A machine learning model of Arctic sea ice motions

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November 21, 2022

Abstract

Sea ice motions play an important role in the polar climate by transporting pollutants, heat, water and salt as well as changing the ice cover. Numerous physics-based models have been constructed to represent the sea ice dynamical interaction with the atmosphere and ocean. Here we propose a new data-driven deep-learning approach which utilizes a convolutional neural network (CNN) to model how Arctic sea ice moves in response to surface winds given its initial ice velocities and concentration a day earlier. Results show that CNN computes the sea ice response with a correlation of 0.82 on average with respect to reality, which surpasses a set of pixel-based predictions, such as persistence (PS), linear regression (LR), random forest (RF), multiple layer perceptrons (MLP) and CICE, a leading physics-based model. The superior predictive skill of CNN suggests the important role played by the connective patterns of the predictors of the sea ice motion.

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5	Key Points:
6	• The response of Arctic sea ice motions to surface winds can be modeled using a
7	convolutional neural network with predictors of the previous-day sea ice velocity
8	and concentration and present-day surface wind.
9	• The superior performance of convolutional neural network suggests the importance
10	of the inter-pixel (across space) dynamical connections of the sea ice motion com-
11	pared to pixel-based baseline models that only consider local interactions.
12	• The success of the convolutional neural network model of sea ice motion holds promise
13	for combining machine learning with physics-based models to simulate sea ice.

Key Points:

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14 Abstract

Sea ice motions play an important role in the polar climate system by transporting pol-15 lutants, heat, water and salt as well as changing the ice cover. Numerous physics-based 16 models have been constructed to represent the sea ice dynamical interaction with the 17 atmosphere and ocean. In this study, we propose a new data-driven deep-learning ap-18 proach which utilizes a convolutional neural network (CNN) to model how Arctic sea ice 19 moves in response to surface winds given its initial ice velocities and concentration a day 20 earlier. Results show that CNN computes the sea ice response with a correlation of 0.8221 on average with respect to reality, which surpasses a set of pixel-based predictions, such 22 as persistence (PS), linear regression (LR), random forest (RF), multiple layer percep-23 trons (MLP) and CICE, a leading physics-based model. The superior predictive skill of 24 CNN suggests the important role played by the connective patterns of the predictors of 25 the sea ice motion. 26

27 Plain Language Summary

Sea ice, the frozen seawater that floats on the ocean, grows in each hemisphere's 28 winter and retreats in the summer but does not disappear in the current climate. The 29 sea ice coverage is discontinuous and thin enough to move with the winds and currents. 30 These movements alter the sea ice cover and transport pollutants, heat, water and salt. 31 Previous studies have advanced mathematical models that represent the physics of how 32 33 sea ice moves in response to surface winds. In this study, we propose a different type of model, called a convolutional neural network (CNN), which is constructed purely from 34 observational data without explicitly accounting for the underlying physical insights. Un-35 like other conventional data-driven models that are trained on each geographic local point 36 independently, CNN takes into account how a given location connects to its neighbors 37 via patterns. The superior performance of CNN suggests that local ice motions depend 38 on a large-scale pattern of surrounding winds and that data-based methods are a promis-39 ing alternative to physics-based models. 40

41 **1** Introduction

As an essential element in polar climate and dynamics, the movement of sea ice dis-42 tributes pollutants, transports heat, water and salt and affects the polar energy budget 43 by modifying the ice cover. The factors that determine sea ice motion include the sea 44 ice inertia, atmospheric and oceanic stress, the Coriolis force, the sea surface tilt and the 45 internal ice force that arises from floe interactions such as collisions, rafting and defor-46 mation. Based on an old rule-of-thumb that sea ice moves with a speed of 2% of the sur-47 face wind speed and 45° to the right of the surface wind direction, Thorndike and Colony 48 (1982) examined the relation between the ice velocity and geostrophic winds and found 49 that for the long-term (multi-month) average, about half of the variance of the ice mo-50 tion is directly related to the surface geostrophic wind, while the other half is due to the 51 mean ocean circulation; on shorter time scales from days to months, geostrophic winds 52 alone can explain more than 70% of the variance of sea ice motion in the central Arc-53 tic region. Particularly, at daily time scale the unexplained variance is even less than 20%. 54 On diurnal time scale or within 400 km of the coasts, however, the ice inertia or coastal 55 stress gradients can be as important as the wind stress, which disqualifies the wind-only 56 approximation. 57

