Mesospheric Q2DW interactions with four migrating tides at 53\$^\circ\$N latitude: zonal wavenumber identification through dual-station approaches

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Abstract

Mesospheric winds from two longitudinal sectors at 53 \circ\$N latitude are combined to investigate quasi-two-days (Q2DWs) and their nonlinear interactions with tides. In a summer 2019 case study, we diagnose the zonal wavenumber \$m\$ of spectral peaks at expected frequencies through two dual-station approaches, a phase differencing technique (PDT) on individual spectral peaks and a least-squares procedure on family-batched peaks. Consistent results from the approaches verify the occurrences of Rossby-gravity modes (m=3 and 4 at periods T= 2.1d and 1.7d), and their secondary waves (SWs) generated from interactions with diurnal, semi-diurnal and quatra-diurnal migrating tides. We further extend the PDT to 2012\$\textendash\$2019, illustrating that Q2DWs exhibit significant interannual variability. Composite analysis reveals seasonal and altitude variations of the Rossby-gravity modes and their SWs. The Rossby-gravity modes maximize in local summer, whereas their 16- and 9.6-hr SWs appear more in winter, potentially originating from Q2DW-tide interactions in the opposite hemisphere.

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Key Points:

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• Multi-station approaches are developed to diagnose zonal wavenumber m of near-14 2d, -16hr, -9.6hr and -6.9hr spectral peaks 15 • Diagnosed are Rossby-gravity modes with m=3, and 4 and their SWs from non-16 linear interactions with 24hr, 12hr, 8hr and 6hr migrating tides. 17 • The Rossby-gravity modes maximize in the local summer, whereas their SWs max-18 imize in the local winter. 19 Index Terms: 3389, 3332, 6994, 3334, 4455 20 Key words: quasi-two-days, atmosphere-ionosphere coupling Rossby-gravity wave, 21 mesosphere, cross-wavelet, zonal wavenumber 22

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23 Abstract

Mesospheric winds from two longitudinal sectors at 53°N latitude are combined to 24 investigate quasi-two-days (Q2DWs) and their nonlinear interactions with tides. In a sum-25 mer 2019 case study, we diagnose the zonal wavenumber m of spectral peaks at expected 26 frequencies through two dual-station approaches, a phase differencing technique (PDT) 27 on individual spectral peaks and a least-squares procedure on family-batched peaks. Con-28 sistent results from the approaches verify the occurrences of Rossby-gravity modes (m=329 and 4 at periods T=2.1d and 1.7d), and their secondary waves (SWs) generated from 30 interactions with diurnal, semi-diurnal, ter-diurnal and quatra-diurnal migrating tides. 31 We further extend the PDT to 2012–2019, illustrating that Q2DWs exhibit significant 32 interannual variability. Composite analysis reveals seasonal and altitude variations of the 33 Rossby-gravity modes and their SWs. The Rossby-gravity modes maximize in local sum-34 mer, whereas their 16- and 9.6-hr SWs appear more in winter, potentially originating 35 from Q2DW-tide interactions in the opposite hemisphere. 36

³⁷ Plain Language Summary

The quasi-two-day wave is the strongest and most widely-studied planetary wave 38 occurring in the mesosphere. Existing observational analyses are based on either single-39 satellite or -station approaches, which suffer from temporal and spatial aliasing, respec-40 tively. The current study implements and develops dual-station approaches to investi-41 gate the mesospheric quasi-two-day wave at 53° N latitude, in a case and a statistical study. 42 Our approaches allow diagnosing both the frequency and zonal wavenumber. In the case 43 study, we diagnosed two Rossby-gravity modes and the secondary waves of the nonlin-44 ear interactions between the Rossby-gravity modes and the migrating tides at periods 45 of 24, 12, 8, and 6hr. While the interactions with the 24 and 12hr tides are expected, 46 those with the 8 and 6hr tides are reported for the first time. In the statistical study, 47 we report the seasonality and altitude variation of the Rossby-gravity modes and their 48 most dominant secondary waves. 49

