Spiral Radiation Pattern of a Relativistic Charged Particle in a Periodic Motion Explained With Bremsstrahlung Asymmetry Parameter -R

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Abstract

The forward peaking radiation pattern of a single particle with an increasing particle velocity is well-established knowledge. Further details of a single particle radiation pattern suggest that a particle also has a backward peaking radiation pattern and two associated asymmetries coming from the Doppler and spiral trajectory bremsstrahlung effects. Relativistic particle under periodic motion emits spiral radiation pattern which is measured as short pulses by the sensors. However, the transition from peaking to spiral radiation pattern as particle transits from discrete to continuous periodic motion is not clear. This paper reports a possible physical asymmetric effect caused by the bremsstrahlung spiral trajectory that could be responsible for the spiral radiation pattern emitted by the periodic particle motion. The bremsstrahlung asymmetry changes within each period that change the radiation intensity symmetry continuously, within a specific minimum and maximum range. This maximum and minimum range is defined by the limits of the bremsstrahlung asymmetry, R. This change in radiation intensity, independent of particle periodicity, emits circular waves of the varying radius that forms a spiral radiation pattern. Hence, the spiral of a periodic motion comes from a parameter, R, that is independent of periodicity. Hence, parameter R can change during a period by preserving the periodicity of the incoming particle motion. Overall, a continuous periodic motion is important in predicting the experimental observations as incoming particle interacts with multiple target particles meaning the same bremsstrahlung process repeats periodically.

Spiral Radiation Pattern of a Relativistic Charged Particle in a Periodic Motion Explained With Bremsstrahlung Asymmetry Parameter - R

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Key Points:

- The Bremsstrahlung asymmetry parameter, R and emitted angular radiation frequency, ω causes a spiral radiation pattern.
- Spiral radiation pattern is a result of connected circular waves of varying radius.
- The Bremsstrahlung asymmetry parameter, R and emitted angular radiation frequency, ω are independent of periodic motion.

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Abstract

The forward peaking radiation pattern of a single particle with an increasing particle velocity is well-established knowledge. Further details of a single particle radiation pattern suggest that a particle also has a backward peaking radiation pattern and two associated asymmetries coming from the Doppler and spiral trajectory bremsstrahlung effects. Relativistic particle under periodic motion emits spiral radiation pattern which is measured as short pulses by the sensors. However, the transition from peaking to spiral radiation pattern as particle transits from discrete to continuous periodic motion is not clear. This paper reports a possible physical asymmetric effect caused by the bremsstrahlung spiral trajectory that could be responsible for the spiral radiation pattern emitted by the periodic particle motion. The bremsstrahlung asymmetry changes within each period that change the radiation intensity symmetry continuously, within a specific minimum and maximum range. This maximum and minimum range is defined by the limits of the bremsstrahlung asymmetry, R. This change in radiation intensity, independent of particle periodicity, emits circular waves of the varying radius that forms a spiral radiation pattern. Hence, the spiral of a periodic motion comes from a parameter, R, that is independent of periodicity. Hence, parameter R can change during a period by preserving the periodicity of the incoming particle motion. Overall, a continuous periodic motion is important in predicting the experimental observations as incoming particle interacts with multiple target particles meaning the same bremsstrahlung process repeats periodically.

Plain Language Summary

High energy particles following circular periodic trajectory emit spiral radiation pattern. Spiral radiation pattern has a changing radius as it expands from its source. A discontinuous single high-frequency radiation pattern has non-uniform radiation intensities known as asymmetries. When these asymmetries are repeated continuously, they cause uniform circular radiation pattern to vary in radius repeatedly. As it is a continuous process, each different sized circular radiation pattern is connected to form a spiral radiation pattern.

