Magnetic anomalies caused by 2D polygonal structures with uniform arbitrary polarization: new insights from analytical/numerical comparison among available algorithm formulations

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Abstract

Since the '60s of the last century, the calculation of the magnetic anomaly caused by 2D uniformly-polarized bodies with polygonal cross-section was mainly performed using the popular algorithm of Talwani & Heirtzler (1962, 1964). Recently, Kravchinsky et al. (2019) claimed errors in the above algorithm formulation, proposing new corrective formulas and questioning the effectiveness of almost sixty years of magnetic calculations. Here we showed, demonstrating analytical equivalence of the two approaches, that Kravchinsky et al.'s formulas simply represent an algebraic variant of those of Talwani & Heirtzler. Moreover, we performed intensive numerical analysis generating a large amount of random magnetic scenarios, involving both changing-shape polygons and a realistic geological model, showing a complete agreement among the magnetic responses of the two discussed algorithms and the one proposed by Won & Bevis (1987). Additionally, we released the source code of the algorithms in Julia and Python languages.

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Key points

- Kravchinsky et al. (2019) claimed errors and omissions in the formulas of Talwani & Heirtzler (1962) for 2D forward magnetic calculation
- Our analysis reveals that Kravchinsky et al.'s formulas are algebraically equivalent to those of Talwani & Heirtzler
- Numerical tests show a complete agreement among the two above formulations and the one by Won & Bevis (1987)

Abstract

Since the ’60s of the last century, the calculation of the magnetic anomaly caused by 2D uniformly-polarized bodies with polygonal cross-section was mainly performed using the popular algorithm of Talwani & Heirtzler (1962, 1964). Recently, Kravchinsky et al. (2019) claimed errors in the above algorithm formulation, proposing new corrective formulas and questioning the effectiveness of almost sixty years of magnetic calculations. Here we showed, demonstrating analytical equivalence of the two approaches, that Kravchinsky et al.’s formulas simply represent an algebraic variant of those of Talwani & Heirtzler. Moreover, we performed intensive numerical analysis generating a large amount of random magnetic scenarios, involving both changing-shape polygons and a realistic geological model, showing a complete agreement among the magnetic responses of the two discussed algorithms and the one proposed by Won & Bevis (1987). Additionally, we released the source code of the algorithms in Julia and Python languages.

Plain Language Summary

Forward magnetic calculation plays a major role in geophysics to model magnetization, location and shape of magnetic anomaly sources in unknown areas investigated by a magnetic survey. One of the most popular approaches to calculate the magnetic anomaly due to two-dimensional bodies is based on the Talwani & Heirtzler’s formulas (1962, 1964), that have been widely used both for scientific and industrial applications from the sixties. Recently, these formulas have been questioned by Kravchinsky et al. (2019), casting doubts on the truthfulness of all the magnetic models and interpretations obtained to date. In order to find a clarification, we examined and compared the two calculation approaches, both from an analytical and numerical point of view. In detail, after rectifying some inaccuracies in Kravchinsky et al.’s formulas, we found complete equivalence between the two discussed formulations, showing that they simply represent two algebraic variants of the same mathematical approach. We also performed intensive numerical tests comparing the results of the two algorithms with a third one proposed by Won & Bevis (1987). The three mathematical approaches gave the same magnetic responses for all the tested models, demonstrating total agreement between the three formulations.

1. Introduction

Modeling of magnetic anomalies is a fundamental tool in exploration geophysics. Since the appearance of early electronic computers, calculation of the magnetic field from models of the subsurface and the related inverse problem have played a major role in investigating the geological framework of unknown areas.

An early mathematical formulation for anomalies due to 2D polygonal structures of uniform polarization is found in Talwani & Heirtzler (1962, 1964). Their algorithm remains the most used and cited to date.

Thanks to its wide applicability, Talwani & Heirtzler's approach has become popular, both for expeditious interpretation of magnetic data and as a forward operator involved in inverse methods. Moreover, the aforementioned 2D formulation can be modified to extended to 3D bodies (Talwani, 1965; Plouff 1975, 1976). More recently, Won & Bevis (1987) proposed an evolution of the original formulation by Talwani & Heirtzler which avoids the use of trigonometric functions, achieving a speed up of the calculation of magnetic anomaly.

Another popular approach, which considers 2D or 3D prism-shaped bodies instead of polygonal-shaped ones, is that proposed in Bhattacharyya (1964). Such approach leads to a formulation where the subsurface is modelled as a set of prismatic bodies, often a set of rectangular cells, characterized by constant magnetic properties.

Despite the fact that in recent years forward calculations have moved toward the computation of magnetic anomalies caused by 3D bodies, hand in hand with the rapid increase in CPU speed, 2D modeling still represents a widely utilized tool to quickly and intuitively gain a better understanding of the subsurface. Moreover, the much lower computational requirements for 2D calculations make them viable for simple interpretations of the magnetic signatures (e.g. in a trial and error approach) and to performing analysis directly on the field (e.g. on a laptop).

Since the introduction of the abovementioned algorithms, the 2D approach developed in the attempt to overcome the simplistic assumption of a uniform polarization in magnetized bodies, trying to consider both demagnetization effects and non-uniform magnetization (Bhattacharyya & Navolio, 1975, 1976; Bhattacharyya & Chan, 1977; Ku, 1977; Blokh, 1980; Mariano & Hinze, 1993; Kostrov, 2007). Unfortunately, the mathematical expressions involved often represent an unrealistic approximation with respect to the experimentally observed spatial variation of rock magnetization. For a more detailed presentation of the main developments in forward magnetic calculation methods, readers are referred to Nabighian et al. (2005) and Kostrov (2007).