50 1 Introduction

The quasi-two-day wave (Q2DW) is perhaps the largest and most widely-studied 51 planetary wave (PW) in the mesosphere. An excellent historical account of observational, 52 theoretical and modeling studies of the Q2DW is given by Tunbridge et al. (2011). Briefly, 53 the Q2DW was first discovered in meteor radar winds (Muller, 1972; Babadshanov et 54 al., 1973), and was later proposed theoretically to be the atmospheric manifestation of 55 the gravest westward-propagating Rossby-gravity normal mode with zonal wavenumber 56 m = 3 (Salby & Roper, 1980; Salby, 1981; Salby & Callaghan, 2001). An alternative ex-57 planation (Plumb, 1983; Pfister, 1985) was that the Q2DW is the response to instabil-58 ity in the mesospheric summer westward jet, with potential zonal wavenumbers m=2-4. 59 It seems likely that the Q2DW is a near-resonant oscillation excited and/or amplified 60 by an instability (Randel, 1994). Within the satellite era, various space-based observa-61 tions have been explored to study the Q2DW (e.g., Huang et al., 2013; Moudden & Forbes, 62 2014; Pancheva et al., 2018), delineating seasonal-latitudinal and interannual variabil-63 ities of Q2DWs with m = 2, 3, and 4 (as summarized by Tunbridge et al., 2011). Insight-64 ful analyses on the Q2DW have also been performed in the context of a global middle 65 atmosphere data assimilation model (Pancheva et al., 2016; Lieberman et al., 2017). 66 An intriguing aspect of the early radar studies was the discovery (e.g., Manson et 67 al., 1982) that nonlinear interaction between the Q2DW and the semi-diurnal tide yields 68 9.6hr and 16hr secondary waves (SWs) which are sometimes observed in the wind spec-69 tra (e.g., Cevolani & Kingsley, 1992; Beard et al., 1999). The underlying theory was fur-70 ther developed (Teitelbaum et al., 1989; Teitelbaum & Vial, 1991) to include longitude 71 dependence of the waves, tide-tide and other PW-tide interactions. The interactions are 72 regulated by the resonance conditions (e.g., Teitelbaum & Vial, 1991; He et al., 2017). 73 Assume that at longitude λ and time t, the disturbance, induced by a zonal traveling 74 wave with an amplitude $\tilde{\alpha}$ and zonal wavenumber m at frequency f, could be represented 75 as, 76

$$\tilde{\Psi}(\tilde{\alpha}, f, m | \lambda, t) := \tilde{\alpha} \tilde{\psi}(f, m | \lambda, t) := \tilde{\alpha} e^{2\pi i (ft + m\lambda)}$$
(1)

Then, two waves, $\tilde{\alpha}_1 \tilde{\psi}_1(f_1, m_1 | \lambda, t)$ and $\tilde{\alpha}_2 \tilde{\psi}_2(f_2, m_2 | \lambda, t)$, might interact nonlinearly and

generate a SW, $\tilde{\alpha}_{SW}\tilde{\psi}_{SW}(f_{SW}, m_{SW}|\lambda, t)$. The resonance conditions specify,

$$\tilde{\psi}_{SW} = \tilde{\psi}_1 \tilde{\psi}_2, \tilde{\psi}_1 \tilde{\psi}_2^*, \tilde{\psi}_1^* \tilde{\psi}_2, \text{or}, \tilde{\psi}_1^* \tilde{\psi}_2^*$$
(2)

⁷⁹ Hereafter, $\tilde{\bullet}^*$, $\Re(\tilde{\bullet})$, and $\arg\{\tilde{\bullet}\}$ denote the conjugate, real part, and argument (in the ⁸⁰ range from 0 to 2π excluding 2π) of a complex value $\tilde{\bullet}$, respectively. Since $\Re(\tilde{\psi}_1\tilde{\psi}_2^*) =$ ⁸¹ $\Re(\tilde{\psi}_1^*\tilde{\psi}_2), \tilde{\psi}_1\tilde{\psi}_2^*$ and $\tilde{\psi}_1^*\tilde{\psi}_2$ represent the same wave, and so do $\tilde{\psi}_1^*\tilde{\psi}_2^*$ and $\tilde{\psi}_1\tilde{\psi}_2$. There-⁸² fore, there are two independent SWs, $\Re(\tilde{\psi}_1\tilde{\psi}_2)$ and $\Re(\tilde{\psi}_1\tilde{\psi}_2^*)$, termed hereafter as upper ⁸³ and lower sidebands (USB and LSB), respectively.

⁸⁴ Equation 2 implies,

$$f_{SW} = f_1 \pm f_2, \text{and}, m_{SW} = m_1 \pm m_2$$
 (3)

According to Equation 3, the interactions between the Rossby-gravity (R-G) modes (Q2DWs with m=3 and 4, Q2DW3 and Q2DW4) and the diurnal and semi-diurnal migrating tides (DW1 and SW2, at f=1 and 2cpd with m=1 and 2, respectively) could generate up to eight SWs. The SWs populate at three periods, 2d, 16hr, and 9.6hr with different m, as displayed in the f-m depiction in Figure 1a. In addition, these three periods are also populated by SWs of interactions between Q2DWs and various non-migrating tides (Forbes & Moudden, 2012).

