequency band. Superimposed signals (basic units) are shown in figure 5(a), figure5(b),and figure 5(c). RRMSE curve to find the best phase combinations is shown in figure 5(d). The target waveform is shown in figure 5(e), in which there is a much higher frequency density between 64Hz and 512Hz. Besides, there could be more than one concern frequency band, such as we can have two and more interested frequency bands, as shown in figure 5(f). In this signal, we have 4 superimposed signal components, total of 27 frequencies, and the spectrum of 27 interested frequencies are well evenly distributed.

3 Real cases

In Jinan, Shandong Province, China, we addressed a field test with the voltage 900v, dipole length 1.8km, and ground resistivity 8Ω. The waveform and spectrum of simulated signal L3-F39-0.25Hz-3072Hz are shown in figure 6(a), in which there are 39 main frequencies, constructed by 3 superimposed units. The lowest frequency is 0.25Hz, the highest frequency 3072Hz. Because of inductive reactance and ground impact, especially for high frequencies, the real transmitted signal and its spectrum are a little different from the designed signal, as shown in figure 6(b). In real transmitted signals, amplitudes of main frequencies below 200Hz maintain the spectral characteristics well, while amplitudes of main frequencies above 200Hz are influenced by the regularity of the earth and inductive reactance. But in general, this kind of pseudo signal can be hired for real applications.

4 Discussion

To find an optimal waveform is essentially an inversion problem. But since there is only one parameter of phase in this design method, in the process of constructing low-order efficient pseudo-random signals, i.e. level 2, level 3, the amount of calculation is not large even through exhaustive methods, so the traversal method is hired to obtain the optimal phase combination. Besides, the RRMSE is the only criterion, so there is no so-called "local minimum". If the RRMSE is small enough, the corresponding phase will be the best phase to apply to design a target waveform. In the case of an odd number of interested frequencies, the superimposed signal will not have zeros, so there will be no zeros in the target pseudo-random waveform after "topping operation", which can be more easily implemented in hardware. So in real cases, the total numbers of interested frequencies would be better odd.

The algorithm of "topping operation" can be understood as a process of dividing the superimposed signal into two parts, one part is the target waveform, and the other part is a clipped part. Adjusting the phase of the square waves can be considered as a process of adjusting the spectrum characteristics of the clipped part and making the target waveform distribution more uniform. Different frequency combinations and different phase split sizes have a certain impact on the uniformity of the target waveform. And for exploration, as long as its uniformity meets the actual needs of exploration, then the corresponding waveform can be an ideal waveform.

The biggest difference from the previous method is that a superimposed signal based on the 2^n sequence superimposed waveform is first constructed. Based on this basic unit, odd multiples such as 3, 5, 7, 9, etc., are used to change the fundamental frequency to obtain new superimposed signals with the misalignment spectrum. Then combine with these superimposed signals that are misaligned to realize the spectrum encryption. In this way, the amount of calculation is greatly reduced, which makes the traversal method possible. And GPU parallel computing can greatly accelerate the process to obtain optimal phases.

Essentially, this method is a process of adjusting the energy distribution of different frequencies in the signal by continuously changing the phase of those units. The rising and falling edges of the optimal signal correspond to the direction of the current in the IGBT bridge circuit. As long as the transition time of the bridge circuit is small enough, we can transmit any waveform we generate.

5 Conclusion

Based on the proposed method in this paper, we can adaptively design waveform for the frequency-domain controlled-source electromagnetic method. All interested frequencies can be combined into a pseudo-random waveform, with the energy of these frequencies evenly distributed. By using such a waveform, exploration efficiency can be greatly improved, since we can obtain all frequencies information simultaneously and do not need to change the transmitter waveform. In the strong interference city area, we only need to extend the acquisition time to obtain a high signal-to-noise ratio data, without considering which frequency acquisition time needs to increase, making exploration more adaptive.

