Mathematics Clouding Climate Sensitivity

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Equilibrium climate sensitivity (ECS) is the global temperature change expected after doubling atmospheric CO2 concentration. This Commentary reviews how Sherwood et al. (2020) used Bayesian statistics and evidence from climate-process physics, historical observations, and earlier proxies to reduce the likely range of ECS from 1.5-4.5 K to 2.6-4.1 K. They may have overestimated ECS by adding non-equilibrium short-term adjustments to the radiative forcing of greenhouse gases and by underestimating the effect of solar irradiance and aerosols. Two alternative periods during the Holocene show that forcing by agents other than CO2 was significant and requires further research.

Mathematics Clouding Climate Sensitivity

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Key Points:

- A recent study used Bayesian statistics to reduce the likely range of equilibrium climate sensitivity by a third.
- This Commentary summarizes and simplifies that mathematical study and analyzes its assumptions and conclusions.
- Further research is recommended on the effective forcing of CO_2 and other agents, especially solar activity and clouds.

Abstract

Equilibrium climate sensitivity (ECS) is the global temperature change expected after doubling atmospheric CO_2 concentration. This Commentary reviews how Sherwood et al. (2020) used Bayesian statistics and evidence from climate-process physics, historical observations, and earlier proxies to reduce the likely range of ECS from 1.5—4.5 K to 2.6—4.1 K. They may have overestimated ECS by adding non-equilibrium short-term adjustments to the radiative forcing of greenhouse gases and by underestimating the effect of solar irradiance and aerosols. Two alternative periods during the Holocene show that forcing by agents other than CO_2 was significant and requires further research.

Plain Language Summary

Sherwood et al. (2020) used complex statistical procedures, instrumental measurements, and pre-historic estimates of climate conditions to refine a common measure of the effect of CO_2 on global temperature, the equilibrium climate sensitivity. Their review is likely to influence future climate policies. This Commentary presents a simplified summary of their procedures, and comments on some assumptions and data that would benefit from further research. It provides additional information that highlights the importance of solar activity and clouds since the last major ice age.

1 Introduction

Greenhouse gases (GHGs) such as water vapour and CO_2 warm the Earth's atmosphere. There are concerns that increasing concentration of CO_2 may cause unacceptably high global temperatures. The equilibrium climate sensitivity (ECS) provides a benchmark prediction of the long-term change in near-surface air temperature after abruptly doubling the concentration of CO_2 from the pre-industrial level (approximately 280 ppm). The confidence limits of ECS have not improved since the National Research Council (NRC, 1979) proposed the likely range was 1.5—4.5 K. The World Climate Research Program commissioned Sherwood et al. (2020, henceforth SW20) to examine ways to reduce this uncertainty before the 6th Assessment Report of the Intergovernmental Panel on Climate Change. SW20 used Bayesian statistics to combine estimates of ECS developed from three independent lines of evidence: theoretical knowledge of the climate process; historic data; and paleoclimate data. Their review is likely to have a major influence on future climate change policies. This Commentary summarizes their arguments and highlights some remaining challenges.

2 Materials and Methods

Early estimates of ECS typically assumed the surface temperature and ice cover remained unchanged to simplify calculations (NRC, 1979). Recently, climate models have estimated 'effective climate sensitivity', S, defined as half the surface temperature change required to restore the energy balance at the top of the atmosphere (TOA) after abruptly quadrupling CO_2, by extrapolating the temperature – energy-imbalance regression established during the first 150 years to a later energy balance. SW20 estimated that ECS was 6% larger than S from a study of 27 simulations involving abrupt changes of 2x to 16xCO_2 (Rugenstein et al., 2020) that showed the median long-term equilibrium temperature change (approximating ECS) for 4xCO_2 simulations from 15 models was 17% larger than

150-year predictions (approximating S), but the 4xCO_2 equilibrium responses were 3 to 18% larger than twice the equilibrium responses to 2xCO_2 (true ECS) from five models (and 94% larger in another).