The importance of SWs arising from Q2DW-tide interactions to atmosphere-ionosphere 92 (A-I) coupling was recently demonstrated using a whole-atmosphere-ionosphere general 93 circulation model (e.g., Gu et al., 2018). Although the Q2DW may or may not penetrate 94 above the mesopause (Salby & Callaghan, 2001), depending on background wind con-95 ditions, a subset of the SWs can propagate well into the E-region (Palo et al., 1999; Nguyen 96 et al., 2016; Gu et al., 2018) and generate electric fields that carry the Q2DW period-97 icity to the F-region (Gu et al., 2018). The point here is that eastward- (westward-) prop-98 agating waves favor propagation into regions of prevailing westward (eastward) zonal-99 mean zonal winds. The presence of SWs in the wind spectrum is significant since they 100 propagate freely as independent oscillations and contribute measurably to the longitude-101 time structure and complexity of the overall dynamics (Gu et al., 2018; Pedatella & Forbes, 102 2012), and to the ionospheric response (e.g., Gu et al., 2018). A major challenge to date 103 has been our inability to unequivocally determine which 9.6h and 16h waves are present 104 in the atmosphere at any given time. As noted above, while SWs at expected frequen-105 cies have been observed using ground-based observations, these observations provide no 106

-4-

information on m of the SWs, and therefore their capability to propagate to higher al-107 titudes. From the vantage point of single-satellite missions in quasi-sun-synchronous or-108 bits, any Q2DW (at f=0.5cpd with any m) and all its SWs from interactions with all 109 migrating tides will be Doppler-shifted to the same frequency f' = |m-0.5| cpd, and 110 therefore cannot be distinguished from each other. The quantity |m-0.5| is also discussed 111 as the apparent or spaced-based zonal wavenumber. Forbes and Moudden (2012) spec-112 ified the apparent wavenumbers of the Q2DW SWs associated with various migrating 113 and non-migrating tides (described also from a somewhat different perspective in Nguyen 114 et al., 2016). 115

In summary, single-satellite or -station approaches suffer from temporal and spatial aliasing (cf, Appendix A in He, Chau, et al., 2020), respectively. In the present paper, we use two dual-station approaches, one developed in the current work and the other called the phase differencing technique (PDT) developed in He, Chau, Stober, et al. (2018), to identify f and ms of diverse SWs arising from Q2DW-tide interactions. Consistent results from the approaches reveal that Q2DW-tide interactions are more diverse than the expectation according to the existing knowledge.

¹²³ 2 Data analysis

According to Equation 1, we define $\tilde{a}(\lambda) := \tilde{\alpha}e^{2\pi im\lambda}$. Assume that $\tilde{a}(\lambda_1)$ and $\tilde{a}(\lambda_2)$ are the observational estimations of $\tilde{a}(\lambda)$ at two longitudes λ_1 and λ_2 . Then, their cross product is,

$$\tilde{c} := \tilde{a}(\lambda_1)\tilde{a}^*(\lambda_2) = |\tilde{\alpha}|^2 e^{im(\lambda_1 - \lambda_2)} := |\tilde{\alpha}|^2 e^{im\lambda_\Delta}$$
(4)

Equation 4 presents the possibility to estimate m,

$$m = \frac{\arg\left\{\tilde{c}\right\} + 2Z\pi}{\lambda_{\Delta}} = \frac{\arg\left\{\tilde{c}\right\}}{\lambda_{\Delta}} + \frac{2\pi Z}{\lambda_{\Delta}} := m_0 + Z\Delta_m \tag{5}$$

Here, $\arg{\{\tilde{c}\}+2\pi Z}$ represents the phase difference between the longitudes, where $Z \in \mathbb{Z}$ is an integer representing a whole-cycle ambiguity. All possible m values are aliases of m_0 , and Δ_m denotes the Nyquist sampling theorem in the spatial domain. Below, we implement the PDT on mesospheric zonal and meridional winds u and v over two sites, Mohe (M, 122°E, 54°N) and the northern part of Germany (G, 13°E, 53°N). The wind over Mohe is detected by a meteor radar (Yu et al., 2013), whereas the wind over Ger¹³⁴ many is derived with echoes from two individual radars at Juliusruh (13°E, 55°N, Hoff-

mann et al., 2010) and Collm (13°E, 51°N, Jacobi, 2012; Lilienthal & Jacobi, 2015) to

expand the data coverage at this longitude. Hourly winds are estimated on an altitude

grid $h=80.5,81.5,\ldots$ 99.5km through the approach presented in Hocking et al. (2001).

Below we present a case and statistical studies in Sections 3 and 4, respectively.