Very recently, Kravchinsky et al. (2019) suggested the evidence of omissions and errors in the formulation of Talwani & Heirtzler (1962, 1964) that would lead to unfitting magnetic anomalies, proposing a modified algorithm to avoid that.

In this paper we compare the original formulations of Talwani & Heirtzler (1962, 1964), Won & Bevis (1987) and the newer Kravchinsky et al. (2019) both from analytical and numerical points of view. For the former, the algorithms have been analyzed in order to highlight algebraic differences and similarities, while for the latter they have been tested and compared in a huge amount of randomly generated scenarios involving both induced and remnant magnetization on shape-changing polygons, to detect possible numerical differences or failing scenarios. In detail, we start by illustrating the three formulations of the algorithms in section 2 and then we discuss in deep the similarities and differences in section 3. We finally show that, after fixing some issues in Kravchinsky et al. (2019), their formulation and that of Talwani & Heirtzler (1962, 1964) are essentially the same algorithm, and that all three algorithms, i.e. including Won & Bevis (1987), produce the same results.

In addition, we release a set of open source codes written in Python and Julia languages illustrating the described algorithms (see section "Algorithms code availability").

2. Algorithm formulations

Let us consider a three-dimensional non-magnetic space in which a body infinitely extended in *y* direction is immersed. The common aim of all formulations is the calculation of the magnetic field of this body upon observation points located along a profile aligned to the *x* direction at a certain height (the positive *z* axis is assumed pointing downward). The starting assumption is that our body can be considered as discretized by an infinite number of uniformly-magnetized elementary volumes with infinitesimal dimensions *dx*, *dy*, *dz*. Within this assumption, the magnetic field associated to the body can be mathematically expressed in terms of a line integral around its periphery, represented in two dimensions as its polygonal cross-section (figure 1a). The specific procedures for each formulation are summarized in the subsections below. For the respective detailed derivations, the reader is referred to Talwani & Heirtzler (1962, 1964), Kravchinsky et al. (2019) and Won & Bevis (1987).

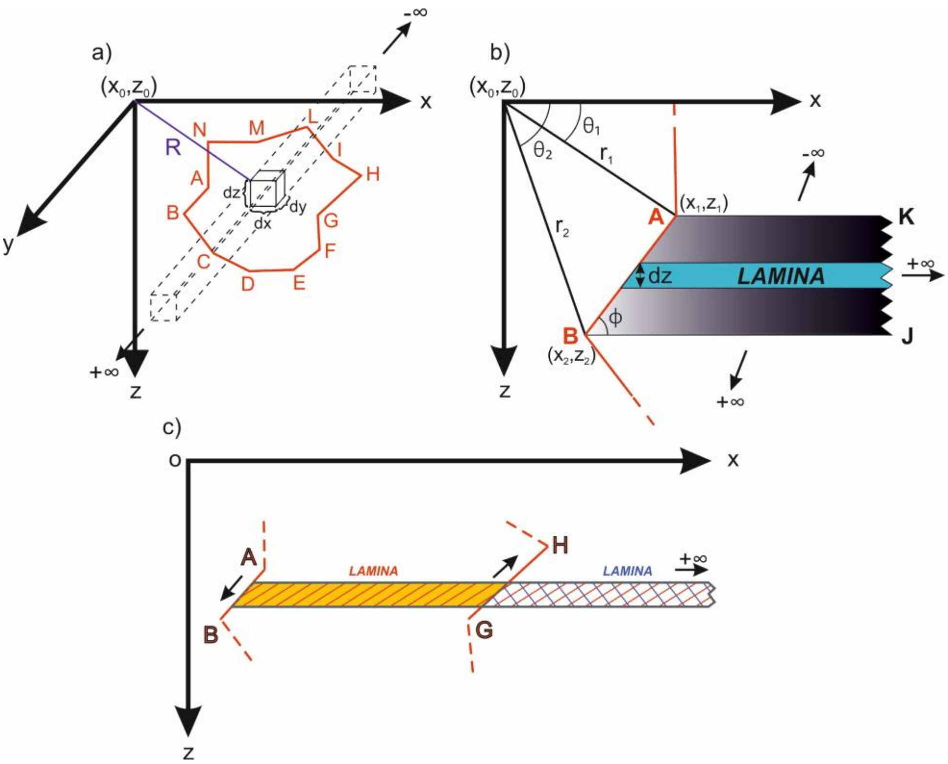

Figure 1 – a) Elementary volume with uniform magnetization immersed in a non-magnetic space. This volume can be extended in the space to define an infinitely elongated body in the y direction. The polygon in red represents the cross-section of this body, that we consider for computing the magnetic field relative to the entire body. Modified from Kravchinsky et al. (2019). b) Sketch of all the parameters involved in the calculation of the vertical V and horizontal H magnetic strength in Talwani & Heirtzler's algorithm. The semi-infinite lamina of thickness dz expands to form a semi-infinite prism with section ABJK built on the side AB of the body. Modified from Kravchinsky et al. (2019). c) Representation of the laminas with thickness dz built along the polygon sides AB and GH drawn in figure a). Moving in a counterclockwise order (black arrows), the lamina along the side AB, defining a semi-infinite infinitesimal prism elongated in the positive x direction (red), provides a positive field, whereas the lamina on the side CD provides a negative field, smaller in absolute value owing to the less extended semi-infinite infinitesimal prism defined along this side (blue). The resulting magnetic anomaly, obtained as scalar sum between the total fields caused by the two laminas, is relative to the area in yellow inside the polygon.

