# Reconstructing Ocean Surface Current Combining Altimetry and Future Spaceborne Doppler Data

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# Abstract

Two methods for the mapping of ocean surface currents from satellite measurements of sea level and future current vectors are presented and contrasted. Both methods rely on the linear and Gaussian analysis frameworkwith different levels of covariance definitions. The first method separately maps sea level and currents with single-scale covariance functions and leads to estimates of the geostrophic and ageostrophic circulations. The second maps both measurements simultaneously and projects the circulation onto 4 contributions: geostrophic, ageostrophic rotary, ageostrophic divergent and inertial. When compared to the first method, the second mapping moderately improves the resolution of geostrophic currents but significantly improves estimates of the ageostrophic circulation, in particular near-inertial oscillations. This method offers promising perspectives for reconstructions of the ocean surface circulation. Even the hourly dynamics can be reconstructed from measurements made locally every few days because nearby measurements are coherent enough to help fill the gaps. Based on numerical simulation of ocean surface currents, the proposed SKIM mission that combines a nadir altimeter and a Doppler scatterometer with a 300 km wide swath (with a mean revisit time of 3 days) would allow the reconstruction of 50% of the near-inertial variance around an 18 hour period of oscillation.

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# Key Points:

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11	•	The Near Inertial Oscillations (NIOs) challenge the mapping of total surface cur-
12		rent from future Spaceborne Doppler data and Altimetry.
13	•	The challenge can be tackled with inversion schemes accounting for the coherency
14		of NIOs, allowing inversion of the current components.
15	•	Altimetry is an essential component to disentangle geostrophy from the other com-
16		ponents of the total surface current.

#### 17 Abstract

Two methods for the mapping of ocean surface currents from satellite measurements of 18 sea level and future current vectors are presented and contrasted. Both methods rely on 19 the linear and Gaussian analysis framework with different levels of covariance definitions. 20 The first method separately maps sea level and currents with single-scale covariance func-21 tions and leads to estimates of the geostrophic and ageostrophic circulations. The sec-22 ond maps both measurements simultaneously and projects the circulation onto 4 con-23 tributions: geostrophic, ageostrophic rotary, ageostrophic divergent and inertial. When 24 compared to the first method, the second mapping moderately improves the resolution 25 of geostrophic currents but significantly improves estimates of the ageostrophic circu-26 lation, in particular near-inertial oscillations. This method offers promising perspectives 27 for reconstructions of the ocean surface circulation. Even the hourly dynamics can be 28 reconstructed from measurements made locally every few days because nearby measure-29 ments are coherent enough to help fill the gaps. Based on numerical simulation of ocean 30 surface currents, the proposed SKIM mission that combines a nadir altimeter and a Doppler 31 scatterometer with a 300 km wide swath (with a mean revisit time of 3 days) would al-32 low the reconstruction of 50% of the near-inertial variance around an 18 hour period of 33 oscillation. 34

# 35 Plain Language Summary

Ocean surface currents are caused by a variety of phenomena that varies at differ-36 ent space and time scales. Here we mainly consider the two dominant contributions. The 37 first is the current resulting from the quasi-equilibrium between the sloping sea level and 38 the Coriolis force, slowly evolving over a few days. The second is also associated with 39 the Coriolis force, but out of equilibrium: oscillating currents caused by rapid changes 40 of the wind with a narrow range of periods around a natural period of oscillation that 41 increase with latitude from 12 hours at the poles. For many applications it is desirable 42 to separate these two contributions, for example to compute transports associated to the 43 slowly evolving component and to evaluate the amount of kinetic energy pumped by the 44 wind, mostly in the fast oscillations. This separation is easy with hourly sampled in situ 45 measurements, but few are available. Here we show that we can perform this separation 46 using satellite passes with measurements of sea level and a swath of surface current vec-47 tors, as can be measured by proposed future satellites. The fast oscillations can be re-48 produced even if data is available every few days, thanks to their spatial patterns and 49 temporal coherence. 50

# 51 **1** Introduction

The ocean surface current, a key variable for many scientific and operational ap-52 plications, is only partially and indirectly observed from space. Altimetry provides the 53 geostrophic component of the current (Fu et al., 1988), which is a dominant contribu-54 tion to surface transport in most of the oceans, effectively resolving wavelengths larger 55 than about 200 km wavelength (Ballarotta et al., 2019). The ageostrophic component, 56 not synoptically observed yet, is locally sampled from drifting buoys (Elipot et al., 2016) 57 or High Frequency Radars near the coasts (Kim et al., 2008). If model estimates for ageostrophic 58 current are available, in particular for the low-frequency part (Rio et al., 2014) the un-59 certainties are still high. Filling this gap with satellite measurements of the total sur-60 face current is the topic of active research, with several emerging concepts of spaceborne 61 Doppler radar for either 1 km resolution local studies such as SEASTAR (Gommenginger 62 et al., 2019) or global mapping at 10 to 30 km resolution with SKIM (Ardhuin, Brandt, 63 et al., 2019, using a 300 km wide swath), STREAM (a new proposal for ESA Earth Ex-64 plorer 11, using a 900 km wide swath) or WaCM (Rodríguez et al., 2019, using a 1200 65 to 1800 km wide swath), see Ardhuin, Chapron, et al. (2019) for a review. Similarly to 66

HF radar, these latter would provide radial components for multiple azimuth angles, from
 which the two-dimensional current vector could be assessed.

As for any satellite observation of a geophysical variable evolving in time and space. 69 an important question is the ability to map the field given the instrument spatial res-70 olution and time revisits. Satellite altimetry offers a very interesting example. Altime-71 ters measure the anomaly of the Sea Surface Height (SSH) that contains the signature 72 of various processes in the ocean spanning over a wide range of scales, some at a much 73 higher frequency than the typical 10-day revisits of the Jason satellite orbits for instance. 74 75 In practice, barotropic tides and response to high frequency winds and pressure turned out to be well handled (Carrère and Lyard (2003), Gille and Hughes (2001)) either from 76 independent or empirical models, allowing accurate reconstructions of the mesoscale eddy 77 field and , when combined with the mean dynamic topography, derived geostrophic cur-78 rents (Le Traon & Dibarboure, 2002) with limited aliasing contamination. The case of 79 total surface current brings new challenges. One of them is related to the signature of 80 Near Inertial Oscillations (NIOs) (D'Asaro, 1985) which translates a natural mode of res-81 onance in the ocean that is excited by winds at the surface. NIOs surface current have 82 average root mean square (RMS) magnitudes of 7-15 cm/s at mid and high latitudes (Elipot 83 et al., 2010; Yu et al., 2019), often comparable to the magnitude of currents in mesoscale 84 eddies. In spite of efforts to understand and model NIOs (Pollard and Millard (1970), 85 D'Asaro (1985), Whitt and Thomas (2015)), the predictability of NIOs is not yet accu-86 rate. Interactions with mesoscale also affect NIO propagation and dispersion which com-87 plicates its modeling (Young & Jelloul, 1997). 88

Relying on independent models of the high-frequency surface current is therefore not yet guaranteed. Although Doppler radar concepts may allow shorter time revisits than altimetry thanks to relatively wide swaths (Rodriguez et al., 2018), they may not directly sample inertial periods, e.g. a 18 hours period at 40° of latitude requires a revisit time of 9 hours for which a swath wider than 2500 km is necessary, which does not appear feasible with a single satellite mission. Therefore the reconstruction of surface current in time and space from space-borne Doppler is a challenge.