Besides, it is easy to design frequency domain log non-uniformly distributed signal adaptively, by putting more energy to much more concern frequencies, making exploration cost-effective. In most cases, waveforms can be designed and constructed in seconds. Furthermore, we can build a waveform library for different exploration purposes.

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the repository. And data archiving is underway. Now data related to this article can be obtained from Supporting Information.

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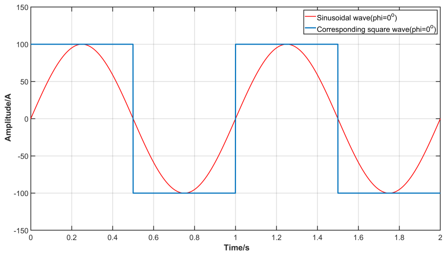
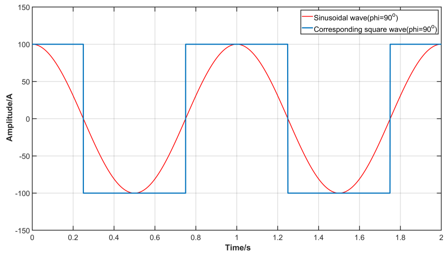
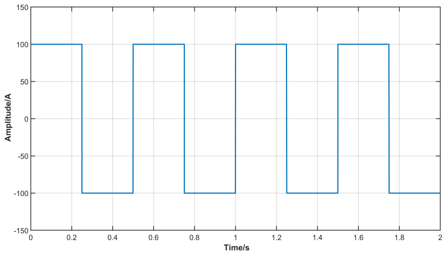


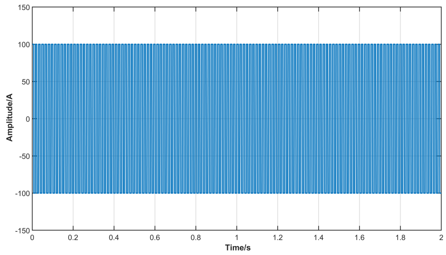
(a) Sinusoidal signal at 0 degrees and corresponding square wave



(b) Sinusoidal signal at 90 degrees and corresponding square wave

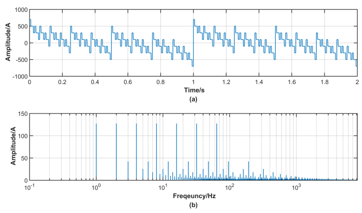


(c) Square wave of 2Hz with phase at 0 degrees

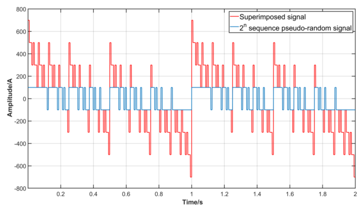


(d) Square wave of 64Hz with phase at 0 degrees

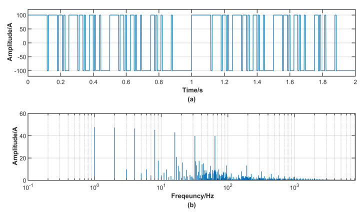
Figure 1: Sinusoidal signal at different phases and corresponding square waves.



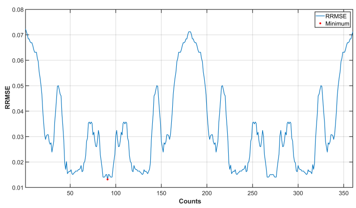
(a) Superimposed signal after superposition based on 7 squares signal in different frequencies at 0 degrees



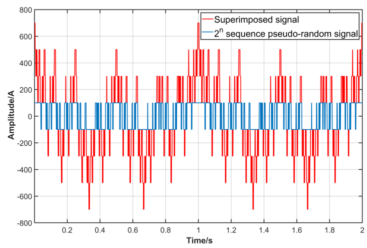
(b) Superimposed signal and its corresponding pseudo random signal