2.1. Talwani & Heirtzler

The formulation of Talwani & Heirtzler (1962, 1964) starts from the definition of the magnetic potential Ω:

$$\Omega = \frac{\vec{M}\, dxdydz\, \vec{R}}{R^3} \tag{1}$$

relative to an elementary volume with uniform magnetization $\vec{M}$ and distance R from the observation point with coordinates (x_0,z_0). Integrating expression (1) from negative to positive infinity in the *y* direction, we achieve the magnetic potential due to an infinitely elongated prism (figure 1a). The vertical *V* and horizontal *H* components of the magnetic strength of this prism can be obtained differentiating its magnetic potential with respect to *x* and *z* directions (the derivative of the magnetic potential along *y* is null not appearing this variable in the expression of the potential; see Talwani & Heirtzler (1962)). Now, integrating *V* and *H* from *x* to positive infinity, we obtain new expressions for *V* and *H*:

$$\mathrm{V} = 2\,\frac{M_x z - M_z x}{(x^2 + z^2)}\,\mathrm{dz} \tag{2}$$

$$\mathrm{H} = 2\,\frac{M_x x - M_z z}{(x^2 + z^2)}\,\mathrm{dz} \tag{3}$$

that are relative to a semi-infinite lamina with thickness *dz* (figure 1b). M_x and M_z represent the components of the magnetization vector $\vec{M}$ along the *x* and *z* axes. $\vec{M}$ is characterized by its own inclination and declination, that differ from that of the Earth magnetic field in the case of a remnant magnetization contribution. In the case of coexistence of induced and remnant magnetization, the resultant magnetization vector is the vectorial sum of both contributions (figure 2a-b).

Now, let us imagine for instance to extend this lamina along the polygon side *AB* shown in figure 1b: integrating (2)-(3) from z_1 to z_2, representing the *z* coordinates of the side vertices taken in a counterclockwise order, *V* and *H* after several steps alter as follow:

$$V = 2 \sin\phi \left[M_x \left\{(\theta_2 - \theta_1)\cos\phi + \sin\phi \ln\left(\frac{r_2}{r_1}\right)\right\} + \\ - M_z \left\{(\theta_2 - \theta_1)\sin\phi - \cos\phi \ln\frac{r_2}{r_1}\right\}\right] \tag{4}$$

$$H = 2\sin\phi\left[M_x\left\{(\theta_2 - \theta_1)\sin\phi - \cos\phi\, ln\left(\frac{r_2}{r_1}\right)\right\} + \right.$$
$$\left. + M_z\left\{(\theta_2 - \theta_1)\cos\phi + \sin\phi\, ln\frac{r_2}{r_1}\right\}\right] \tag{5}$$

with:

$$r_1 = \sqrt{x_1^2 + z_1^2} \quad ; \quad r_2 = \sqrt{x_2^2 + z_2^2} \tag{6}$$

and the following angles:

$$\theta_1 = tan^{-1}\left(\frac{z_1}{x_1}\right) \; ; \; \theta_2 = tan^{-1}\left(\frac{z_2}{x_2}\right) \tag{7}$$

$$\phi = cot^{-1}\left(\frac{x_1 - x_2}{z_2 - z_1}\right) \tag{8}$$

Notice that x_1, x_2, z_1, z_2 are respectively the x and z coordinates of the side vertices taken in a counterclockwise order with respect to an observation point (x_0, z_0) where the magnetic anomaly is calculated (figure 1b).

Equations (4)-(5) represents the vertical and horizontal components of the magnetic strength due to a semi-infinite prism with section *ABKJ* (K and J located at infinity) built on the side *AB* (figure 1b). These equations can be rewritten in a simplified fashion as:

$$V = 2(M_x Q - M_z P) \tag{9}$$

$$H = 2(M_x P + M_z Q) \tag{10}$$

in which the terms P and Q are:

$$P = \frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}\, ln\frac{r_2}{r_1} + \frac{z_{21}^2}{z_{21}^2 + x_{21}^2}(\theta_2 - \theta_1) \tag{11}$$

$$Q = \frac{z_{21}^2}{z_{21}^2 + x_{21}^2}\, ln\frac{r_2}{r_1} - \frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}(\theta_2 - \theta_1) \tag{12}$$

where:

$$x_{21} = x_2 - x_1 \quad ; \quad z_{21} = z_2 - z_1 \tag{13}$$

In the case $z_{21} = 0$, both P and Q become zero and therefore the side provides no magnetic contribution.

Now, the total field scalar anomaly is obtained as vectorial projection of V and H along the direction of the Earth magnetic field as follows:

$$T = V \sin I + H \cos I \sin(P - D) \tag{14}$$

where I and D are respectively the inclination and declination of the Earth magnetic field, whereas P is the angle between the Geographic North and the profile direction along with the magnetic field of the body is computed (figure 2a).

Finally, since the polygon consists of n sides, the overall total field scalar anomaly is computed by summing over the contributions T relative to each side in a counterclockwise order. A simplified representation of the physical meaning of the latter operation is illustrated in figure 1c.