1 Introduction

Bremsstrahlung high-frequency radiation pattern of a charged particle peaks in the forward direction. This peaking scales with the inverse of the Lorentz factor depending on the particle velocity (Koch & Motz, 1959). Moreover, peaking of radiation intensity was predicted to be Doppler and bremsstrahlung asymmetric and occurs in the backward direction as well as forward direction (Yucemoz & Füllekrug, 2020). These predictions are based on a single particle following a specific trajectory only once and interacting with only one particle. When a relativistic particle follows a periodic circular trajectory, continuously interacts with a target particle, and radiates, radiation pattern was shown to be spiral (Tsien, 1972). However, the detail of the radiation pattern transition is not quite well understood. The transition process is quite important to understand as it helps understand how the particle behaves and radiates. A particle with a peaking radiation pattern coming from the only one bremsstrahlung interaction transforms into a spiral radiation pattern when the particle continuously interacts with other target particles periodically. Spiral radiation pattern was found to be associated with the bremsstrahlung asymmetry, R which occurs as a result of a particle following spiral particle trajectory. Spiral is an important particle trajectory where the particle follows a curved path of the varying radius within a single mean free time, τ. This means that the bremsstrahlung asymmetry, R is one of the two-parameter that changes within a single period without affecting the periodicity of the continuous radiation process of an incoming particle. The second parameter that is independent of particle motion periodicity is the emitted angular radiation frequency, ω which scales with the particle acceleration. These two parameters are the only parameters that cause a spiral radiation pattern by changing the radius of the emitted circular wave constantly without affecting the periodicity of the particle motion. Emitted angular radiation frequency, ω is significant in explaining spiral radiation patterns emitted by circular motion. The circular motion has a constant bremsstrahlung asymmetry, R parameter. As R is constant, it cannot contribute to the spiral radiation generation in the case of circular particle motion. However, particle acceleration, hence the emitted angular radiation frequency, ω is not constant and varies along the circular particle path. Varying particle acceleration is the property of a periodic particle motion. Varying emitted angular radiation frequency, ω causes spiral radiation pattern in periodic circular particle trajectory.

2 Theory of Transition from Asymmetric Forward-Backward Radiation to Spiral Radiation pattern

The radiation pattern for a charged particle following a spiral trajectory (Eq.1) is predicted to be dipole at non-relativistic and asymmetric forward-backward peaking at relativistic particle speed (Yucemoz & Füllekrug, 2020).

$$r(t) = \frac{(t^R)^2 b^R (\omega^{'})^R cos(\theta_{n,r(t)})^R c}{\tau^{2R} c^R \omega' cos(\theta_{n,r(t)})} - \frac{at}{\tau}, \tag{1}$$

A charged particle following periodic motion emits spiral radiation pattern (Tsien, 1972).

To understand how spiral radiation can be emitted by the particle when it already emits a peaking radiation pattern, following a spiral trajectory can be understood by transforming a peaking radiation pattern into a periodic radiation pattern. This would mean that the particle is now following the original spiral radiation pattern defined in equation (1) but repeats in periodic intervals. This periodic interval is in every mean free time, τ. The transformation from a discrete radiation pattern into a periodic continuous radiation pattern can be achieved using the Fourier series.

Discrete peaking radiation pattern for one mean free time, τ period is found to be (Yucemoz & Füllekrug, 2020, p.18, Eq.36),

$$\begin{aligned}\frac{d^2I}{d\omega\Omega_{rad}} = \frac{z^2e^2(\gamma\omega(S_{SpecialR} - \beta S_{SpecialR} cos(\theta_{n,\beta}))^2}{4\pi^2 c\epsilon_0}\Bigg| sin(\theta_{n,\beta})\Bigg[-\frac{s_{fv}s_f z(s_{ft})^{1.461} 4.365\times 10^{26}}{c} \\ \Bigg[\pi^{1/2}2^{-(1/2)\nu_1}\alpha^{-\nu_1-1}e^{-\frac{y^2\alpha^{-2}}{8}} \times D_{\nu_1}(2^{-1/2}\alpha^{-1}y)\Bigg] + \\ \frac{s_{fv}zs_f(s_{ft})^{1.5}1.565\times 10^{27}}{c}\Bigg[\pi^{1/2}2^{-(1/2)\nu_2}\alpha^{-\nu_2-1}e^{-\frac{y^2\alpha^{-2}}{8}} \times D_{\nu_2}(2^{-1/2}\alpha^{-1}y)\Bigg]\Bigg]\Bigg|^2,\end{aligned} \tag{2}$$

Equation two needs to be simplified before the Fourier series representation by eliminating the coefficient of modulus as it only scales and sharpens the pattern. The coefficient of modulus represents part of the Doppler effect. However, the effect is still acting on the radiation pattern as the Doppler shift is also embedded in parameters α and y.