Using daily sea ice velocities recovered from passive microwave satellite images, Kimura and Wakatsuchi (2000) showed that a linear relation based on the surface geostrophic winds with a speed reduction factor generally explains 70% to 90% of the sea ice velocity variance over the Arctic, except along some coastal regions. The spatial variation of the speed reduction factor of the sea ice motion relative to the surface winds depends on the internal ice stress gradient, which depends on ice thickness and concentration. Generally speaking, thicker and higher sea ice concentration result in a greater internal ice
 stress gradient, leading to a greater speed reduction. For example, sea ice moves at 2%
 of the surface wind speed over the seasonal ice zones but at less than 0.8% over the in-

⁶⁷ terior Arctic (Kimura & Wakatsuchi, 2000).

Similar conclusions were drawn for Antarctic sea ice from other studies. For example, in the Weddell Sea, nearly 70% to 95% of the variance of daily ice drift velocity can be linearly related to the wind velocity, except when the wind speed is below 3.5 m/s (Kottmeier et al., 1992). In addition, statistically significant relations between the sea ice velocity and local winds were detected from observations in most sectors over the Antarctic (Holland & Kwok, 2012).

Several studies have come up with equations-based models of varying complexity
that utilize a wind-ice relation to model sea ice motion. A simple linear relational rule
was described by Thorndike and Colony (1982) as follows:

$$\begin{bmatrix} U\\V \end{bmatrix} = F\begin{bmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u\\v \end{bmatrix} + \begin{bmatrix} \bar{c_u}\\ \bar{c_v} \end{bmatrix},$$
(1)

⁷⁷ where (U,V) and (u,v) are the sea ice velocity and wind velocity, respectively, for the ⁷⁸ same day; F and θ are the speed reduction factor and turning angle, respectively; (\bar{c}_u, \bar{c}_v) is the daily mean surface ocean current velocity. Park and Stewart (2016) introduced ⁸⁰ a more complex nonlinear analytical model that calculates the ice velocity given that the ⁸¹ surface winds are known and the ocean geostrophic currents are weak.

Physics-based models compute the sea ice velocity from the ice momentum equation (Hibler, 1979), which is primarily driven by surface stresses from winds and ocean currents. Specifically, the sea ice velocity tendency is modeled as

$$m\frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \vec{\sigma} + \vec{\tau_a} + \vec{\tau_o} - \hat{k} \times mf\mathbf{u} - mg\nabla H_o, \tag{2}$$

where $\vec{\sigma}$, $\vec{\tau_a}$ and $\vec{\tau_o}$ are the internal stress tensor, atmosphere stress, and ocean stress, respectively; m is the combined mass of ice and snow per unit area; H_o is the sea surface height; and \mathbf{u} represents (u, v). The last two terms on the right hand side are the Coriolis force and sea surface tilt term. The parameterization for the atmosphere and ocean stresses contains the sea ice concentration as a multiplicative factor in order to accommodate the free drift scenario in low ice concentration regions.

Though having achieved a certain amount of success, physics-based models face chal-91 lenges in practice, which include, but are not limited to, justification for assumptions about 92 the sea ice rheology and the requirement for high-resolution observations to verify the 93 large set of parametrizations. In this study, our goal is to model the sea ice response in 94 an alternative approach that is data-driven and free of physical assumptions. Specifically, 95 we aim to construct a convolutional neural network (CNN) model, in order to find how 96 Arctic sea ice responds to the future surface winds in general given its current sea ice 97 velocity and concentration. The trained CNN model has been shown to outperform the 98 persistence and a set of pixel-based baseline predictions. The practice here aims to re-99 cover as much of the general relation between the surface winds and the Arctic sea ice 100 motion as possible from the limited measurement. 101

The structure of the paper is as follows: Section 2 describes the concept and architecture of the CNN as well as the data used for training and testing; section 3 documents the path we took to select the input predictors for the CNN model; section 4 concludes with the training results and discussions.