The focus of this paper is to explore this reconstruction challenge in simulation, taking the practical example of the SKIM Doppler concept combined with altimetry, using basic and improved mapping methods accounting for the physical properties of NIOs. The skills of the reconstruction will be evaluated quantitatively for both geostrophic and ageostrophic components in the North Atlantic basin.

#### <sup>101</sup> 2 Reconstruction methods

#### 102 2.1 Background on linear analysis

The different mapping approaches explored in this study are all derived from the linear and Gaussian mapping framework reviewed below. We assume a state to estimate, noted  $\mathbf{x}$ , and partial observations, noted  $\mathbf{y}$ , that can be related to the state with a linear operator  $\mathbf{H}$  such as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\epsilon} \tag{1}$$

where  $\epsilon$  is an independent signal (e.g. observation error) not related to the state. If we define **B** the covariance matrix of **x** and **R** the covariance matrix of  $\epsilon$ , both variables being assumed Gaussian, then the linear estimate is written:

$$\mathbf{x}^{\mathbf{a}} = \mathbf{B}\mathbf{H}^{\mathbf{T}}(\mathbf{H}\mathbf{B}\mathbf{H}^{\mathbf{T}} + \mathbf{R})^{-1}\mathbf{y}.$$
(2)

This formulation, known as Optimal Interpolation, requires the inversion of a matrix of the same size as the observation vector **y**. When the number of observations exceeds the size of the state to resolve, it can be interesting to use an equivalent formulation given <sup>113</sup> by the Sherman-Morrison-Woodbury transformation, allowing an inversion in state space,

114 with a matrix of the size of the state vector  $\mathbf{x}$ ,

$$\mathbf{x}^{\mathbf{a}} = (\mathbf{H}^{\mathbf{T}}\mathbf{R}^{-1}\mathbf{H} + \mathbf{B}^{-1})^{-1}\mathbf{H}^{\mathbf{T}}\mathbf{R}^{-1}\mathbf{y}.$$
(3)

This second formulation is particularly useful when considering transformed states expressed in orthogonal bases (see section 2.3.1 where **B** becomes diagonal and the whole system gets easier to invert). If we note **K** the linear operator such as  $\mathbf{x}^{\mathbf{a}} = \mathbf{K}\mathbf{y}$  from Eq. (2) or Eq.(3), the covariance matrix of analysis error is given by:

$$\mathbf{B}^{\mathbf{a}} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{B} \tag{4}$$

<sup>119</sup> This latter can be used to characterize the uncertainty of the solution.

<sup>120</sup> 2.2 Basic mappings

In the basic mapping approaches, we perform separate mappings of the SSH and total surface current involving simple covariance functions in the **B** matrix defined separately for each variable. Then the geostrophic component is given by the derivation of SSH maps and the ageostrophic component by subtraction of the geostrophic component to the total current.

# 2.2.1 For SSH: the standard Aviso/CMEMS mapping

To map the SSH, we first map the Sea Level Anomaly (SLA), defined in reference 127 to the long-term mean. We followed, as in the standard Aviso/CMEMS mapping, a ba-128 sic formulation derived from Eq.2. The observation vector  $\mathbf{y}$  is the SLA observations, 129 noted  $\mathbf{h}^{\mathbf{o}}$ . The state vector  $\mathbf{x}$  is the gridded SLA. The observation operator  $\mathbf{H}$  (a tri-linear 130 interpolator transforming the gridded state SLA to equivalent along-track SLA) is not 131 written explicitly: in practice, the matrices **BH**<sup>T</sup> and **HBH**<sup>T</sup>, representing the covari-132 ance of the signal in the (grid, obs) and (obs, obs) spaces, are directly written with the 133 analogical formula of the covariance model as described in (Pujol et al., 2016). The  $\mathbf{R}$ 134 matrix, for representativity and instrumental errors, is assumed diagonal. Since the co-135 variance of SLA is assumed to vanish beyond a few hundreds of kilometers in space and 136 beyond 10 to 20 days in time ((Le Traon & Dibarboure, 2002)), separate inversions are 137 performed locally selecting observations over time and space windows adjusted to these 138 values. In practice, the number of observations being limited to less than 1000, the in-139 version in observation space is computationally manageable. Details on the map produc-140 tion are given in (Pujol et al., 2016). From the SLA maps, the SSH maps are given with 141 the addition of the long-term mean subtracted before the mapping. 142

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#### 2.2.2 For total surface current: a bi-variate weighted least square

The total surface current has different covariance structures than SSH, and does not benefit from the long history of developments with operational systems (at least when the measurements are scattered in space and azimuth angles as in the SKIM concept). As a first level processing, we therefore choose a basic filter, where eq.(3) is applied locally to solve for a single current vector  $\mathbf{x} = [u, v]^T$  from the radial velocity observations nearby within a time-space radius. In this context, **B** can be considered as infinite, and eq. (3) reduces to the following bivariate least square formula:

$$[u, v]^T = (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{R}^{-1} \mathbf{u_r^o},$$
(5)

where  $\mathbf{u}_{\mathbf{r}}^{\mathbf{o}}$  are the radial velocity observations. The **R** matrix represents the covariances of  $\mathbf{u}_{\mathbf{r}}^{\mathbf{o}}$  errors, namely the representativity error and Doppler measurement error, both assumed diagonal. Note that  $\mathbf{R}^{-1}$  can also be called the weight matrix **W**, the weights

being the inverse of observation error variances (set to  $0.2^2$  (m/s)<sup>2</sup> for the problem con-147 sidered in section 3, which is an optimal value also including representativity errors). These 148 weights also take into account the time and space radius, set to 40 km and 10 days in 149 section 3, with a Hamming window. Finally, the observation operator  $\mathbf{H}$  can be writ-150

ten from the vector of  $\mathbf{u_r^o}$  azimuth angles  $\theta_1 \dots \theta_p$  as follows, 151

$$\mathbf{H} = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) \\ \vdots & \vdots \\ \cos(\theta_p) & \sin(\theta_p) \end{bmatrix}$$
(6)

where p is the size of the observation vector. Since **B** tends to infinity, the covariances 152

of analysis error  $\mathbf{B}^{\mathbf{a}}$  given by the limit of Eq. (4) are written as: 153

$$\mathbf{B}^{\mathbf{a}} = \mathbf{H}^{\mathbf{T}} \mathbf{R}^{-1} \mathbf{H}.$$
 (7)

A geometrical illustration of the solution is shown in Figure 1, with the ellipses of the 154 solution uncertainty given by the  $\mathbf{B}^{\mathbf{a}}$  matrix of size (2×2). Note that at least two ob-

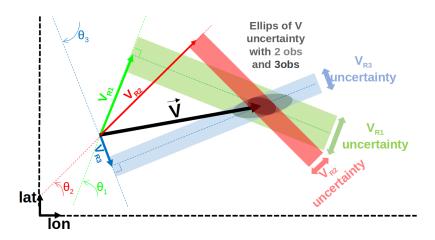


Figure 1: Scheme of the basic surface current mapping algorithm based on a bi-variate weighted least square, from at least two radial Doppler observations at different azimuth  $\theta$ (here there are 3 observations  $V_{R1}$ ,  $V_{R2}$ ,  $V_{R3}$ ).