SW20 chose to estimate S (using equations shown in Supplementary Information Table S1) because its timeframe was relevant to predictions of warming this century. They assumed an energy balance at the TOA between the radiative response, ΔR, from a warmer climate system (characterized by the forced change in surface temperature, ΔT) and the change in the radiative forcing, ΔF, that caused the warming. SW20 also assumed that ΔR was a linear function of ΔT, expressed as $\lambda \times \Delta T$, where λ was the climate feedback parameter. That assumption ignored non-linear relationships between temperature and several feedback components, especially Planck radiation which depends on the 4^{th} power of T (Petty, 2006) and water vapour which has an exponential relationship. They inflated ΔF by using effective radiative forcing (ERF) which included rapid adjustments that may not be relevant at equilibrium but they did not include adjustments for slower changes to surface temperature and sea ice (IPCC, 2013). Their estimate of λ assumed only six feedback components were relevant and were entirely independent. The energy balance equation assumed the response from each component did not depend on the forcing source. However, SW20 did include error terms to address feedback dependence on global temperature and regional changes in warming patterns.

SW20 used Bayesian statistics to produce improved (posterior) estimates of the probability distribution function (PDF) of S by multiplying the initial PDF (prior) by the normalized likelihood calculated from new information, E, generated by Monte Carlo sampling from estimated PDFs. This process depends on independence of the prior and E (Annan & Hargreaves, 2020; Gelman & Robert, 2013) but that was not always confirmed. To demonstrate transparency, assumptions were identified more than 100 times. Most were justified, but some were supported by statements such as 'this "stratospheric adjustment" is well-understood'. They noted the unavoidable risk of bias in selecting data but argued that their expert knowledge minimized this risk.

3 Three lines of evidence

The first line of evidence that SW20 used to develop S was the current knowledge of the climate process. That required calculating the radiative forcing from CO_2 doubling, ΔF_{2xCO2}, and λ. They determined ΔF_{2xCO2} was 4 ± 0.3 Wm^{-2} by adding 'rapid' adjustments for changes in the stratosphere (0.9 Wm^{-2}), troposphere (0 Wm^{-2}), and surface albedo (0.2 Wm^{-2}) to the instantaneous radiative forcing (2.9 Wm^{-2}). The rapid stratospheric adjustment accounted for reduced radiation lost to space from the stratosphere because of initial cooling caused by doubling CO_2 concentration and the positive stratospheric temperature gradient. Although a similar tropospheric adjustment due to a negative gradient would have subtracted 0.7 Wm^{-2}, this was completely offset by forcing attributed to water vapour and clouds (SW20, Figure 3).

The feedback parameter, λ, was calculated as a linear sum of six components (the Planck effect, tropospheric water vapor and lapse rate, various cloud effects, surface albedo, stratospheric water vapour, and atmospheric composition). SW20 discussed each component in detail, drawing attention to the complexity and uncertainties associated with cloud feedback. Most of the PDFs for the feedback terms (SW20, Table1) depended to some extent on global climate models (GCMs). The resulting normally distributed PDF for λ was -1.30 $\pm$ 0.44 Wm^{-2}. This uncertainty may not adequately cover unidentified co-dependence. SW20 concluded that major errors or omissions were unlikely because interannual variability in globally averaged TOA net radiation implied that λ was in this range. However, they noted that emergent constraints developed from the current climate system generally produced smaller values of λ. They pointed out that the two methods were not entirely independent and there was no procedure to combine the various constraints to improve their preferred PDF which placed S_{proc} in the range 2.3—4.6 Wm^{-2} with a median value of 3.1 Wm^{-2}.