¹³⁹ 3 A case study

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The zonal wind u and meridional wind v between 1 June and 15 September 2019 140 are used to calculate the Lomb-Scargle spectra, yielding complex amplitudes for the Mohe 141 and German radar systems \tilde{a}_u^M , \tilde{a}_v^M , \tilde{a}_u^G , and \tilde{a}_v^G , at each individual altitude. The am-142 plitudes are used to calculate $\langle |\tilde{a}_u^2| + |\tilde{a}_v^2| \rangle$ averaged between G and M, for which the am-143 plitudes below the significance level $\alpha = 0.05$ are set to zero. The average is displayed 144 in Figure 2a. At h > 96 km in Figure 2a the spectrum is noisy, potentially due to the rel-145 atively low density of the meteor distribution and high plasma density. Therefore, the 146 current work focuses mainly on the altitude below 96km. To implement Equation 4, we 147 calculate the cross product $\tilde{c}_u := \tilde{a}_u^{M*} \tilde{a}_u^G$ and $\tilde{c}_v := \tilde{a}_v^{M*} \tilde{a}_v^G$, and calculate the altitude 148 average $\tilde{c} := \langle \tilde{c}_u + \tilde{c}_v \rangle_{80 < h < 96 \text{km}}$ displayed in Figure 2b. To inspect the details in Fig-149 ure 2b, we develop a representation for the spectrum in Section 3.1, with which we es-150 timate m of spectral peaks using two approaches. The results are summarized in Fig-151 ure 1b and Table 1 and explained below. 152

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3.1 Spectral periodic table

We divide the $\tilde{c}(f)$ spectrum in Figure 2b into 0.5cpd-wide pieces, as denoted by 154 the color bars on the top of Figure 2b, indexed as ' $N\pm$ ', i.e., '0+', '1-', '1+',...,'3+'. Each 155 of the pieces is zoomed into one row in Figure 2c. Note that in Figure 2c the x-axis rep-156 resents $|\delta f| := |f - \lfloor f \rceil|$, namely, the distance between f and its nearest integer $\lfloor f \rceil$. We 157 call the representation of Figure 2c as the spectral periodic table. According to Equa-158 tion 3, SWs of all potential interactions between a wave at given frequency $f_2 > 0$ and 159 all tides at $f_1=1$ cpd, 2cpd,... will share the same $|\delta f| = |f_2 - \lfloor f_2 \rceil|$ and therefore are lo-160 cated at the same column in the spectral periodic table. 161

3.2 The tidal signatures

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At the top of Figure 2c, there are short horizontal bars in black, blue and red, corresponding to the three maximum peaks of $P(|\delta f|) := \prod_{N\pm} |\tilde{c}(|\delta f|)|$. Each of these bars represents a 0.015cpd-wide δf interval, narrower than the spectral frequency resolution $2\sigma_f \sim 2/T_w = 2/106d = 0.019$ cpd. Here, $T_w = 106d$ is the window width. For example, the black bar corresponds to $\delta f = 0 \pm 0.0075$ cpd.

At $\delta f = 0 \pm 0.0075$ cpd in each row of Figure 2c, we search for a maximum spectral 168 peak, displayed as the vertical black lines in Figures 2b-c and specified in black in Ta-169 ble 1. The \tilde{c} values of these maximum peaks are input into Equation 5 to estimate m, 170 resulting in m^{PDT} listed in black in Table 1. According to Equation 5, all solutions m^{PDT} 171 comprise aliases denoted by $Z\Delta_m$, where $\Delta_m=3.31$ is determined by our radar separa-172 tion. At f = 1.0 cpd and when Z=0, the PDT estimation is $m^{PDT}=0.95+3.31Z=0.95$ 173 when Z=0. At $f = 2.0, 3.0, \text{ and } 4.0 \text{ cpd}, m^{PDT} = 1.09, 3.46, \text{ and } 3.56 \text{ when } Z = 0,1, \text{ and}$ 174 1, respectively. The nearest integers of m^{PDT} are $\lfloor m^{PDT} \rfloor = 1, 2, 3, \text{ and } 4$, as listed in 175 the column m^{f} in Table 1. Therefore, we explain the corresponding spectral peaks as 176 migrating solar tides, DW1 SW2, TW3, and QW4 (diurnal, and semi-, ter-, and quatra-177 diurnal westward traveling tides with m=1, 2, 3, and 4). 178