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servations at different  $\theta$  angles are necessary to ensure invertibility of eq.(5), which is also 156 illustrated on Figure 1. This is why the time-space radius must be carefully set with re-157 spect to the observation sampling. 158

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# 2.2.3 Geostrophic and ageostrophic current gridded maps

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The geostrophic current 
$$(\mathbf{u}_{\mathbf{g}}, \mathbf{v}_{\mathbf{g}})$$
 is directly derived from the mapped SSH

$$\begin{cases} u_g = -\frac{g}{f_c} \frac{\partial SSH}{\partial y} \\ v_g = \frac{g}{f_c} \frac{\partial SSH}{\partial x} \end{cases}$$
(8)

where  $f_c$  is the Coriolis frequency, which is a function of latitude. The ageostrophic es-161

timates  $(\mathbf{u}_{\mathbf{ag}}, \mathbf{v}_{\mathbf{ag}})$  are obtained by substraction to the total surface current  $(\mathbf{u}, \mathbf{v})$  of sec-162 tion 2.2.2, 163

$$\begin{cases} u_{ag} = u - u_g \\ v_{ag} = v - v_g \end{cases}$$

$$\tag{9}$$

These geostrophic and ageostrophic current estimates will be considered as the basic mapping solutions in section 4.

# 2.3 Improved mapping

The improved mapping also relies on linear analysis framework but with extended state, extended observation vector and multivariate covariances. For practical reasons, the inversion problem is framed in a reduced sub-component space such as to accommodate the number of observations in large spatio-temporal windows. This is particularly interesting to handle multiple signals of various scales in time and/or space.

172 **2.3.1 Formulation** 

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We consider an extended state vector  $\mathbf{x}$  composed by N physical components (e.g. geostrophy, low and high frequency ageostrophy as proposed in section 2.3.2):

$$\mathbf{x} = (\mathbf{x}_1^{\mathbf{T}}, \dots, \mathbf{x}_N^{\mathbf{T}})^{\mathbf{T}}.$$
(10)

Each component  $\mathbf{x}_{\mathbf{k}}$  contains the surface topography and surface current variables to be resolved in the grid space, noted  $\mathbf{x}_{\mathbf{k}} = (\mathbf{h}_{\mathbf{k}}^{\mathbf{T}}, \mathbf{u}_{\mathbf{k}}^{\mathbf{T}}, \mathbf{v}_{\mathbf{k}}^{\mathbf{T}})^{\mathbf{T}}$ . The key aspect of the method is a rank reduction of the state vector, through a sub-component decomposition, such as  $\mathbf{x}_{\mathbf{k}}$  can be written as:

$$x_{k} = \begin{bmatrix} \Gamma_{k,h} \\ \Gamma_{k,u} \\ \Gamma_{k,v} \end{bmatrix} \eta_{k} = \Gamma_{k} \eta_{k}$$
(11)

where  $\eta_{\mathbf{k}}$  is the reduced state vector for component k,  $\Gamma_{\mathbf{k},\mathbf{h}}$ ,  $\Gamma_{\mathbf{k},\mathbf{u}}$ , and  $\Gamma_{\mathbf{k},\mathbf{v}}$  are the sub-177 component matrices in topography and currents. Note that for some components, one 178 of the block can be set to zeros (e.g. if ageostrophy component is considered with zero 179 contribution on SSH). Their concatenation is called  $\Gamma_{\mathbf{k}}$  which is the matrix transform-180 ing the reduced state vector in the grid space for topography and currents. In practice. 181  $\Gamma_{\mathbf{k}}$  will be a wavelet decomposition of the time-space domain, with elements of appro-182 priate temporal and spatial scales to represent the component k. These wavelet scales, 183 and their specified variance set with a diagonal matrix noted  $\mathbf{Q}_{\mathbf{k}}$ , will define the equiv-184 alent covariance model  $\mathbf{B}_{\mathbf{k}}$  in the grid space for component k: 185

$$\mathbf{B}_{\mathbf{k}} = \boldsymbol{\Gamma}_{\mathbf{k}} \mathbf{Q}_{\mathbf{k}} \boldsymbol{\Gamma}_{\mathbf{k}}^{\mathrm{T}} \tag{12}$$

The observation vector  $\mathbf{y}$  is also extended to the observed surface topography and surface current noted  $\mathbf{y} = (\mathbf{h^{oT}}, \mathbf{u_r^{oT}})^{\mathrm{T}}$ . Then, if  $\mathbf{H_k}$  is the observation operator for component k (from grid space to observation space), we note  $\mathbf{G_k} = \mathbf{H_k}\Gamma_k$  the sub-component matrix expressed in observation space. In these conditions, the observation vector  $\mathbf{y}$  is the sum of all component contributions plus the unexplained signal  $\epsilon$  (instrument error and representativity),

$$\mathbf{y} = \sum_{\mathbf{k}=1}^{\mathbf{N}} \mathbf{G}_{\mathbf{k}} \eta_{\mathbf{k}} + \epsilon \tag{13}$$

If we use the notation  $\eta = (\eta_1^{\mathbf{T}}, ..., \eta_{\mathbf{N}}^{\mathbf{T}})^{\mathbf{T}}$  for the concatenation of the sub-component state vectors, and  $\mathbf{G} = (\mathbf{G}_1, ..., \mathbf{G}_{\mathbf{N}})$ , then we have,

$$\mathbf{y} = \mathbf{G}\boldsymbol{\eta} + \boldsymbol{\epsilon} \tag{14}$$

Applying the same transformation from eq. (1) to eq. (3), to the reduced state vector  $\eta$ , the global solution is written:

$$\eta^{\mathbf{a}} = (\mathbf{G}^{\mathbf{T}}\mathbf{R}^{-1}\mathbf{G} + \mathbf{Q}^{-1})^{-1}\mathbf{G}^{\mathbf{T}}\mathbf{R}^{-1}\mathbf{y}$$
(15)

where **Q** is the covariance matrix of  $\eta$ , expressed as the concatenation of the diagonal matrices **Q**<sub>k</sub> for each component. Finally the solution in the reduced-space projects into the grid space with the following relation:

$$\mathbf{x}^{\mathbf{a}} = \mathbf{\Gamma} \eta^{\mathbf{a}} \tag{16}$$

In practice, in order to solve for Eq. (15), each block of **G** is directly filled from 199 the analytical expression of the reduced-space elements (wavelets in section 2.3.2) con-200 stituting the columns of the matrix. Also, in many situations, the  $(\mathbf{G^TR^{-1}G} + \mathbf{Q^{-1}})$ 201 matrix, noted A hereafter, would be too large to be inverted (and even written) explic-202 itly. We use a pre-condition conjugate gradient method to solve for  $\eta = \mathbf{A}^{-1}\mathbf{z}$  where 203  $\mathbf{z} = \mathbf{G}^{T} \mathbf{R}^{-1} \mathbf{y}$  is a vector of reduced-state size computed initially from  $\mathbf{G}$  and the ob-204 servation vector y. The algorithm involves many iterations of  $\mathbf{A}\eta$  computations for up-205 dated  $\eta$ . Note that if **A** is too large to be written explicitly, the result **A** $\eta$  can still be 206 computed in two steps from a matrix multiplication of  $\mathbf{G}$  then of  $\mathbf{G}^{\mathbf{T}}$ . Once the conver-207 gence for the solution  $\eta$  is reached, the projection in physical grid space given by Eq. (16) 208 is applied sequentially, by summing the analytical expression of the wavelets applied to 209 grid coordinates (the columns of  $\Gamma$ ), separately for each component k. 210

As in any inversion based on linear analysis, the result strongly relies on the choice of covariance models, here defined by the reduced elements of each component. The choices of these elements are discussed in the following section.