The second line of evidence compared historical observations from a base period (1861-1889) with the present (2006-2018). SW20 selected this base period because it had low CO_2, little volcanism, and enough reliable measurements. They based changes in global temperature (ΔT = 1.03 $\pm$ 0.14 K) on measurements of surface air temperatures (SAT) and adjusted sea surface temperatures (SST) from GCMs. The energy imbalance (ΔN = 0.6 $\pm$ 0.3 Wm^{-2}) was from model estimates of ocean heating and was assumed to be independent of ΔF. The median value of ΔF (1.83 Wm^{-2}) was calculated by Monte Carlo sampling from ERFs for GHGs, aerosols, ozone, land use albedo, volcanic activity, stratospheric water vapor, contrails, black carbon, and solar irradiance (Supplementary Information, Table S2). SW20 used 'expert judgement' to select -0.08 Wm^{-2} (median -1.179 Wm^{-2}) as the preferred value for aerosol forcing from a wide range of published estimates. The change in solar irradiance obtained from Lean (2018) was an order of magnitude larger than their estimate (0.017 Wm^{-2}). The PDF for S_{hist} had a most likely value of 2.5 K and a median of 3.1 K for a non-Bayesian analysis or a median of 4.3 K for a Bayesian analysis. The larger Bayesian estimate resulted from selection of a uniform prior (Annan & Hargreaves, 2020; Zwickl & Holder; 2004).

SW20 identified several unresolved issues. They suggested that S_{hist} may underestimate S because radiative feedbacks became less negative as equilibrium was approached due to warming pattern effects. They applied sensitivity tests to assess the impact of using different aerosol forcing estimates, a different base period (1850-1900), and unadjusted SST. SW20 emphasized that neither the global energy budget approach nor fitted dynamical models provided a purely observational constraint on S_{hist}. Furthermore, the models did not provide warming patterns that resembled observations. They noted that atmosphere-only model simulations produced values of S_{hist} in the range 1.6–2.1 K, which agreed well with global energy budget constraints, but were considerably lower than values from $4xCO_2$ simulations using the same models. SW20 accounted for the pattern effect in their maximum likelihood prediction of 3.8 Wm^{-2}. They considered that the observed 1 K of historical warming provided strong evidence of S >1.5 K, but this may have underestimated the contribution from solar activity. Significant uncertainty remained regarding surface air warming and ocean heat uptake.

The third line of evidence came from paleoclimate records from the Last Glacial Maximum (LGM; 19 to 23 ka) and the mid-Pliocene Warm Period (mPWP; 3.3 to 3.0 Ma). These periods were selected because they were respectively several degrees cooler and warmer than the present and their temperatures were relatively stable over periods measured in millennia. Although SW20 considered the stability indicated equilibrium conditions, there were large centennial to millennial temperature variations during both periods (IPCC, 2013) in addition to longer variations that correlated with orbital parameters (Climate Data, 2020). SW20 also estimated S from the Paleocene-Eocene Thermal Maximum (PETM, about 56 Ma) but did not use this in their final calculations.

To estimate S_{LGM}, SW20 assumed climate change was reversible and calculated back in time from their pre-industrial base period to the LGM resulting in negative numbers for the temperature increase, ΔT (-5 ± 2 K) and ΔF (-8.43 ± 2 Wm^{-2}). ΔF was the sum of ERFs for CO_2 (that actually increased from 190 to 284 ppm), CH_4 (375 to 808 ppb), N_2O (200 to 273 ppb), ice sheets (including sea level change), vegetation, and dust that were -2.27, -0.57, -0.28, -3.2, -1.1, and -1 Wm^{-2} respectively. The ERFs for the GHGs were the stratosphere-adjusted radiative forcings from Etminan et al. (2016, Table 1) increased by 5% to account for tropospheric and albedo adjustments. SW20 increased the ERF for CH_4 by an additional 45% to account for effects on stratospheric water vapour and ozone. They followed Hegerl et al. (2007) in setting the radiative forcing from shrinking ice sheets at -3.2 ± 0.7 Wm^{-2}, despite this being almost incompatible with more recent estimates (e.g. -3.6 to -5.2 Wm^{-2} by Braconnot & Kageyama, 2015). Using the energy balance equation, SW20 found climate sensitivity was 2.4 K. They regarded this as a quasi-equilibrium estimate, 'ECS', and reduced it by almost 6%. They applied another correction for non-linear feedback responses. Their Bayesian analysis proposed that 2.5 K was the most likely value of S_{LGM}. This would be larger if the temperature change was >5 K. It could be smaller if ice sheet and sea level forcing was larger, solar irradiance (independent of orbital effects) change was approximately 1 Wm^{-2}, or reverse causation resulted from drawdown of GHGs in the cooler oceans.