Moreover, further simplification allows parabolic cylinder function, $D_v(z)$ to be eliminated as it replicates and only adds details to the pattern predicted by the exponential factor. In addition, one of the terms inside the modulus can also be neglected, as it is identical to the first term. Lastly, by neglecting all the constants multiplying the exponential function in the first term, the simplified equation (2) can be written as,

$$\frac{d^2I}{d\omega\Omega_{rad}} = \left| sin(\theta_{n,\beta}) e^{-\frac{y^2\alpha^{-2}}{8}} \right|^2, \tag{3}$$

Where, α is $\alpha^2 = \frac{b^R(\gamma\omega(S_{SpecialR} - \beta S_{SpecialR} cos(\theta_{n,\beta})))^R (sin(\theta_{n,\beta}))^R}{(\tau^R)^2 c^R}$ $[s^{-2}]$, and y is $y = \frac{\gamma\omega(S_{SpecialR} - \beta S_{SpecialR} cos(\theta_{n,\beta})) sin(\theta_{n,\beta}) a}{c\tau}$ $[\mathrm{s}^{-1}]$.

Equation 3 is the simplified version of equation 2. However, it still contains all the physical information of bremsstrahlung asymmetry, Doppler effect, and preserves peaking radiation pattern. All the physical details are preserved to predict, simulate, and explain the spiral radiation pattern of a constant period coming from the initial peaking pattern. A constant period is a time it takes for a particle to complete its one bremsstrahlung spiral particle trajectory.

Either case, as the parameters α and y involve both mean free time, τ, and the $\theta_{n,\beta}$, they should be substituted into the equation (3) and simplified.

Substituting and simplifying equation 3 gives,

$$\frac{d^2I}{d\omega\Omega_{rad}} = \left| sin(\theta_{n,\beta}) e^{\frac{1.391\times10^{-18} a^2 \omega^2 \tau^{2R-2} e^{19.519R} S^2_{SpecialR} b^{-R} (-\beta^R+1)^{0.5R} sin(\theta_{n,\beta})^{2-R} (-cos(\theta_{n,\beta})\beta+1)^{2-R} (\omega S_{SpecialR})^{-R}}{\beta^{2-R}}} \right|^2, \tag{4}$$

Collecting all factors of the Solid angle, $\theta_{n,\beta}$ together leads to,

$\beta_2 = \frac{1.391\times10^{-18} a^2 \omega^2 \tau^{2R-2} e^{19.519R} S^2_{SpecialR} b^{-R} (-\beta^R+1)^{0.5R} (\omega S_{SpecialR})^{-R}}{\beta^2 - R}$.

Where, $\omega^{'} = \gamma\omega(S_{SpecialR} - \beta S_{SpecialR} cos(\theta_{n,\beta}))$.

Periodic radiation pattern can be written as Fourier series of $n = 1$. Where, $n = 1, 2, 3, 4...\infty$

$$\frac{d^2I}{d\omega\Omega_{rad}} = \left| \int_0^{2\pi} sin(\theta_{n,\beta}) e^{\beta_2 sin(\theta_{n,\beta})^{2-R} (-cos(\theta_{n,\beta})\beta+1)^{2-R}} d\theta_{n,\beta} + \sum_{n=1}^{\infty} \left[a_n cos(n\theta_{n,\beta}) + b_n sin(n\theta_{n,\beta}) \right] \right|^2, \tag{5}$$

Factors of cosine and sine terms are $a_n = \int_0^{2\pi} sin(\theta_{n,\beta}) e^{\beta_2 sin(\theta_{n,\beta})^{2-R} (-cos(\theta_{n,\beta})\beta+1)^{2-R}} cos(\theta_{n,\beta}) d\theta_{n,\beta}$ and $b_n = \int_0^{2\pi} sin(\theta_{n,\beta}) e^{\beta_2 sin(\theta_{n,\beta})^{2-R} (-cos(\theta_{n,\beta})\beta+1)^{2-R}} sin(\theta_{n,\beta}) d\theta_{n,\beta}$.

At relativistic speeds "$\beta \approx 1$", where bremsstrahlung asymmetry becomes significant, equation 5 can be written as:

$$\frac{d^2I}{d\omega\Omega_{rad}} = \left| \int_0^{2\pi} sin(\theta_{n,\beta}) e^{\beta_2 (i+1)^{2-R}} d\theta_{n,\beta} + \sum_{n=1}^{\infty} \left[a_n cos(n\theta_{n,\beta}) + b_n sin(n\theta_{n,\beta}) \right] \right|^2, \tag{6}$$

Therefore, radiation intensity of a particle is,

$$\frac{d^2I}{d\omega\Omega_{rad}} \approx \Bigg| \Big[- e^{\beta_2(1+i)^{2-R}} cos(\theta_{n,\beta}) \Big]_0^{2\pi} + \Big[- \frac{1}{2} e^{\beta_2(1+i)^{2-R}} cos^2(\theta_{n,\beta}) \Big]_0^{2\pi} cos(\theta_{n,\beta}) + \Big[\frac{1}{2} e^{2i\beta_2(1+i)^{-R}} (\theta_{n,\beta} - sin(\theta_{n,\beta}) cos(\theta_{n,\beta})) \Big]_0^{2\pi} sin(\theta_{n,\beta}) \Bigg|^2, \tag{7}$$