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3.3 The Q2DW4 family

Similarly, at $|\delta f| = 0.4275 \pm 0.0075$ cpd, denoted by the blue bar on the top of Fig-180 ure 2c, we search for the maximum peak in each row of Figure 2c, above a threshold value 181 $\sqrt{|\tilde{c}|} > 1.1 \text{m/s}$ that is the QW4 tidal amplitude from the previous subsection. Six max-182 ima are searched, displayed as the vertical blue lines in Figures 2b-c, whose frequencies 183 and \tilde{c} values (denoted as $\tilde{c}_{N\pm}$) are specified in blue in Table 1. $\tilde{c}_{N\pm}$ are used to estimate 184 the corresponding wavenumber $m_{N\pm}$ through PDT according to Equation 5, resulting 185 in $m_{N\pm}^{PDT}$ specified in Table 1. To deal with the ambiguities represented by Z in $m_{N\pm}^{PDT}$, 186 below we present a family-batched estimation of $m_{N\pm}$. According to Equation 4, 187

$$\tilde{c}_{N\pm}/|\tilde{c}_{N\pm}| = e^{im_{N\pm}\lambda_{\Delta}} \tag{6}$$

Here, $m_{N\pm}$ might not be completely independent but generate each other by interacting nonlinearly with the tides identified in Section 3.2, and therefore are regulated by the resonance conditions in Equation 3. In the most extreme case, only one wave is in-

- dependent and all the others are associated SWs. In this single-independent-wave case,
- only one wavenumber is independent and $m_{N\pm}$ can be represented as:

$$m_{N\pm} = N \pm m_{0\pm}$$
 (7)

¹⁹³ Substituting Equation 7 into Equation 6, yielding,

$$\tilde{c}_{N\pm}/|\tilde{c}_{N\pm}| = e^{i(N\pm m_{0\pm})\lambda_{\Delta}} \Rightarrow e^{-iN\lambda_{\Delta}}\tilde{c}_{N\pm}/|\tilde{c}_{N\pm}| = e^{\pm im_{0\pm}\lambda_{\Delta}}$$
(8)

which is an overdetermined system comprised of only one unknown m_{0+} and six equations. We first estimate the least-squares solution of $e^{im_{0+}}$ denoted as \tilde{e}_{LS} , and then estimate m_{0+} through an optimization $m_{0+}^f = \underset{\hat{m}}{\operatorname{argmax}} \Re(\tilde{e}_{LS}^* e^{i\hat{m}})$, subject to: $\hat{m} \in [-5, -3, ..., 5]$. Here, we use the superscript f to distinguish the family-batched estimations $m_{N\pm}^f$ from the PDT estimations $m_{N\pm}^{PDT}$.

Solving the optimization yields the estimation $m_{0+}^f = -3$. Other $m_{N\pm}^f$ are calculated according to Equation 7, listed in blue in the column ' m^f ' in Table 1. At each of the six estimations, $m_{N\pm}^f$ is compared with $m_{N\pm}^{PDT}$. The comparison results are listed in the column ' $m^f \approx m^{PDT}$ ' in Table 1, in which checkmarks ' \checkmark ' denote consistencies between $m_{N\pm}^f$ and $m_{N\pm}^{PDT}$ whereas the cross mark ' \times ' denotes an inconsistency. The consistency is claimed if $\exists Z: \lfloor m_{N\pm}^{PDT}(Z) \rceil = m_{N\pm}^f$ which equals to,

$$\delta m_{N\pm} := \min |m_{N\pm}^{PDT} - m_{N\pm}^{f}| < 0.5 \tag{9}$$

Among the six estimations, five estimations exhibit consistency, among which the one at f=0.58cpd could be explained as the R-G mode Q2DW4 (e.g., Salby & Callaghan, 2007). Accordingly, we explain the remaining four as the SWs of tide±Q2DW4 interactions as specified in the last column in Table 1. (Here, '+' and '-' represent the USBand LSB-generating interactions, respectively.) The blue symbols in Figure 1b present the five consistent estimations in the f-m plane.

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3.4 The Q2DW3 family

Further, we implement the above peak identification, least-squares estimation, and optimization, at $|\delta f| = 0.4775 \pm 0.0075$ cpd denoted by the red bar on the top of Figure 2c. The results are displayed as the vertical red lines in Figures 2b-c and specified in red

in Table 1. The family-batched estimation m^{f} and the individual PDT estimation m^{PDT} 215 are consistent for all six peaks, denoted as the red symbols in Figure 1b. Among the six 216 peaks, the one at f=0.485 cpd is explained as the Q2DW R-G mode with $m^{f}=3$ (Q2DW3, 217 e.g., Salby & Roper, 1980). Therefore, the remaining five peaks are explained in terms 218 of migrating tides \pm Q2DW3 interactions as specified in the last column in Table 1. 219

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To our knowledge, the interactions of the R-G modes with TW3 and QW4 are reported here for the first time. 221

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4 2012–2019 composite analysis