SW20 used a similar time-reversed process to estimate S_{mPWP}. They set ΔT at 3 ± 1 K noting a high level of uncertainty. They stated that CO_2 had dominated forcing in the mPWP without providing any justification. They calculated ΔF to be 2.2 ± 0.6 Wm^{-2} based on CO_2 changing from 375 to 284 ppm and adding 40% for other GHGs. They added a further 50% to cover all other unmeasured forcing agents, possibly including changes in ice sheets, vegetation, orbital cycles, and tectonic events. This proportion was only one third the additional forcing calculated for LGM which was dominated by ice sheet changes. They attributed none of the apparent warming to the collisional tectonism that closed the Isthmus of Panama (O'Dea et al., 2016) and redirected a warm East-West equatorial current northward to form the Gulf Stream (Ruddiman, 2014). SW20 also ignored possible variations in solar irradiance. They concluded that the maximum likelihood estimate of S_{mPWP} was 3.2 K. They discussed how it could be larger if CO_2 concentration was lower or temperature change was higher but did not consider that it could have been smaller if CO_2 contributed a smaller fraction of the forcing.

Their estimate of S_{PETM} was based on less reliable knowledge of conditions from further in the past. The temperature rise of 5 $\pm$ 2 K was based on SST proxies covering the subsequent fall in temperature to the Early Eocene. SW20 assumed the rise was caused by a rapid increase in CO_2 concentration from 900 to 2400 ppm inferred from a spike in the $\delta^{13}C$ ratio. They applied the same correction (40%) for other GHGs that they used for mPMP. This significantly overestimated the effect of CO_2 if the ratio changed because of a sudden influx of CH_4 (that has >20 times the global warming potential of CO_2). Despite acknowledging that paleogeography, global temperatures, and vegetation were probably different, no specific forcings were included. A climate state correction was added to the variance of the feedback parameter which was assumed to be the same as the present. After making a quasi-equilibrium adjustment, their maximum likelihood estimate of S_{PETM} was 2 K.

Combining the three lines of evidence through Bayesian statistics, SW20 estimated the baseline probability distribution for S had a median value of 3.1 K and a 66% range from 2.6 to 3.9 K. They assumed a uniform prior on S but noted that was only justifiable if feedback elements were small or negatively correlated. Annan & Hargreaves (2020) analyzed this process and showed that choosing a uniform prior on S would significantly increase the median and upper bound of the estimated range of S. SW20 identified other uncertainties relating to the effects of surface warming, lags in ocean warming, clouds, aerosols, and missing or varying feedback mechanisms.

The consistency of the results from three lines of evidence could have resulted from co-dependence or choice of similar climatic events. SW20 examined potential co-dependence in detail. They justified the use of GCMs in each line of evidence because they were used for different purposes, i.e.: to constrain feedback for the process line; to assess the historical pattern effect; and to estimate forcing for paleo-reconstructions. They also claimed co-dependence would be mitigated by the different relationships to CO2 concentration and the multiple lines of evidence used to establish key feedback mechanisms. A similar argument was applied to the radiative transfer functions and cloud physics, whereas aerosol errors were thought to be buffered. Their statistical tests showed that their treatment of the warming pattern effects had not produced significant co-dependence in the historical and process lines.