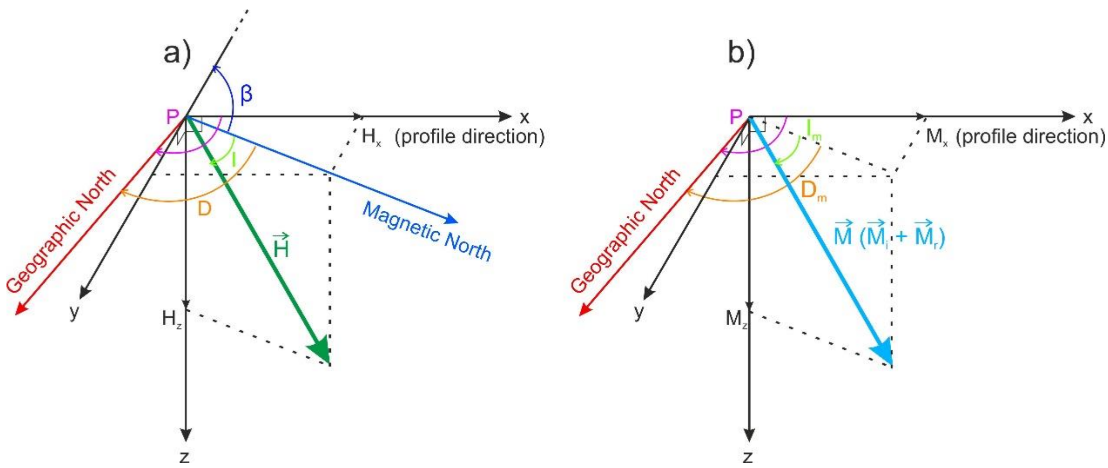

Figure 2 – a) Total Earth magnetic field $\vec{H}$, characterized by changing inclination I and declination D as a function of profile location upon Earth surface. I is defined as the angle made by $\vec{H}$ with the horizontal plane and D as the angle between the Magnetic and the Geographic Norths. In detail, I may vary from ± 90° (respectively at North and South Poles) to 0° (at the equator) and D from 180° to -180°. The angle P define the orientation of the profile direction (x axis) along with the computation of the magnetic field is performed. β is the angle between the magnetic north and the negative direction of the body elongation (-y). I, D, P are taken clockwise, whereas β counterclockwise. b) Total magnetization vector $\vec{M}$, defined as the vectorial sum of induced $\overrightarrow{M_\iota}$ and remnant $\overrightarrow{M_r}$ magnetization. It is characterized by own inclination I_m and declination D_m. In the case of induced magnetization solely, then $I_m = I$ and $D_m = D$.

2.2. Kravchinsky et al.

Kravchinsky et al. (2019) suggested the existence of some mathematical omissions and errors in the original formulation of Talwani & Heirtzler (1962, 1964), with consequently possible failure of magnetic anomaly calculations. Nevertheless, this new formulation derives closely from that of Talwani & Heirtzler (1962, 1964), starting from the definition of the magnetic potential in the case of SI units (modified by a factor $\frac{1}{4\pi}$ with respect to (1)). The mathematical derivation partially differs during the integration from z_1 to z_2 leading to the Talwani & Heirtzler's corresponding equations (2)-(3), owing to a different definition of x, leading to the following new modified terms P and Q (Kravchinsky et al., 2019):

$$\mathrm{P} = \frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}\, ln\frac{r_2}{r_1} + \delta \frac{z_{21}^2}{z_{21}^2 + x_{21}^2}\,(\alpha_2 - \alpha_1) \tag{15}$$

$$Q = \frac{z_{21}^2}{z_{21}^2 + x_{21}^2}\, ln\frac{r_2}{r_1} - \delta \frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}\,(\alpha_2 - \alpha_1) \tag{16}$$

where the meanings of r_1, r_2, x_{21}, z_{21} are defined in (6)-(13).

Now recalling (13), then

$$g = \frac{x_{21}}{z_{21}} \tag{17}$$

and

$$\delta = \begin{cases} -1, & x_1 < g z_1 \\ 1, & x_1 > g z_1 \end{cases} \tag{18}$$

$$\alpha_1 = \tan^{-1}\left(\frac{\delta(z_1 + g x_1)}{x_1 - g z_1}\right) \tag{19}$$

$$\alpha_2 = \tan^{-1}\left(\frac{\delta(z_2 + g x_2)}{x_2 - g z_2}\right) \tag{20}$$

The new relations (15)-(16) appear very similar to the previous equations (11)-(12), where the major difference seems to be related to the angles α_1 and α_2 in place of Talwani's θ_1 and θ_2 (cfr. eqs (7)-(19)-(20)). In addition to a different expression, α_1 and α_2 present also a δ term in order to take into account an absolute value appearing during the derivation in both the denominators in the arguments of the arctangents (cfr. Kravchinsky et al., 2019 – Supporting Information).

Now, the computation of the vertical and horizontal components of the magnetic strength V and H is achieved by means of the following equations:

$$V = \frac{1}{2\pi}(M_x Q - M_z P) \tag{21}$$

$$H = \frac{1}{2\pi}(M_x P + M_z Q) \tag{22}$$

which differs from Talwani & Heirtzler's ones only of a factor $\frac{1}{4\pi}$ owing to the utilization of SI instead of emu units.

At this point, the computation of the scalar total field magnetic anomaly of the entire body should be carried out using eq. (14) for each polygon side in a counterclockwise order as for Talwani & Heirtzler's algorithm. On the contrary, the authors (Kravchinsky et al., 2019) specify a clockwise order that, from a physical point of view, corresponds to having a semi-infinite polygon in the opposite x direction built for each side, resulting in a negative scalar total field (-T) contribution. In the supporting information we explain how such issue has been corrected.