¹⁰⁶ 2 Methodology and Data

¹⁰⁷ CNN is a class of deep-learning neural network first introduced to classify the hand-¹⁰⁸ written documents in the field of computer vision (LeCun et al., 1998) and later mostly applied to image analysis. Inspired by the biological process whereby animals visualize
 images through the connectivity between neurons, CNN breaks-down an image composed
 of complex patterns into smaller and simpler pattern feature blocks by employing a math ematical operation called "convolution". The basic architecture of a CNN consists of the
 following elements:

- an input layer that is a tensor with dimensions of (number of image samples)×(image height)×(image width)×(number of input predictors/channels),
- **convolutional layers** that convolve the original images with kernels and activation functions into smaller-sized features maps,
 - **pooling layers** that further reduce the dimensions of the feature maps via down sampling, and
- a fully connected layer that, via multiple layer perceptrons, fully connects the flattened (i.e., collapsing a multi-dimensional array into one dimension) output tensor from the last convolutional layer to the terminal output of the CNN.

Our objective is to predict present-day sea ice velocity (ui_1, vi_1) as accurately as possible with a parsimonious set of predictors as features input to the CNN. We investigate the importance of a list of candidate input predictors:

• present-day surface wind velocity (ua_1, va_1) ,

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- previous-day sea ice velocity (ui_0, vi_0) , and
- previous-day sea ice concentration (c_0) and thickness (s_0) .

All of the predictors except sea ice thickness are readily available from a combi-129 nation of satellite-based observations and atmospheric reanalysis from 1990 to 2018. Un-130 fortunately, satellite-based observations of thickness prior to 2019 have high uncertainty, 131 are discontinuous in time, and are completely unavailable in summer. Because we deemed 132 observations of thickness to be insufficient for our purpose, we first test the importance 133 of predictors with data sets taken exclusively from a climate model by constructing a se-134 ries of CNN models with various combinations of the predictors listed above. Fortunately, 135 the process of this feature exploration, as will be discussed in Section 3, reveals a com-136 bination lacking s_0 as one of the optimal feature input combinations. Therefore, we re-137 construct the CNN model based on a data sets available from satellite-based observa-138 tions and atmospheric reanalysis. 139

For climate model output, we use one historical simulation of the Community Earth System Model version 2 (CESM2)(Danabasoglu et al., 2020) in the Coupled Model Intercomparison Project 6 (CMIP6). The resolution is a nominal 1° in all components, which, for sea ice variables, is a 320x384 grid, with a global average resolution of 64 km approximately, and a finer resolution of 3 to 11 km over the Arctic. The 850-hPa winds are remapped to the sea ice grid to make consistent pixel-by-pixel alignments.

For reanalysis and satellite-based observations, we use the 10-m surface winds from the Japanese 55-year Reanalysis (Kobayashi et al., 2015), which has a resolution of 47 km on average in the Arctic, Polar Pathfinder Version 4 Daily Sea Ice Motion Vectors (Tschudi et al., 2019) and passive microwave sea ice concentrations from the NASA team (Cavalieri et al., 1996). The ice data are regridded from their 25-km EASE-grid to the JRA55 grid for consistent pixel-wise alignments.