4 Two alternative events

This Commentary presents two alternative climatic events: a period of cooling from the mid-Holocene Warm (mHW, about 7 ka) to the Little Ice Age (LIA, about 0.3 ka) and a 150-year period of warming from the LIA to the base period of SW20. During the first period, CO_2 concentration (Monnin, 2006) was negatively correlated with global temperature (Masson et al., 2000; Vinther et al., 2009). Using GHG forcings based on Etminan, et al. (2016) and assumptions similar to those made by SW20 for the historic period, the most likely values of S_{mHW} and S_{LIA} are estimated to be 12.15 K and 1.37 K, respectively (Supplementary Information, Table S2). This divergence would be reduced if forcing by GHGs was smaller, if larger (possibly unidentified) negative forcings affected both periods, and/or ΔN was strongly positive during S_{mHW}. These two events and several centennial-scale temperature fluctuations during the Holocene show that recent climatic changes may be influenced more by solar irradiance, cloud cover, volcanic activity, or aerosol concentrations than CO_2.

5 Discussion

A factor that directly affected all their results is the magnitude of ΔF_{2xCO2}. This included a rapid adjustment larger than 30% which may be irrelevant after a few years. The radiative balance in the stratosphere would be restored within 40 days (Maycock et al., 2011) and the Brewer-Dobson circulation (Butchart, 2014) would reduce any transient thermal anomalies within months, essentially restoring the original thermal gradient. The rapid response in the troposphere would be eliminated more quickly by stronger convective systems. Near the new energy balance, most of the atmosphere would have slightly raised temperatures and the tropopause would be higher. Satellite measurements may eventually establish changes in the infrared energy lost from the Earth, but they have been frustrated by excessive instrumental noise for wavelengths >14 microns (Bantges et al., 2016).

The results of SW20 depended on many assumptions, approximations, and model outputs. Although they thoroughly analyzed the issues and apparently reduced the uncertainty of ECS, their main conclusion – that ECS values <2 K are unlikely – remains inconsistent with many recent estimates based on energy balance (IPCC, 2013; Forster et al., 2016). This may indicate they have overestimated S by including rapid adjustments in ΔF_{2xCO2} or underestimated changes in solar irradiance, ice albedo, cloud cover or CO_2 solubility in the oceans. Evidence from the Holocene suggests that agents other than CO_2 have been responsible for much of the climate change and require further research.

Acknowledgments and Data

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- Data were not created for this research.Other affiliations for any author that may be perceived as having a conflict of interest with respect to the results of this paper.

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Earth's Future

Supporting Information for

Mathematics Clouding Climate Sensitivity

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Tables S1 to S2

Introduction

Table S1 shows the equations (Main Equations) that SW20 used to define S in each of the climatic events they evaluated in the three lines of evidence. It also shows the more important Supporting Equations that they used to develop the feedback term or to generate a likelihood function for their Bayesian analysis. A brief description of each term is included.

Table S2 shows the results for the historic line of evidence from SW20 Table 4. It shows an alternative calculation for that period using some more realistic values and the values used to calculate S from two additional periods during the Holocene.

Line of evidence	Main equation	Supporting equation
1. Process	$S_{proc} = \frac{-\Delta F_{2\times CO_2}}{\lambda}$	$\lambda = \Sigma\lambda_i$
2. Historic	$S_{hist} = \frac{\Delta F_{2xCO_2} \times \Delta T}{\Delta F - \Delta N}$	$\Delta T = -(\Delta F - \Delta N)/(\lambda - \Delta\lambda)$
3. Paleoclimate (a) Last Glacial Maximum	$S_{LGM} = \frac{-\Delta F_{2\times CO_2}}{\lambda}$	$\Delta T = \frac{-\left(-0.57\Delta F_{2\times CO_2} + \Delta F'\right)}{\frac{\lambda}{1+\zeta} + \frac{\alpha}{2}\Delta T}$
3. Paleoclimate (b) mid-Pliocene Warm Period	$S_{mPWP} = \frac{-\Delta F_{2\times CO_2}}{\lambda}$	$\Delta T = \frac{-\Delta F_{2\times CO_2} \times ln[C]\left(1 + f_{CH_4}\right)(1 + f_{ESS})}{\frac{\lambda}{(1+\zeta)}}$
3. Paleoclimate (c) Paleocene-Eocene Thermal Maximum	$S_{PETM} = \frac{-\Delta F_{2\times CO_2}}{\lambda}$	$\Delta T = \frac{-\Delta F_{2\times CO_2} \times ln[C]\left(1 + f_{CH_4}\right)}{\frac{\lambda}{(1+\zeta)} + \beta}$