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# 2.3.2 Application to simultaneous mapping of geostrophy, low and high frequency ageostrophy

We propose to apply the above formulation for four components (N = 4), considering that the surface current is dominated by geostrophy, low frequency ageostrophy (splitting the low-frequency flow in rotationnal and divergent components for practical reasons) and high frequency ageostrophy, for which specific wavelet bases will be defined.

221 2.3.2.1 Geostrophy

Geostrophy is the component that has a signature on both topography and currents, where to expect some synergy between the altimetry and doppler observations. We define here the gridded variable  $H_1$  to resolve, and the corresponding gridded geostrophic current field  $(U_1, V_1)$  writes

$$\begin{cases} U_1 = -\frac{g}{f_c} \frac{\partial H_1}{\partial y} \\ V_1 = \frac{g}{f_c} \frac{\partial H_1}{\partial x} \end{cases}$$
(17)

The proposed reduced state for geostrophy is based on an element decomposition of  $H_1$ , expressed by  $\Gamma_{1,h}$  with wavelets of various wavelength and temporal extensions. This will allow to approximate the standard covariance models used in altimetry mapping, accounting for specific variations with wavelength and time. A given p element of the decomposition  $\Gamma_{1,h}$  is expressed as follows:

$$\Gamma_{1,h}[i,p] = \cos(k_{x,p}(x_i - x_p) + k_{y,p}(y_i - y_p) + \Phi_p) * f_{tap}(\frac{x_i - x_p}{L_{x_p}}, \frac{y_i - y_p}{L_{y_p}}, \frac{t_i - t_p}{L_{t_p}})$$
(18)

where *i* is a given grid index of coordinates  $(x_i, y_i, t_i)$ . For the ensemble of *p*,  $\Phi_p$  is alternatively 0 and  $\pi/2$ , such as all subcomponents are defined by pairs of sine and cosine functions to allow the phase degree of freedom.  $k_{x,p}$  and  $k_{y,p}$  are zonal and meridional wavenumbers respectively, set to vary in the mappable mesoscale range (between 80km to 800km for the problem considered in section 3, with a spacing inversely proportional to the wavelet extensions, allowing to represent a signal of any intermediate wavelength).  $(x_p, y_p, t_p)$  are the coordinates of a space-time pavement. The function  $f_{tap}$  localizes the sub-component in time and space (at scales  $L_{t_p}$ ,  $L_{x_p}$  and  $L_{y_p}$  respectively) as geostrophy has local extension of covariances. It is expressed as:

$$f_{\rm tap}(\delta x, \delta y, \delta t) = \begin{cases} \cos(\pi \delta x/2) \times \cos(\pi \delta y/2) \times \cos(\pi \delta t/2), & \text{for } (|\delta x|, |\delta y|, |\delta t|) < (1, 1, 1) \\ 0, & \text{elsewhere} \end{cases}$$
(19)

In practice, for the problem considered in section 3,  $L_{x_p}$  and  $L_{y_p}$  will be set to 1.5 the wavelength of element p and  $L_{t_p}$  to the decorrelation time scale of Aviso maps, on the order of 10 days in this region. Then, the same element p of the decomposition has also an expression in geostrophic current (through the geostrophic relation Eq. 8) written in the  $\Gamma_{1,\mathbf{u}}$  and  $\Gamma_{1,\mathbf{v}}$  matrices:

$$\begin{cases} \Gamma_{1,u}[i,p] = -g \left( \partial \Gamma_{1,h}[i,p] / \partial y_i \right) / f_c, \\ \Gamma_{1,v}[i,p] = g \left( \partial \Gamma_{1,h}[i,p] / \partial x_i \right) / f_c \end{cases}$$
(20)

As an illustration,  $\Gamma_{1,h}[:,p]$  is shown on Figure 2 upper left panel, in plain color and  $\Gamma_{1,u}[:,p]$ ,  $\Gamma_{1,v}[:,p]$  in arrows. Here, this  $p^{\text{th}}$  component has a dominant wavelength  $\lambda = \frac{2\pi}{\sqrt{k_{x,p}^2 + k_{y,p}^2}}$  in a given direction. The lower-left panel represents the temporal extension of the sub-component set by  $L_{t_p}$ . The whole time-space domain is paved with similar

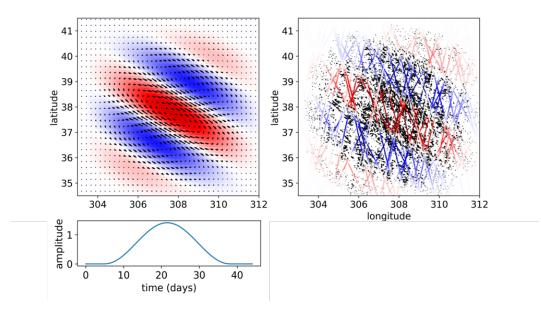


Figure 2: Illustration of a sub-component belonging to the mesoscale geostrophic component. The upper-left panel represents its expression in the grid space (= a column of the  $\Gamma_1$  matrix), in color for the topography ( $\Gamma_{1_h}$ ) and arrows for the current ( $\Gamma_{1_u}$  and  $\Gamma_{1_v}$ ), as a function of space. The lower-left panel represent the temporal modulation. The right panel represents the same sub-component in observation space considering altimetry tracks and scattered radial velocity observations at various azimuth angles, noted  $G_1$  declined in  $G_{1_h}$  (colors) and  $G_{1_{u_r}}$  (arrows).

sub-components, along coordinates  $(x_p, y_p, t_p)$  for wavelengths between 80km and 800km 249 spanning in all directions of the plane. The ensemble can be seen as a wavelet basis. Fi-250 nally, each sub-component p is assigned an expected variance in the  $\mathbf{Q}_1$  matrix, consis-251 tent with the power spectrum observed from altimetry at the corresponding wavelength 252 with isotropy assumption. 253

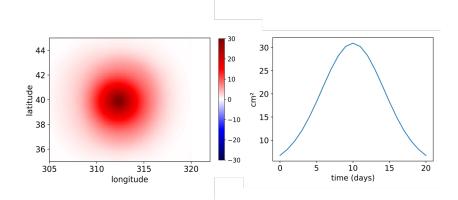


Figure 3: Illustration of the representor  $\Gamma_{\mathbf{1}_{h}}[\mathbf{i}, :]\mathbf{Q}\Gamma_{\mathbf{1}_{h}}$  for a given point *i* on the timespace grid (312°E,40°N, 10 days) represented as a function of space at 10 days (left panel) and as a function of time at 312°E,40°N.