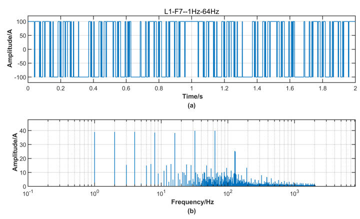
(c) 2^n sequence pseudo random signal with phase at 0



(d) L1-F7 related root mean square error curve of interested frequencies

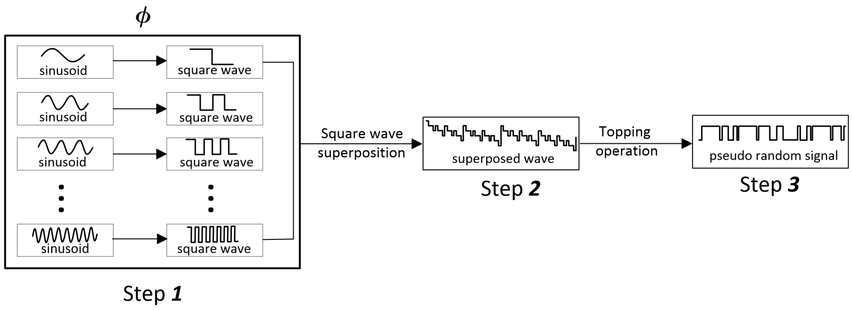


(e) Superimposed signal with phase at 90 degrees and its corresponding signal

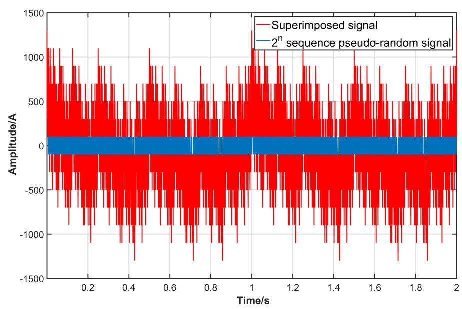


(f) 7 frequencies 2^n sequence pseudo random signal with phase at 90 degrees

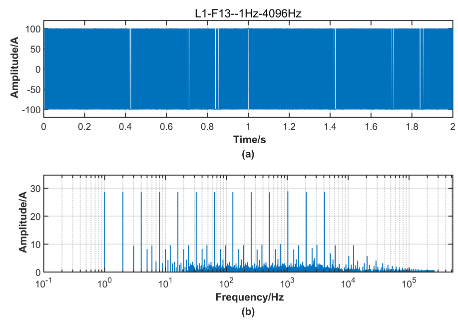
Figure 2: 7 frequencies 2^n sequence pseudo random signal respectively with phase at 0, 90 degrees and its spectrum.



(a) Workflow of the optimal waveform construction

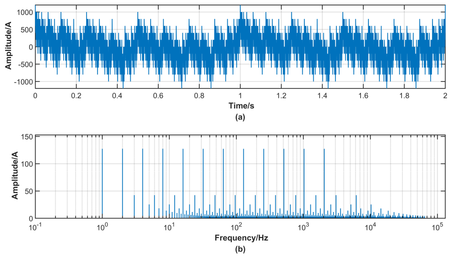


(b) Superimposed 13 frequency signal with phase at 51 degrees and its corresponding target signal