2.3. Won & Bevis

Won & Bevis (1987) proposed a faster approach to compute the magnetic anomaly thanks to the substitution of trigonometric functions with simpler relations referred to the vertex coordinates of the polygon (e.g. Grant & West, 1965). Moreover, this formulation allows to perform magnetic calculation even in the case of side vertices crossing the x axis (we have extended this possibility to the other two algorithms in the code attached to this paper).

However, the theory behind this algorithm differs from that previously examined, being the formulation derived by means of the Poisson relation (Won & Bevis, 1987). This relation links the gravitational attraction to the scalar magnetic potential of a body, taking advantage from the fact that some similarities are observed between them. For instance, both have magnitude that is inversely proportional to the square of the distance to the relative sources (Blakely, 1995). The Poisson relation can be differentiated to obtain the magnetic strength vector $\vec{H}$ as follow:

$$\vec{H} = \frac{M}{G\rho}\frac{\partial}{\partial\alpha}\overrightarrow{g'} \tag{23}$$

where M is the magnetization module, G the gravitational constant, ρ the body density and $\overrightarrow{g'}$ the gravitational attraction related to the body.

The term $\frac{\partial}{\partial\alpha}$ is defined as follows:

$$\frac{\partial}{\partial\alpha} \equiv sinI_m\frac{\partial}{\partial z} + sin\beta cosI_m\frac{\partial}{\partial x} \tag{24}$$

where I_m represents the inclination of the magnetization vector and β the strike of the body measured counterclockwise from magnetic north to the negative y axis (figure 2a). This relation is used to achieve the magnetic strength components of a polygon side along *x* and *z*, that are:

$$H_z = \frac{M}{G\rho}\left(sinI_m\frac{\partial g'_z}{\partial z} + cosI_m sin\beta\frac{\partial g'_z}{\partial x}\right) \tag{25}$$

$$H_x = \frac{M}{G\rho}\left(sinI_m\frac{\partial g'_x}{\partial z} + cosI_m sin\beta\frac{\partial g'_x}{\partial x}\right) \tag{26}$$

where g'_x and g'_z are the *x* and *z* components of the gravitational attraction of the body, defined as:

$$g'_x = 2G\rho X \;;\; g'_z = 2G\rho Z \tag{27}$$

with *X* and *Z* representing line integrals along the polygon side (refer to Won & Bevis (1987) for details).

Recalling (6)-(7)-(13)-(17), the partial derivatives of *X* and *Z* in respect to *x* and *z* are respectively:

$$\frac{\partial X}{\partial x} = \frac{x_{21}z_{21}}{x_{21}^2 + z_{21}^2}\left[\frac{1}{g}(\theta_1 - \theta_2) - ln\frac{r_2}{r_1}\right] + P \tag{28}$$

$$\frac{\partial X}{\partial z} = -\frac{{x_{21}}^2}{x_{21}^2 + z_{21}^2}\left[\frac{1}{g}(\theta_1 - \theta_2) - ln\frac{r_2}{r_1}\right] + Q \tag{29}$$

$$\frac{\partial Z}{\partial x} = -\frac{x_{21}z_{21}}{x_{21}^2 + z_{21}^2}\left[(\theta_1 - \theta_2) + \frac{1}{g}ln\frac{r_2}{r_1}\right] + Q \tag{30}$$

$$\frac{\partial Z}{\partial z} = -\frac{{x_{21}}^2}{x_{21}^2 + z_{21}^2}\left[(\theta_1 - \theta_2) + \frac{1}{g}ln\frac{r_2}{r_1}\right] - P \tag{31}$$

where now:

$$P = \frac{x_1z_2 - x_2z_1}{x_{21}^2 + z_{21}^2}\left[\frac{x_1x_{21} - z_1z_{21}}{r_1^2} - \frac{x_2x_{21} - z_2z_{21}}{r_2^2}\right] \tag{32}$$

$$Q = \frac{x_1z_2 - x_2z_1}{x_{21}^2 + z_{21}^2}\left[\frac{x_1z_{21} + z_1x_{21}}{r_1^2} - \frac{x_2z_{21} + z_2x_{21}}{r_2^2}\right] \tag{33}$$

As in the previous derivations, the total field scalar anomaly of the side is obtained as a projection of *V* and *H* onto the Earth magnetic field:

$$\mathrm{T} = H_z \mathrm{sinI} + H_x \mathrm{cosIsin} \tag{34}$$

Contrary to the two preceding algorithms, now the computation of the total field scalar magnetic anomaly of the body using (34) should be carried out in clockwise order.

3. Discussion

3.1. Analytical results

The three formulations discussed in this paper share the same purpose, i.e. to calculate the magnetic anomaly due to a body with uniform magnetization and polygonal section. Among these, those of Talwani and Heirtzler (1962, 1964) and Kravchinsky et al. (2019) present similar derivations, with some differences. In principle, Kravchinsky et al. (2019) addressed some mathematical errors and omissions in the original derivation of Talwani & Heirtzler, revealing some inconsistencies in magnetic anomaly calculation. However, after analyzing the two formulations, some inaccuracies in Kravchinsky et al. (2019) have been found. These issues are related to i) the order of calculation around the polygon sides (clockwise/counterclockwise), ii) the use of the cosine theorem formula and iii) the projection of *V* and *H* along the Earth magnetic field vector. A detailed discussion of these findings is provided in the supporting information of this paper.

Regarding to the corrections brought to Talwani and Heirtzler (1962, 1964) by Kravchinsky et al. (2019), they mainly concern: a) a modification of the definitions of the angles θ_1 and θ_2 and b) an addition of a δ term in order to account for an absolute value which appears in their derivation.