For a given point i on the time-space grid (312°E,40°N, day 10), the representor 254  $\Gamma_{1,\mathbf{h}}[\mathbf{i},:]\mathbf{Q}\Gamma_{1,\mathbf{h}}$  is plotted on Figure 3, shown as a function of space (left panel) and as 255 a function of time (right panel). It illustrates the equivalent covariance function, which 256 is quite similar to what is currently used for altimetry mapping with OI inverted in ob-257 servation space. 258

As mentioned in section 2.3.1, the inversion involves the construction of  $G_1$  ma-259 trix (see Eq.13), whose  $p^{\text{th}}$  column is represented on the right panel of Figure 2 consid-260 ering altimetry tracks and scattered radial velocity observations at various azimuth an-261 gles described later in section 3. Here, the arrows are the projection of the  $(\Gamma_{1,\mu}[:,p],\Gamma_{1,\mu}[:$ 262 , p] along the various instrument azimuth angles and the colored dots the bilinear in-263 terpolation at nadir altimetry coordinates. 264

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#### 2.3.2.2 Low-frequency ageostrophy: rotational part

The low-frequency ageostrophy cannot be reduced a priori, as for geostrophy, to 266 a single potential scalar field. However, if working with the zonal and meridional cur-267 rent  $\mathbf{U}$  and  $\mathbf{V}$  would be a first option to build the reduced state, we decided to work with 268 the rotational and divergent current functions as they are scalars fields more likely to 269 have isotropic features than the directional variables U and V. Assuming isotropy of the 270 scalar fields practically allows easier reduced space decomposition. This paragraph deals 271 with the rotational flow, defined by a gridded variable  $\mathbf{P}$  (potential) to resolve, such as 272 the gridded SSH and surface current  $(\mathbf{H}_2, \mathbf{U}_2, \mathbf{V}_2)$  for this component are written: 273

$$\begin{cases} H_2 = 0\\ U_2 = -\frac{\partial P}{\partial y}\\ V_2 = \frac{\partial P}{\partial x} \end{cases}$$
(21)

 $H_2 = 0$  since we consider no contributions of P on SSH. A reduced state is considered 274 275

for **P**, constructed with single time/space window elements allowing the representation

of the field down to a certain regularity in time and space. The proposed decomposition is much simpler than for topography, because the scales involved are larger in space and we did not find the necessity to tune the covariance model beyond to get acceptable results. The reduced state is represented in the time-space domain by the following  $\Gamma_{2,P}$ matrix:

$$\Gamma_{2,P}[i,p] = f_{tap}(\frac{x_i - x_p}{L_x}, \frac{y_i - y_p}{L_y}, \frac{t_i - t_p}{L_t})$$
(22)

where  $f_{\text{tap}}$  is defined by Eq. 19. Here again,  $(x_p, y_p, t_p)$  are the coordinates to a space and time pavement. In practice,  $L_x$ ,  $L_y$  and  $L_t$  correspond to the decorrelation scales (in time and space) of the reduced space. Using Eq 21, the matrices  $\Gamma_{2,h}$ ,  $\Gamma_{2,u}$  and  $\Gamma_{2,v}$ are written:

$$\begin{cases} \Gamma_{2,h}[i,p] = 0\\ \Gamma_{2,u}[i,p] = -\frac{\partial\Gamma_{2,P}[i,p]}{\partial y_i}\\ \Gamma_{2,v}[i,p] = \frac{\partial\Gamma_{2,P}[i,p]}{\partial x_i} \end{cases}$$
(23)

As an illustration, the  $p^{\text{th}}$  column of  $(\Gamma_{2,u}, \Gamma_{2,v})$  is represented on Figure 4. Here

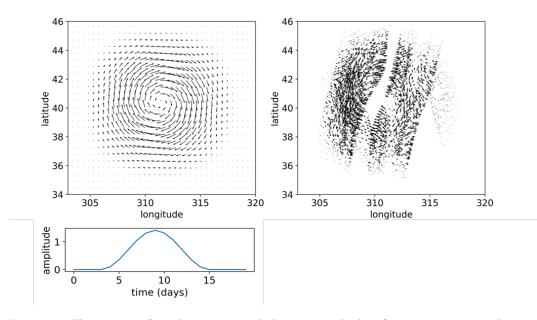


Figure 4: Illustration of a sub-component belonging to the low-frequency ageostrophic rotational component. The upper-left panel represents its expression in the grid space (= a column of the  $\Gamma_2$  matrix), the arrows for ( $\Gamma_{2_u}$  and  $\Gamma_{2_v}$ ), as a function of space. The lower-left panel represent the temporal modulation. The right panel represents the same sub-component in observation space, noted  $\mathbf{G}_{2,u_r}$ .

<sup>285</sup> 

again, the whole time-space domain is paved with similar wavelet sub-components along 286 coordinates  $\mathbf{x}_{\mathbf{p}}, \mathbf{y}_{\mathbf{p}}$  and  $\mathbf{t}_{\mathbf{p}}$ . The equivalent covariance model obtained from Eq.12, not 287 shown, is overall similar to what is shown on Fig.2 for geostrophy, with a more basic spa-288 tial function only driven by  $L_x$  and  $L_y$ . It is set larger in space and shorter in time, aim-289 ing to capture large and more rapid signals than geostrophy. Targeting shorter scales in 290 space would not be compatible with the observation dataset considered. In practice, for 291 the problem considered in section 3, they will be set to 400km and 5 days in space and 292 time, ensuring enough observations to resolve the total current at this space/time scale. 293

The  $p^{\text{th}}$  column of  $\mathbf{G}_2$  matrix involved in the inversion is shown on the right panel of Figure 4 for illustration. It represents the projection of the sub-component vector field at the location and azimuth angle of all observations in the local domain.

297 2.3.2.3 Low-frequency ageostrophy: divergent part

The divergent part is handled exactly the same way as the rotational part, except that the gridded field to resolve is a Solenoidal function  $\mathbf{S}$ , such as the gridded SSH and currents  $(\mathbf{H}_3, \mathbf{U}_3, \mathbf{V}_3)$  for this component are written:

$$\begin{cases} H_3 = 0\\ U_3 = -\frac{\partial S}{\partial x}\\ V_3 = -\frac{\partial S}{\partial y} \end{cases}$$
(24)

Here again, we consider no contributions of  $\mathbf{S}$  on SSH. Following the same reduced-state

decomposition for S than for P, with a matrix noted  $\Gamma_{3,S}$ , the matrices  $\Gamma_{3,h}$ ,  $\Gamma_{3,u}$  and  $\Gamma_{3,v}$  writes

$$\begin{cases} \Gamma_{3,h}[i,p] = 0\\ \Gamma_{3,u}[i,p] = -\frac{\partial\Gamma_{3,S}[i,p]}{\partial x_i}\\ \Gamma_{3,u}[i,p] = -\frac{\partial\Gamma_{3,S}[i,p]}{\partial u_i} \end{cases}$$
(25)

The  $p^{\text{th}}$  column of  $(\Gamma_{3,u}, \Gamma_{3,v})$  is represented on the left panel of Figure 5 for illustration, as well as the  $p^{\text{th}}$  column of the  $\mathbf{G}_2$  matrix involved in the inversion.

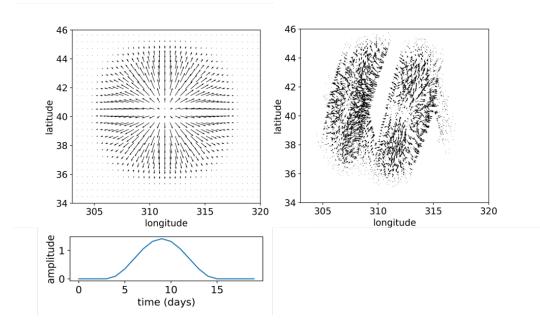


Figure 5: Illustration of a sub-component belonging to the low-frequency ageostrophic divergent component. The upper-left panel represents its expression in the grid space (= a column of the  $\Gamma_3$  matrix), the arrows for ( $\Gamma_{3_u}$  and  $\Gamma_{3_v}$ ), as a function of space. The lower-left panel represent the temporal modulation. The right panel represents the same sub-component in observation space, noted  $G_{3_{ur}}$ .