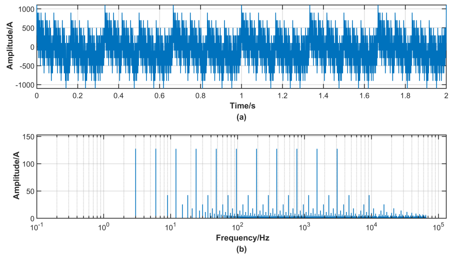


(c) 13 frequency 2^n sequence pseudo random signal at best phase point

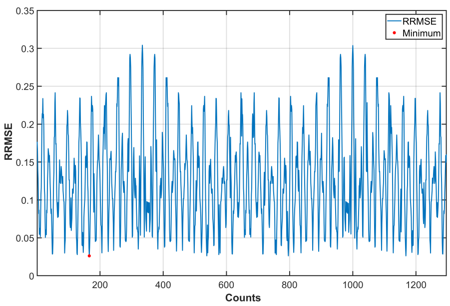
Figure 3: Workflow of the optimal waveform construction and optimal transmitter waveform sample.(a) In step *1*, a set of sinusoidal signals with frequencies increasing by multiples of 2 or 2^n are created first and then are put into corresponding square waves. All these frequencies share same phase ϕ. In step *2*, we add all these square waves together to get a basic unit. High level will contain more than one superposed wave. In step *3*, by "topping operation" mentioned in this paper, we get a set of candidate transmitter waveforms. Different ϕ in sinusoidal signals will lead to different waveforms. By calculating RRMSE of interested frequencies at different ϕ, we can get in which phase the transmitter waveform is optimal. (b) It shows a example of optimal L1-F13–1Hz-4096Hz transmitter waveform and its superimposed signal. (c) It shows the optimal L1-F13–1Hz-4096Hz transmitter waveform and its spectrum.



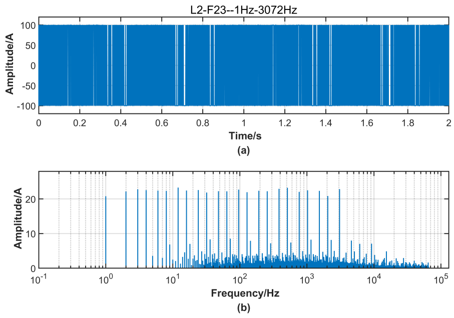
(a) 1-2048Hz superimposed signal with phase at 40 degrees(marked as S1)



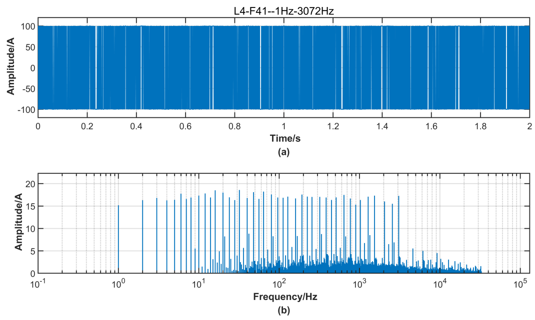
(b) 3-3072Hz superimposed signal with phase at 210 degrees(marked as S2)



(c) L2-F23-1Hz-3072Hz RRMSE curve

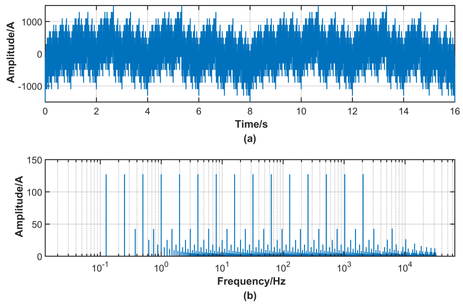


(d) L2-F23-1Hz-3072Hz optimal transmitter waveform

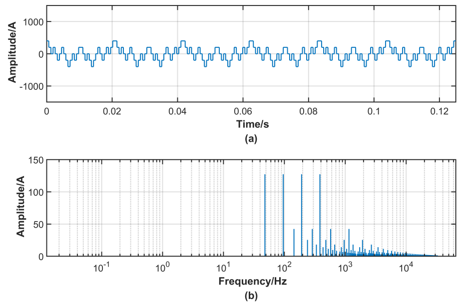


(e) L4-F41-1Hz-3072Hz optimal transmitter waveform and its spectrum

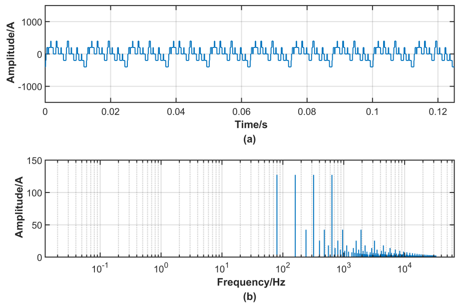
Figure 4: The process to construct L2-F23-1Hz-3072Hz optimal transmitter waveform and higher level waveform example.