In the following, we illustrate how the two algorithms, after removing the inaccuracies in Kravchinsky et al. (2019), can be reconciled to a single approach, showing their equivalence from an analytic point of view. For this purpose, let us rewrite expressions (19)-(20) substituting the term δ with an absolute value at both the denominators in the argument of the arctangents, since $x_1 - gz_1 = x_2 - gz_2$ (refer to the supporting information of Kravchinsky et al. (2019) for an explanation):

$$\alpha_1 = tan^{-1}\left(\frac{z_1 + gx_1}{|x_1 - gz_1|}\right) \tag{35}$$

$$\alpha_2 = tan^{-1}\left(\frac{z_2 + gx_2}{|x_2 - gz_2|}\right) \tag{36}$$

where the term g is the same than the one presented in eq. (17). Then, by cancelling out x_1 and x_2 both in the numerator and denominator of the argument of the respectively arctangents α_1 and α_2, we can distinguish two cases:

1. if $x_1 - gz_1 = x_2 - gz_2 > 0$ (that is $\delta = 1$ in (19)-(20)):

$$\alpha_1 = \tan^{-1}\left(\frac{z_1 + gx_1}{x_1 - gz_1}\right) = \tan^{-1}\left(\frac{\frac{z_1}{x_1} + g}{1 - \frac{z_1}{x_1}g}\right) \tag{37}$$

$$\alpha_2 = tan^{-1}\left(\frac{z_2 + gx_2}{x_2 - gz_2}\right) = tan^{-1}\left(\frac{\frac{z_2}{x_2} + g}{1 - \frac{z_2}{x_2}g}\right) \tag{38}$$

2. if $x_1 - gz_1 = x_2 - gz_2 < 0$ (that is δ = -1 in (19)-(20)):

$$\alpha_1 = tan^{-1}\left(-\frac{z_1 + gx_1}{x_1 - gz_1}\right) = tan^{-1}\left(-\frac{\frac{z_1}{x_1} + g}{1 - \frac{z_1}{x_1}g}\right) \tag{39}$$

$$\alpha_2 = tan^{-1}\left(-\frac{z_2 + gx_2}{x_2 - gz_2}\right) = tan^{-1}\left(-\frac{\frac{z_2}{x_2} + g}{1 - \frac{z_2}{x_2}g}\right) \tag{40}$$

Now, using the mathematical relation combining sums of arctangents in a unique arctangent expression:

$$\tan^{-1}\mathrm{A} + \tan^{-1}\mathrm{B} = \begin{cases} tan^{-1}\left(\frac{A+B}{1-AB}\right), & AB < 1 \\ tan^{-1}\left(\frac{A+B}{1-AB}\right) + (sign\ of\ A)\pi, & AB > 1 \end{cases} \tag{41}$$

then we can rewrite (37)→(40) as:

$$\alpha_1 = \tan^{-1}\left(\frac{g + \frac{z_1}{x_1}}{1 - g\frac{z_1}{x_1}}\right) = \tan^{-1}(g) + \tan^{-1}\left(\frac{z_1}{x_1}\right) \tag{42}$$

$$\alpha_2 = tan^{-1}\left(\frac{g + \frac{z_2}{x_2}}{1 - g\frac{z_2}{x_2}}\right) = tan^{-1}(g) + tan^{-1}\left(\frac{z_2}{x_2}\right) \tag{43}$$

when the product $\frac{z_1}{x_1}g = \frac{z_2}{x_2}g < 1$, and:

$$\begin{aligned} \alpha_1 &= \tan^{-1}\left(-\frac{g + \frac{z_1}{x_1}}{1 - g\frac{z_1}{x_1}}\right) = -\tan^{-1}\left(\frac{g + \frac{z_1}{x_1}}{1 - g\frac{z_1}{x_1}}\right) \\ &= -\tan^{-1}(g) - \tan^{-1}\left(\frac{z_1}{x_1}\right) + (sign\ of\ g)\pi \end{aligned} \tag{44}$$

$$\alpha_2 = \tan^{-1}\left(-\frac{g+\frac{z_2}{x_2}}{1-g\frac{z_2}{x_2}}\right) = -\tan^{-1}\left(\frac{g+\frac{z_2}{x_2}}{1-g\frac{z_2}{x_2}}\right) = -\tan^{-1}(g) - \tan^{-1}\left(\frac{z_2}{x_2}\right) + (sign\ of\ g)\pi \qquad (45)$$

when $\frac{z_1}{x_1}g = \frac{z_2}{x_2}g > 1$.

As it is apparent, the terms $tan^{-1}\left(\frac{z_1}{x_1}\right)$ and $tan^{-1}\left(\frac{z_2}{x_2}\right)$ are exactly equivalent to the expressions of θ_1 and θ_2 in Talwani & Heirtzler (1962, 1964) (see figure S4 in the supporting information for details concerning the angles involved in Kravchinsky et al. (2019) formulation). Hence, cancelling out each term $tan^{-1}(g)$, in the case (1) the difference $\alpha_2 - \alpha_1$ will always be algebraically the difference $\theta_2 - \theta_1$, whereas in (2) $\alpha_2 - \alpha_1$ will be equal to $-(\theta_2 - \theta_1)$. Recalling now the expressions (11)-(12) for P and Q in Talwani & Heirtzler (1962, 1964),

$$P = \frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}\, ln\frac{r_2}{r_1} + \frac{z_{21}^2}{z_{21}^2 + x_{21}^2}\, (\theta_2 - \theta_1) \qquad (11)$$