306	2.3.2.4 High-frequency ageostrophy (NIO)
307	This component stands for the near inertial motions featuring very distinct oscil-
308	lating patterns near the inertia frequency $f_c$ . It is possible to define a reduced space made
309	of two gridded fields to resolve <b>A</b> and <b>B</b> , slowly varying in time and space with the grid-
310	ded SSH and current field $(H_4, U_4, V_4)$ expressed as follows:

$$\begin{cases} H_4 = 0\\ U_4 = A\cos(-2\pi f_c t) + B\sin(-2\pi f_c t)\\ V_4 = A\sin(-2\pi f_c t) - B\cos(-2\pi f_c t) \end{cases}$$
(26)

Here again, we assume no contribution on SSH. This current field oscillates near the inertia frequency, with a coherency related to the time variations of **A** and **B**. Note that the distinct **A** and **B** allow the degree of freedom on the phase of the NIOs. The reduced space for **A** and **B** is defined by the following  $\Gamma_{4_{\rm A}}$  and  $\Gamma_{4_{\rm B}}$  identical matrices, giving:

$$\Gamma_{4,A}[i,p] = \Gamma_{4,B}[i,p] = e^{-\frac{|t_i - t_p|^q}{\tau^q}} f_{tap}(\frac{x_i - x_p}{L_x}, \frac{y_i - y_p}{L_y}, 0)$$
(27)

The time decay is not set with the  $f_{tap}$  function, but with an exponential of degree q which seemed to better represent actual time perturbations of the oscillations. For the problem considered in section 3,  $L_x$  and  $L_y$  will be both set to 250km, q at 2 and  $\tau$  at 3 days. These values were optimized to fit the covariance properties of the NIOs signal in the reference simulation. Using Eq. (26), the matrices  $\Gamma_{4,h}$ ,  $\Gamma_{4,u}$  and  $\Gamma_{4,v}$  writes:

$$\begin{cases} \Gamma_{4,h}[i,p] = 0\\ \Gamma_{4,u}[i,p] = \Gamma_{4,A}[i,p]\cos(-2\pi f_c t) + \Gamma_{4,B}[i,p]\sin(-2\pi f_c t_i)\\ \Gamma_{4,v}[i,p] = \Gamma_{4,A}[i,p]\sin(-2\pi f_c t) - \Gamma_{4,B}[i,p]\cos(-2\pi f_c t_i) \end{cases}$$
(28)

The  $p^{\text{th}}$  column of  $(\Gamma_{4,u}, \Gamma_{4,v})$  is represented on the left panels of Figure 6 for illustration. The arrows on the upper panel indicate a spatially coherent pattern of NIOs, actually rotating in time as indicated by the time-modulation on the lower panel.

Finally, the  $p^{\text{th}}$  column of the  $\mathbf{G_4}$  matrix involved in the inversion is shown on the right panel of Figure 6. The arrows indicate multiple directions are the observations span over different times in the local domain of the sub-component.

## 326 **3** Observing System Simulation Experiments

#### 3.1 The reference scene

Ocean circulation numerical models provide realistic scenes of ocean variability, use-328 ful to assess the impact of existing and future observing systems. In this study, we used 329 the outputs of a high-resolution  $(1/60^{\circ})$  in the horizontal) simulation at hourly frequency, 330 the NEMO NATL60-CJM simulation further described in (Amores et al., 2018). This 331 simulation, forced with hourly winds, allows the resolution of a wide spectrum of pro-332 cesses at ocean surface, from basin to sub-mesoscales and from annual to hourly scales 333 including NIOs, in the North Atlantic region. This simulation does not include tidal forc-334 ing, but as discussed in the conclusion, this should not impact our analysis. The SSH 335 and surface current in the first layer constitute our ground-truths in the experiments span-336 ning over the year 2012. 337

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# 3.2 Synthetic observations from instrument simulators

The instrument simulators are based on existing software used to generate synthetic observations. They perform a sampling, in time and space, of the reference scene over the satellite view along the orbit, and generate a realistic measurement error, either instrumental or geophysical.

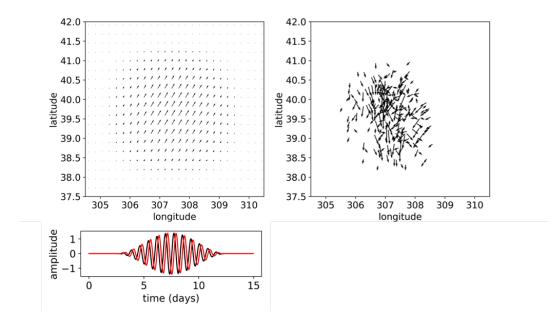


Figure 6: Illustration of a sub-component belonging to the high-frequency ageostrophic component. The upper-left panel represents its expression in the grid space (and corresponds to a column of the  $\Gamma_4$  matrix), the arrows for ( $\Gamma_{4_u}$  and  $\Gamma_{4_v}$ ), as a function of space. The lower-left panel represent the temporal modulation for the zonal u (black) and meridional v (red) components. The right panel represents the same sub-component in observation space, noted  $\mathbf{G}_{4_{un}}$ .

#### 343 3.2.1 The altimetry simulator

The altimetry simulator (Gaultier et al., 2016) in its nadir version was used in this study to simulate a constellation of 5 altimeters on different orbits (two Jason-like and three Sentinel3-like). The model SSH was sampled at 1 Hz posting (approximately 6 km) along these orbit tracks over 1-year. An instrumental error of 3cm RMS at 1Hz was applied to all satellites following a random Gaussian law to simulate the white-noise plateau. An illustration of the altimetry dataset is shown on the top panels of Figure 7.

350

#### 3.2.2 The Doppler simulator

The Doppler simulator for the SKIM concept, called 'SKIMulator' (Gaultier, 2019a, 351 2019b), was developed in the context of SKIM phase A studies by ESA (ESA, 2019). The 352 simulator was applied on the first-layer vector current of the reference field, providing 353 radial current vectors along multiple azimuth angles of the rotating beams as illustrated 354 by the green arrows on Figure 8. An instrument error is applied, accounting for radar 355 noise and Doppler processing errors such as the error in the surface wave Doppler retrieval, 356 as further described in (F. Ardhuin et al., 2019). The total error is on the order of 5-10 357 cm/s. The overall swath, of 270km width, samples any given point of the Ocean at or-358 bit repeat (12days), and according to the latitude, the ascending/descending and over-359 lapping swaths allow several revisits at different intervals. 360

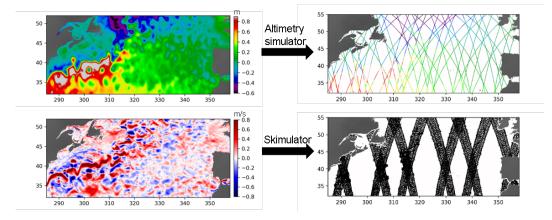


Figure 7: Schematic showing the instrument simulator sampling (altimetry on the top, doppler current Skimulator on the bottom) from the reference scene on the left to the sampled data with instrument error added on the right. Three-day worth of synthetic data are shown on the right panels.