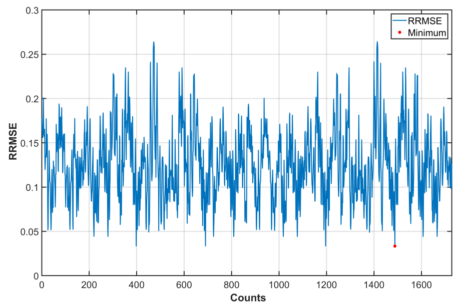
(a) 0.125-2048Hz superimposed signal component with phase at 300 degrees



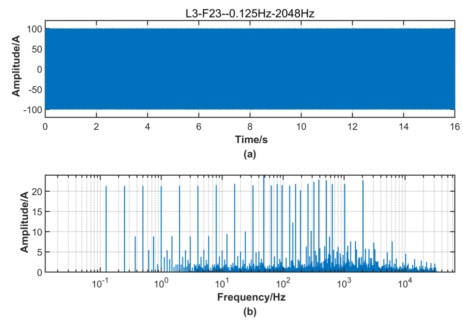
(b) 48-384Hz superimposed signal component with phase at 90 degrees



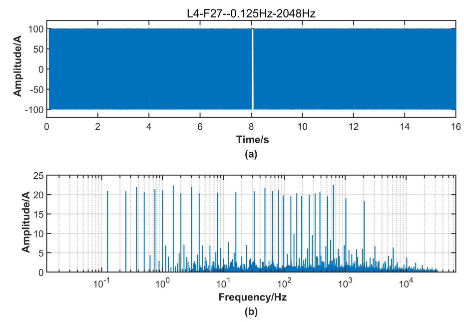
(c) 8-64Hz superimposed signal component with phase at 330 degrees



(d) L3-F23-0.125Hz-2048Hz RRMSE curve

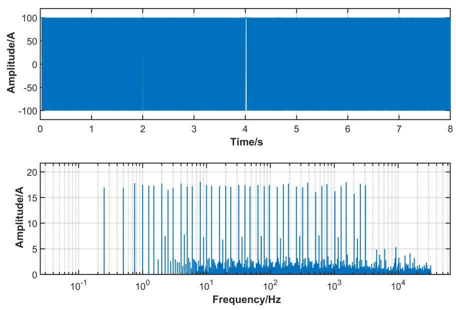


(e) L3-F23-0.125Hz-2048Hz optimal transmitter waveform and its spectrum

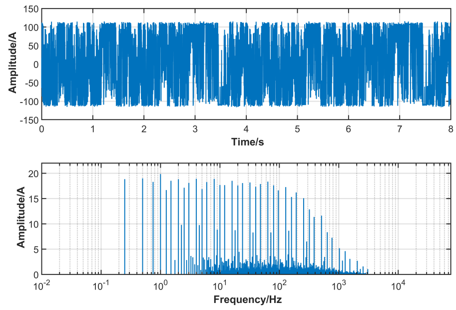


(f) L4-F27-0.125Hz-2048Hz optimal transmitter waveform and its spectrum

Figure 5: The process to construct L3-F23-1Hz-3072Hz optimal transmitter waveform and higher level waveform example.



(a) L3-F39-0.25Hz-3072Hz designed pseudo random signal (simulated with amplitude 100A)



(b) L3-F39-0.25Hz-3072Hz real transmitter waveform (dipole length 1.8km)

Figure 6: L3-F39-0.25Hz-3072Hz simulated and real field transmitter waveform and spectrum.