The previous section implements the PDT in a case study through Lomb-Scargle 223 spectral analysis. In the current section, we replace the Lomb-Scargle spectral analysis 224 with cross wavelet (CWL) analysis (following, e.g., He, Yamazaki, et al., 2020; He, Chau, 225 Hall, et al., 2018), to implement PDT to a statistical analysis using the data between 226 January 2012 and December 2019. We first calculate the Gabor wavelet (Torrence & Compo, 227 (1998) of the u and v components over Germany and Mohe at each altitude h, resulting 228 in amplitudes $\tilde{W}_{(f,t)}^{G,u,h}$, $\tilde{W}_{(f,t)}^{G,v,h}$, $\tilde{W}_{(f,t)}^{M,u,h}$, and $\tilde{W}_{(f,t)}^{M,v,h}$. These amplitudes are the obser-229 vational estimations of $\tilde{a}(\lambda)$ in Equation 4, and are functions of t and f. Then, the cross 230 product $\tilde{C} := \tilde{W}_{(f,t)}^{G*} \tilde{W}_{(f,t)}^M$, namely, the CWL spectrum, is the observational estima-231 tion of \tilde{c} in Equation 4. We sum the CWL spectra of the u and v components, yielding 232 $\tilde{C}_{u+v} := \tilde{C}_u + \tilde{C}_v$. Below, in Sections 4.1 and 4.2 we inspect the f-t depiction of altitude-233 averaged \tilde{C}_{u+v} and h-t depiction of \tilde{C}_{u+v} at discrete frequencies, respectively. 234

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4.1 Altitude-averaged CWL spectrum

Figure 3a displays $\langle \tilde{C}_{u+v} \rangle_{80 < h < 96 \text{km}}$, in which the darkness represents the magni-236 tude while the color hue denotes its phase. The magnitude exhibits an annual variation, 237 maximizing in local summer (July–September) with dim spectral peaks in local winter. 238 The $\langle \hat{C}_{u+v} \rangle_h$ in Figure 3a is composited as a function of the month, displayed in 239 Figure 3b. As indicated by the black dashed line in Figure 3b, the composited spectrum 240 maximizes largely earlier at a higher frequency. Below, we mainly focus on the frequency 241 range Δf_{Q2DW} :=[0.40 0.60cpd] where the strongest spectral peaks occur. The strongest 242 two peaks are the red one at f=0.57-0.60 cpd (T=40.0-42.1 hr) and the blue at f=0.48-0.50 cpd 243 (T=48.0-50.0hr), both of which appear in July. The redness and blueness are associated 244 with m=4 and 3, respectively. These two peaks can be explained as R-G modes Q2DW4 245

and Q2DW3, because both their f and m are consistent with the theoretical expectations [T,m] = [1.7d, 4] and [2.1d, 3] (cf, Salby & Callaghan, 2001). Except for the red and blue peaks, Figure 3b is dominated mainly by magenta associated with m=-3 (Q2DE3), including a peak at f=0.52-0.55cpd in July and the smeared region within f=0.35-0.50cpd in August. The smeared Q2DE3 signature could be explained as the LSB of the interaction between DW1 and Q2DW4 at f=0.57-0.60cpd, whereas the Q2DE3 peak at f=0.52-0.55cpd might not be excited in-situ as no Q2DW4 signature appears at the required frequency.

The above four Q2DW signatures exhibit significant interannual variabilities, as 253 revealed in Figure 3a. The Q2DW3 signature occurred in summers of 2012, 2013, 2016, 254 and 2019; the Q2DW4 occurred in 2012, 2013, 2017, and 2019; the Q2DE3 signature at 255 f=0.35-0.50 cpd occurred weakly in 2012, 2014, 2015, 2016, and 2017; and the Q2DE3 256 signature at f=0.50-0.55 cpd occurred in 2012, 2014, 2015, 2016, and 2017. Besides, also 257 between f=0.45cpd and 0.55cpd occurred the green peak in 2012, 2014, 2018 and 2019, 258 which is not visible in the composited spectrum. The greenness suggests m=2, (e.g., Tun-259 bridge et al., 2011). 260