# 361 4 Results

#### 362

# 4.1 Reconstruction of Geostrophic and Ageostrophic current

The reconstructed geostrophic current (zonal and meridional) compares well with the reference (geostrophic component derived from the reference *SSH*) as suggested by Figure 9 for both basic and improved mapping. Minor differences appear with slightly finer structures in the second case with error fields slightly reduced (3rd versus 5th rows on the figure). This will be quantified in section 4.2

The left three panels of figure 10 shows the same snapshots (reference, basic and 368 improved mapping) for the ageostrophic component. The reference ageostrophic field on 369 the top is the reference total current minus the reference geostrophic current. Here, as 370 opposed to geostrophy, the fields are fairly different. The temporal evolution of these fields 371 is shown for a selected location on the right panel. First, we note the reference current 372 is composed of periodic fluctuations of approximate near-inertial frequency on top of a 373 slower signal. The spatial scales of the dominent patterns is larger than the mesoscale 374 eddies, probably linked to the atmospheric wind field patterns. Estimated ageostrophic 375 field with basic mapping clearly fails on several aspects. By construction, inertial mo-376 tions are not resolved since they occur at a much higher frequency than the filtering scales 377 of the basic mapping so the time series (blue line) does not feature oscillations. Further-378 more, the low frequency component does not seem accurate. Given the small number 379 of Doppler instrument revisits (as represented by the grey diamonds on the right panel) 380 the estimation suffers from aliasing. For instance, between days 15 and 30, the obser-381 vations happen to occur primarily near the maxima of the oscillations for the zonal com-382 ponent, leading to overestimation of the zonal component at low-frequency (blue curve) 383 in this particular case. However, the estimated ageostrophic field with improved map-384 ping is fairly different. It does resolve inertial motions, and succeeds in capturing, to a 385 large extent, their modulation and phase. The reconstruction capability is based on the 386 degrees of freedom of the signal with respect to the number of independent observations. 387 Since the spatial patterns of our "true" NIOs are quite large and their temporal exten-388 sion exceeds a few inertial periods, a large volume in time/space can be constrained with 389 the Doppler observations. From the reconstructed series (red line on the right panel), 390 it is also clear that the low frequency variations of the ageostrophy current is better re-391 solved, the aliasing issue being now mitigated. 392

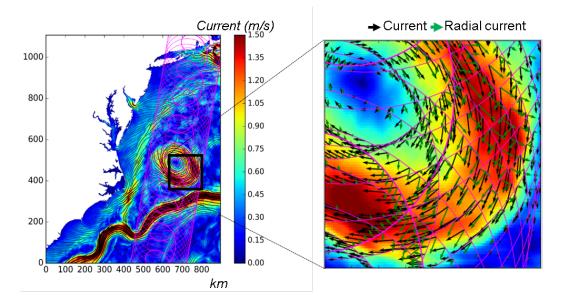


Figure 8: Detailed view of the SKIMulator outputs showing, with respect to the reference 2D current vectors in black, the observed radial current along the satellite azimuth angles in green.

We illustrate on Figure 11 the total current(represented by local Lagrangian trajectories) obtained with the two methods. The effect of resolving inertial motions clearly shows up on the Lagrangian trajectories, looping like actual drifter trajectories when the near inertial current amplitudes exceeds the low-frequency current.

397

#### 4.2 Reconstruction skills as a function of spatial and temporal scales

We propose a quantitative analysis of the reconstructions, both in the spatial and 398 temporal spectral domain. This will validate the results discussed above and further shed light on the reconstruction skills as a function of spatio-temporal scales. The analysis 400 is based on the spectral ratio of the error over the true signal, computed along spatial 401 or temporal sections of the domain. For spatial analysis, the computation is similar to 402 what was proposed in (Ballarotta et al., 2019) to assess the effective resolution of Sea 403 Level Anomaly products, but on the velocity in the normal direction of the section. For 404 temporal analysis, the rotary spectra are considered for the spectral ratio, leading to two 405 separate estimates in the clockwise and counter-clockwise directions. These ratios r are 406 represented under the form of a percent scores 100\*(1-r) summarized on Figures 12 407 and 13 for the different runs and components. As suggested by the upper panels of Fig-408 ure 12, for geostrophic reconstructions, the improvements from basic mapping (green curves) 409 to improved mapping (red curves) are sensible at all scales, especially below 150km. If 410 we consider 50% as a reasonable threshold, then the resolving capabilities of the altime-411 ter reconstruction for zonal and meridional current is about 110km, and 90km with SKIM 412 combination. This is still a fair improvement for a single instrument added to an exist-413 ing 5-instrument constellation. From this experiment, the Doppler observations would 414 therefore improve the geostrophic component reconstruction even though altimetry is 415 already efficient to capture this particular component. An additional experiment was led 416 with improved mapping from Doppler observations only, represented in blue on the fig-417 ure. The performances are not as good as with the combination, but do exceed those of 418

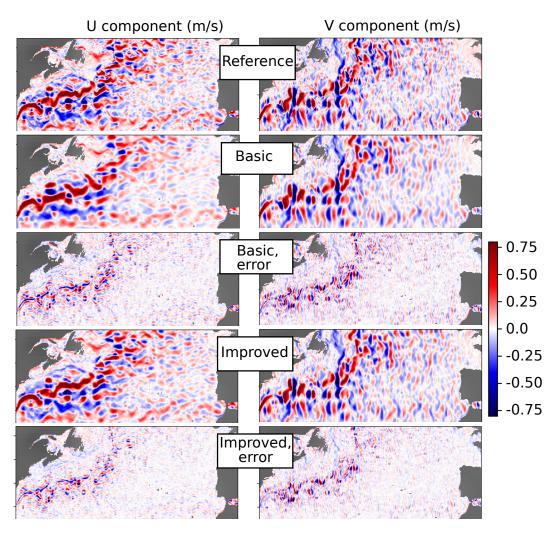


Figure 9: Snapshots of the geostrophic component (m/s) in the zonal (left column) and meridional (right column) directions, with differences (errors). The upper row is the reference, the second and third rows are the basic reconstructions and errors (w.r.t. reference) respectively. The fouth and fith rows are the same as second and third for the improved mapping.

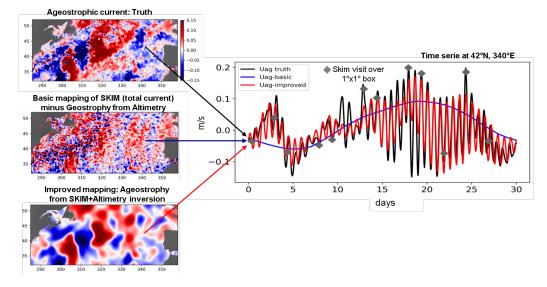


Figure 10: Reconstruction of the zonal ageostrophic component compared to the reference, in m/s. Upper left panel: snapshot of true (reference) ageostrophic zonal current. Middle left panel: reconstruction from basic mapping. Lower left panel: reconstruction from improved mapping. Right panel: time series of the reference (black), basic mapping (blue) and improved mapping (red) at 340°E, 42°N as a function of time over a month.

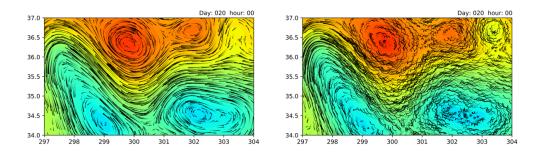


Figure 11: SSH (plain color) and local Lagrangian trajectories (black lines) of the surface current resolved with basic (left) and improved (right) mapping.

altimetry only at small scales (below 250km). At large scales, the ambiguity with ageostrophy, in absence of altimetry, certainly explains the lower performances.

We also note minor differences between the zonal and meridional performances: at large scales beyond 250km, the meridional component seems slightly better resolved with altimetry and SKIM, meaning that the zonal gradients of SSH would be more accurate at these large scales. With SKIM, the design was indeed found to perform better for alongtrack azimuth angles on swath average (Gaultier, 2019b) resulting in slighly better meridional currents on global average.