We use the data sets that span 1990 to 2018 and split it into a training set (1990 to 2014), a validation set (2015 to 2016) and a testing set (2017 to 2018). The architecture of the CNN is shown in Fig.1: the input is composed of five channels, which are the present-day surface zonal and meridional velocities, the previous-day ice zonal and meridional velocities, and the previous-day ice concentration over the Arctic field of size 40×640. In this example, the input combination does not include thickness. The input is then pro-



Figure 1. The CNN architecture. In each layer, the numbers in red are the dimensions of each channel, i.e. (image height)×(image width), and the numbers in blue specify the number of input predictors/channels.

cessed through five consecutive layers repeating a block unit: a 2D convolutional layer, a LeakyReLU (Leaky Rectified Linear Unit) layer and a 2D max-pooling layer. The output of this five-time repeated block unit is then passed to a 20% drop-out layer before getting flattened to a one-dimensional (1D) vector with a length of 1152. The final step is to regress the flattened 1D vector to another 1D layer of a size 51200, which is the concatenated 1D vector of two flattened 40×640 images that represent the present-day zonal and meridional ice velocities.

The kernel size for the 2D convolutions and max poolings are (2,3) or (2,2), where 165 the latter is used for later layers since the image size is shrinking as the convolution pro-166 gresses, and strides = (1,1) uniformly for all convolutions and max poolings. The con-167 volutional layers all use a linear activation followed by a LeakyReLU added as a non-168 linear activation with a negative slope coefficient, $\alpha = 0.1$. A root mean square error nor-169 malized by the standard deviation, i.e. the second term in Equation (4), is used as the 170 loss function for optimization with the Adam optimizer. The total number of trainable 171 parameters is 59,132,828. 172

Implemented in Python with the Tensorflow/Keras library (https://keras.io), the 173 training takes 50 epochs with a batch size of 365 days on 9862 daily frames (from 1990 174 to 2016), the last two years of which (2015 to 2016) are used for validation. Then a fresh 175 new set of daily frames (2017 and 2018) are used for final testing. To avoid obtaining 176 accidental results due to fixed test sets, we have also performed a set of randomizations 177 by shuffling the years for training, validation and testing. For example, one of the ran-178 domized set uses 1992 to 2016 for training, 2017 2018 for validation, and 1990 to 1991 179 for testing, etc. The results confirm that the prediction skill is not sensitive to the ran-180 domization and maintains stable metric scores. Therefore, here we present one set of train-181 ing only by focusing on the results obtained using 1990 to 2016 for training and valida-182 tion and 2017 to 2018 for final testing. 183

	M1	M2	M3	M4	M5	M6	M7
ua_1							
ui_0							
$vi_0\ c_0$							
<i>s</i> ₀			\checkmark				
Corr PPP	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	0.829	0.813 0.413	0.8001	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	0.787	0.828 0.437

Table 1. Seven CNN models, i.e. M1, M2, ..., M7 with different input combinations constructed for comparisons. The prediction skill evaluations of testing using Corr and PPP for the seven models are listed on the bottom rows.

To evaluate the CNN performance, we set-up four baseline models for comparison: 184 persistence (PS), linear regression (LR), random forest (RF) and multiple layer percep-185 tron (MLP). PS predicts the present-day sea ice velocity the same as that of the previous-186 day. LR, RF and MLP all regress the present-day sea ice velocity on the five input pre-187 dictors used by CNN by using each time snapshot at each pixel as an independent sam-188 ple for fitting. It is worth noticing that one advantage of CNN over these baseline mod-189 els lies in the fact that instead of processing each pixel individually, CNN processes them 190 in blocks that distill the neighboring connections into useful feature information. 191

As for skill metrics, the Pearson correlation (Corr),

$$corr_{x,y} = \frac{\sum_{i}^{n} (x_{i} - \bar{x})(y_{i} - \bar{y})}{\sqrt{\sum_{i}^{n} (x_{i} - \bar{x})^{2}} \sqrt{\sum_{i}^{n} (y_{i} - \bar{y})^{2}}},$$
(3)