Figures 3c-d present the same plots as Figure 3b but for periods near 16hr and 9hr, 261 namely, frequency ranges of the SWs. Given that in Figures 3b Q2DWs occur mainly 262 within the $\Delta f_{Q2DW} = [0.40 \ 0.60 \text{cpd}]$, we focus hereafter on $\Delta f_{Q2DW} + 1 \text{cpd}$ and $\Delta f_{Q2DW} + 2 \text{cpd}$. 263 In these ranges, the spectra maximize annually during December–January, when the south-264 ern hemispheric Q2DW maximizes (e.g., Salby & Callaghan, 2001). The SWs associated 265 with the southern hemispheric Q2DWs-tides interactions might extend to northern hemi-266 spheric mid-latitudes. In Figure 3, SWs are weaker in the summer than that in the win-267 ter, possibly due to the preference of the southern hemispheric Q2DW towards periods 268 between 48–52hr and m=3 (e.g., Tunbridge et al., 2011), whereas the northern hemispheric 269 Q2DW tends to spread out in period and zonal wavenumber. This suggests that the south-270 ern hemispheric SWs would be more repeatable from year to year. 271

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4.2 Altitude variation

Figure 4 displays the composited spectra of \tilde{C}_{u+v} as a function of the month and altitude at the frequencies illustrated by the horizontal gray dashed lines in Figures 3b–d. Figures 4a and 4b capture the Q2DW3 and Q2DW4, respectively. While the Q2DW3 occurs mainly above 90km, the Q2DW4 peak extends to a lower altitude. Such a differ-

-10-

ence could be observed in the geopotential amplitudes at southern hemispheric mid-latitude
in a theoretical computation (Figures 6 vs. 10 in Salby & Callaghan, 2001). The computation suggests that the Q2DW4 is strongly distorted by the mean wind and does not
penetrate much across the mesopause whereas the Q2DW3 extends upward into the thermosphere.

In Figure 1, there are four near-16hr SWs (m=-2, -1, 4, and 5), and four near-9hr SWs (m=-1, 0, 5, and 6). Among the potential near-16hr SWs, the m=4 in Figure 4c is the strongest one. In the case of near-9hr SWs, the m=0 appears in Figure 4f while m=-1 and 6 occur in Figure 4e. Both the near-16hr and -9hr SWs maximize in the local winter. In August–September in Figures 4c–d, near-16hr SWs occur only at high altitudes.

288 5 Summary

Mesospheric winds, from two longitudinal sectors at 53°N, are combined to inves-289 tigate the Q2DW in a summer 2019 case study and an 8-year statistical study. In the 290 case study, PDT is first implemented to diagnose zonal wavenumber m of the Lomb-Scargle 291 cross-spectral peaks at T = 24, 12, 8, and 6hr, suggesting migrating tides, DW1, SW2, 292 TW3, and QW4. Then, we arrange the cross-spectrum into a periodic table to batch spec-293 tral peaks of each expected Q2DW and all its potential SWs into one family. For each 294 family, we estimate m through two approaches, the PDT on individual peaks and a family-295 batched estimation. Consistent estimations of the approaches suggest two families, namely 296 Q2DW3 with its five SWs, and Q2DW4 with its four SWs. These SWs entail tidal waves 297 of DW1, SW2, TW3, and QW4, among which TW3 and QW4 are reported for the first 298 time as the parent waves. 299

In the statistical study, cross-wavelet analysis is implemented to the wind obser-300 vations between 2012 and 2019. The cross-wavelet spectra are composited into frequency-301 month (f-t) depictions in three period ranges near-2d, -16hr and -9.6hr and altitude-month 302 (h-t) depictions at discrete frequencies. The near-2d spectrum exhibits significant inter-303 annual variabilities and seasonal variation, which maximizes in July at f=0.57-0.60 cpd, 304 0.48-0.50 cpd, and 0.50-0.55 cpd (T=40.0-42.1 hr, 48.0-50.0 hr, and 48.0-43.6 hr) associ-305 ated with m=3, 4, and -3, first two of which are R-G modes. The near-16hr and -9.6hr 306 spectra maximize in local winter when the near-2d spectra are weak locally and max-307

- imize in the southern hemisphere, suggesting that SWs of Q2DW-tide interactions in the
- ³⁰⁹ southern hemisphere extend into northern hemispheric mid-latitudes. The Q2DWs and
- their SWs exhibit various altitude distributions: (1) in summer, the Q2DW4 peak ex-
- tends to low altitudes than Q2DW3, and (2) the summer near-16hr SWs are distributed
- at a higher altitude than those in the winter. Given the potential relevance to global atmosphere-
- ionosphere coupling, the mechanisms underlying the height and seasonal-latitudinal be-
- haviors of SWs revealed here warrant study by global models.

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Figure 1: (a) Distribution of expected Q2DWs and associated secondary waves from nonlinear interactions with tides in f-m plane, (b) waves detected in 2019 summer at 53°N consistently through two approaches (cf, Section 3).