For ageostrophic reconstructions (lower panels of Figure 12), more sensitive differences were found as expected. Indeed, with the basic mapping, only the largest scales

are partially resolved. Because of aliasing issues discussed in 4.1, the portion of resolved 429 signal is weak, of about 15% (zonal) to 30% (meridional) beyond 1000km wavelength. 430 However, the reconstruction with improved mapping exceeds 45% above 500km, where 431 most of the inertial energy is. Note that, by construction of the sub-components for ageostrophic 432 current, we do not aim to resolve scales below 300km. This could be explored, but do-433 ing so with this observing system would be a challenge because of high-temporal frequen-434 cies at short spatial scales. The dashed lines, showing the NIO contribution only, indeed 435 suggest that most of the improvements owe to the inertial part. The experiment with 436 Doppler observations only (blue curve) also brings interesting results. The drop in per-437 formances, especially at large scales, suggests the importance of an altimetry constella-438 tion the better separate the geostrophic contribution and therefore better estimate the 439 ageostrophic component as well. 440

The score evaluations in the time-frequency domain (Figure 13) bring additional 441 elements, in particular about the low-frequency ageostrophy, by comparing the plain (NIO+ 442 low-frequency ageostrophy) with the dashed (NIO only) lines on the lower panels. The 443 low-frequency ageostrophy is indeed an essential component, allowing a recontruction 444 score above 50% to 60% beyond a week period. We also found (not shown) that the ro-445 tational part was dominent over the divergent part, which is not surprising since the low 446 frequency wind should be directly related to low-frequency wind, mostly rotational. The 447 inertial peak appears also clearly on the scores at around 16 hours in the clockwise di-448 rection (lower-left panel). For geostrophy (upper panels) the time window does not al-449 low to fully resolve the eddy time band (mostly beyond a month) where scores would 450 reach the values found in the spatial analysis. However, the relative scores are consis-451 tent, we note that the relative improvement between the two methods (green versus red) are high between 5 and 10 days, suggesting that the time aliasing mitigation is efficient. 453 We also note no significant differences between clockwise and counter-clockwise direc-454 tions, as expected since quasi-geostrophic motions have similar energy for the two com-455 ponents of their rotary spectra. 456

#### 457 5 Conclusions and perspectives

This study demonstrated, in principle, the possibility to disentangle and map var-458 ious components of the ocean surface current from partial observations of the surface dy-459 namic topography and current. This was achieved thanks to a specific treatment of the 460 covariance structures used in the mapping. Indeed, for mid-latitudes, the time revisits 461 of proposed spaceborne instruments for surface current measurements exceeded half the 462 inertial periods, where a large part of the signal energy is. Basic mapping algorithms, 463 acting as a low-pass filter, not surprisingly fail in resolving those signals and also introduce strong aliasing. The improved mapping presented here performs well thanks to the 465 spatial and temporal coherence of high-frequency signals, long enough with respect to 466 observation sampling. However, several additional tests (not shown) also show that in-467 creasing the time sampling, with a wider swath such as proposed in the WaCM or STREAM 468 design, or a constellation of several SKIM-like satellites can resolve a much larger frac-469 tion of the NIOs variance even if it comes with higher instrumental noise. The present 470 work therefore should help in the identification of trade-offs for the optimization of Doppler 471 scatterometer designs and orbit choice. In general altimetry is an essential source of ob-472 servations in addition to Doppler scatterometers, in particular to disentangle the sur-473 face current components. 474

The results of the reconstruction method considered in this study probably depends on the basis of sub-components chosen. This latter have been constructed manually with a wavelet basis approach, accounting for coherent structures seen in the different component of the flow considered. This method has the limitation to project observations on prescribed bases, requiring a priori knowledge of the signal characteristics (**G** matrix) and statistics (**Q** matrix). Also, potential interactions between the components, for in-

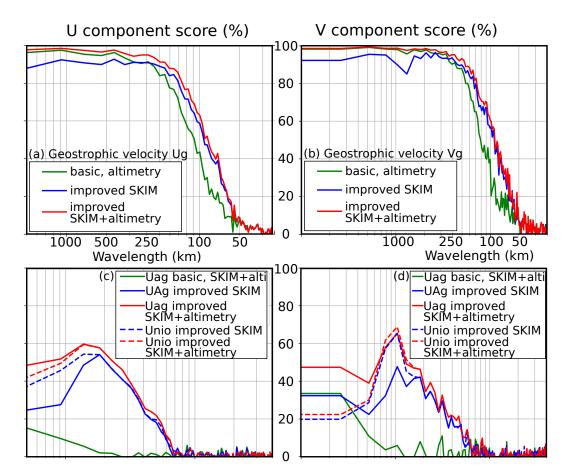


Figure 12: Performances as a function of spatial wavelength computed, in percent, from the ratio of the reconstruction error spectrum by the true signal spectrum. 100% means a full reconstruction with no errors.(a) Scores for geostrophic zonal current with basic mapping of altimetry (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM + Altimetry combined (red).(b) same for meridional current. (c) scores for ageostrophic zonalcurrent with basic mapping of SKIM + Altimetry combined (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM + Altimetry combined (red). The dashed lines represent the contribution of near-inertial current only. (d) same for meridional current.

stance the impact of mesoscale eddies on inertial oscillations through relative vorticity fluctuations, is not yet accounted. We also acknowledge that tidal currents have not been considered is this experiment as the reference run is tide-free. However, dedicated analyses presented in (F. e. a. Ardhuin, 2019) suggest that tidal current may be well handled thanks to accurate barotropic models and favorable orbit aliasing. Baroclinic tidal currents, not always phase locked (Zaron, 2019) may also be a challenge, but they are probably dominated by shorter scales with a minimal interaction with inertial oscillations.

The practical applicability of the present result strongly depends on the realism of the surface current field, in particular its ageostrophic component. A preliminary analysis of drifter pairs, which will be reported elsewhere, suggests that half of the velocity variance is contained in covariances with scales larger than 100 km (Xiaolong Yu, personal communication 2019). We thus expect that the present approach is qualitatively

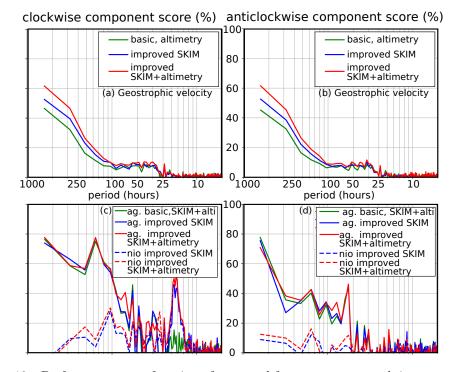


Figure 13: Performances as a function of temporal frequency computed, in percent, from the ratio of the reconstruction error rotary spectrum by the true signal error rotary spectrum. (a) Scores for geostrophic clockwise current with basic mapping of Altimetry (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM + Altimetry combined (red). (b) same for counter-clockwise current. (c) scores for ageostrophic clockwise current with basic mapping of SKIM + Altimetry combined (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM + Altimetry combined (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM + Altimetry combined (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM + Altimetry combined (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM (green), with improved mapping of SKIM (blue) and with improved mapping of SKIM (green), with improved mappi

valid, and that there may also be a chance to successfully invert some near inertial current from the drifters alone in regions of high drifter density like subtropical Gyres.

#### 496 Acknowledgments

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All data used in this study (reference fields, synthetic observations and gridded analysis) are available on the following repository: [ongoing, see 'materials for reviewers']

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