Simplified,

$$\frac{d^2I}{d\omega\Omega_{rad}} \approx \Big| \pi e^{\beta_2(1+i)^{2-R}} sin(\theta_{n,\beta}) \Big|^2, \tag{8}$$

3 Results

In this section, the radiation patterns of a particle undergoing a continuous particle collision following a spiral trajectory with a period of mean free time, τ for each collision is predicted using equation 8. The mean free time, τ that remains constant after each particle-particle collision makes the whole journey periodic.

As can be seen in equation 8, the exponential coefficient of sine is made up of parameters bremsstrahlung asymmetry, R, mean free time, τ, emitted radiation frequency, ω, scaled particle velocity, β, relative speed between a particle and an observer frame of reference, $S_{SpecialR}$ and spiral trajectory properties a and b. When an incoming particle collides periodically with many target particles, all these parameters remain constant. However, bremsstrahlung asymmetry parameter, R, relative speed between a particle and an observer frame of reference, $S_{SpecialR}$ and emitted angular radiation frequency, ω are independent of the particle's periodic motion. Parameters R and ω only change within each spiral trajectory followed by incoming particle.

Changing bremsstrahlung asymmetry, R, relative speed between a particle and an observer frame of reference, $S_{SpecialR}$ and emitted angular radiation frequency, ω, changes the radius of the emitted circular wave without affecting the periodicity of the multiple particle-particle interactions. Parameter, R, relative speed between a particle and an observer frame of reference, $S_{SpecialR}$ and ω transforms circular wave into multiple circular waves with different radius. Overall, when detected by sensors, this effect is measured as a spiral radiation pattern.