¹⁹³ and Potential Prognostic Predictability (PPP),

$$PPP = 1 - \frac{\sqrt{(y_i - x_i)^2}}{\sqrt{(x_i - \bar{x})^2}},$$
(4)

are used to quantify the model skill. For a given sample size n of a random variable x194 indexed with i, the skill of its prediction y can be quantified as (1) the covariance be-195 tween the prediction and the truth scaled by their individual standard deviations (i.e., 196 Corr) and (2) the the percentage of the true standard deviation explained by y (i.e., PPP). 197 The "-"s in Equation (3) and (4) denote the sample mean. Both Corr and PPP have 198 a range from 0 to 1, with 1 being a perfect skill and 0 no skill. The difference between 199 the metrics is that Corr quantifies how much x and y vary together regardless of their 200 relative magnitudes to each other, while PPP takes the predicted magnitude into account 201 for the accuracy. 202

²⁰³ **3** Feature exploration

To explore the effectiveness of the candidate predictors in a nonlinear CNN model, we train seven CNN models with different input combinations using the historical simulations of CESM2 with the resulting prediction skill evaluations, as shown in Table 1. A comparison across the seven models draws the following conclusions:

• M3 and M5 are better than M4, indicating that in the absence of the sea ice ve-208 locity persistence (u_i, v_i) as predictor, including the sea ice concentration as pre-209 dictor boosts the prediction accuracy. 210 • M3 and M5 are no different in terms of rounded metrics, indicating that there is 211 no added benefit to the prediction from including thickness as predictor. 212 • Both M2 and M3 are better than M4, meaning that combining predictors of ei-213 ther ice conditions of concentration and thickness $(c_0 \text{ and } s_0)$ or velocity persis-214 tence $(u_{i_0} \text{ and } v_{i_0})$ with surface winds $(u_{a_1} \text{ and } v_{a_1})$ enhances the prediction ac-215 curacy. 216 • M2 is slightly better than M3, indicating that the concentration and thickness might 217 be correlated with the ice velocity field. This is further supported by the fact that 218 adding ice conditions to M2 does not significantly improve the prediction as shown 219 in the comparison between M1 and M2 220

For the remainder of our study, we select the best two combinations with the least number of predictor inputs, M2 and M7, for training using the observational and reanalysis data.

224 4 Results

By training M2 and M7 using JRA reanalysis data for surface winds and satellitederived data for ice velocities and conditions, we find that M7 is slightly better (by 5%) than M2 in terms of the prediction metrics. Therefore we adopt M7 as our final CNN model and present its results here.

From the learning curve (Fig.2c,d), we see that the minimisation of the loss function is saturated at around 50 epochs with approximately 50% of the standard deviation unexplained, which corresponds to a correlation of 0.88 and a PPP of 0.5 approximately with respect to the truth. The training and validation curves closely link to each other to show improvement with the number of epochs, confirming that the model does not suffer from overfitting.

The prediction skill are evaluated in two fashions: local-wise and pattern-wise. Localwise evaluation measures how well the model predicts the time series at a given pixel and presents a spatial distribution of the prediction metrics, as shown in Fig.2a,b. Patternwise evaluation, on the other hand, quantifies how well the prediction recovers the spatial pattern for each predicted field, quantified by computing Corr or PPP on each field flattened as a 1D vector and averaged over an interval such as a month (Fig3).

For local-wise evaluation, we evaluate the predicted sea ice velocity in terms of speed 241 and angle. Figure 2a shows the spatial distribution of the averaged difference between 242 the predicted and true speed. We see that most of the prediction errors of the CNN model 243 manifest underestimating the speed by 5 cm/s or so (approximately 10% of the average 244 speed). In general, most of the underestimations occur away from the central Arctic and 245 are greatest (up to 25 cm/s) near the sea ice edge in the Greenland and Barents seas. 246 The significant deviations in the angle prediction, on the other hand, are mostly along 247 the Siberian coast of the Arctic with an angular deviation of up to 100° (i.e. $\cos^{-1}(-0.2) \approx$ 248 100°) from the truth (Fig.2b). On average, the angular error by CNN prediction over 249 the interior of the central Arctic is approximately 32° (i.e. $\cos^{-1}(0.85) \approx 32^{\circ}$). 250

For pattern-wise evaluation, four baseline models (PS, LR, RF and MLP) are used for comparison. As shown in Fig.3, CNN outperforms all the baseline models that make pixel-based predictions, indicating the advantage gained from CNN's nonlinearity and the neighboring pixel connections. Specifically, the fact that CNN outperforms persistence (PS) confirms the influence of surface winds and ice conditions on the sea ice dynamic response.