Diagnoses from the two approaches are consistent at some peaks but inconsistent at the others, denoted by the solid and dashed lines, respectively. The $\delta f := |f - \lfloor f \rceil|$, namely, the distance between f and its nearest integer $\lfloor f \rceil$. In (b,c), the vertical blue, red and black lines indicate three families of specpanel and labeled as (0+2, -1)... Each of the pieces is zoomed in and displayed in (c) as one row labeled on the very left. In (c) the x-axis represents spectrum $\tilde{c} := \langle \tilde{a}_n^M * \tilde{a}_n^G + \tilde{a}_n^M * \tilde{a}_n^G \rangle_h$, and (c) a periodic table representation of (b). (b) is divided into pieces denoted by the color bars on the top of the tral peaks. Peaks from the same family share the same $|\delta f|$ value, and are used to diagnose the m through two approaches and specified in Table 1. Figure 2: (a) Lomb-Scargle spectra $\langle |\tilde{a}_v^2| \rangle_{M,G}$, averaged between the two longitudes, Mohe and Germany, (b) altitude average of the crosswhite horizontal white lines in (a) indicate the spectrum used for calculating (b). See Section 3 for details.

$N\pm$	f (cpd)	$T \ (hr)$	$\sqrt{ ilde{c} }\ ({ m m/s})$	m^{PDT}	m^f	$\frac{\delta m}{({\rm Eq.}\ 9)}$	$\begin{array}{c} m^{PDT}\approx m^{f} \\ (\delta m < 0.5) \end{array}$	potential wave*
$1\pm$	1.000	24.0	9.3	0.95 + 3.31Z	1	0.05	\checkmark	24hr migrating tide (DW1)
$2\pm$	2.000	12.0	20.6	1.89 + 3.31Z	2	0.11	\checkmark	12hr migrating tide (SW2)
$3\pm$	2.995	8.0	1.6	0.15 + 3.31Z	3	0.46	\checkmark	8hr migrating tide (TW3)
$4\pm$	4.000	6.0	1.1	0.25 + 3.31Z	4	0.44	\checkmark	6hr migrating tide (QW4)
0+	0.425	56.5	4.3	0.54 + 3.31Z	-3	0.23	\checkmark	DW1–Q2DW4
1–	0.580	41.4	5.5	1.14 + 3.31Z	4	0.45	\checkmark	Q2DW4
2–	1.570	15.3	1.7	2.65 + 3.31Z	5	0.96	×	DW1+Q2DW4
2+	2.430	9.9	1.5	2.57 + 3.31Z	-1	0.26	\checkmark	TW3–Q2DW4
3–	2.565	9.4	2.1	2.74 + 3.31Z	6	0.05	\checkmark	SW2+Q2DW4
3+	3.420	7.0	1.1	0.25 + 3.31Z	0	0.25	\checkmark	QW4-Q2DW4
0+	0.485	49.5	4.4	2.72 + 3.31Z	3	0.28	\checkmark	Q2DW3
1–	0.520	46.2	2.0	0.93 + 3.31Z	-2	0.38	\checkmark	$DW1-Q2DW3^{**}$
1 +	1.475	16.3	1.3	1.00 + 3.31Z	4	0.31	\checkmark	$DW1+Q2DW3^{***}$
2-	1.520	15.8	2.8	2.55 + 3.31Z	-1	0.25	\checkmark	SW2–Q2DW3
2+	2.475	9.7	1.5	1.57 + 3.31Z	5	0.12	\checkmark	SW2+Q2DW3
3–	2.530	9.5	1.9	0.34 + 3.31Z	0	0.34	\checkmark	TW3-Q2DW3

Table 1: Wavenumber estimation through two approaches: m^{PDT} vs. m^f

*In the last column, the symbols '-' and '+' represent the LSB- and USB-generating

interactions between the waves at the flanks of the symbols, respectively.

 $\ast\ast$ also known as Q2DE2

*** also explained as SW2–Q2DE2 interaction



Figure 3: (a) 2012–2019 composited $\langle \tilde{C}_u + \tilde{C}_v \rangle_h$, namely, the sum of the cross-wavelet spectra of u and v wind components between the two longitudes, averaged in the altitude range $h \in [80,96\text{km}]$ and frequency range $f \in \Delta f := [0.3 \ 0.7\text{cpd}]$. (b) the composited spectrum of (a). (c,d) same plots as (b) but in the frequency ranges $\Delta f + 1$ and $\Delta f + 2$, respectively. (a) and (b) share the same color-code map, in which the dotted, solid and dashed black isolines denote amplitudes at $\sqrt{|\tilde{C}|} = 3$, 4, and 5m/s.



Figure 4: Multi-year composited $\tilde{C}_u + \tilde{C}_v$, as a function of month and altitude at six periods indicated by the horizontal dashed lines in Figures 3b–d.