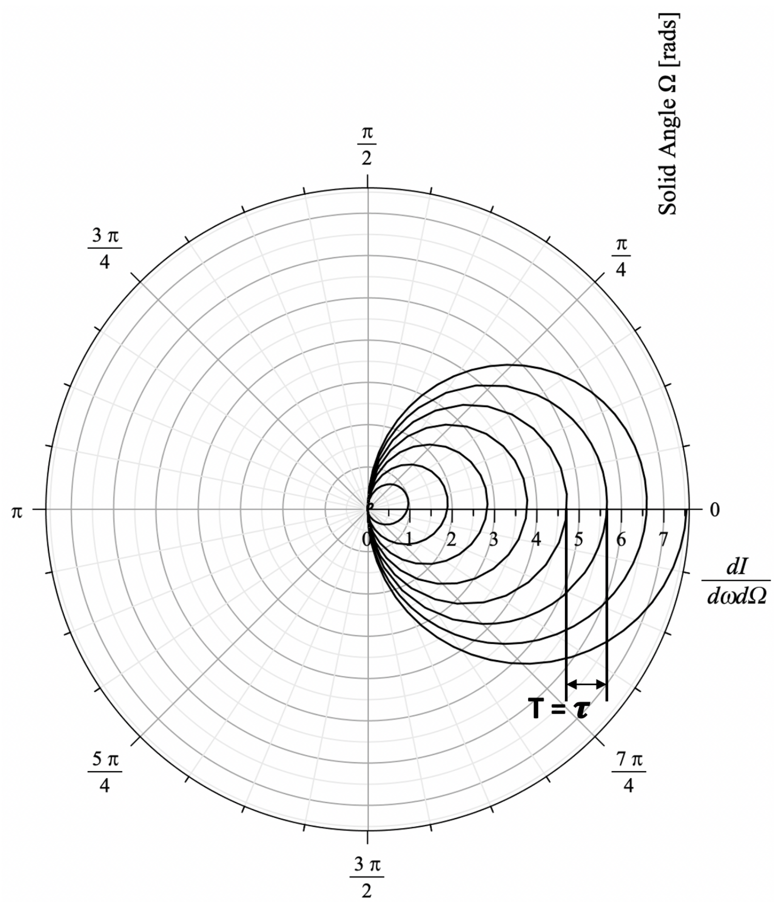

Figure 1. Radiation patterns of a single relativistic particle following a spiral trajectory at periodic intervals of mean free time, τ is predicted by derived equation 8. Radiation intensity values are arbitrary and do not reflect the actual observations. Prediction assumes relativistic speed, $\beta \approx 1$ and Fourier series index, $n = 1$. The spiral radiation pattern is formed by the multiple, changing radius circular wave interference. When a particle is in periodic motion, the emitted radiation intensity is expected to be uniform around the particle. This is the result of a periodic motion, as parameters defining the particle motion should not change to preserve periodicity. The possibility of the spiral radiation pattern in a periodic motion means there should be some parameters that can change to different values without affecting the particle period and periodic motion. These parameters are found to be bremsstrahlung asymmetry parameter, R, emitted angular radiation frequency, ω and relative speed between particle and observer frame of reference, $S_{SpecialR}$. parameters R and ω vary as a result of a defined spiral particle trajectory. As they vary as a result of spiral trajectory and as the spiral trajectory is constant and same when an incoming particle interacts continuously with different particles, ω and R are cannot affect periodicity of continuous particle-particle interaction. On the other hand, continuous variation of speed between two particle and observer frame of references, triggers and affects the magnitude of emitted radiation by particle. This is causes parameter ω to vary and initiate spiral radiation pattern. In addition, parameter ω also varies as a result of Doppler effect, which in turn contributes to the generation of spiral radiation pattern.

Periodic spiral particle trajectory could be one spiral right after another. Alternatively, periodic spiral particle trajectory could be a spiral combined with a linear motion until the next particle-particle interaction, presented in figure 2.

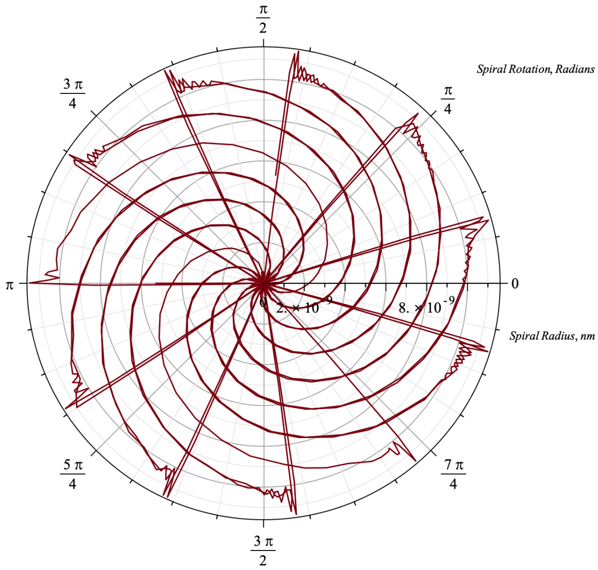

Figure 2. Peridoic particle following spiral combined with linear trajectory at each mean free time, τ intervals.

4 Discussion & Conclusion

The particle's radiation pattern is not only dependent on periodic motion but is also dependent on the trajectory it follows. Hence, a particle following a spiral trajectory in a periodic motion with a period of τ can radiate a spiral radiation pattern similar to a particle following a circular periodic motion. A single particle following a spiral trajectory in periodic motion emits a spiral radiation pattern. Spiral trajectory has a property that its bremsstrahlung asymmetry index, R is not constant and changes with time. The reason for a transition from forward-backward peaking to spiral radiation pattern is that, when particle starts to repeat the bremsstrahlung process continuously with a period of mean free time, τ, factor $e^{-\frac{y^2\alpha^{-2}}{8}}$ in equation (2) overtakes, and starts producing circular radiation. Produced circular radiation is similar to the circular waves produced on the surface of the water closer to its disturbance. However, when these circular waves are affected by the bremsstrahlung asymmetry, R, relative speed between a particle and an observer frame of reference, $S_{SpecialR}$ and emitted angular radiation frequency, ω, the radius of emitted circular waves changes. This in turn causes circular waves to be spiral waves, which are measured by the radio observer as pulses.

In summary, firstly, the spiral radiation pattern is caused by the bremsstrahlung spiral trajectory that changes, parameter, R to create a spiral. The second cause of spiral radiation pattern is the Doppler effect and particle acceleration that changes the emitted angular radiation frequency, ω. Finally, another affecting factor is the relative speed between a particle and an observer frame of reference, $S_{SpecialR}$ that again causes the circular wave to change radius for spiral radiation pattern.

On the other hand, the spiral trajectory has limits in terms of the number of different radii that it consists of. This puts a limit or a range of values that parameters bremsstrahlung asymmetry, R and emitted angular radiation frequency, ω can achieve. This implies that, during a single period τ, the particle is limited in the number of spirals that it can generate. Afterward, the radiation pattern either should become uniform or the second period starts with the generation of a new set of the same spiral radiation pattern.

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