$$Q = \frac{z_{21}^2}{z_{21}^2 + x_{21}^2}\, ln\frac{r_2}{r_1} - \frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}\, (\theta_2 - \theta_1) \qquad (12)$$

and the homologous (15)-(16) in Kravchinsky et al. (2019),

$$P = \frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}\, ln\frac{r_2}{r_1} + \delta\frac{z_{21}^2}{z_{21}^2 + x_{21}^2}\, (\alpha_2 - \alpha_1) \qquad (15)$$

$$Q = \frac{z_{21}^2}{z_{21}^2 + x_{21}^2}\, \ln\frac{r_2}{r_1} - \delta\frac{z_{21}\, x_{21}}{z_{21}^2 + x_{21}^2}\, (\alpha_2 - \alpha_1) \qquad (16)$$

we can observe that the formulations differ for another term δ multiplying the difference $\alpha_2 - \alpha_1$. If we are in the case (2), eq. (44)-(45), we have δ = -1, then the difference $\alpha_2 - \alpha_1$ again leads back to $\theta_2 - \theta_1$. Hence, contrary to what pointed out by the authors, we have demonstrated that the formulation of Kravchinsky et al. (2019) does not differ from that of Talwani & Heirtzler (1962, 1964), rather it simply represents an algebraic variant, leading to identical results in terms of computed magnetic anomalies. For this reason, either formulations can be considered as a single approach and used without any distinctions. Regarding the Won & Bevis' (1987) formulation, it is not easily comparable in details to the other two from an analytical point of view, being derived from different assumptions and theoretical approach. However, in the following section we compare it from a numerical point of view in order to understand whether its calculated magnetic response is always in agreement with that of the other algorithms.

3.2. Numerical results

Two different numerical tests have been implemented using the above algorithm formulations (i.e. considering our rectified version for that of Kravchinsky et al. (2019)) in order to achieve two different purposes, namely i) to detect possible issues and irregularities in magnetic anomaly computation in a wide variety of random magnetic scenarios and ii) to assess the results in a more realistic geological context upon sane magnetic scenarios, more helpful for geophysical applications.

The former purpose has been accomplished by means of a random changing-shape generation of up to five polygons repeated for 1000000 iterations (i.e. magnetic scenarios). In detail, both induced $\overrightarrow{M_i}$ and $\overrightarrow{M_r}$ remnant magnetizations changing have been limited in a range between 0 and 50 A/m, their inclination and declination respectively between -90° and 90° and between -180° and 180°. One hundred observation points have been located evenly spaced at a constant clearance of 10 meters (toward up) along a profile 100 meters long. Figure 4a shows one of these iterations (for the relative frequency of the magnetic properties tested see figure S5 in the supporting information of this paper).

During the test, a huge amount of combinations between the above magnetic properties have been sampled, showing in all cases full agreement between the three algorithms. Moreover, none anomalous or failing magnetic computations have been detected.

For what concern the second purpose, it has been carried out for the same number of iterations in a more realistic geological context like that modelled in Armadillo et al. (2020). The bodies modelled are three polygons with fixed geometries, representing a horst tectonic structure (figure 4b). In this test, the random variation of both induced and remnant magnetization has been restricted up to 5 A/m, representing a realistic value for geophysical studies. The range of variation for the others magnetic properties is the same of the former test. The external bodies extend respectively up to -100000 and 100000 meters in *x* direction to avoid “border effects”. Being these bodies represented by the same lithotype, we have assigned to them the same induced $\overrightarrow{M_i}$ and remnant $\overrightarrow{M_r}$ magnetization and the respective inclinations and declinations. For geological consistency, the central body is characterized by induced magnetization with same inclination and declination of that of the lateral bodies, but different module $|\overrightarrow{M_i}|$. In addition, it is characterized by different remnant magnetization module, inclination and declination. During each iteration, the magnetic properties randomly are changed following the rules described. One thousand observation points have been located evenly spaced at a constant clearance of 100 meters (toward up) along a profile 15000 meters long. Figure 4b presents one iteration relative to this analysis.