Table S1. Critical equations used by Sherwood et al., 2020. Terms are defined below.

S_x	Effective Climate Sensitivity, S (for specific line of evidence, x)
ΔF, $\Delta F'$	Forcing at top of atmosphere (total, or all sources except CO_2)
ΔF_{2xCO_2}	Forcing due to doubling CO_2 concentration
ΔN	Energy imbalance at top of atmosphere
ΔT	Forced change in near-surface temperature
$\Delta\lambda$	Feedback correction for pattern effect
λ, λ_i	Feedback (total, or for component i)
ζ	Correction to convert S to Equilibrium Climate Sensitivity, ECS
α	Correction for uncertainty in feedback
β	Correction for state dependence
f_{CH4}, f_{EES}	Correction for forcing due to CH_4 and N_2O, or ice sheets and vegetation
$ln[C]$	Logarithm of ratio of CO_2 concentrations / logarithm 2

Parameter	Mid-Holocene to Warm (~7 ka)		Little Ice Age (~1700) to		Pre-industrial (1861-1889) to The present (2006-2018)			
						Alternate	SW20	
ΔT (K)		*-1.2*[a]		*0.2*[b]		*1.03*[c]	*1.03*[c]	
ΔN	0.4	-0.2	0.2[c]	0[c]	0.2[c]	0.6[c]	0.6[c]	0.8[c]
ΔF		-0.395		0.584		2.268	1.83[c]	
CO_2 (ppm)	*262*[b]	0.315[d]	*278*[b]	0.188[d]	*288*[b]	1.655[d]	1.731[c]	*392*[e]
CH_4 (ppm)	*600*[b]	0.079[d]	*700*[b]	0.059[d]	*780*[b]	0.531[d]	0.969[c]	*1825*[e]
N_2O (ppm)	*260*[b]	0.041[d]	*272*[b]	-0.003[d]	*270*[b]	0.165[d]		*325*[e]
Troposphere O_3		0		0.1[c]		0.348[c]	0.348[c]	
Stratosphere O_3		0		0		-0.050[c]	-0.050[c]	
Aerosols		0		-0.2[c]		-0.667[c]	-1.179[c]	
Land use		0		0		-0.106[c]	-0.106[c]	
Stratosphere H_2O		0		0		0.064[c]	0.064[c]	
Black C on snow		0		0		0.020[c]	0.020[c]	
Contrails		0		0		0.048[c]	0.048[c]	
Solar irradiance		-0.8[f]		0.5[f]		0.2[f]	0.017[c]	
Volcanics		-0.03[g]		-0.06[g]		0.06[g]	-0.113[c]	
Energy balance estimation of S (K)		*24.62*		*1.37*		*2.47*	*3.11*[c]	

Table S2. Measurements and calculated values used to estimate S for Holocene scenarios (Wm^{-2} except numbers in italics. Their units are shown in column 1)

Data sources:
(a) –Vinther et al. (2009)
(b) – IPCC (2013)
(c) – Sherwood et al. (2020). Values from SW20 Table 4 are medians which differ from mean/mode in non-Gaussian distributions, e.g. ΔF, aerosols and S.
(d) – Etminan et al. (2016)
(e) – Dlugokencky (2020)
(f) – Lean (2018)
(g) – Kobashi et al. (2017)