Figure 2. The top row shows the mean prediction errors in terms of (a) speed and (b) angle averaged over the test data set. Specifically, (a) shows the difference of the predicted speed and the true speed in cm/s with negative values associated with underestimating the speed; (b) shows the cosine of θ , the angle between the predicted velocity and the true velocity, with a $\cos\theta$ closer to 1 associated with a smaller angular deviation from the truth and a negative $\cos\theta$ associated with an angular deviation of greater than 90°. Both (a) and (b) are calculated for a given pixel at each time and temporally averaged over the two testing years. The bottom row shows (c) loss function and (d) Corr of prediction in the training and validation sets as indicated by the legends. The metrics are computed on each flattened 1-D sample frame.

Figure 3. The averaged (a) Corr and (b) PPP for different models over each calendar month for the daily frames of the test set. The averages are computed as pattern-wise evaluation for each frame given a month. The mean and standard deviation (std) of Corr and PPP over the entire test set for each model are presented in the legends in the form of "mean+/-std".

In addition, we also compute the Corr and PPP for the daily prediction of sea ice 257 motion from a "stand-alone" sea ice simulation that is forced by prescribing the JRA55 258 reanalysis. The output is from the CICE5, the 5th version of the Los Alamos Sea Ice Model 259 (Hunke et al., 2015), which is also the sea ice component of CESM2. However, we find 260 a Corr of 0.27 and PPP of -0.47, very low compared to the predictions of the CNN model. 261 This is very likely resulted from the difference in sampling between the CICE5 model 262 and verification data sets, which makes the comparison not really legitimate: the CICE5 263 daily output is a snapshot at an instant in time, while the Polar Pathfinder ice motion 264 is a daily composite composed of mosaic satellite images on a given day (Tschudi et al., 265 2019). In summary, CNN predicts the sea ice motions in response to the surface winds 266 with the highest stable predictive skills compared to all the other pixel-based baseline 267 models. 268

Furthermore, we also developed a CNN model to predict the sea ice motion sim-269 ulated by the CICE5 model on its hourly timestep. In this case, (ua_1, va_1) were the present-270 hour winds from JRA55 (used to force CICE5) and (u_i, v_i) and c_0 were the previous-271 hour output simulated by CICE5. Because the CICE5 model's motion fields lack mea-272 surement error and the timestep was so small, we expected this CNN model trained on 273 hourly CICE5 motion would yield an improved Corr and PPP over the results for pre-274 dicting daily satellite-based motion. However, it turns out that the finer temporal res-275 olution only leads to less than 6% of improvement in the prediction skill. This result sug-276 gests that an upper bound might exist for a data-driven model to learn the relationships 277 that predict sea ice motion. 278

279 5 Conclusions

In this study, we construct a deep-learning model, CNN, to predict how the Arctic sea ice responds to a given surface wind. Specifically, it takes the previous-day sea ice velocity and concentration as well as the present-day surface wind as input features

and predicts the present-day sea ice velocity. The result of prediction is evaluated against

four pixel-fitted baseline models which are PS, LR, RF and MLP. It follows that CNN

²⁸⁵ outperforms all four baseline models in terms of Corr and PPP as the accuracy metrics.

The superior performance of CNN over the pixel-based models suggests the importance of connecting the neighboring local dynamics over the Arctic when modeling the sea ice

rheology. The CNN model is also far superior at predicting the observed motion than

the standard CICE5 model, which suggests there may be promise for combining a machine-

learning method with a physics-based model.

291 Acknowledgments

All data used in this study are publicly available. For climate model output, we use one

historical simulation (r1i1p1f1) of the Community Earth System Model version 2 (CESM2)(Danabasoglu

et al., 2020) in the Coupled Model Intercomparison Project 6 (CMIP6). The 10-m sur-

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