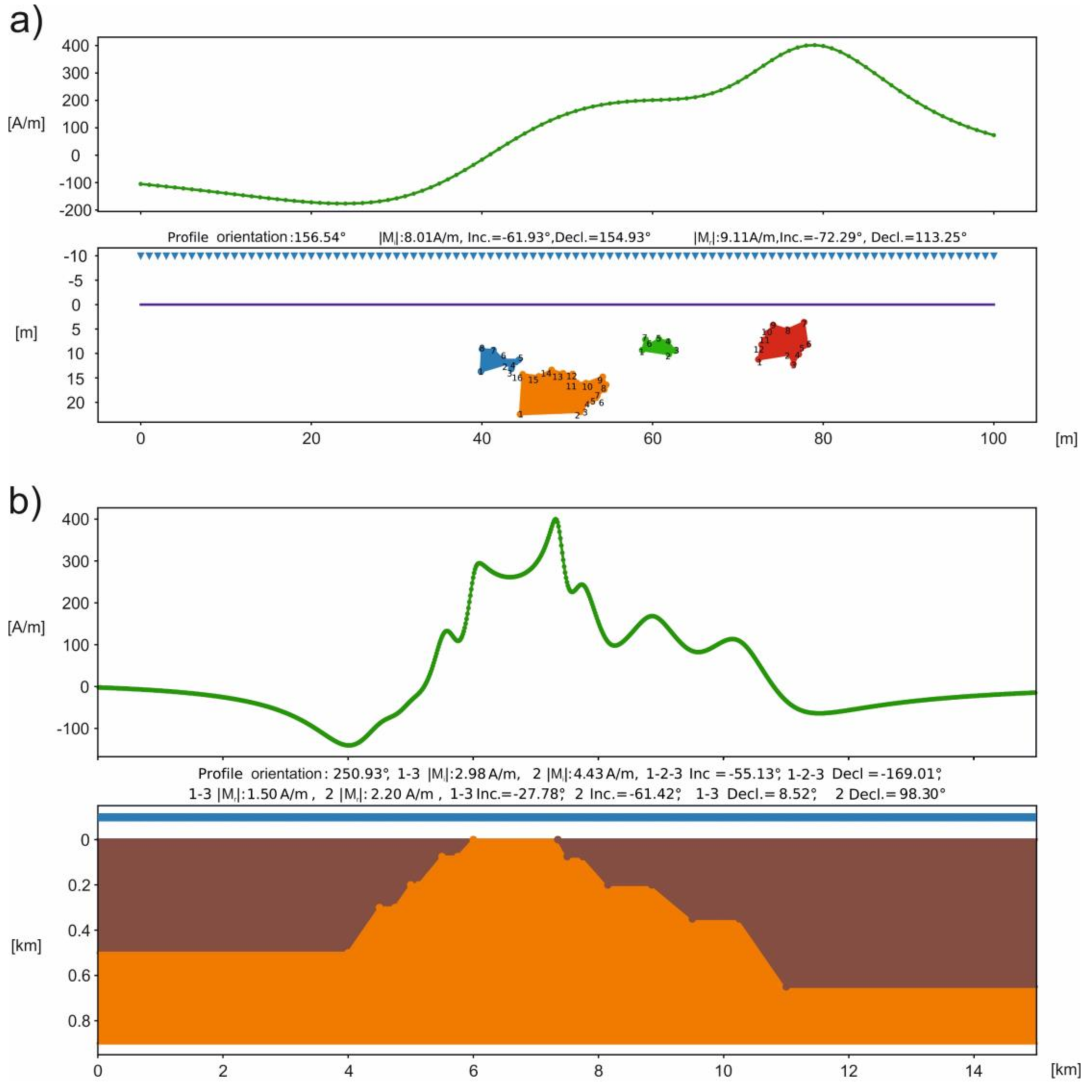

Figure 3 – a) Magnetic responses (green curves) due to four randomly generated polygons upon a magnetic scenario using the three algorithms of Talwani & Heirtzler (1962, 1964), Won & Bevis (1987) and Kravchinsky et al. (2019). In this forward model, representing an iteration of the former numerical test described in the main text, $|\overrightarrow{M_i}|$ *and* $|\overrightarrow{M_r}|$ *represent the induced and remnant magnetization vector modules, whereas the abbreviation Inc. and Decl. their inclinations and declinations respectively. The numbers around each polygon described the order and verse of calculation performed on its segments. The inverted triangles depict the observation points where the magnetic anomaly is calculated. b) Forward magnetic model representing an iteration of the second numerical test in the case of a geological horst structure upon a random magnetic scenario. The body in orange (body 2) represents the horst body, surrounded by two other identical bodies (bodies 1 and 3 from left to right) for geological consistency. For the meaning of the abbreviations refer to the caption of the above figure a).*

Even in this test, in all sampled cases there has been full agreement between the results of all the algorithms, with differences in magnetic anomalies in each observation point next to the machine precision of the computer utilized for these tests.

As result of both our numerical and analytical tests, we might confirm that the three formulations lead to the same results and no algorithm is advantageous over the other two, showing always to operate correctly and without abnormal behaviors. Moreover, the speed up in magnetic calculation originally obtained by Won & Bevis (1987) no longer has any advantage considering the much higher computing power of modern computers.

4. Conclusions

In this paper, we have reviewed and compared the available formulations used to compute the magnetic anomaly caused by a 2D uniformly-polarized body with polygonal section, both from an analytical and a numerical point of view. During the analytical analysis we have demonstrated that the formulation of Kravchinsky et al. (2019) does not differ from that of Talwani & Heirtzler (1962, 1964), being simply an algebraic variant. Indeed, the angle differences $\alpha_2 - \alpha_1$ in Kravchinsky et al. (2019) reduces in all cases to the difference $\theta_2 - \theta_1$ in Talwani & Heirtzler (1962, 1964). In addition, we have revealed and fixed some inaccuracies in Kravchinsky et al. (2019), that are: i) the order of calculation around the polygon sides, ii) the use of the cosine theorem formula and iii) the projection of V and H along the Earth magnetic field vector, leading in case ii) often immediate termination of the algorithm during the numerical tests. During these tests, we have generated a huge number of magnetic scenarios in two different ways and purposes, namely i) to investigate possible irregularities in magnetic anomaly computation for random-changing polygon numbers and geometries and ii) to evaluate the utilization of the algorithms in realistic geological/tectonic context like that presented in Armadillo et al. (2020). In all cases, the three algorithms have behaved in the same manner without criticality, computing in all the sampled scenarios the same magnetic anomaly response. For this reason, the reader is free to follow the three approaches described without any preference.

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Algorithms code availability

The source code and documentation for the three algorithms discussed above is provided in the programming languages Julia and Python as packages on the following GitHub repositories:

- https://github.com/inverseproblem/Mag2Dpoly.jl
- https://github.com/inverseproblem/pyMag2DPoly

The codes are available only through the above GitHub repositories or upon request addressed to the corresponding author.

This publication has to be referred with any use of the code.

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